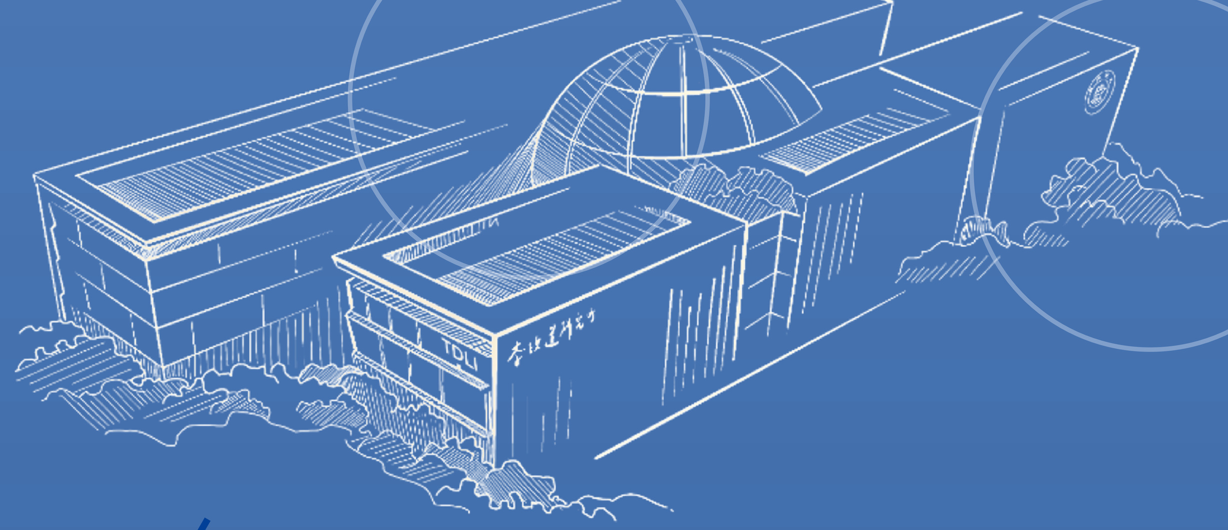
The $U(1)_{L_\mu - L_\tau}$ Model Meets the $(g-2)_\mu$ and MNT Scattering Again

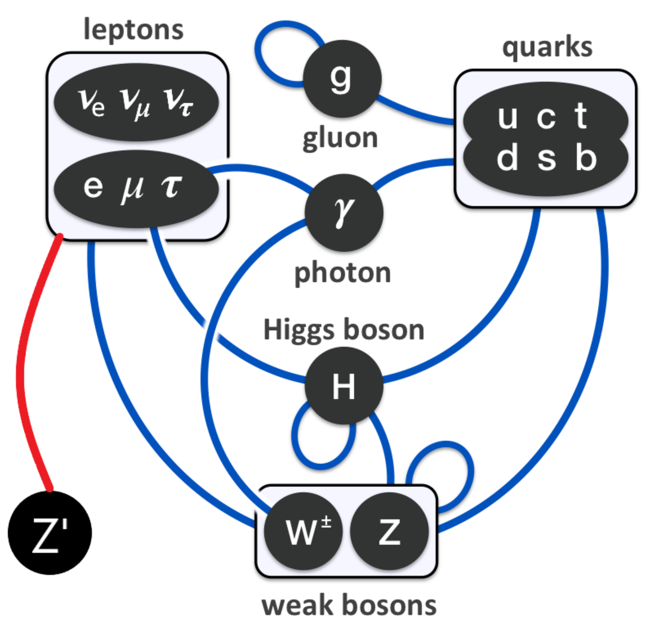
Ming-Wei Li   limw2021@sjtu.edu.cn

In collaboration with X.-G. He, A. Cheek, and X. Chu.

Tsung-Dao Lee Institute, Shanghai Jiao Tong University



1. The Simple Extensions to the SM



Simplest gauge group is a $U(1)$ extension of the SM. One of the simplest extensions is the $U(1)_{L_\mu - L_\tau}$ model [1],

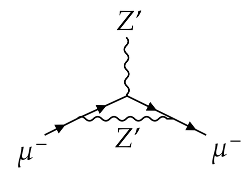
$$\mathcal{L}_{Z'} = -\frac{1}{4}(Z')_{\mu\nu}(Z')^{\mu\nu} + \frac{1}{2}m_{Z'}^2 Z'_\mu Z'^\mu - \tilde{g}\left(\bar{\ell}_\mu\gamma^\mu\ell_\mu - \bar{\ell}_\tau\gamma^\mu\ell_\tau + \bar{\mu}_R\gamma^\mu\mu_R - \bar{\tau}_R\gamma^\mu\tau_R\right)Z'_\mu .$$

Then we can write the new interaction terms directly,

$$-\tilde{g}(\bar{\mu}\gamma^\mu\mu - \bar{\tau}\gamma^\mu\tau + \bar{\nu}_\mu\gamma^\mu P_L\nu_\mu - \bar{\nu}_\tau\gamma^\mu P_L\nu_\tau)Z'_\mu \, .$$

2. Contribution to $(g-2)_\mu$

The new interaction would introduce the additional positive contribution to the muon anomalous magnetic moment,

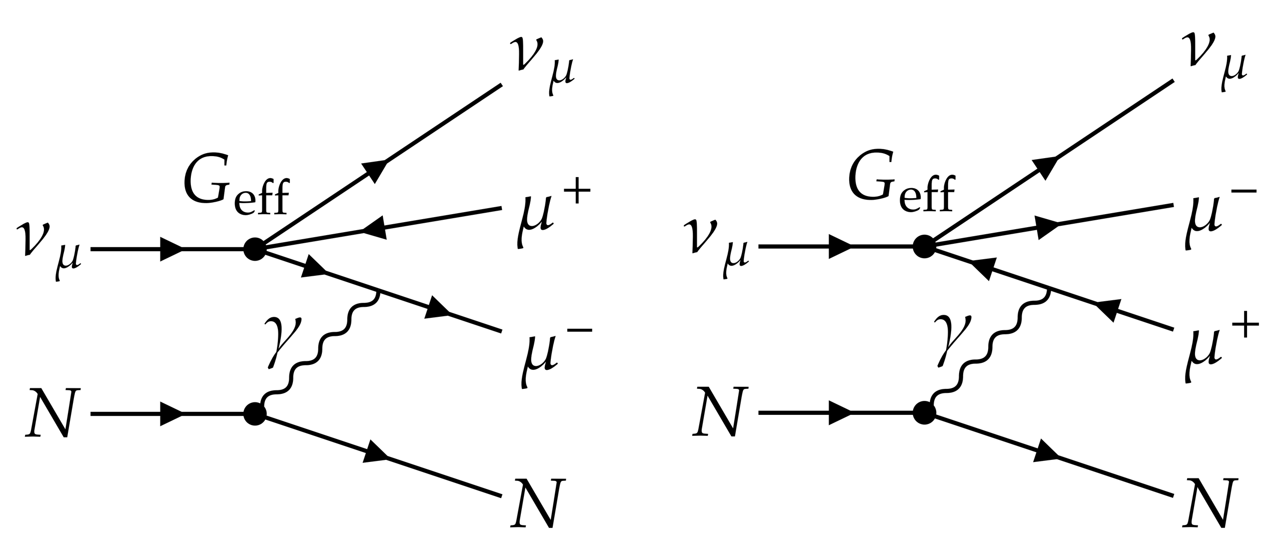


$$\Rightarrow \quad \Delta a_\mu^{Z'} = \frac{\tilde{g}^2}{8\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\int_0^1 \frac{2x^2(1-x)dx}{1-x+(m_\mu^2/m_{Z'}^2)x^2}$$

In the light and heavy mass limits, with the $(g-2)_\mu$ data [2, 3], we can derive that

$$\begin{cases} \Delta a_\mu^{Z'} \approx \frac{\tilde{g}^2}{8\pi^2} & m_{Z'} \ll m_\mu \\ \Delta a_\mu^{Z'} \approx \frac{\tilde{g}^2}{12\pi^2}\frac{m_\mu^2}{m_{Z'}^2} & m_{Z'} \gg m_\mu \end{cases} \xrightarrow{\frac{m_\mu}{m_{Z'}} \ll 1} \begin{cases} \Delta a_\mu^{2021} = (251 \pm 59)\times 10^{-11} \\ \Rightarrow \frac{\tilde{g}^2}{m_{Z'}^2} = (2.66 \pm 0.63)\times 10^{-5}\,\text{GeV}^{-2} \end{cases} \quad \begin{cases} \Delta a_\mu^{2025} = (39 \pm 64)\times 10^{-11} \\ \Rightarrow \frac{\tilde{g}^2}{m_{Z'}^2} = (4.14 \pm 6.79)\times 10^{-6}\,\text{GeV}^{-2} \end{cases}$$

3. Introduction to MNT $\nu_\mu N \to \nu_\mu \mu^- \mu^+ N$



Considering the heavy gauge bosons, the effective interactions are given by

$$\mathcal{L}_{\text{SM}}^{\text{eff}} = -\sqrt{2}G_F\left(\bar{\nu}_\mu\gamma^\mu P_L\nu_\mu\right)\left(\bar{\mu}\gamma_\mu\frac{1+4\sin^2\theta_W+\gamma^5}{2}\mu\right), \quad \mathcal{L}_{Z'}^{\text{eff}} = -G_{Z'}\left(\bar{\nu}\gamma^\mu P_L\nu\right)\left(\bar{\mu}\gamma_\mu\mu\right)$$

where $G_F = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2}$, $G_{Z'} = \frac{\tilde{g}^2}{m_{Z'}^2}$. Both of them has contribution to muon neutrino trident (MNT) scattering $\nu_\mu N \to \nu_\mu\mu^-\mu^+ N$. Due to the γ^5 trace property, we can derive that

$$\sum_{\text{spins}}|\mathcal{M}|^2 = \sum_{\text{spins}}|\mathcal{M}_V|^2 + \sum_{\text{spins}}|\mathcal{M}_A|^2 \;\Rightarrow\; R_{\text{trident}} \equiv \left.\frac{\sigma_{\text{SM}+Z'}}{\sigma_{\text{SM}}}\right|_{\text{trident}} = \frac{1+\left(1+4\sin^2\theta_W + \frac{\sqrt{2}G_{Z'}}{G_F}\right)^2}{1+\left(1+4\sin^2\theta_W\right)^2}.$$

4. Constraints from MNT Scattering

From $(g-2)_\mu$ results, we can derive that

$$G_{Z'}^{2021} = (2.66 \pm 0.63)\times 10^{-5}\,\text{GeV}^{-2}\,, \quad G_{Z'}^{2025} = (4.14 \pm 6.79)\times 10^{-6}\,\text{GeV}^{-2}\,.$$
$$(G_F = 1.1664\times 10^{-5}\,\text{GeV}^{-2}\,)$$

Then we can compute the ratio of the trident cross section R_{trident},

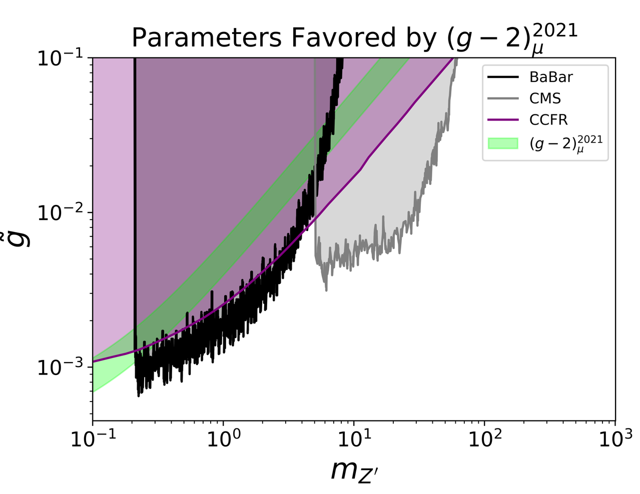
$$R_{\text{trident}}^{2021} = 5.94 \pm 1.08\,, \quad R_{\text{trident}}^{2025} = 1.47 \pm 0.87\,.$$

Comparing with experimental measurements from trident

$$\frac{\sigma_{\text{CHARM-II}}}{\sigma_{\text{SM}}} = 1.58 \pm 0.57\,, \quad \frac{\sigma_{\text{CCFR}}}{\sigma_{\text{SM}}} = 0.82 \pm 0.28\,, \quad \frac{\sigma_{\text{NuTeV}}}{\sigma_{\text{SM}}} = 0.72^{+1.73}_{-0.72}$$
$$\Rightarrow \sigma_{\text{exp}}/\sigma_{\text{SM}} = 0.96 \pm 0.25\,,$$

For $U(1)_{L_\mu - L_\tau}$ model, the new $(g-2)_\mu$ result $R_{\text{trident}}^{2025}$ is consistent with trident experiment, while the $R_{\text{trident}}^{2021}$ is clearly disfavored.

Due to the constraints from muon neutrino trident scattering, the mass $m_{Z'}$ is tightly constrained and cannot be too large.



5. Effects on $\mu^-\mu^+ \to \tau^-\tau^+$

For the $U(1)_{L_\mu - L_\tau}$ model, $\mu^-\mu^+ \to \tau^-\tau^+$ will be the most consequential,

$$\mathcal{M} = \left(\frac{4\pi\alpha_{\text{em}}}{s} - \frac{G_{Z'}m_{Z'}^2}{q^2 - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}}\right)(\bar{\mu}\gamma^\mu\mu)(\bar{\tau}\gamma_\mu\tau) + \frac{\sqrt{2}G_F m_Z^2}{q^2 - m_Z^2 + im_Z\Gamma_Z}(\bar{\mu}\gamma^\mu(g_V - g_A\gamma_5)\mu)(\bar{\tau}\gamma_\mu(g_V - g_A\gamma_5)\tau)$$

where $g_V = -\frac{1}{2} + 2\sin^2\theta_W$ and $g_A = -\frac{1}{2}$ and

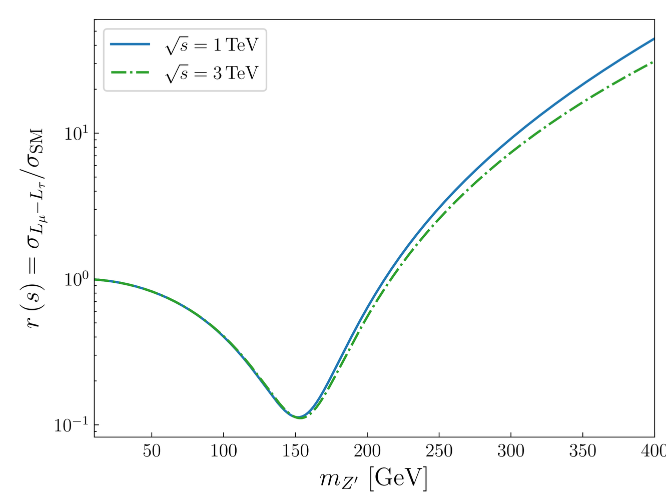
$$\begin{cases} \Gamma_{Z'\to\mu^-\mu^+} = \Gamma_{Z'\to\tau^-\tau^+} = \frac{G_{Z'}m_{Z'}^3}{12\pi} \\ \Gamma_{Z'\to\nu_\mu^-\nu_\mu^+} = \Gamma_{Z'\to\nu_\tau^-\nu_\tau^+} = \frac{G_{Z'}m_{Z'}^3}{24\pi} \end{cases} \Rightarrow \Gamma_{Z'} = \frac{G_{Z'}m_{Z'}^3}{4\pi}$$

Neglecting muon and tauon masses, we obtain the cross-section as

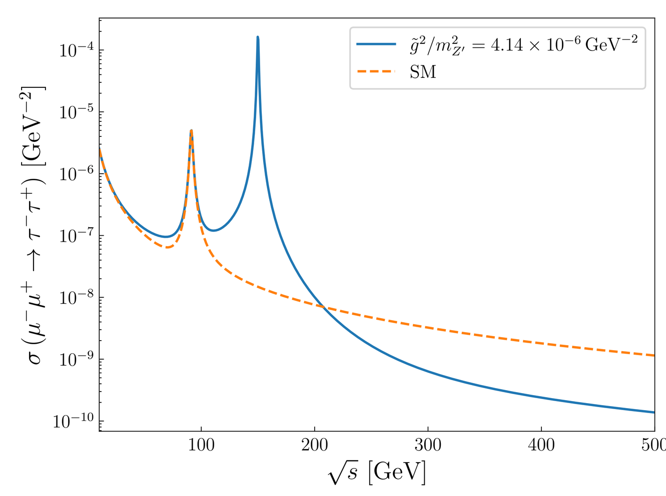
$$\sigma_{\mu-\tau}^{\text{SM}+Z'} = \frac{4\pi\alpha_{\text{em}}^2}{3s} + \frac{G_{Z'}m_{Z'}^2\left((G_{Z'}m_{Z'}^2 - 8\pi\alpha_{\text{em}})s + 8\pi\alpha_{\text{em}}m_{Z'}^2\right)}{12\pi\left((s-m_{Z'}^2)^2 + m_{Z'}^2\Gamma_{Z'}^2\right)} + \frac{G_F^2 m_Z^4 s}{6\pi}\frac{(g_V^2 + g_A^2)^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$
$$+ \frac{2\sqrt{2}\alpha_{\text{em}}G_F m_Z^2 g_V^2(s-m_Z^2)}{3\left((s-m_Z^2)^2 + m_Z^2\Gamma_Z^2\right)}\left(1 - \frac{G_{Z'}m_{Z'}^2 s}{4\pi\alpha_{\text{em}}}\frac{s - m_{Z'}^2 + \frac{m_Z m_{Z'}\Gamma_Z\Gamma_{Z'}}{s-m_Z^2}}{(s-m_{Z'}^2)^2 + m_{Z'}^2\Gamma_{Z'}^2}\right)$$

6. Muon Collider Signatures

We use the central value of Δa_μ^{2025} (taking $G_{Z'} = 4.14\times 10^{-6}\,\text{GeV}^{-2}$) to plot the cross section ratio vs. $m_{Z'}$.



Then setting $m_{Z'} = 150\,\text{GeV}$, we observe an additional resonance.



It provides a potential probe for the $U(1)_{L_\mu - L_\tau}$ model in future experiments.

Summary

1. $U(1)_{L_\mu - L_\tau}$ introduces the new interaction mediated by Z', which affects MNT and $\mu^-\mu^+ \to \tau^-\tau^+$.
2. Under the $(g-2)_\mu^{2021}$ data, only a light Z' was allowed due to the constraints from MNT experiments.
3. As the recent $(g-2)_\mu^{2025}$ result is consistent with the SM prediction, heavier Z' masses are now allowed.
4. A future muon collider could provide a promising opportunity to probe the $U(1)_{L_\mu - L_\tau}$ model.

Reference

[1] X. G. He, G. C. Joshi, H. Lew and R. R. Volkas, Phys. Rev. D **43** (1991), 22-24 doi:10.1103/PhysRevD.43.R22

[2] D. P. Aguillard *et al.* [Muon g-2], [arXiv:2506.03069 [hep-ex]].

[3] R. Aliberti *et al.* [arXiv:2505.21476 [hep-ph]].

李政道研究所 Tsung-Dao Lee Institute