



Gallium Anomaly: An Unsolvable Puzzle?

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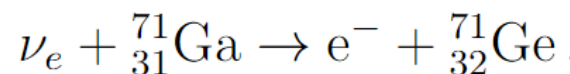
MAYORANA School 2025
19-25 June



History of the Gallium Anomaly



- Since the 1990s, the **SAGE** and **GALLEX** experiments have been studying solar neutrinos through the neutrino capture reaction on a gallium target:



Two deficits in **electron neutrinos** were observed:

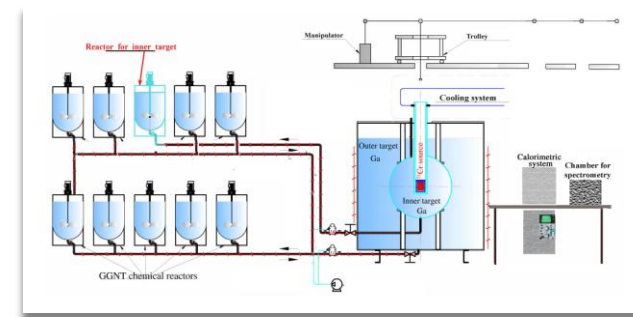
- One related to the flux of solar neutrinos → resolved through the formalism of solar neutrino oscillations into other flavors.
- The other related to neutrinos from the source used in the experiments' calibrations → still not understood today, it is known as the «Gallium anomaly».



2022: First results from **BEST**

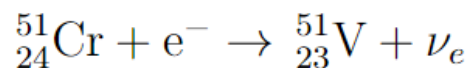


$$R = \frac{N_{\text{exp}}}{N_{\text{cal}}} \approx 0.8$$



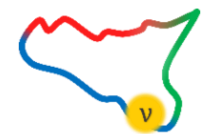
Phys. Rev. Lett. 128 (23 giu. 2022)

- Chromium produces neutrinos via electron capture.



- Confirms the anomaly and increases its significance to approximately 5σ .

Ground state cross section



The process under examination is an inverse beta decay (IBD), $\nu_e + n \rightarrow e^- + p$ which in its simplest form is:

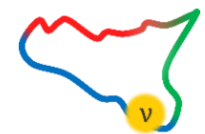
- The cross section for the ground state (gs) is expressed as:

$$\sigma_{\text{gs}} = \frac{G_F^2 \cos^2 \theta_C}{\pi} p_e E_e \mathcal{F}_n(Z, E_e) g_A^2 \tilde{B}_{\text{GT}}^{(\nu, e)}(\text{gs}) \frac{[1 + g_{v,b}](\nu, e)}{[1 + g_{v,b}]_{\text{EC}}} [1 + \epsilon_q]$$

Diagram illustrating the components of the ground state cross section formula:

- $G_F^2 \cos^2 \theta_C$: Fermi constant
- π : Momentum e
- $p_e E_e$: Energy e
- $\mathcal{F}_n(Z, E_e)$: Fermi function
- g_A^2 : CKM Matrix el.
- $\tilde{B}_{\text{GT}}^{(\nu, e)}(\text{gs})$: Nuclear Matrix el.
- $\frac{[1 + g_{v,b}](\nu, e)}{[1 + g_{v,b}]_{\text{EC}}}$: Radiative corrections
- $[1 + \epsilon_q]$: Interference between G-T strength and weak magnetism

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- In my work, I focused on the Fermi function and the nuclear matrix element.

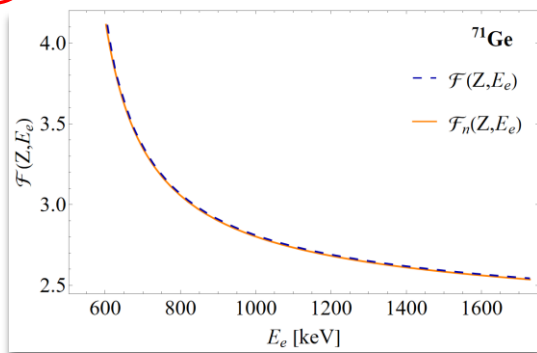
Fermi function

- $\mathcal{F}_n(Z, E_e)$ is obtained by solving Dirac equation using DHFS method.

$$\mathcal{F}_n(Z, E_e) = F_0(Z, E_e) L_0(Z, E_e) U(Z, E_e) S(Z, E_e)$$

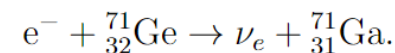
$$F_n(Z, E_e) = \lim_{r \rightarrow 0} \frac{f_1^2(r) + g_{-1}^2(r)}{r^2}$$

Asintotically free electronic wave functions



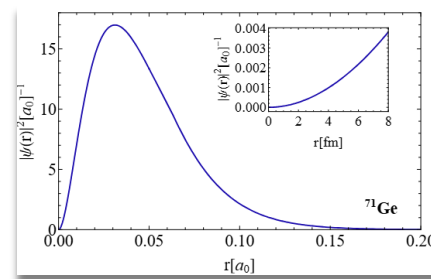
Nuclear matrix element

- The nuclear matrix element is obtained from the reverse process, which is an electron capture (EC):



$$\omega = \frac{\ln 2}{t_{1/2}} = \frac{G_F^2 \cos^2 \theta_C}{\pi} g_A^2 \tilde{B}_{\text{GT}}^{(\nu, e)}(\text{gs}) [1 + \epsilon_q]_{\text{EC}} f_{\text{EC}}$$

${}^{71}\text{Ge}$ half-life ~ 11.46 d



Phase space factor of electron capture

$$f_{\text{EC}} = |\psi_{1s}|^2 E_{\nu, 1s}^2 [2(1 + \epsilon_o^{1s})(1 + \frac{L+M}{K})]$$

Bound state electron wave function

$$\tilde{B}_{\text{GT}}^{(\nu, e)}(\text{gs}) \propto \frac{1}{(ft_{1/2})_{\text{EC}}}$$

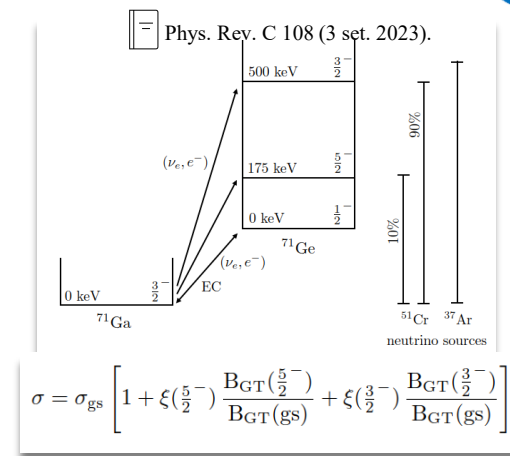
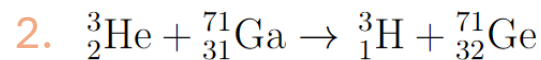
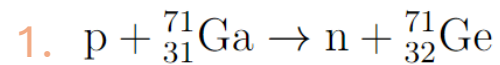
$f_{\text{EC}} [(h/m_e c)^3]$	$\tilde{B}_{\text{GT}}^{(\nu, e)}(\text{gs})$
0.0219(3)	0.0887(13)

Total cross section and interpretation with sterile neutrinos



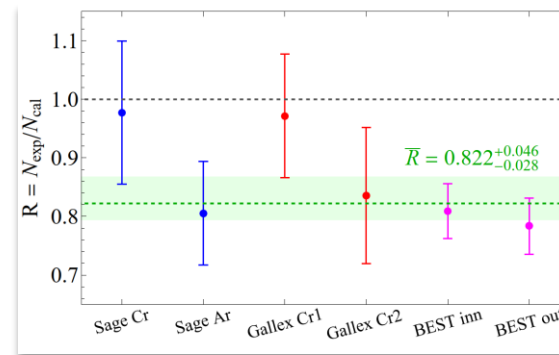
$$\sigma_{\text{gs}} = \frac{\ln 2}{(ft_{1/2})_{\text{EC}}} \frac{[1 + \epsilon_q]_{\text{EC}}}{[1 + \epsilon_q]_{\text{EC}}} \frac{[1 + g_{v,b}]_{(\nu,e)}}{[1 + g_{v,b}]_{\text{EC}}} p_e E_e \mathcal{F}_n(Z, E_e)$$

- The ratios $\frac{B_{\text{GT}}(5/2^-)}{B_{\text{GT}}(\text{gs})}$ and $\frac{B_{\text{GT}}(3/2^-)}{B_{\text{GT}}(\text{gs})}$ are obtained experimentally by studying:



The increase in the cross section is of the order of 5% and 9%, respectively.

Source	$\sigma_{\text{gs}} [\times 10^{-45} \text{ cm}^2]$	$\sigma_{(\text{p},\text{n})} [\times 10^{-45} \text{ cm}^2]$	$\sigma_{({}^3\text{He}, {}^3\text{H})} [\times 10^{-45} \text{ cm}^2]$
${}^{51}\text{Cr}$	5.55 ± 0.08	$5.86^{+0.28}_{-0.09}$	6.03 ± 0.18
${}^{37}\text{Ar}$	6.65 ± 0.09	$7.06^{+0.35}_{-0.11}$	7.33 ± 0.22



Possible interpretations of the anomaly

Possibility of electron neutrino oscillations into sterile neutrinos on scales of a few meters. 

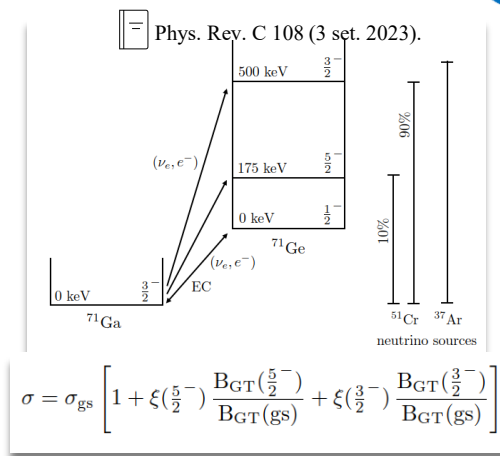
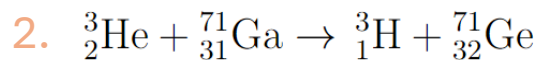
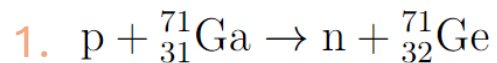
This hypothesis has not been completely ruled out yet but is under strong tension due to the lack of experimental evidence.

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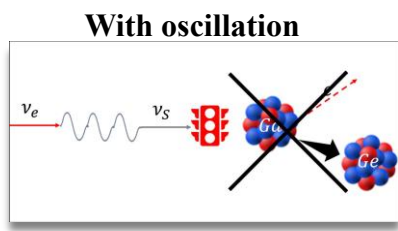
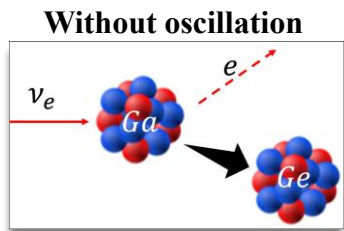


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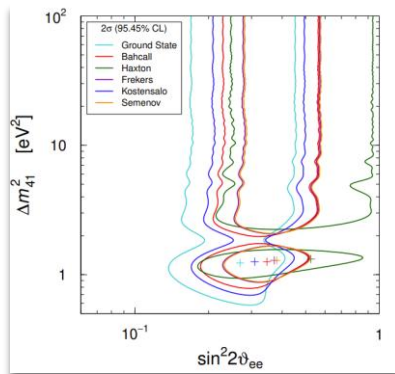


$$P_{ee} \simeq 1 - \sin^2 2\theta_{ee} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

- The anomaly can be explained by short-baseline neutrino oscillations in the 3+1 scenario.

Suggested parameter space :

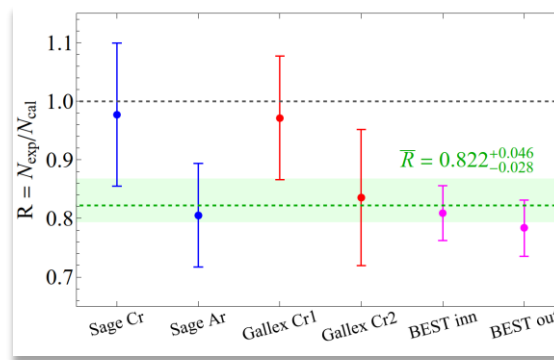
- $\Delta m_{41}^2 \sim 1 \text{ eV}^2$.
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J. High Energ. Phys. 2022, 164

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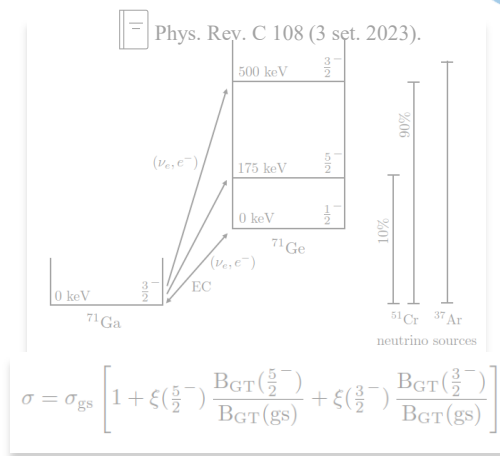
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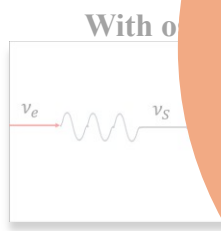
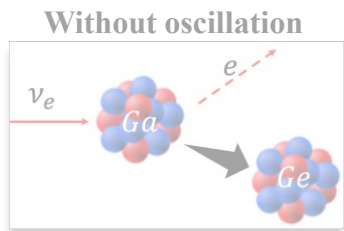


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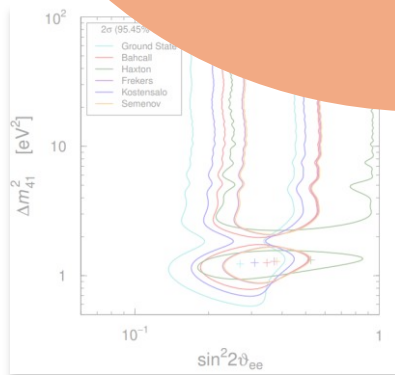
Thank you!!

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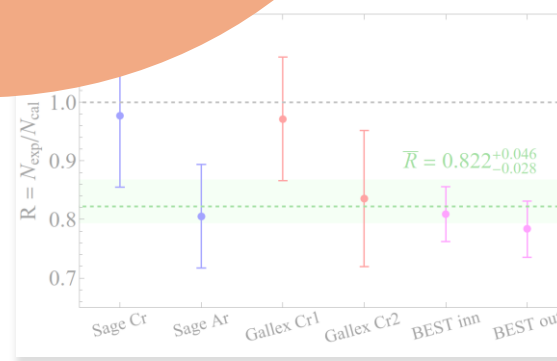
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J. High Energ. Phys. 2022, 164



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