

MAYORANA International School
Modica
June 19-25 2025

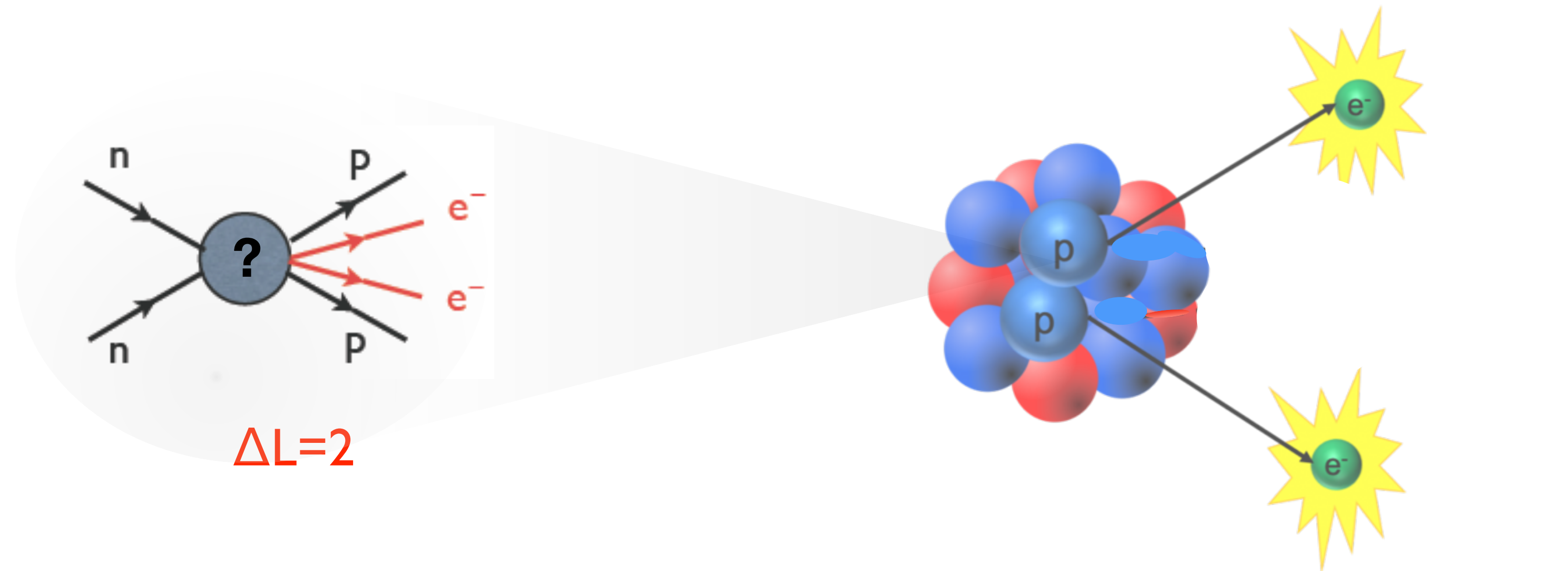
Neutrinoless double beta decay and Lepton Number Violation - I

Vincenzo Cirigliano

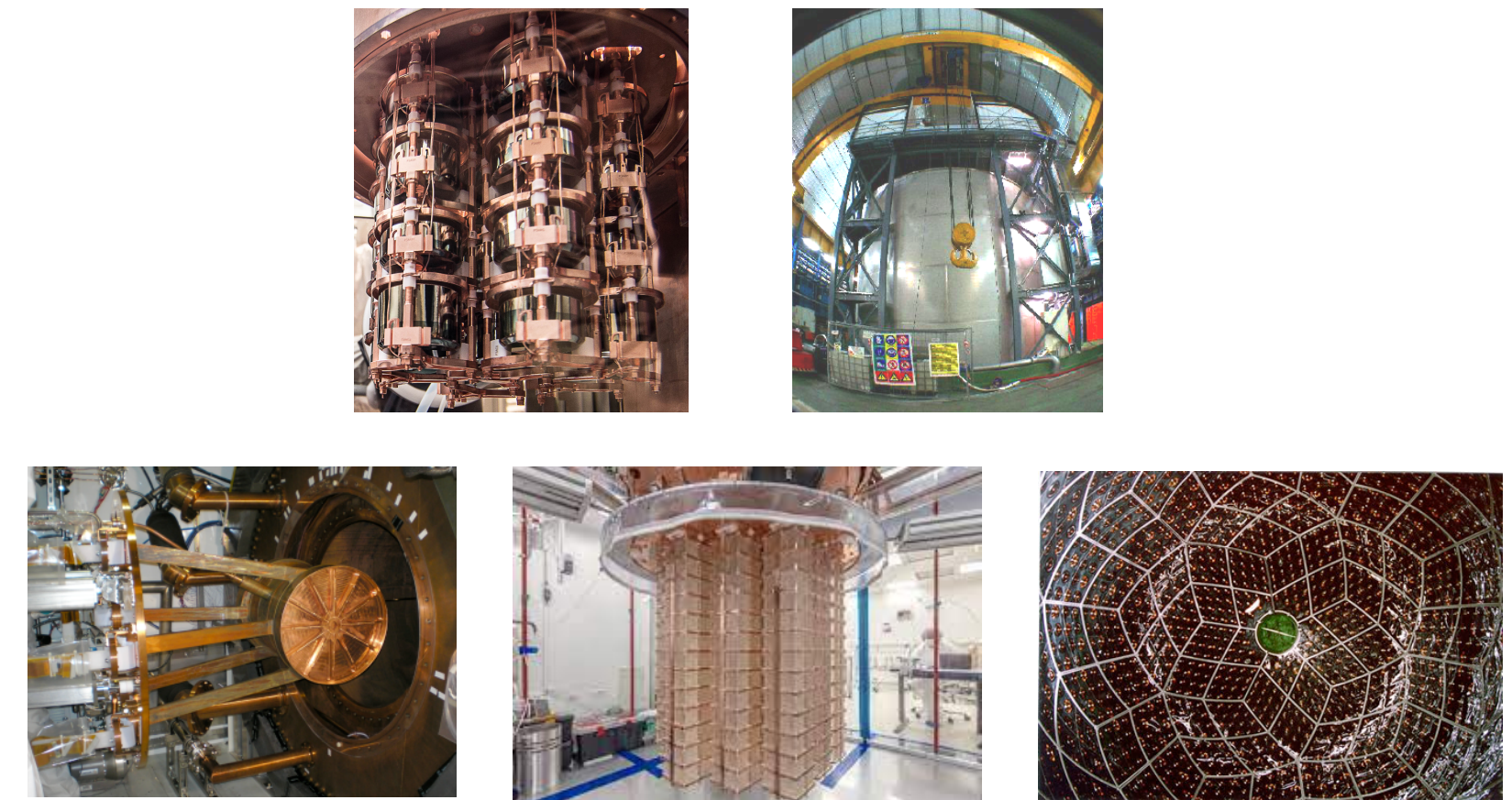


Goal of these lectures

Introduction to **neutrinoless double beta decay** ($0\nu\beta\beta$),
its **significance** (nature of neutrino mass & baryon asymmetry),
and the **discovery potential** of current experimental searches

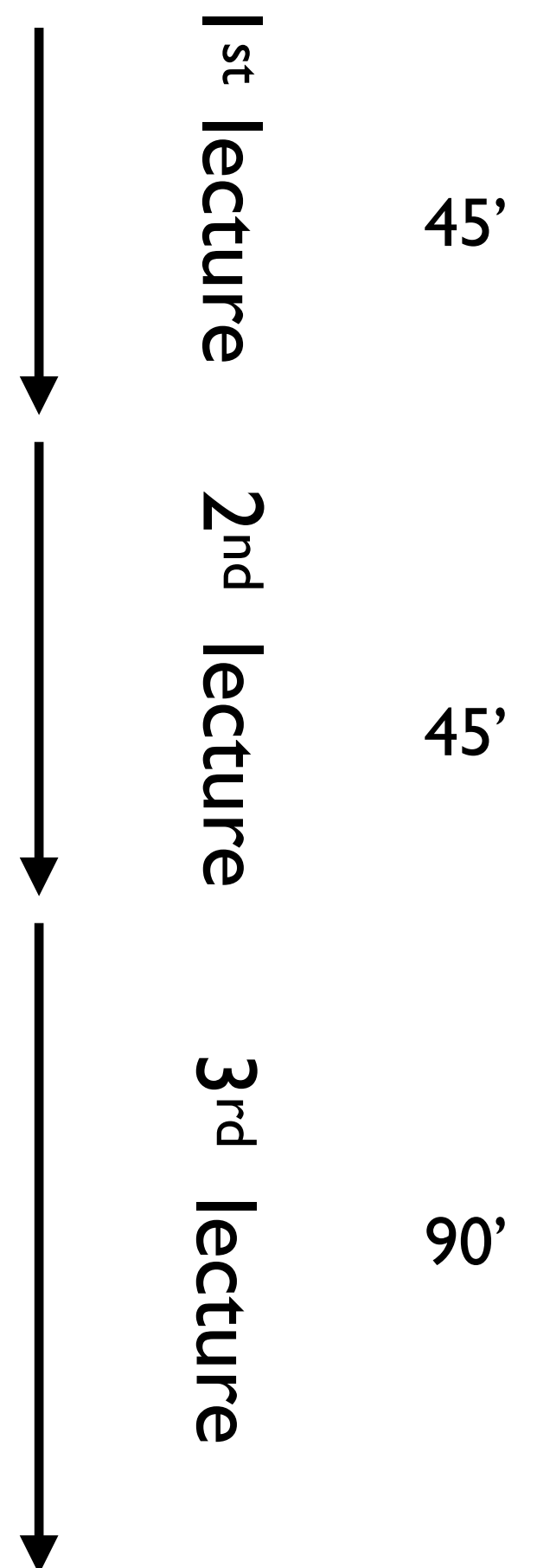


$$(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$$



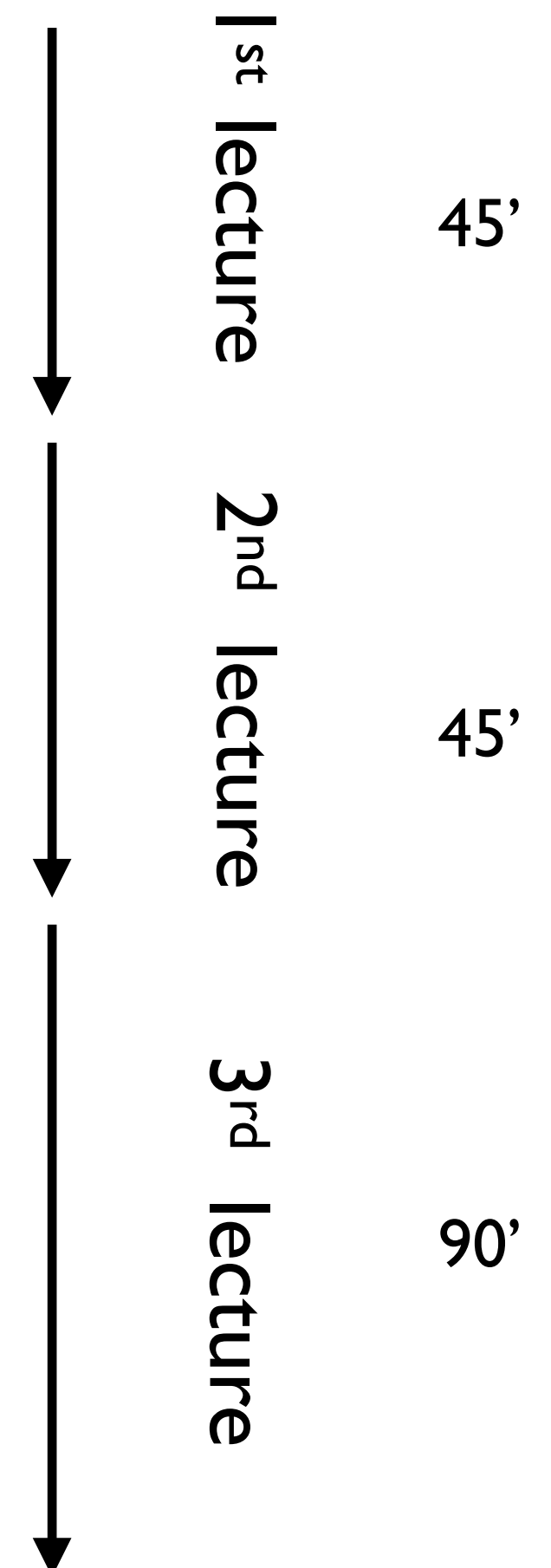
Plan for the lectures

- Significance of neutrinoless double beta decay & connection to big questions
 - Origin and nature of neutrino mass
 - The baryon asymmetry of the universe
 - Discovery potential of $0\nu\beta\beta$ — overview
- *End-to-end* Effective Field Theory for Lepton Number Violation (LNV) and $0\nu\beta\beta$
 - $0\nu\beta\beta$ from **high-scale see-saw** (LNV @ dim 5 = 3 light ν exchange mechanism)
 - $0\nu\beta\beta$ from **(multi)TeV-scale dynamics** (LNV @ dim 7, 9, ...)
 - $0\nu\beta\beta$ from **sterile neutrinos**
- Conclusions and outlook



Plan for the lectures

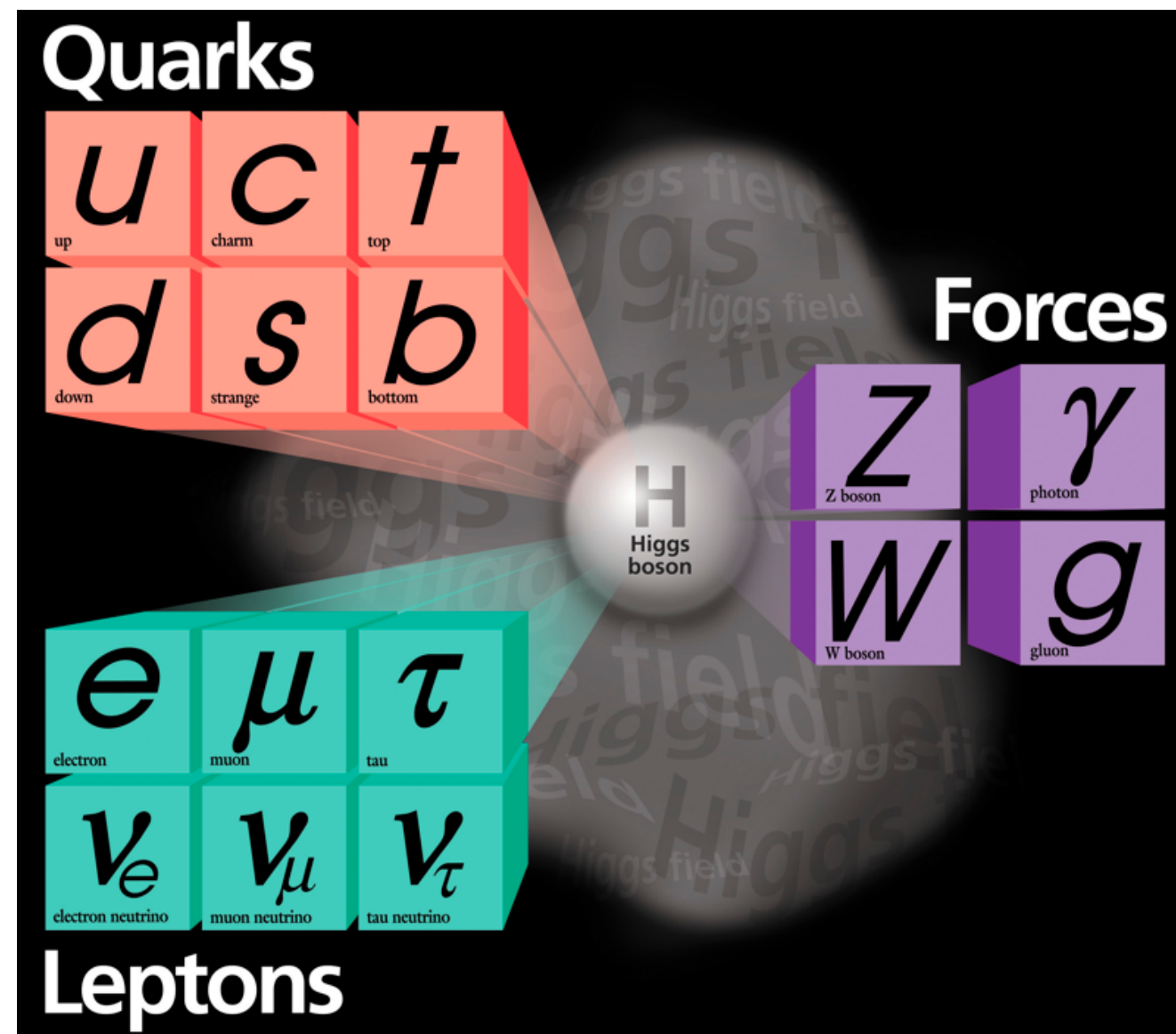
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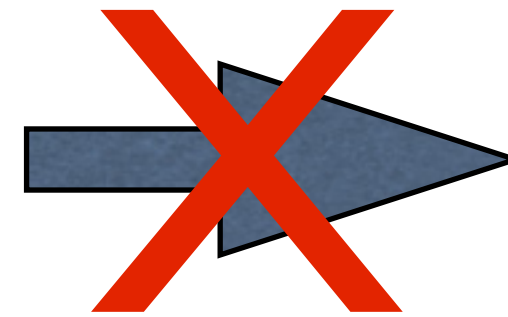
Special thanks to collaborators on these topics:
W. Dekens, J. de Vries, M. Graesser, M. Hoferichter, E. Mereghetti, S. Pastore, M. Piarulli,
S. Urrutia-Quiroga, U. van Kolck, A. Walker-Loud, R. Wiringa

Context: open questions in subatomic physics

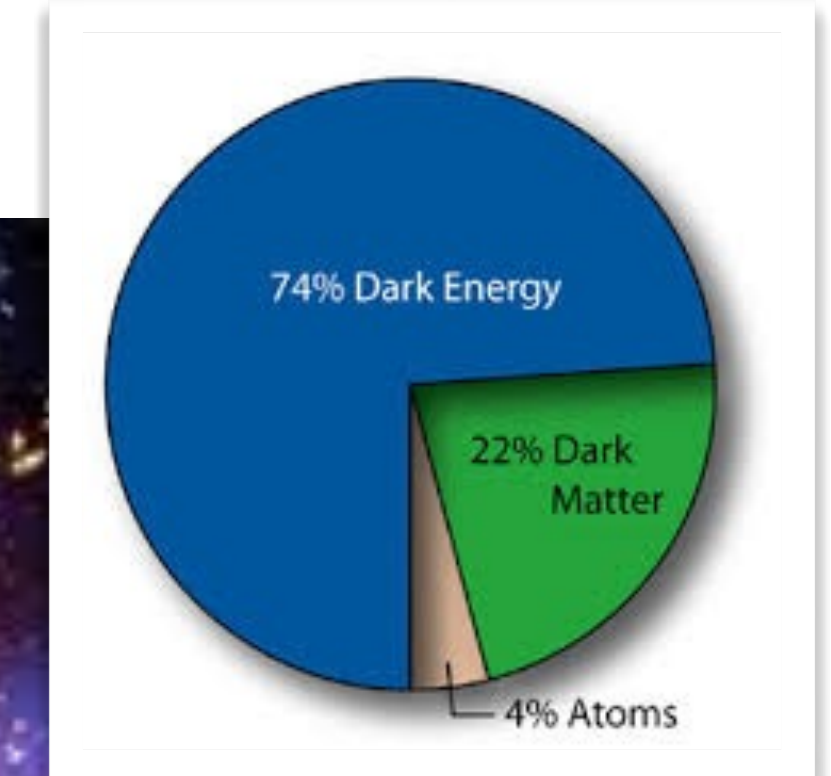
- The Standard Model encodes our knowledge of nature's building blocks and interactions, but it is incomplete!



Credit: Fermilab



Credit: X-ray: NASA/CXC/CfA/M.Markevitch et al.; Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al.

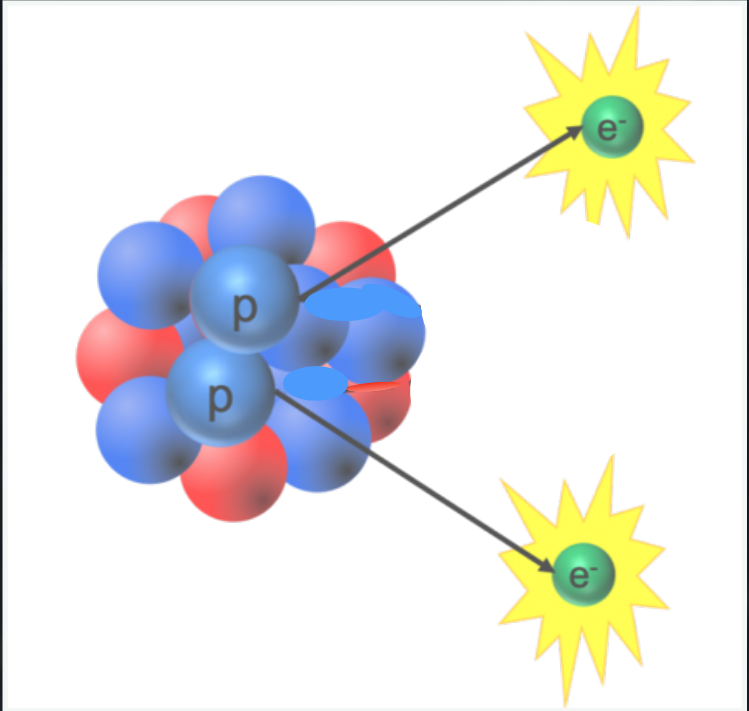


No Neutrino Mass, no Baryon Asymmetry, no Dark Matter, no Dark Energy

Addressing these shortcomings requires physics beyond the Standard Model (BSM)

Context: open questions in subatomic physics

- The Standard Model encodes our knowledge of nature's building blocks and interactions, but it is incomplete!

$$(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$$


$0\nu\beta\beta$ decay plays a prominent role in the quest for new physics by addressing two major questions related to shortcomings of the Standard Model

The diagram illustrates the $0\nu\beta\beta$ decay process. On the left, a cluster of blue spheres (protons) and red spheres (neutrons) represents an initial nucleus. Two arrows point from this cluster to two separate yellow starburst shapes, each labeled with e^- , representing the emission of two electrons. The background of the slide is a dark, starry space.

Demonstrate Majorana nature of massive neutrinos (neutrino=antineutrino) $\Rightarrow$
Shed light on the origin of neutrino mass

Demonstrate that an excess of matter over antimatter can be created in an elementary process $\Rightarrow$
Point to baryogenesis via leptogenesis

$0\nu\beta\beta$ decay: significance

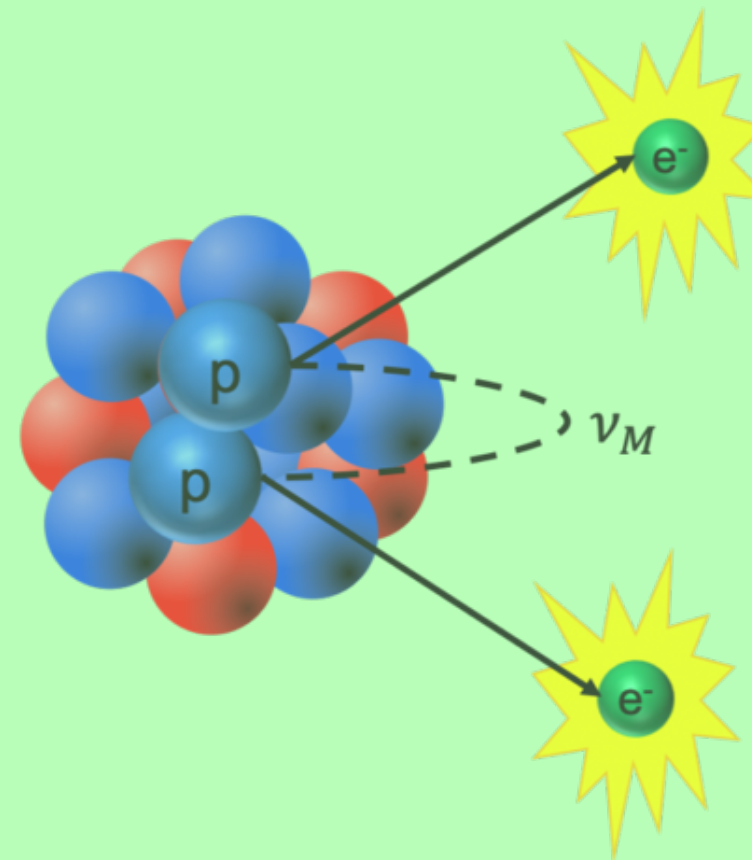
The neutrino and its mysteries

Nature of massive neutrinos:
is the neutrino its own antiparticle?

A cosmic mystery

How did matter survive
the big bang?

Nuclear $0\nu\beta\beta$ decay

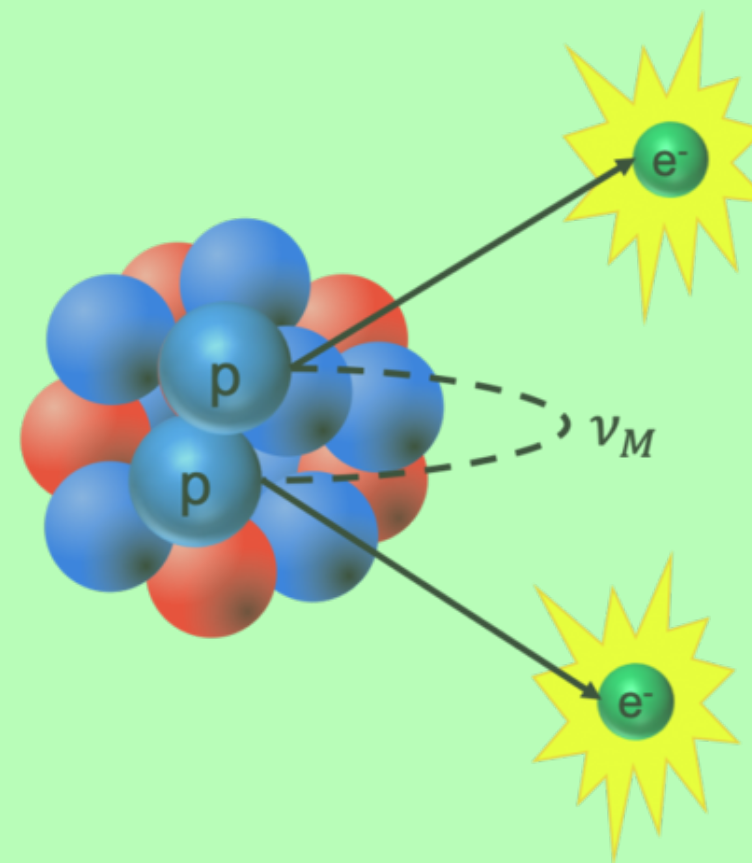


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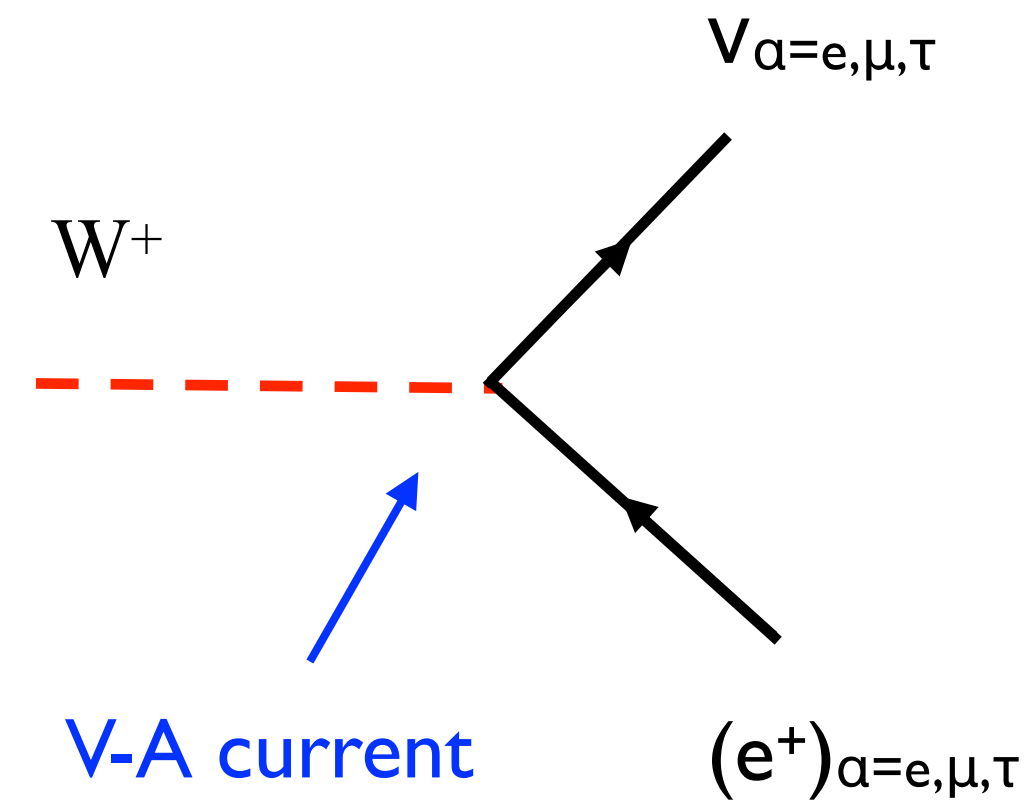
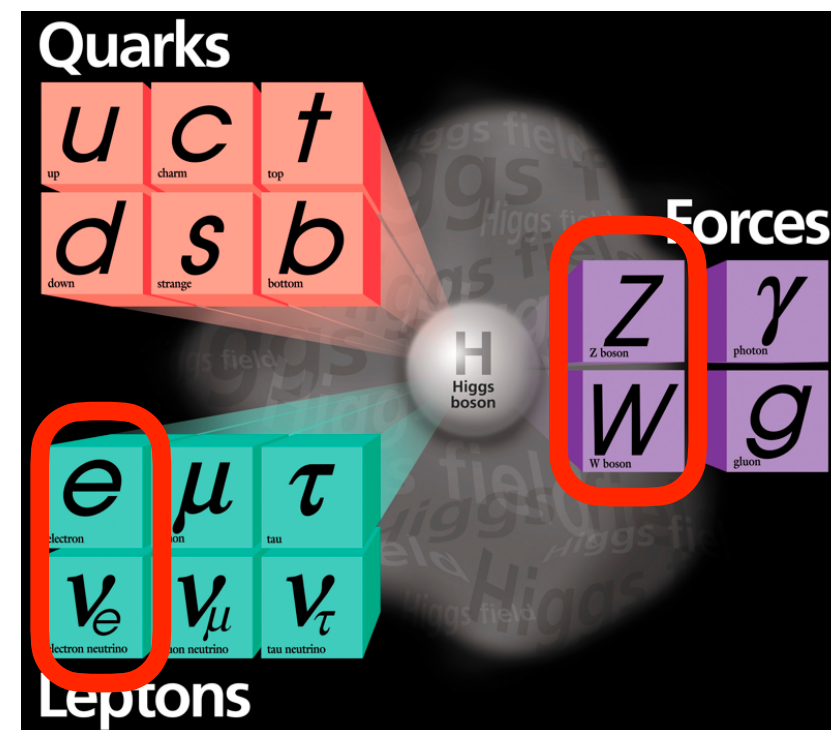
Nuclear $0\nu\beta\beta$ decay



Neutrinos

- **Elusive particles:** feel only the weak force, form a “weak isospin doublet” with electrons

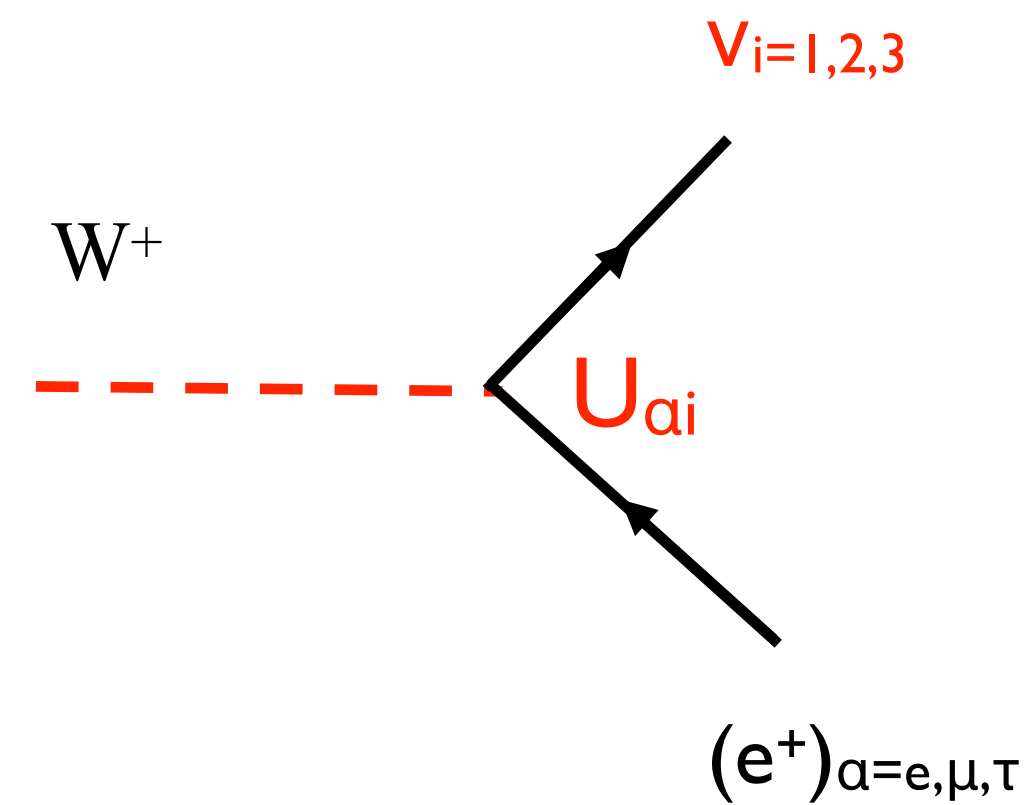
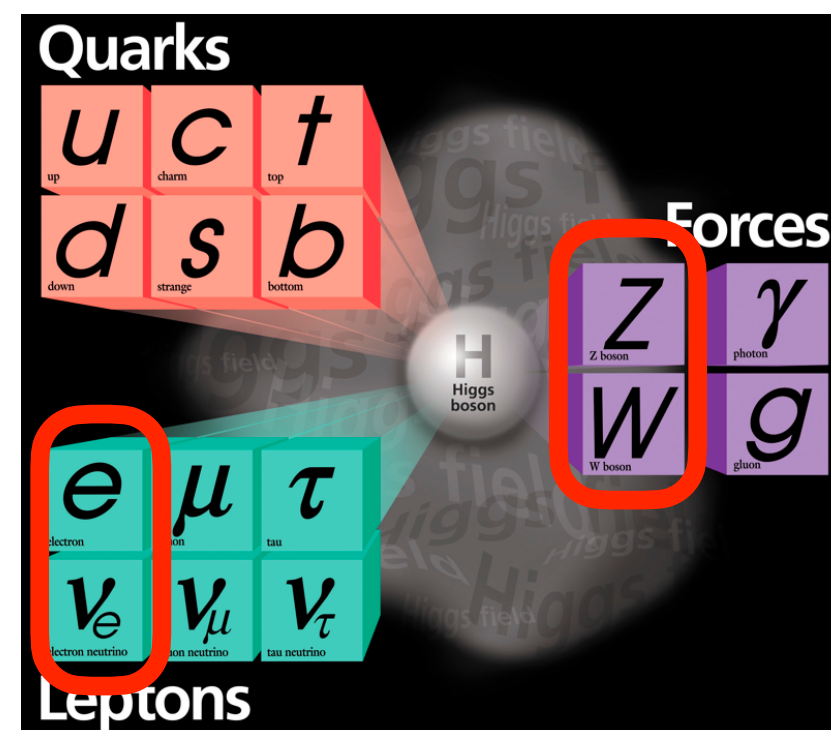
$$L_L^\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}$$



Neutrinos

- **Elusive particles:** feel only the weak force, form a “weak isospin doublet” with electrons
- Massive neutrinos produced in a given interaction (“flavor”) state can “**oscillate**” into another flavor through QM interference

$$L_L^\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}$$



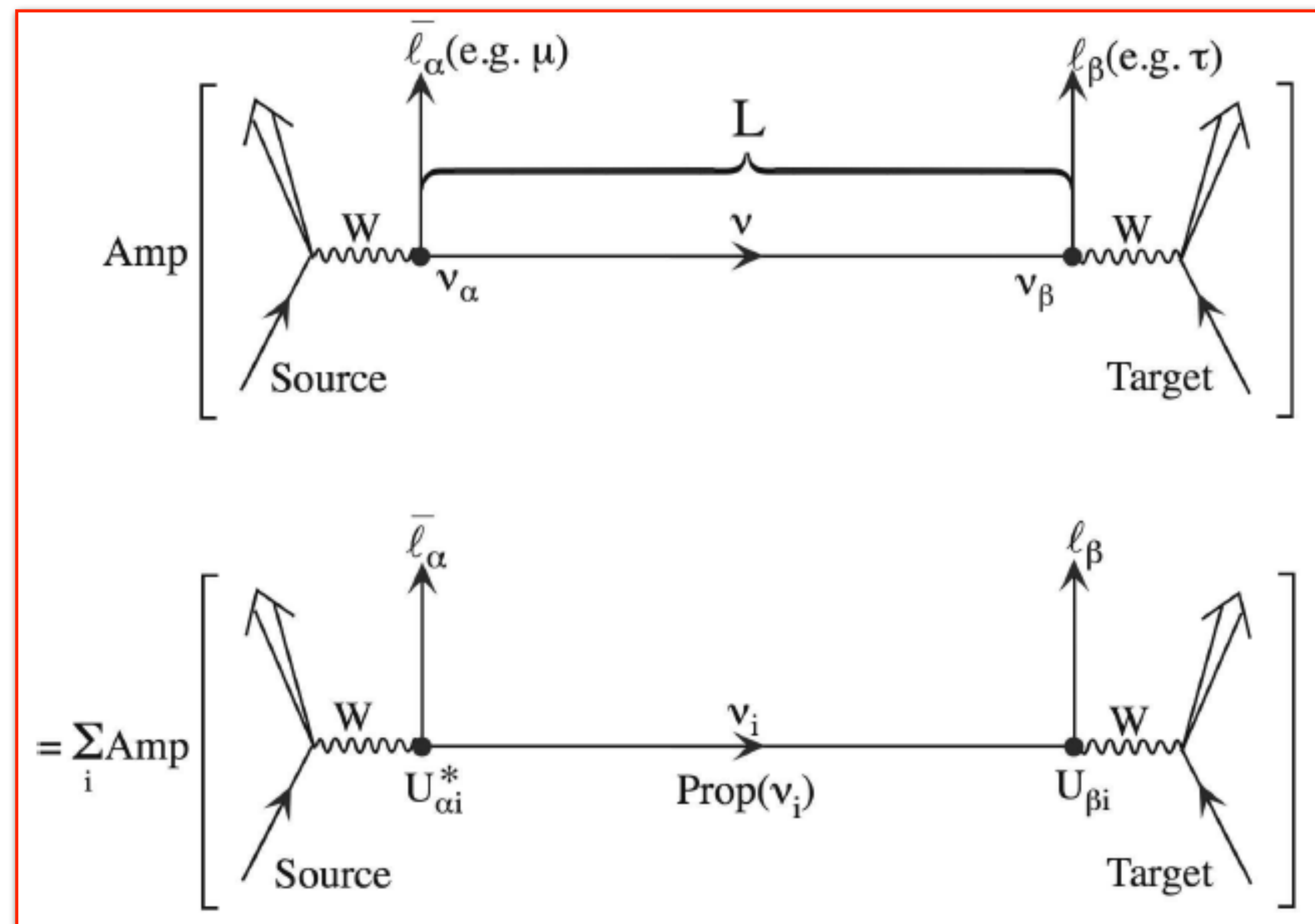
$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

Neutrinos

- **Elusive particles:** feel only the weak force, form a “weak isospin doublet” with electrons
- Massive neutrinos produced in a given interaction (“flavor”) state can **“oscillate”** into another flavor through QM interference

$$L_L^\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}$$

Image credit: B. Kayser



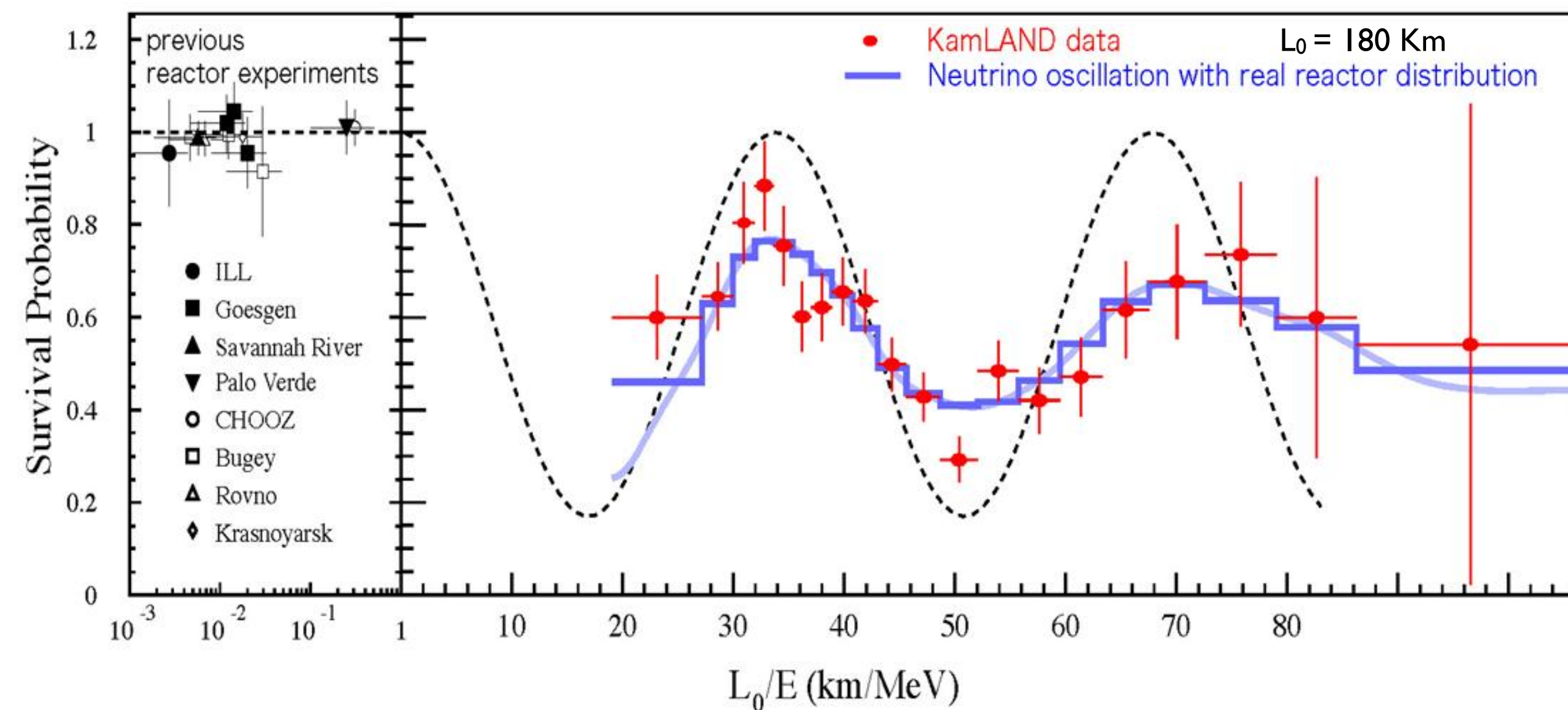
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$$L_L^\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}$$

KAMLAND Reactor electron anti-neutrino survival probability



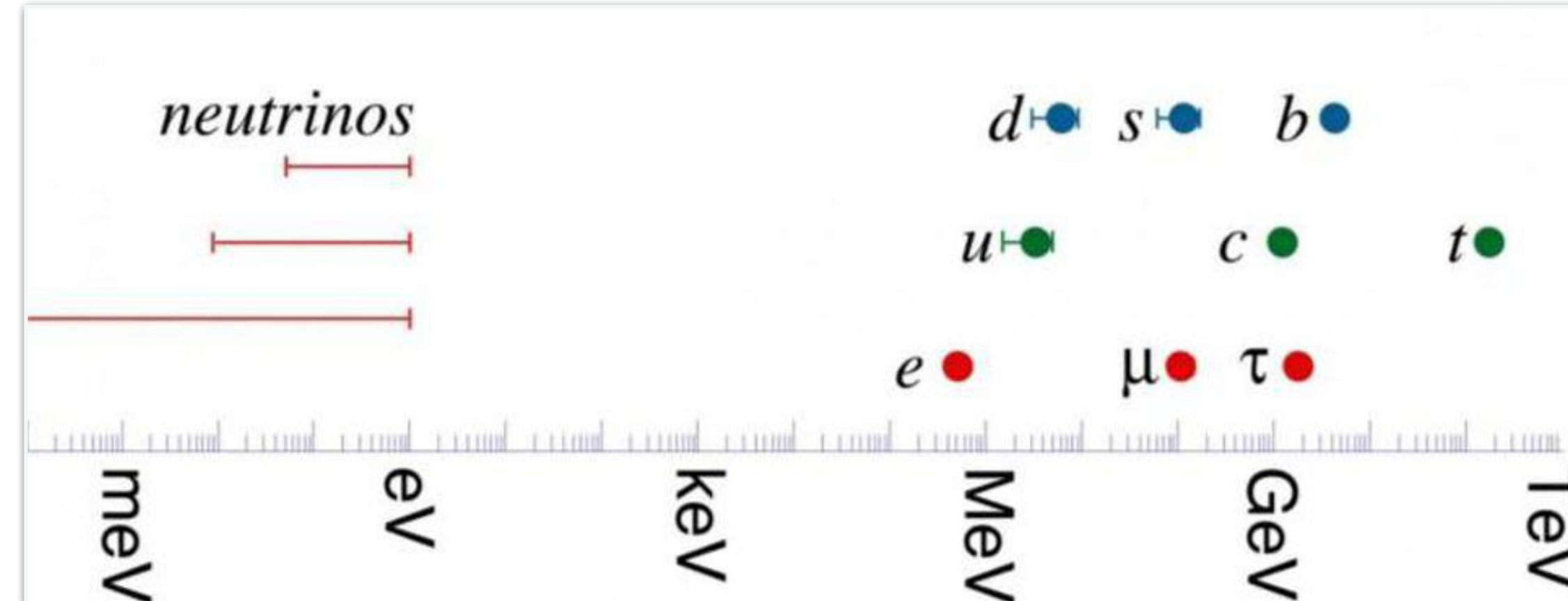
$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Neutrinos

$$L_L^\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}$$

- **Elusive particles:** feel only the weak force, form a “weak isospin doublet” with electrons
- Massive neutrinos produced in a given interaction (“flavor”) state can “**oscillate**” into another flavor through QM interference
- Neutrinos have masses and they are tiny compared to other fermion’s masses!



H. Murayama

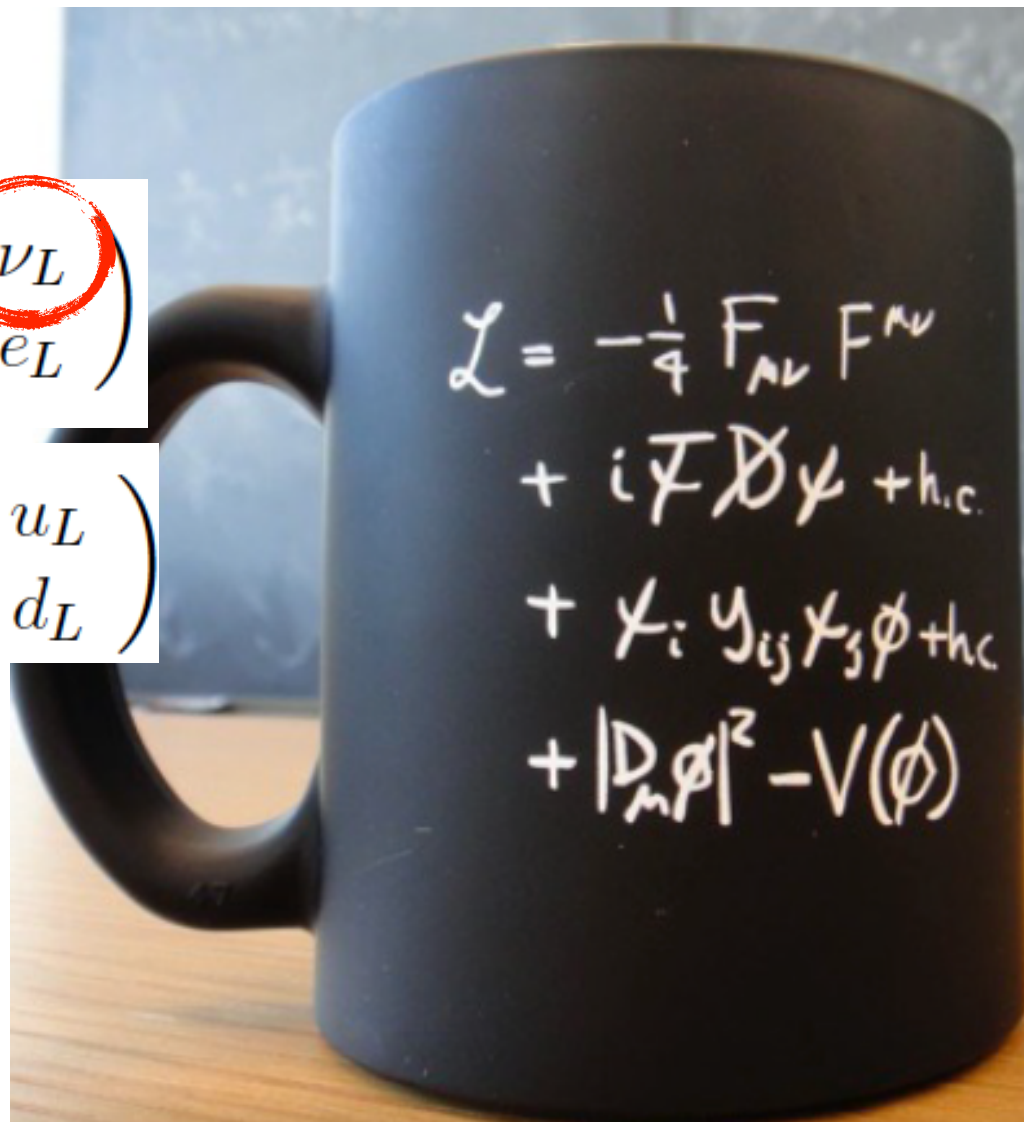
So what's the big deal?

Neutrino mass = new physics

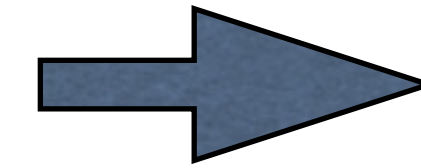
The Standard Model

$$\psi_i = \begin{pmatrix} \ell_L \\ e_R \\ q_L \\ u_R \\ d_R \end{pmatrix}_i \quad \begin{matrix} \longrightarrow \\ \longrightarrow \end{matrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$i=1,2,3$



Credit: CERN



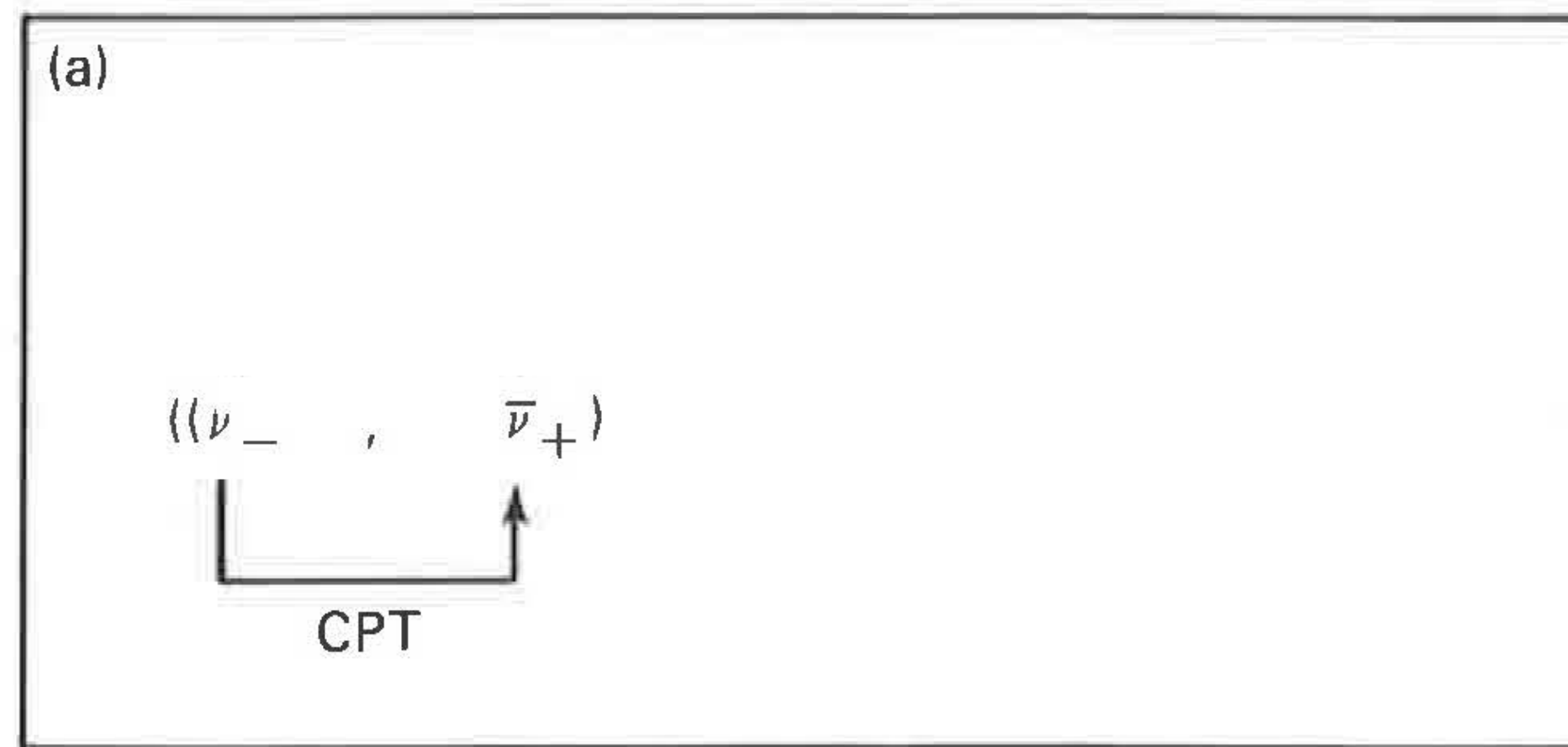
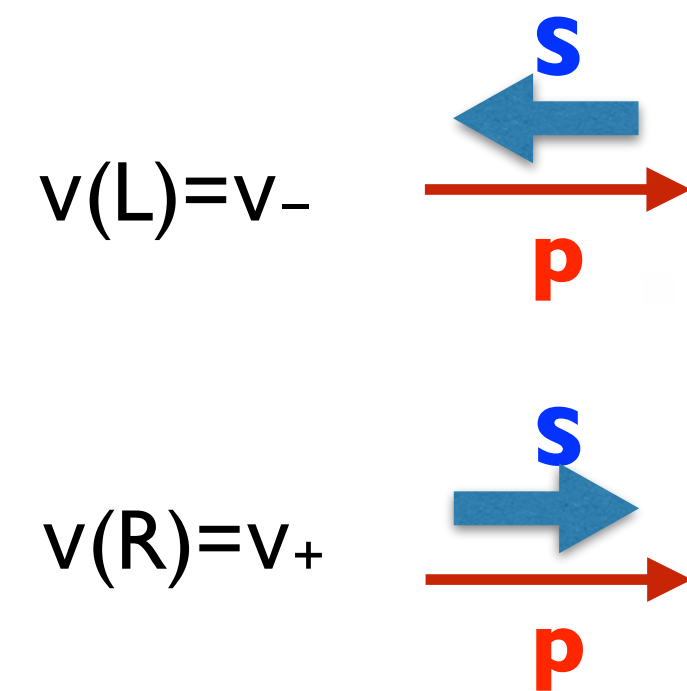
No
neutrino
mass

We currently don't even know what's the quantum mechanical nature of *massive* neutrinos!
(= we don't know what is the form of the neutrino mass term to be added to the SM Lagrangian)

Neutrino mass and symmetries

- Lorentz invariance $\Rightarrow$ two options: Dirac or Majorana

B. Kayser 1984

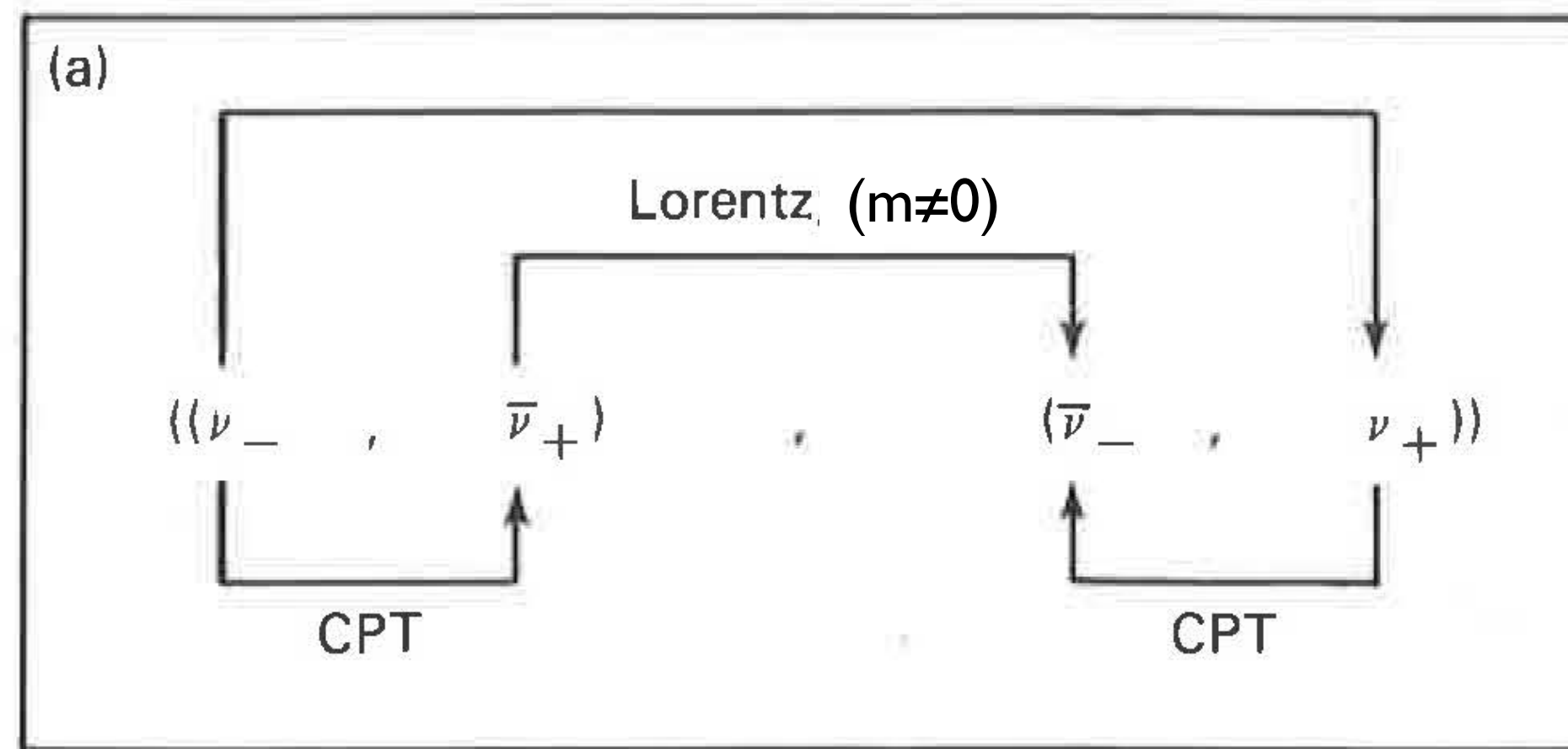
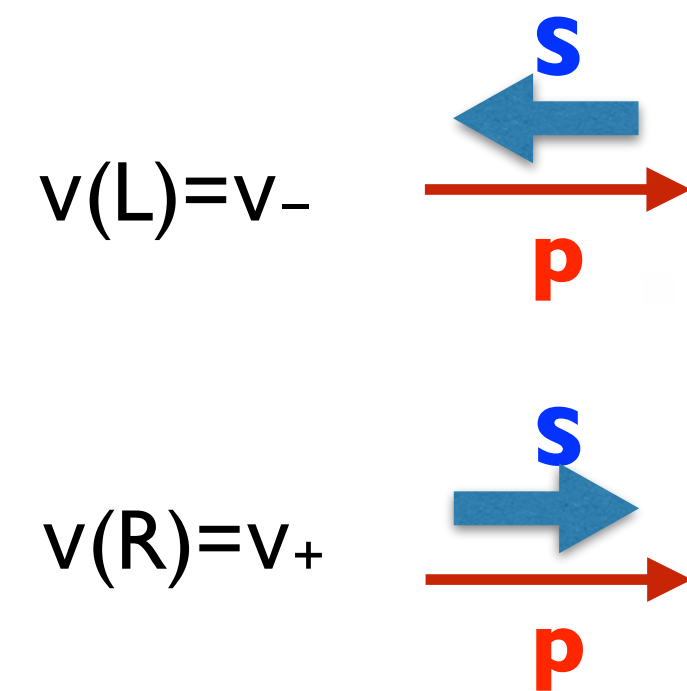


Dirac:
4 states

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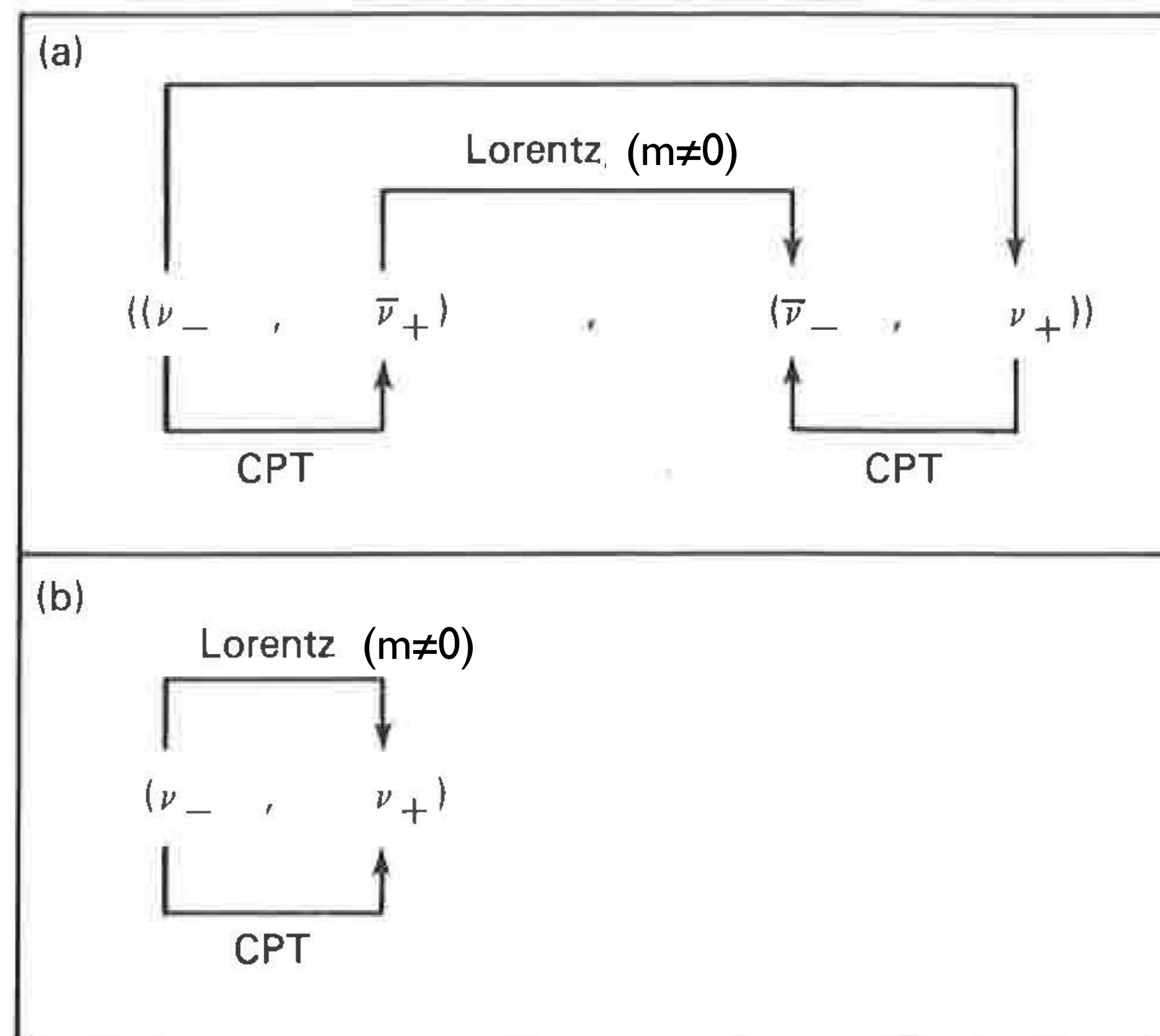
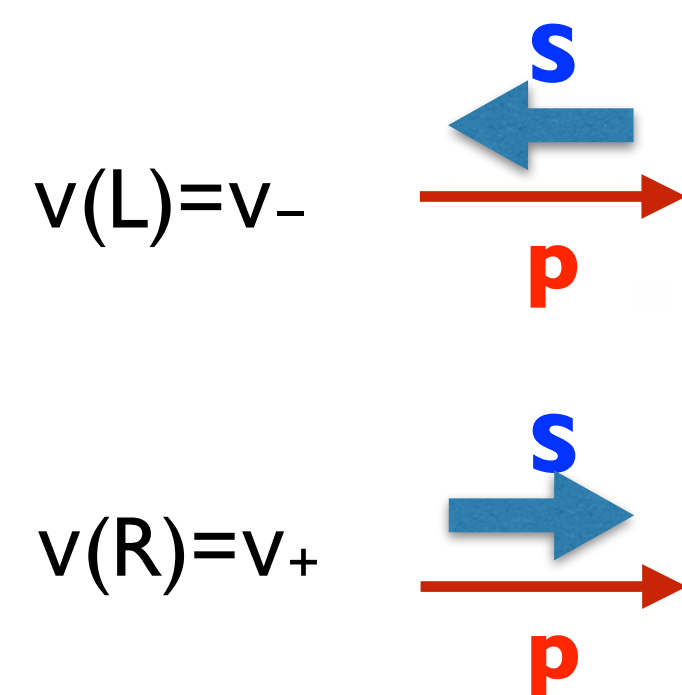


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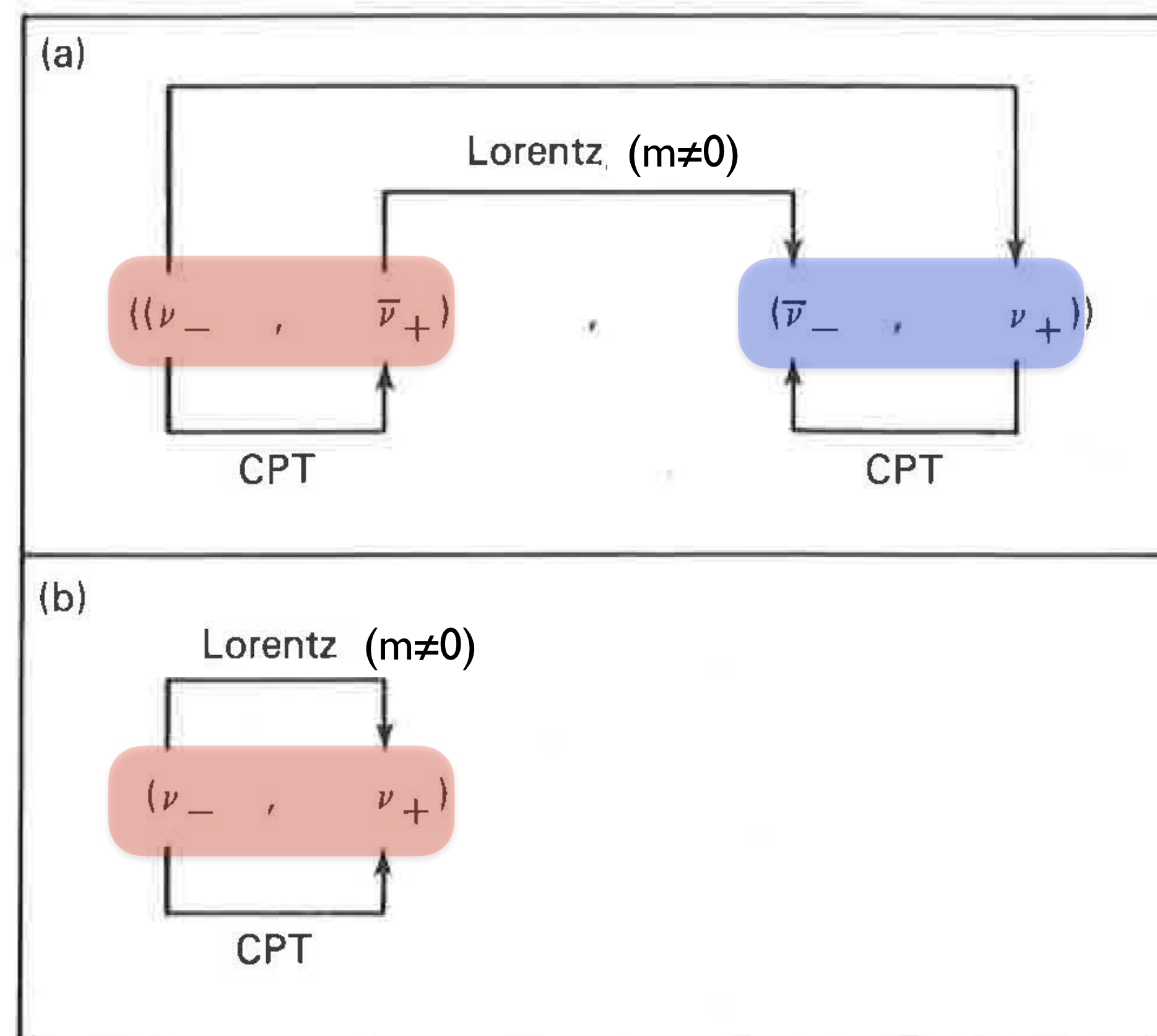
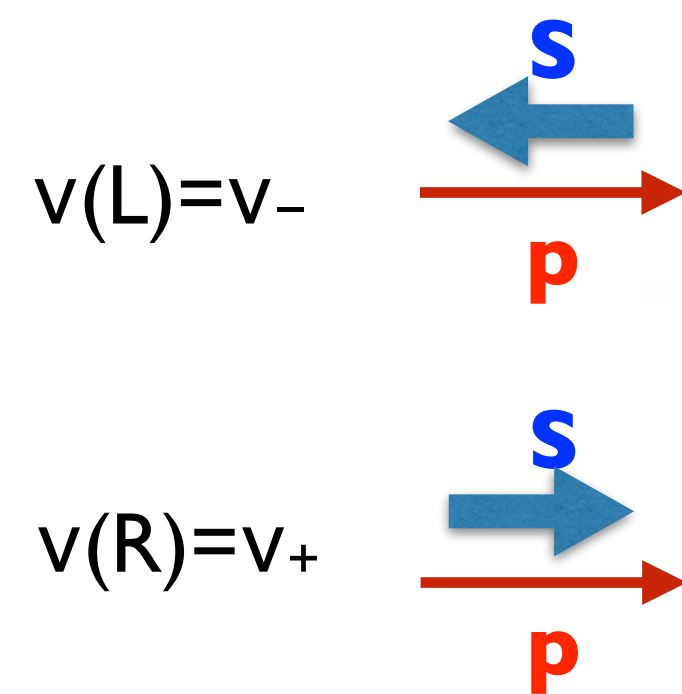
Majorana:
2 states ($\bar{\nu}_+ = \nu_+$)

Only possible if there no
internal quantum number
that flips sign under "C"

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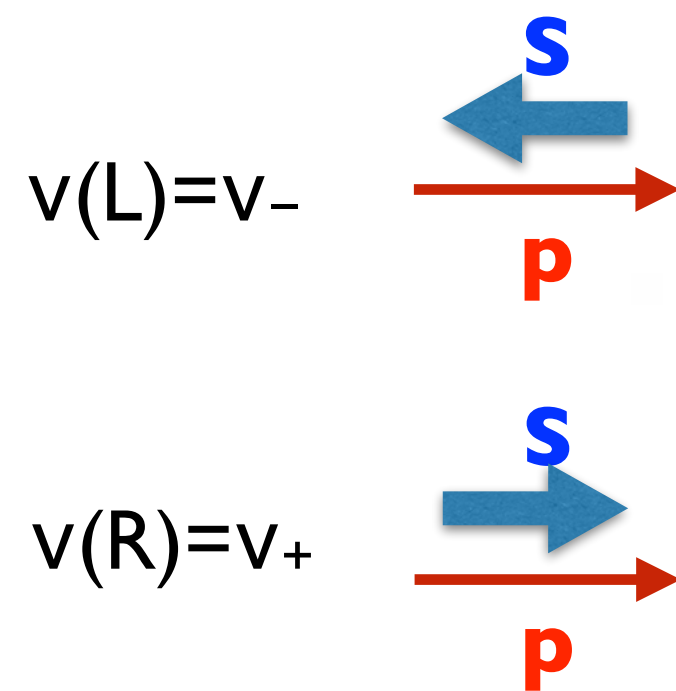
$\mathbf{V_L(x)}$

$\mathbf{V_R(x)}$

Neutrino mass and symmetries

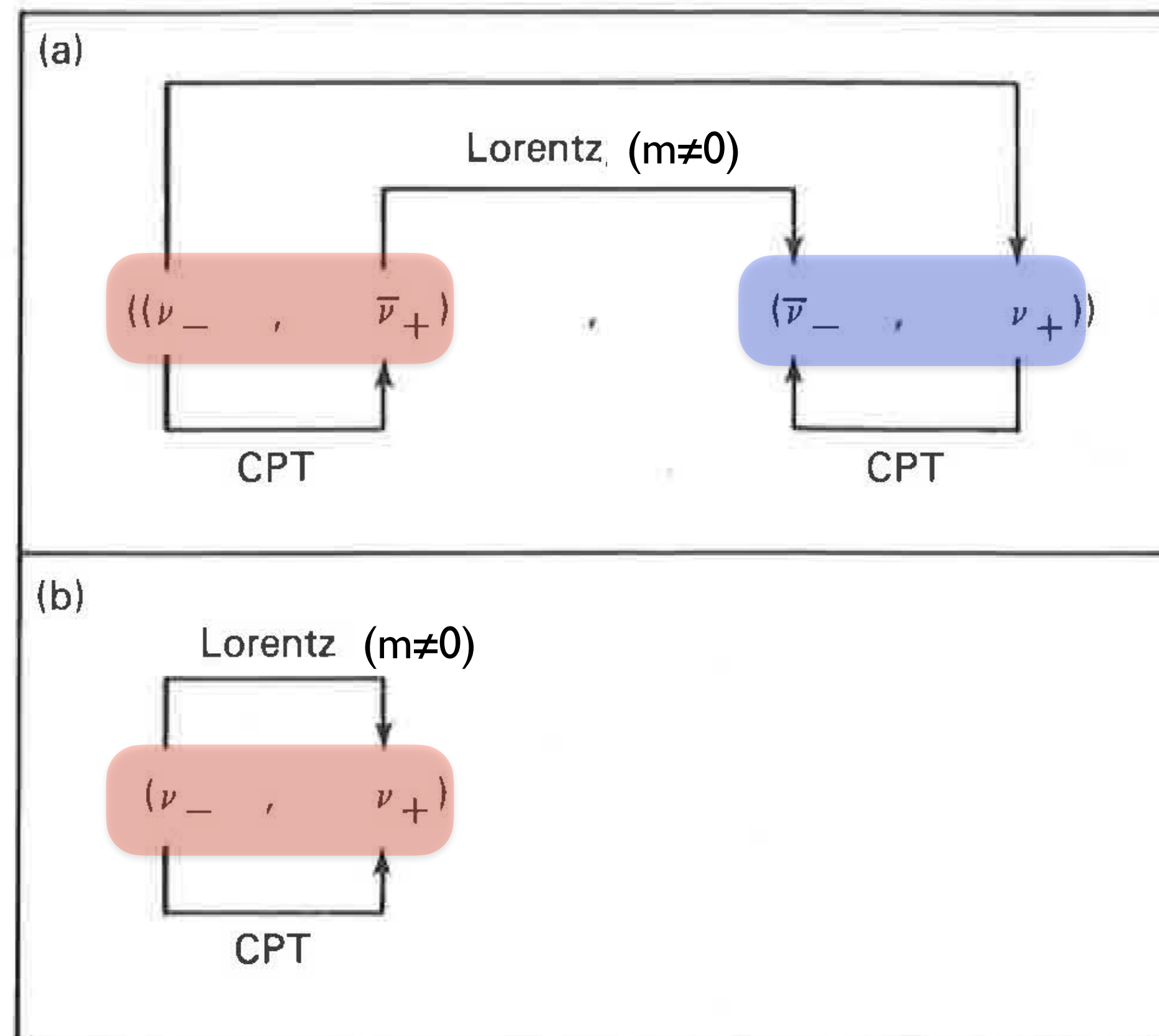
- Lorentz invariance $\Rightarrow$ two options: Dirac or Majorana

B. Kayser 1984



Up to effects of $O(m_\nu/E_\nu)$, only these states participate in SM (V-A) weak interactions

v_- = will be pair-produced with e^+
 $\bar{v}_+$ = will be pair-produced with e^-



Dirac:
4 states

Majorana:
2 states ($\bar{v}_+ = v_+$)

Only possible if there no internal quantum number that flips sign under “C”

$\mathbf{v_L(x)}$: takes part in weak interactions

$\mathbf{v_R(x)}$: no interactions in the SM

Neutrino mass and symmetries

- Lorentz invariance $\Rightarrow$ two options: Dirac or Majorana

Recall: can build two Lorentz-invariant bilinears from spin-1/2 fields $\psi_{L,R}$

Dirac mass:

$$m_D \bar{\psi}_L \psi_R + \text{h.c.}$$

Majorana mass:

$$m_M \psi_L^T C \psi_L + \text{h.c.}$$

$$\psi_{L/R} = \frac{1 \mp \gamma_5}{2} \psi$$

$$C = i\gamma_2\gamma_0$$

$$\psi^c = C\bar{\psi}^T = i\gamma_2\psi^*$$

Neutrino mass and symmetries

- Lorentz invariance $\Rightarrow$ two options: Dirac or Majorana

Dirac mass:

$$m \bar{\nu}_L \nu_R + \text{h.c.} = m \bar{\nu} \nu$$

$$\nu = \nu_L + \nu_R$$



Majorana mass:

$$m \nu_L^T C \nu_L + \text{h.c.} = m \bar{\nu} \nu$$

$$\nu = \nu_L + \nu_L^c = \nu^c$$



Both options written this way clash with the Standard Model particle content and symmetries

Neutrino mass and symmetries

- Lorentz invariance $\Rightarrow$ two options: Dirac or Majorana
- Lorentz and weak isospin $[SU(2)_w]$ invariance $\Rightarrow$ need **new degrees of freedom**

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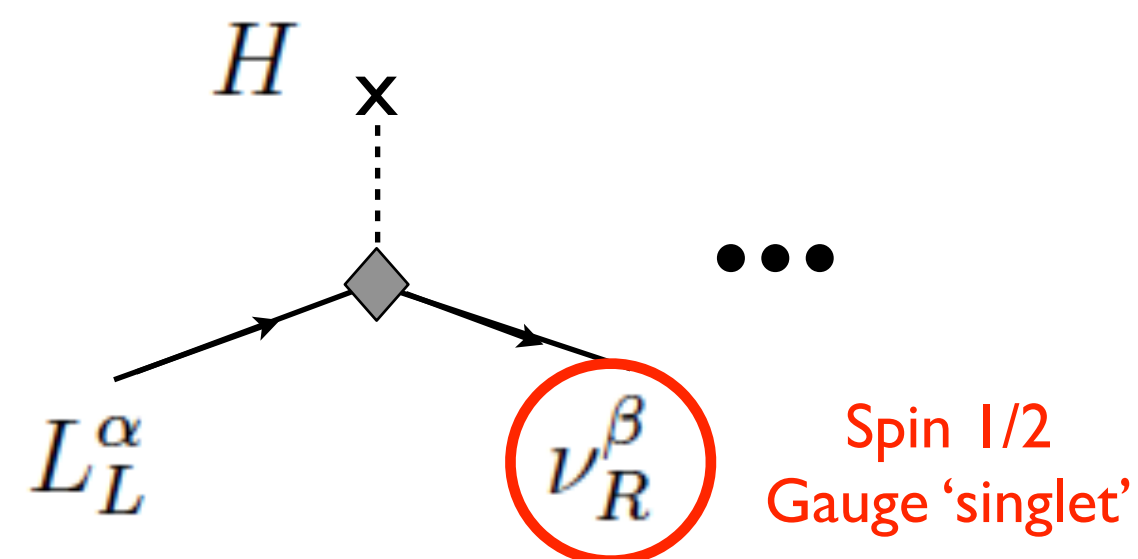
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$$\bar{L}^\alpha \tilde{H} \nu_R^{\alpha'}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\tilde{H} = \epsilon H^* = \begin{pmatrix} h^{0*} \\ -h^- \end{pmatrix}$$

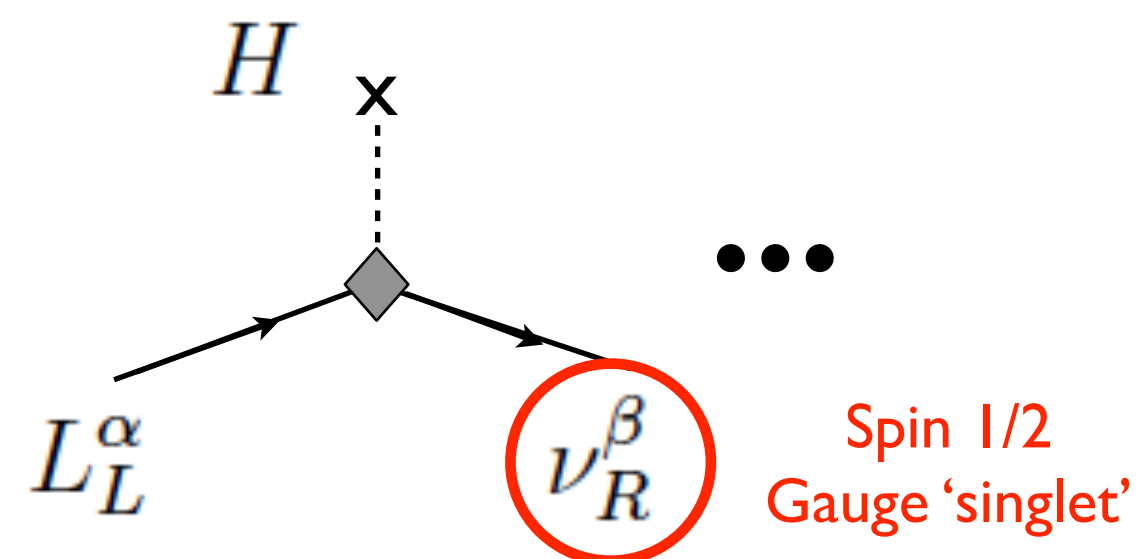
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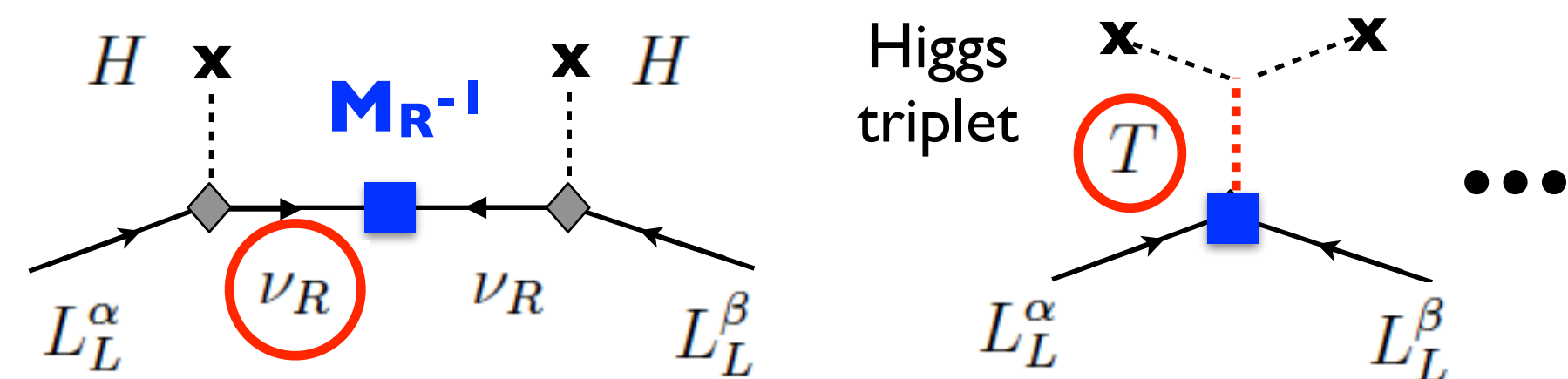
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$$L_\alpha^T C \epsilon H H^T \epsilon L_{\alpha'}$$

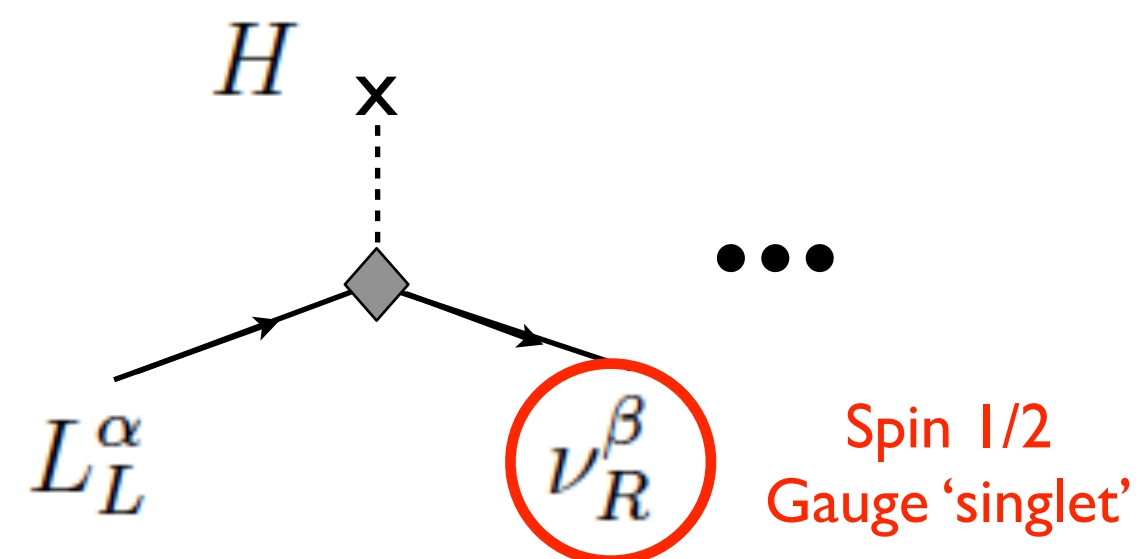
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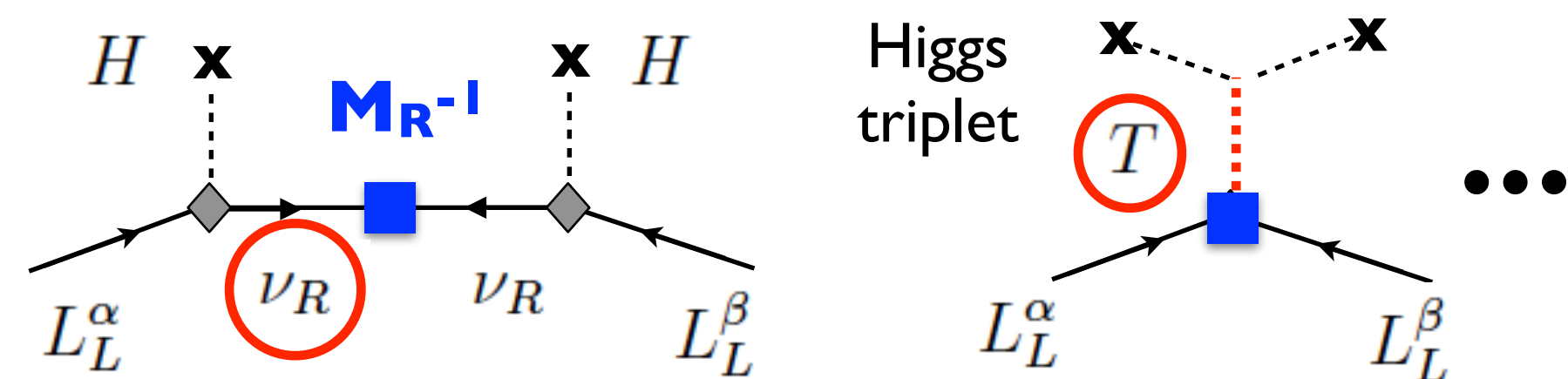
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Majorana mass:

$$m \nu_L^T C \nu_L + \text{h.c.} = m \bar{\nu} \nu$$

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- Violates $L_{e,\mu,\tau}$, conserves L

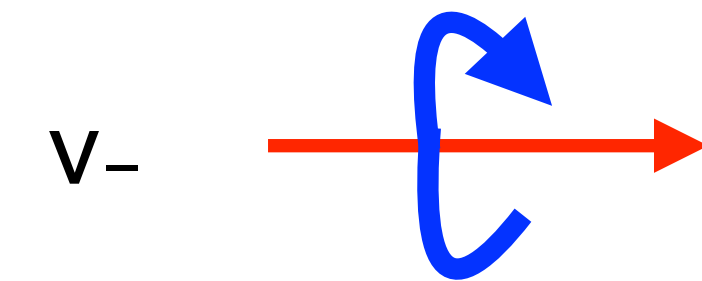
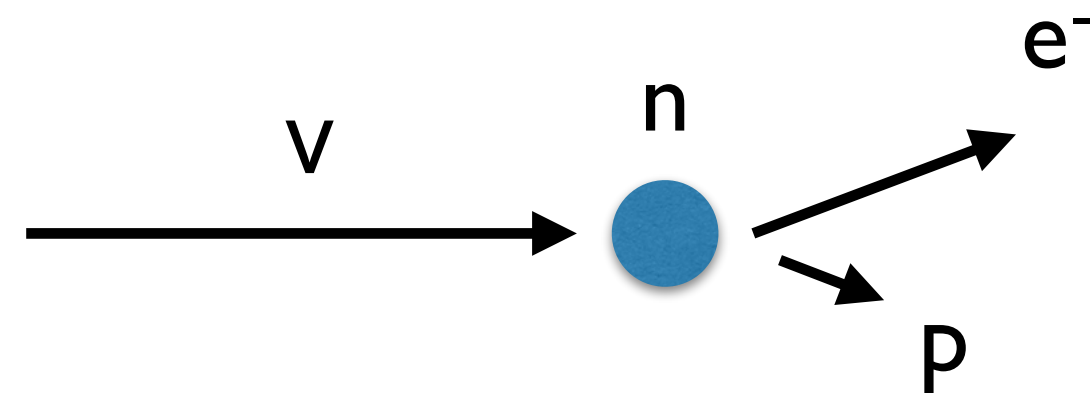
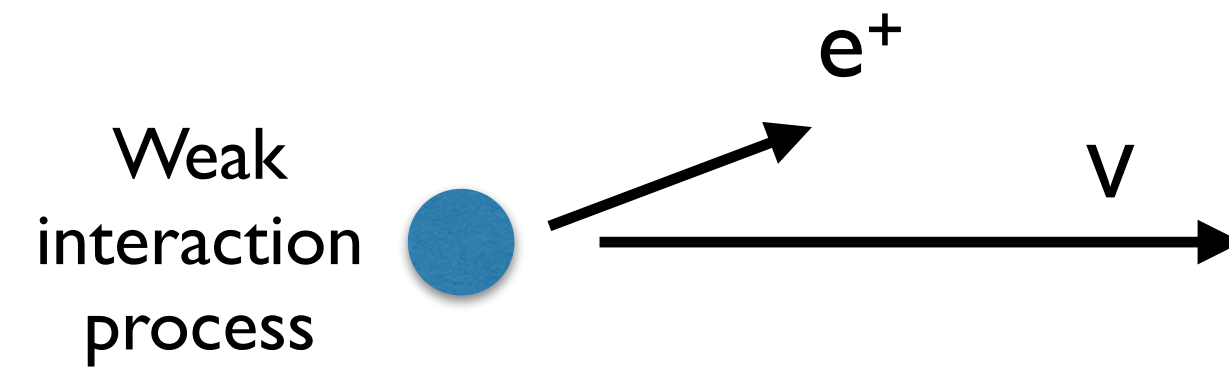
Violates $L_{e,\mu,\tau}$ and L ($\Delta L=2$)

Crucial for
experimental
probes!

A complementary point of view

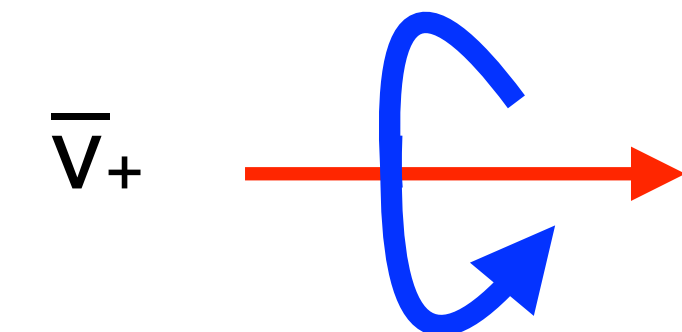
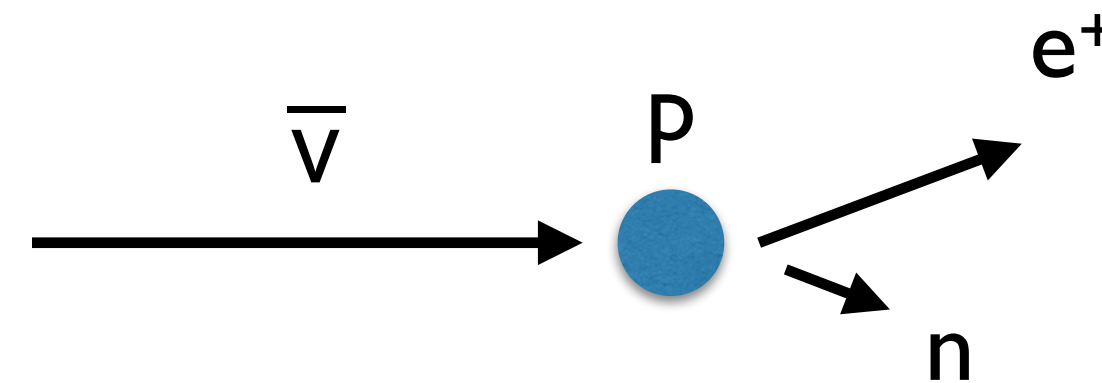
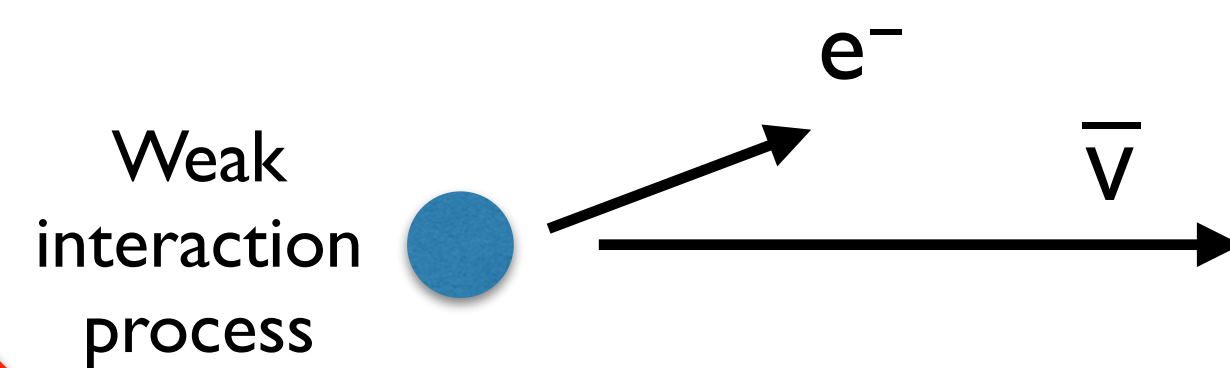
Neutrino (ν): emitted with e^+ , when interacts with matter can transform into e^-

Left-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = -1/2$



Anti-neutrino ($\bar{\nu}$): emitted with e^- , when interacts with matter can transform into e^+

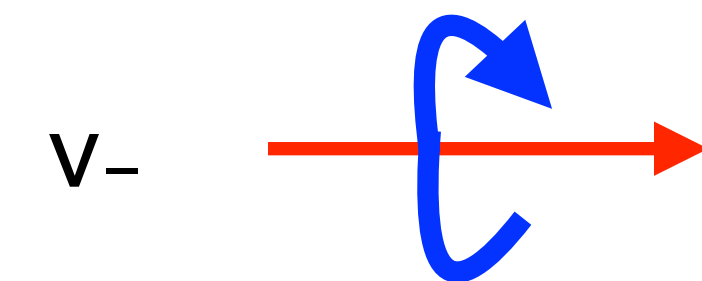
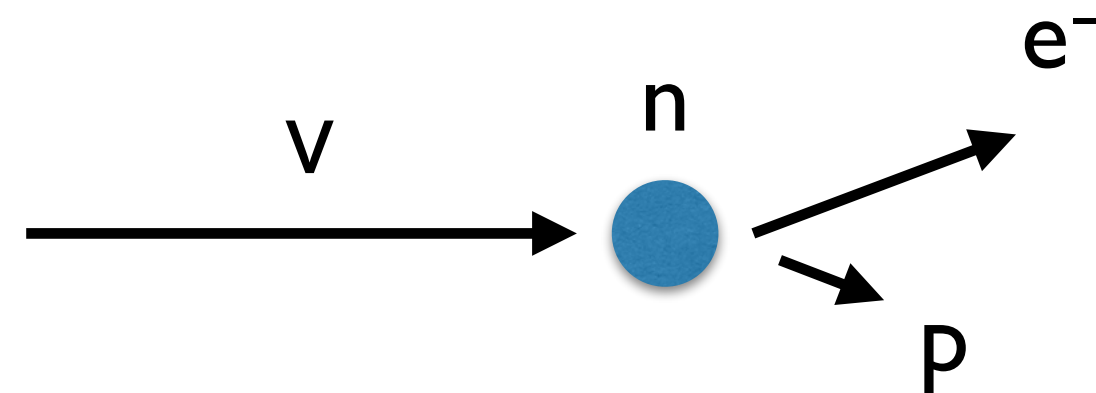
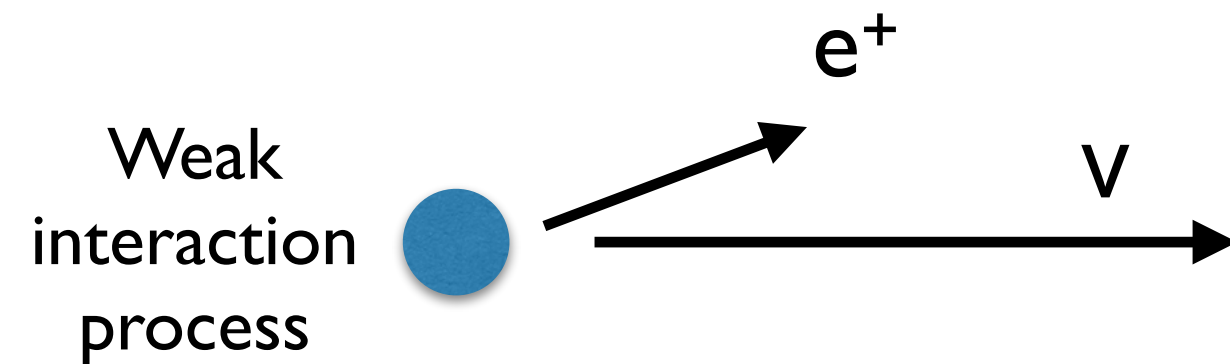
Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



A complementary point of view

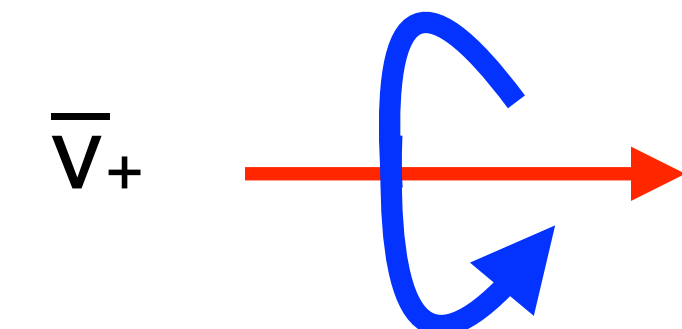
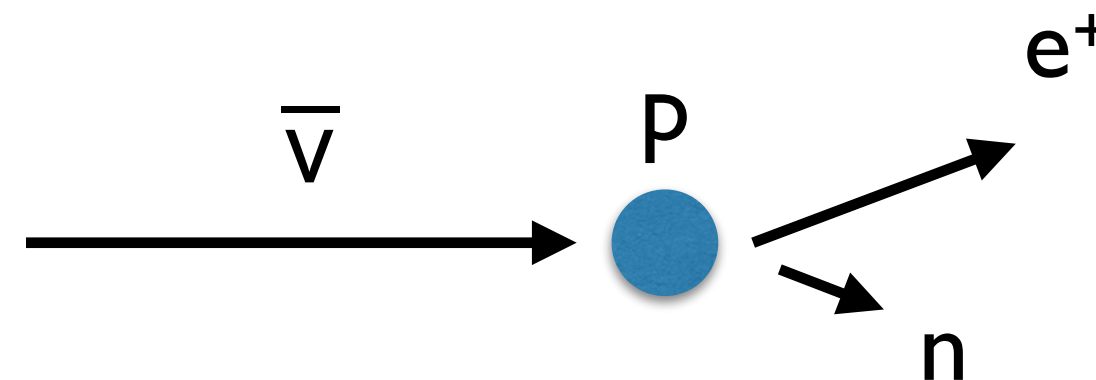
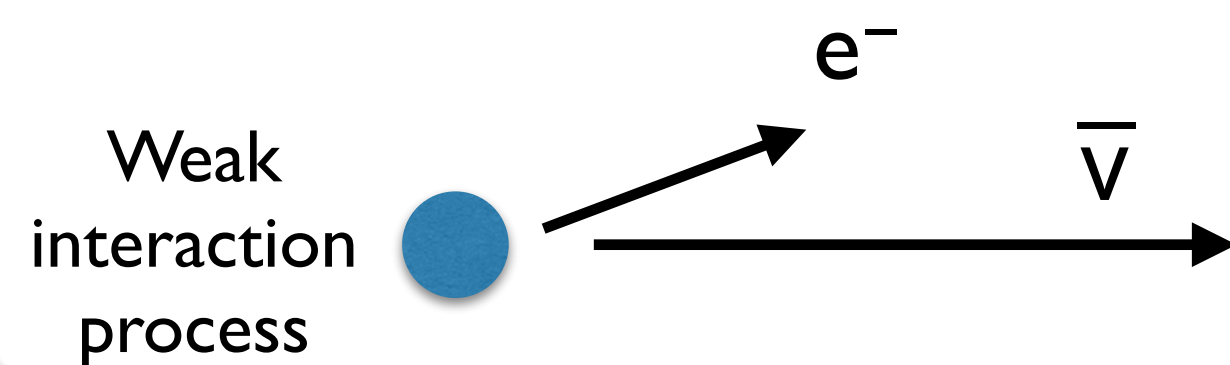
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Anti-neutrino ($\bar{\nu}$): emitted with e^- , when interacts with matter can transform into e^+

Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



Are these two different spin states of the same particle?

No!

Neutrino $\neq$ anti-neutrino

4 states:

$\nu_- \quad \bar{\nu}_+ \quad \nu_+ \quad \bar{\nu}_-$

Participate in weak interactions

Yes!

Neutrino = antineutrino

2 states: $\nu_- \quad \nu_+$

A new type of spin=1/2 fermion!



Dirac

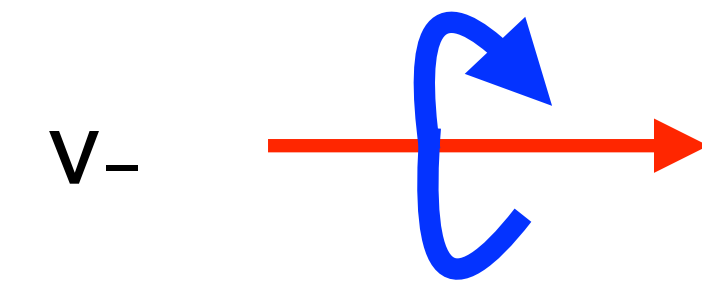
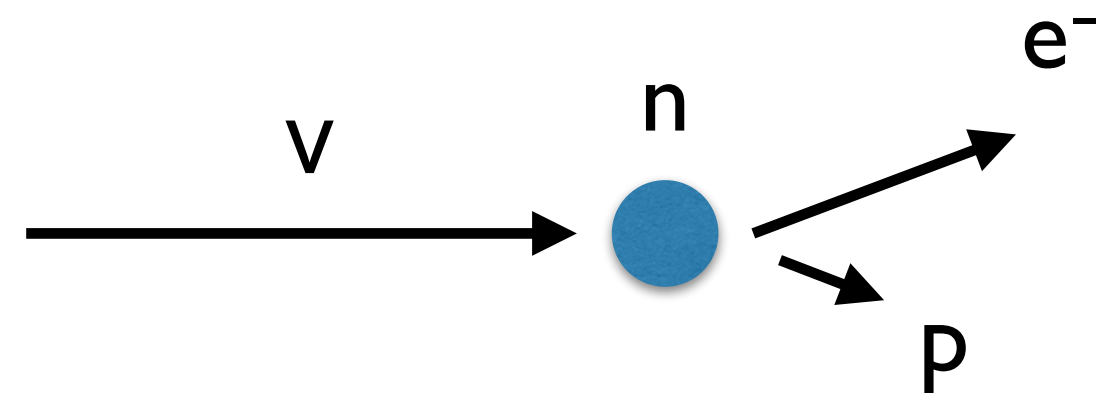
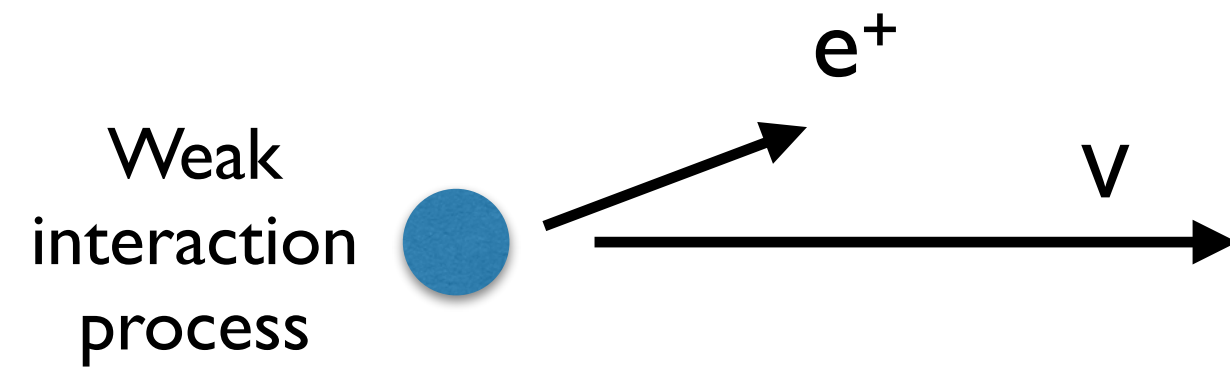


Majorana

A complementary point of view

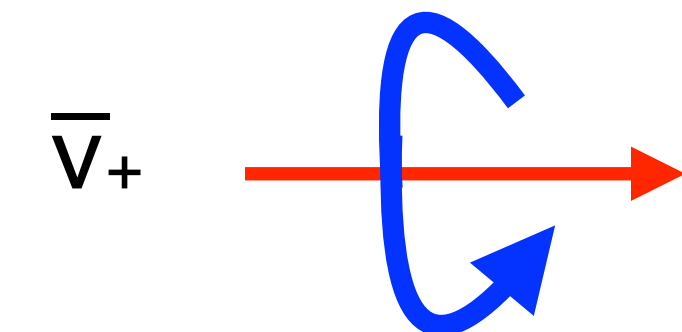
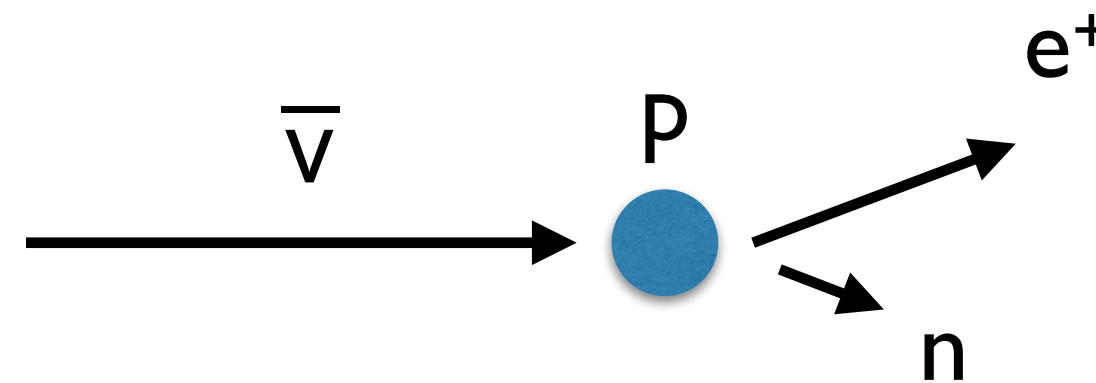
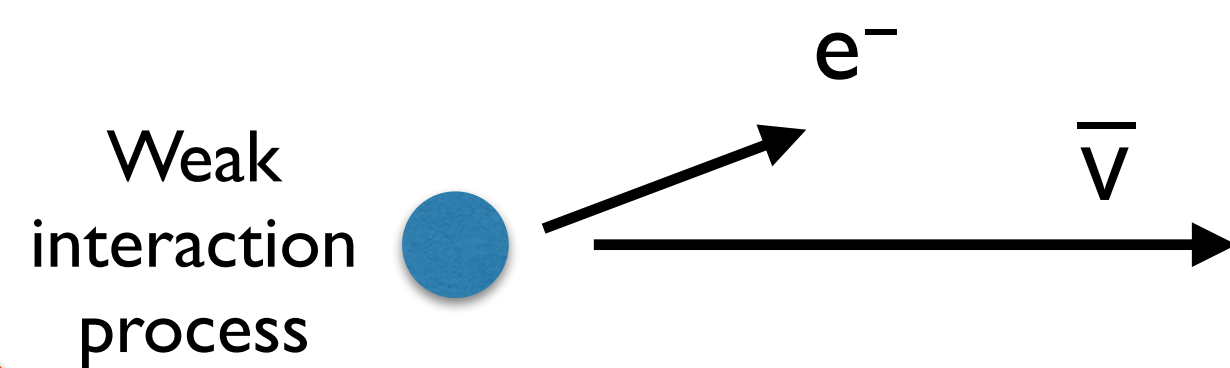
Neutrino (ν): emitted with e^+ , when interacts with matter can transform into e^-

Left-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = -1/2$



Anti-neutrino ($\bar{\nu}$): emitted with e^- , when interacts with matter can transform into e^+

Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



Can assign conserved lepton number

4 states:
 $\nu_- \quad \bar{\nu}_+ \quad \nu_+ \quad \bar{\nu}_-$

Participate in
weak interactions

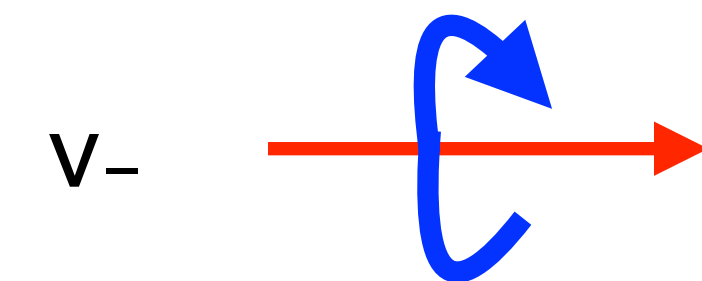
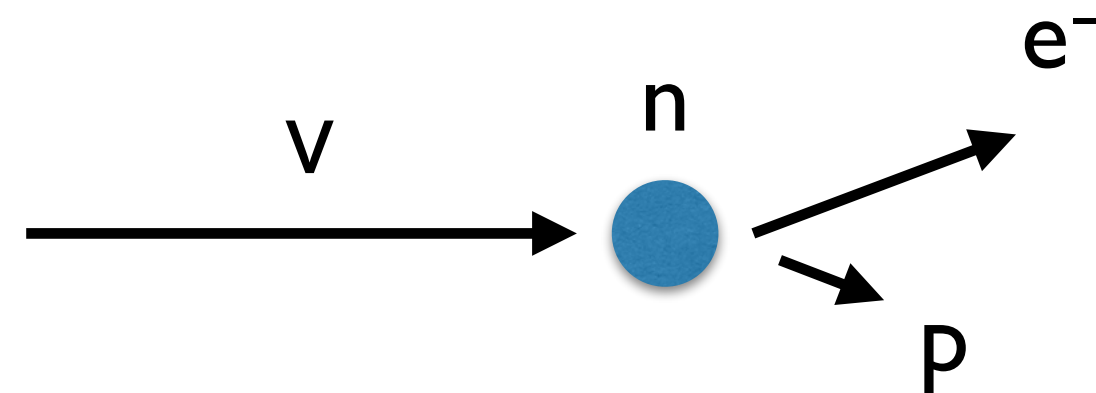
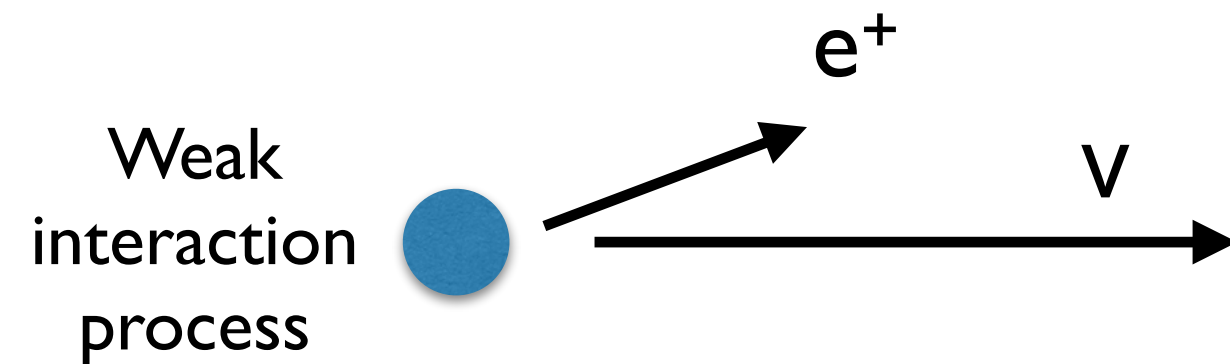
Cannot assign a conserved lepton number

2 states: $\nu_- \quad \nu_+$

A complementary point of view

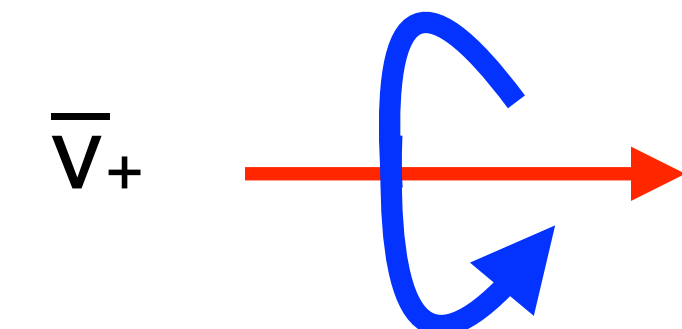
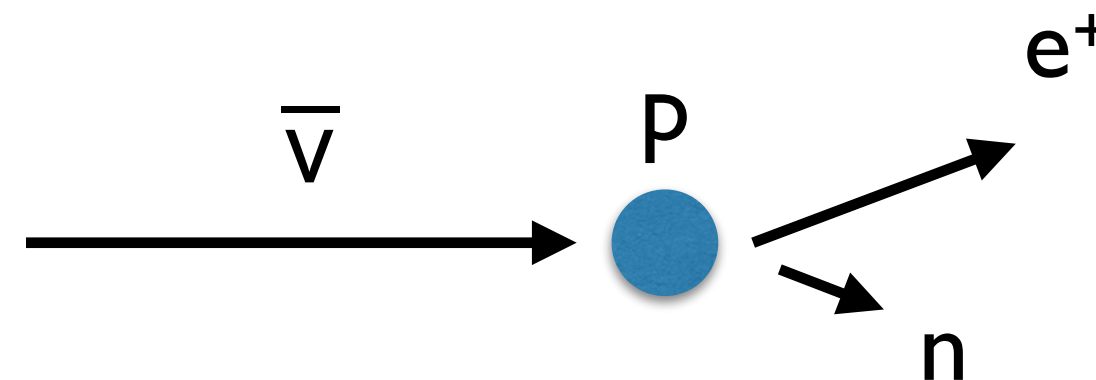
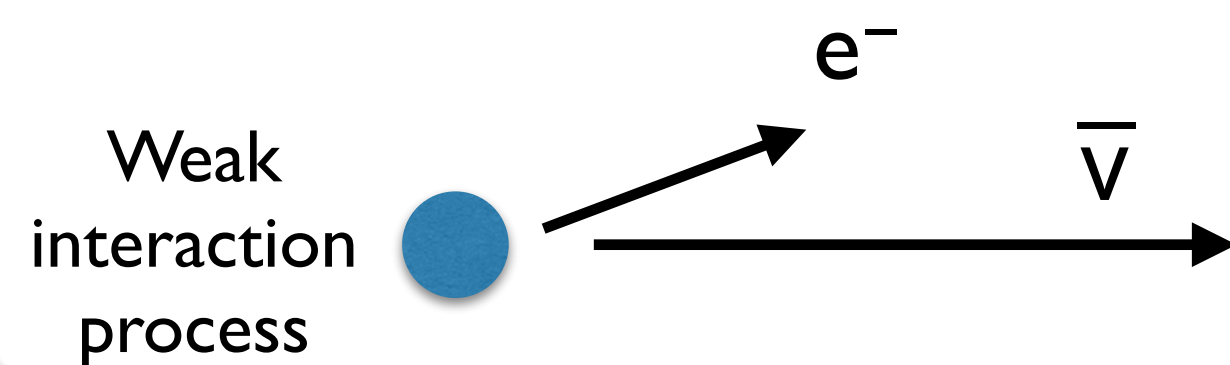
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Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



For $m = 0$ the two options are not distinguishable: Lepton Number = helicity

Can assign conserved lepton number

4 states:
 $\nu_- \quad \bar{\nu}_+ \quad \nu_+ \quad \bar{\nu}_-$

Participate in
weak interactions

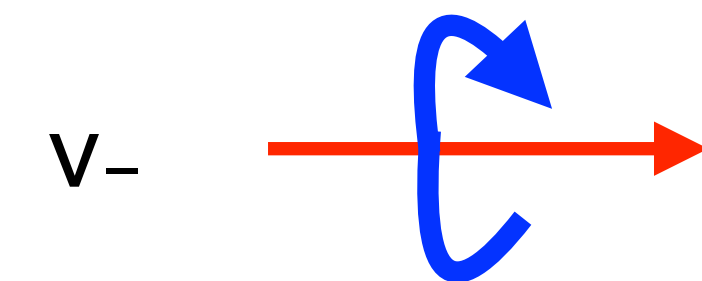
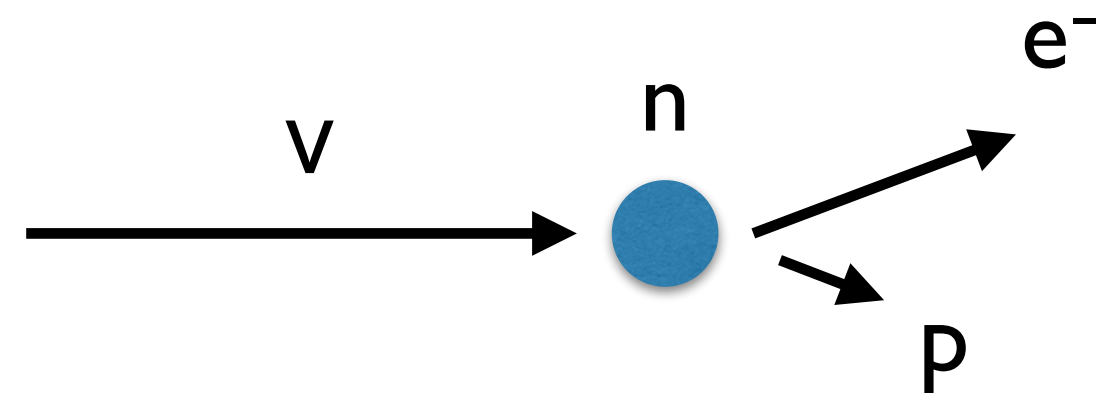
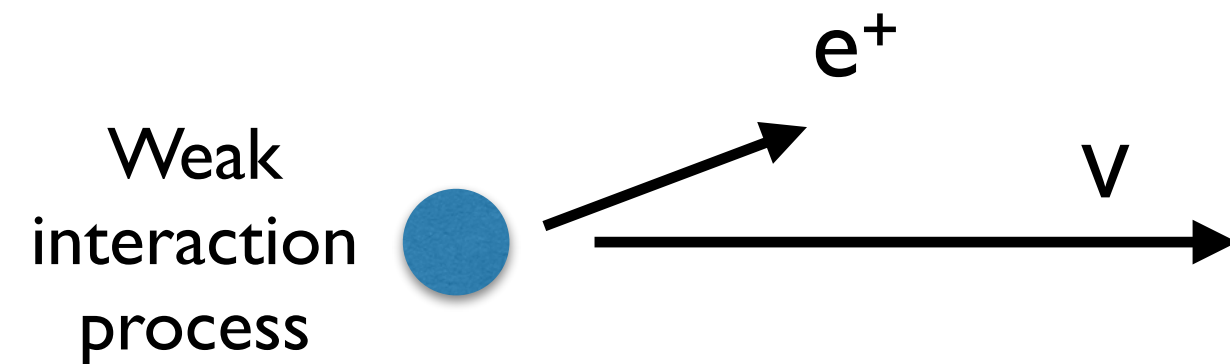
Cannot assign a conserved lepton number

2 states: $\nu_- \quad \nu_+$

A complementary point of view

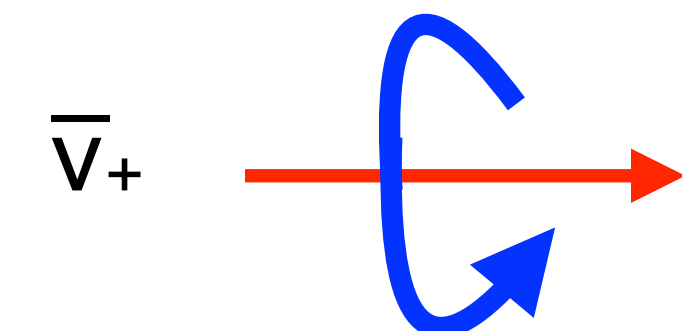
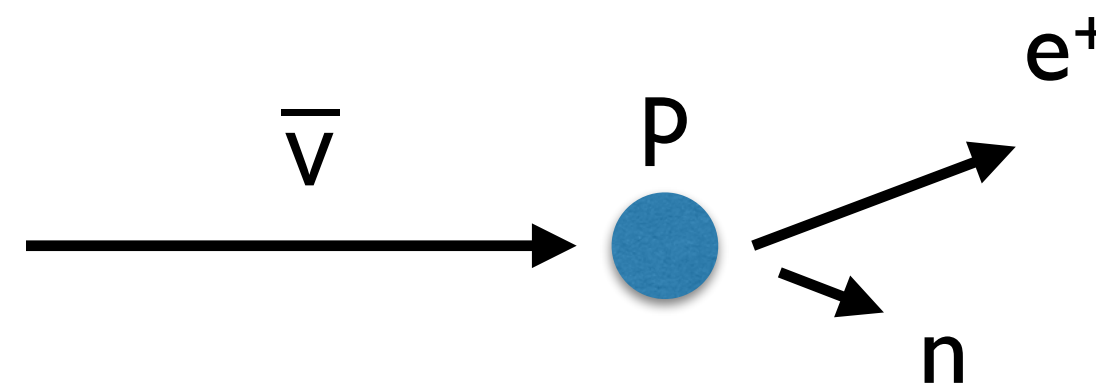
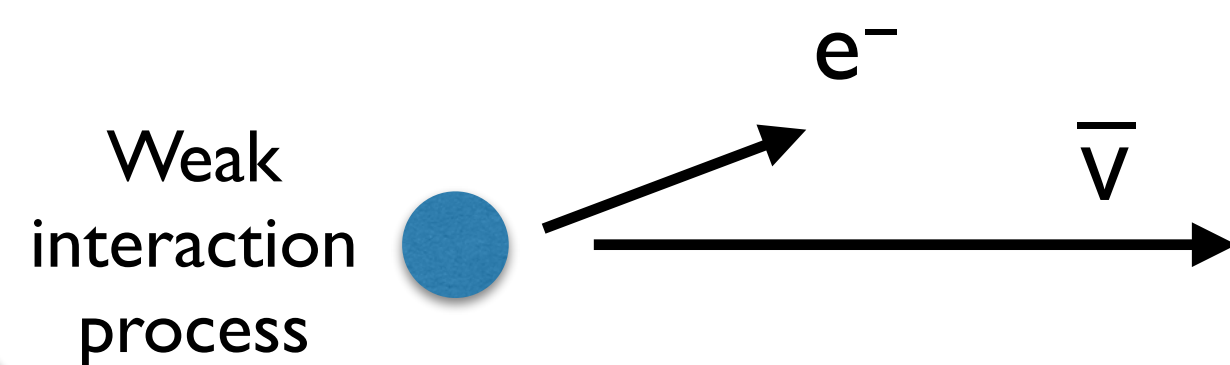
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Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



For $m \neq 0$ the distinction between Dirac and Majorana *matters*.
The Majorana option blurs the notion of matter and antimatter!

Can assign conserved lepton number

4 states:
 $\nu_- \quad \bar{\nu}_+ \quad \nu_+ \quad \bar{\nu}_-$

Participate in
weak interactions

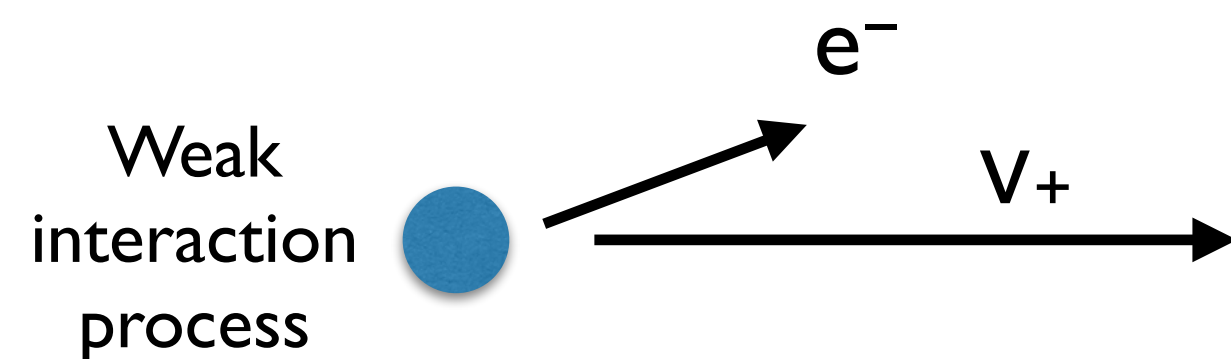
Cannot assign a conserved lepton number

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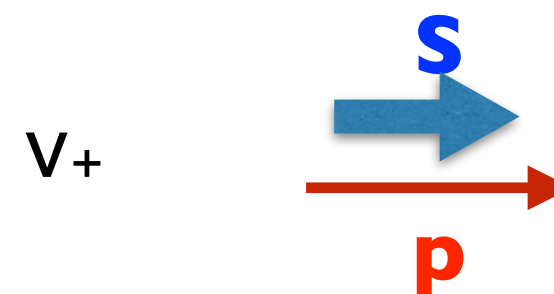
Distinguishing Dirac from Majorana?

Majorana
case

'Anti-neutrino' ($\bar{\nu}$): emitted with e^-

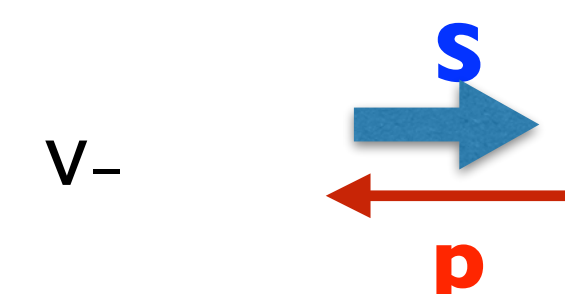


Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$



If $m \neq 0$,
Lorentz boost

Left-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = -1/2$



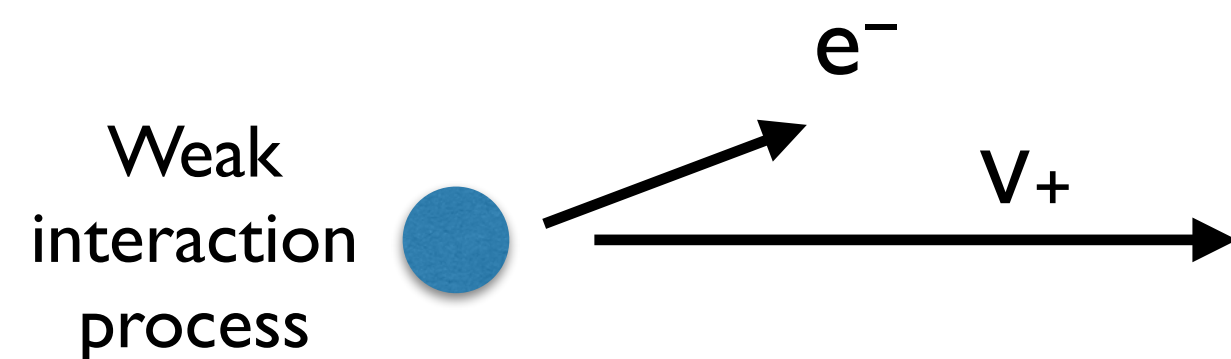
For a massive particle, helicity is frame-dependent $\Rightarrow$

The emitted massive (anti)neutrino can be in both $V_{+/-}$ states and hence can transform into both e^+ and e^- !

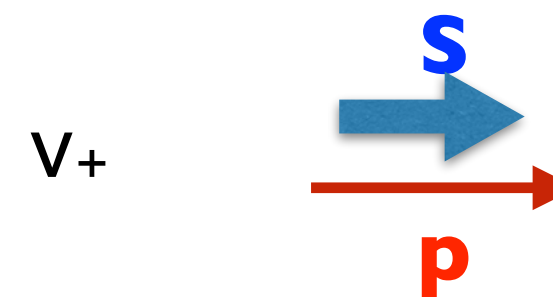
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Majorana case

'Anti-neutrino' ($\bar{\nu}$): emitted with e^-

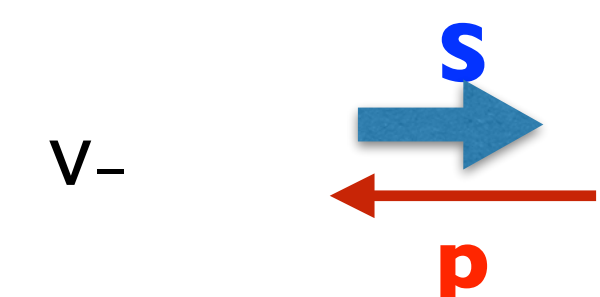


Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$

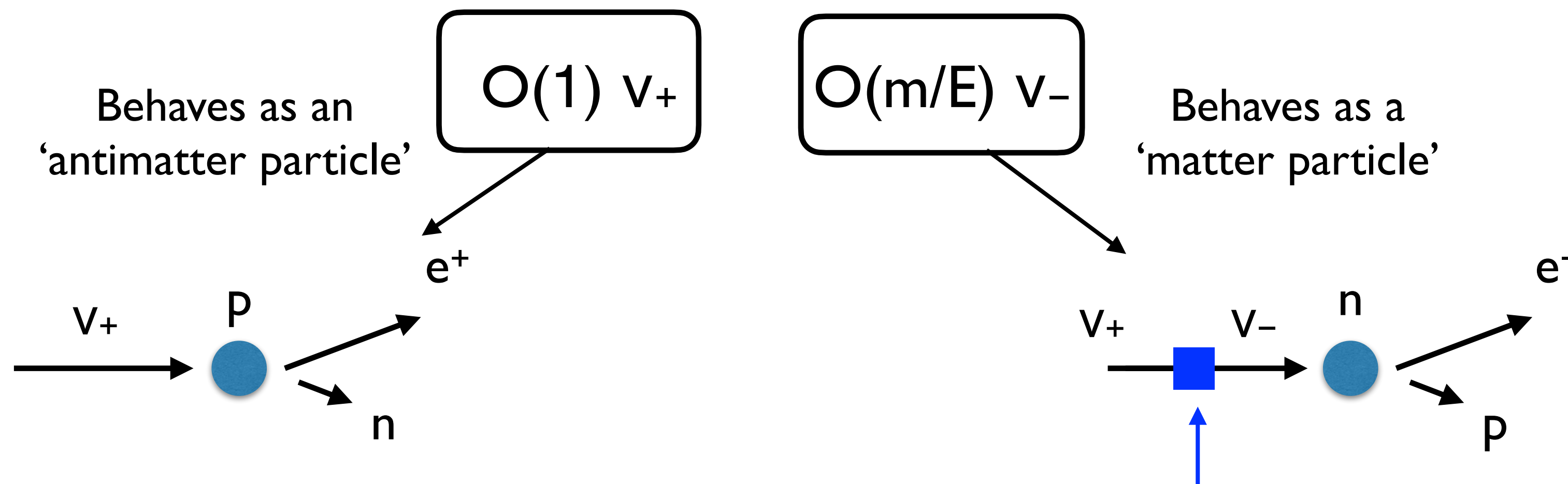


If $m \neq 0$,
Lorentz boost

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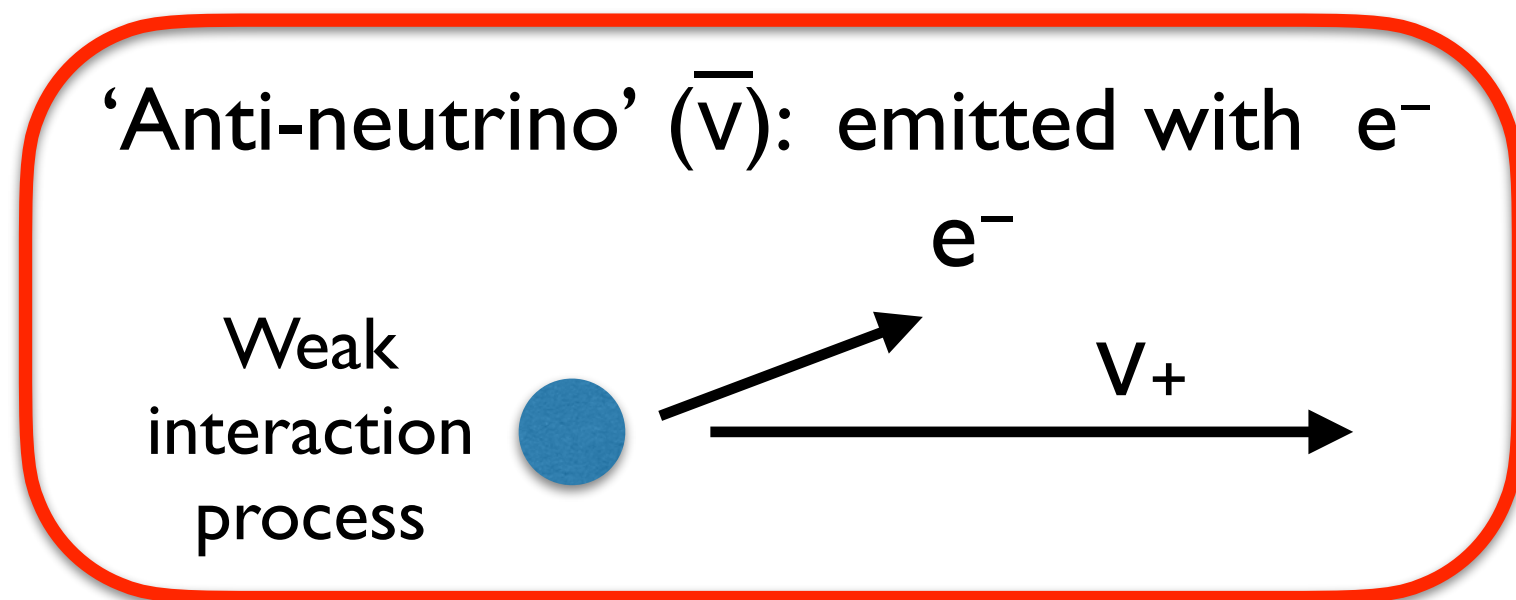
- Weak interactions produce massive 'antineutrino' in both helicity states



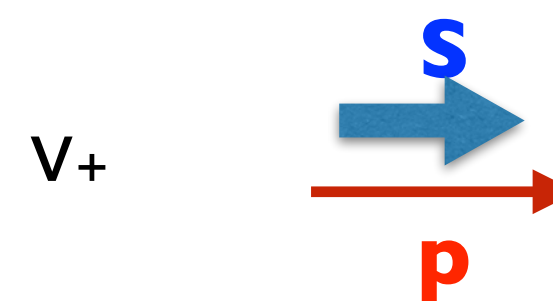
Perturbatively, the two helicity states mix through mass insertion

Distinguishing Dirac from Majorana?

Majorana case

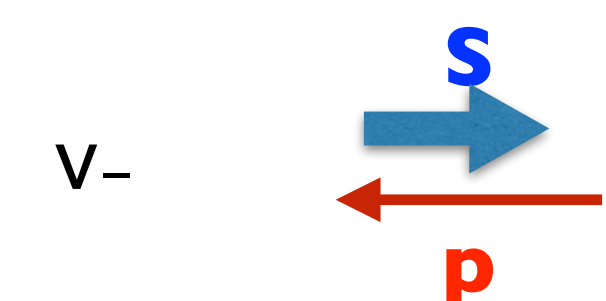


Right-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = +1/2$

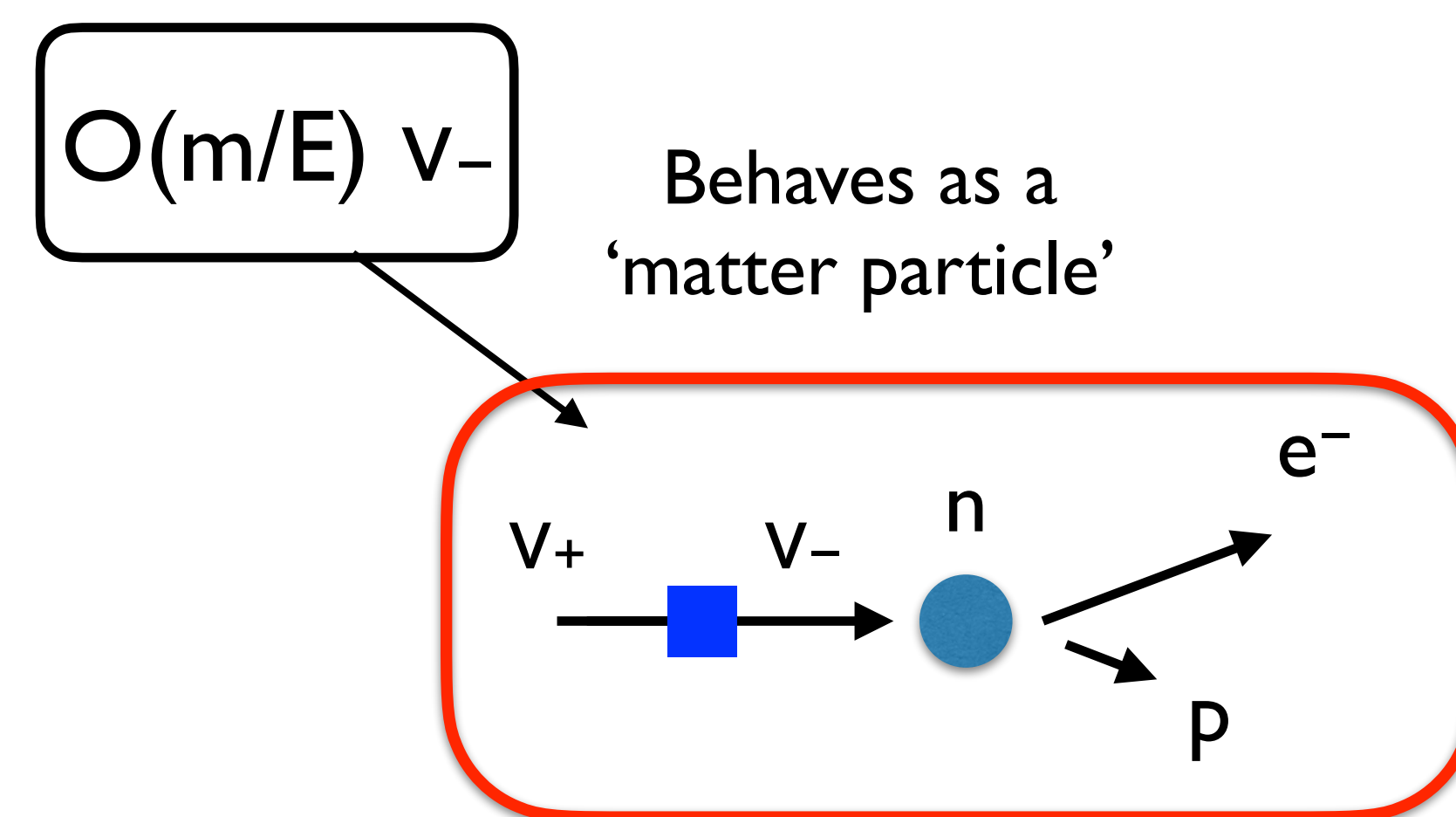
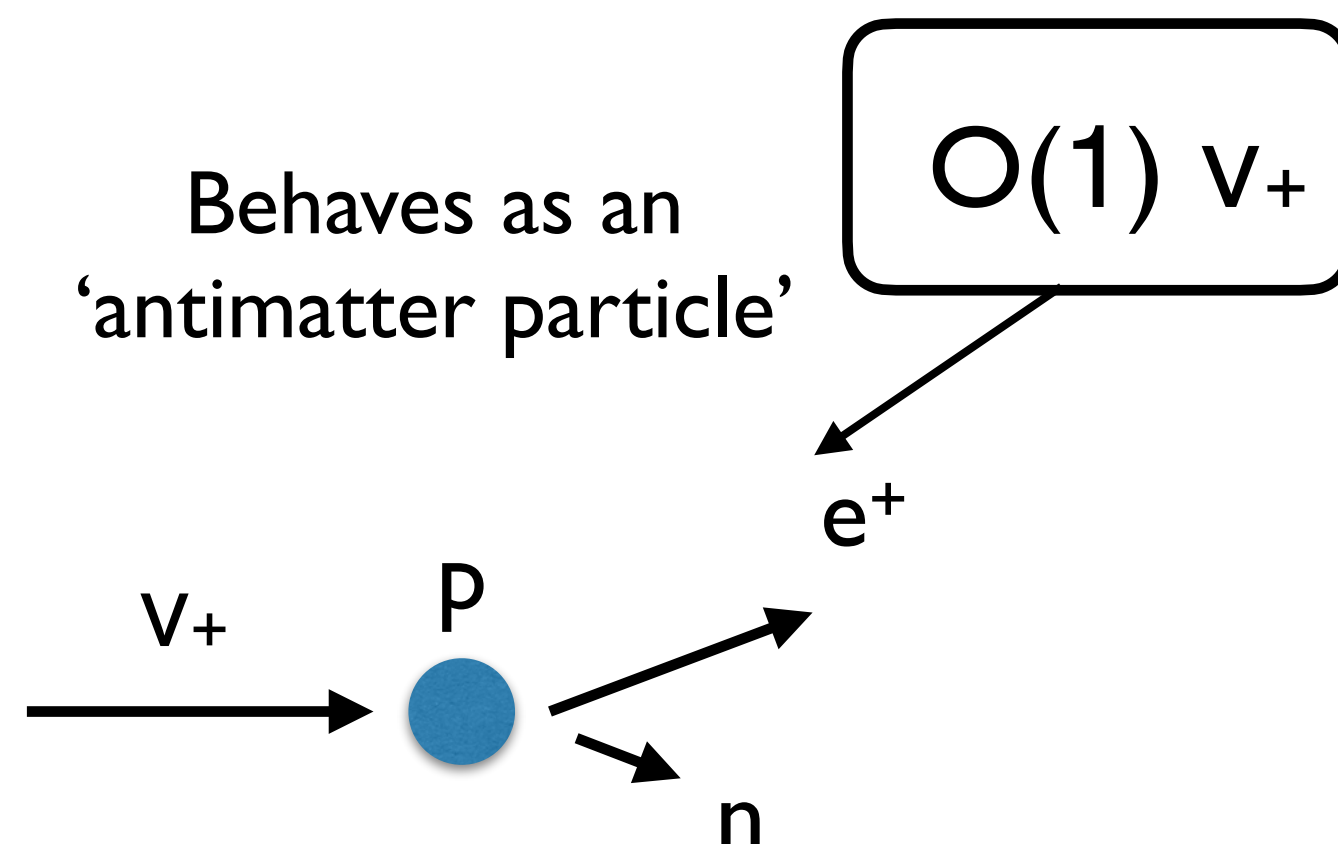


If $m \neq 0$,
Lorentz boost

Left-handed: $\mathbf{S} \cdot \hat{\mathbf{p}} = -1/2$



- Weak interactions produce massive 'antineutrino' in both helicity states



- To detect the Majorana signature (e^- in final state) need to overcome the m/E factor

Non-relativistic neutrinos

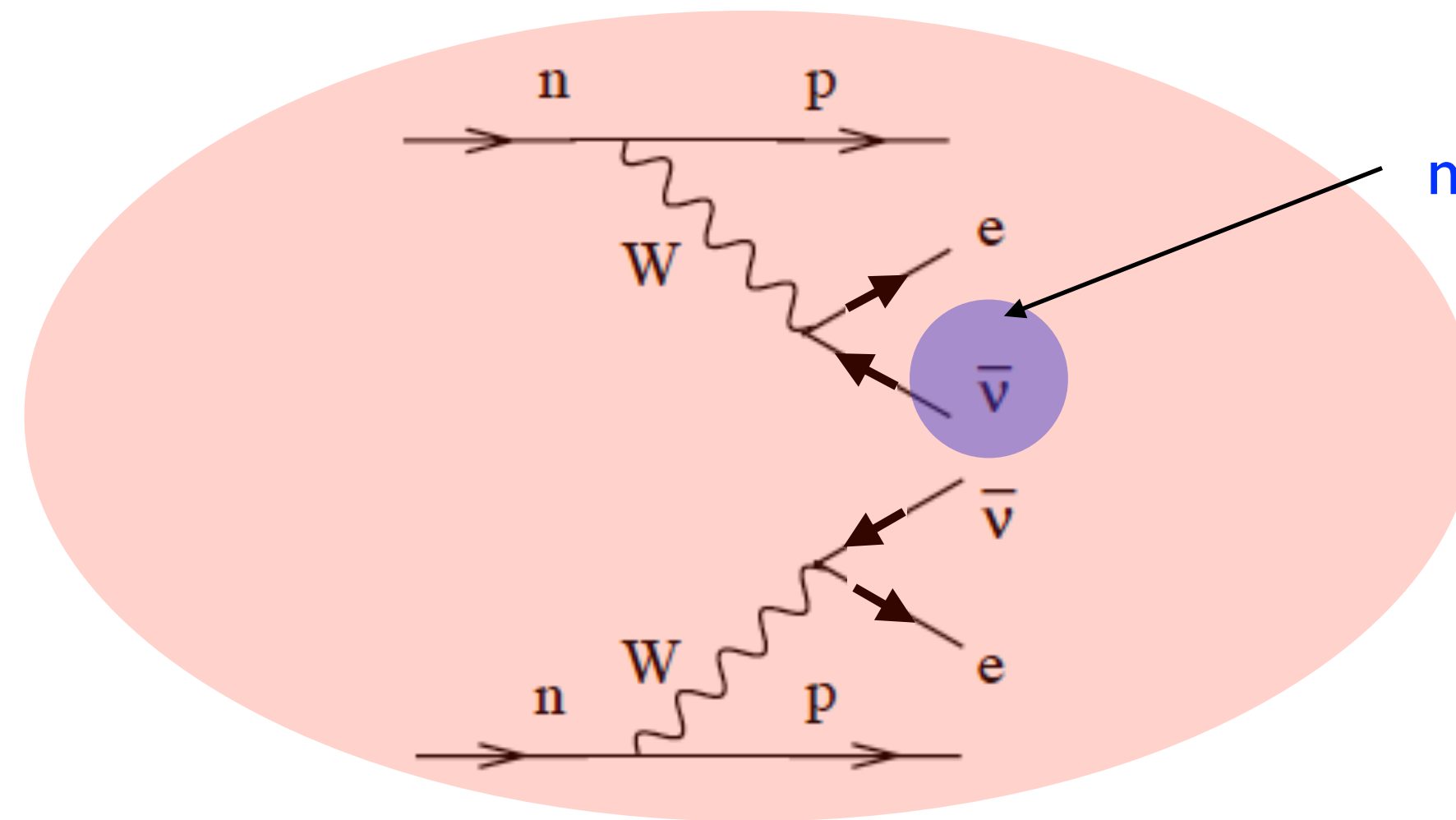
Avogadro's number: double beta decay!

$0\nu\beta\beta$ decay is the arbiter

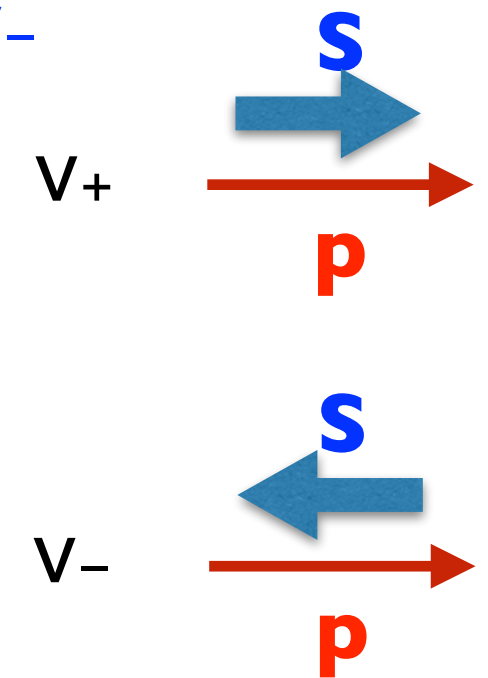
- If neutrinos are Majorana particles, a *virtual anti-neutrino* can convert into a *neutrino* and mediate $0\nu\beta\beta$



W. H. Furry, 1939



This is just V_+ , which via mass insertion turns into V_-

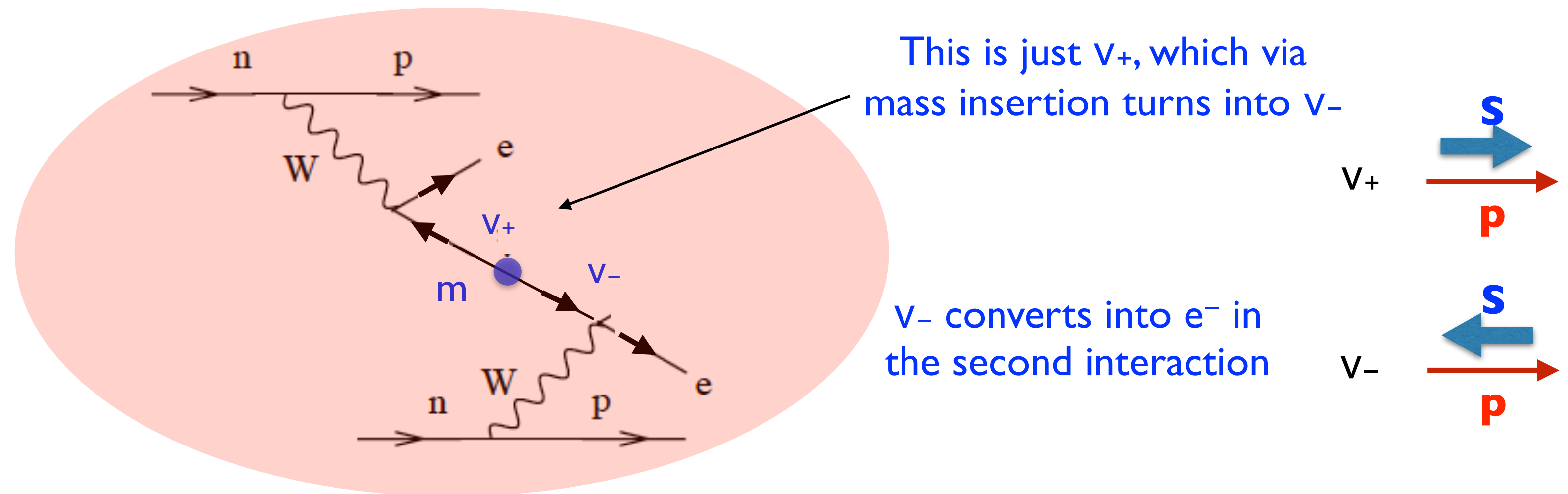


$0\nu\beta\beta$ decay is the arbiter

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W. H. Furry, 1939



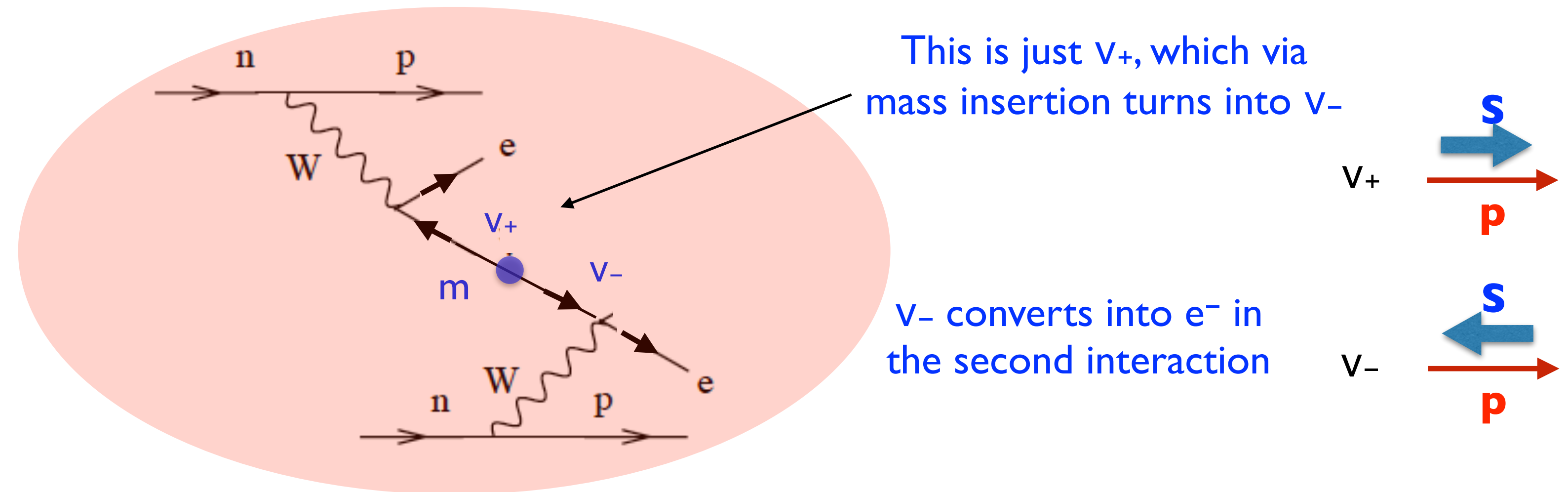
“Subject to the usual limitations on the meaning of such language, one can say that a (virtual) neutrino is emitted together with one of the electrons and reabsorbed when the other electron is emitted.”

$0\nu\beta\beta$ decay is the arbiter

- If neutrinos are Majorana particles, a *virtual anti-neutrino* can convert into a *neutrino* and mediate $0\nu\beta\beta$



W. H. Furry, 1939



- Key point: in $0\nu\beta\beta$ Lepton Number changes by two units. Majorana ν exchange is just one possible mechanism. Furry understood this:

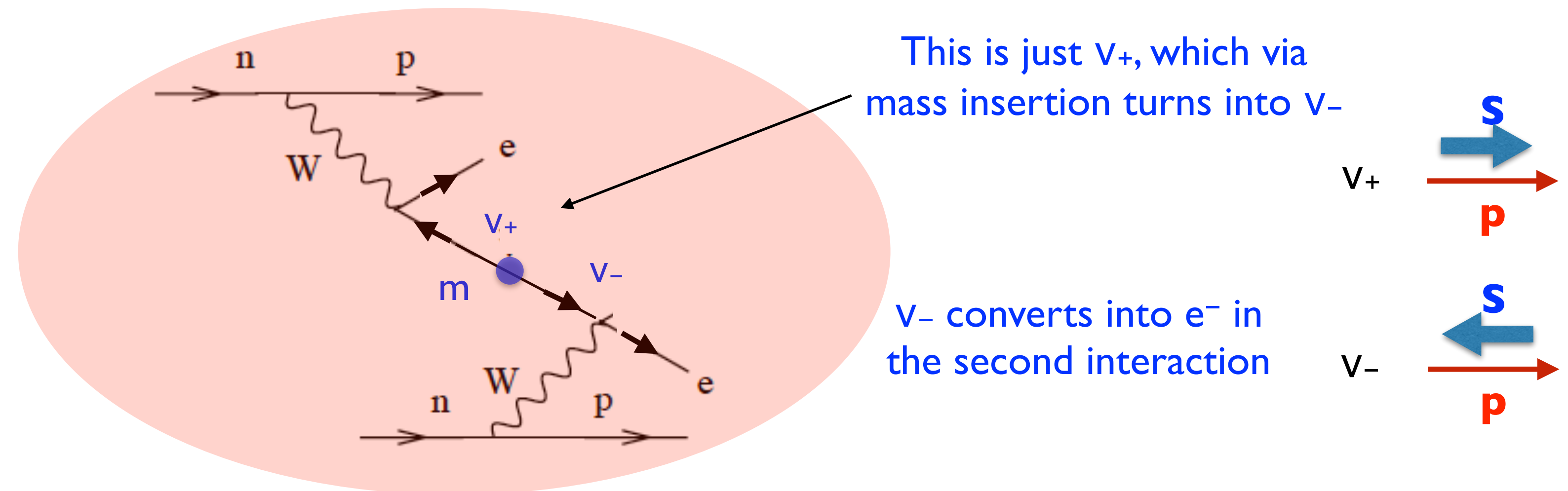
“The Majorana form of the theory is not the only one that permits this new form of disintegration [...]. The Majorana theory provides, so to speak, a canonical form.”

$0\nu\beta\beta$ decay is the arbiter

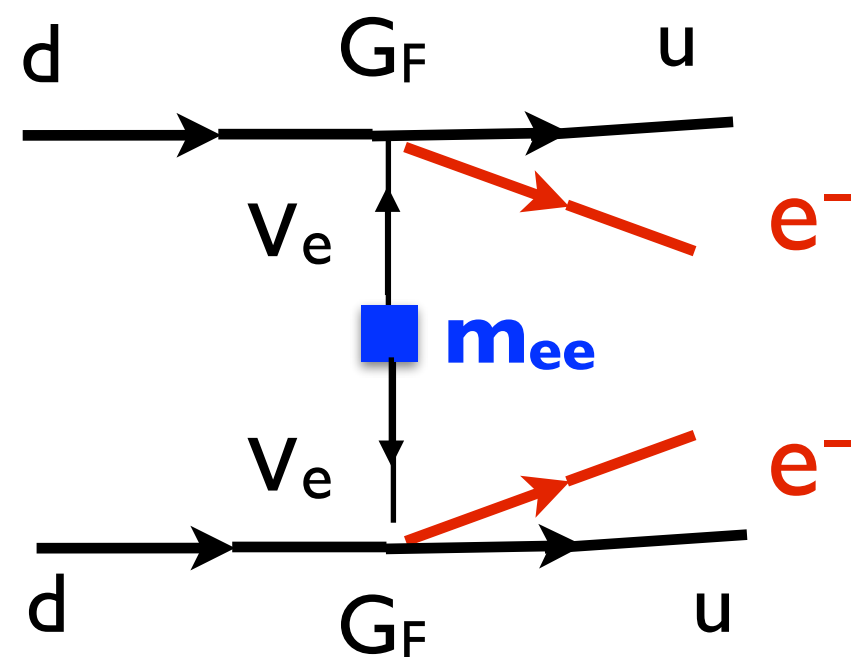
- If neutrinos are Majorana particles, a *virtual anti-neutrino* can convert into a *neutrino* and mediate $0\nu\beta\beta$



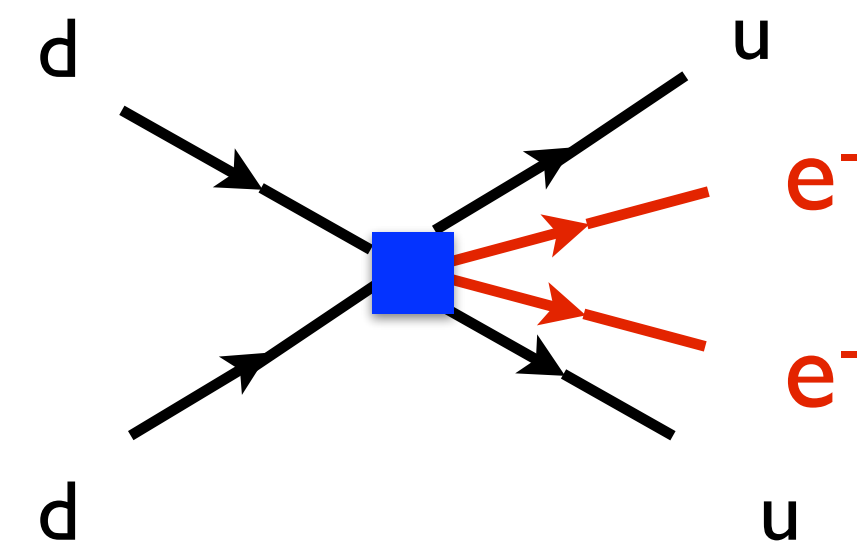
W. H. Furry, 1939



- Modern viewpoint on Lepton Number Violation:



but also



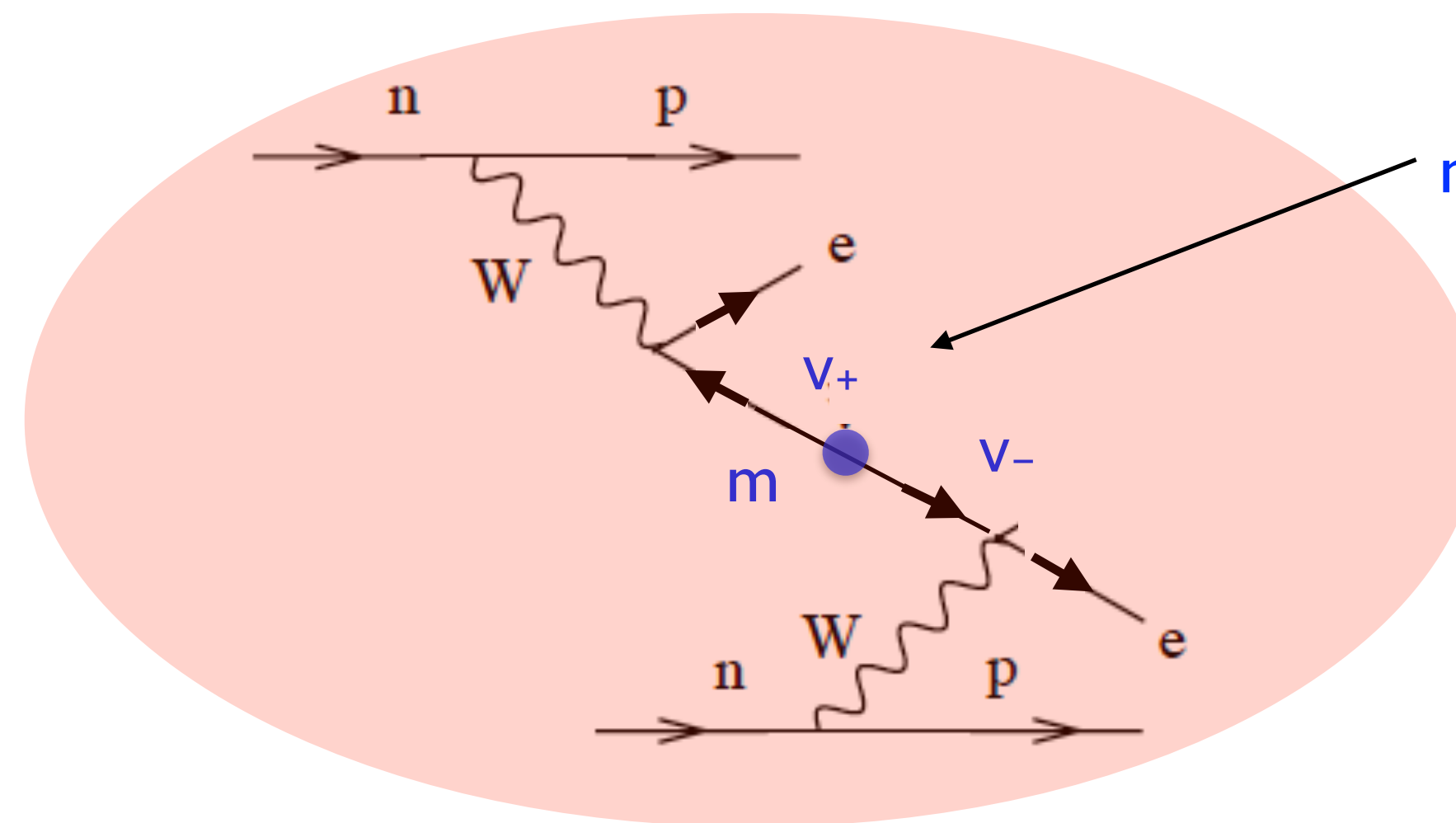
Exchange of heavier neutrinos or other Majorana particles. At low-energy induce six-fermion operator $\sim 1/\Lambda^5$

$0\nu\beta\beta$ decay is the arbiter

- If neutrinos are Majorana particles, a *virtual anti-neutrino* can convert into a *neutrino* and mediate $0\nu\beta\beta$

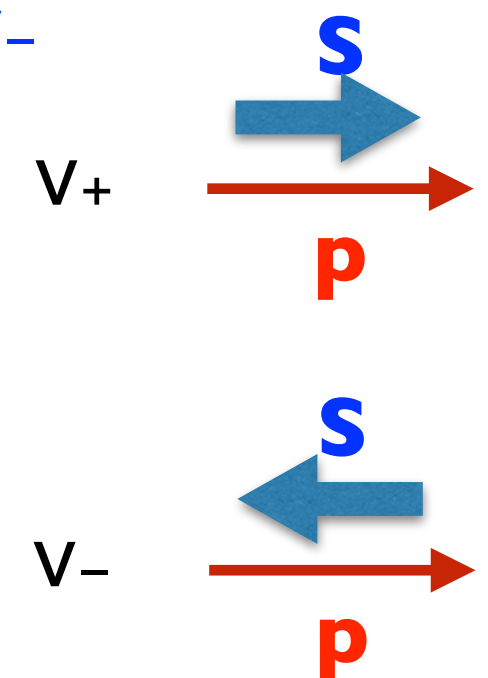


W. H. Furry, 1939

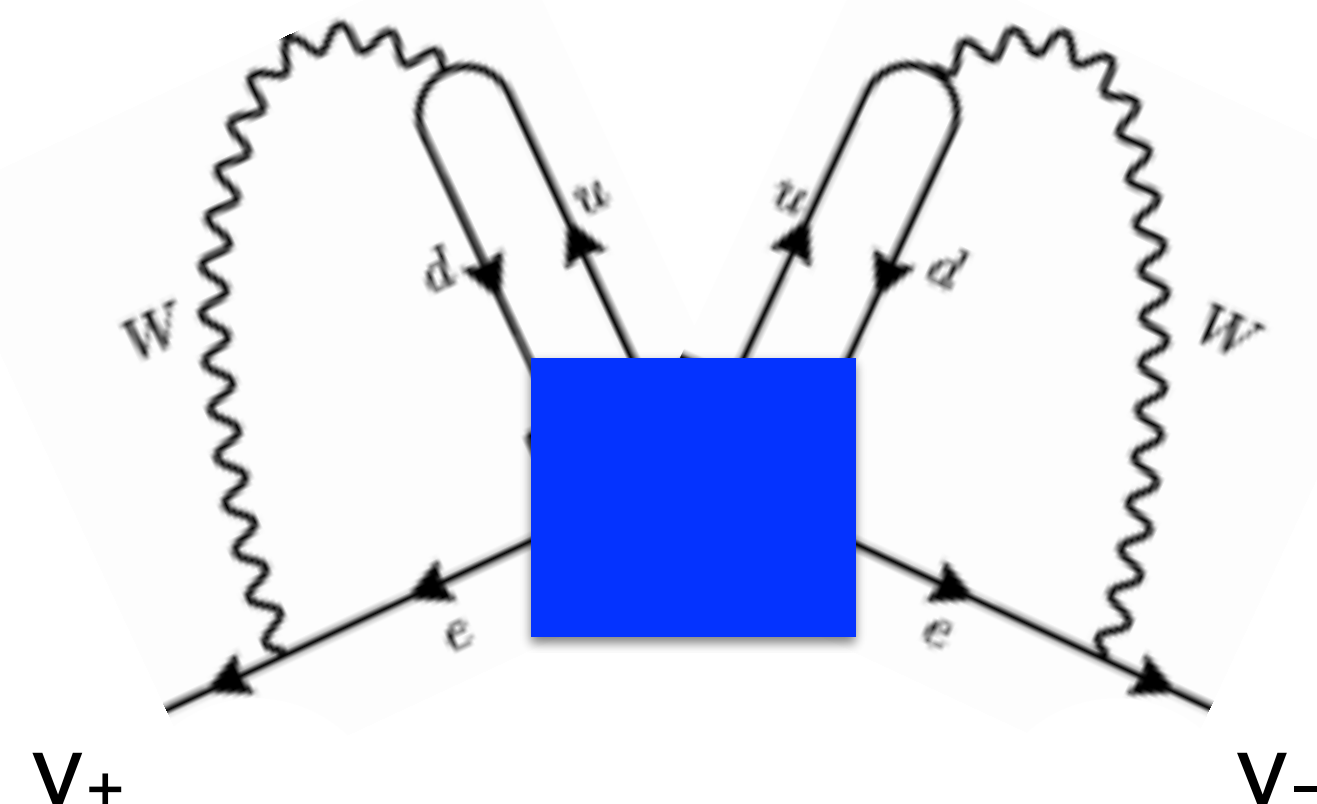
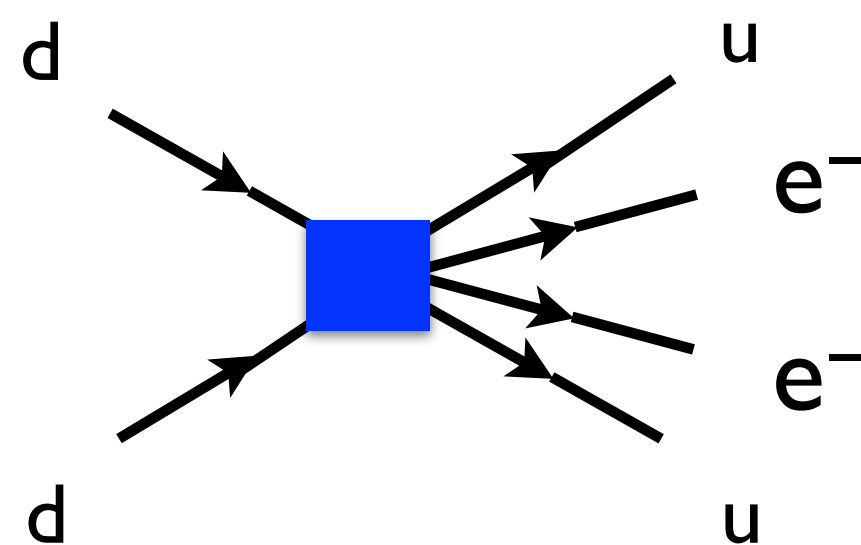


This is just V_+ , which via mass insertion turns into V_-

V_- converts into e^- in the second interaction



- If $0\nu\beta\beta$ decay happens, through *quantum mechanical fluctuations* a V_+ can convert into $V_- \Rightarrow$ hallmark of Majorana ν !



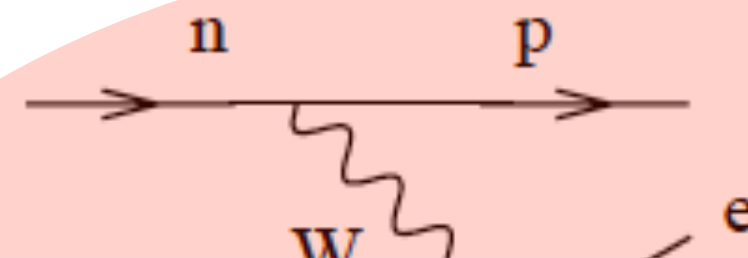
Schechter-Valle 1982

$0\nu\beta\beta$ decay is the arbiter

- If neutrinos are Majorana particles, a *virtual anti-neutrino can convert into a neutrino* and mediate $0\nu\beta\beta$



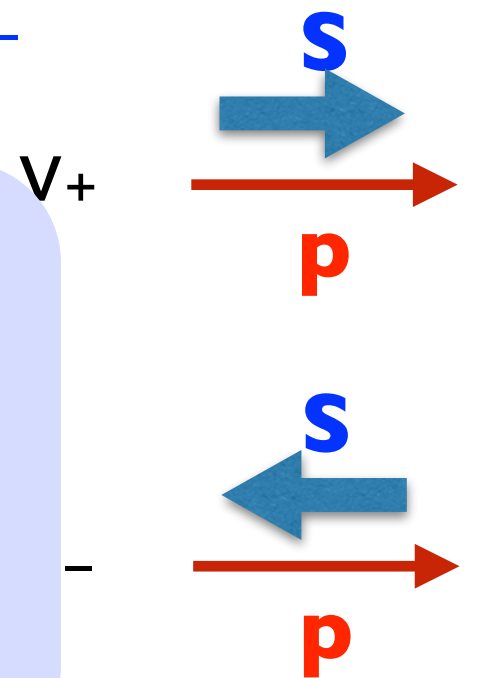
W.H.



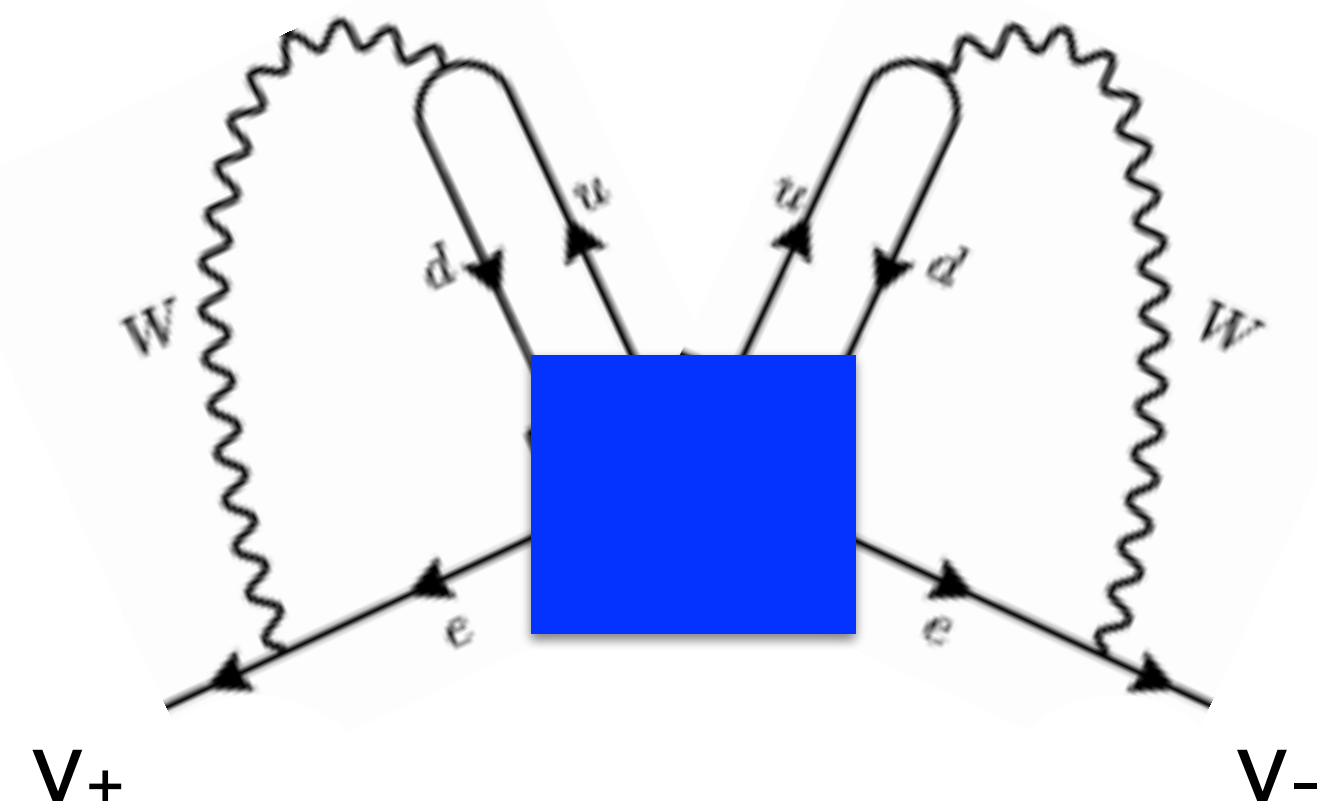
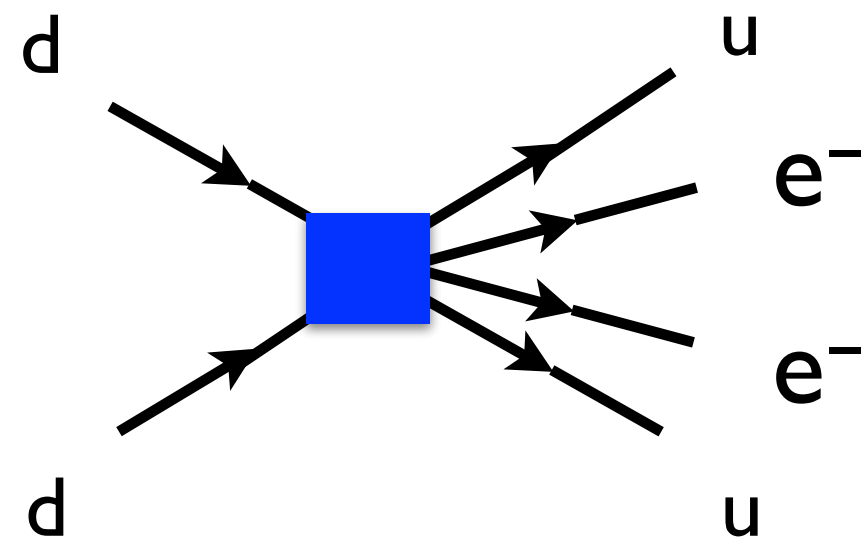
This is just V_+ , which via mass insertion turns into V_-

It's a two-way arrow:

Neutrino is a Majorana fermion $\iff 0\nu\beta\beta$ decay happens at some rate



- If $0\nu\beta\beta$ decay happens, through *quantum mechanical fluctuations* a V_+ can convert into $V_- \Rightarrow$ hallmark of Majorana ν !



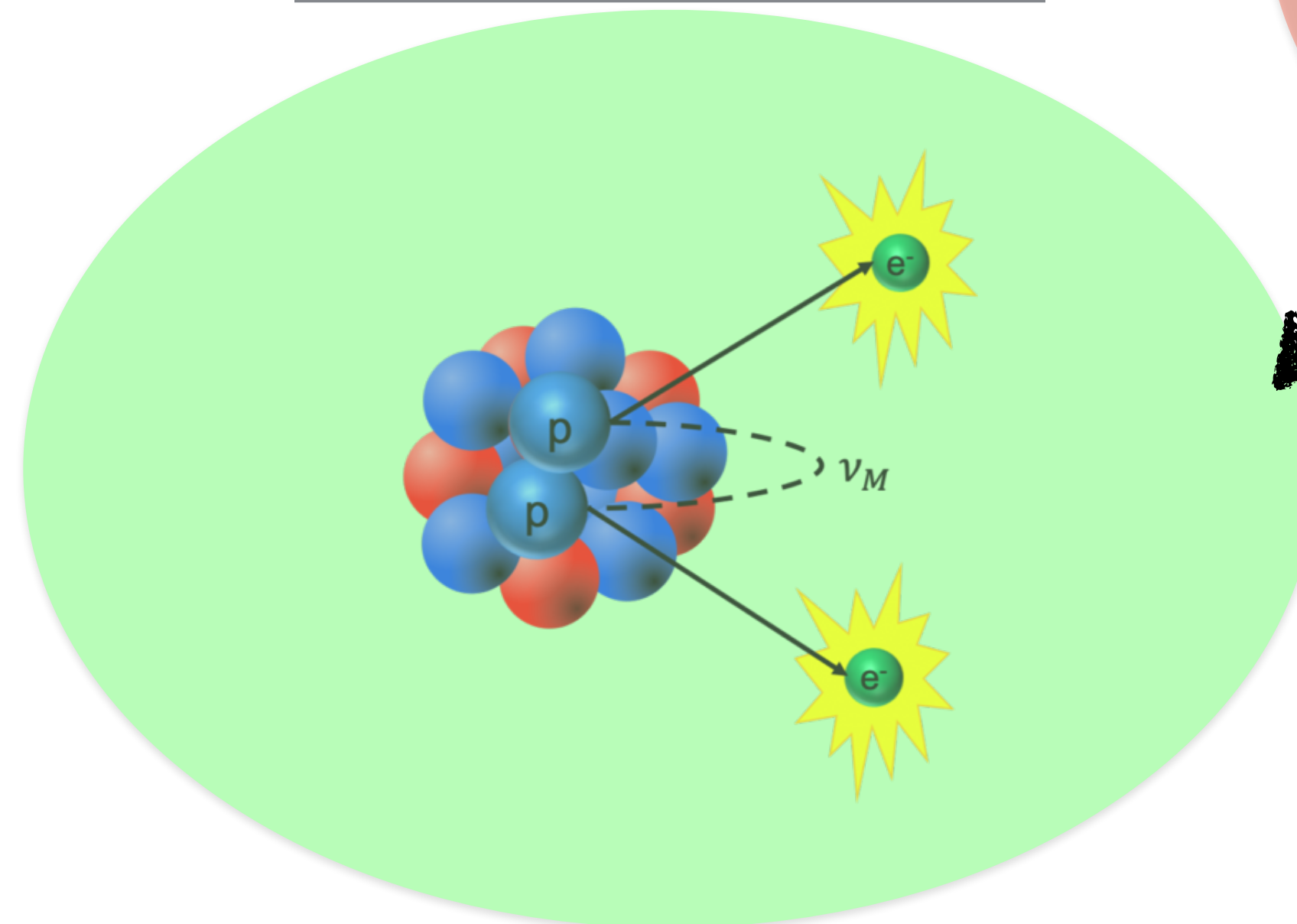
Schechter-Valle 1982

$0\nu\beta\beta$ decay: significance

A cosmic mystery

How did matter survive
the big bang?

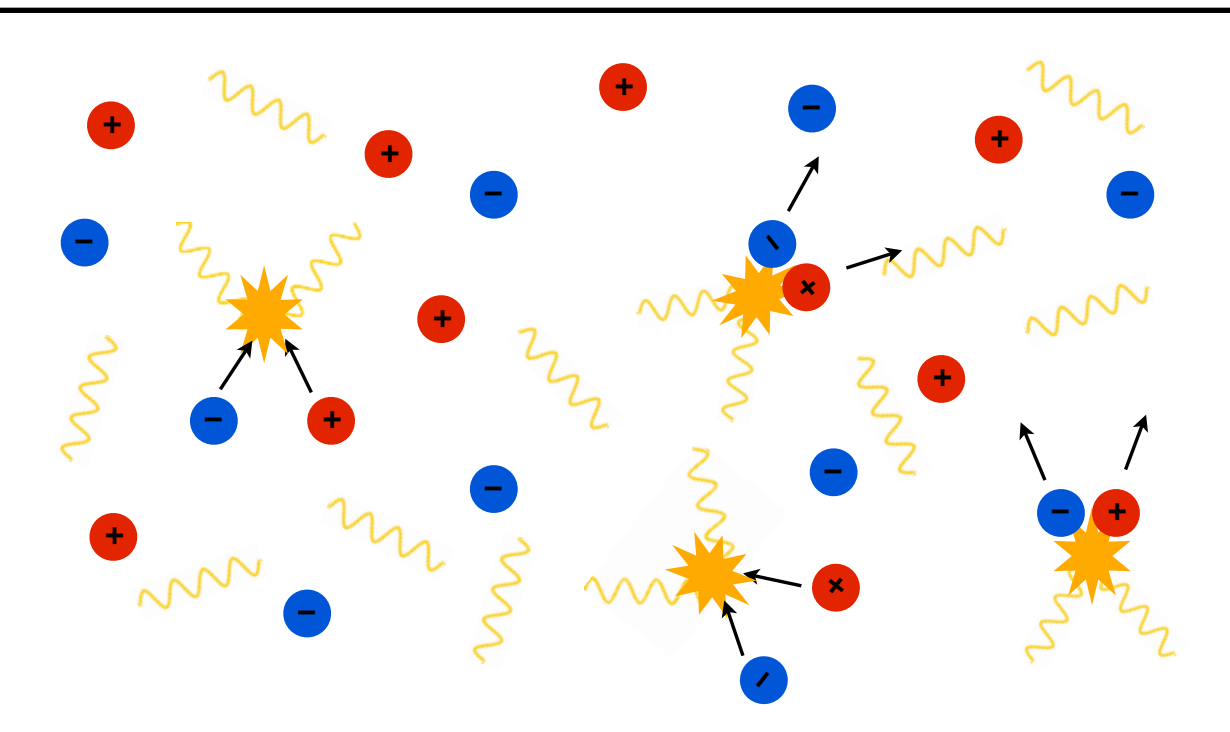
Nuclear $0\nu\beta\beta$ decay



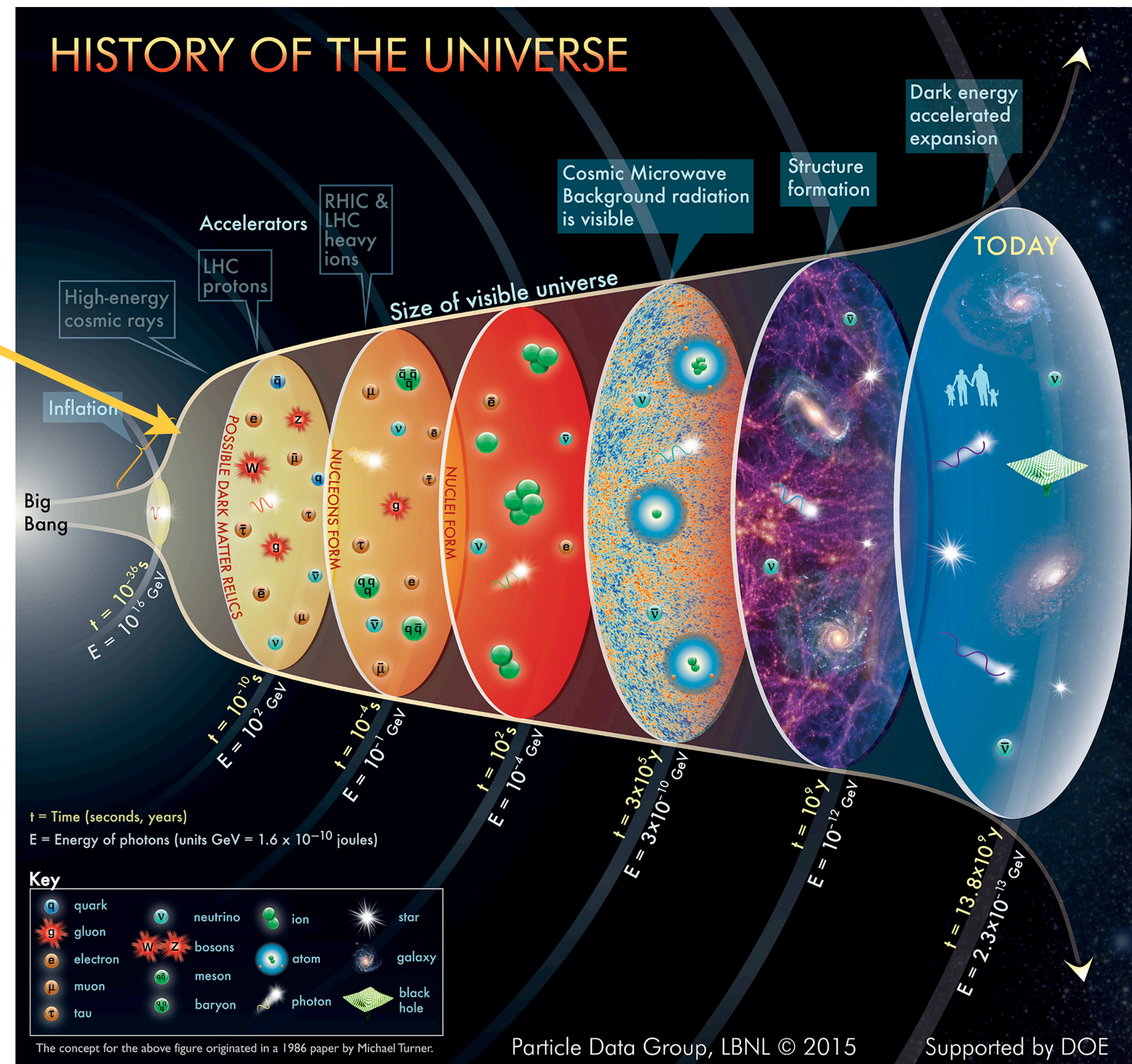
What's the origin of matter in the universe?

Equal number of particles and antiparticles right after the big bang

$t = 0.000001 \text{ s}$



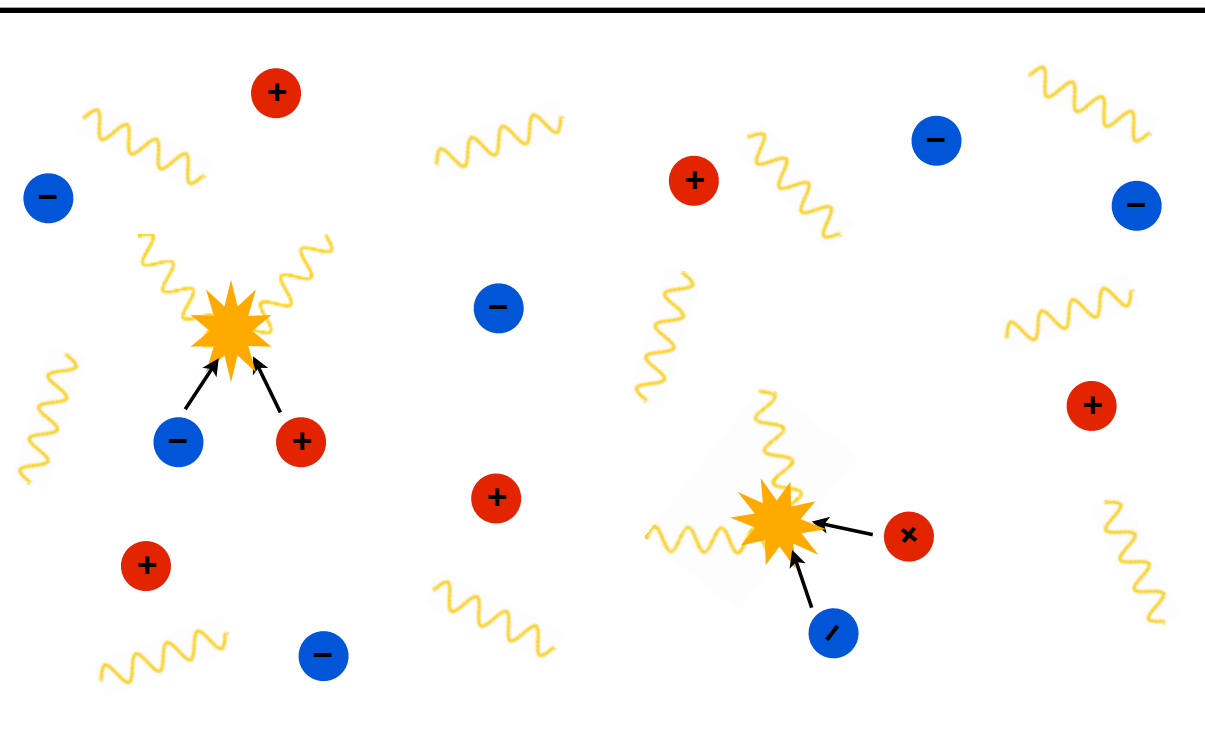
As the universe expands and cools, particle-antiparticle annihilation takes over: end up with just radiation!



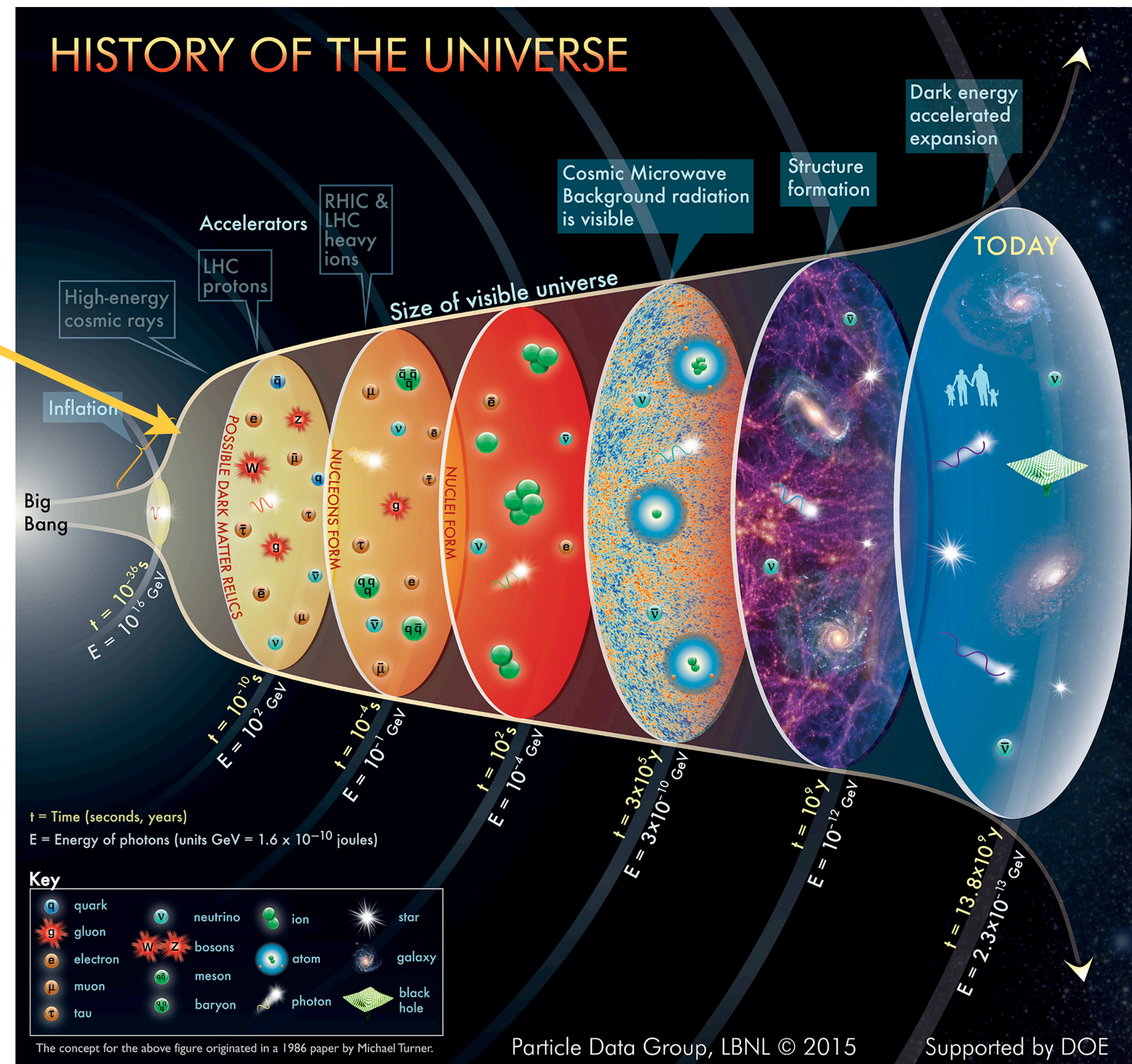
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$t \sim 0.00001s$



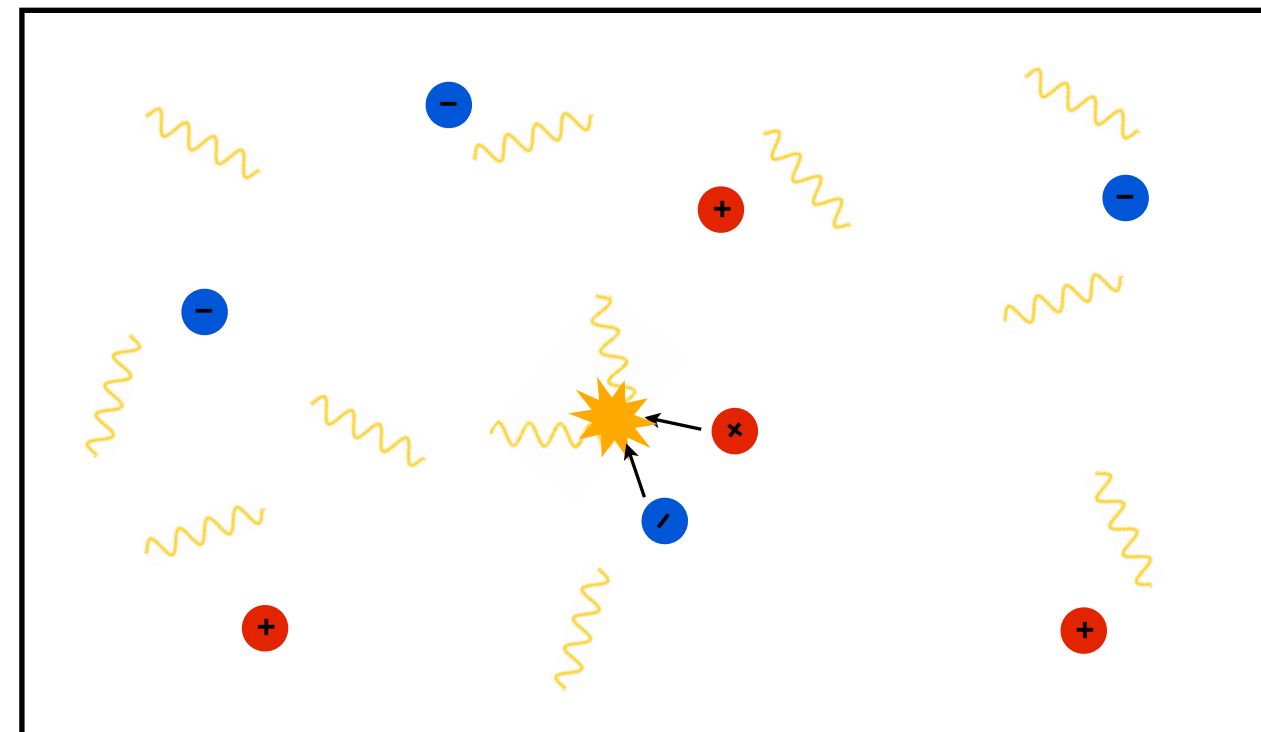
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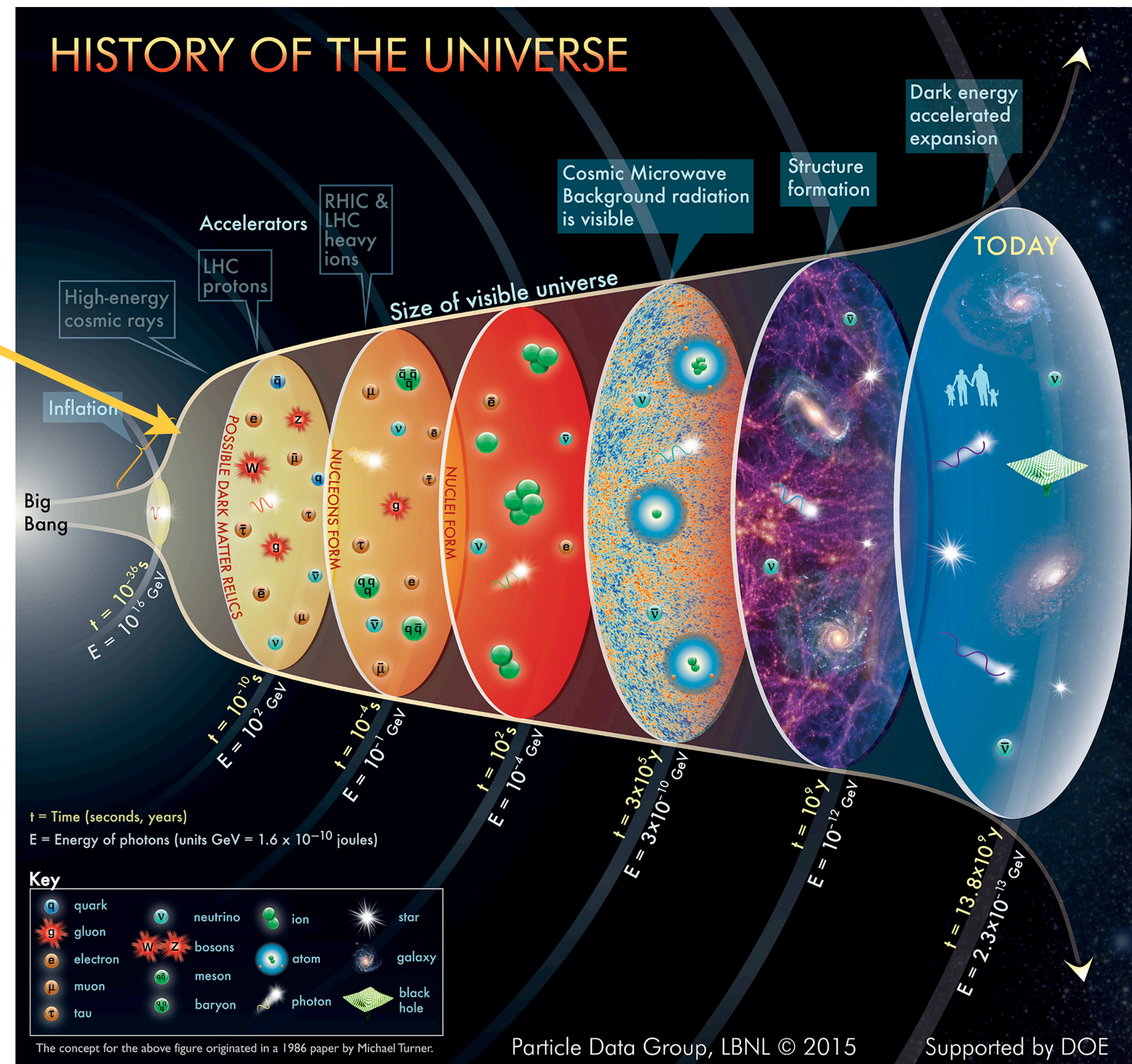
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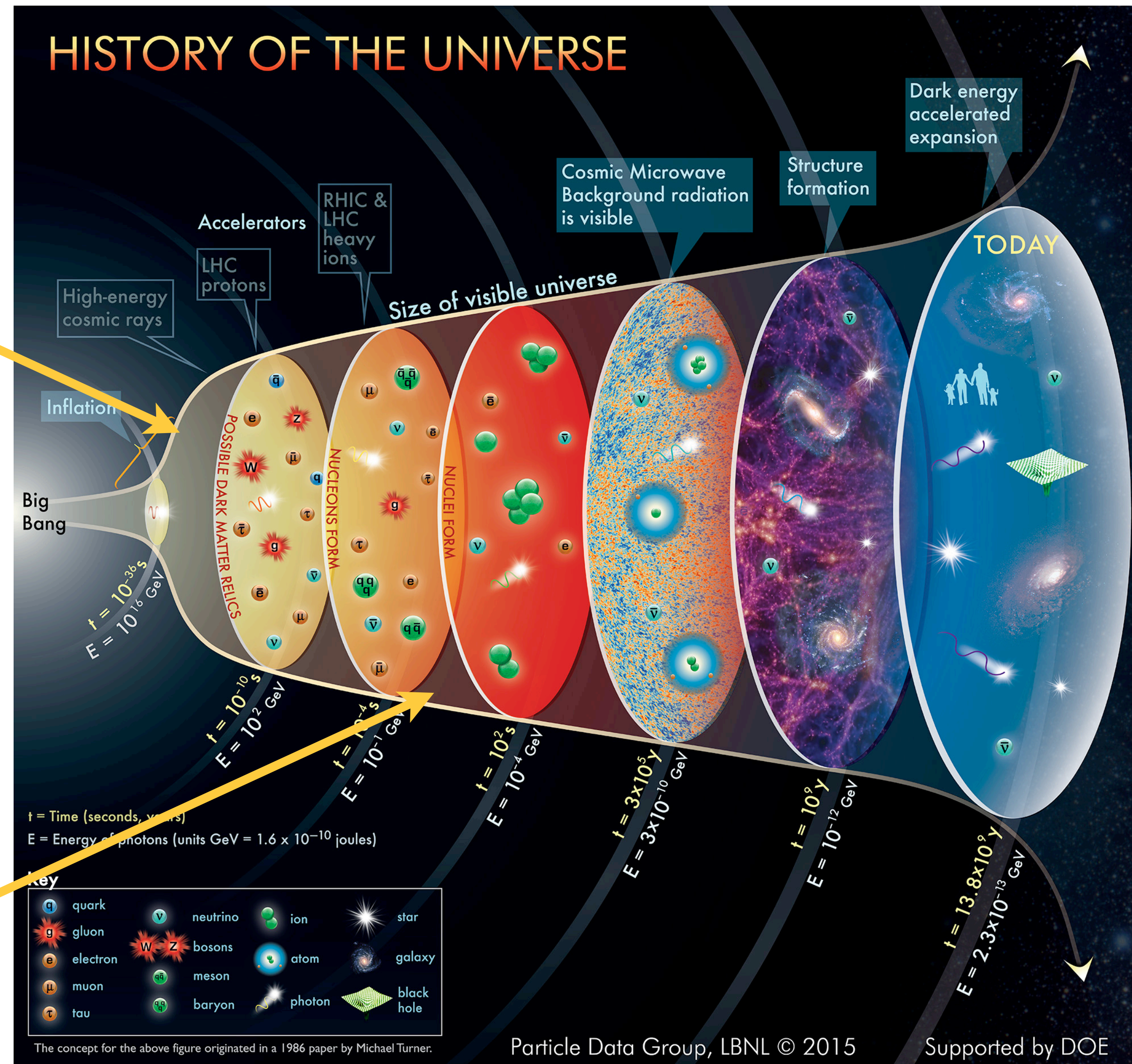
As the universe expands and cools, particle-antiparticle annihilation takes over: end up with just radiation!



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$t \sim 0.003 \text{ s}$



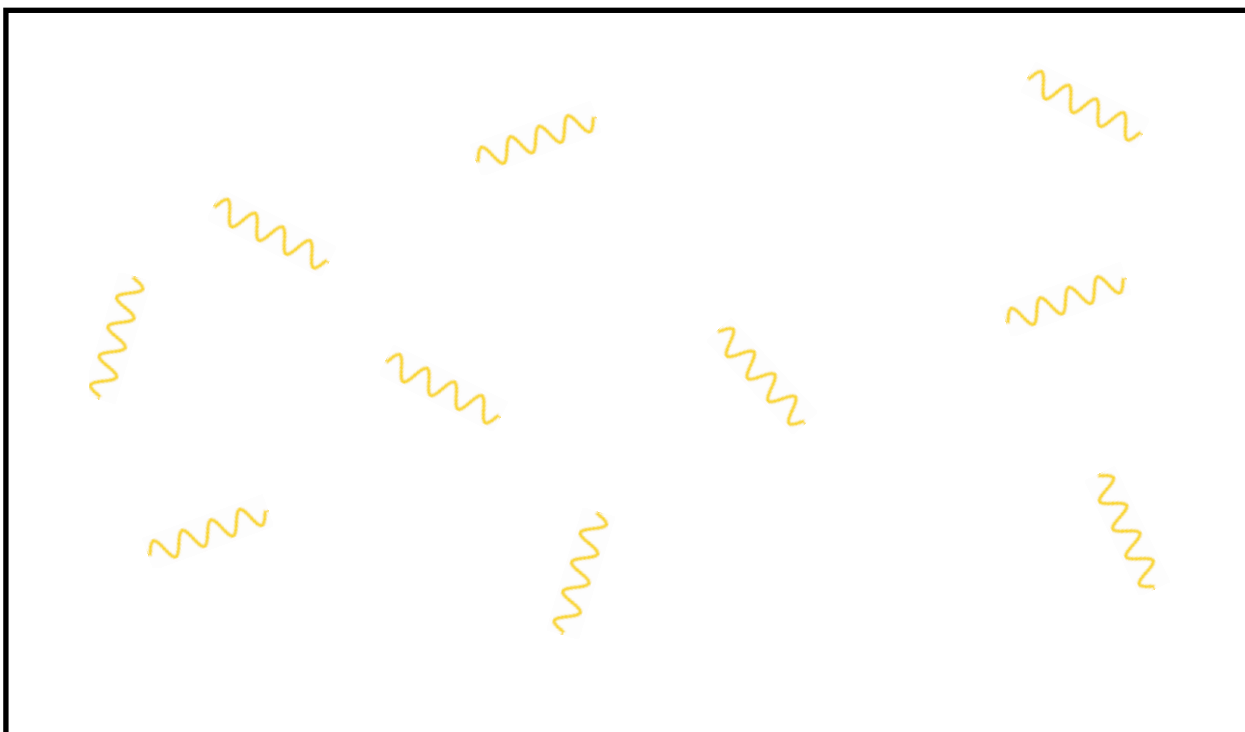
As the universe expands and cools, particle-antiparticle annihilation takes over: end up with just radiation!

$$n_B/n_\gamma = n_B/n_\gamma \sim 10^{-18}$$

What's the origin of matter in the universe?

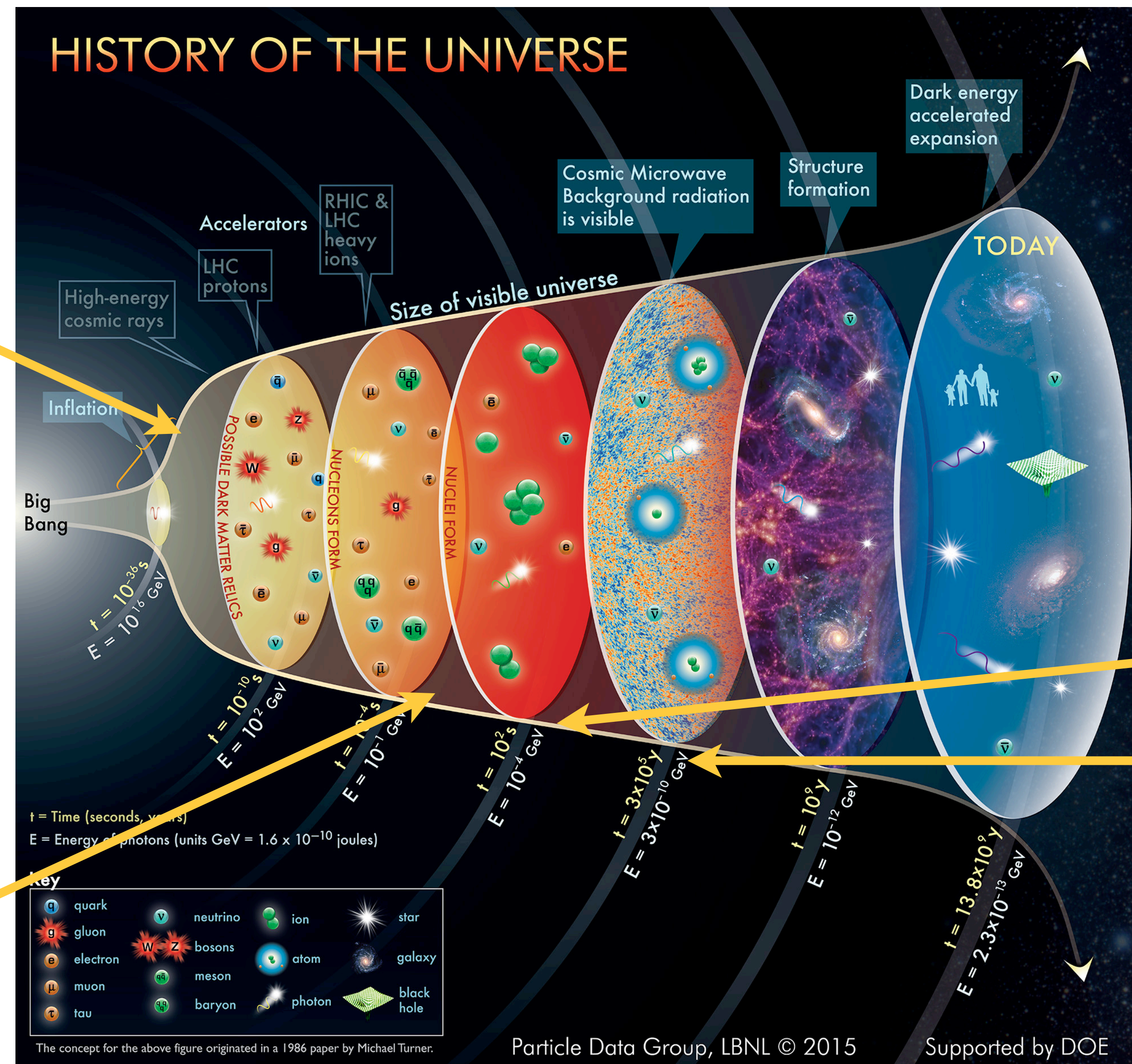
Equal number of particles and antiparticles right after the big bang

$t \sim 0.003 \text{ s}$



As the universe expands and cools, particle-antiparticle annihilation takes over: end up with just radiation!

$$n_B/n_\gamma = n_{\bar{B}}/n_\gamma \sim 10^{-18}$$



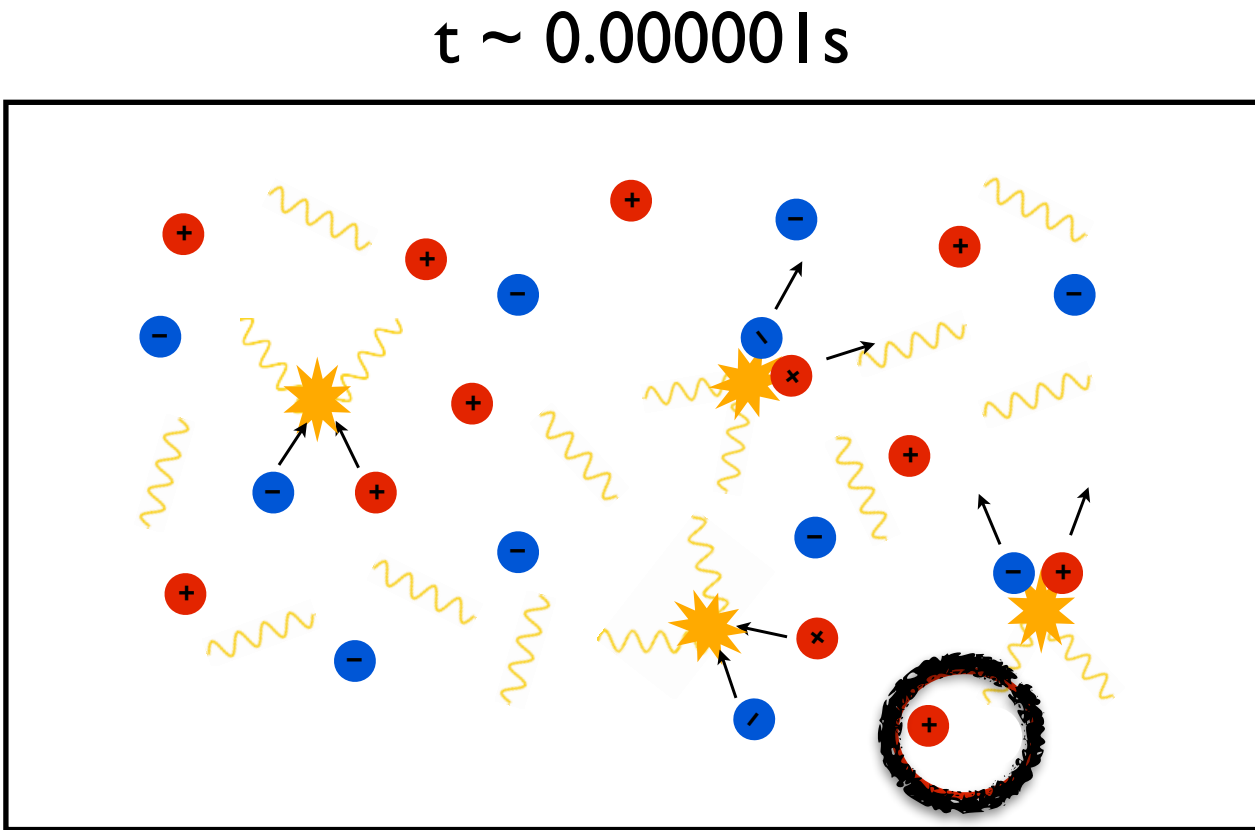
But cosmological observations require a non-zero matter-antimatter asymmetry!

$$\eta \equiv (n_B - n_{\bar{B}})/n_\gamma$$

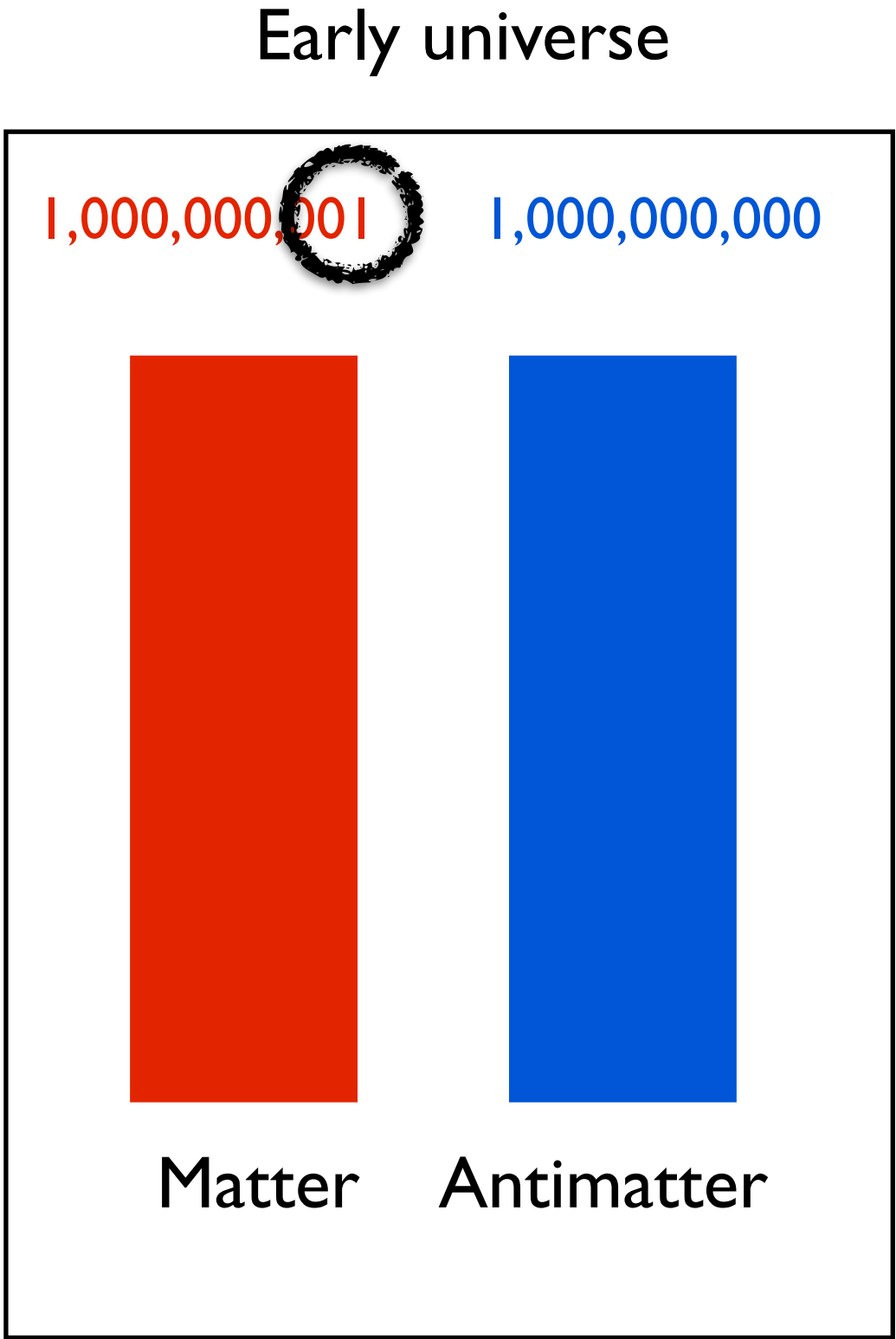
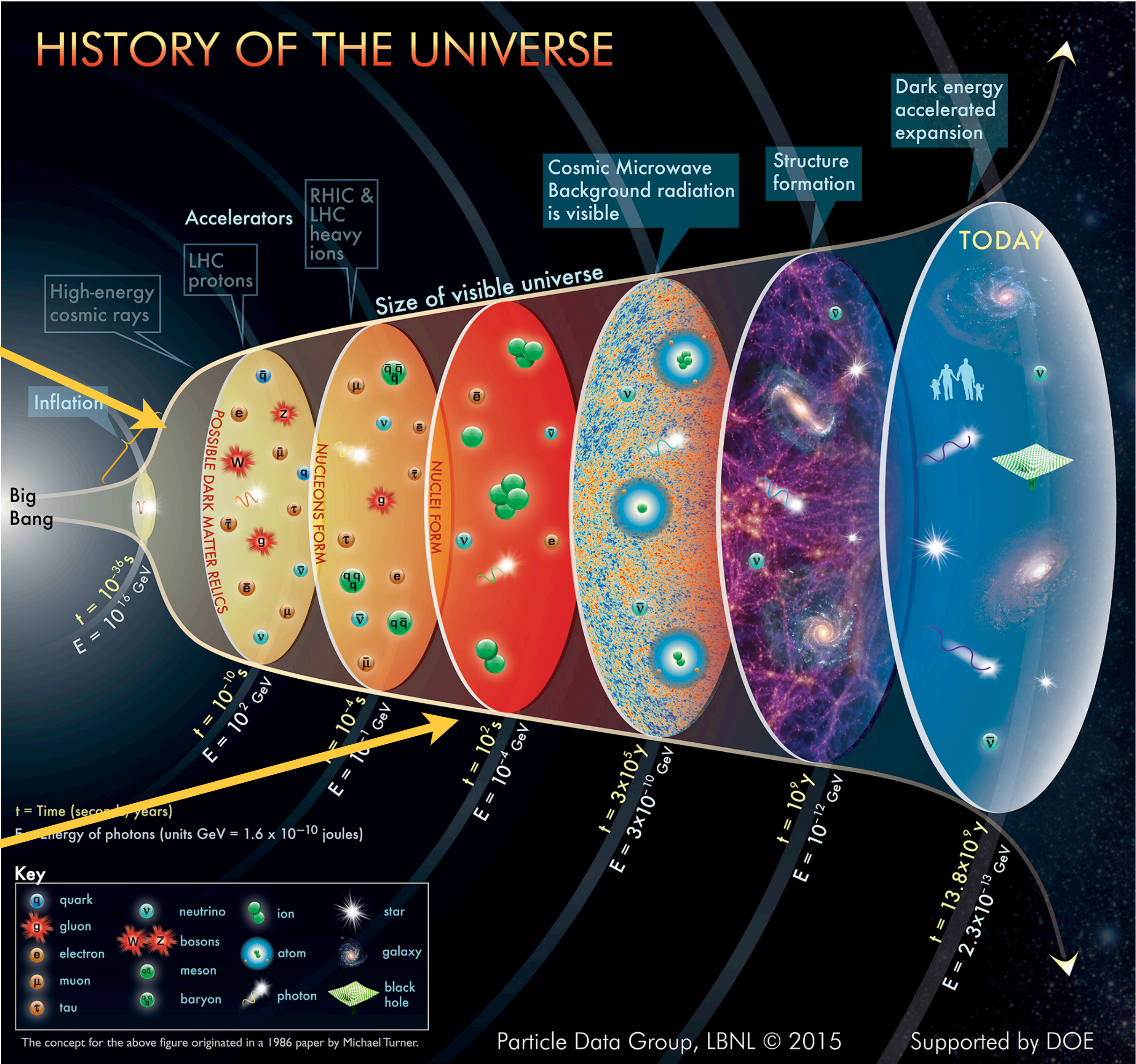
Big Bang Nucleosynthesis ($t \sim 3 \text{ min}$) and the Cosmic Microwave Background ($t \sim 300,000 \text{ yr}$) point to $\eta \sim 6 \times 10^{-10}$

What's the origin of matter in the universe?

Equal number of particles and antiparticles right after the big bang



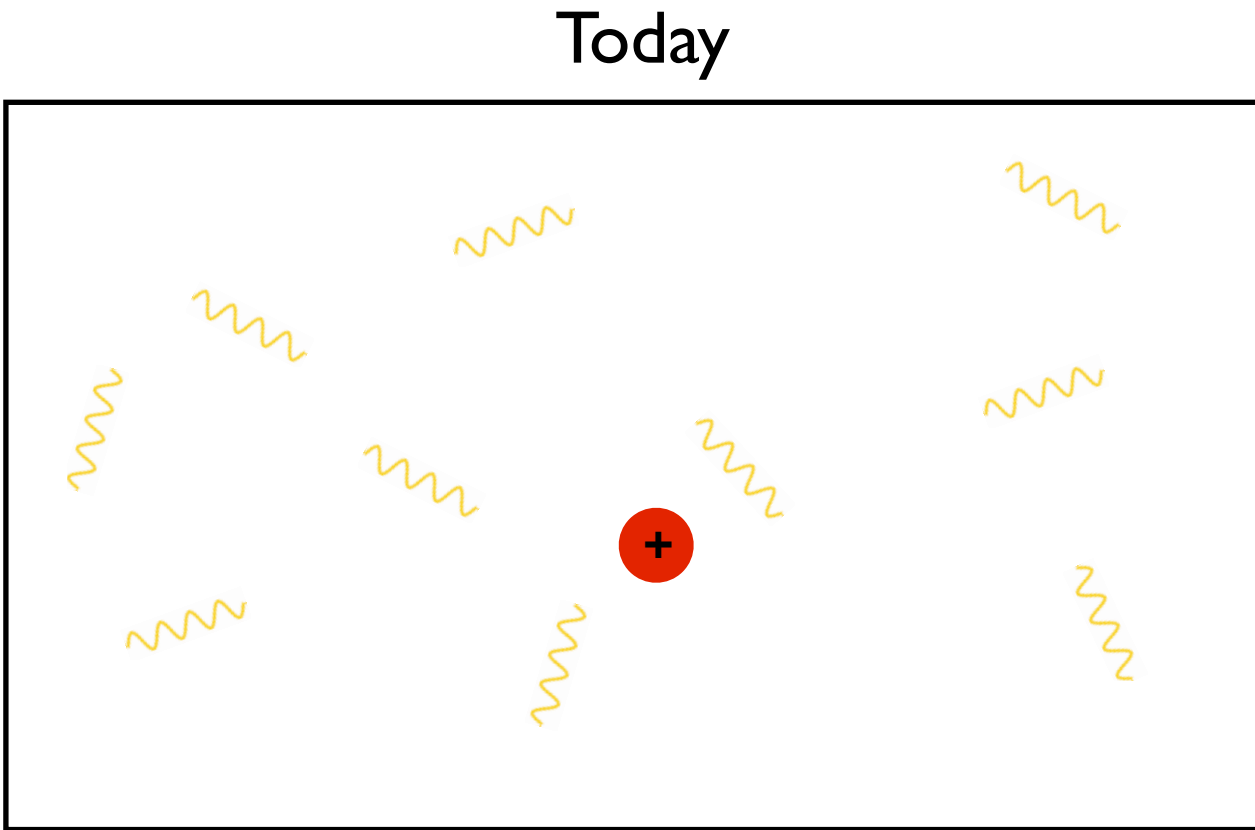
To obtain $O(1)$ protons per cubic meter today, early on need a tiny imbalance of $+$ over $-$



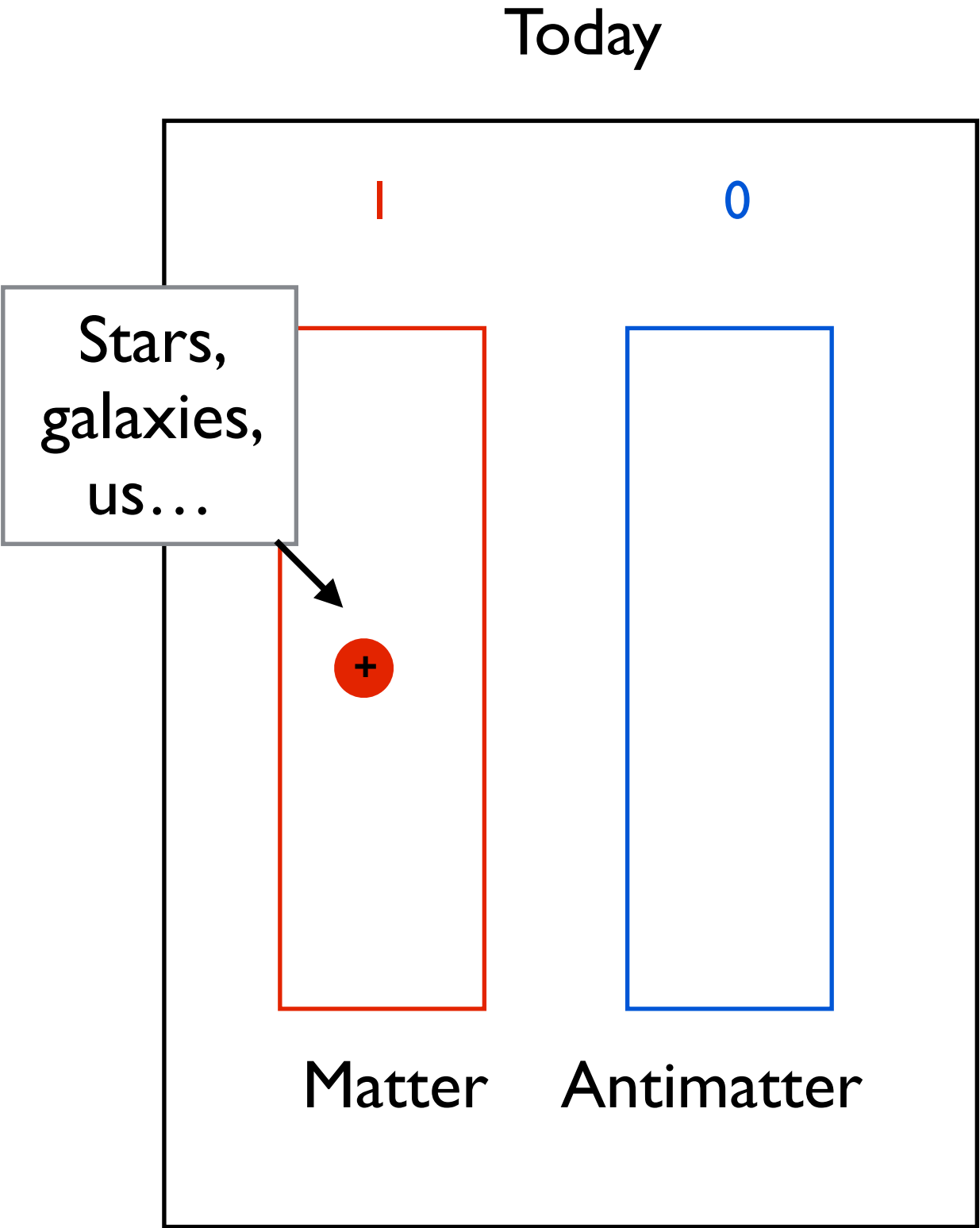
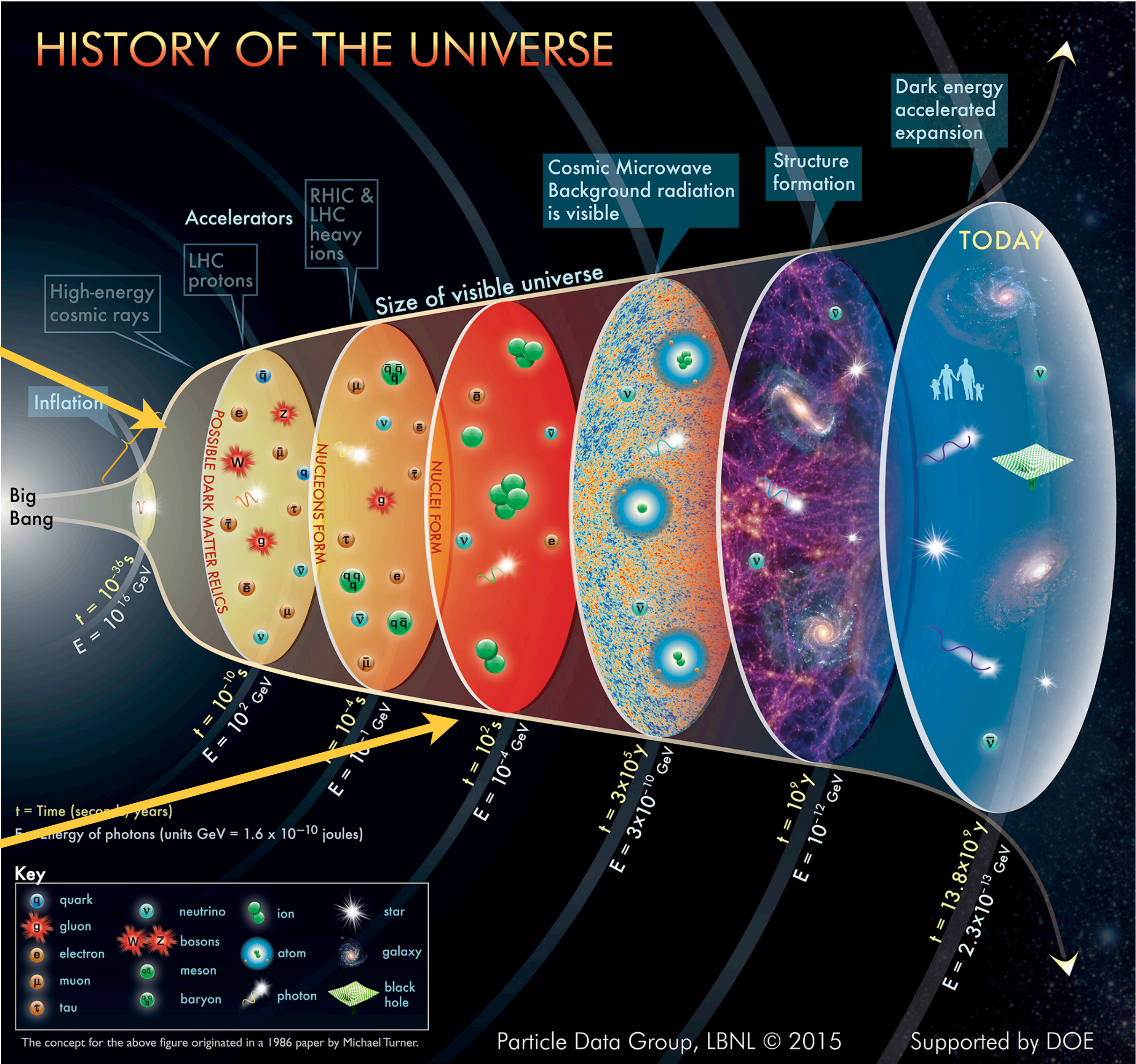
Credit: H. Murayama

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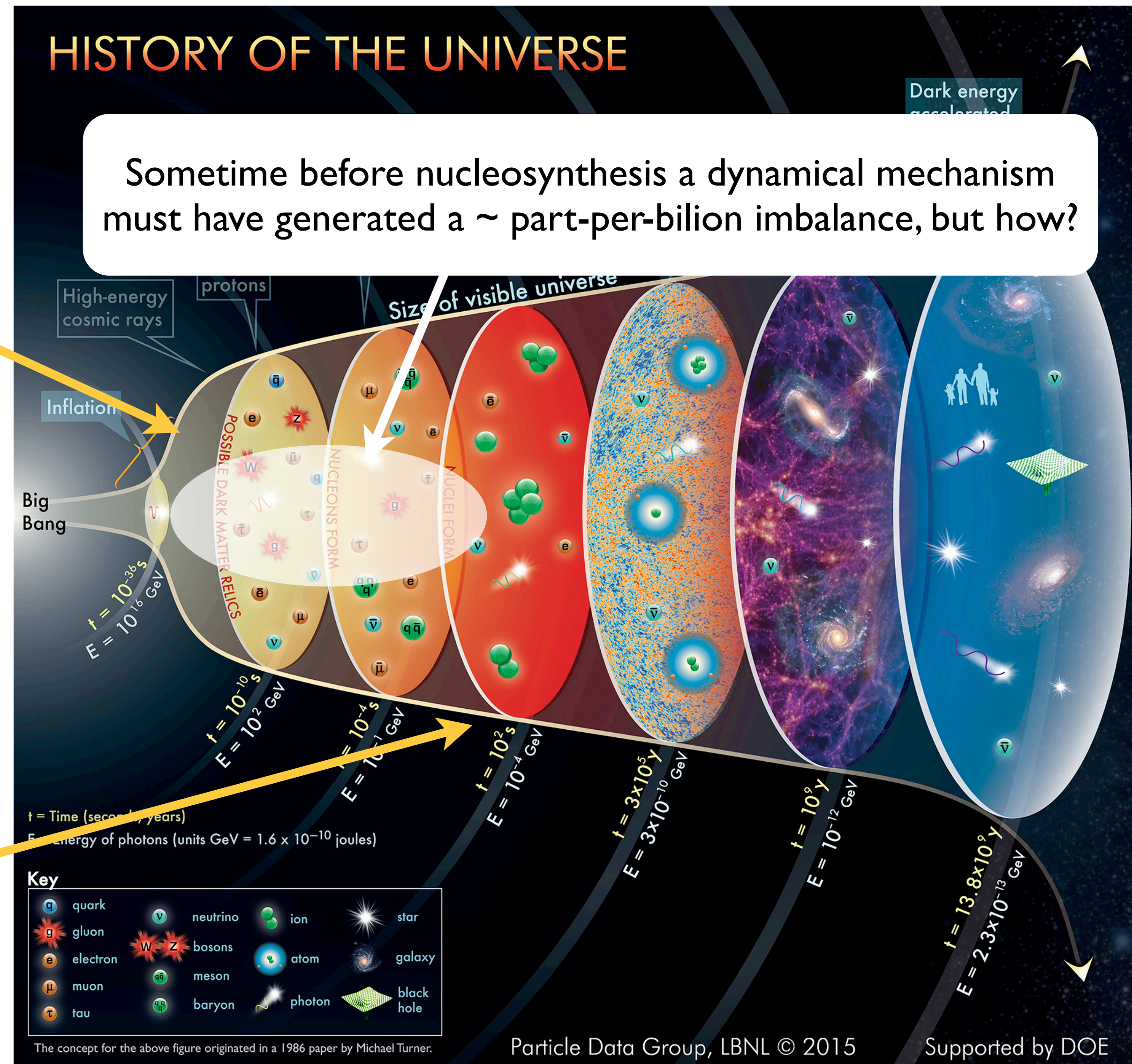
What's the origin of matter in the universe?

Equal number of particles and antiparticles right after the big bang

Today

Sometime before nucleosynthesis a dynamical mechanism must have generated a $\sim$ part-per-billion imbalance, but how?

To obtain $O(1)$ protons per cubic meter today, early on need a tiny imbalance of $+$ over $-$



Today

Stars,
galaxies,
us...

1

0

Matter

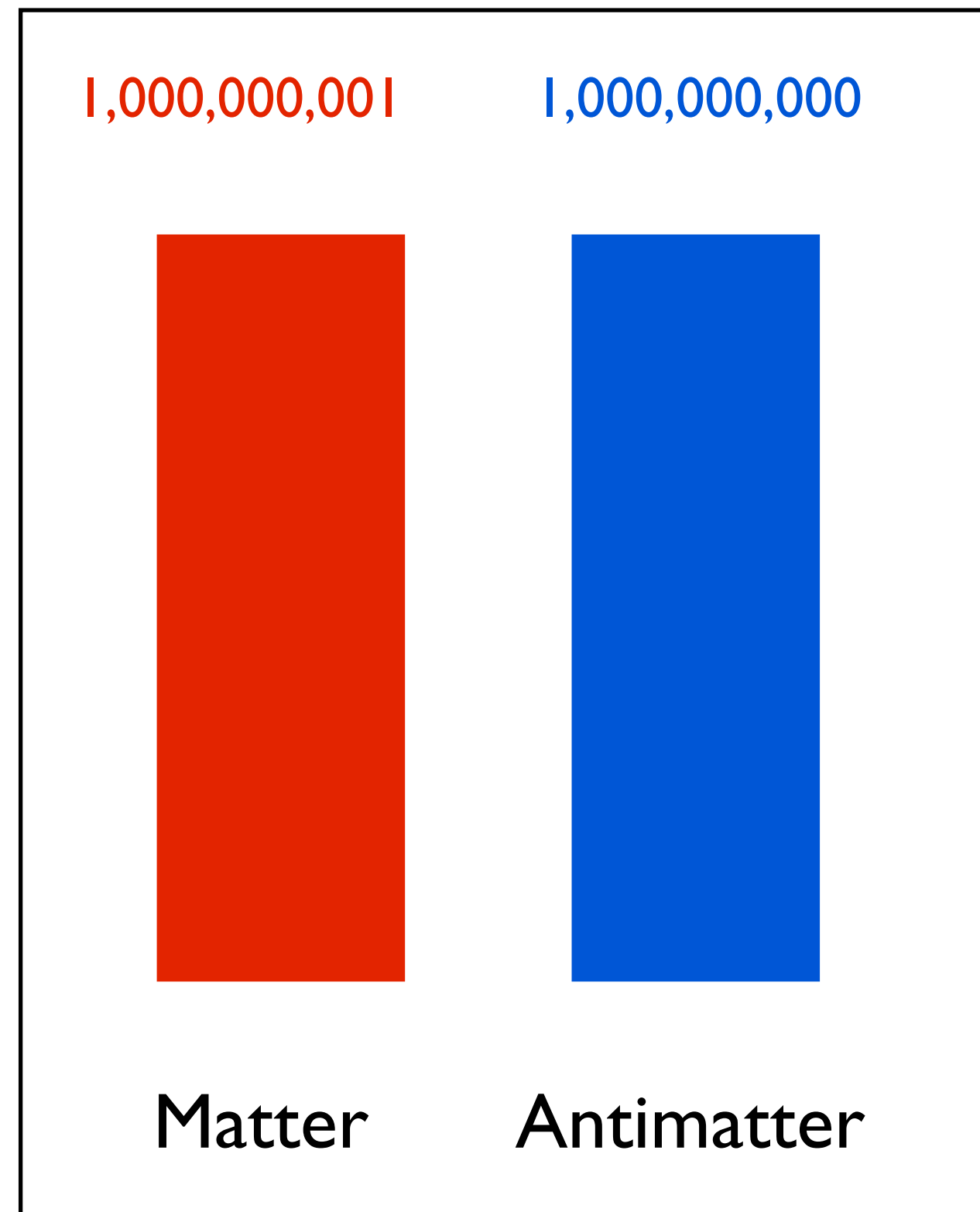
Antimatter

Credit: H. Murayama

Ingredients for a lopsided universe



Andrei Sakharov, 1967



Credit: H. Murayama

#1. Processes that “create matter” [B, L violation]

$$A \rightarrow B$$

*# of particles – # of antiparticles
is different in A and B*

#2. “Asymmetrically” (faster than corresponding antimatter-creating process) [$\mathcal{C}$, $\mathcal{CP}$]

$$A \rightarrow B$$

$\neq$

$$\bar{A} \rightarrow \bar{B}$$

#3. “Irreversibly” (faster than matter annihilating inverse process)

$$A \rightarrow B$$

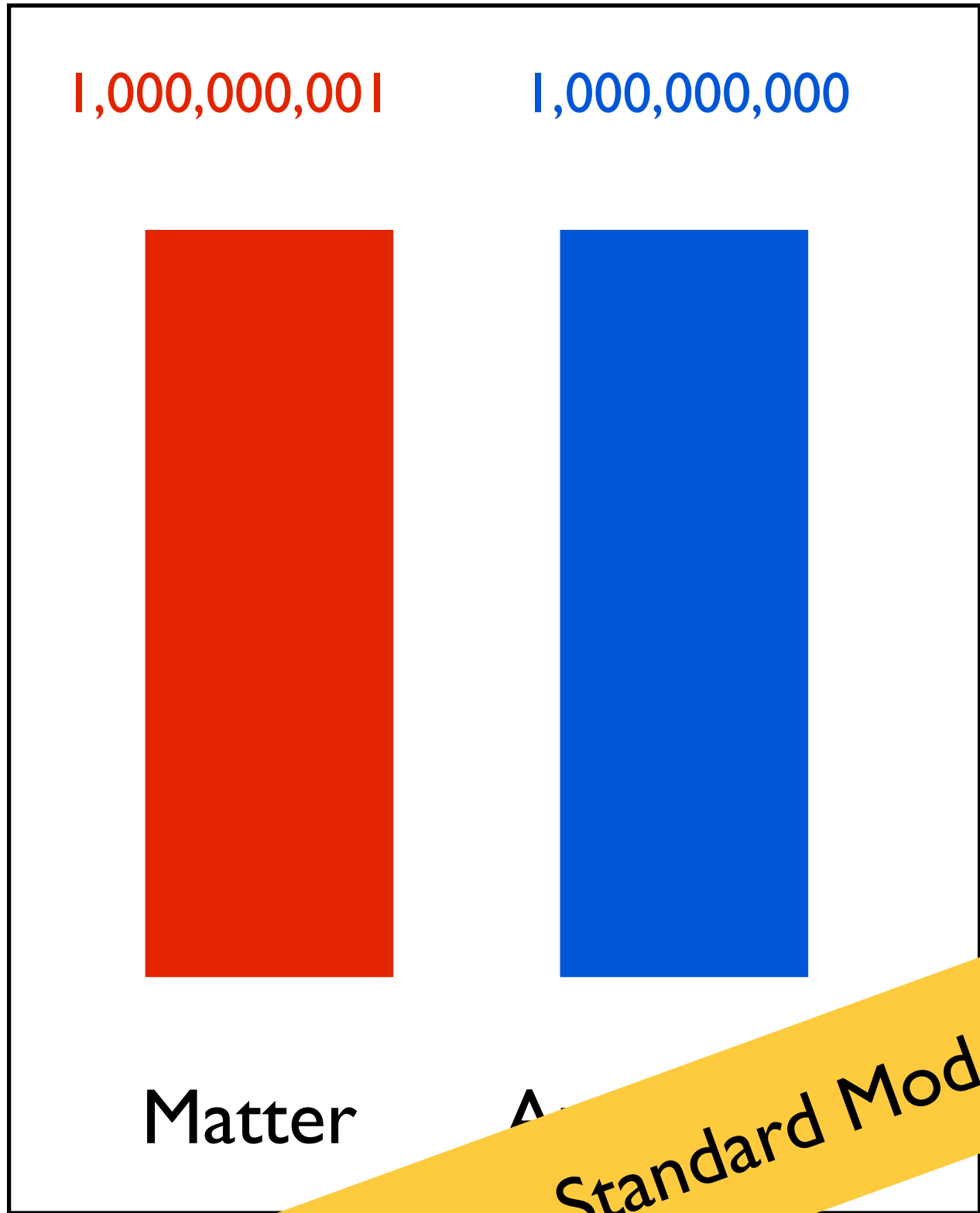
$\neq$

$$B \rightarrow A$$

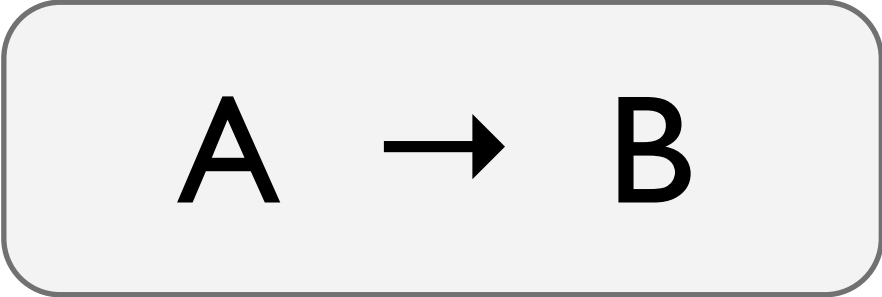
Ingredients for a lopsided universe



Andrei Sakharov, 1967



#1. Processes that “create matter” [B, L violation]

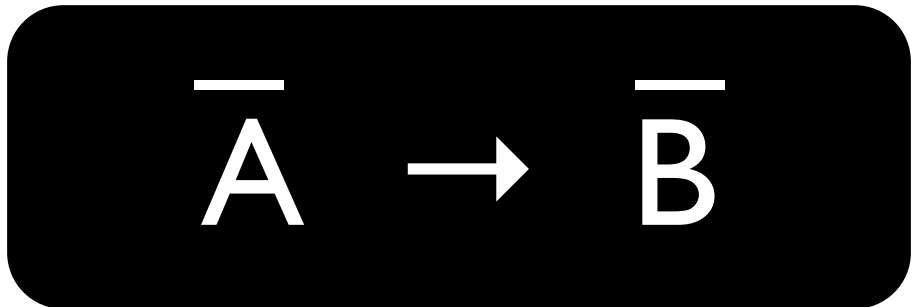


of particles – # of antiparticles is different

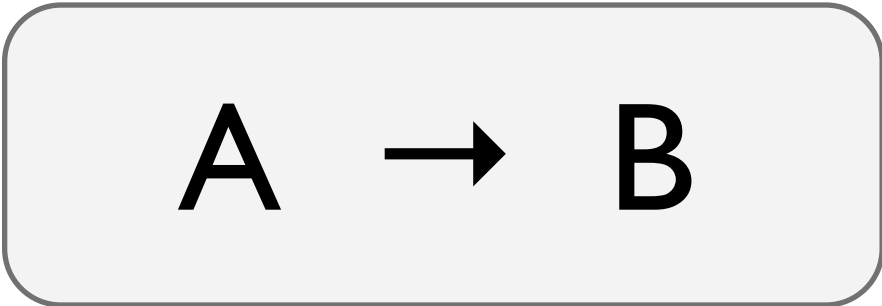
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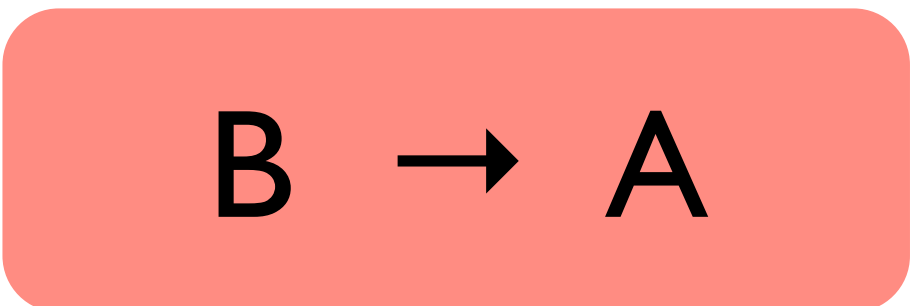
$\neq$



#3. “Irreversibly” (faster than matter annihilating inverse process)



$\neq$



The Standard Model doesn't have all the ingredients to generate the asymmetry: need new physics!

How does $0\nu\beta\beta$ decay help?



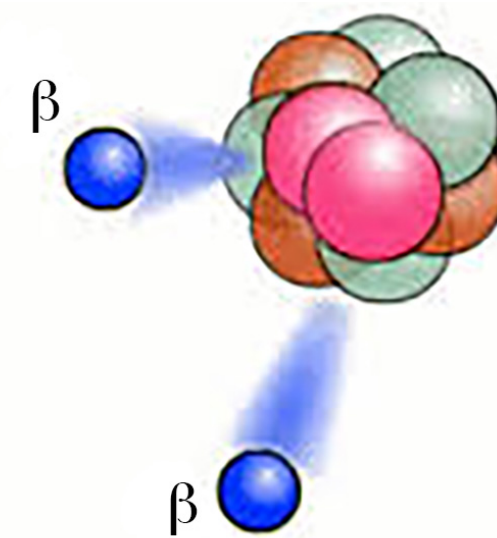
Andrei Sakharov, 1967

#1. Processes that “create matter”

$$A \rightarrow B$$

of particles – # of antiparticles
is different in A and B

$0\nu\beta\beta$ decay is a matter-creating process!



$$(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$$

Before: $N + Z$ nucleons, no antiparticles

After: $N + Z$ nucleons *plus two electrons*, no antiparticles

1,000,000,001

1,000,000,000



Matter

Antimatter

Credit: H. Murayama

How does $0\nu\beta\beta$ decay help?



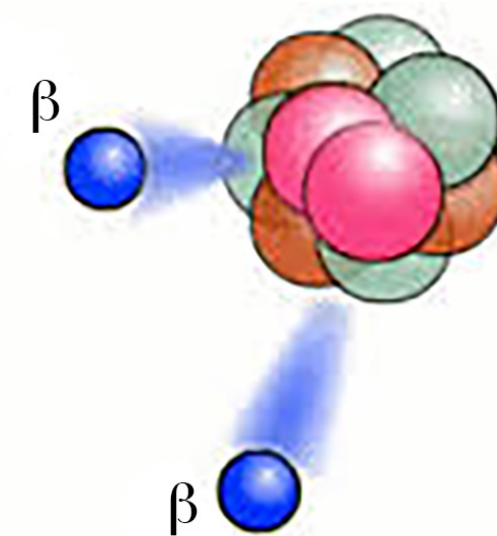
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This is deeply related to the Majorana nature:
neutrino = anti-neutrino

1,000,000,001

1,000,000,000



Matter



Antimatter

Credit: H. Murayama

How does $0\nu\beta\beta$ decay help?



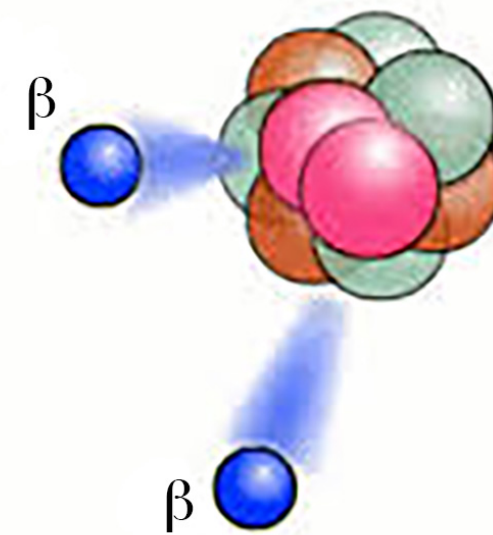
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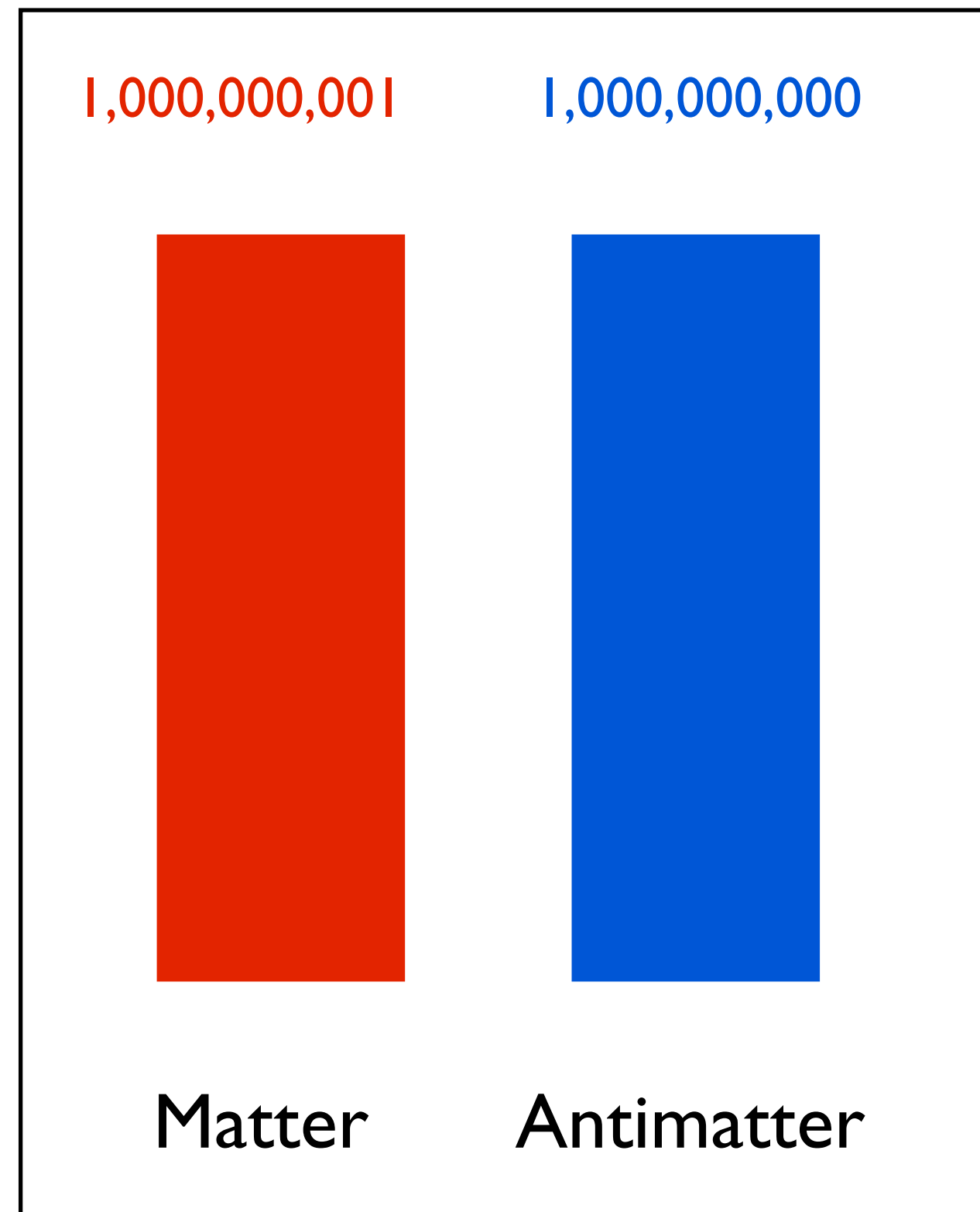
Credit: H. Murayama

But there's more! The same physics could be responsible for both $0\nu\beta\beta$ decay and for generating the matter excess in the universe through the *leptogenesis mechanism*

How does $0\nu\beta\beta$ decay help?



Andrei Sakharov, 1967



Credit: H. Murayama

- #1. Processes that “create matter”
 - #2. “Asymmetrically” (faster than corresponding antimatter-creating process)
 - #3. “Irreversibly” (faster than matter annihilating inverse process)
- $0\nu\beta\beta$ directly address first condition
 - **Explicit models** of Majorana neutrino mass satisfy the other two conditions, as well: **baryogenesis via leptogenesis**

Fukugita-Yanagida 1987

The simplest mass model

- Simple / natural option: add three R-handed neutrinos ν_{Ri} (gauge singlets = no interaction)

$$\mathcal{L}_{\nu SM} = \mathcal{L}_{SM} + i\bar{\nu}_R \not{\partial} \nu_R - \left(\frac{1}{2} \nu_R^T C M_R \nu_R + \bar{\ell} Y_\nu \nu_R \tilde{\varphi} + \text{h.c.} \right)$$

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- Dirac neutrinos: $M_R = 0$. Same as quarks & charged leptons, except for tiny ($O(10^{-10})$) Yukawa couplings

$$Y_e = V_{eL}^\dagger Y_e^{\text{diag}} V_{eR}$$

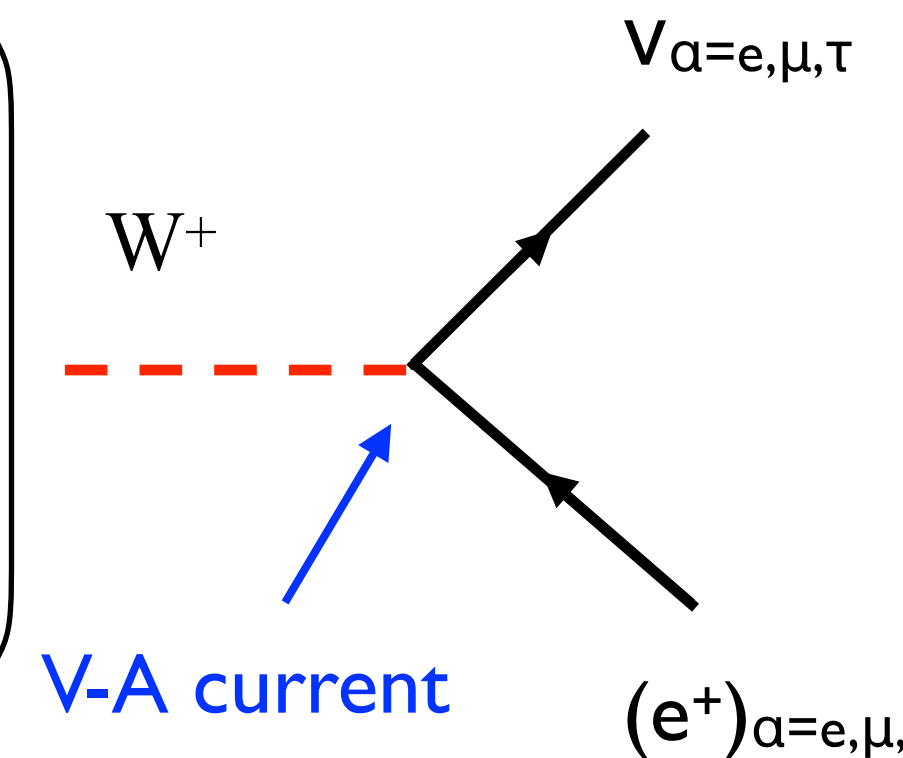
$$Y_\nu = V_{\nu L}^\dagger Y_\nu^{\text{diag}} V_{\nu R}$$

$\Rightarrow$

$$\frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L^\alpha \gamma^\mu U^{\alpha i} \nu_L^i$$

$$U = V_{eL} V_{\nu L}^\dagger$$

Unitary mixing in
Charged Current
vertex: 3 angles, 1 phase



The simplest mass model

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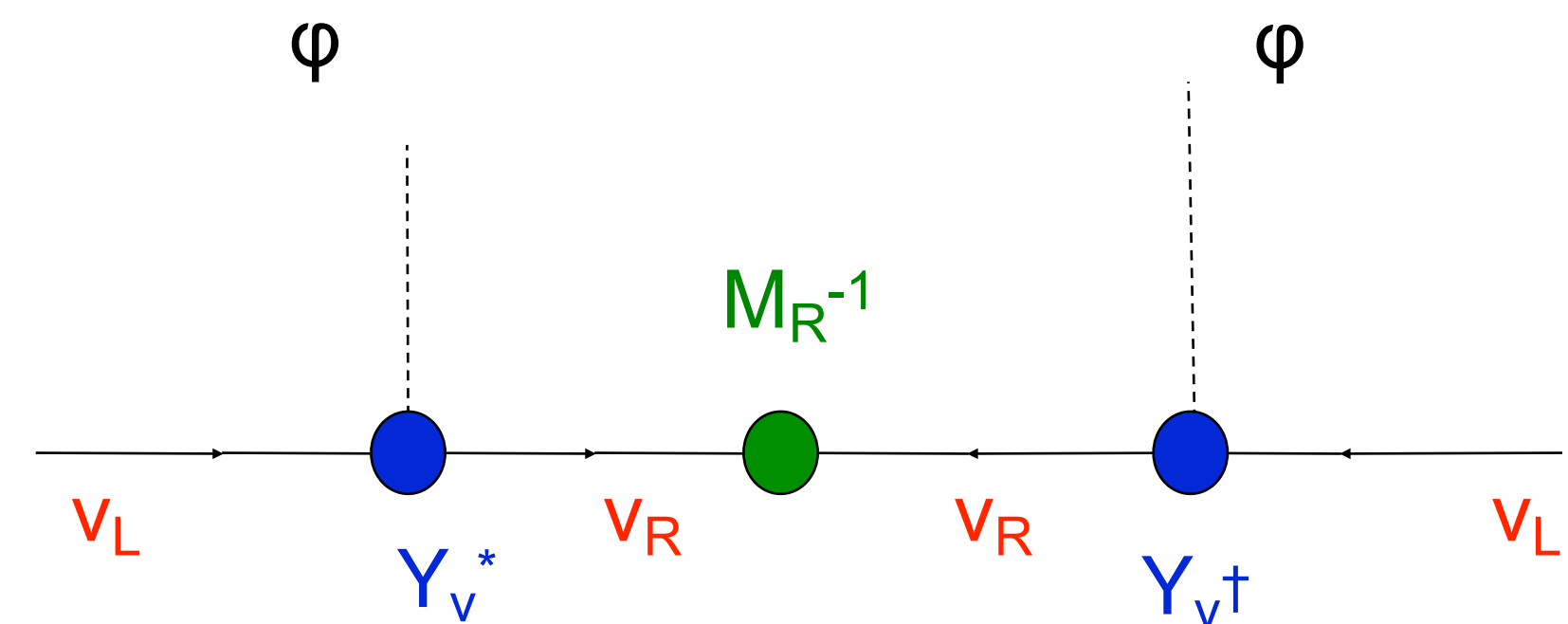
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- Majorana neutrinos: $M_R \neq 0 \Rightarrow$ L not conserved & 6x6 mass matrix for $\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$: six Majorana ($\nu = \nu^c$) eigenstates
- If $M_R \gg v Y_\nu$: 3 light ($\nu_L \rightarrow \nu_i$) and 3 heavy ($\nu_R \rightarrow N_i$) eigenstates

$$\mathcal{L}_{\nu SM} \supset -\frac{1}{2} \nu_L^T C m_\nu \nu_L$$

Seesaw
mechanism
(Type I)

$$m_\nu = v^2 Y_\nu^* M_R^{-1} Y_\nu^\dagger$$



The simplest mass model

- Simple / natural option: add three R-handed neutrinos ν_{Ri} (gauge singlets = no interaction)

$$\mathcal{L}_{\nu SM} = \mathcal{L}_{SM} + i\bar{\nu}_R \not{\partial} \nu_R - \left(\frac{1}{2} \nu_R^T C M_R \nu_R + \bar{\ell} Y_\nu \nu_R \tilde{\varphi} + \text{h.c.} \right)$$

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$$Y_e = V_{eL}^\dagger Y_e^{\text{diag}} V_{eR}$$

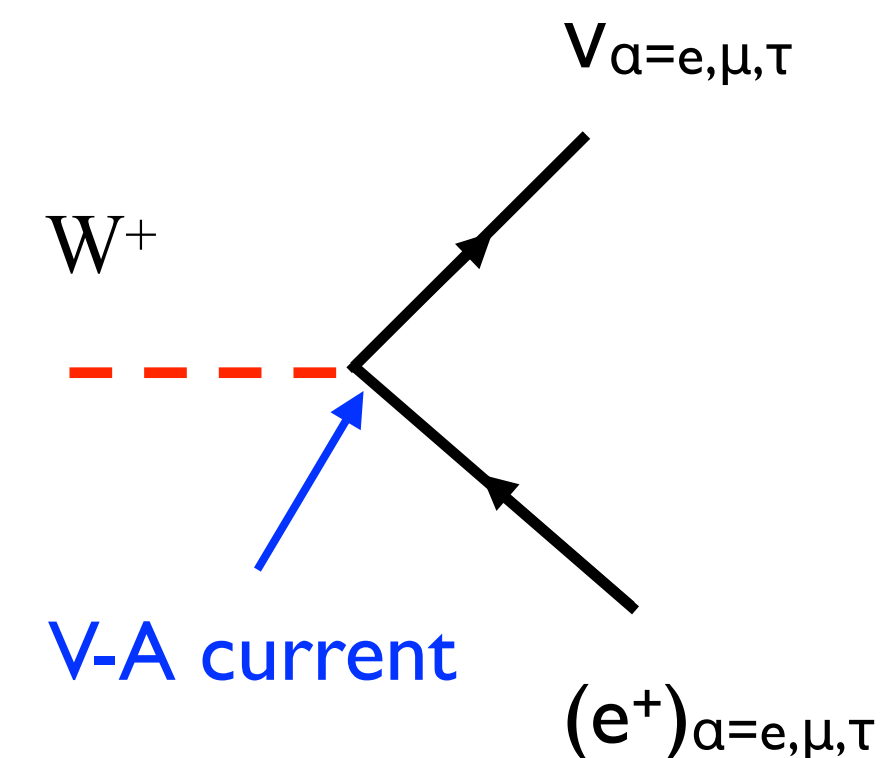
 $\Rightarrow$

$$\frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L^\alpha \gamma^\mu U^{\alpha i} \nu_L^i$$

$$U = V_{eL} V_{\nu L}^\dagger$$

Unitary mixing in CC vertex: 3 angles, **1+2** phases

$$U = U_{\text{Dirac}} \times \begin{pmatrix} 1 & & \\ & e^{i\alpha_1} & \\ & & e^{i\alpha_2} \end{pmatrix}$$

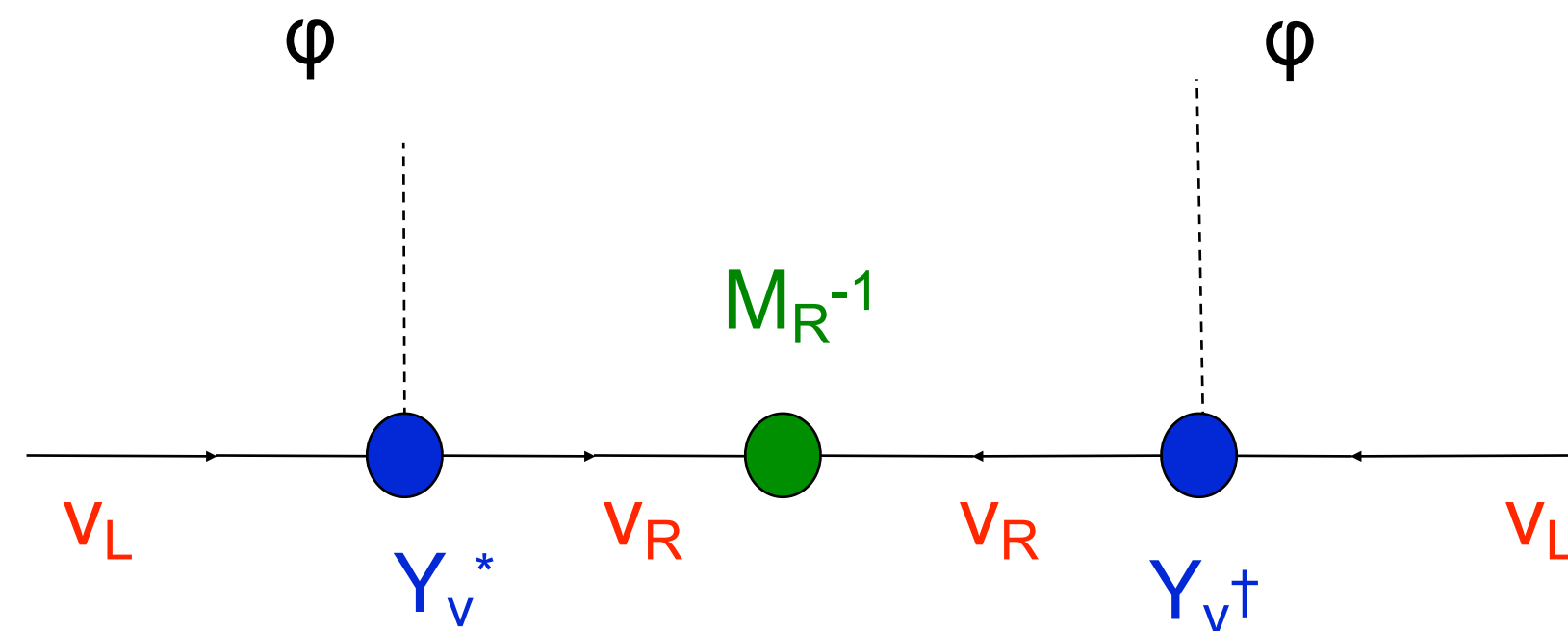


The simplest mass model

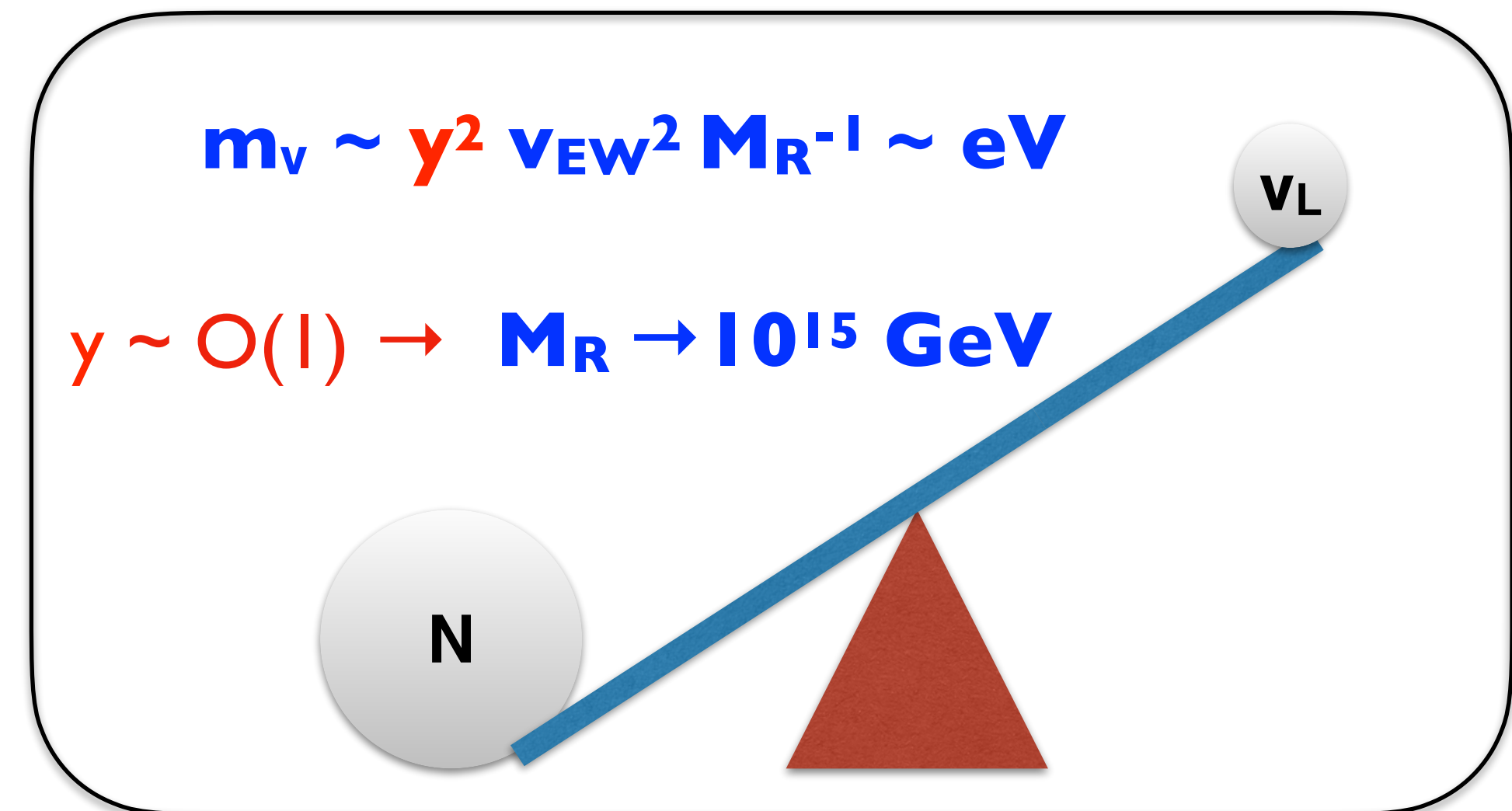
Seesaw mechanism summary:

Heavy (Majorana) singlets are introduced to give mass to the light neutrinos in a way consistent with Lorentz and weak interaction symmetries

They also provide an avenue to generate the baryon asymmetry...

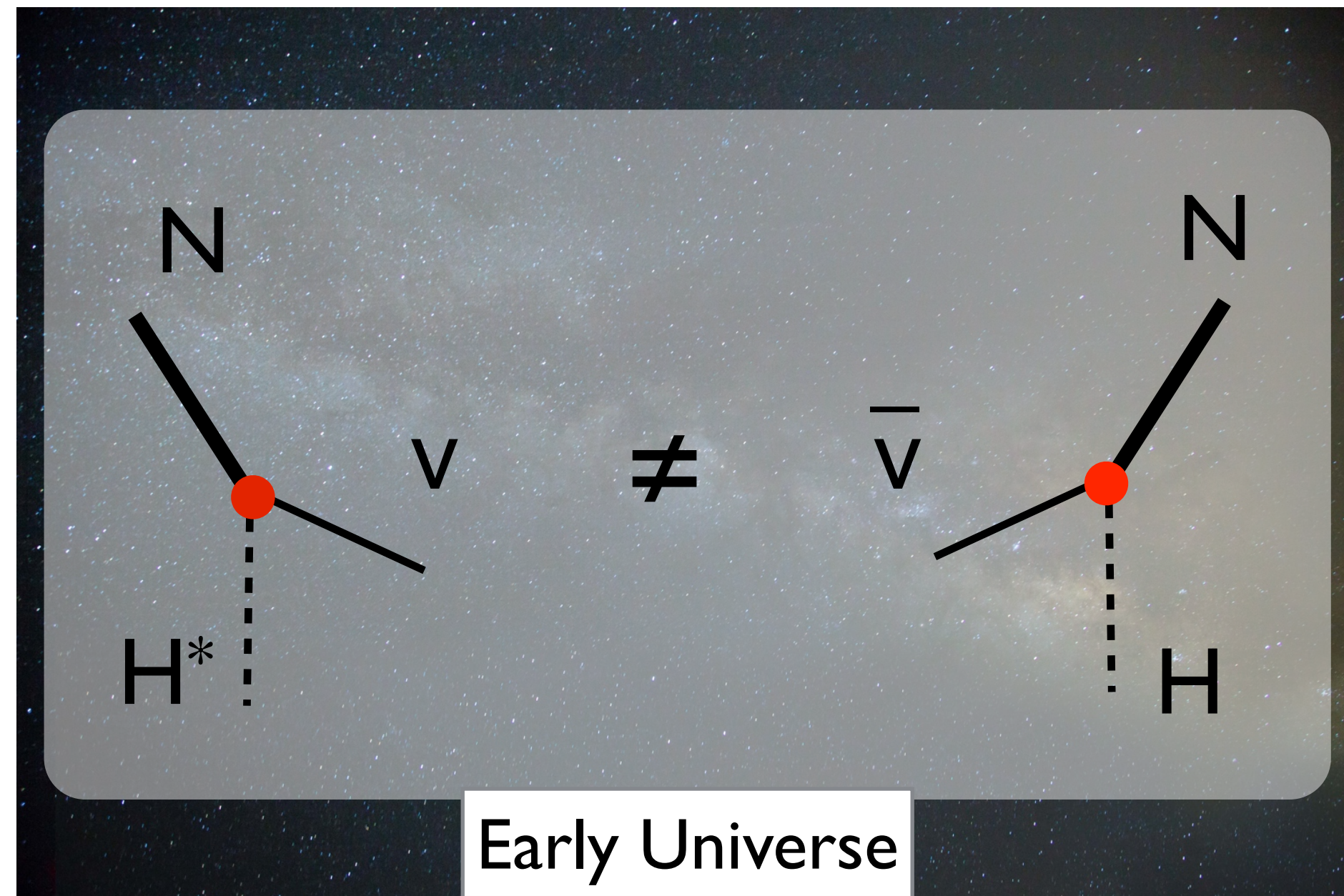


Minkowski 1977,
Gell-Mann, Ramond, Slansky 1979, ...



Leptogenesis and $0\nu\beta\beta$: a tantalizing connection

Fukugita-Yanagida 1987



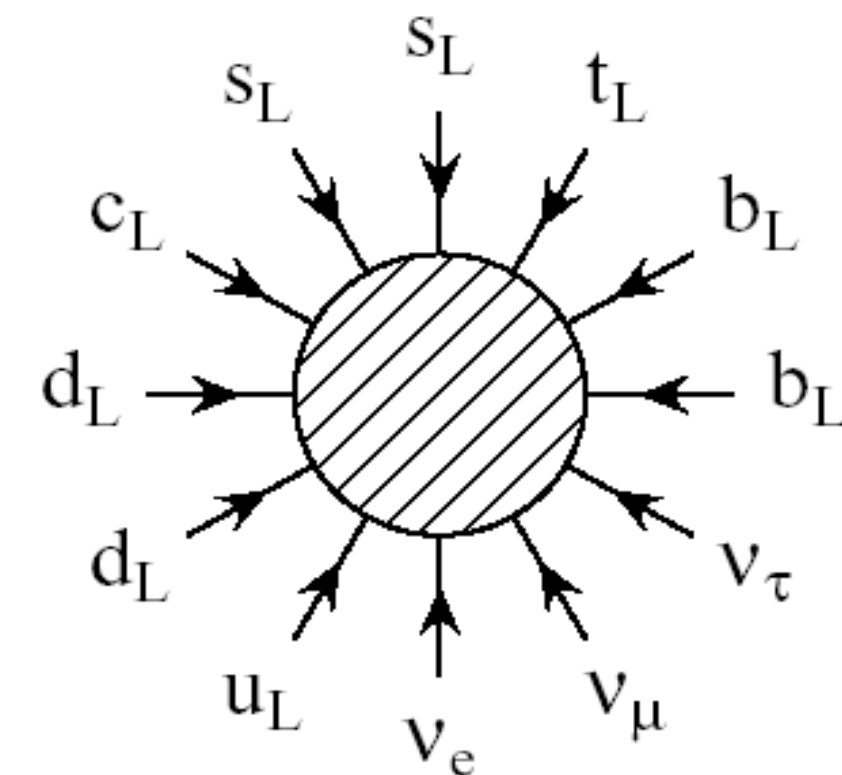
Heavy neutrinos (N) play key role in generating the matter-antimatter asymmetry by decaying into (anti)neutrinos and Higgs (H) particles '**asymmetrically**' and '**slowly**'

Baryogenesis via Leptogenesis

1) CP- and L- violating out-of-equilibrium decays of heavy N $\Rightarrow n_L$

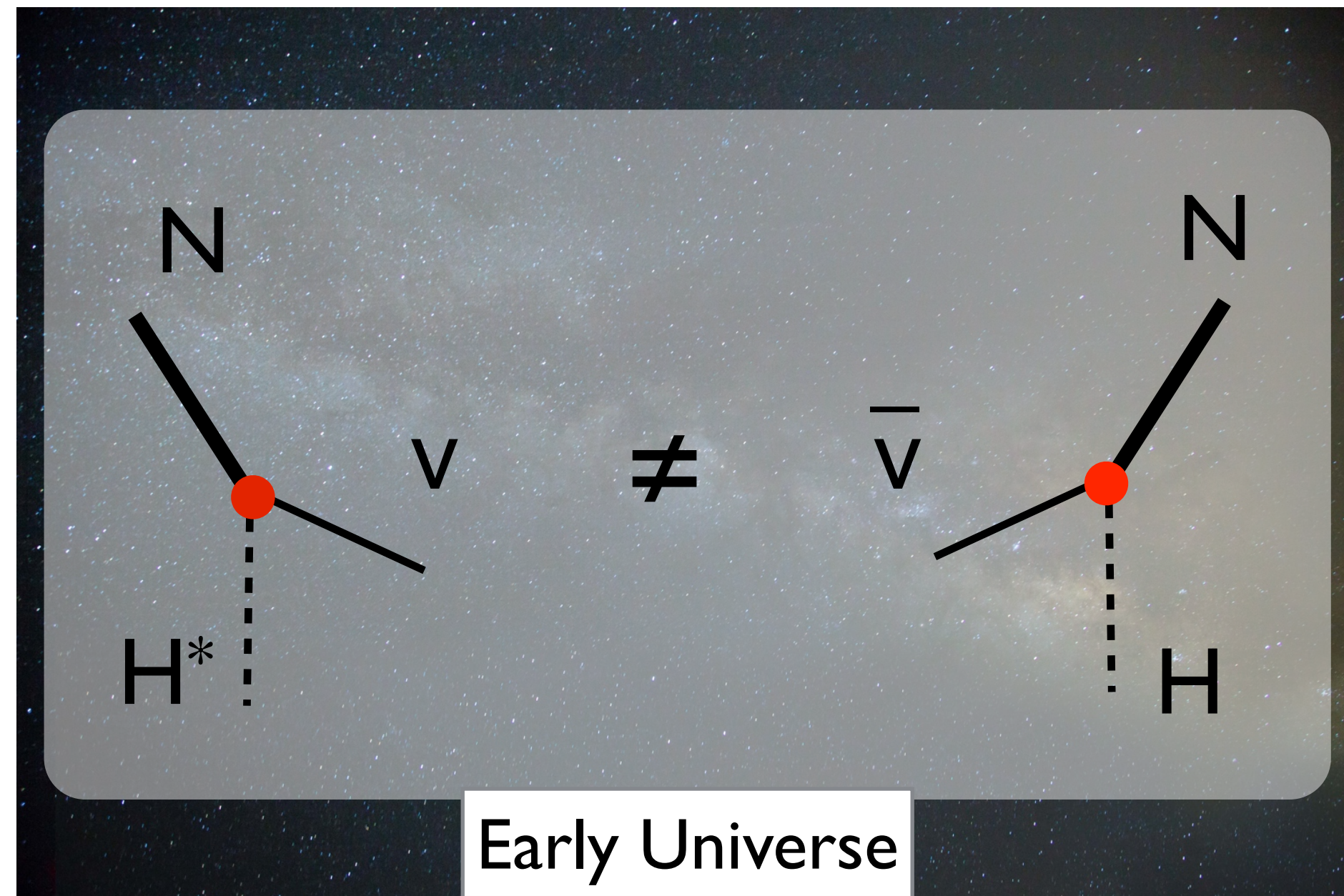
$$\Gamma(N \rightarrow H^* \ell) \neq \Gamma(N \rightarrow H \bar{\ell})$$

2) Electroweak sphalerons $\Rightarrow n_B = \# n_L$

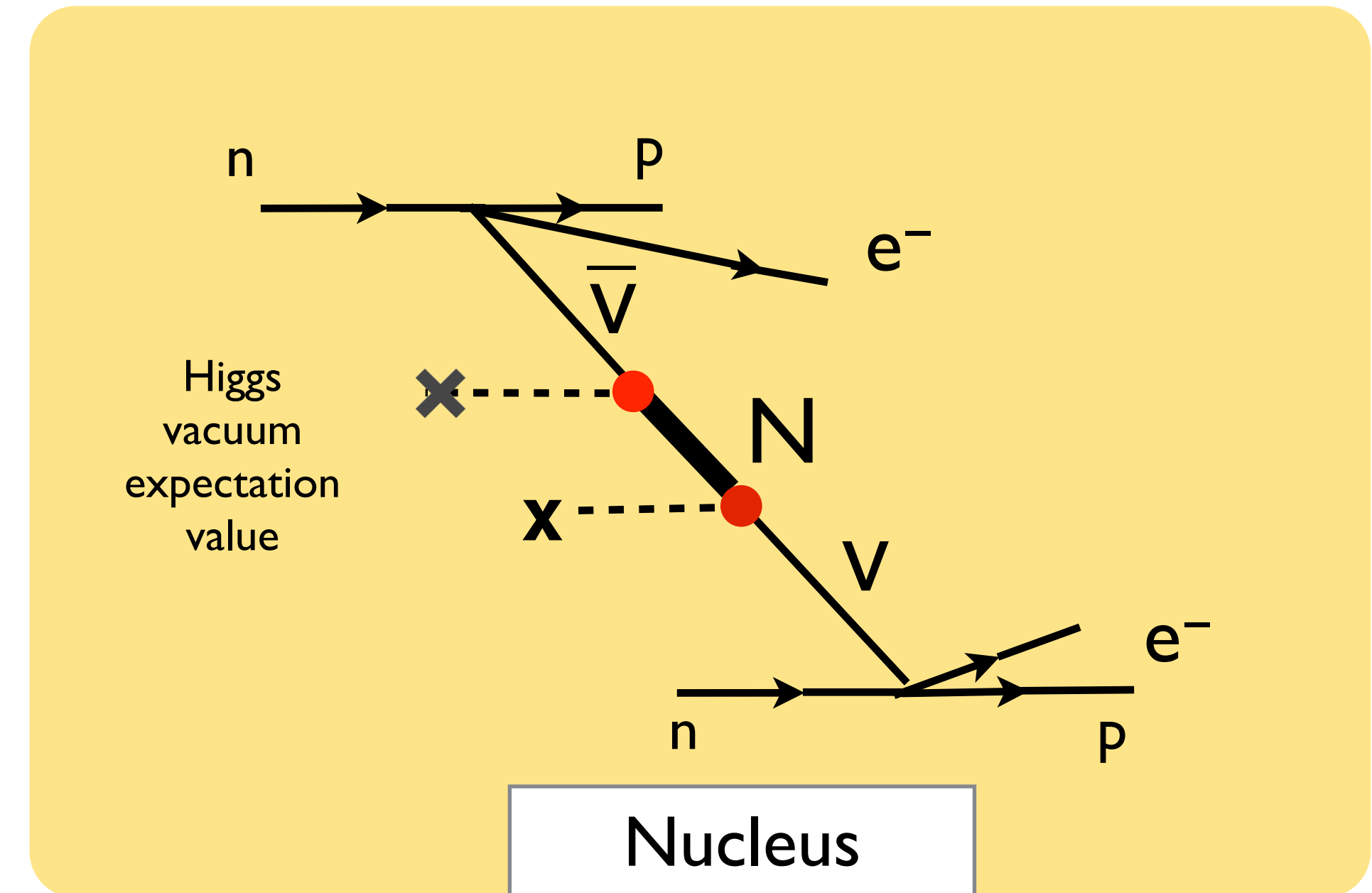


Leptogenesis and $0\nu\beta\beta$: a tantalizing connection

Fukugita-Yanagida 1987



Heavy neutrinos (N) play key role in generating the matter-antimatter asymmetry by decaying into (anti)neutrinos and Higgs (H) particles 'asymmetrically' and 'slowly'



In $0\nu\beta\beta$ decay, through the lens of Quantum Mechanics, we probe within a nucleus the same interactions that operated in the early universe**

** An anti-neutrino scatters off the Higgs field vacuum expectation value (VEV) and becomes N , then N scatters off the Higgs VEV and becomes a neutrino

$0\nu\beta\beta$ decay: summary of significance

The neutrino and its mysteries

Demonstrate Majorana nature of massive neutrinos
(neutrino=antineutrino)

A cosmic mystery

Demonstrate that an excess of matter over antimatter can be created in an elementary process
Point to baryogenesis via leptogenesis

Nuclear $0\nu\beta\beta$ decay

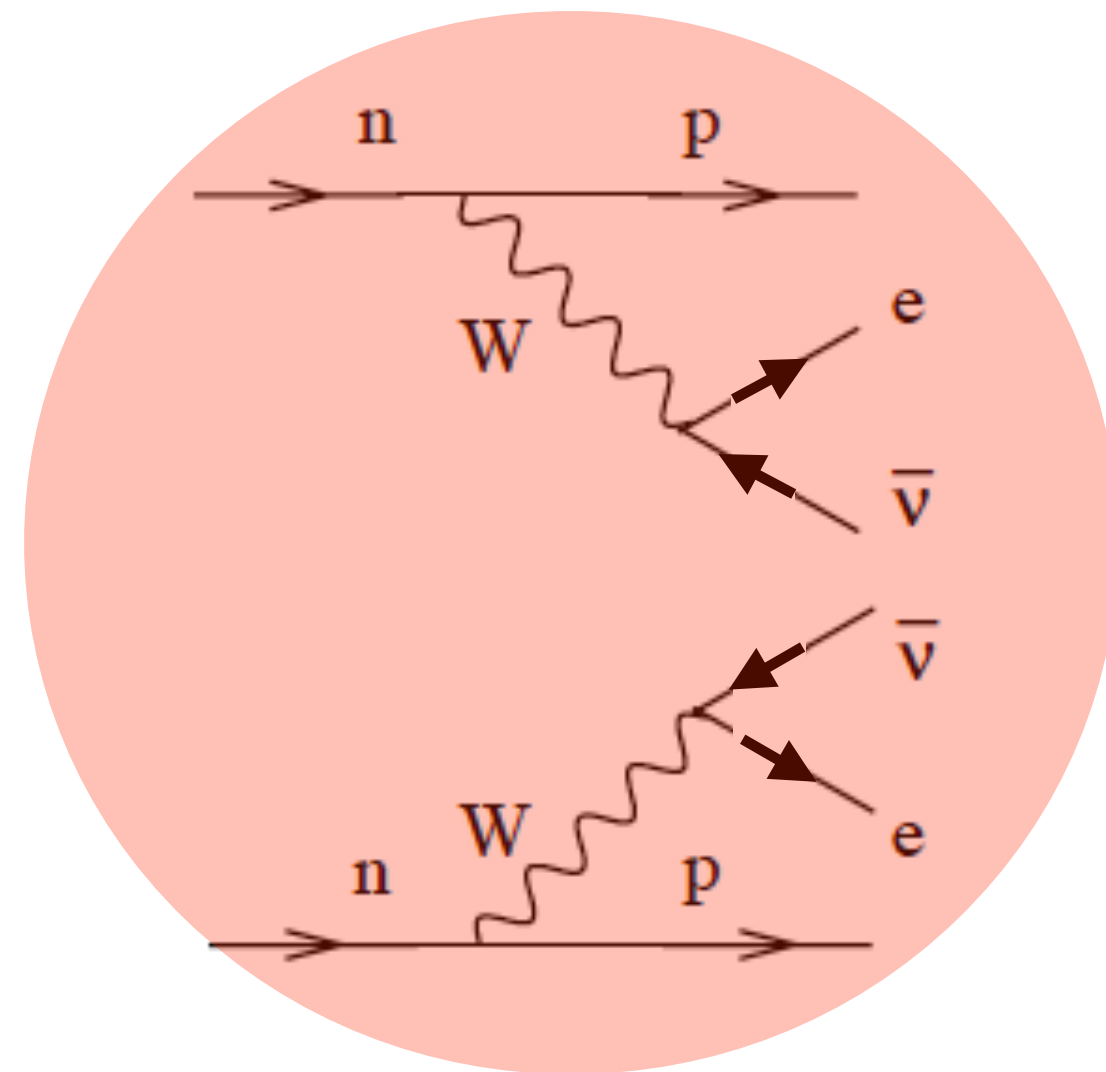
A 'matter-creating' nuclear process whose observation would have far reaching implications

The quest is on...

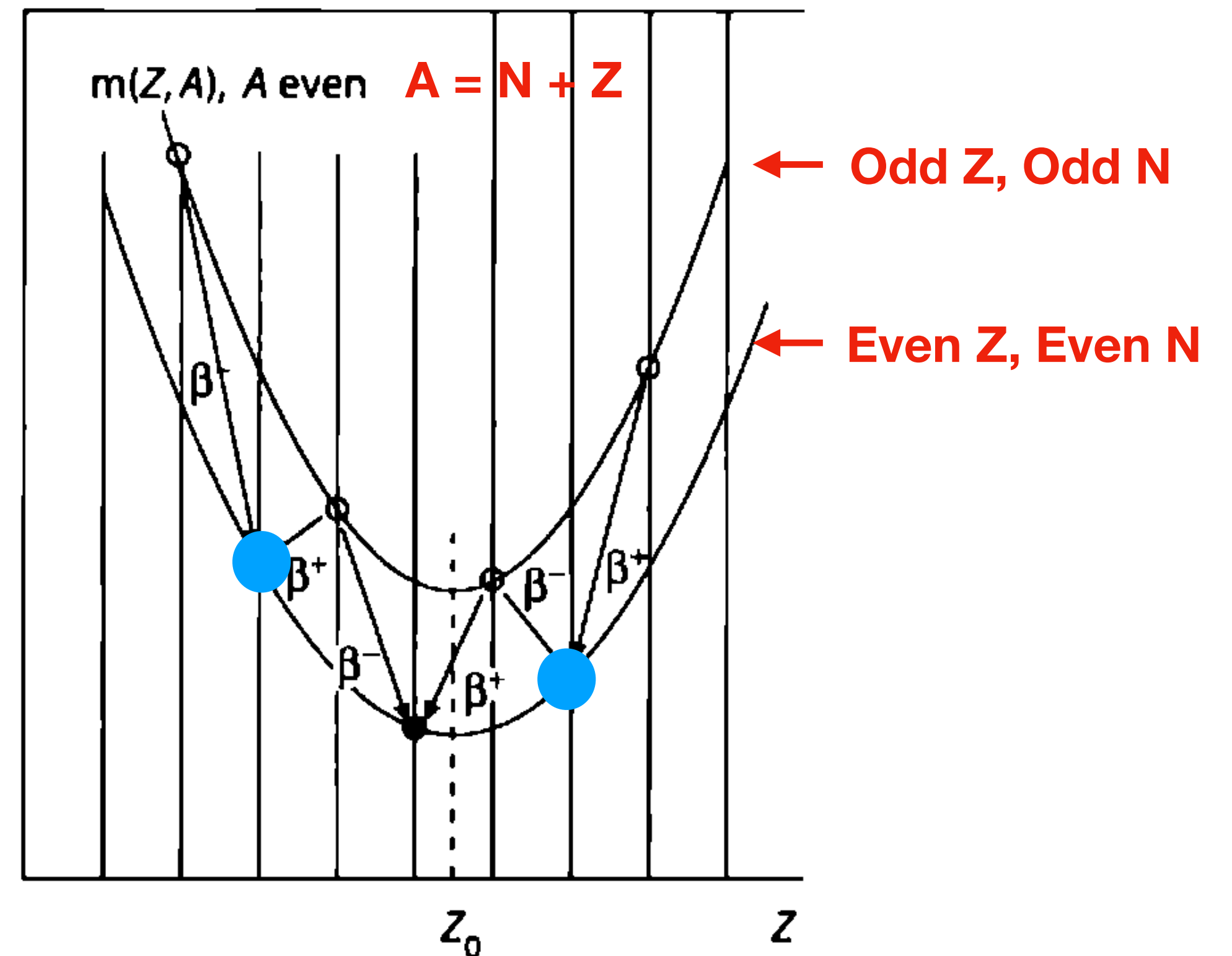
- For certain even-even nuclei (^{48}Ca , ^{76}Ge , ^{136}Xe , ...), single β decay is energetically forbidden $\rightarrow \beta\beta$ decay
- $2\nu\beta\beta$ is the rarest process ever observed, with $T_{1/2} \sim 10^{21}$ years



M. Goppert
Mayer, 1935

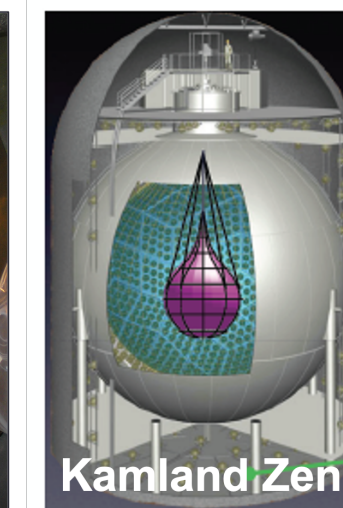
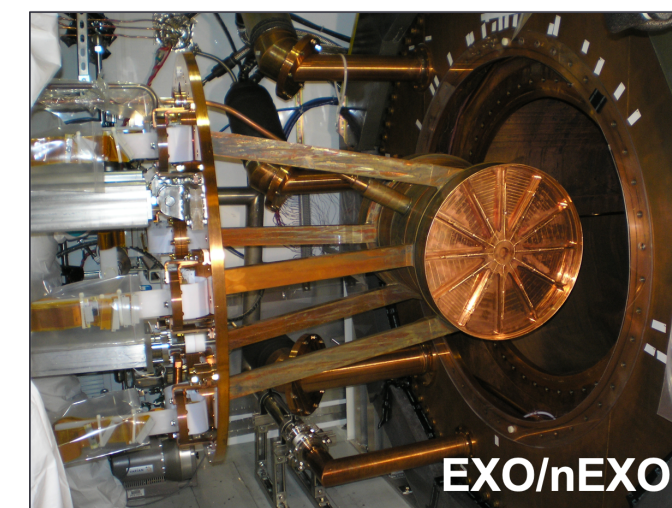
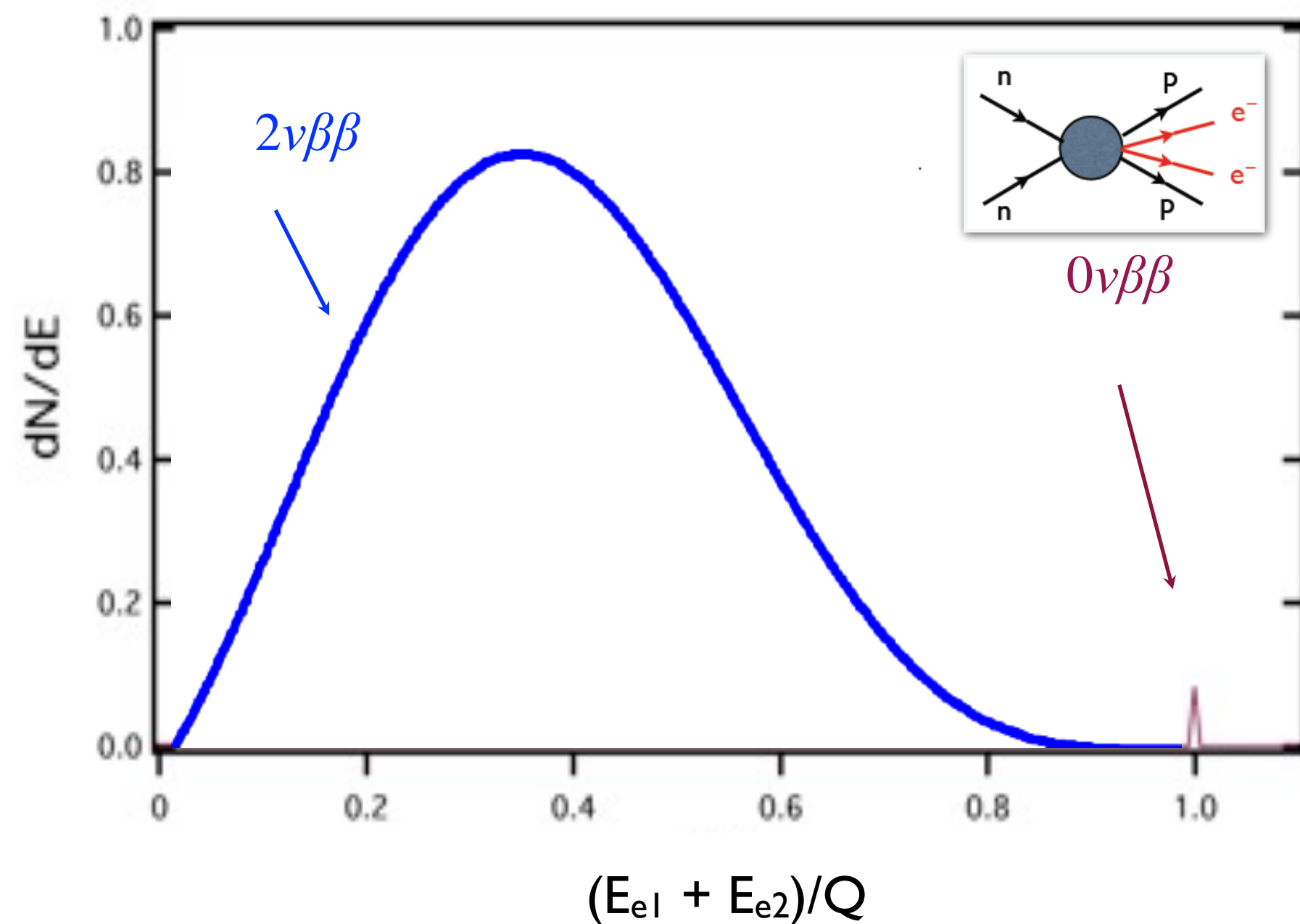


Credit: C. Bertulani's book



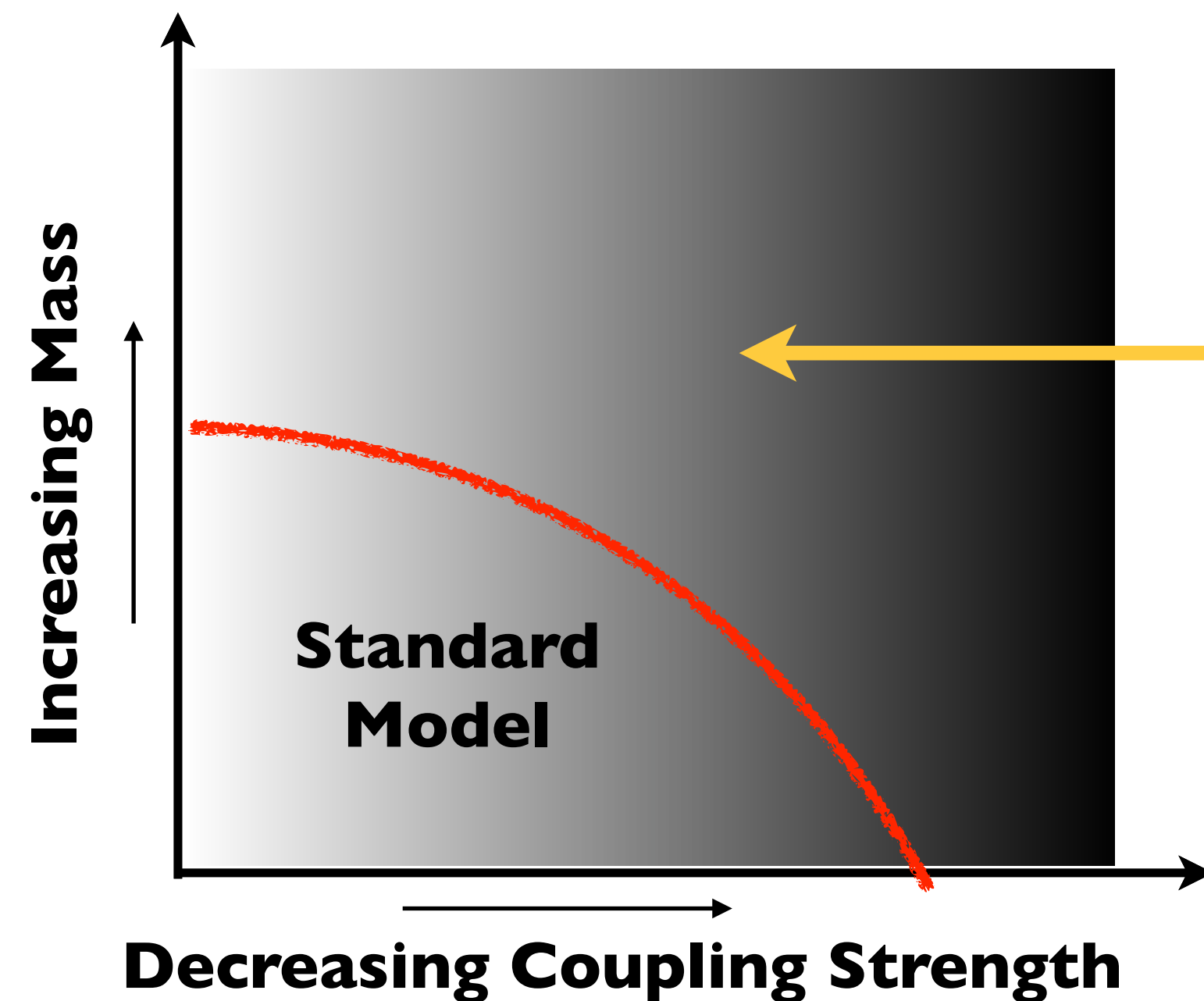
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- For certain even-even nuclei (^{48}Ca , ^{76}Ge , ^{136}Xe , ...), single β decay is energetically forbidden $\rightarrow \beta\beta$ decay
- $2\nu\beta\beta$ is the rarest process ever observed, with $T_{1/2} \sim 10^{21}$ years
- Several “ton-scale” experiments with different isotopes and technologies are searching for $0\nu\beta\beta$, with sensitivity up to $T_{1/2} \sim 10^{28}$ yr, which is 10^{18} times the age of the universe!



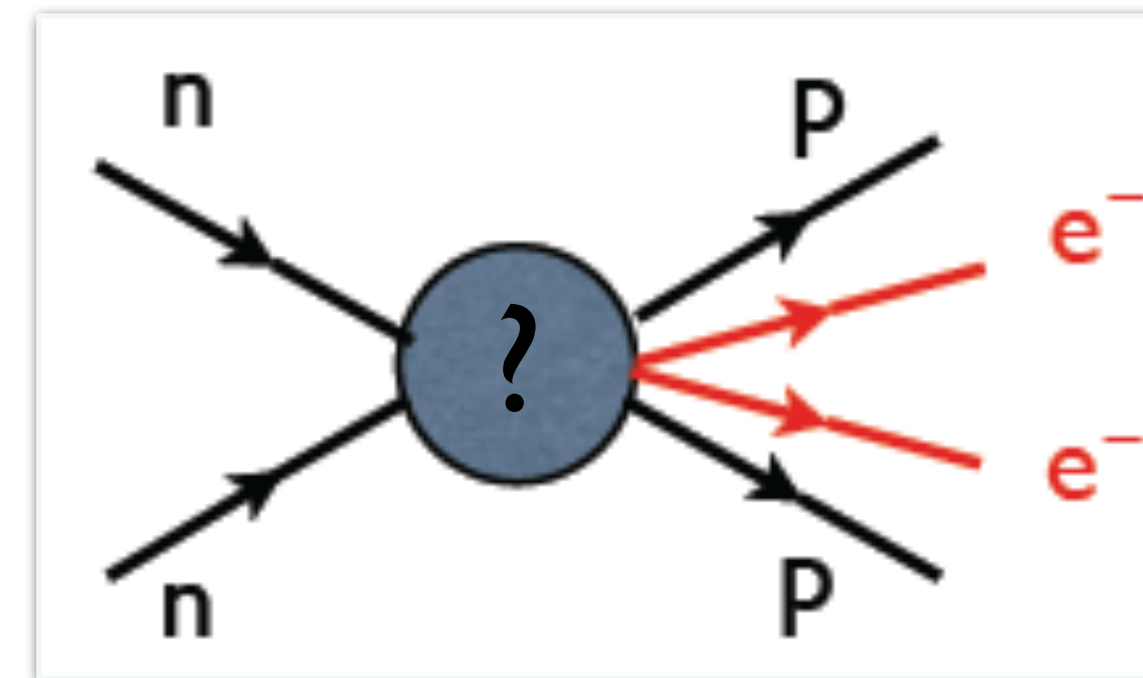
$0\nu\beta\beta$ decay: broad discovery potential

- Ton-scale $0\nu\beta\beta$ searches can discover Lepton Number Violation from a broad variety of mechanisms that involve different mass scales and interaction strengths



Somewhere out here there must be new physics responsible for neutrino masses

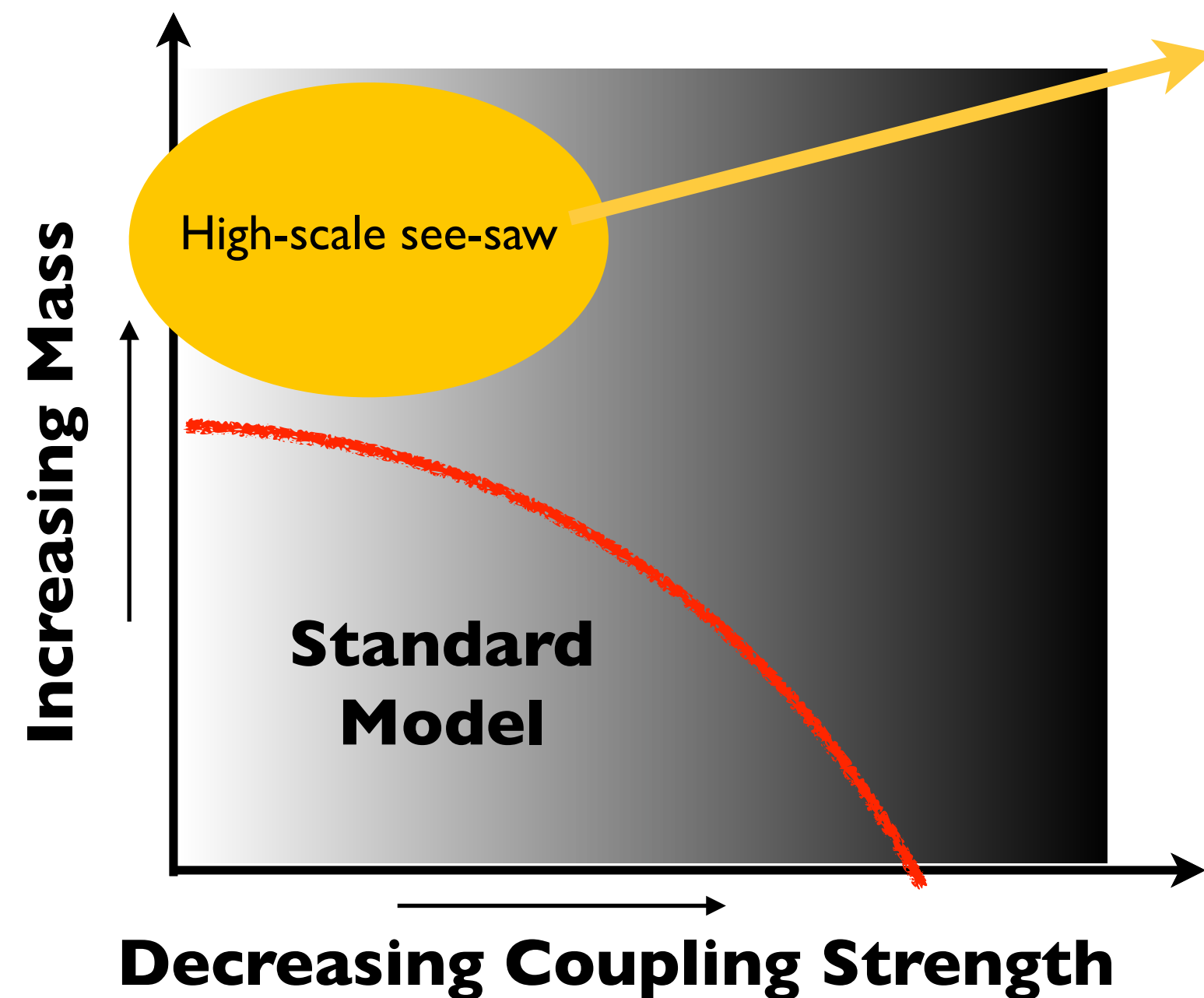
If Lepton Number is not conserved most of this uncharted territory can be explored only by $0\nu\beta\beta$ decay



Backup

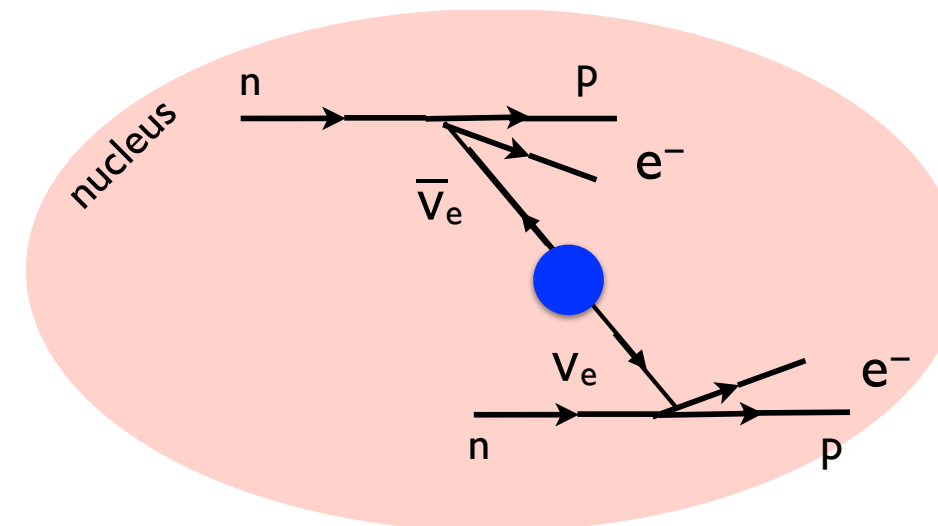
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LNV up to 10^{15} GeV scale:

$$m_{\beta\beta} \sim (v_{ew})^2 / \Lambda$$

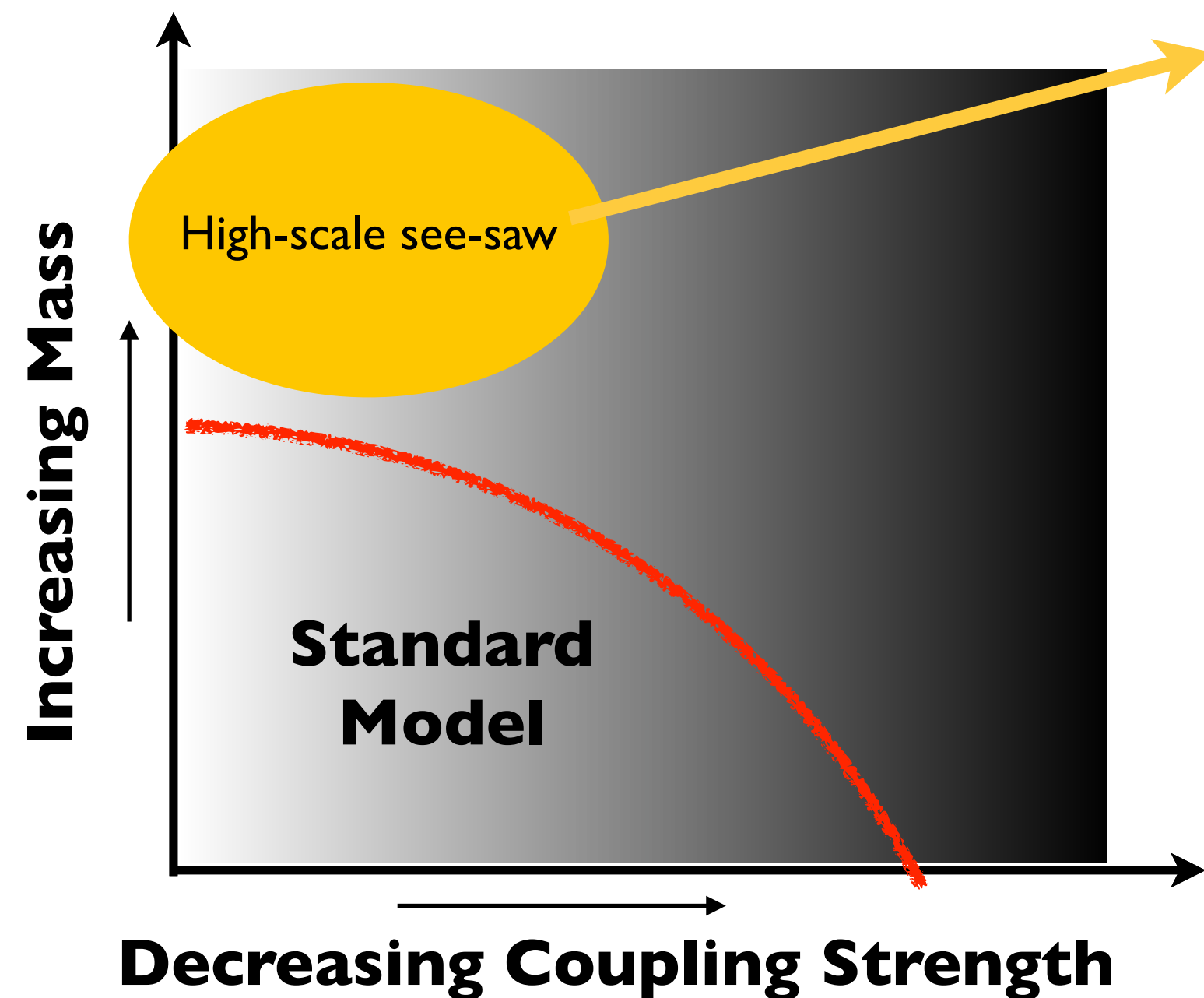


$$\langle m_{\beta\beta} \rangle^2 = \left| \sum U_{ei}^2 m_{\nu i} \right|^2$$

Half-life is related to neutrino mass: concrete discovery targets & falsifiable correlations with other probes of m_ν

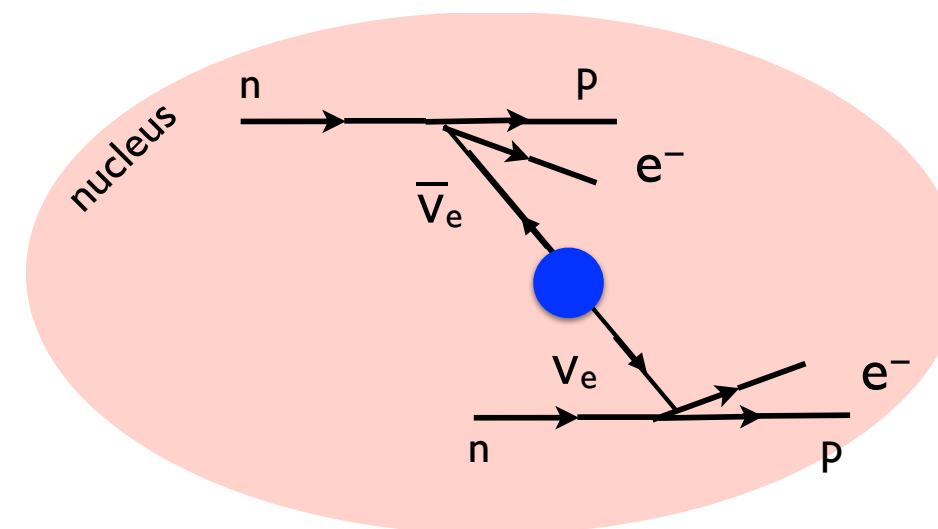
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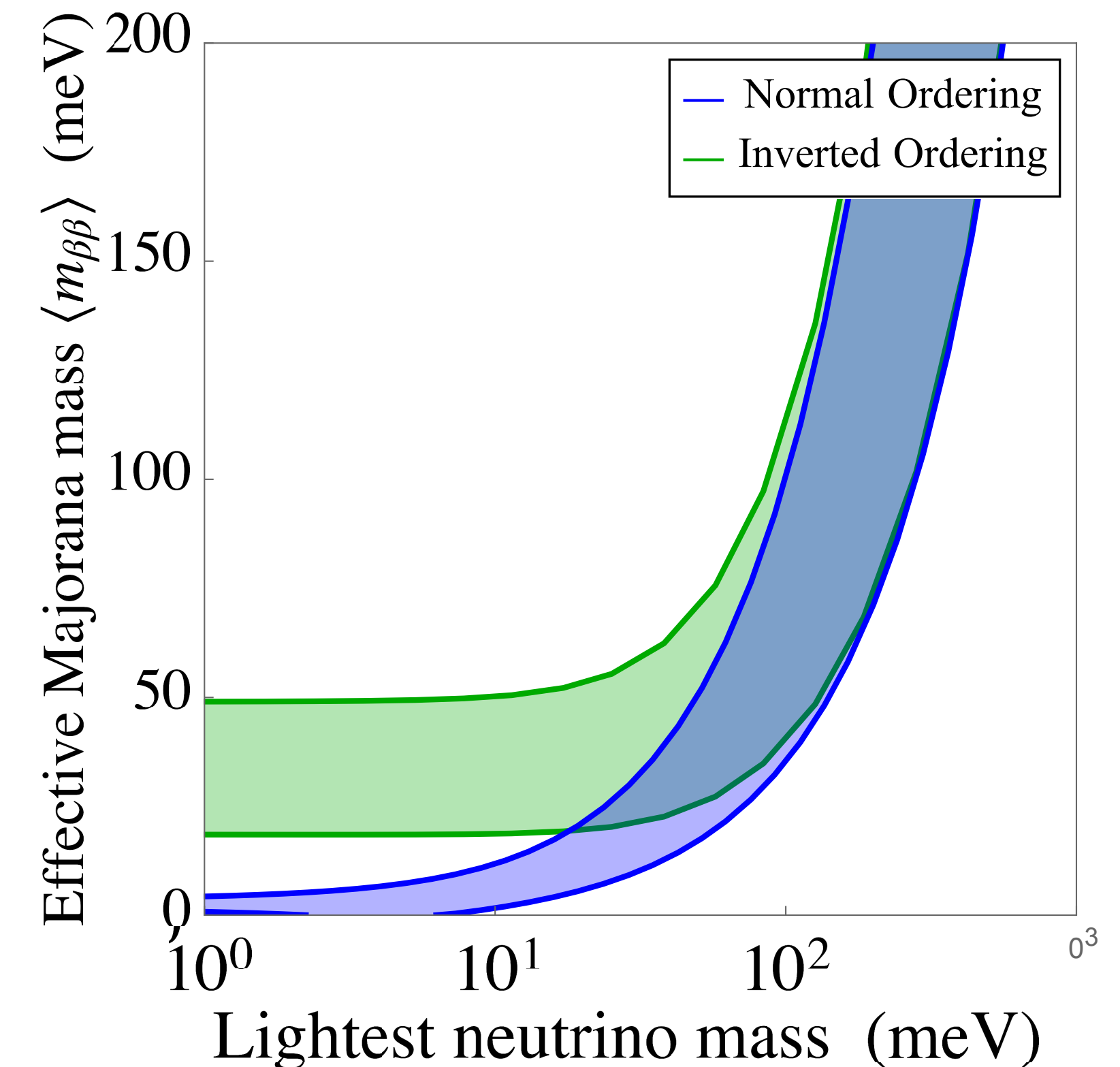


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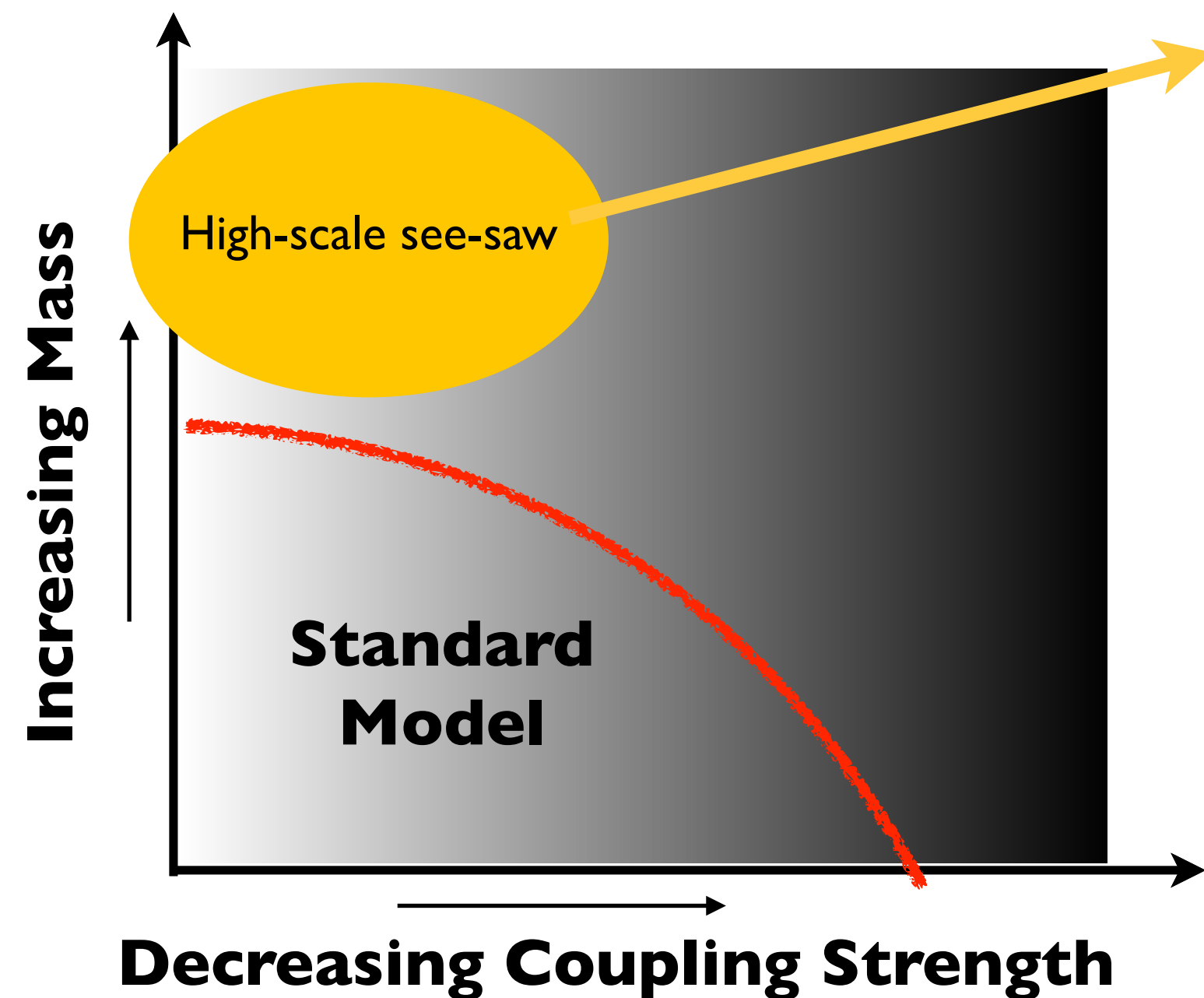
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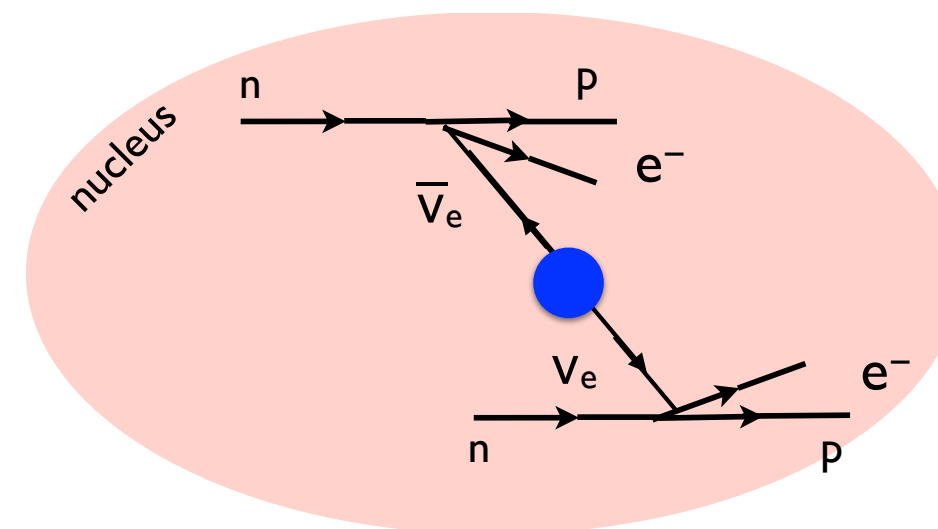
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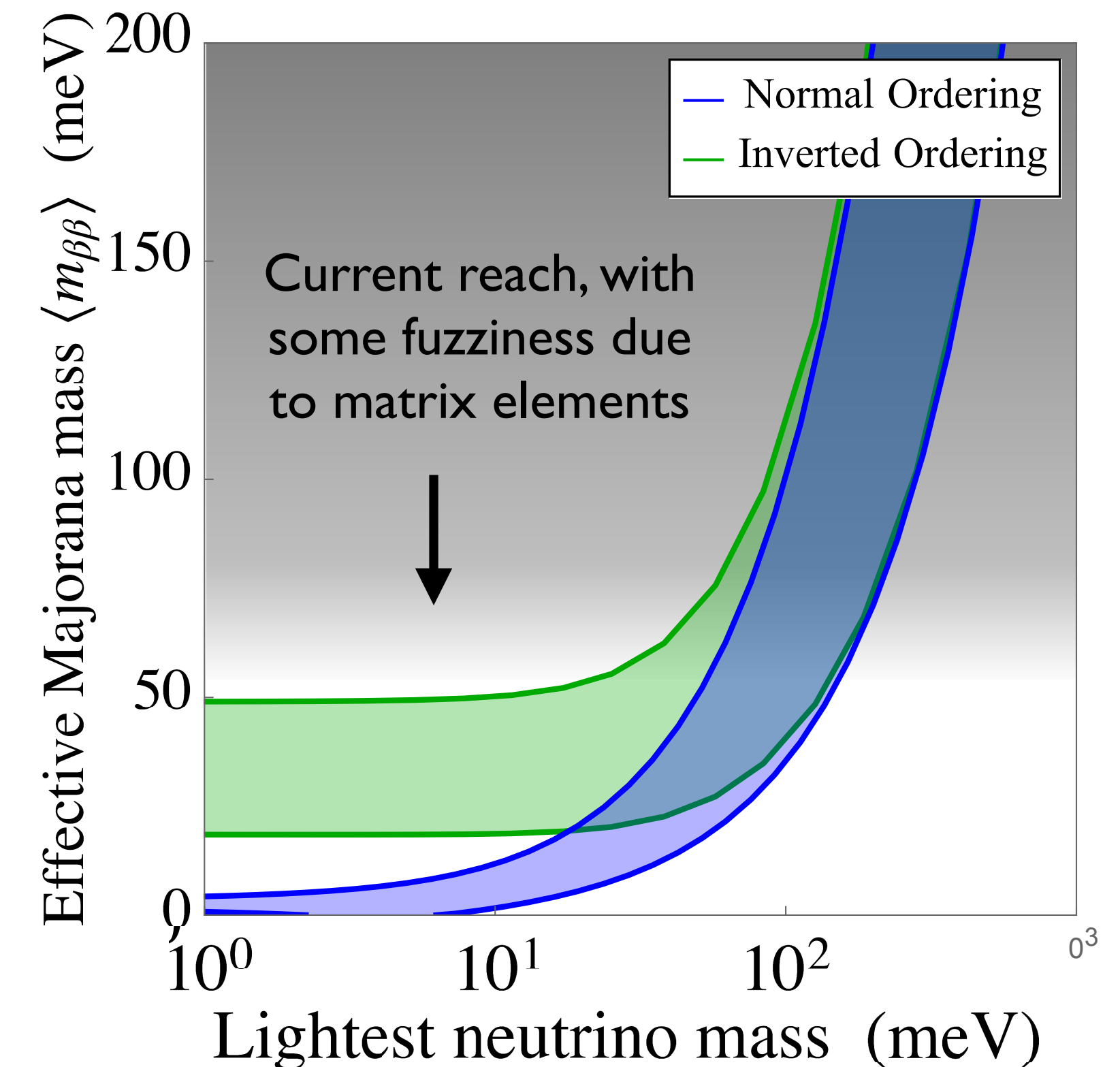


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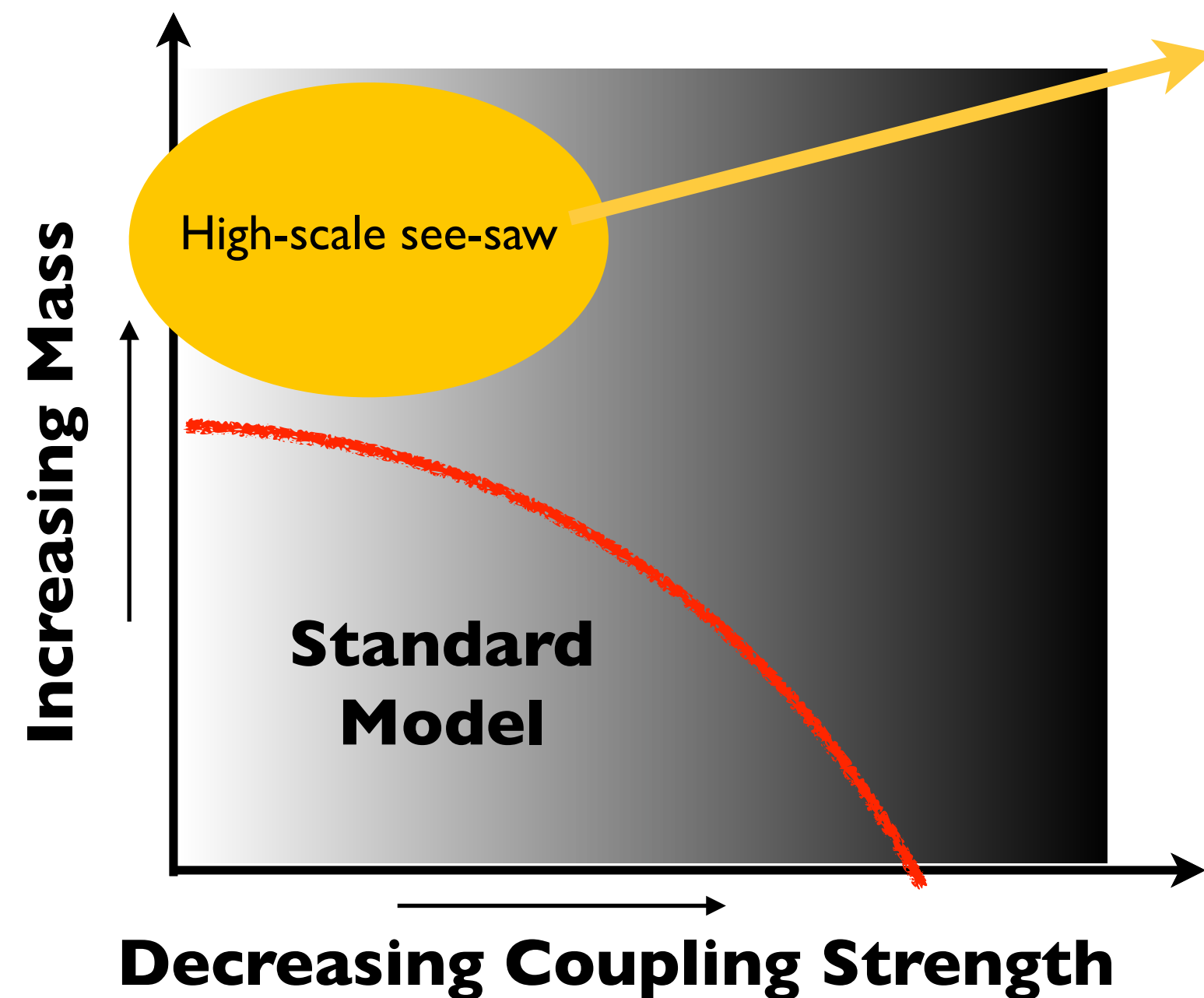
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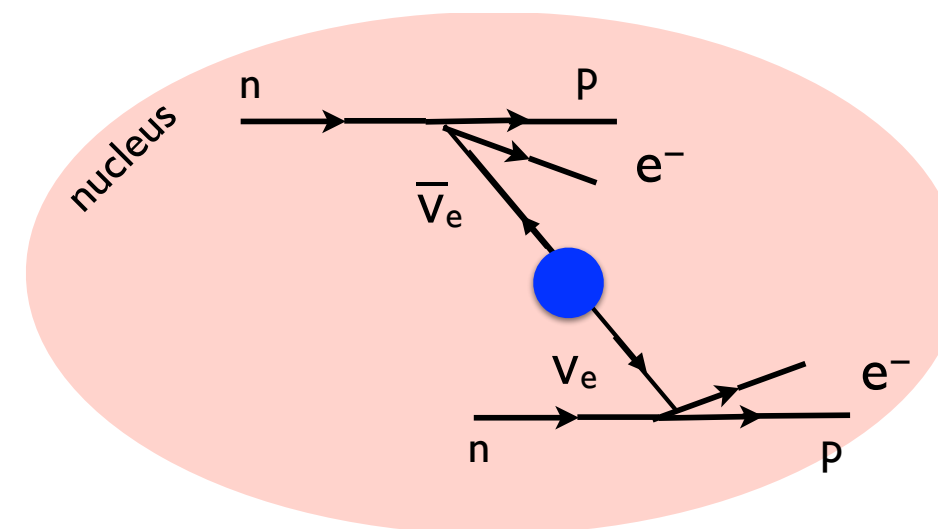
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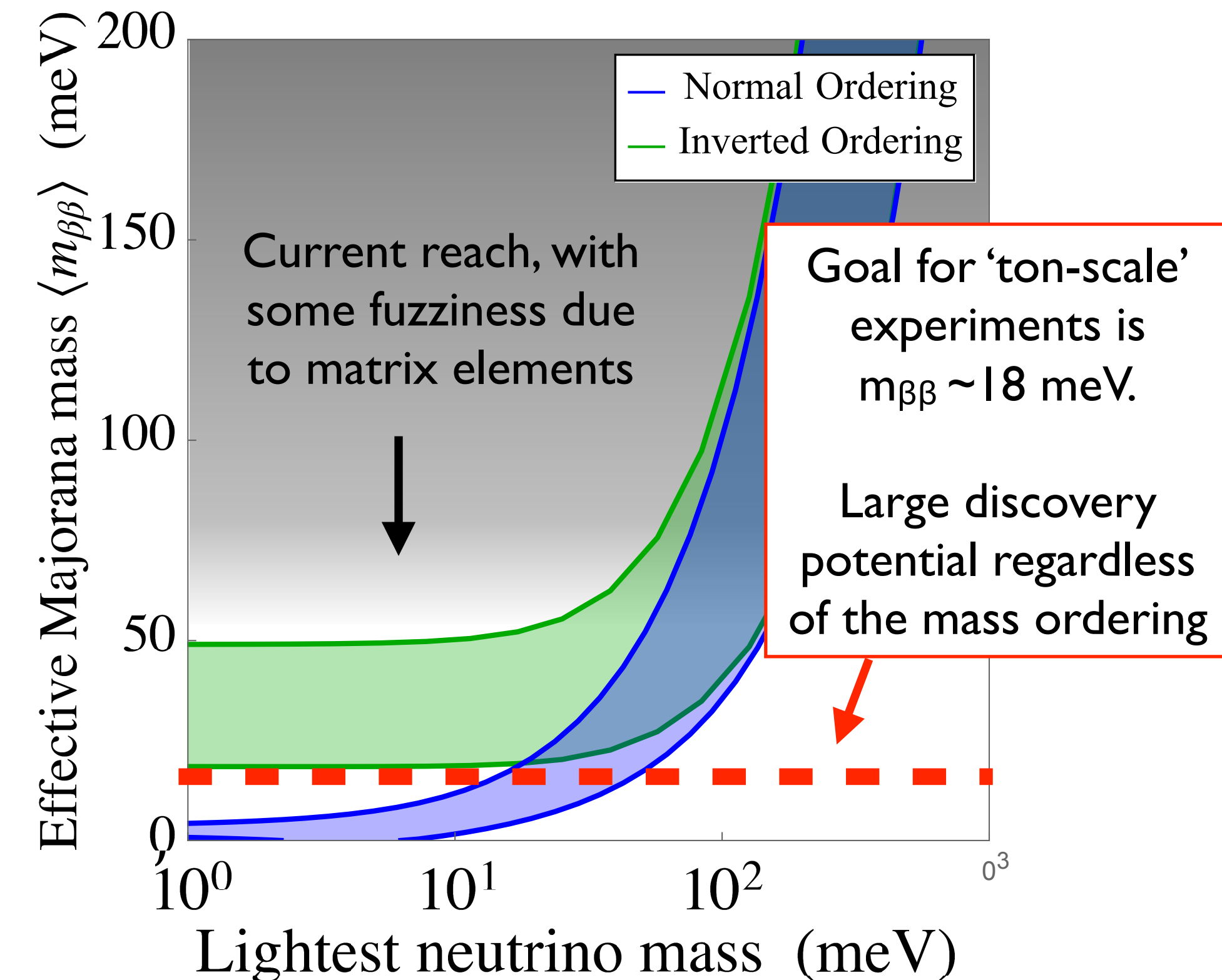


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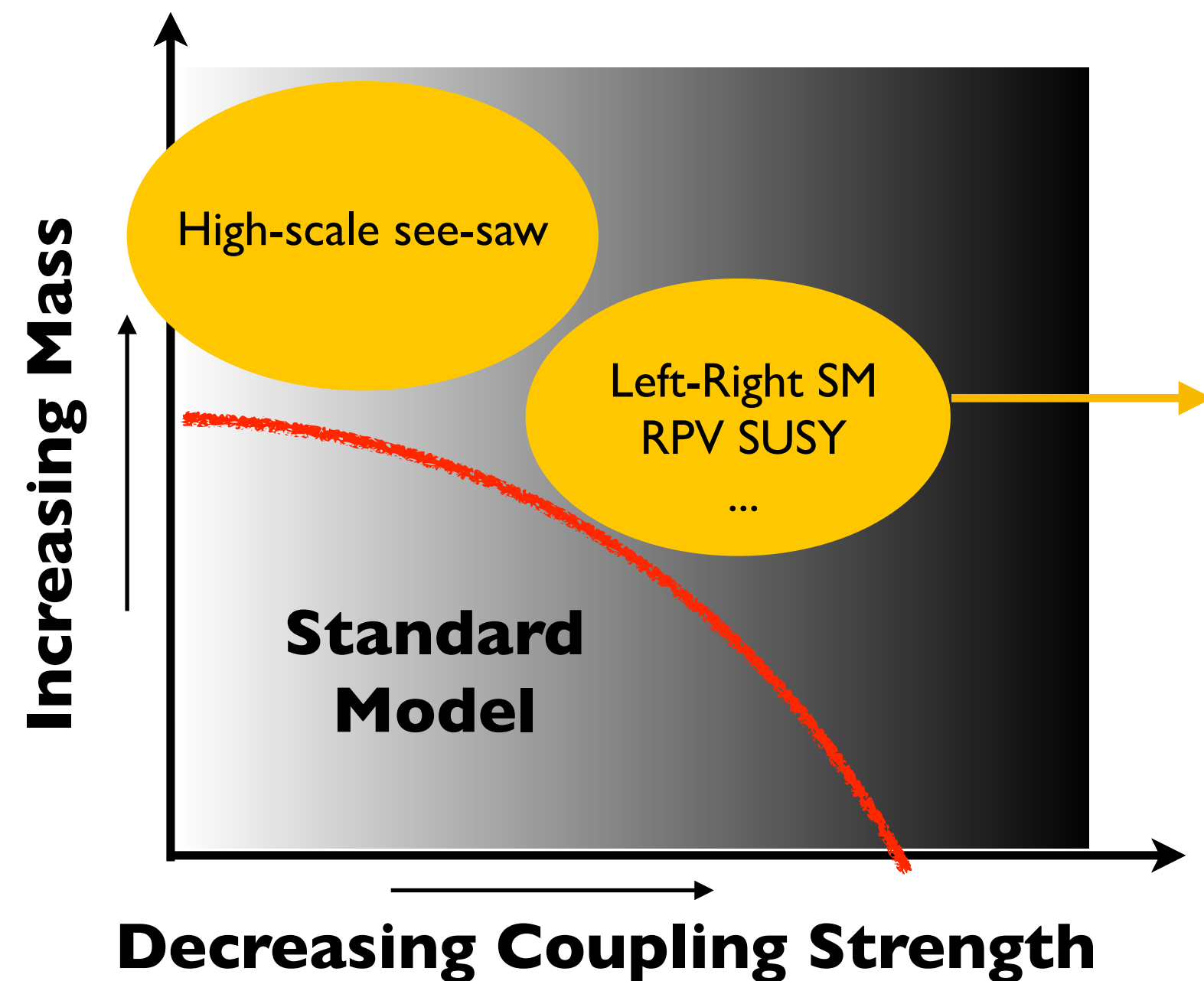
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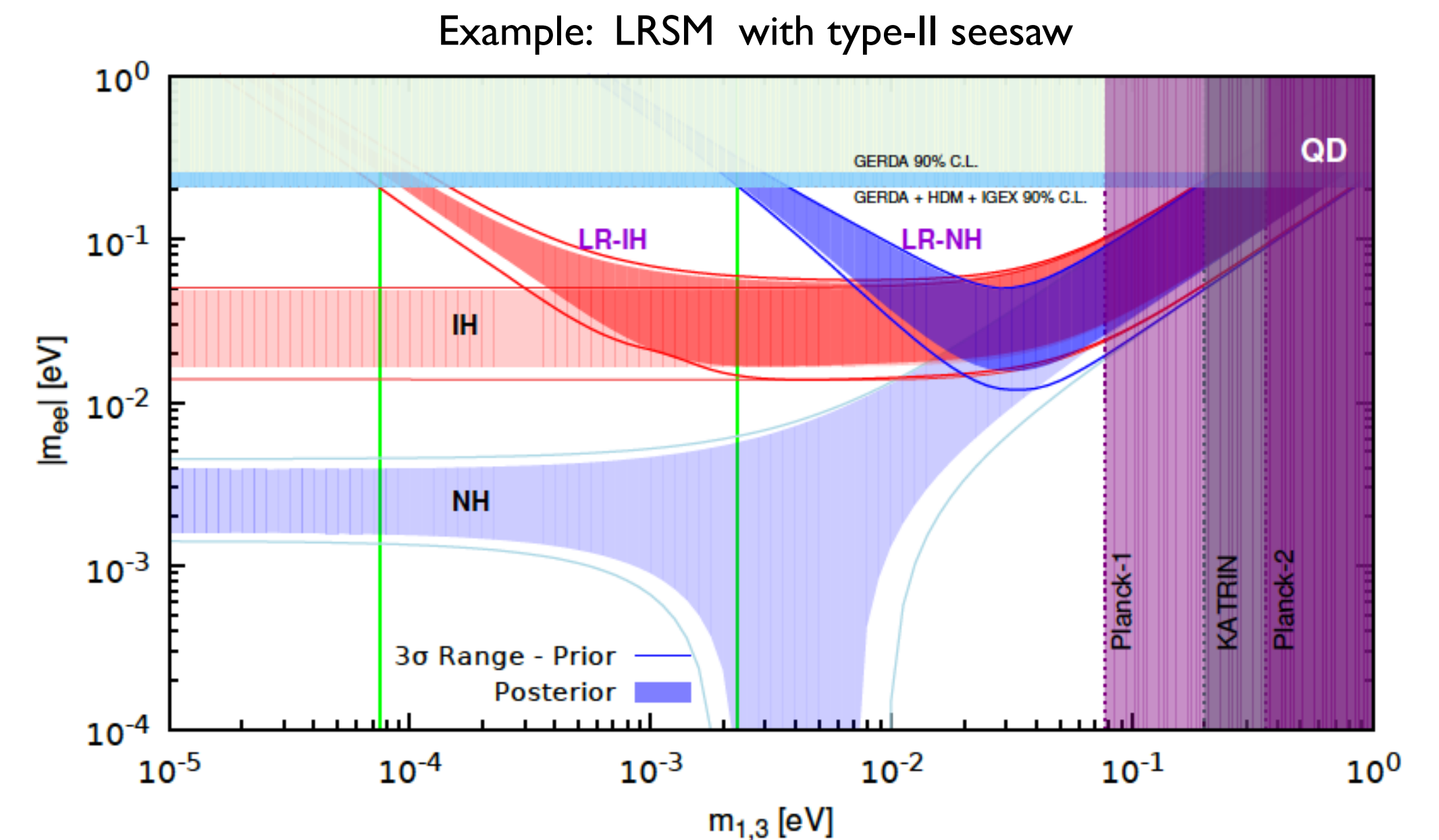
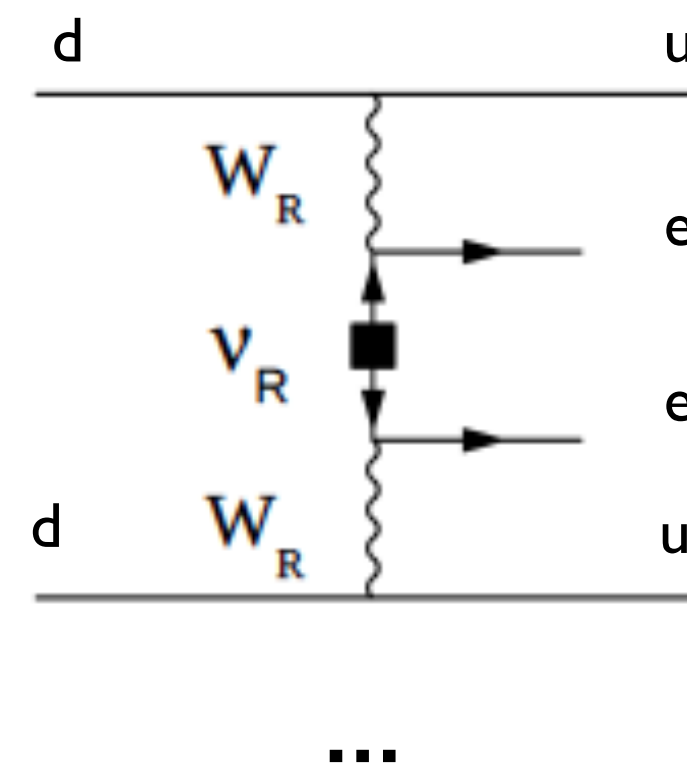
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LNV @ the TeV scale

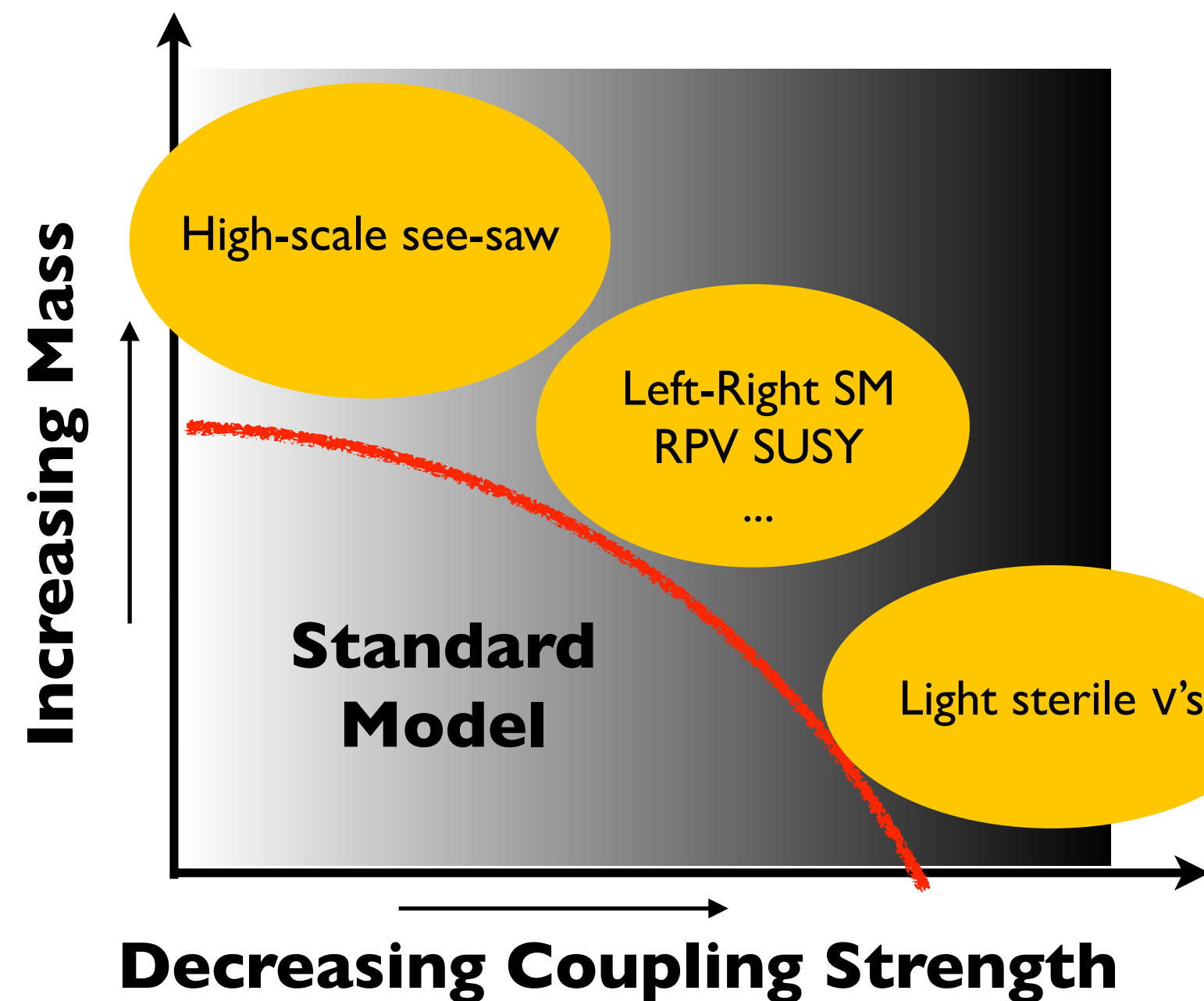


Tello-Nemevesek-Nesti-Senjanovic-Vissani 1011.3522
 Ge-Lindner-Patra 1508.07286
 Li, Ramsey-Musolf, Vasquez 2009.01257
 ...

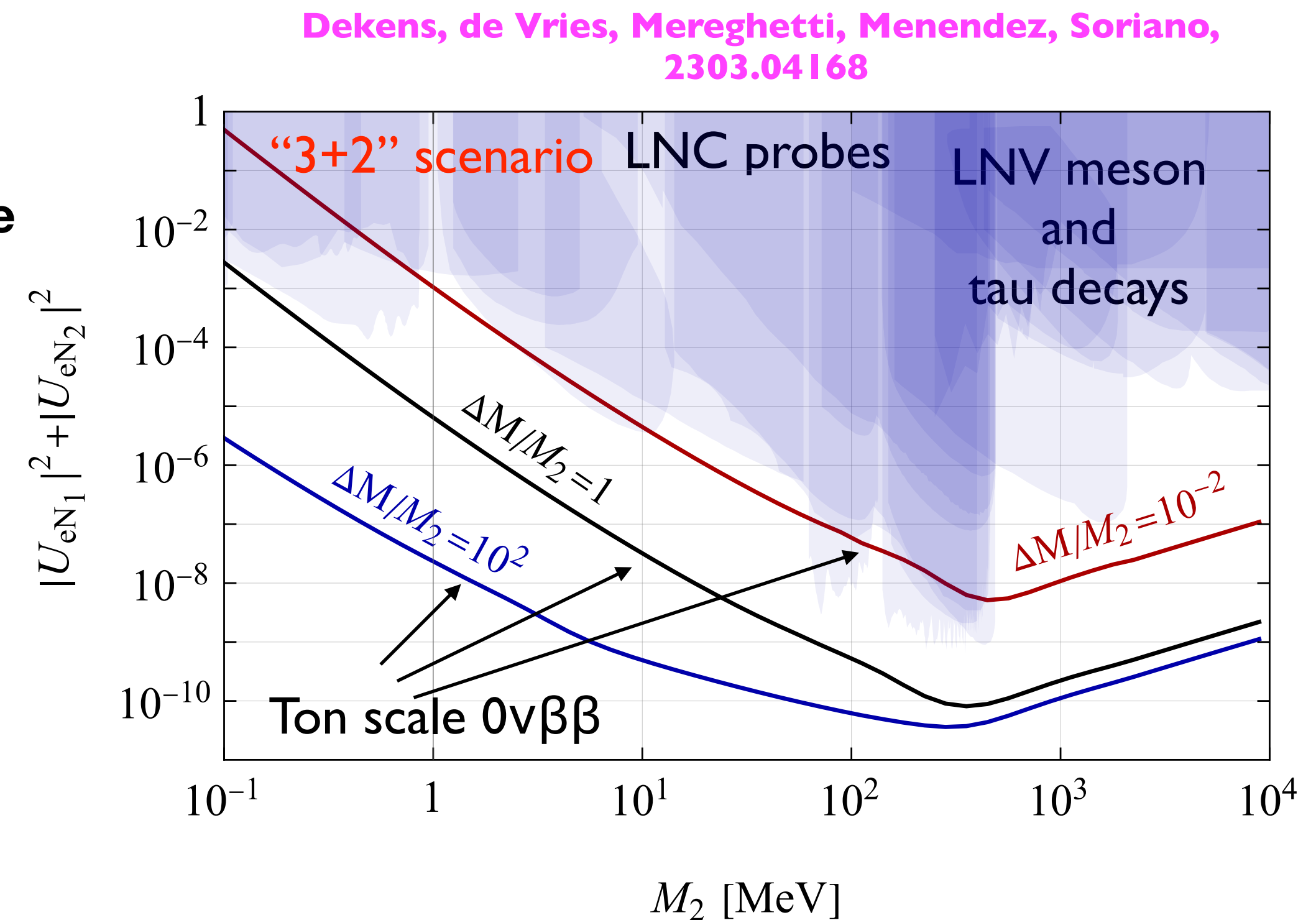
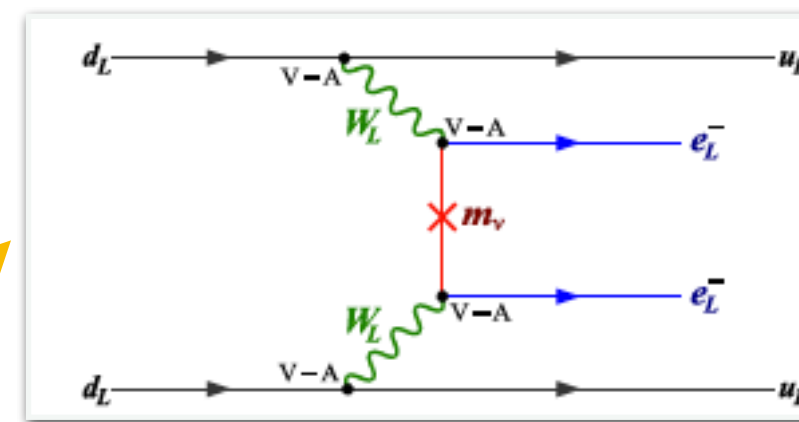
Contributions to $0\nu\beta\beta$ not directly related to the exchange of light neutrinos:
 within reach of planned experiments & possibly correlated with signal at LHC in $pp \rightarrow ee jj$

$0\nu\beta\beta$ decay: broad discovery potential

- Ton-scale $0\nu\beta\beta$ searches can discover Lepton Number Violation from a broad variety of mechanisms that involve different mass scales and interaction strengths



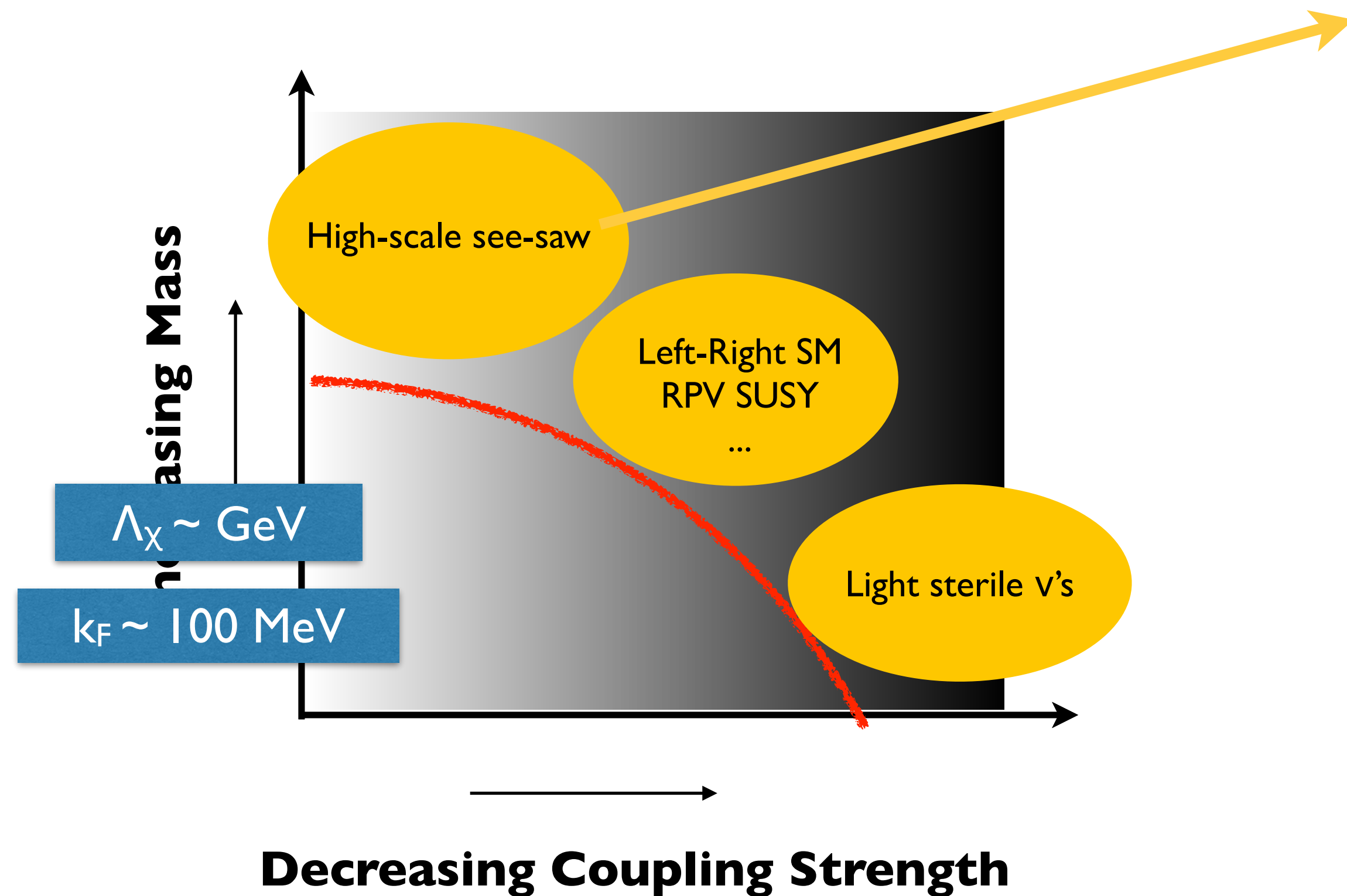
LNV @ the MeV-GeV scale



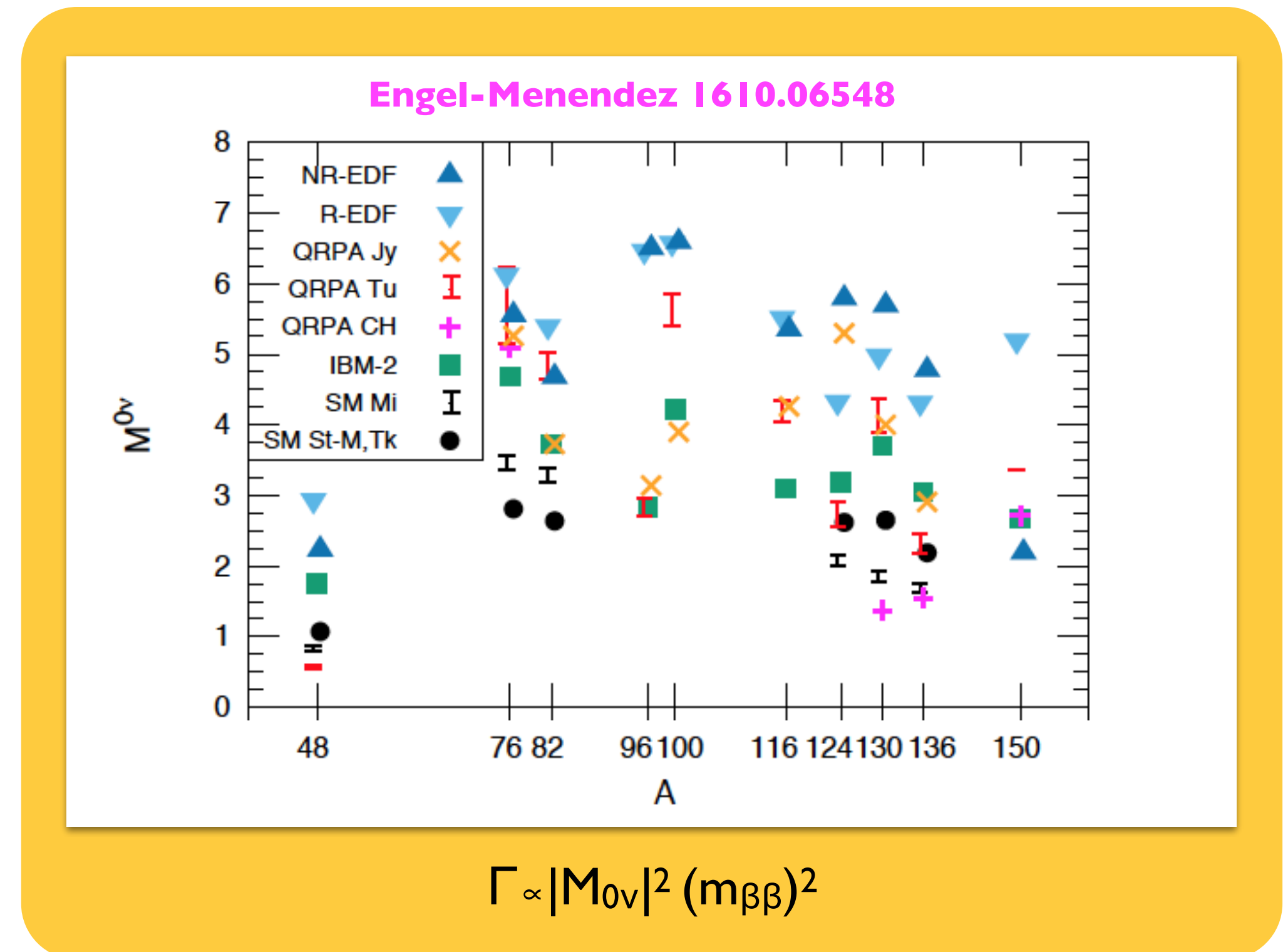
Contributions to $0\nu\beta\beta$ not directly related to the exchange of light neutrinos:
 $0\nu\beta\beta$ decay is an extremely competitive probe of ν_R 's — plenty of opportunity for discovery

$0\nu\beta\beta$ decay: theoretical challenges

- Connecting sources of LNV to nuclei is a multi-scale problem.

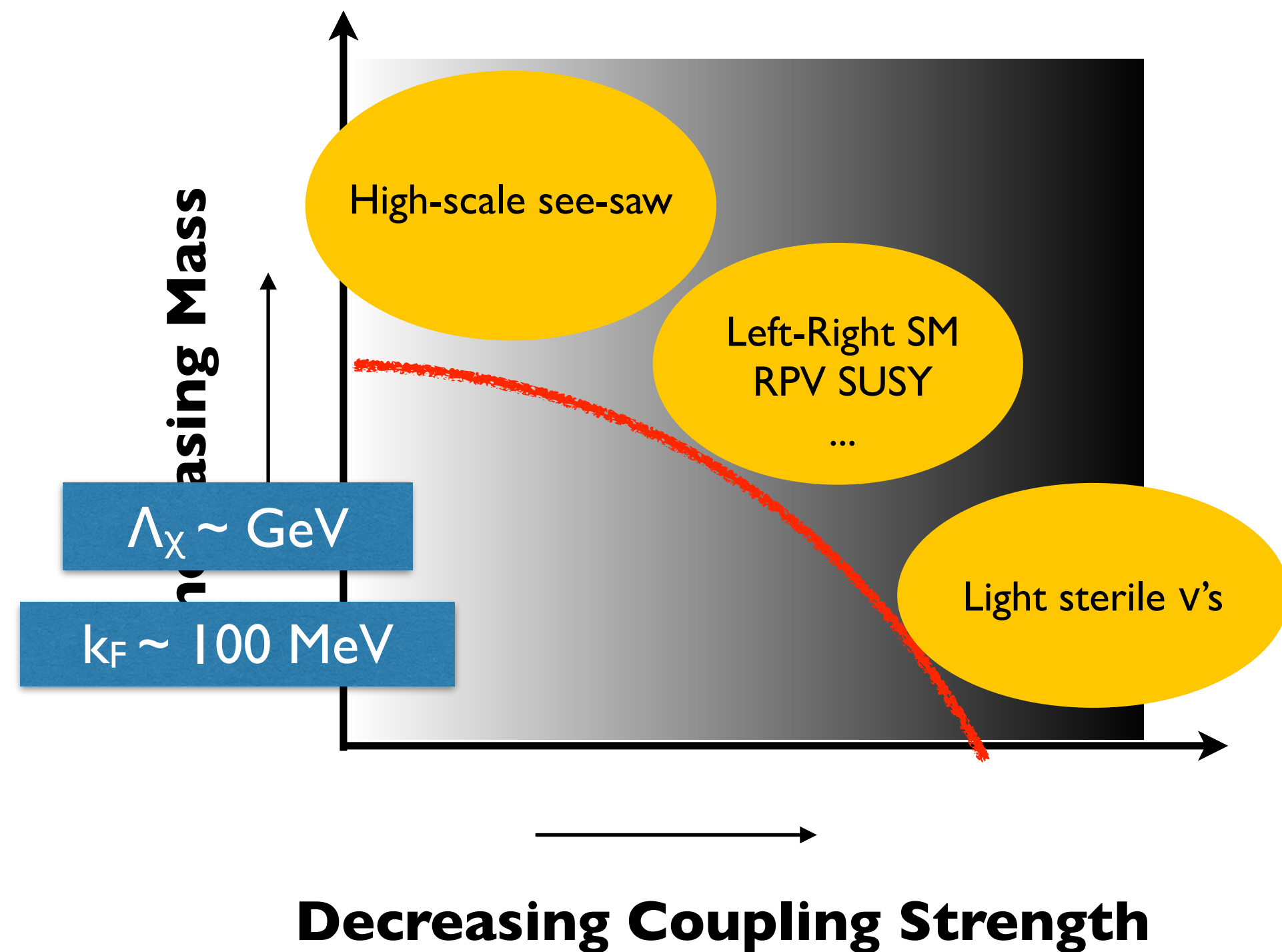


$\mathcal{O}(1)$ theoretical uncertainty in matrix elements hinders the interpretation of a positive or null experimental signal

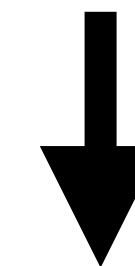


$0\nu\beta\beta$ decay: theoretical challenges

- Connecting sources of LNV to nuclei is a multi-scale problem. Best tackled through a **tower of EFTs**** coupled to **lattice QCD** and **ab-initio nuclear many-body** calculations to achieve controlled uncertainty



** Effective Field Theory:
exploit separation of scales & use appropriate
degrees of freedom at each scale



LN
parameter SMEFT LEFT Chiral EFT

$$(T_{1/2})^{-1} \propto (g_{\text{LNV}})^2 (m_W/\Lambda)^A (\Lambda_\chi/m_W)^B (k_F/\Lambda_\chi)^C$$

White papers [2203.21169](#) & [2207.01085](#) and refs therein

The Standard Model

- Gauge group:

$$SU(3)_c \times SU(2)_w \times U(1)_Y$$

$$\psi'(x) = e^{ig_s \alpha_A(x) \frac{\lambda_A}{2}} \psi(x)$$

$$\psi'(x) = e^{ig' \gamma(x) Y} \psi(x)$$

$$\psi'(x) = e^{ig \beta_a(x) \frac{\sigma_a}{2}} \psi(x)$$

Fundamental
representation
(color triplets and
weak doublets)

- Building blocks: gauge bosons

		$SU(3)_c \times SU(2)_w \times U(1)_Y$ representation
gluons:	$G_\mu^A, \quad A = 1 \cdots 8,$ $G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f_{ABC} G_\mu^B G_\nu^C.$	$(8, 1, 0)$
W bosons:	$W_\mu^I, \quad I = 1 \cdots 3,$ $W_{\mu\nu}^I = \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \epsilon_{IJK} W_\mu^J W_\nu^K$	$(1, 3, 0)$
B boson:	$B_\mu,$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$	$(1, 1, 0)$
Gauge transformation:		$W_{\mu\nu}^I \frac{\sigma^I}{2} \longrightarrow V(x) \left[W_{\mu\nu}^I \frac{\sigma^I}{2} \right] V^\dagger(x)$ $V(x) = e^{ig\beta_a(x) \frac{\sigma_a}{2}}$

- Building blocks: fermions and Higgs

$$\psi = \begin{pmatrix} q \\ \ell \\ u \\ d \\ e \end{pmatrix}$$

SU(3) _c x SU(2) _w x U(1) _Y representation: (dim[SU(3) _c], dim[SU(2) _w], Y)		SU(2) _w transformation
$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1,2,-1/2)	$l \rightarrow V_{SU(2)} l$
$e = e_R$	(1,1,-1)	
$q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	(3,2,1/6)	$q \rightarrow V_{SU(2)} q$
$u^i = u_R^i$	(3,1,2/3)	
$d^i = d_R^i$	(3,1,-1/3)	
$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	(1,2,1/2)	$\varphi \rightarrow V_{SU(2)} \varphi$

$$\tilde{\varphi} = \epsilon \varphi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix} \qquad (1,2,-1/2) \qquad \tilde{\varphi} \rightarrow V_{SU(2)} \tilde{\varphi}$$

$\epsilon = i\sigma_2$

Left- and right-handed fermions have different gauge charges

- The SM Lagrangian: all operators of dimension ≤ 4 that respect gauge and Lorentz symmetry

Homework: work out mass dimension of fields

- Spinor: $[\Psi]=3/2$
- Scalar and vector: $[\phi] = [V_\mu] = 1$

- The SM Lagrangian: all operators of dimension ≤ 4 that respect gauge and Lorentz symmetry

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$D_\mu = I \partial_\mu - ig_s \frac{\lambda^A}{2} G_\mu^A - ig \frac{\sigma^a}{2} W_\mu^a - ig' Y B_\mu$$

$$\begin{aligned} \mathcal{L}_{\text{Gauge}} = & -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^I W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \sum_{i=1,2,3} \left(i\bar{\ell}_i \not{D} \ell_i + i\bar{e}_i \not{D} e_i + i\bar{q}_i \not{D} q_i + i\bar{u}_i \not{D} u_i + i\bar{d}_i \not{D} d_i \right) \end{aligned}$$

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U(3) for each gauge multiplet, e. g. $q_i \rightarrow M_{ij} q_j$, $M \in U(3)$

No notion of “flavor”: three identical copies

- The SM Lagrangian: all operators of dimension ≤ 4 that respect gauge and Lorentz symmetry

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$$+ \sum_{i=1,2,3} \left(i\bar{\ell}_i \not{D} \ell_i + i\bar{e}_i \not{D} e_i + i\bar{q}_i \not{D} q_i + i\bar{u}_i \not{D} u_i + i\bar{d}_i \not{D} d_i \right)$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - \lambda (\varphi^\dagger \varphi - v^2)^2 \xrightarrow{\text{EWSB}}$$

$$\langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle \tilde{\varphi} \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\tilde{\varphi} = \epsilon \varphi^*$$

- The SM Lagrangian: all operators of dimension ≤ 4 that respect gauge and Lorentz symmetry

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

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$$\begin{aligned} \langle \varphi \rangle &= \begin{pmatrix} 0 \\ v \end{pmatrix} \\ \langle \tilde{\varphi} \rangle &= \begin{pmatrix} v \\ 0 \end{pmatrix} \\ \tilde{\varphi} &= \epsilon \varphi^* \end{aligned}$$

$U(3)^5$ symmetry broken by Yukawa couplings $\mathbf{Y}_{e,u,d}$: flavor physics & fermion masses

Fermion masses in the SM

$$\mathcal{L}_{\text{Yukawa}} = \bar{e}_L Y_e e_R \left(v + \frac{h}{\sqrt{2}} \right) + \bar{d}_L Y_d d_R \left(v + \frac{h}{\sqrt{2}} \right) + \bar{u}_L Y_u u_R \left(v + \frac{h}{\sqrt{2}} \right) + \text{h.c.}$$

$$\varphi = \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix}$$

- Fermion mass matrices diagonalized by bi-unitary transformation

$$Y_f = V_{fL}^\dagger Y_f^{\text{diag}} V_{fR} \quad f = e, d, u \quad \longrightarrow \quad m_{f,i} = v \left(Y_f^{\text{diag}} \right)_{ii}$$

- Higgs coupling to fermions is flavor-diagonal and proportional to mass

$$\mathcal{L}_{\text{Yukawa}} = \sum_{f=e,d,u} m_f \bar{f} f \left(1 + \frac{h}{\sqrt{2}v} \right) \quad f = f_L + f_R$$