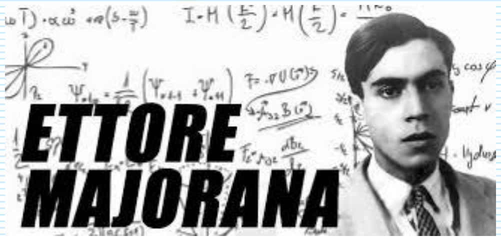
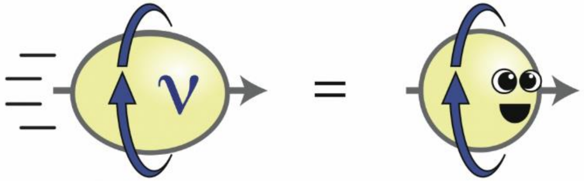
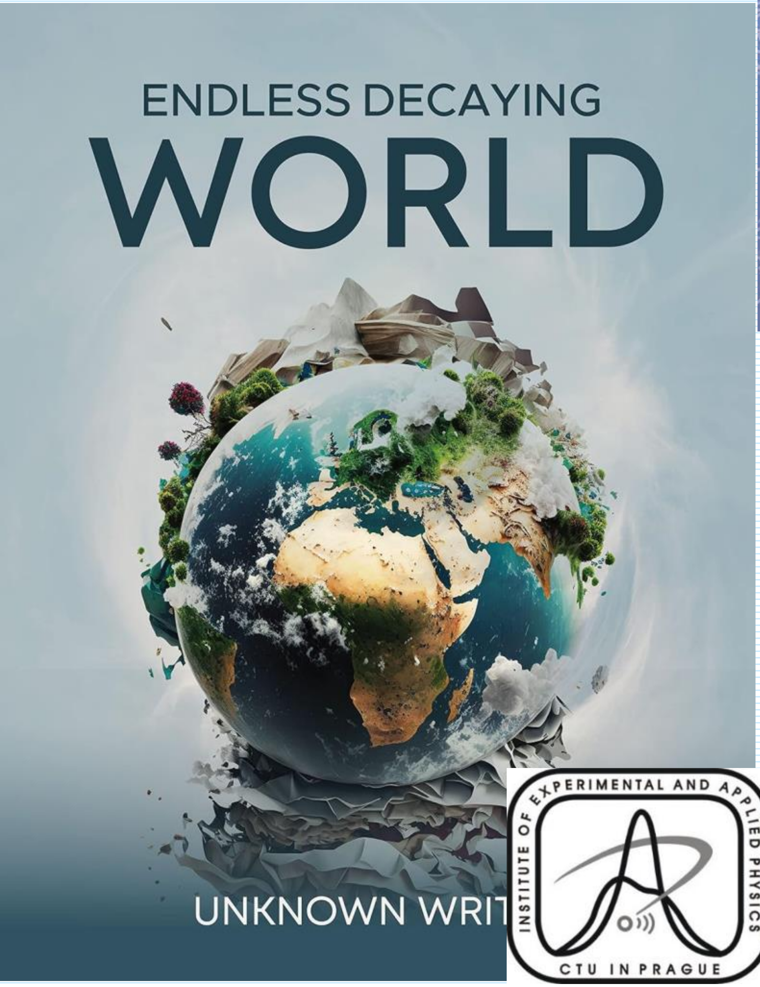
Multi-Aspect Young ORiented Advanced Neutrino Academy

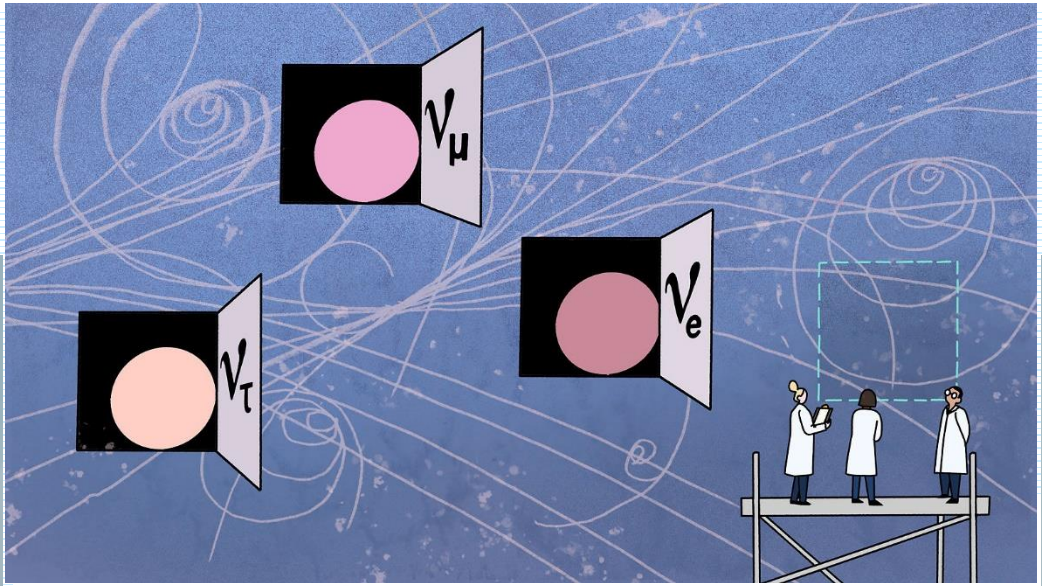
MAYORANA Workshop

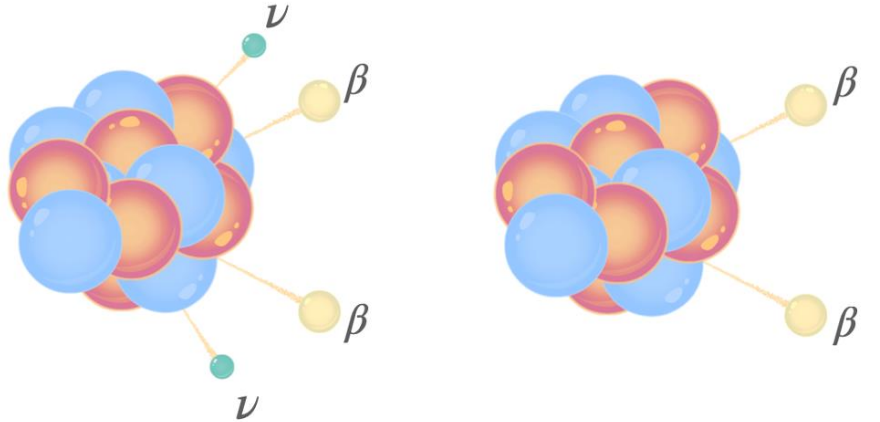
Modica (Sicily-Italy), June 16 to 18, 2025

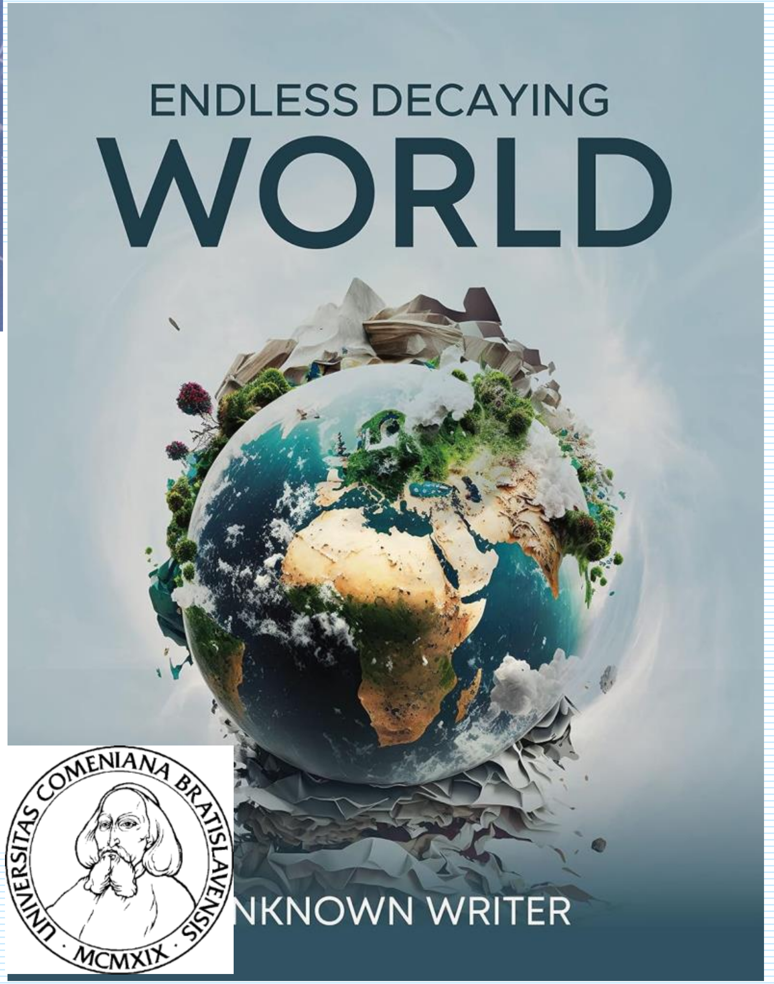








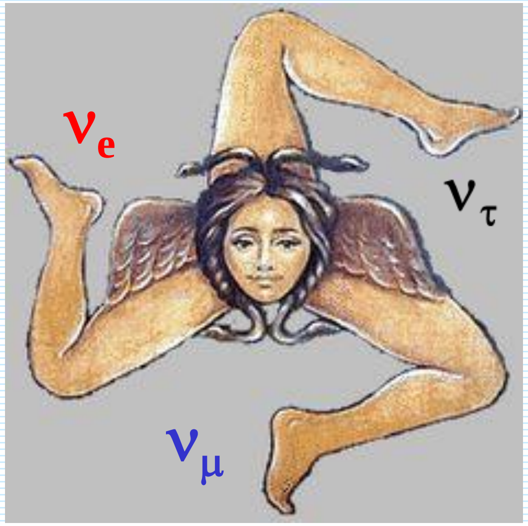




Two-neutrino double-beta decay:

A key for the $0\nu\beta\beta$ NMEs problem

Fedor Šimkovic



OUTLINE

I. Introduction (a connection of $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs)

II. A derivation of the $2\nu\beta\beta$–decay half-life revisited
(the r ole of a summation of over contributions through the states of intermediate nucleus)

III. A calculation of the $2\nu\beta\beta$–decay NMEs within a schematic model
(role of the spin-isospin symmetry restoration)

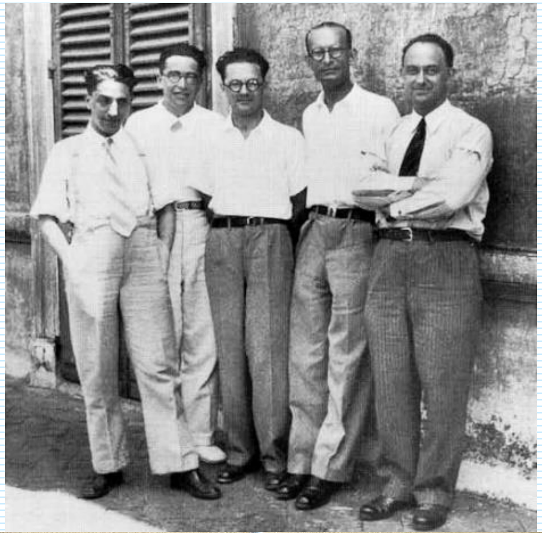
IV. The semi-empirical formula for $2\nu\beta\beta$-decay NMEs

V. Improved description of the $2\nu\beta\beta$-decay
(the effect of lepton energies in energy denominators, widths of intermediate nuclear states, effect of $p_{1/2}$ wave states, …)

VI. New physics beyond the SM

VII. Outlook

Acknowledgments: A. Faessler, P. Vogel, O. Nitescu, ...




Ragazzi di Via Panisperna
In questa strada al civico 89, negli anni 30 i ricercatori
D'Agostino, Fermi (Nobel 1938), Amaldi,
Majorana, Rasetti, Pontecorvo, Segrè (Nobel 1959)
accedevano all'istituto di Fisica, dove insieme
aprirono l'era nucleare al mondo.

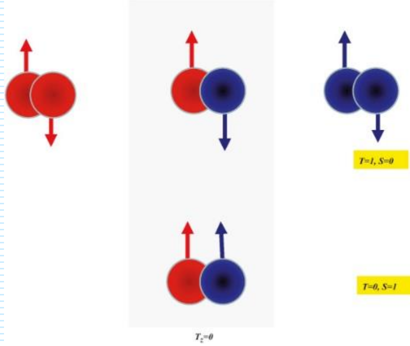
The Boys of Via Panisperna
The entrance in this street at number 89, was used in the 1930s by researchers
D'Agostino, Fermi (Nobel 1938), Amaldi,
Majorana, Rasetti, Pontecorvo and Segrè (Nobel 1959)
to reach the Institute of Phisics, where together
they opened the world up to the nuclear age.

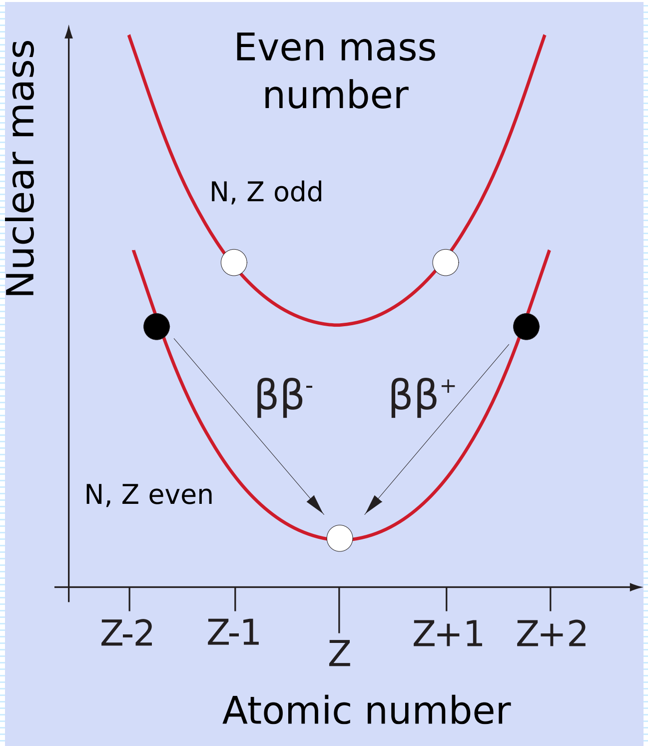

Bruno Pontecorvo

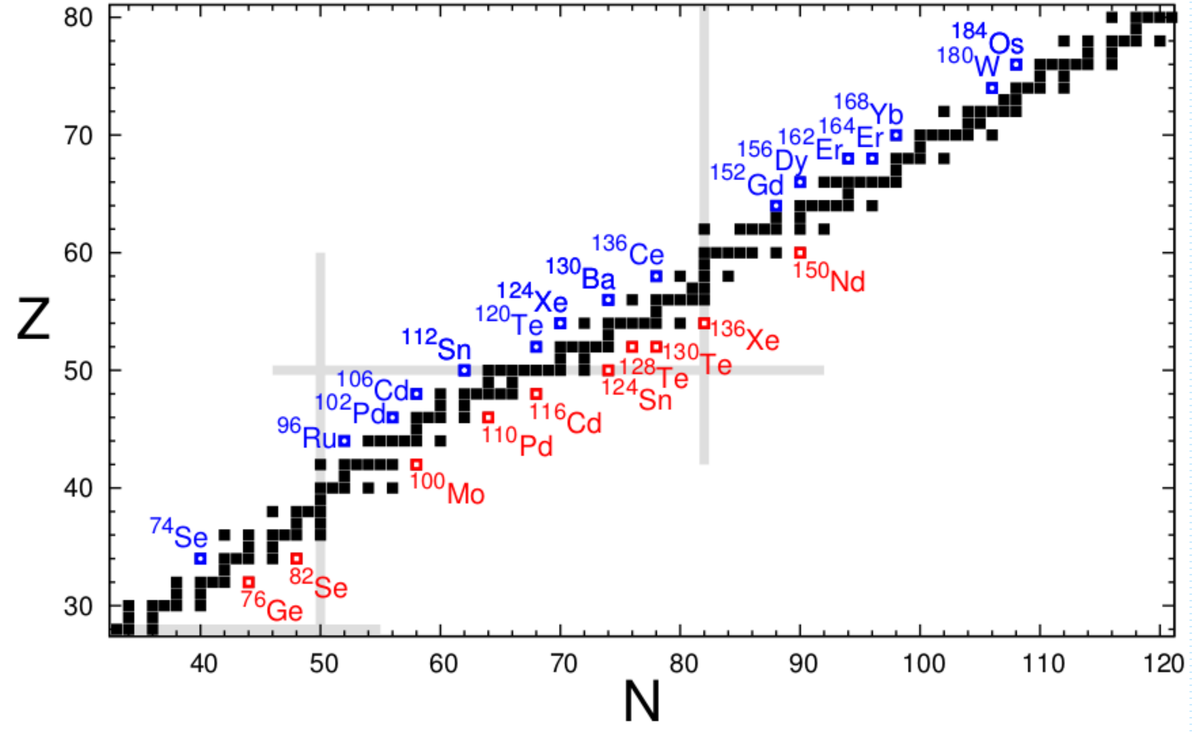
edor Sim


Ettore Majorana

Nuclear double-β decay
(even-even nuclei, pairing int.)







Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$

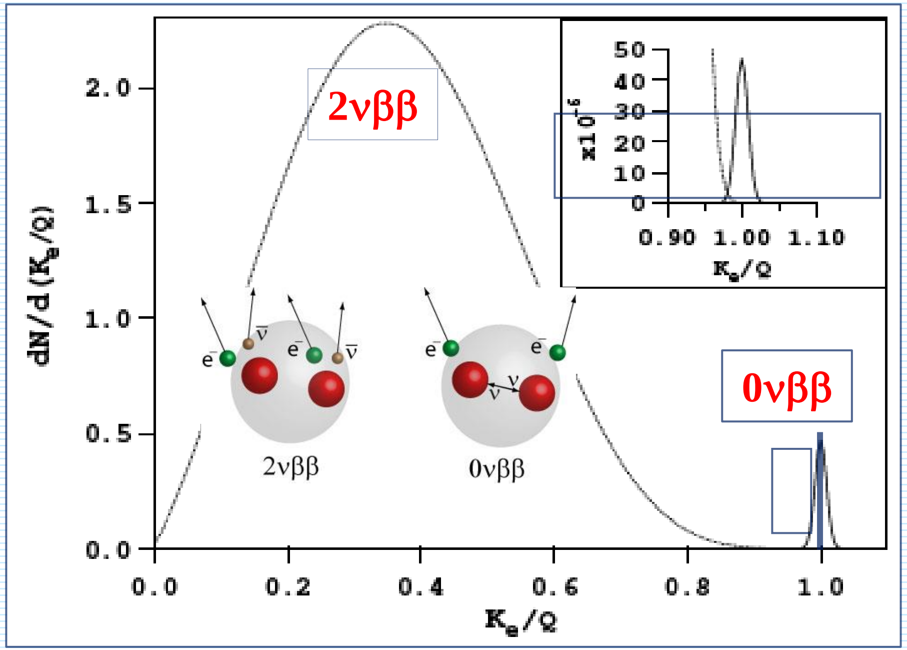
Goepert-Mayer – 1935. 1st observation in 1987

Nuovo Cim. 14, 322 (1937) Phys. Rev. 56, 1184 (1939)

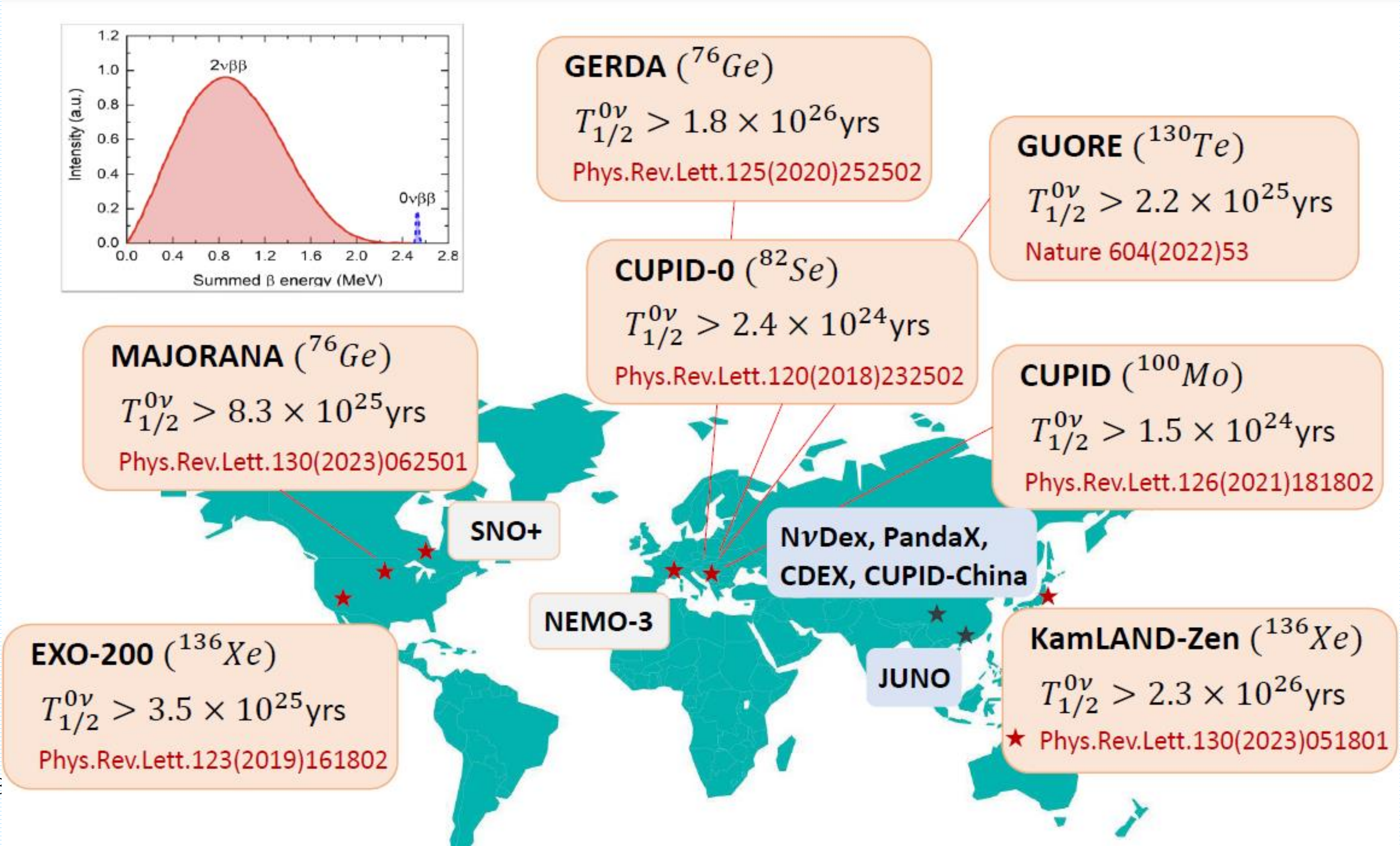
Neutrinoless double-β decay – LN violated

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ (Furry 1937)

Not observed yet. Requires massive Majorana ν's



0νββ experiments – a worldwide competition of ideas and underground physics technologies



NνDEx

No neutrino-Double-beta Decay Experiment

High-pressure TPC chamber

$0\nu\beta\beta$ decay isotopes and experiments

Candidates	$Q_{\beta\beta}$(MeV)	N.A. (%)
$^{48}Ca \rightarrow ^{48}Ti$	4.268	0.187
$^{76}Ge \rightarrow ^{76}Se$	2.039	7.8
$^{82}Se \rightarrow ^{82}Kr$	2.998	8.8
$^{96}Zr \rightarrow ^{96}Mo$	3.356	2.8
$^{100}Mo \rightarrow ^{100}Ru$	3.034	9.7
$^{110}Pd \rightarrow ^{110}Cd$	2.017	11.7
$^{116}Cd \rightarrow ^{116}Sn$	2.813	7.5
$^{124}Sn \rightarrow ^{124}Te$	2.293	5.8
$^{130}Te \rightarrow ^{130}Xe$	2.528	34.1
$^{136}Xe \rightarrow ^{136}Ba$	2.458	8.9
$^{150}Nd \rightarrow ^{150}Sm$	3.371	5.6

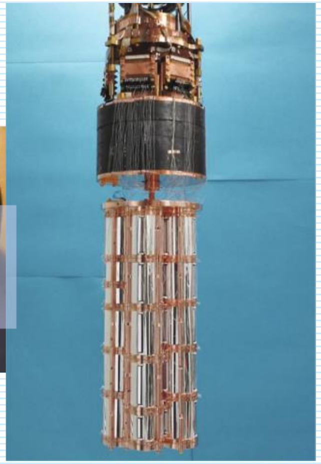
CANDLE
CaF scintillating crystal

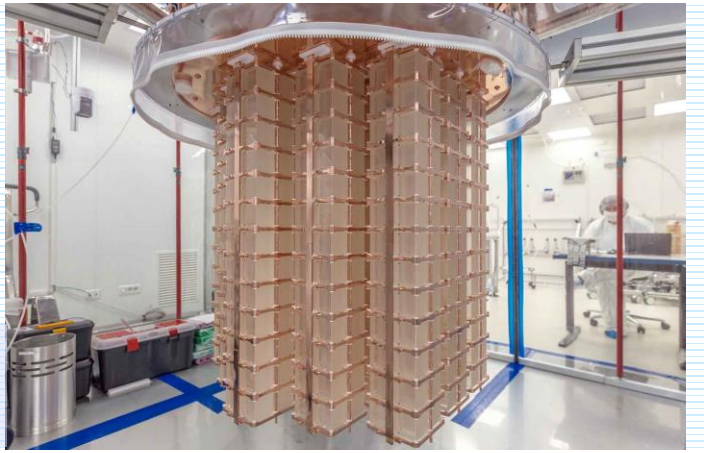
SuperNEMO
Se source foil

GERDA, MAJORANA
Ge crystal

CUPID-0
ZnSe scintillating crystal

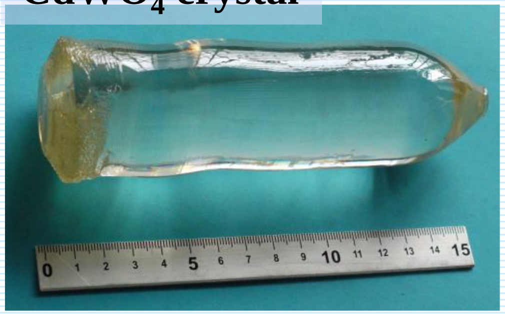
SuperNEMO
Se source foil



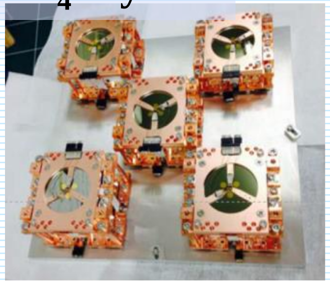


CUORE
TeO$_2$ crystal

Aurora
CdWO$_4$ crystal



Amore
CaMoO$_4$ crystal



EXO, KamLAND-Zen
Liquid Xe

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

$0\nu\beta\beta$-decay

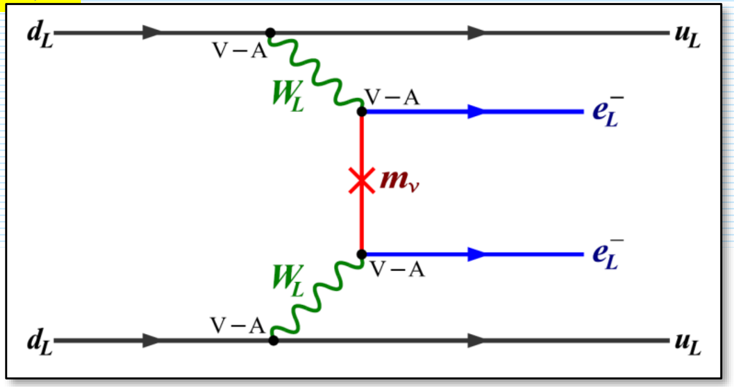
(LNV at ≈GUT scale, exchange of three light ν)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; g_A^4 \; \left|M^{0\nu}_{\nu}\right|^2 \; G^{0\nu}$$

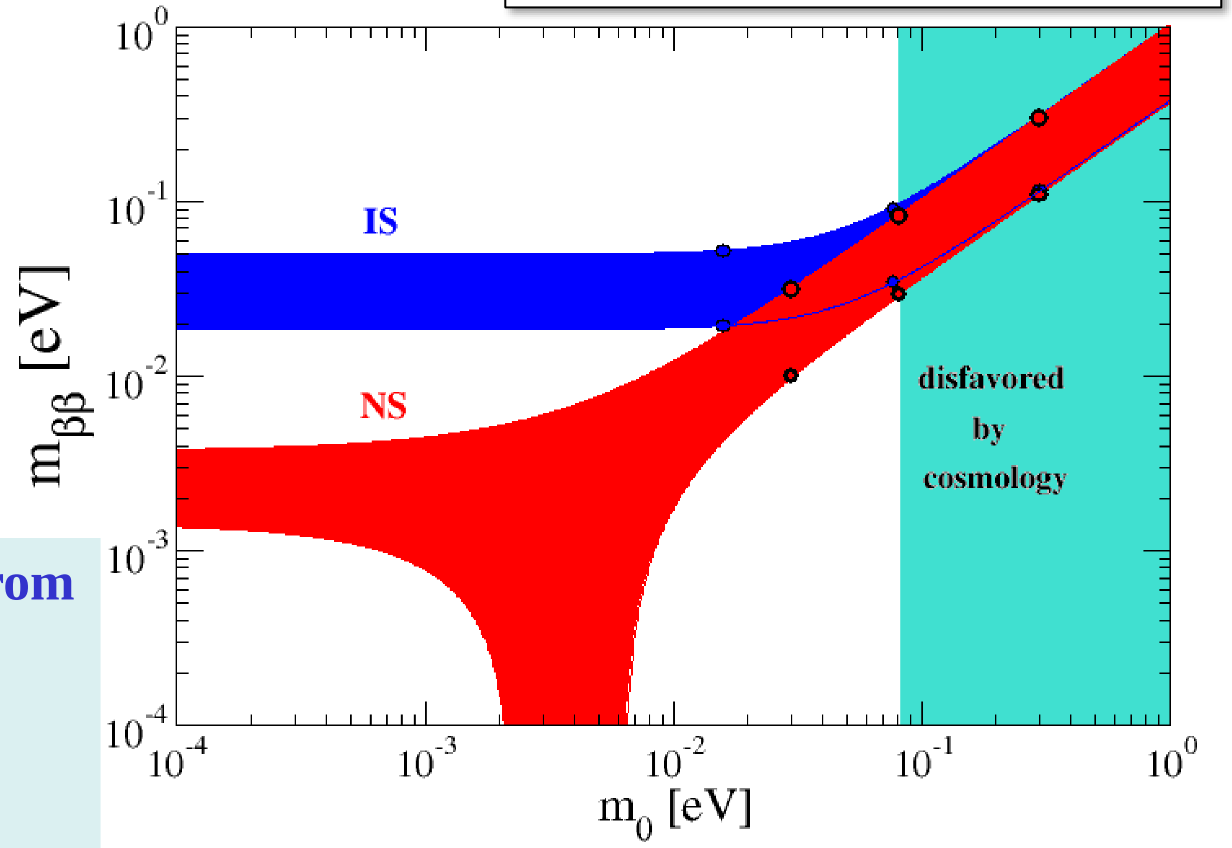
Phase space factor well understood

?

***NME** must be evaluated using tools of nuclear theory*



$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$



Constraint from cosmology

$\Sigma = m_1 + m_2 + m_3$
< 0.90 eV
< 0.26 eV (Planck coll.)
< 0.12 eV

Contrary, the constraint from $0\nu\beta\beta$-decay (KLZ)

$m_{\beta\beta} < 0.036$-0.156 eV

implies

$\Sigma < 0.12$ eV

$0\nu\beta\beta$-decay NME
status 2023(25)

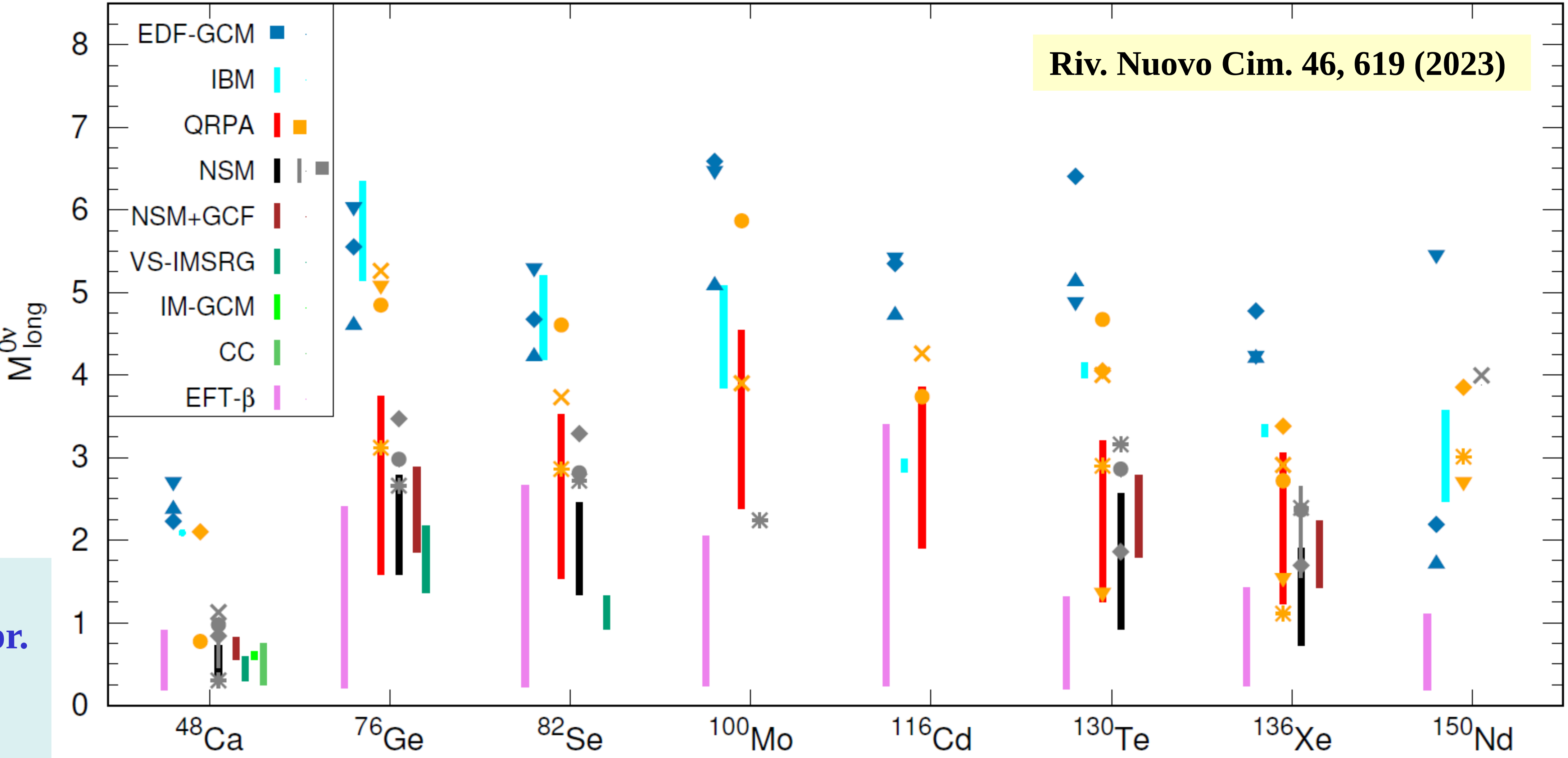
The nuclear w. f. of
(A,Z), (A,Z+1)*, (A,Z+2)
Many-body methods
of choice:

The $0\nu\beta\beta$ nuclear transition operators
(F, GT, and tensor type):

! Isospin, and spin-isospin symmetries
($M_{Fcl}\approx 0$, M_{GTcl} strongly suppressed):

All
models
missing
essential
physics

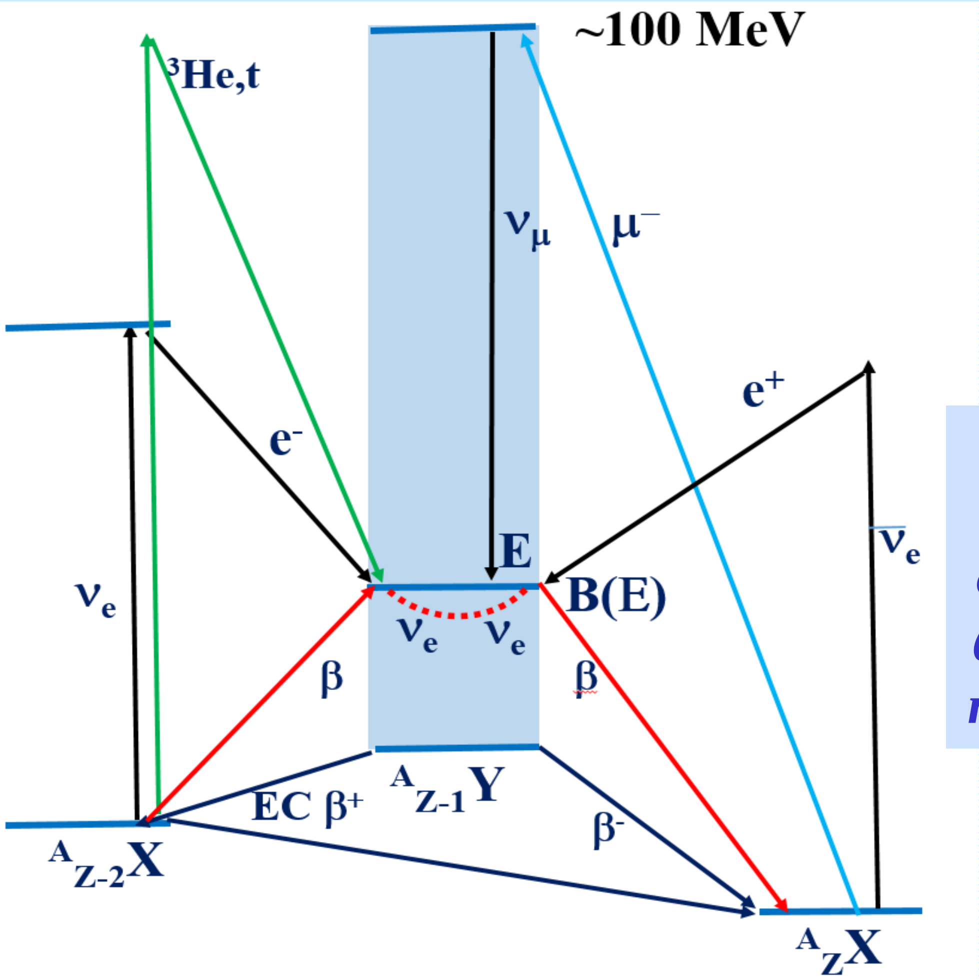
Impossible
to assign
rigorous
uncertainties



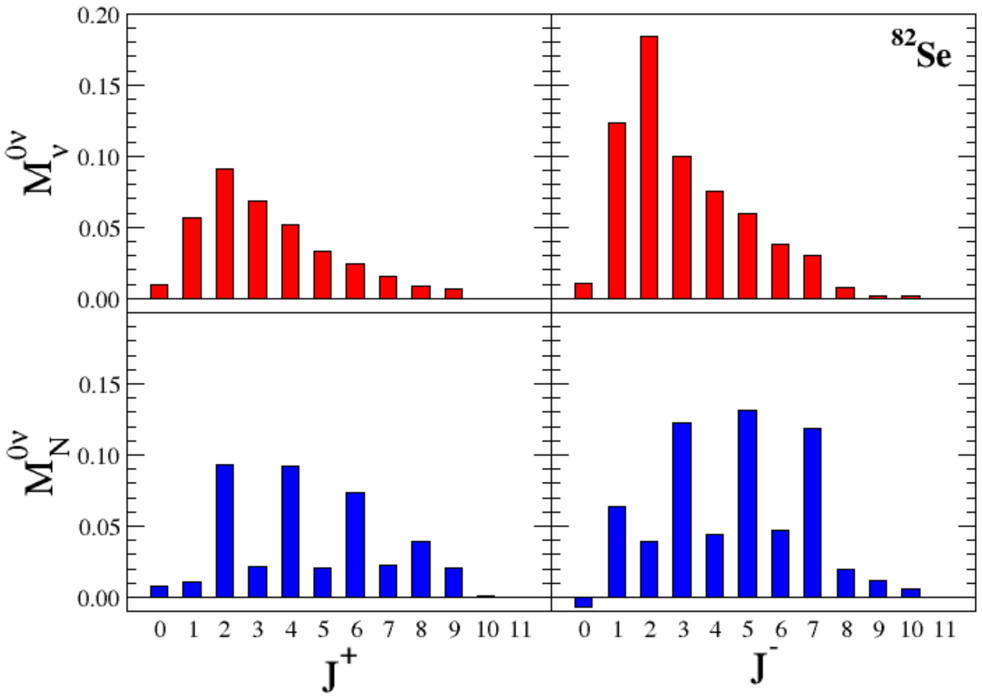
Differencies:
Many-body appr.
Size of the m.s.
residual int.

Supporting nuclear physics experiments
(Measurements still not conclusive for 0νββ NME)

- ✓β-decay, EC and 2νββ decay
- ✓μ-capture
- ✓(π^+, π^-)
- ✓ $(^3He,t)$, $(d,^2He)$, transfer reactions
- ✓γ-ray spectroscopy, γγ-decay
- ✓A promising experimental tool:
  Heavy-Ion Double Charge-Exchange



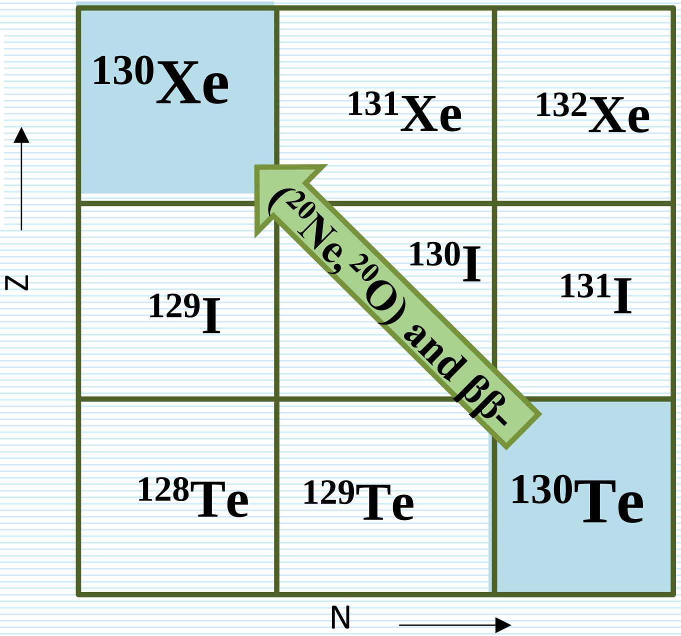
Multipole decomposition of light and heavy 0νββ-decay NMEs normalized to unity

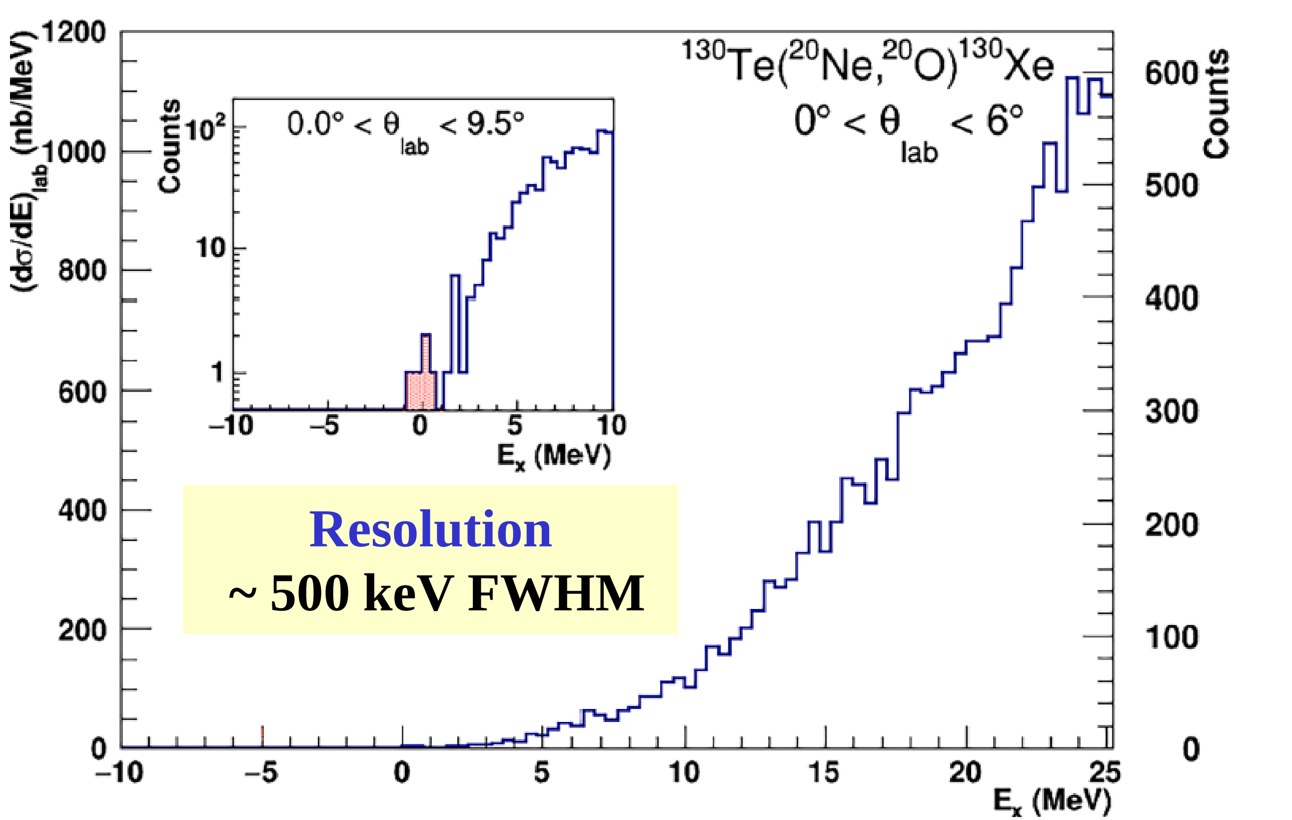


Higher multipoles are populated mostly due to large ν-momentum transfer

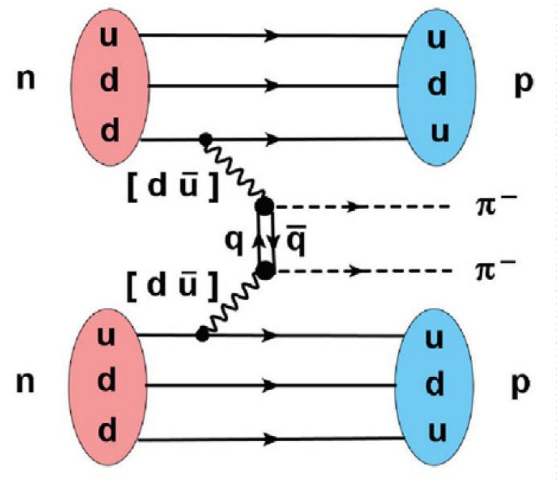
Fedor Simkovic

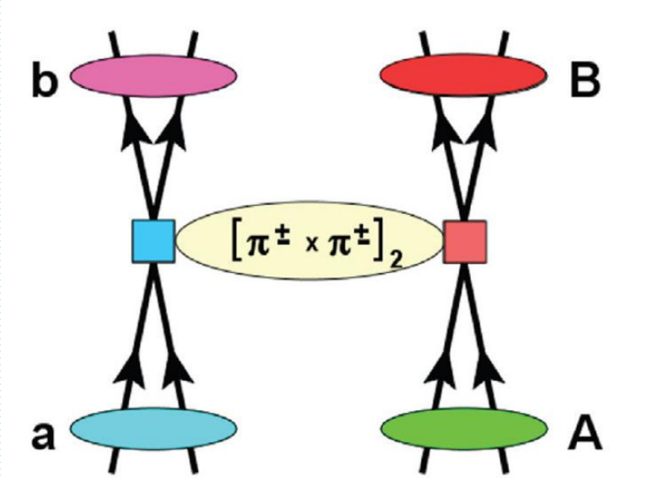
The ^{130}Te(^{20}Ne,^{20}O)^{130}Xe DCE reaction





- g.s. → g.s. transition can be isolated
- Absolute cross section measured





State (MeV)	Counts	Absolute coss section (nb)	Cross section 95% limit (nb)
g.s. (0^+) + 2^+ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest

A connection between closure 2νββ and 0νββ GT NMEs (and NMEs of DCE reactions)

$$M_{GT}^{0\nu} = \int_0^\infty H_{GT}^{0\nu}(r) C_{GT-cl}^{2\nu}(r) dr$$

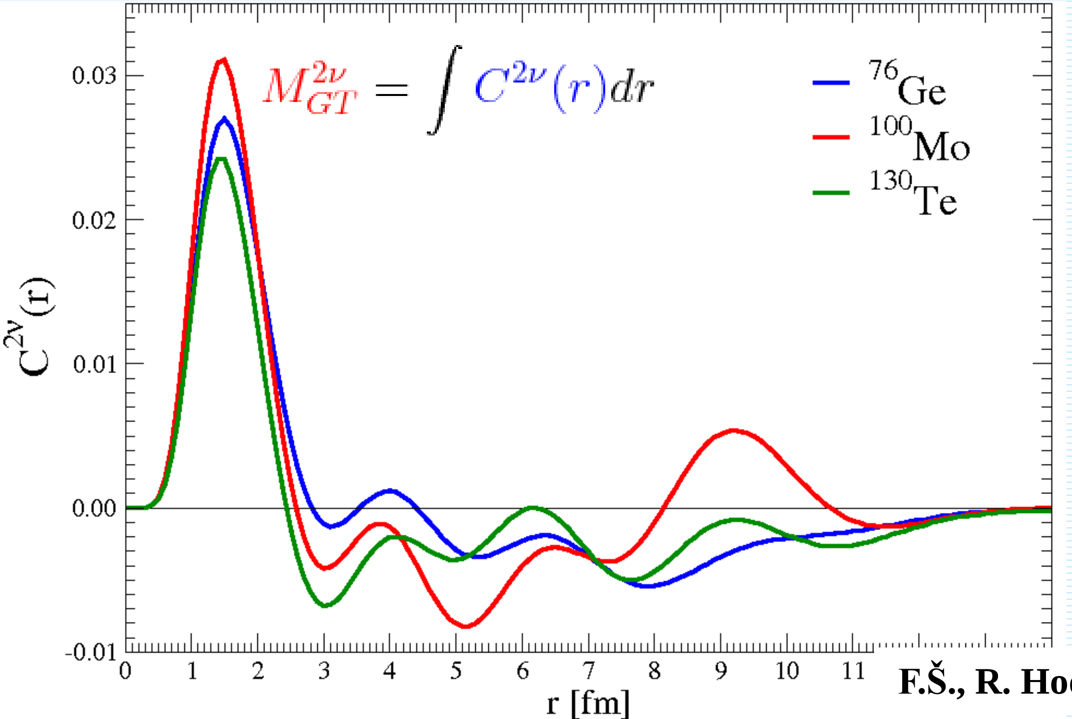
$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f_{FNS}^2(q^2) g_{HOT}(q^2) dr$$

0νββ
2n-potential is well determined

$$M_{GT-cl}^{2\nu} = \int C_{GT-cl}^{2\nu} dr$$

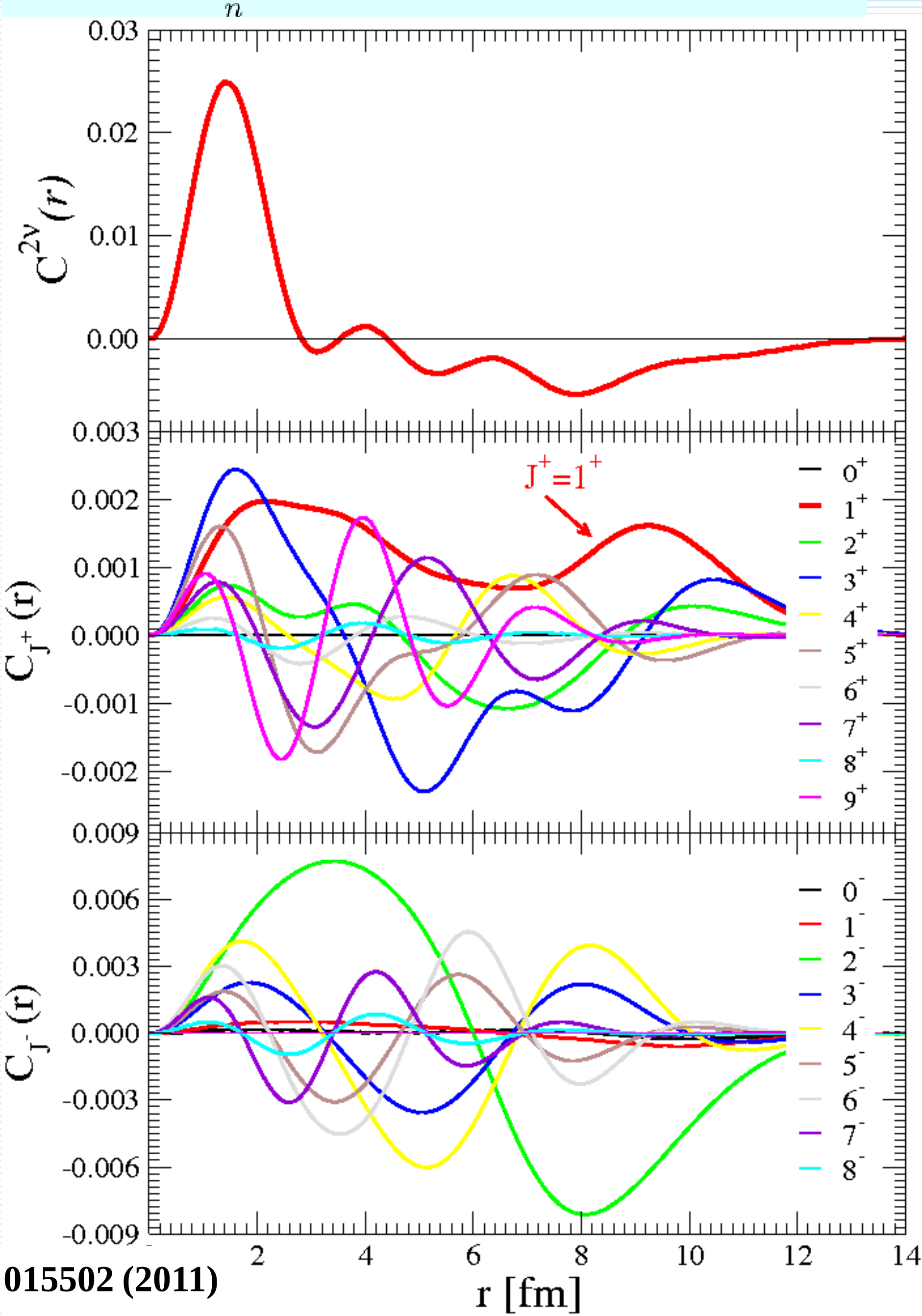
$$C_{GT-cl}^{2\nu} = \sum_{J^\pi} C_{GT-cl}^{2\nu}(J^\pi)$$

$$M_{GT-cl} = \sum_n \langle 0_f^+ \parallel \tau^+\sigma \parallel 1_n^+\rangle\langle 1_n^+ \parallel \tau^+\sigma \parallel 0_i^+\rangle$$



for $J^\pi = 1^+$
$\int C_J(r)\, dr = M_{GT}^{2\nu}$

for $J^\pi \neq 1^+$
$\int C_J(r)\, dr = 0$



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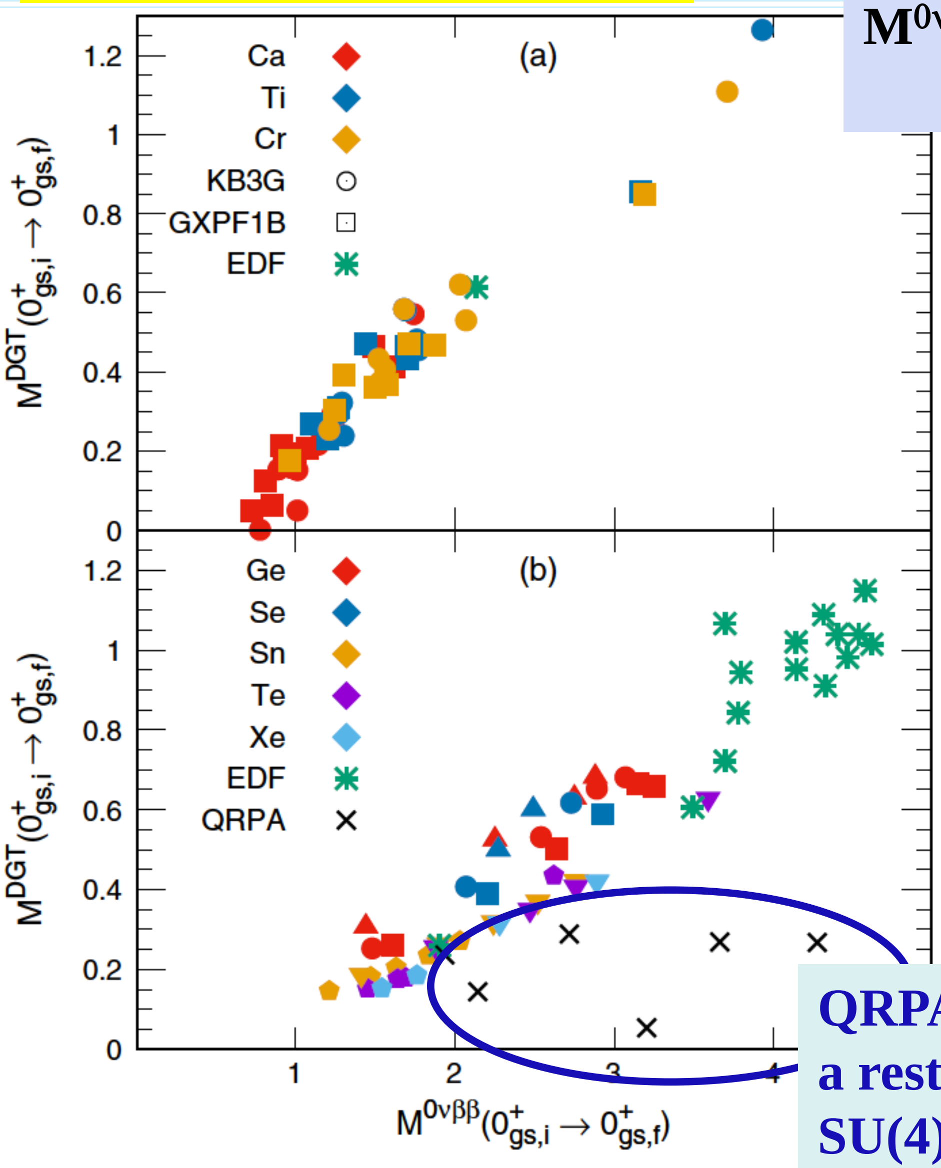
F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

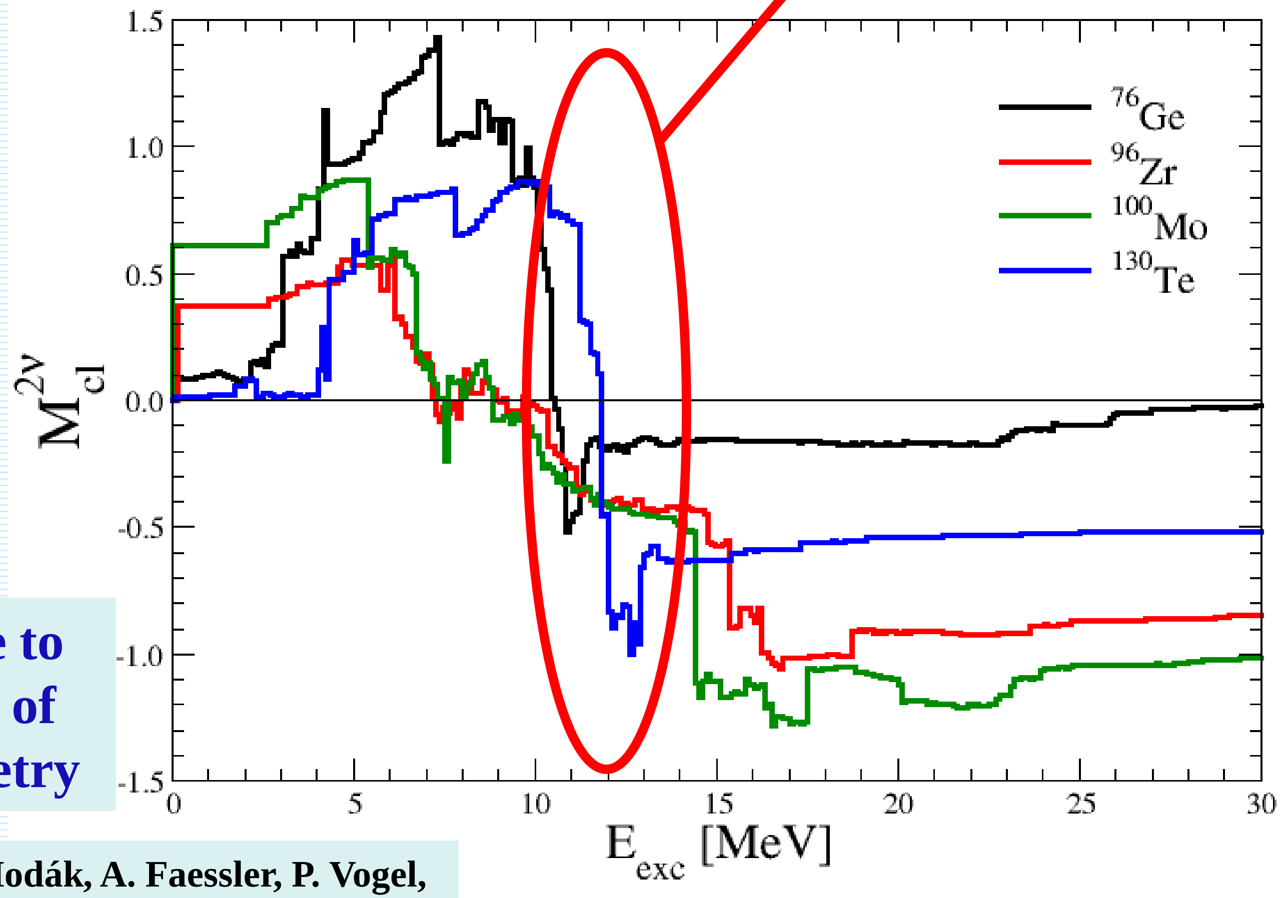
ISM, EDF : $M^{0\nu} \propto M^{2\nu}_{GT\text{-}cl}$

$M^{DGT} \equiv M_{GT\text{-}cl}$ – only 1^+
$M^{0\nu}$ - contribution from many J^{π} (!)

QRPA: There is no proportionality between $0\nu\beta\beta$- and $2\nu\beta\beta$-decay NMEs
(change of sign at GTR level)



QRPA - close to a restoration of SU(4) symmetry



ISM: N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

2νββ-decay NME

Weak interaction Hamiltonian

$$\mathcal{H}^{\beta}(x) = \frac{G_F}{\sqrt{2}} 2\left[\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x)\right] j_\alpha(x) + h.c.$$

Phys. Atom. Nucl. 62 (1999) 585

2νββ-decay amplitude

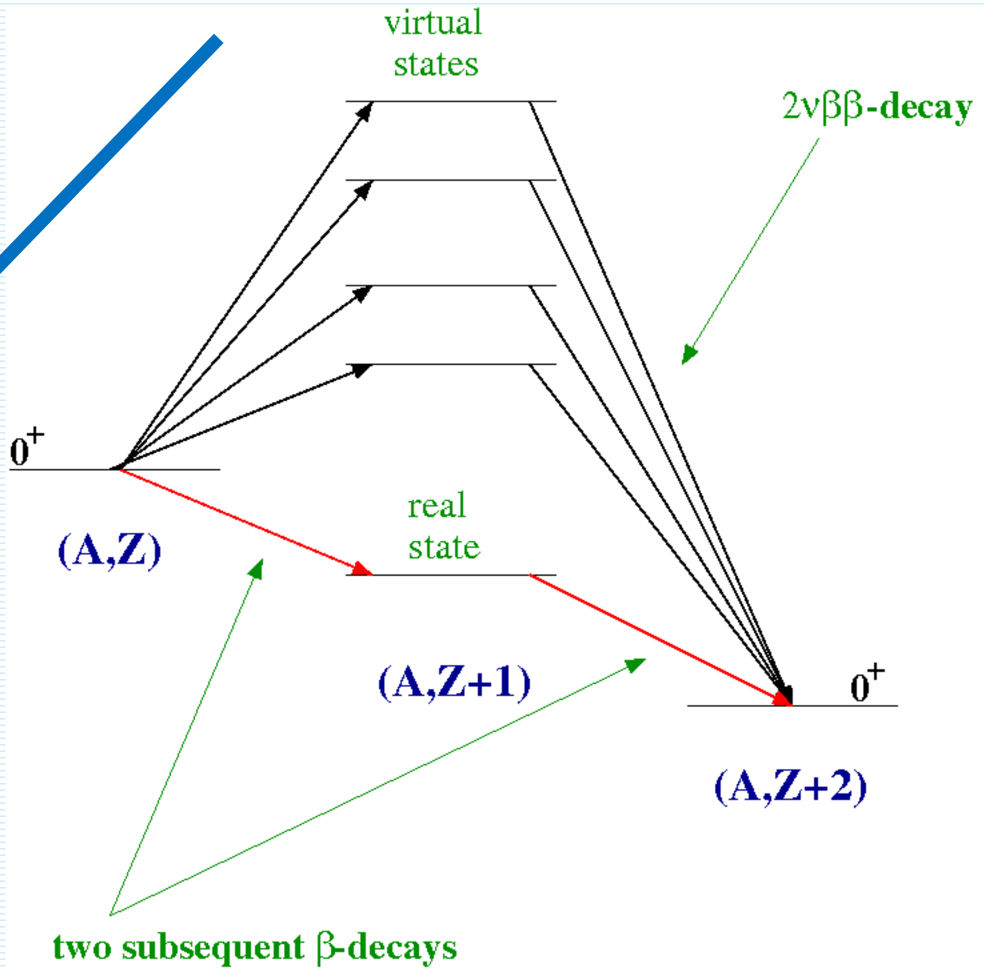
Hadron part of amplitude

$$J_{\mu\nu}(p_1,p_2,k_1,k_2) = \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2} {}_{out}\langle p_f | T(J_\mu(x_1)J_\nu(x_2)) | p_i \rangle_{in}\, dx_1 dx_2$$

$$< f|S^{(2)}|i> = \frac{(-i)^2}{2}\left(\frac{G_F}{\sqrt{2}}\right)^2 L_{\mu\nu}(p_1,p_2,k_1,k_2) J_{\mu\nu}(p_1,p_2,k_1,k_2) - (p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2)(k_1 \leftrightarrow k_2)$$

$$\begin{aligned} T(J_\mu(x_1)J_\nu(x_2)) &= J_\mu(x_1)J_\nu(x_2) & (two\ \beta - decays) \\ &+ \Theta(x_{20}-x_{10})[J_\nu(x_2), J_\mu(x_1)] & (2\nu\beta\beta - decay) \end{aligned}$$

F.Š., G. Pantis, Phys. Atom. Nucl. 62 (1999) 585



A sum over intermediate nuclear states represents a sum over all meson and γ-exchange correlations of two β-decaying nucleons inside nucleus

The two-nucleon β-decays in 2νββ-decay are connected through strong and EM interactions

$$S^{(2)} = -\frac{(-i)^2}{2} 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \times$$
$$\int N[\overline{e}_L(x_1)\gamma\nu_{eL}(x_1)\ \overline{e}_L(x_2)\gamma\nu_{eL}(x_2)] \times$$
$$T\left[j_\alpha(x_1)\ j_\beta(x_2)\ exp\left\{-i\int \mathcal{H}^{\mathrm{str+em}}(x)dx\right\}\right]\ dx_1\ dx_2$$

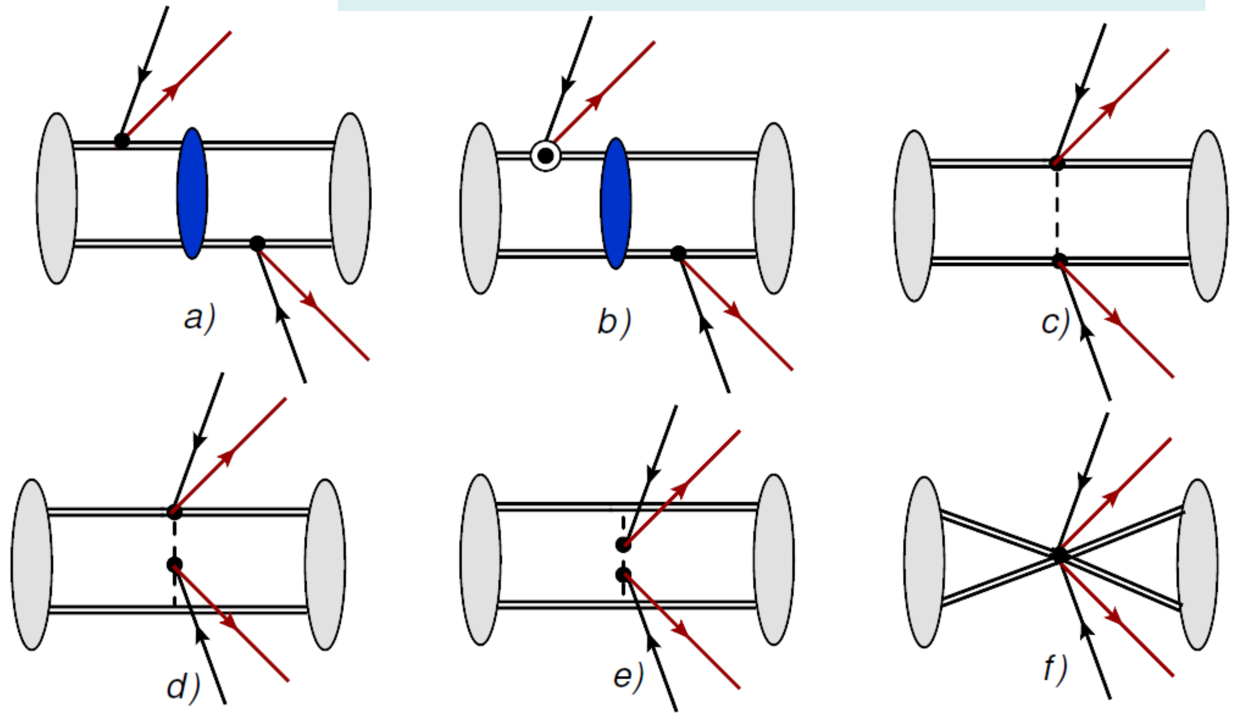
$$[j_\alpha(x_1), j_\beta(x_2)] =$$
$$[\overline{p}(x_1)\gamma_\alpha(1+\gamma_5)n(x_1), \overline{p}(x_2)\gamma_\alpha(1+\gamma_5)n(x_2)]$$
$$= 0$$

c, d, e pion-exchange contributions (and maybe also f) are already included ín a and b diagrams

$$J_\alpha(x) = U^\dagger(x_0,0)\ j_\alpha(x)\ U(x_0,0)$$

$$U(t,t_0) = T\left(exp\left[-i\int_{t_0}^{t} \mathcal{H}^{str+em}(t)dt\right]\right)$$

$$[J_\alpha(x), J_\beta(y)] \neq 0$$



Due to $0^+ \rightarrow 0^+$ nuclear transition

$$J^{2\beta 2\nu}_{\mu\nu}(p_1, p_2, k_1, k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k}$$
$$\times 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \;\; k = 1, 2, 3,$$

Nuclear current (impulse approx.)

$$J_\alpha(0, \vec{x}) = \sum_n \tau_n^+ (\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k})\delta(\vec{x} - \vec{x}_n)$$

2νββ-decay NME in time integral representation

$$M_{GT} = \frac{i}{2}\int_0^\infty (e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}) M_{AA}(t) dt$$

with

$$M_{AA}(t) = < 0_f^+ | \frac{1}{2}[A_k(t/2), A_k(-t/2)] | 0_i^+ >$$

time dependent axial current

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \qquad A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k, \;\; k = 1, 2, 3.$$

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H, A_k(0)]...]]}^{n \;\; times}$$

Completeness: $\Sigma_n |n><n| = 1$

$$< A' | J_\alpha(x_1) J_\beta(x_2) | A > \;=\; \sum_n < A' | J_\alpha(0, \vec{x}_1) | n >< n | J_\beta(0, \vec{x}_2) | A > \times$$
$$e^{-i(E' - E_n)x_{10}} e^{-i(E_n - E)x_{20}}$$

integration over time variable

$$\int_0^\infty e^{-iat} dt \Rightarrow \lim_{\epsilon \to 0} \int_0^\infty e^{-i(a - i\epsilon)t} dt = \lim_{\epsilon \to 0} \frac{-i}{a - i\epsilon}$$

Standard form of 2νββ-decay NME

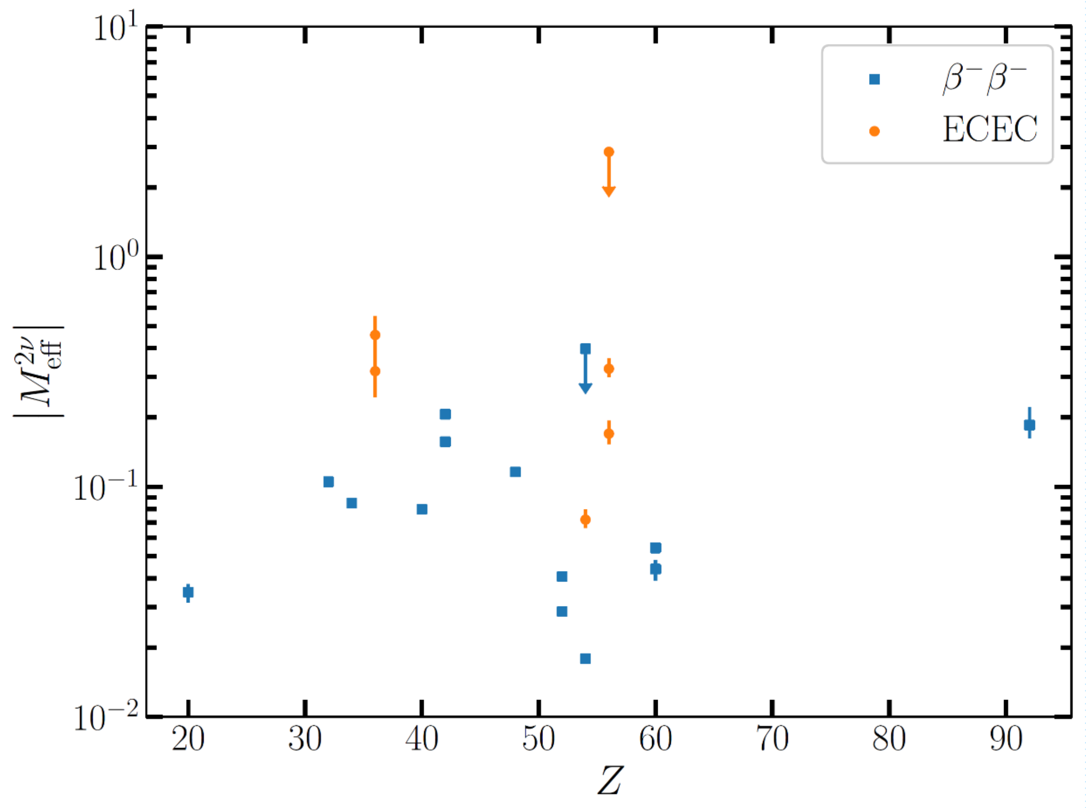
$$M_{GT} = \sum_n \frac{< 0_f^+ | A(0)_k | 1_n^+ >< 1_n^+ | A(0)_k | 0_i^+ >}{E_n - E_i + \Delta}$$

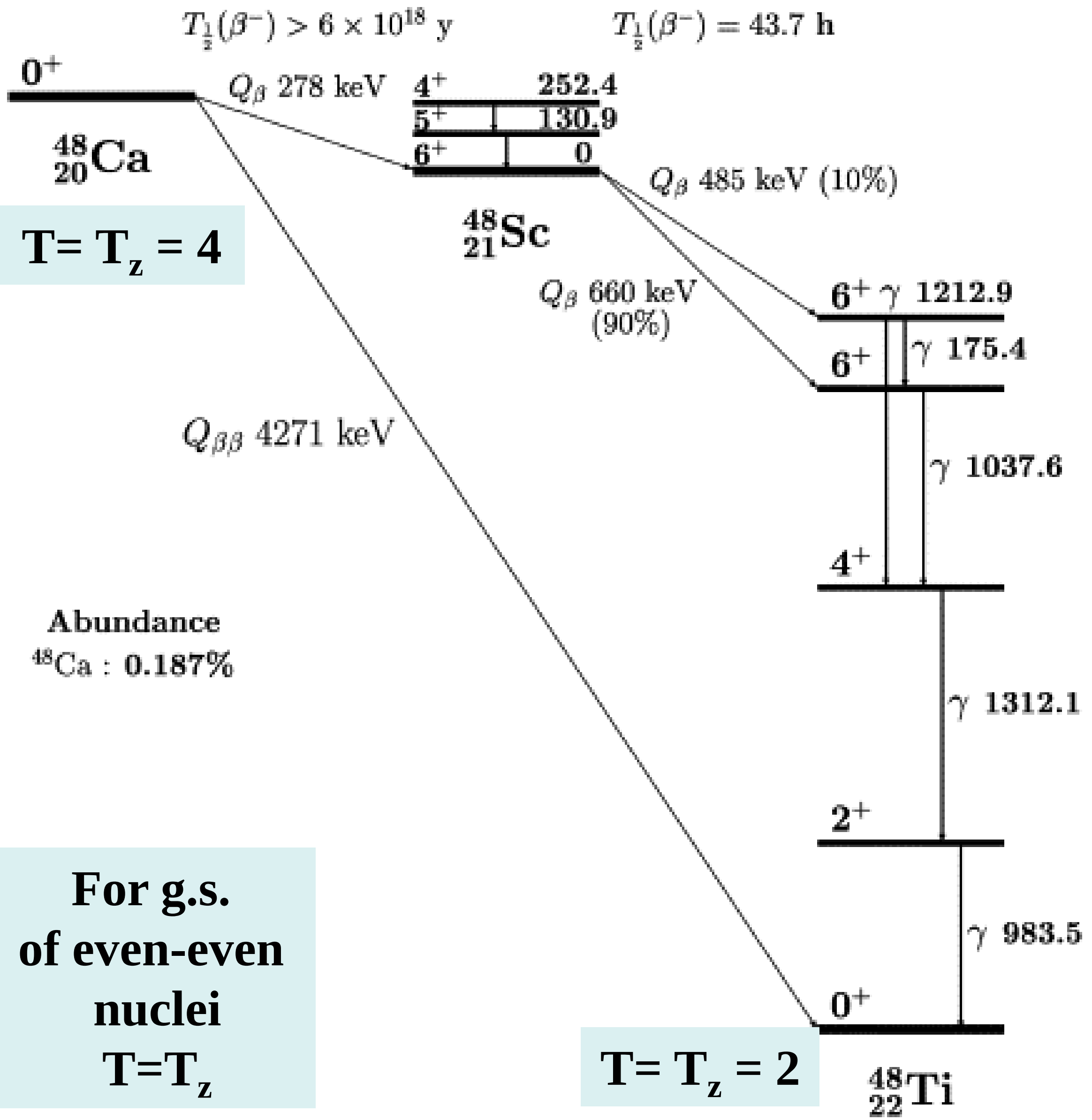
$\Delta = (E_i - E_f)/2$, stands for sum of lepton energies

Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.
Explaining 2νββ-decay is necessary but not sufficient





T= T$_z$ = 4

For g.s. of even-even nuclei T=T$_z$

T= T$_z$ = 2

QRPA calculations at the forefront: i) importance of pairing int.; ii) importance of isospin and spin-isospin symmetry restoration (isoscalar and isovector residual int.); iii) importance of relative deformation of the initial and final nuclei

$M^{2\nu}_{F\text{-}cl} = 0$

The DBD Nuclear Matrix Elements and the SU(4) symmetry restoration)

$M^{2\nu}_{GT\text{-}cl} = 0$

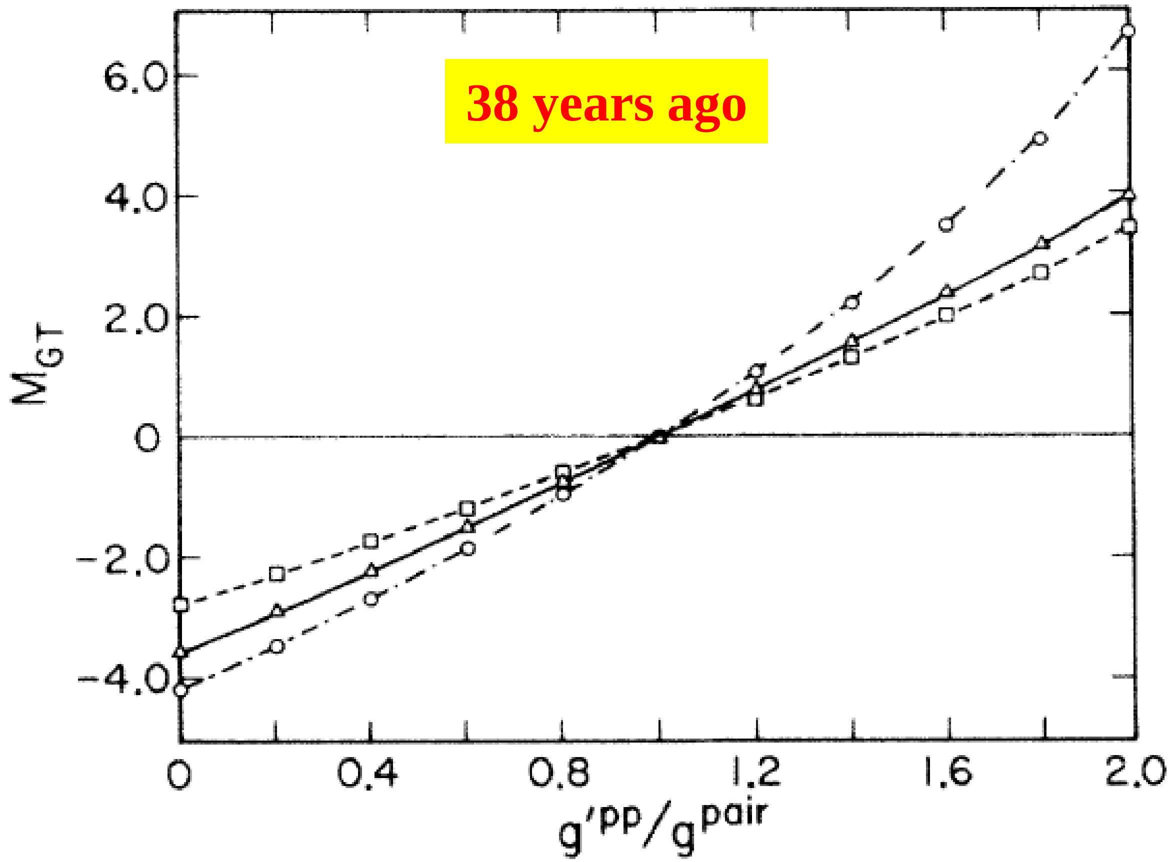
Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects
P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11
E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53
…

The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior?
Is it an artifact of the QRPA?



Study of
$2\nu\beta\beta$ NME
within the schematic model

s.p. mean-field

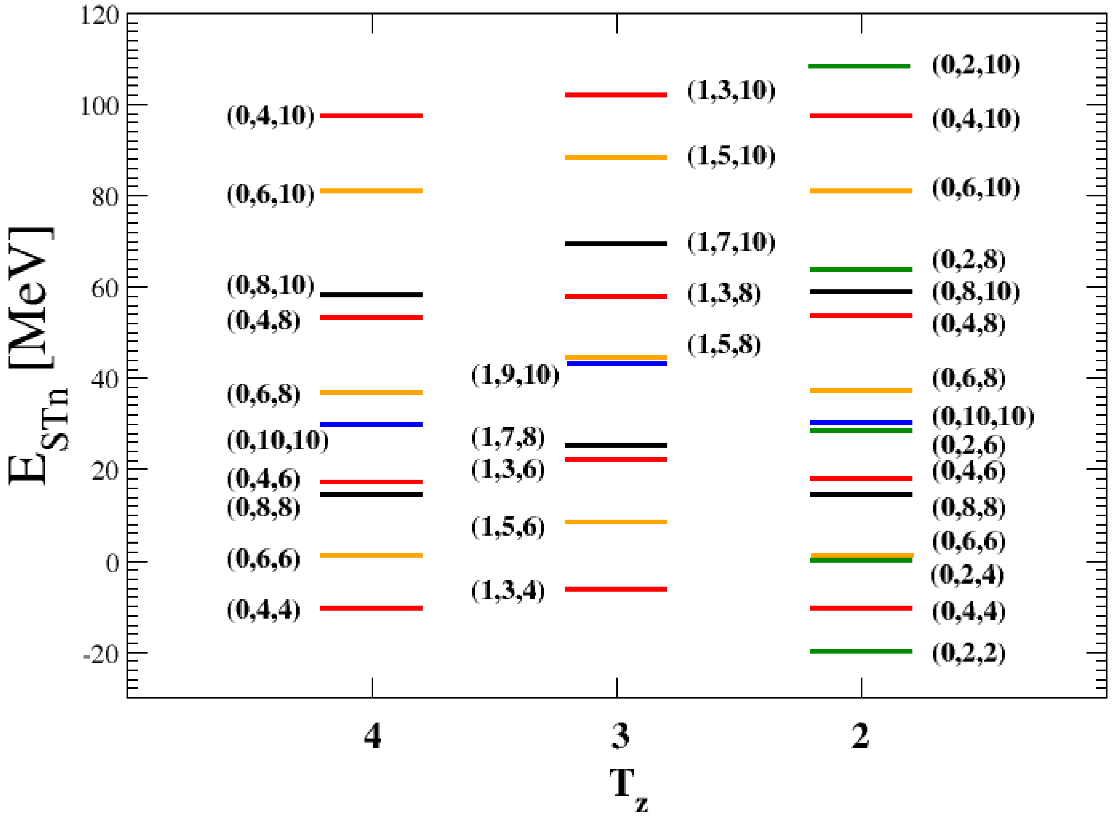
Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^{\dagger}_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^{\dagger}_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+\,(g_{pair} - g^{T=0}_{pp}) \sum_{M_S=-1,0,1} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g^{T=1}_{pp})A^{\dagger}_{0,1}(0,0)A_{0,1}(0,0)\,.}$$

H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)
$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0)
$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)
g_{ph}- strength of particle-hole force



Energies of excited states for the case of conserved SU(4) symmetry
M_F=0, M_{GT}=0 (see SU(4) multiplets)

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

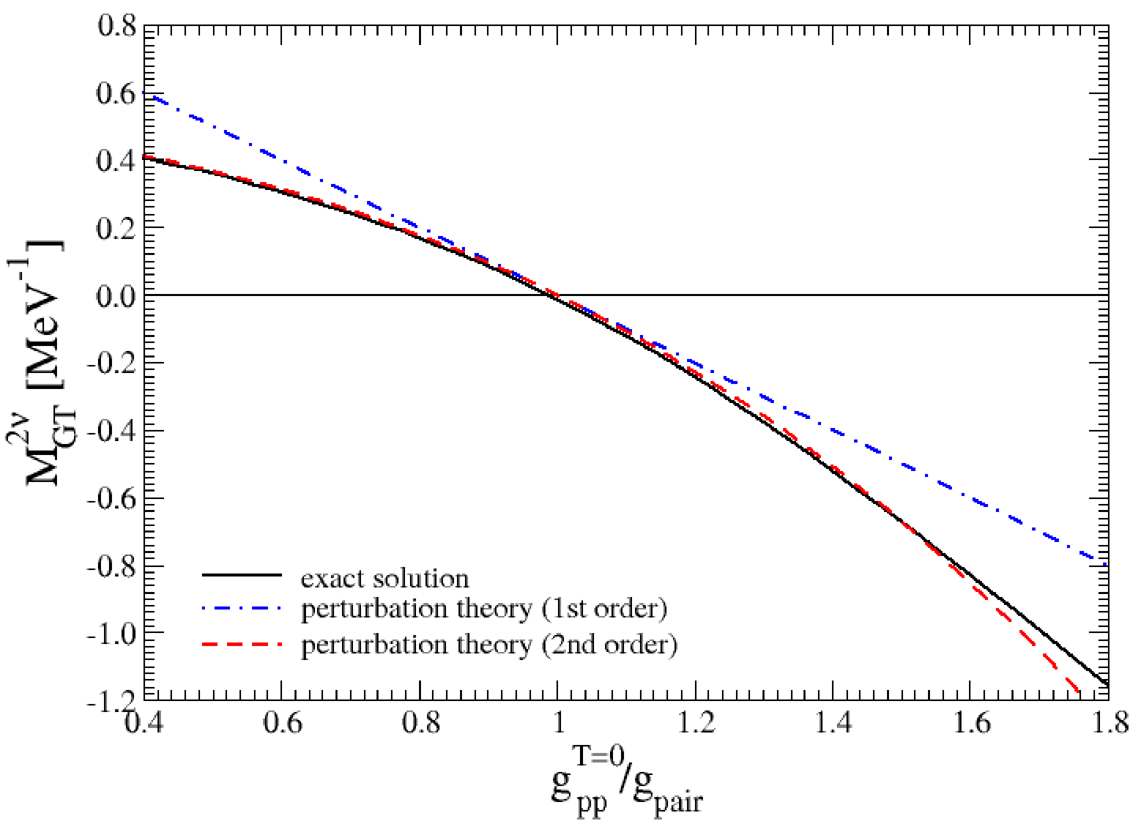
M_F and M_{GT} do not depend on the mean-field part of H

M_F and M_{GT} are governed by a weak violation of the SU(4) symmetry by the article-particle interaction of H

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\left(g_{pair} - g_{pp}^{T=1}\right)}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

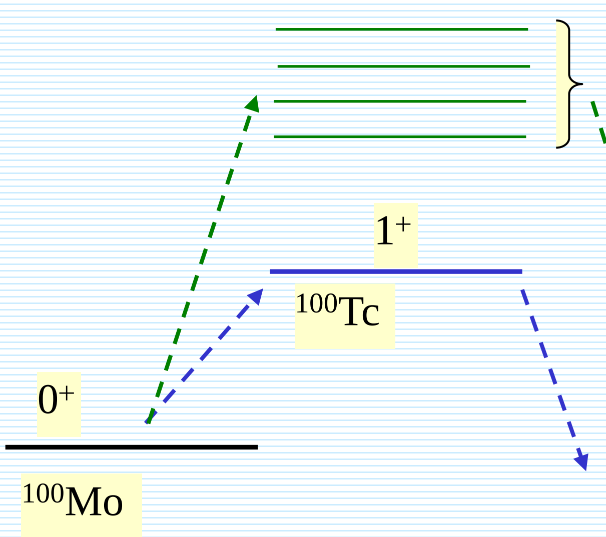
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}}\left\{\frac{\left(g_{pair} - g_{pp}^{T=0}\right)}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}\left(g_{pair} - g_{pp}^{T=1}\right)}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}\right\}$$

M_{GT} up to the second order of perturbation theory due to the violation of the SU(4) symmetry by the particle-particle interaction of H



Results confirm the dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA

Single State Dominance (^{100}Mo, ^{106}Cd, ^{116}Cd, ^{128}Te …)



HSD, higher levels contribute to the decay

SSD, 1^+ level dominates in the decay
(Abad et al., 1984, Ann. Fis. A 80, 9)

Isotope	f.s.	$T_{1/2}$(SSD)[y]	$T_{1/2}$(exp.)[y]
		$2\nu\beta^-\beta^-$	
^{100}Mo	$0_{g.s.}$	7.3 10^{18}	7.07 10^{18}
	0_1	4.2 10^{20}	6.1 10^{20}
^{116}Cd	$0_{g.s.}$	1.1 10^{19}	2.63 10^{19}
^{128}Te	$0_{g.s.}$	1.1 10^{25}	2.49 10^{24}
		EC/EC	
^{106}Cd	$0_{g.s.}$	>4.4 10^{21}	>4.7 10^{20}
^{130}Ba	$0_{g.s.}$	5.0 10^{22}	2.2 10^{21}

$$M^K_{GT} = \sum_m \left(\frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + e_{10} + \nu_{10}} + \frac{M^i_m(1^+)M^f_m(1^+)}{E_m - E_i + e_{20} + \nu_{20}} \right)$$

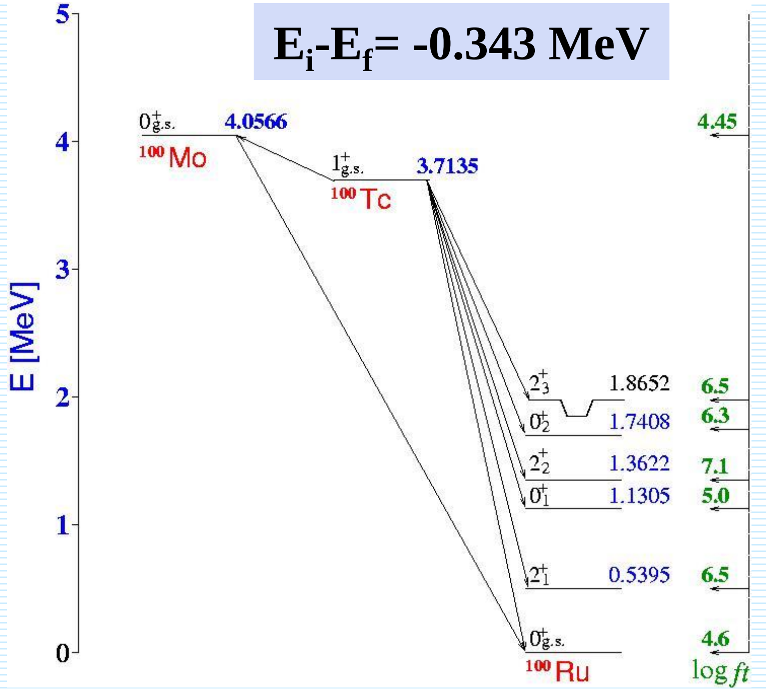
$$\Rightarrow \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + e_{10} + \nu_{10}} + \frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + e_{20} + \nu_{20}} \Rightarrow 2\frac{M^i_1(1^+)M^f_1(1^+)}{E_1 - E_i + \Delta}$$

SSD

common approx.

$$M^K_{GT} = M^L_{GT}(\nu_{10} \leftrightarrow \nu_{20})$$

$$e_{10} + \nu_{10} \approx e_{20} + \nu_{20} \approx (E_i - E_f)/2 \equiv \Delta$$



Domin, Kovalenko, F.Š., Semenov, NPA 753, 337 (2005)

Fedor Simkovic

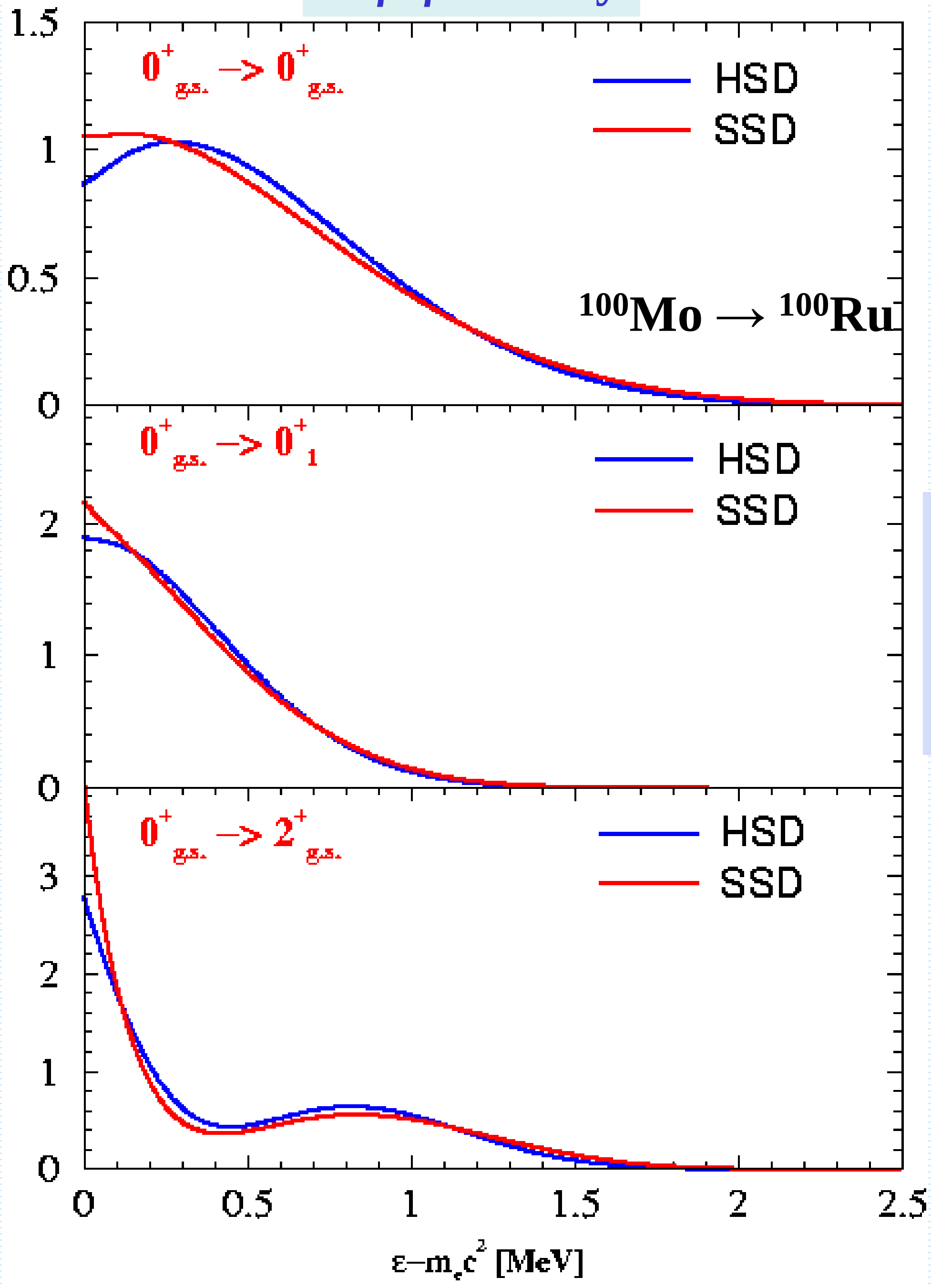
Study of differential characteristics

E_1-E_i ≈ 0 or neg. ⇒ sensitivity to lepton energies in energy Denominators

⇒ SSD and HSD offer different differential characteristics

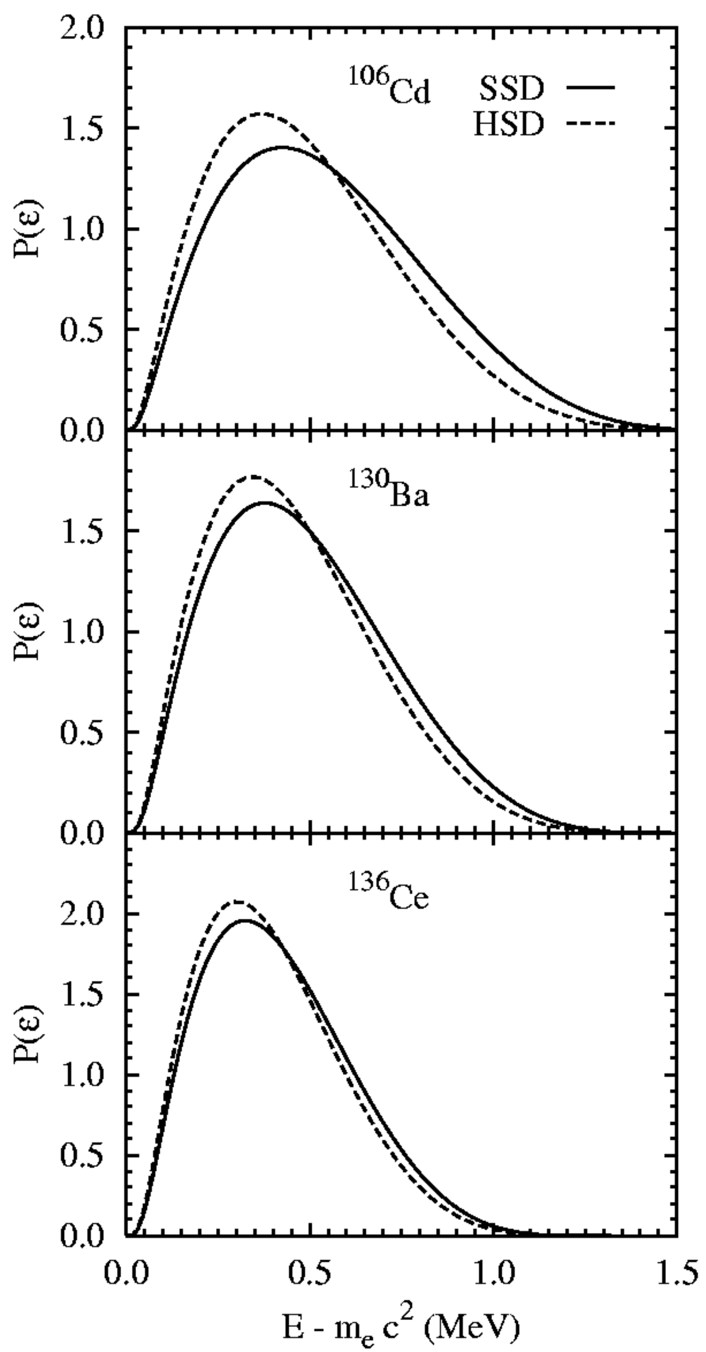
F.Š., Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

2νβ⁻β⁻–decay



Do not depend on M^iM^f

2νEC/β⁺–decay

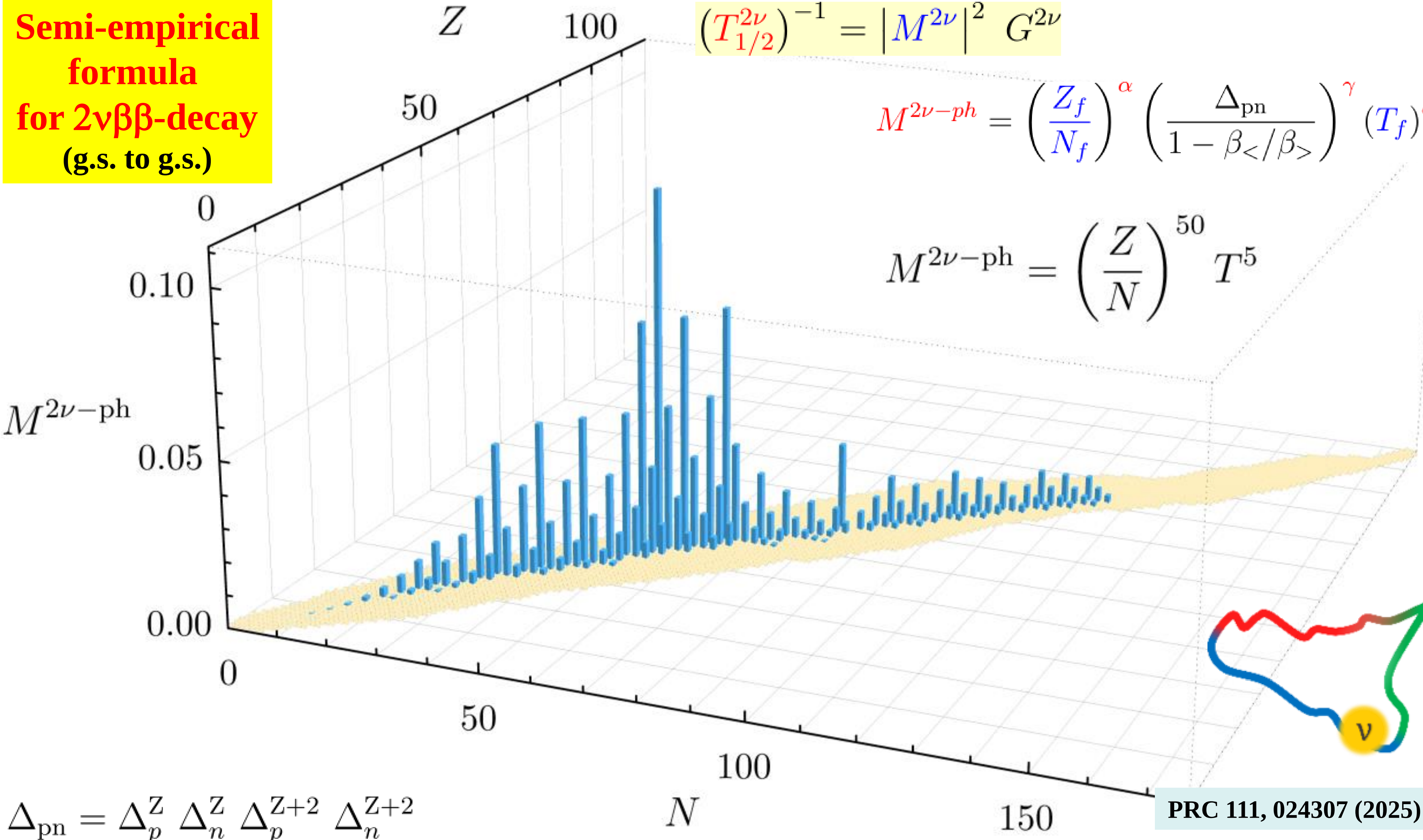


Semi-empirical formula for 2νββ-decay
(g.s. to g.s.)

$$\left(T_{1/2}^{2\nu}\right)^{-1} = \left|M^{2\nu}\right|^2 \; G^{2\nu}$$

$$M^{2\nu-ph} = \left(\frac{Z_f}{N_f}\right)^{\alpha} \left(\frac{\Delta_{\mathrm{pn}}}{1-\beta_{<}/\beta_{>}}\right)^{\gamma} (T_f)^{\sigma}$$

$$M^{2\nu-\mathrm{ph}} = \left(\frac{Z}{N}\right)^{50} T^5$$



$$\Delta_{\mathrm{pn}} = \Delta_p^{Z} \; \Delta_n^{Z} \; \Delta_p^{Z+2} \; \Delta_n^{Z+2}$$

PRC 111, 024307 (2025)

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - P_i)^2}{\sigma_i^2}$$

There is no reliable calculation of the *2νββ-decay NMEs yet*

PRC 111, 024307 (2025)

Nucleus	$M^{2\nu-\mathrm{th}}$ QRPA	QRPA	IBM	IBM	IBM	NSM	NSM	PHFB	FSQP	ET	$M^{2\nu-\mathrm{exp}}$	$M^{2\nu-\mathrm{ph}}$ SEF	SSD
Fitted													
^{48}Ca	–	0.016	0.069	0.024	0.045	–	0.039	–	–	–	0.0314 ± 0.0030	0.022	–
^{76}Ge	–	0.063	0.083	0.018	0.085	–	0.097	–	0.083	0.085	0.0987 ± 0.0010	0.105	–
^{82}Se	–	0.058	0.072	0.024	0.063	0.099	0.105	–	0.103	0.156	0.0828 ± 0.0005	0.081	–
^{96}Zr	–	0.133	0.058	0.054	0.034	–	–	0.092	0.072	–	0.0770 ± 0.0040	0.063	–
^{100}Mo	0.105	0.251	0.197	0.157	0.045	–	–	0.164	0.154	0.179	0.2019 ± 0.0016	0.199	0.174
^{116}Cd	0.112	0.049	0.089	0.069	0.031	–	–	–	0.088	0.137	0.1142 ± 0.0027	0.120	0.148
^{130}Te	0.057	0.053	0.035	0.010	0.038	0.027	0.036	0.059	0.027	0.034	0.0265 ± 0.0003	0.028	–
^{136}Xe	0.036	0.030	0.056	0.022	0.032	0.024	0.021	–	–	–	0.0174 ± 0.0002	0.016	–
^{150}Nd	–	–	0.077	0.054	0.017	0.069	–	0.048	–	–	0.0527 ± 0.0019	0.032	0.023
^{238}U	–	–	–	–	0.023	–	–	–	–	–	0.0550 ± 0.0110	0.026	–
χ^2/N	5170	2100	2845	2828	1773	470	686	3442	480	4774	–	30	233
Predicted													
^{110}Pd	0.167	–	–	0.022	0.041	–	–	–	0.233	0.211	< 2.61	0.147	–
^{124}Sn	0.023	–	–	–	0.034	0.037	–	–	–	–	–	0.029	–
^{128}Te	0.051	0.063	0.022	0.018	0.044	0.013	0.049	0.058	0.030	0.050	0.0366 ± 0.0007	0.047	0.015
^{134}Xe	0.063	–	–	–	–	–	–	–	–	–	< 1.25	0.033	–

Improved description of the $0\nu\beta\beta$–decay rate (a way to fix g_A^{eff})

PRC 97, 034315 (2018)

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons. Explaining $2\nu\beta\beta$-decay is necessary but not sufficient

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon^2_{K,L}}$$

$$\epsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$
$$\epsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

Taylor expansion $$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

We get

$$\left[T^{2\nu}_{1/2}\right]^{-1} \simeq (g_A^{\rm eff})^4 \left|M_{GT-1}\right|^2 \left[G^{2\nu}_0 + \xi_{31} G^{2\nu}_2 + (\xi_{31})^2 G^{2\nu}_{22} + \left(\frac{1}{3}(\xi_{31})^2 + \xi_{51}\right) G^{2\nu}_4 + \frac{1}{3}\xi_{31}\xi_{51} G^{2\nu}_{42} + \frac{2}{3}\xi_{31}\xi_{51} G^{2\nu}_6\right]$$

$$\xi_{31} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$
$$\xi_{51} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

$$M^{2\nu}_{GT-1} = \sum_n M_n(0^+) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2}$$
$$M^{2\nu}_{GT-3} = \sum_n M_n(0^+) \frac{4\ m_e^3}{(E_n(1^+) - (E_i + E_f)/2)^3}$$
$$M^{2\nu}_{GT-5} = \sum_n M_n(0^+) \frac{16\ m_e^5}{(E_n(1^+) - (E_i + E_f)/2)^5}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM)

The running sum of the 2νββ–decay NMEs (QRPA)

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

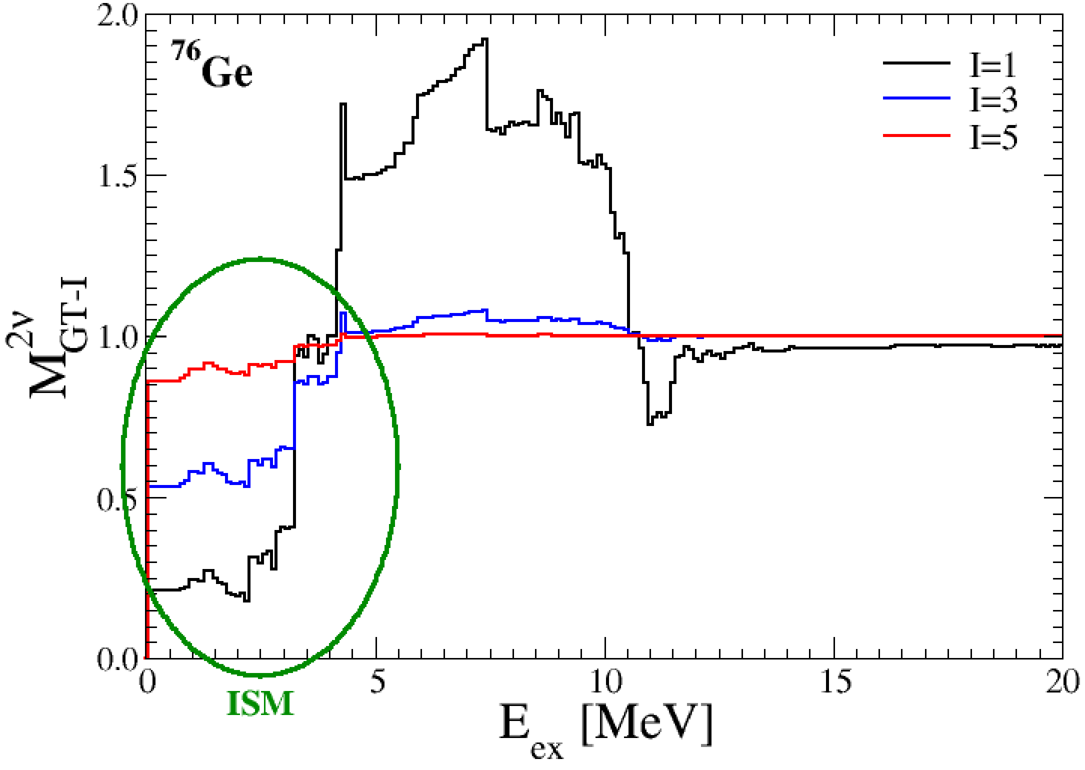
$$\xi^{2\nu}_{13} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$

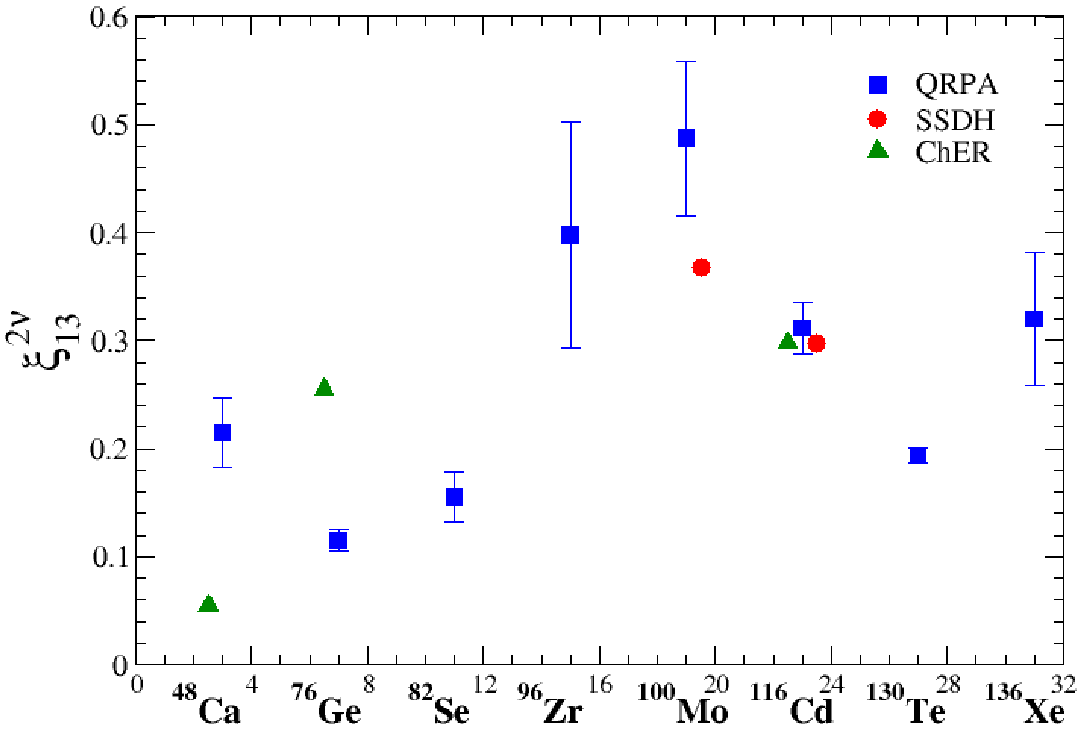
$$\xi^{2\nu}_{15} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

ξ_{13} tell us about importance of higher lying states of int. nucl.

HSD: $\xi_{13}\approx 0$

$\xi_{13}\approx 1$ (large)
Possible because of a large cancellati
of contributions through lower and
higher-lying states of (A,Z+1) nucleu





ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

F.Š., Šmotlák, Semenov, J. Phys. G, 27, 2233, 2001

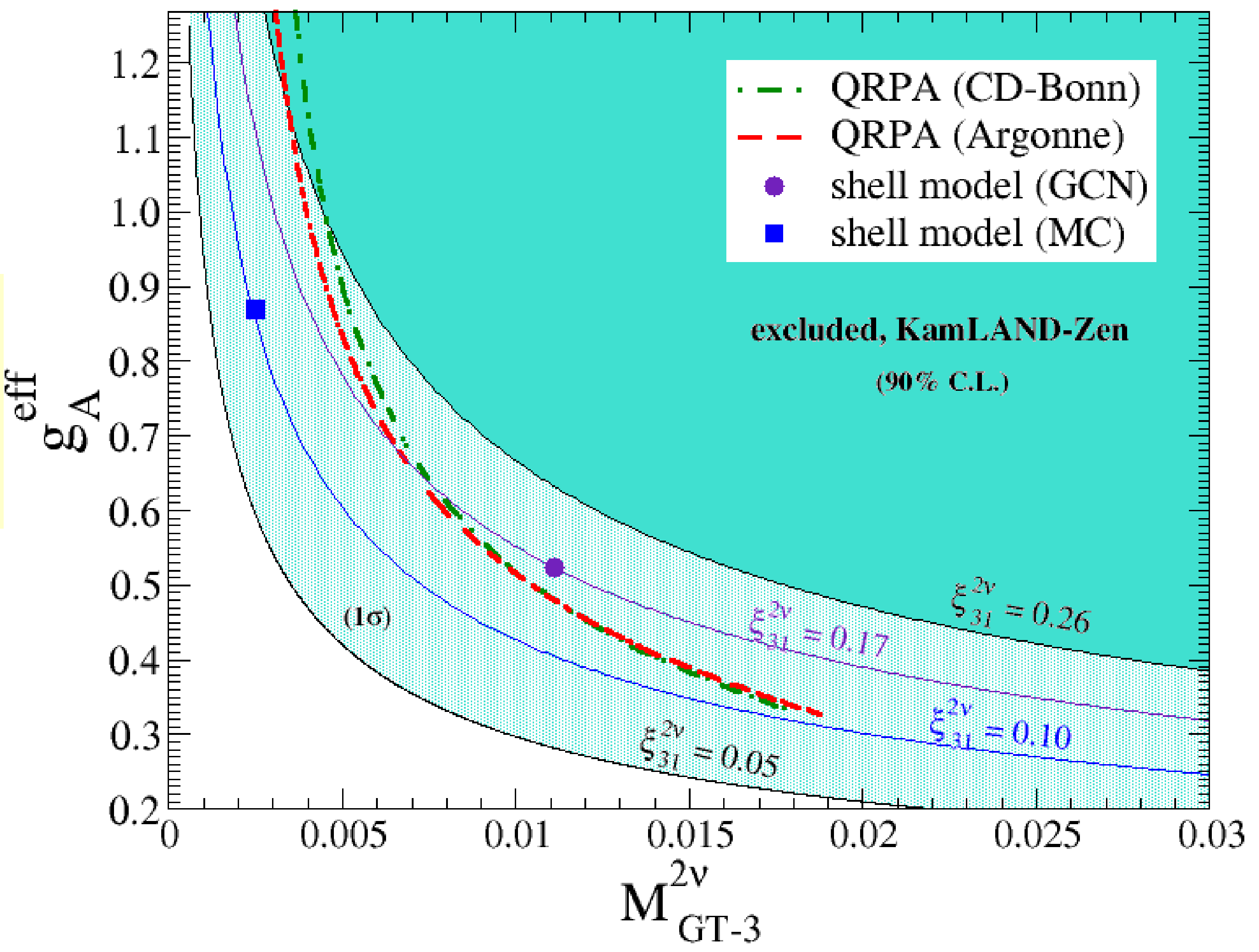
KamLAND-Zen Exp. : ξ_{13}<0.26 (^{136} Xe)

ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

The g_A^{eff} can be deterimed with measured half-life, ratio of NMEs ξ_{31} and calculated NME, dominated by transitions through low lying states of the intermediate nucleus.

$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|}\frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)}}$$

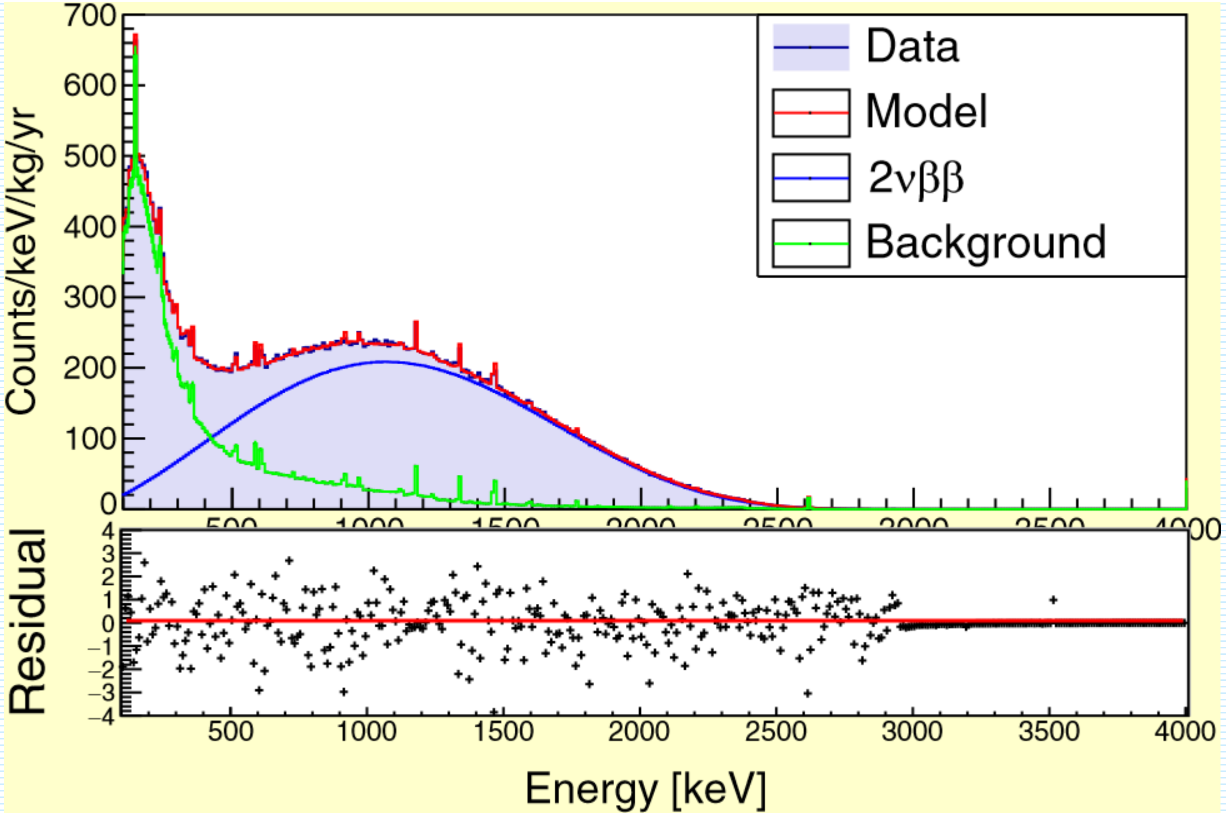
$M_{GT\text{-}3}$ have to be calculated by nuclear theory - ISM

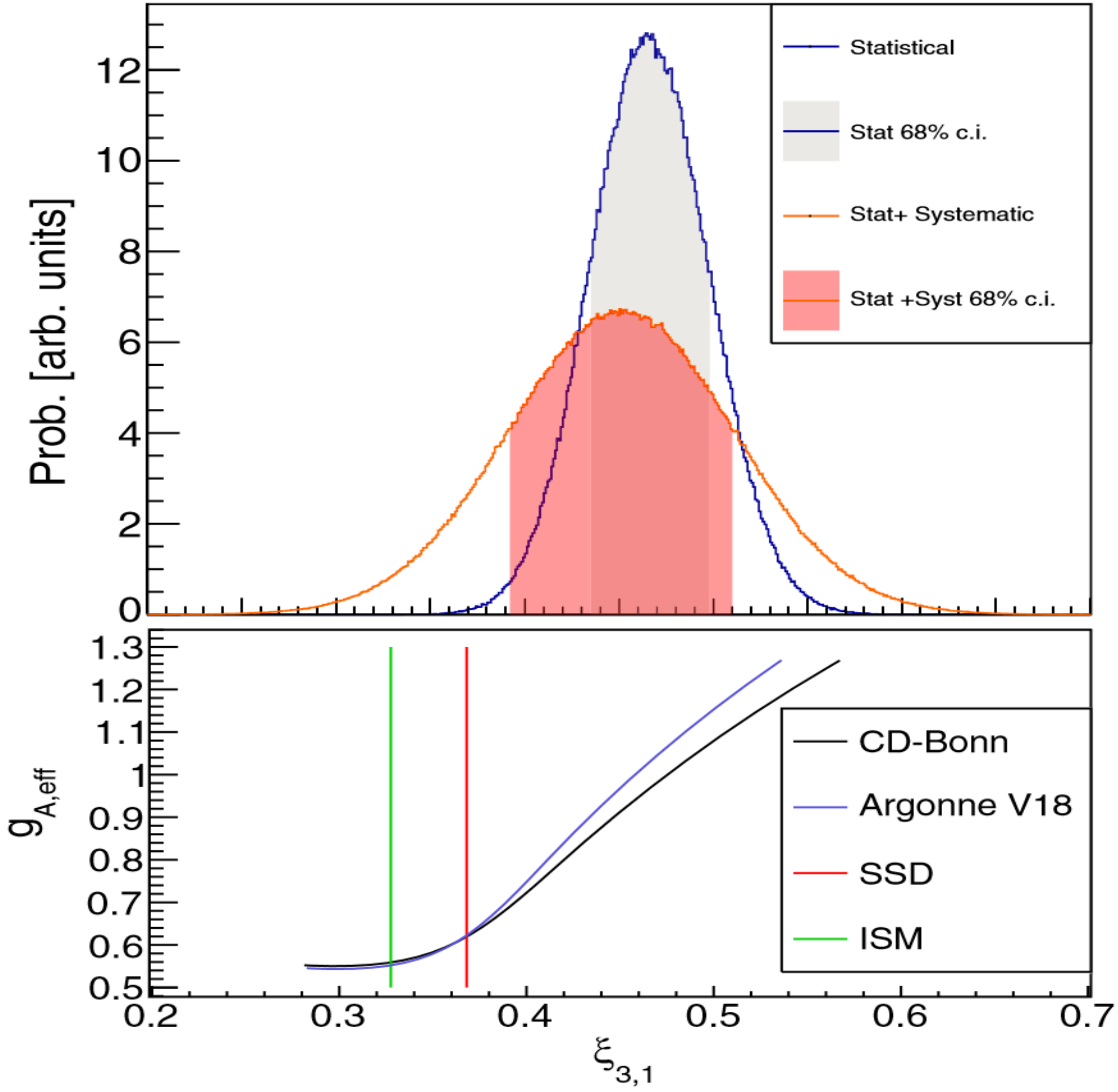


KamLAND-Zen Coll. (+J. Menendez, F.Š.), Phys.Rev.Lett. 122, 192501 (2019)

CUPID-Mo Exp. : ξ_{13}=0.45±0.03 (stat) ±0.05 (syst) (^{100} Mo)

Phys.Rev.Lett. 131, 162501 (2023)





$T_{1/2}$ = [7.07±0.02(stat)±0.11(syst)] x 10^{18} yr

ξ_{51}/ξ_{31} = 0.364-0.368 (QRPA), 0.367 (SSD), 0.349 (ISM)

g_A^{eff} (pnQRPA) = 1.0 ±0.1(stat)±0.2(syst)

g_A^{eff} (ISM) = 1.11 ±0.03(stat)±0.05(syst)

CUORE Exp. (^{130}Te)

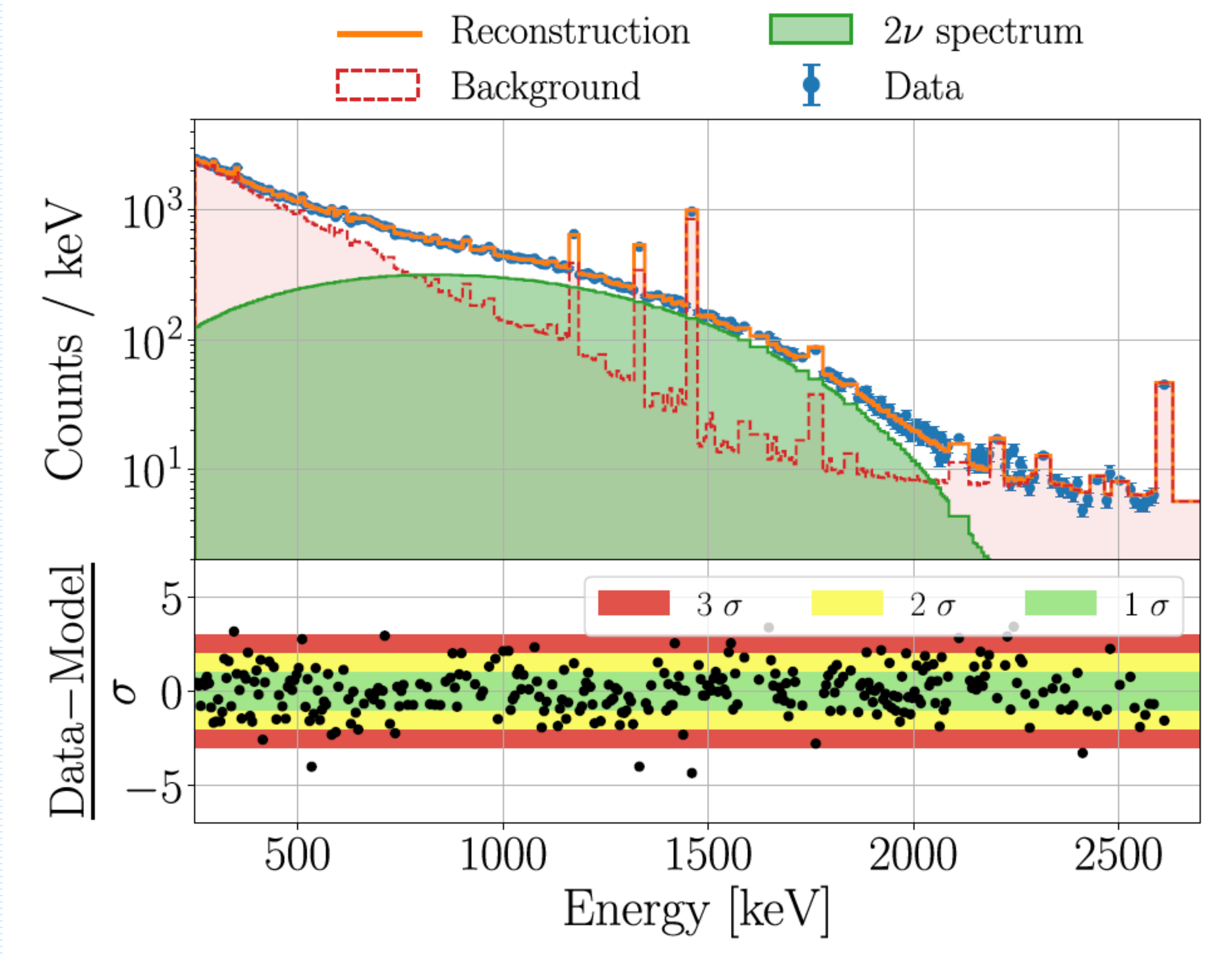
arXive: 2503.24137 [nucl-ex]

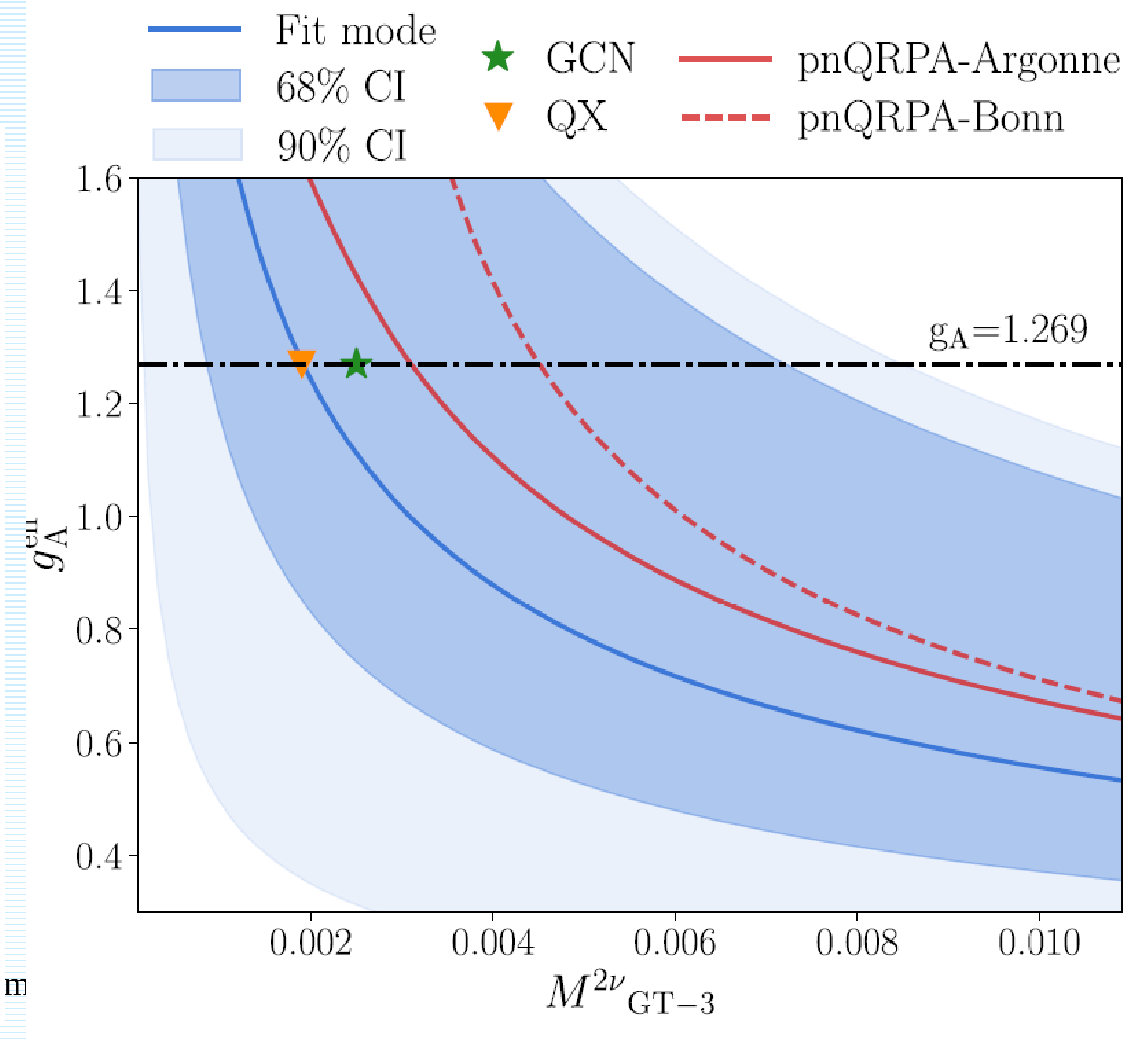
$$T^{2\nu}_{1/2} = (9.32^{+0.05}_{-0.04}\text{stat.}^{+0.07}_{-0.07}\text{syst.}) \times 10^{20}\ \text{yr}$$

$$\xi_{31} = 0.01^{+0.31}_{-0.01}$$

$$\xi_{51} = 1.46^{+0.33}_{-0.62}$$

It does not make sense(!?)
We are confused …





The role of the width of the states of the intermediate nucleus
($E_n \to E_n + i\,\Gamma_n/2$)

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \left(g_A^{\text{eff}}\right)^4 \left|M_{GT}^{R1}\right|^2 \left[\left(1 + \xi_{IR11}{}^2\right) G_0^{2\nu} + \left(\xi_{RR31} + \xi_{IR11}\xi_{IR31}\right) G_2^{2\nu} + \frac{1}{3}\left(\xi_{IR31}{}^2 + \xi_{RR31}{}^2\right)\left(G_{22}^{2\nu} + G_4^{2\nu}\right)\right]$$

$$\xi_{RR31} = \frac{M_{GT}^{R3}}{M_{GT}^{R1}} \qquad \xi_{IR11} = \frac{M_{GT}^{I1}}{M_{GT}^{R1}} \qquad \xi_{IR31} = \frac{M_{GT}^{I3}}{M_{GT}^{R1}}$$

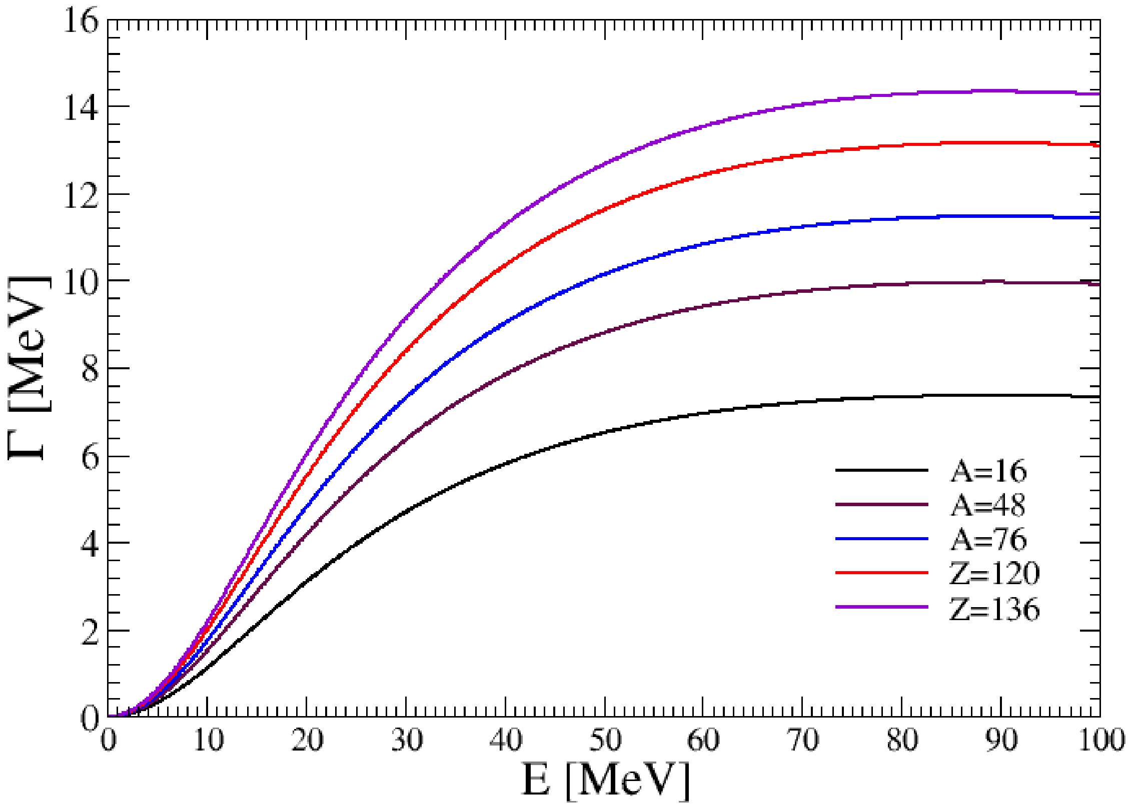
$$M_{GT}^{R1} = m_e \sum_n \frac{M_n D_n}{(D_n^2 + \Gamma_n^2/4)}$$

$$M_{GT}^{R3} = 4m_e^3 \sum_n \frac{M_n D_n\,(D_n^2 - 3\Gamma_n^2/4)}{(D_n^2 + \Gamma_n^2/4)^3}$$

$$M_{GT}^{I1} = m_e \sum_n \frac{M_n \Gamma_n/2}{(D_n^2 + \Gamma_n^2/4)}$$

$$M_{GT}^{I3} = 4m_e^3 \sum_n \frac{M_n \Gamma_n/2\,(3D_n^2 - \Gamma_n^2/4)}{(D_n^2 + \Gamma_n^2/4)^3}$$

$$D_n = E_n - (E_i + E_f)/2$$

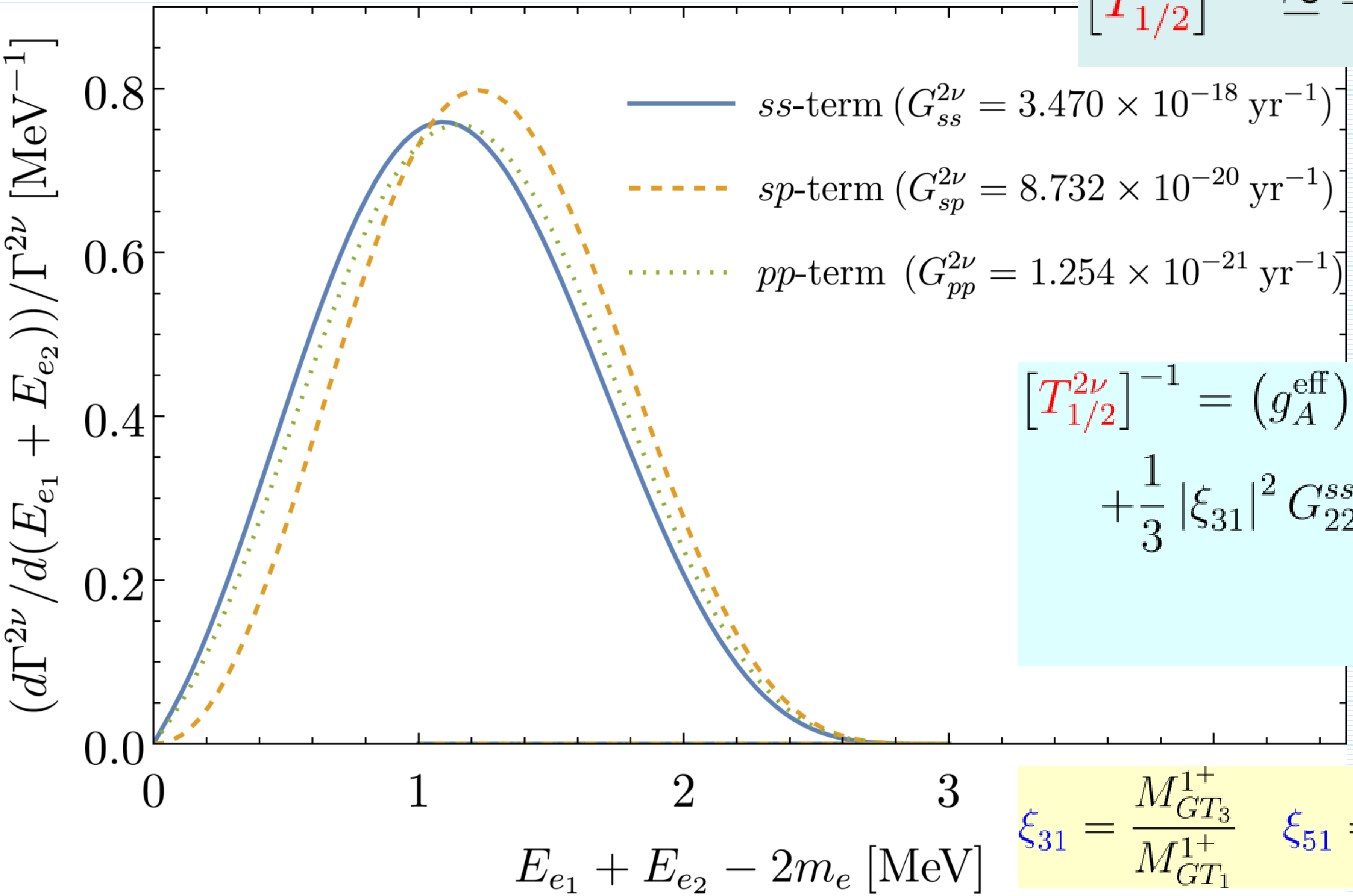


We found that ξ_{IR11} and ξ_{IR31} are negligibly small

Effect of the electron $p_{1/2}$-wave state in $2\nu\beta\beta$-decay (preliminary)

$$A^{2\nu} = A^{ss} + A^{p_{1/2}p_{1/2}} + \cdots$$

$$\left[T^{2\nu}_{1/2}\right]^{-1} \simeq \frac{\Gamma^{ss}_0 + \Gamma^{ss}_2 + \Gamma^{ss}_4 + \Gamma^{sp_{1/2}}_0}{\ln(2)}$$



$$\left[T^{2\nu}_{1/2}\right]^{-1} = \left(g^{\rm eff}_A\right)^4 \left|M^{ss}_{GT_1}\right|^2 \Big(G^{ss}_0 + \Re\{\xi_{31}\}G^{ss}_2 + \frac{1}{3}\left|\xi_{31}\right|^2 G^{ss}_{22} + \left(\frac{1}{3}\left|\xi_{31}\right|^2 + \Re\{\xi_{51}\}\right) G^{ss}_4 + G^{sp_{1/2}}_0 \, \xi_{p_{1/2}s}\Big)$$

$$\xi_{31} = \frac{M^{1^+}_{GT_3}}{M^{1^+}_{GT_1}} \qquad \xi_{51} = \frac{M^{1^+}_{GT_5}}{M^{1^+}_{GT_1}} \qquad \xi_{p_{1/2}s} = \frac{M^{0^-} + M^{1^-}}{M^{1^+}_{GT_1}}$$

$2\nu\beta\beta$ probes New Beyond SM Physics

All 100 kg- and ton-class $0\nu\beta\beta$ experiments can also study a diverse range of exotic phenomena, e.g. through spectral distortion in $2\nu\beta\beta$. Future searches will probe the $2\nu\beta\beta$ with high statistics about 10^5-10^6 events.

Common subjects:

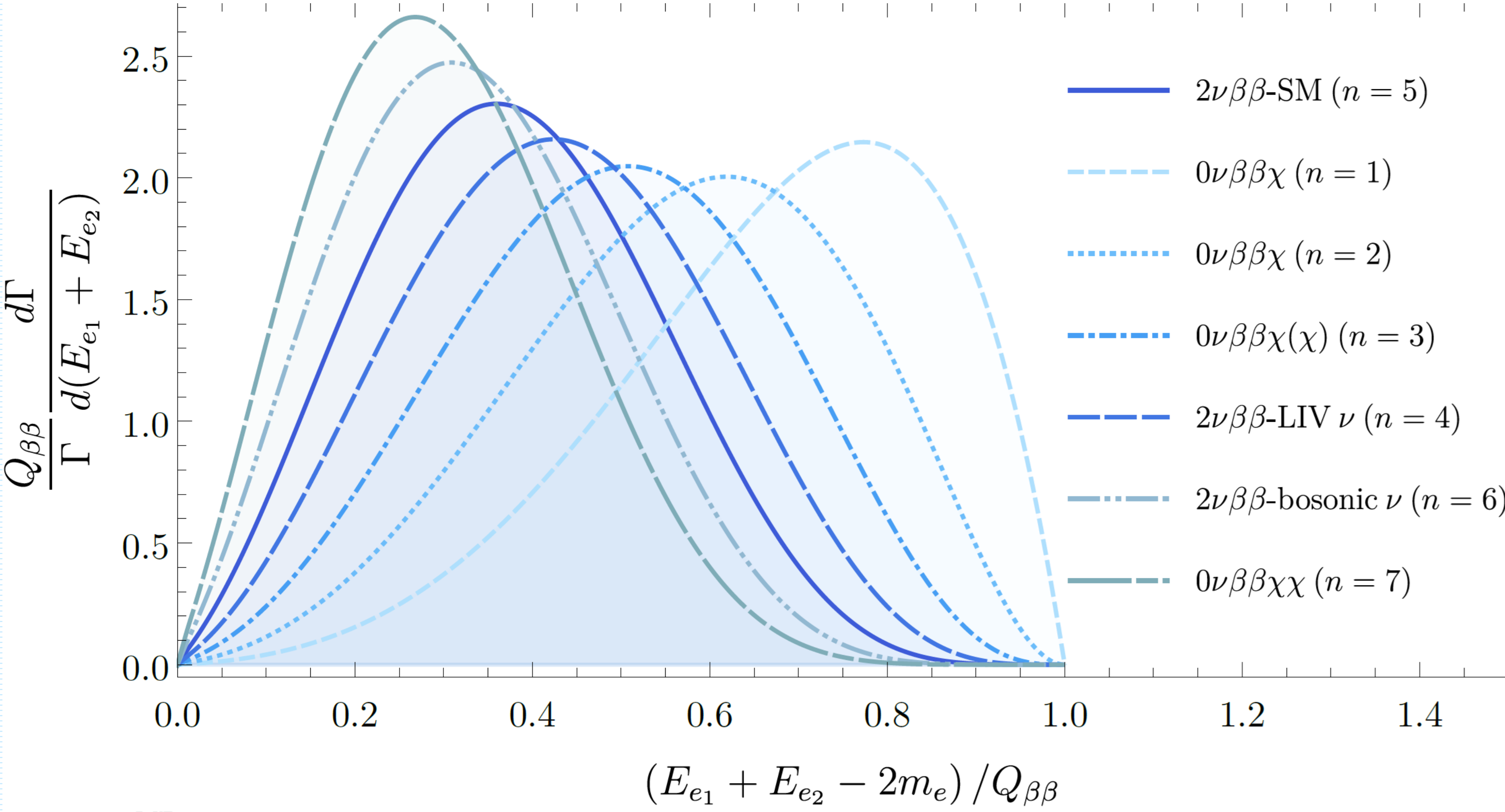
Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance violation

Recent subjects:

Lepton-number conserving right-handed currents
(PRL 125 (2020) 17, 171801)

Neutrino self-interactions
(PRD 102 (2020) 5, 051701)

Sterile neutrino and light fermion searches through energy end point
(PRD 103 (2021) 5, 055019;
PLB 815 (2021) 136127)



$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q-\varepsilon_1-\varepsilon_2)^n\ [p_1\varepsilon_1 F(\varepsilon_1)]\ [p_2\varepsilon_2 F(\varepsilon_2)]$$

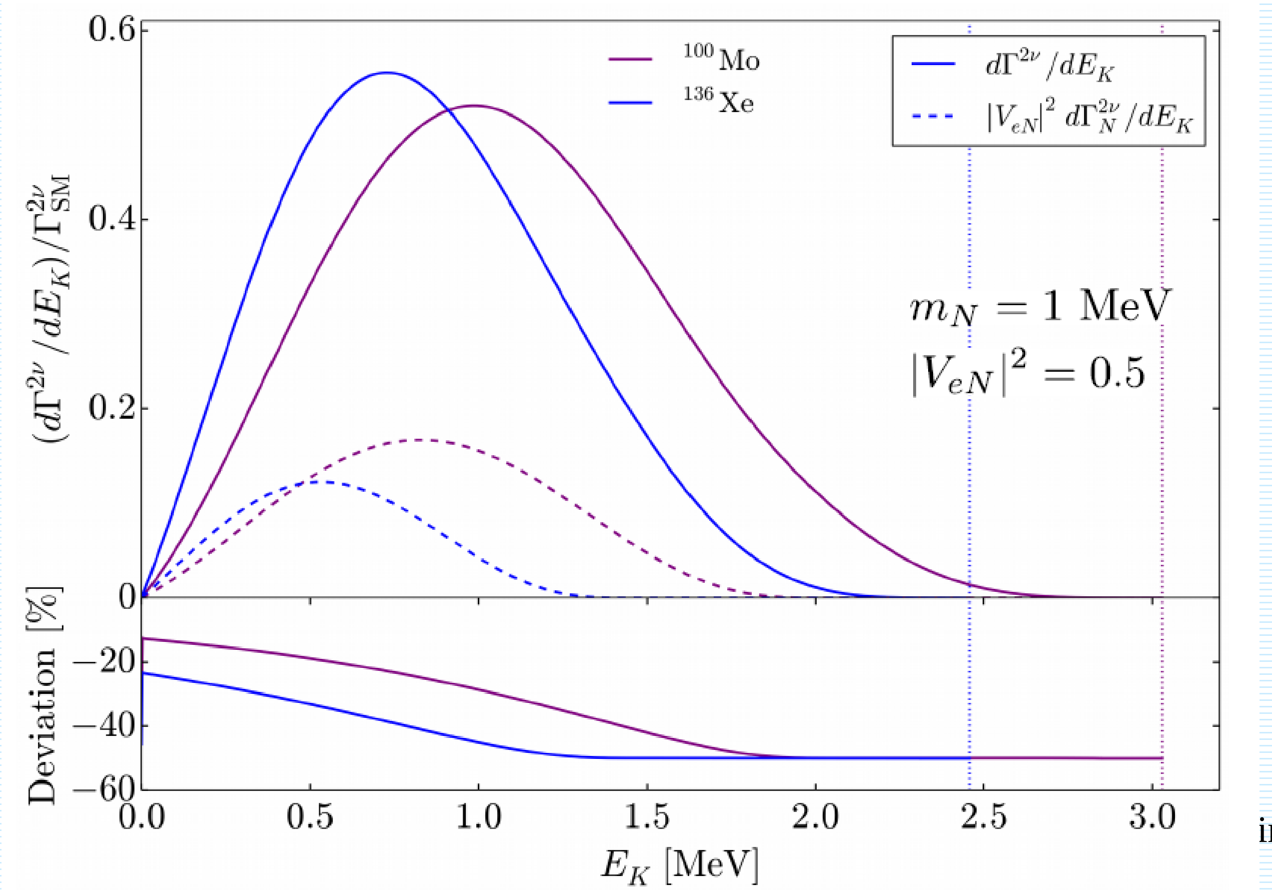
Spectral index n

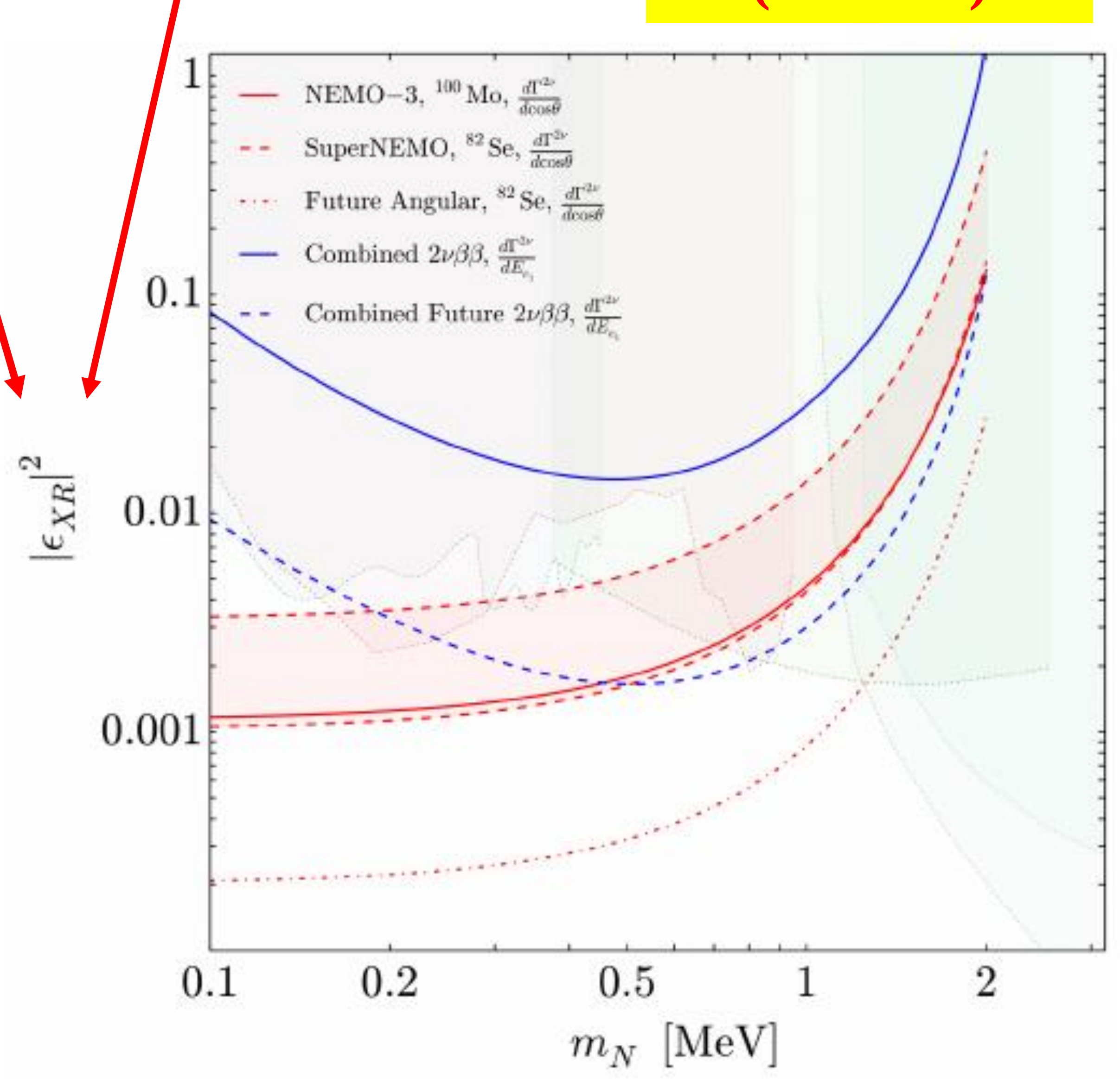
$$\mathcal{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1+\delta_{\mathrm{SM}}) j_L^{\mu} J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \text{h.c.}$$

Exotic 2νββ Sterile Neutrino (or HNL)

Consider either mixing of sterile ν with active ν, or right-handed currents

Phys.Rev.D 103 (2021) 055019

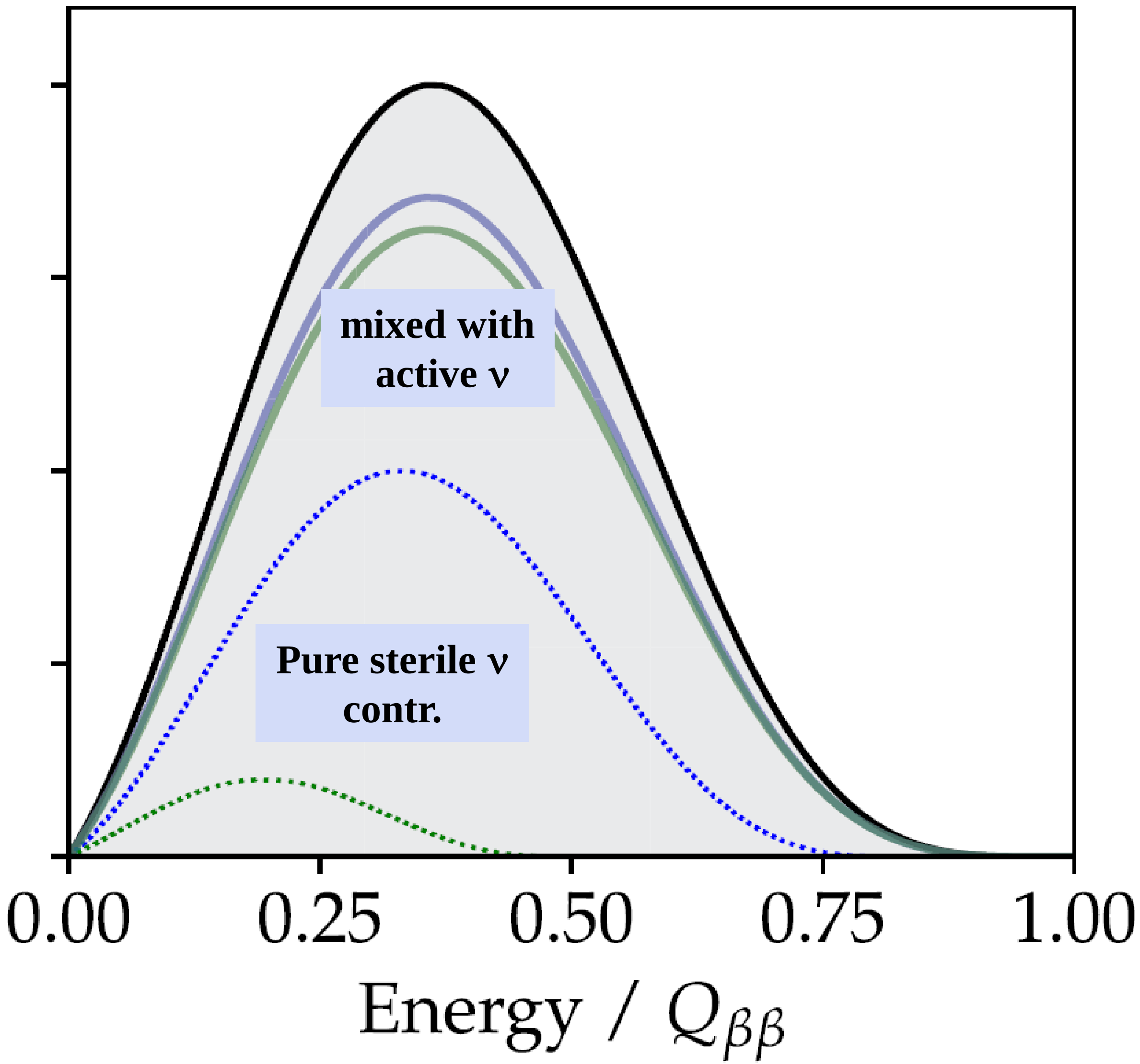


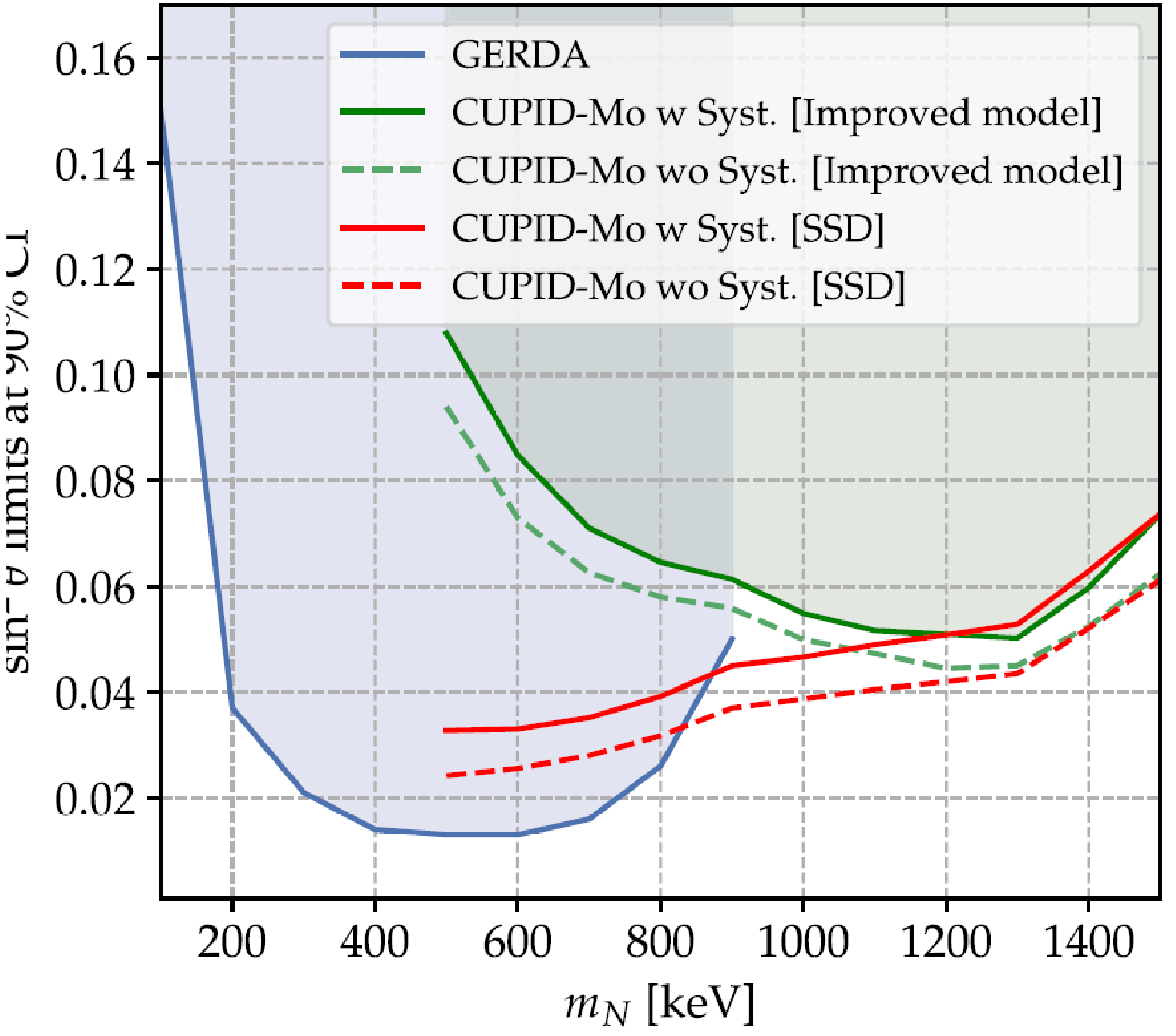


2νββ probe – sterile neutrino search
CUPID-Mo

m_N= 0.5 MeV and 1.5 MeV, $\sin^2\theta$= 0.01

Eur. Phys. J. C 84, 925 (2024)






NEUTRINO
NEUTRINO
NEUTRINO


THANK YOU!


A MOST INTERESTING
PROBLEM

Fedor S


ν
Time flies when
you are having fun.
Albert Einstein
quotefancy