Boson Stars, Primary photons and Phase Transitions

G. Kozlov

❑ Instead of Introduction (what do we know)

- ✓ ***Observational evidence Universe mysterious substance – Dark Matter (DM)***

- ✓ ***Particle Physics of DM & Origin in early Universe*** **very little known up to now**

- ✓ **Historically, favored DM Thermal relic production**

- ✓ **DM thermalized with SM Plasma in early Universe.** ***Equilibrium? Fluctuations?***

- ✓ **DM Relic *abundance*** **Interactions "freeze-out"**

↓ ↘

DM heavier (increasing) ***Large interaction strength (decreasing)***

❖ $\boldsymbol{m_D} < \boldsymbol{O(100\,TeV)}$ $partial\ wave\ unitarity\ violation; th.\ equilibrium,\ stab.$

Griest –Kamionkowski (G-K) bound, 1990

- ➢ **Cosmo FOPT may alter the expansion rate of the Universe**

- ➢ **Cosmo FOPT may trigger an Electro-Weak FOPT**

Candidates to DM

- MACHO's (boson stars, neutron stars, …, black holes,…)

○ WIMP $m_X \sim 0.03 - 0.3\, TeV,\quad \Omega_{WIMP} = 0.1\Omega_{DM}$

$m_X \sim 0.1 - 1\, TeV,\quad \Omega_{WIMP} = \Omega_{DM} \sim 26.8\%$

$(neutralino, m_X \sim$ 0.03 - 5 TeV)

▪ $\boldsymbol{\nu},\quad \boldsymbol{\Omega_\nu \sim 0.3\,\%}$ ***very small contribution to DM***!

❑ Axion, $m_a < 0.67\, eV$ $(Planck\ exp. 2016 - 2018)$

➢ Dark photon $\boldsymbol{\bar{\gamma}}$, $\varepsilon\ (\boldsymbol{\bar{\gamma}} - \gamma\ kinetic\ mixing)\ , m_{\bar{\gamma}} < 1\, GeV$

➢ **Dark fermion $\bar{f}$**, "Darkonium", $\boldsymbol{e^+e^- \to \gamma\, \Upsilon_D \to \gamma\, \bar{\gamma}\, \bar{\gamma}\, \bar{\gamma}}$

$\boldsymbol{0.05 < m_{\Upsilon_D} < 9.5\, GeV, 0.001 < m_{\bar{\gamma}} < 3.16\, GeV,}$ BABAR (2022)

❑ **Scalar DM, e.g., dilaton/ "Glueball", $\boldsymbol{m_G \sim O(\Lambda_{confinement})}$**

❑ Motivations for MACHO

- *Modern scenarios, macroscopic Bose-like DM objects broach the Q.:*

DM scalar role & an impact to development of *cosmo-inhomogeneities*

- *Related to GR bound state composed of DM ϕ lighter than spin-1/2 DM χ*
- $\bar{\chi}\chi \rightarrow \phi\phi$, $\mathbb{Z}_2$ *symmetry protects stable* $\boldsymbol{\chi}$
- *Interactions* $\sim(1+\phi/\phi_0)m_\chi\bar{\chi}\chi$, $m_\chi < O(100\,TeV)$, $\phi_0 \sim O(1\,TeV)$
- *Thermal bath, ϕ bounded by GR (+ gauge fields, + Higgs)* *GK 2023*

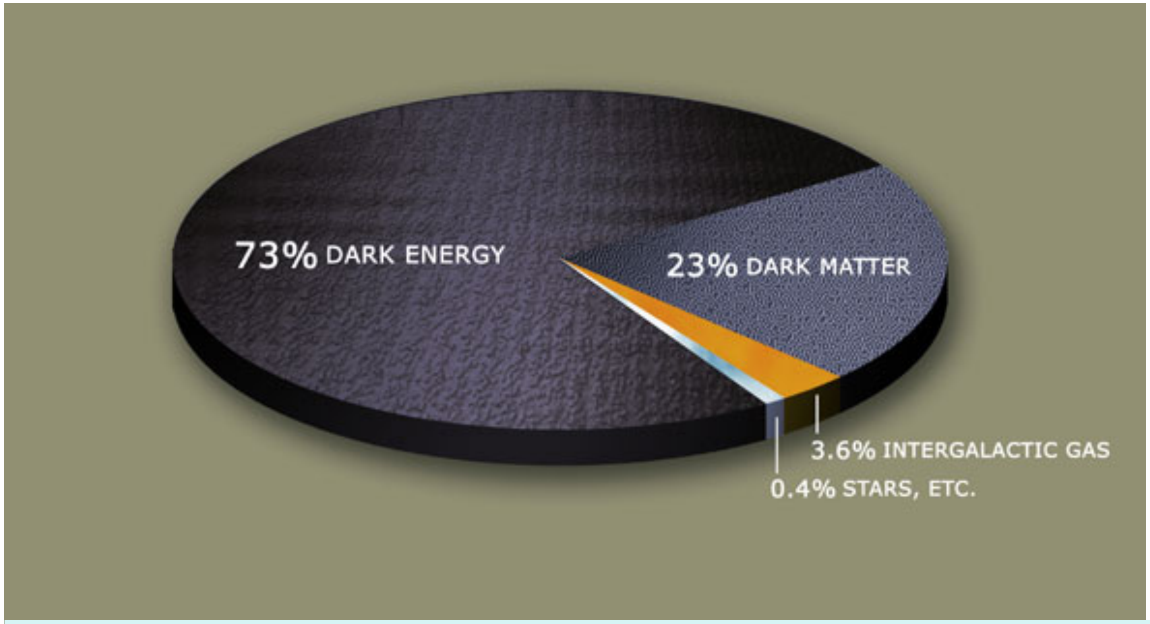
✓ ***Universe driven by scalar DM minimally coupled to GR,*** **dynamical approach**

✓ $$\frac{L}{\sqrt{-g}} = \frac{1}{2}R\,\zeta_\phi|\phi|^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi^* + L_D,$$

$$M_{Pl}^2 \sim \zeta_\phi\,|\phi|^2, \quad \Lambda_{cut} \sim M_{Pl}/\sqrt{\zeta_\phi}$$

✓ ***"Cross-over" free DM*** $\leftrightarrow$ $CC \Rightarrow$ ***fluctuations of ϕ excitations*** (T, ϱ, n)

Energy budget. Universe. Symmetries.



The subjects of the balance between visible matter & hidden matter in the Universe

"Missing mass/energy" – what are that?
New fields, particles, forces, ...? How to find out? Where is the ***symmetry***?

From History: first "dark matter" problem occurred at the **nuclear level**, and eventually new particles, **neutrons**, were identified as a source of a "hidden mass" – and immediately with the **new force of nature**, ***the strong interaction force.***

Hidden World *(what do we know)*

Known $\begin{cases} \textit{Galactic Moving} \\ \textit{Cosmic Microwave radiation} \end{cases}$

Observation in Universe (expansion) up to stars moving in galaxies can not be explained by ordinary matter

↓

HYPOTHESIS

NON-DETECTING DM **(GALAXIES ARE EMBEDDED INTO SPHERICAL HALO OF DM)**

DM: $\boldsymbol{\Omega_{DM} = 26.8 \pm 0.014}$ % *PLANCK COLLABORATION (2018)*

Ordinary matter: $\sim$ **5%** $\boldsymbol{constitutes\ only\ of\ E_{Univ}\ content}$

comprised ≥ 10 $elementary\ particles$

- ***Admit:*** **DM composed of different kinds of *fundamental entities***

&

Gravitationally clustered into macroscopic lumps – *e.g., Boson stars to display the universality*

Dark matter Nature (Unknown)

Widespread viewpoint

- Ultra-light axions (CP problem solved) Peccei, Quinn, 1977

- **WIMP,** $m \geq GeV$ *lightest SUSY, Neutralino* Jungman et al., 1996

- **Ultra-light bosons,** $\ll eV$
  gravitationally form macroscopic BEC Suarez et al.\Li et al., 2014

✓ **DM:** ***NO INTERACTION with EM sector***

- *No radiation*
- *No absorption*
- *No scattering*

❖ ***DM exploration:***

- ***DM production at accelerators,*** ***lifetime ??***
- ***DM interaction with baryonic matter*** ***portal?***
- ***Matter detection*** $\begin{cases} annihilation\ of\ DM \\ decays\ of\ DM \end{cases}$

○ ***What/*****where** ***is a SYMMETRY/*** **are the SYMMETRIES** ***?***

❖ Conformal symmetry

Max. space-time group symmetry *(SUSY no considered)*

SSB: $G_{conf}(d) \times G_{int} \to \Pi$ $(vector\ subgroups\ of\ x_\mu\ \&\ internal\ symmetries)$

$$factor\ space:\ \boldsymbol{g_{conf}} = \boldsymbol{e^{ix_\mu P^\mu}}\ \ \boldsymbol{e^{iB_\mu(x)K^\mu}}\ \ \boldsymbol{e^{iD\phi(x)}}$$

translations SCT Dilatation

Scalar DM, DILATON field $\boldsymbol{\phi}$ with $\langle\phi\rangle \neq 0$ $breaks\ conformal\ invariance\ at\ f > v$

$\widehat{D}\phi = d_\phi \phi,\ \widehat{K_\mu}\phi = 2x_\mu d_\phi \phi$ **No direct action of SCT on SDM $\boldsymbol{\phi}$.**

$\boldsymbol{Through\ x_\mu\ only}$

- **Vector field** $\boldsymbol{B_\mu}(x)$ $not\ necessary\ to\ describe\ any\ local\ fluctuations\ in\ vacuum$

$B_\mu(x) \sim f^{-1}\partial_\mu\phi(x)$ *Dark Photon field massive* *Non-primary operator* GK (2019)

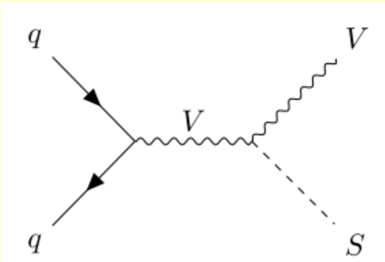
“Inverse Higgs condition”. To express the excessive field in terms of the “physical” one

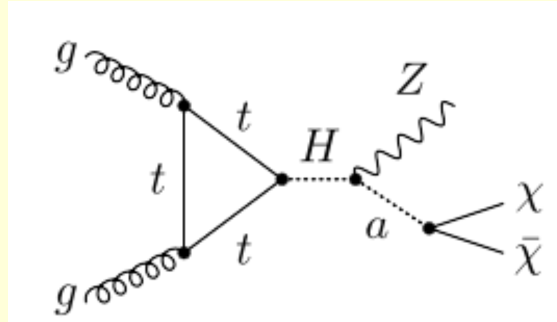
E. Ivanov, V. Ogievetsky (1975)

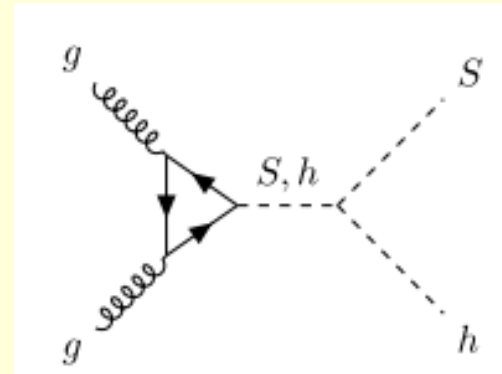
❑ Earth laboratory: *Search for Dark Matter Candidates*

The presence of DM can only be revealed by an imbalance in the transverse momentum in the detector measured as missing transverse momentum

An effective approach to DM within the *LHC Dark Matter Forum, ATLAS & CMS*

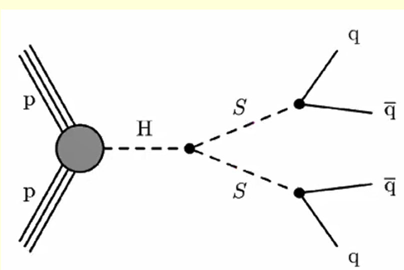


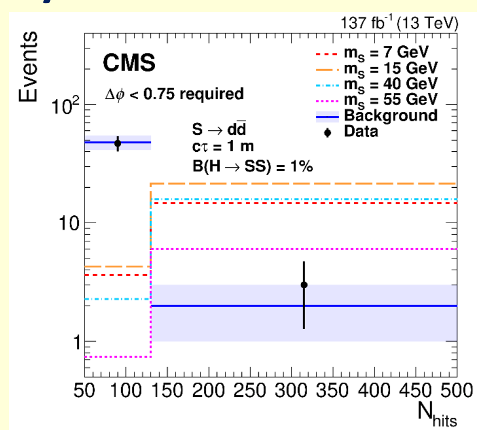


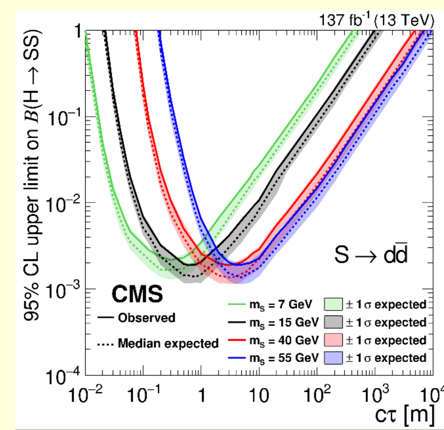


This exciting tool opens up a new program of searches for *LLPs* (Scalar DM, S) in a wide variety of theoretical models

$$h \rightarrow SS \rightarrow 4l \ or \ 4q$$







❑ LHC data used & need more

- $H \to ss,\ L \sim \lambda\alpha^2|H|^2 s^2 \ldots$

CMS Coll., *Phys. Rev. D99 (2019) 112003*

- *Higgs portal coupling constraints*

CMS Coll., *Phys. Lett. B793 (2019) 520*

- $H \to LLP \to \tau^+\tau^-\ (40 - 55\ GeV)$

CMS Coll., *Phys. Rev. Lett. 127 (2021) 261804*

- $\tau_h = 1.6 \cdot 10^{-22}\ sec,\ \ Higgs\ decay\ 95\%\ C.L.$
- $\Gamma_h = 3.2{+2.4}{-1.7}\ MeV$

CMS Coll., *Nature Phys. 18 (2022) 1329*

- ✓ *Higgs portal* $m_\phi \sim 0.1\ GeV\ - 0.5\ m_h$

CMS Coll., *Euro Phys. J. C83 (2023) 933*

❑ CMB radiation + perturbations

- Shed light on the composition of the Universe at recombination
- Planck satellite searched for: CMB spectrum's inhomogeneities

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Universe is flat (surprise) & homogeneous

- Inflation theory. Cosmological perturbations from quantum fluctuation of a single scalar field (scalar DM, dilaton)

- CMB spectrum measurement: $\Omega_{DM} \sim 0.26\, \Omega_{Univ}^{budget}$
  Evidence comes from the GR interactions of DM

➢ Universe evolution balance:

$$\frac{3}{8\pi} H^2 M_{Pl}^2 = \rho_{SM} + \rho_{DM} + \rho_{med} + \rho_{PBH}$$

- ***Boson star (BS)*** *forms a BEC in ground state,*
  Uncertainty Principle does keep the **BS** from collapsing

Opposite **FS**: stability achieved by equilibrium between the Fermi pressure and GR

❑ Main focus

Feebly interacting Dark Matter (DM) with SM @ high T and ρ_{DM}

DM Physics & Cosmology

↑

LHC data + Astro-Particle data

Early Universe: free q's, g's, DM species (different spins)

- $T\ cooled \rightarrow cosmological\ CP,\ Phase\ transition\ (PT)\ \leftrightarrow Symmetry\ breaking$

Separation of unification Fundamental Interactions

❑ ***Boson stars (BS's):*** ***SCALAR (s=0)*** ***/Colpi et al., 1986/*** ***PROCA (s=1)*** ***/Brito et al., 2016/***

↘ ↙

$DM\ \ \chi\ (s = 1/2)$

Stable $\mathbb{Z}_2$ $Symmetry, Thermal\ relic\ production$

- $T_c \sim O(MeV\ - TeV)$ ***a few µsec after Big Bang***

Scalar Dark Matter (SDM) + SM confined into BS (scale symmetry broken down)

- ***No observable signatures for physical processes (inside the BS) operated***
- ***Indirect astro-cosmological observations leave BS formation unknown yet***

Scalar Stars/Vector Stars

Bosonic - like Stars

⇓

Gravitationally-bound/Potentially-bound
bosonic structures in context of the DM search

DM ingredients/components

Boson star (BS), s=0 | *Proca star (s=1)*

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \Pi_{S/V}\right)$$

$\Pi_S \sim g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi, \dots)$

$\Pi_V \sim -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\mu^2 A_\mu A^\mu$

φ: *dilaton, ..., SDM*

A_μ: *Hidden vectors, Dark photon, ...*

$M^{BS} \sim M_{Pl}^2/\mu_s$ Lighter than M^{PS} compare $\mu_V \ll \mu_s$ $M^{PS} \sim M_{Pl}^2/\mu_V$ Heavier than M^{BS}

$M_{max}^{BS} \sim 10^{58}$ $GeV > M_\odot$ *GK (2023)* Ligo-Virgo GW signal 190521, PRL 2021

❑ Scalar Boson star. Lifetime.

- ***BS existence longevity*** is governed **by principles of symmetry**

- Lifetime $\boldsymbol{\tau_{BS}}$ *depends on* $\boldsymbol{\tau_O}$ *in* $\sim \frac{\xi m^2}{\Lambda} \boldsymbol{H^+ H(x) O(x)}$, $\quad O(x) = \sum_k c_k \varphi_k(x)$

- $\boldsymbol{\tau_H \ll \tau_O}$, **hidden scalar tower** ***GK (2022)***

- In the approximate $\mathbb{Z}_2$ *symmetry* $\boldsymbol{O(x)}$ *is the* **LLP** if $\xi \ll \frac{\Lambda v}{m^2}$

➢ $\boldsymbol{\tau_{BS} \sim \tau_O = \xi^{-2} \frac{(2v\Lambda)^2}{m\, m_h^3} \tau_h}$, $GK(2023)$ $\quad 7.7 \cdot 10^{-23} < \tau_h < 1.3 \cdot 10^{-21}$s

LLPs: $h \to \boldsymbol{OO} \to 2\tau^+ \, 2\tau^-, c\tau > 40\, m, m_{LLP} = 40\, GeV \quad CMS\ (2021)$

➢ $\boldsymbol{\tau_{BS} \sim \tau_O > 1.3 \cdot 10^{-7}\, s}$, $\quad \xi < 4 \cdot 10^{-2}$, $\quad \Lambda \sim O(M_{NP} \sim 10^5 \text{ TeV})$

▪ Boson star Minimal model.

BS is massive scalar object in the asymptotic flat space-time

BS field $X(x) \in \{h(x), O(x)\}$

$\downarrow$ Higgs

$\downarrow$ $\sum_{k=1}^{N} c_k \phi_k(x)$ ***hidden scalar tower***

$$L = \frac{1}{2i}\left[\left(\partial_\mu X\right)^2 - \left(\partial_\mu X^*\right)^2\right] + D_\mu X(D^\mu X^*) + \frac{1}{2i}(v^{*2}X^{*2} - v^2X^2)$$

$\phi \to a\phi,\quad h \to a^{-1}h \qquad D_\mu = \partial_\mu + igB_\mu \quad$ ***(DP)***

$$X(x) = \frac{\omega\phi(x) + i\kappa h(x)}{\sqrt{2}}$$

$$h(x) = \frac{\omega}{\kappa}\frac{Im\,(a^2v^2)}{Re(v^2)}\left[\phi(x) + f x_\mu B^\mu(x)\right] + C(x), \qquad (\Delta + Re\,v^2)C(x) = 0$$

✓ ADPM: ***DP*** $B_\mu(x) = m_{DP}^{-1}(const\, m_{DP}^{-2}\,\partial^2 - 1)\partial_\mu\phi(x)$ $\quad GK\ (2021)$

$$\left[P_\mu, \phi(x)\right] = -2ifB_\mu(x); \quad \left[P_\mu, h(x)\right] = -i\partial_\mu h(x); \ \theta(k^0)\phi(k)|\mathbf{0}\rangle = 0$$

vacuum

❖ BS formation. Mechanism. Fields + GR

Formation: spatial uniformity, topology, defects, electric fluxes

- $\frac{L}{\sqrt{-g}} = \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + L_D, \ G = M_{Pl}^{-2}$

- **Minimally coupling to GR:** $M_{Pl}^2 \sim \xi_\phi |\phi|^2, \ \xi_\phi \sim O(10^{-30})$

$$L_D = -\frac{1}{2}(\partial_\mu B_\nu)^2 + \frac{\theta}{2} \partial_\mu B_\nu \partial^\nu B^\mu + |(\partial_\mu + igB_\mu)\phi|^2 - \alpha(|\phi|^2 - \phi_0^2)^2 - f|\phi|^4$$

- $B_\mu(x) \to B_\mu(x) + \partial_\mu \Lambda(x), \quad \phi(x) \to e^{-ig\Lambda(x)} \phi(x), \ \partial^2 \Lambda(x) = 0$

Gen. curr. $\overline{k_\mu} \equiv k_\mu + \mathrm{P}_\mu = \partial^\nu B_{\mu\nu} + (\theta - 1)\partial_\mu \partial_\nu B^\nu, \ \theta \neq 1$ *nontrivial automorphism*

$\hookrightarrow$ ***phantom vector***

$$[Q_R, X(y)] = \left[\int \partial^i B_{oi}(\vec{x}) t_R(\vec{x}) d^3\vec{x}, X(y)\right] = -qX(y), t_R \subset S(\mathcal{R}^4), t_R = const, |\vec{x}| \leq R$$

Solution: $gB_\mu = \frac{1}{2g} \frac{\overline{k_\mu}}{\varphi^2} - \partial_\mu \eta, \ \phi(x) = \varphi(x) e^{i\eta(x)}$, $R = \frac{4f\phi_0^2}{\xi_\phi}, \ f \neq 0$

❖ ELECTRIC FLUX

- *Flux configuration:* $\Phi = \int B_{\mu\nu}(x) d\sigma^{\mu\nu} = \oint B_{\mu}(x) dx^{\mu}$

  $\hookrightarrow \Delta x^{\mu}\Delta x^{\nu}\ (\mu \neq \nu)\ in\ S(\mathcal{R}^4)$ space

At cosmological scale $\Phi \approx \frac{\theta - 1}{2g^2} \oint \partial_\mu (\partial \cdot B) \frac{1}{\varphi^2} dx^\mu - \frac{2\pi n}{g} \rightarrow -\frac{2\pi n}{g}$

winding topological number (integer)

- Number configurations of the FT $N(R) = V l^{-3} e^{\frac{sR}{l}},\ V \subset S(R^3)$

✓ Singularity at the center of the FT $\vec{\nabla} \times \vec{\nabla} \eta = 2\pi n\, \delta(\vec{x})\, \delta(\vec{y})\, \vec{e_z}$ $,\eta = n\,\theta_{az}$

Scale hierarchy/expansion $k_s = \frac{\sqrt{2\alpha}}{g} \begin{cases} < 1, & Attractive \\ > 1, & Repel,\ \ Star\ expansion\ (in\ size) \end{cases}$

- **Squeezing** $\varphi(r_s) \sim 0,\ \bar{B}(r_s) \sim 0,\ \ r_s \rightarrow 0;\ \theta = 0,\ \ NO\ Phantom\ vector$

- **Expansion** $\varphi(r_s) \rightarrow \phi_0,\ \bar{B}(r_s) \rightarrow \frac{n}{g},\ r_s \rightarrow \infty$

❖ *BS stability, decays*

- ✓ $\boldsymbol{A\ core\ of\ BS\ is\ hottest\ \&\ densest\ regions\ in\ the\ Universe}$
- ✓ $\boldsymbol{BS\ fluctuate, a\ best\ factory\ of\ HE\ particles\ beyond\ SM}$
- ✓ $\boldsymbol{BS\ unstable, decays\ to\ primary\ \gamma' s, \overline{\gamma'} s\ (dark\ photons)}$
- ✓ $Model\ dependent$

❑ *Boson Stars*. Phase transition. Observation

- **BS:** PT @ finite T is identified through ***observables*** (measured quantities)

- **Primary (direct) photons** registration at the detector.
  - *"Primary" operator means not a derivative of another operator*

- **Theory*****: primary photons induced by Conformal Anomaly***

  are in fluctuating regime

- **Bose-Einstein correlations of primary photons:**
  space-time distribution of hot matter prior to freeze-out *&*
  the size of the primary photons source GK (2011)

➢ *Origin of primary photons escape:*

Hidden scalars, **dilatons** *are warm, have BEC into compact* ***BSs****:*

exp.***: direct detection through primary photons.*** GK (2022-23)

❑ *Primary photons from **BS***

- In exact scale symmetry:

Hidden scalar – SM: $L = \frac{\phi}{f}\left(\Theta^{\mu}_{\mu\ tree} + \Theta^{\mu}_{\mu\ anom}\right)$

-$\sum_q m_q \bar{q} q + \cdots \hookleftarrow$ $\hookrightarrow$ ***in contrast to SM***

➢ Hidden scalars (***SDM)*** couple to **primary** $\boldsymbol{\gamma\gamma}$ *or* gg ***even before*** *running any SM in the loop* $\Rightarrow$ ***trace*** $\boldsymbol{\Theta^{\mu}_{\mu\ anom}}$

DP ↰

$$\sim -\boldsymbol{\alpha b_{EM} F^2_{\mu\nu} - \alpha_s \sum_i b_{oi} \left(G^a_{\mu\nu}\right)^2 - \bar{\varepsilon} F_{\mu\nu} B^{\mu\nu}}$$

Primary (direct) $\gamma' s$ *radiated by* ***BS*** *through the decays of* ***SDM***

- Indications? Observables? What's happed @ PT? ***BS*** stability?

❑ *Boson Stars stability.* *Thermal scenario*

○ ***BS*** *(N quantum states) in stat. equilibrium* $Z_N = Spe^{-H\beta},\ \beta = T^{-1}$

$$H = \sum_{1\le j\le N} H(j) = \sum_f F(f) b_f^{+} b_f = \sum_f F(f)\, n_f \quad (\text{in } f - \text{repres'n})$$

$$F(f) = E(f) - \mu\, Q(f),\ \ b_f \to b_f = a_f + r_f$$

random fluctuation

➢ Probability to form the ***BS*** *@ finite* ***T***

$$P(\bar\rho) = \sum_{N=1}^{\infty} Z_N \bar\rho^N \to \prod_f \left[1 - \bar\rho\ e^{-F(f)\beta}\right]^{-1},\ \ \bar\rho = \frac{\rho}{\rho_{BS}}$$ *SDM density*

$$\frac{P(\bar\rho)}{\bar\rho^N} = \sum_{N'=0}^{\infty} \frac{Z_{N'}\ \bar\rho^{N'}}{\bar\rho^N},\ \ 0 < \bar\rho < r_c,\ r_c \ge 1\ convergence\ radius$$

➢ **Minimization of the probability to formation of the *BS***

$$\frac{d}{d\bar\rho}\left[P(\bar\rho)\ \bar\rho^{-N}\right]_{\bar\rho=\bar\rho_0} = 0,\quad \bar\rho_0\ critical\ value, \textbf{BS is formed}, stable$$

❑ *BS*. *Thermo-statistical contribution*

- $\sum_f ln\left[1 - \bar{\rho} e^{-F(f)\beta}\right] = N K_{DM}(\bar{\rho}),$ *large enough N*

$$K_{DM}(\bar{\rho}) \sim \nu \int ln\left[1 - \bar{\rho} e^{-F(f)\beta}\right] df\,, \qquad \nu = \frac{\Omega_{BS}}{N} = const$$

➢ Condition to condensed formation of ***BS*** in the early Universe

$$\sum_f \bar{n}_f = \sum_f \frac{1}{\bar{\rho}_0^{-1} e^{F(f)\beta} - 1} = N \begin{cases} \boldsymbol{N \to \infty} \;\Rightarrow\; \boldsymbol{light}\, SDM, \\ \bar{\rho}_0 \to 1, very\ high\, T \end{cases}$$

↓

ground state of $\bar{\rho} = \rho/\rho_{BS}$ ($\bar{\rho}_0$ can be extracted/ estimated)

Essential *changes* $T \to \infty$, $\bar{\rho}_0 \to 0$; *induced by strong* $B^s \sim \frac{\mu_s^2}{e} \sim O(10^{20})T$ **Early U.**

➢ *Above* B^s *the scale symmetry should be restored.*

➢ *At lower T the strong* B^s *may exist in the* **late Universe** *in the vicinity of the magnetized Black Holes*

❖ *Warm & cold BSs*

$$\left.\begin{array}{l}\textbf{Warm}\ BS\ \ \bar{\rho}_0 e^{\mu Q\beta} < 1\ (T > T_c) \\ \textbf{Cold}\ BS\ \ \bar{\rho}_0 e^{\mu Q\beta} \sim 1\ (T < T_c)\end{array}\right\}\quad \sum_f \bar{n}_f = \sum_f \frac{1}{\bar{\rho}_0^{-1} e^{F(f)\beta} - 1} = N$$

- ➢ ***Warm BS,*** $SDM\ \mu_s > 2\pi\beta/(\nu B)^{-2/3}$, B=2,612…, $\quad v = \Omega_{BS}/N$
- ➢ $\mu_s \to 0$ ***when*** $T \to \infty$ ***or*** $\Omega_{BS} \to \infty$, $N \to 0$ ***symmetry restored***

❑ Warm ***BS*** $\quad T_c = \dfrac{2\pi}{\mu_s}(2{,}612\ldots\ v)^{-2/3}, \quad SDM \quad \mu_s \neq 0$

✓ $\nu \sim \left(\dfrac{M^*}{M_{Pl}^2}\right)^3 \dfrac{1}{N} = const\ @\ N \to \infty,\ \Omega_{BS} \to \infty$

- $M^* \sim \dfrac{M_{Pl}^2}{\mu_s}$, $\Omega_{BS} \sim \left(\dfrac{M^*}{M_{Pl}^2}\right)^3$, $p \sim \mu_s$, $M_{Pl} \approx 1.2 \cdot 10^{19}\ GeV$

❖ Cold *BS*

❖ Cold ***BS*** when $T < T_c,\ \bar{\rho}_0 \to 1$ ***BS*** is formed already

❖ The small values of the ***SDM*** momenta, $|p| \leq \delta$, are most important for Bose-Einstein condensation

$$\lim_{\delta\to 0,\ N\to\infty} \frac{1}{\Omega} \sum_{|p|\leq\delta} \bar{n}_p = \frac{1}{v}\left[1 - \left(\frac{\beta_c}{\beta}\right)^{3/2}\right]$$

- Only part $\sim(\beta_c/\beta)^{3/2}$ *of the* ***SDM*** *distributed inside* ***BS***

Result:

- *Low T*. The ***SDM*** condensate $\sim \left[1 - (\beta_c/\beta)^{3/2}\right]$ large value
- *High T*. *Th*e condensates stay almost close to zero.

❖ *Fluctuations of scalars inside the BS* – T dependent

Event-by-event fluctuation of the **SDM** *density @ T in* ***BS*** *volume* $V < \Omega_{BS}$

$$\frac{\langle (n_V - \langle n_V \rangle)^2 \rangle}{\langle n_V \rangle} - 1 = \frac{\sqrt{2}\, v}{\pi^2} (\mu_s T)^{\frac{3}{2}} \int_0^\infty \frac{x^2 dx}{\left(\bar{\rho}_0^{-1} e^{-\mu Q \beta} e^{x^2} - 1\right)^2}$$

$\downarrow V/\Omega_{BS}$

- *Increase sharply if* $T \to \mu Q / ln(1/\bar{\rho}_0)$

▪ ***PT*** *approached (vicinity of* **CP**), ***Formation of the BS***

$$\frac{\langle (n_V - \langle n_V \rangle)^2 \rangle}{\langle n_V \rangle} - 1 \sim \frac{4}{\sqrt{\pi}\, 2{,}612 \ldots.} \int_0^\infty \frac{x^2 dx}{\left(e^{x^2} - 1\right)^2}$$

❑ *No free parameters: neither* ***SDM*** *mass* μ_s, *nor* T.

✓ **Non-monotonous rising if** $\bar{\rho}_0 \sim O(1)$ **@** $\mathbf{T} \sim T_c$, $(N \to \infty)$ **!**

✓ ***Infinitely increasing behavior! Phase transition.***

✓ ***BS may explode.***

Registrations. Observations

❑ ***BS.*** By CI, the beta functions including ALL states (CFT + SM) **vanish.**

If $\left.\begin{matrix} QCD \\ EM \end{matrix}\right\} \in \boldsymbol{conformal\ sector} \rightarrow \sum_{light} b_o = -\sum_{heavy} b_o\ (Higgs)$

quark-lepton conformal condition

- **The *SDM* mass splits *light* and *heavy* *(q's & l's)* states !**

$$\frac{\beta(g)}{2g}\left(G^a_{\mu\nu}\right)^2 \rightarrow \frac{\alpha_s}{8\pi} b_o^{light}\left(G^a_{\mu\nu}\right)^2, \qquad b_o^{light} = -11 + \frac{2}{3} n_L$$

$m_\phi \sim O(\Lambda) \rightarrow\ n_L = 3$: $COUPLING\ STRENGTH \sim \frac{gg\phi}{ggh} \sim 14!$ *increase*

Low-energy eff. $\left\langle \gamma\gamma \middle| b_0^{light} \alpha_s \left(G^a_{\mu\nu}\right)^2 \middle| 0 \right\rangle = -\left\langle \gamma\gamma \middle| b_{EM} \alpha \left(F_{\mu\nu}\right)^2 \middle| 0 \right\rangle, \vec{q} \approx 0$

- **Primary photons emission:**

$-8/3$ ↰

$$\Gamma(\phi \rightarrow \gamma\gamma) \cong \left(\frac{\alpha F_{anom}}{4\pi}\right)^2 \frac{m_\phi^3}{16\pi f^2}, \quad F_{anom} = -\left(\frac{2n_L}{3}\right)\left(\frac{b_{EM}}{b_0^{light}}\right)$$

❑ BS. SU(N) Hidden sector

Soni (2016)

➢ ***Direct coupling.*** *"Glueball" ϕ.*

$$\frac{1}{M_{cut}^4}\left(H_{\mu\nu}\right)^2\left(F_{\alpha\beta}\right)^2 \rightarrow \frac{N m_\phi^3}{M_{cut}^4}\phi\left(F_{\alpha\beta}\right)^2$$ *"EM portal"*

$SU(N)_{hidden\ gauge\ group}$

❑ **Primary $\gamma' s$ *emission* (*direct point – like in BS*):**

$$\Gamma(\phi \rightarrow \gamma\gamma) = \frac{1}{4\pi} m_\phi N^2 \left(\frac{m_\phi}{M_{cut}}\right)^8$$

the value N for a self-interacting ϕ: $N \approx Max\left[\left(0.1\, GeV/m_\phi\right)^{3/4}, 2\right]$

➢ **Combined result [*conformal anomaly* $\leftrightarrow SU(N)_{hidden}$]:**

$$M_{cut} > 3.4\, GeV, \qquad \Lambda = 330\, MeV$$

$m_\phi \sim O(\Lambda)$

$$M_{cut} > 5.2\, GeV, \qquad \Lambda = 500\, MeV$$

GK 2021

❑ *Primary photons. Observation.*

BSs unstable, showers of $\gamma\gamma,\ \bar{\gamma}\bar{\gamma},\ \bar{\gamma}\gamma, \ldots$ $(conformal\ EM\ anomaly)$

- **Measurement of $\gamma's$ *escape* $-$ *decisive way to observe* & *differentiate* primary $\gamma's$ and *ordinary* $\gamma's$ $(\pi^0 \to \gamma\gamma, \ldots)$**

- **Fluctuation rate of *primary* $\gamma's$ *production in approximate conformal sector* (*proximity to PT*)** $f \sim O(\Lambda), f_\pi \approx 0.3\Lambda$

$$r_{\gamma\gamma} = 1 + \frac{BR(\pi^0 \to \gamma\gamma)}{BR(\phi \to \gamma\gamma)} = 1 + m_\pi^3 \left(\frac{6}{F_{anom}}\right)^2 \xi^3$$

❑ Abundant $\gamma's$ *escape*: $r_{\gamma\gamma} \to \infty$ *as* $\xi(T \to T_c) \to \infty$

and with $n_L \to 0$, *and* $N_f \to N_f^c$, $m_\phi \approx \left(1 - \frac{N_f}{N_f^c}\right)^{\frac{1}{2}} \Lambda$

N_f^c separates ***conformal phase*** from the one with the ***chiral symmetry***

✓ **Result:** ***Fluctuation of the BSs in the proximity to* IRFP**

❖ Dark photons. SDM Decay. Observations.

- **BS formation,** $V(S, Higgs) + Gravity$, *instability* ***SDM decays***

- $S \to \gamma\bar{\gamma} \to \gamma l\bar{l}$, $\sim C_A \Lambda^{1-d}\, \bar{l}\, \gamma^{\mu}\gamma^{5} l\, \partial_{\mu} S$, $\gamma - \bar{\gamma}$ *interference*, $\sim \varepsilon^2$ *mixing*

Lifetime $\bar{\gamma}$, $\tau_{\bar{\gamma}} \sim \left(\frac{\Lambda}{\tilde{\mu}}\right)^{2(d-1)} \frac{1}{C_A^2\, \varepsilon^2\, \tilde{\mu}}$ *mean displacement of the vertex*

- $\tau_{\bar{\gamma}} \geq 10^{+8} sec$! , *if* $d \to 2$, $C_A \sim O(1)$, $\Lambda \sim O\left(M_{pl} \sim 10^{19}\, GeV\right)$

Candidate to DM *(almost stable)*

Exp. Constraints ↷

BABAR (2017) & *NA64 (2019)* $\varepsilon \leq 10^{-3}$ $\tilde{\mu} \leq 8\, GeV$

$\tau_{\bar{\gamma}} \geq 10^{+8} sec$ Compared to $\tau_{Universe} \sim 10^{+17} sec$

❖SDM. Dark Photon. Registration I

$gg \to \phi \to \gamma\gamma, \gamma\bar{\gamma}, \bar{\gamma}\bar{\gamma}$ $\left(\frac{33}{2} - n_L\right) \sim O(10)$ Compared to that of the $gg \to H$

$$For\ exp.: \ \Gamma_{\gamma\gamma} = 2\left(\frac{\mu}{8\pi}\right)^3 \left(\frac{\alpha b_{EM}}{f}\right)^2 \leq [0.013\ -0.24]\ KeV$$

$b_{EM} = -\frac{80}{9}$, $\mu < 2m_W$, $\sim [60 - 160]\ GeV$ $f \geq 3\ TeV$ $LHC \begin{cases} di - jets \\ di - photons \end{cases}$

For exp.: $\sim\varepsilon^2$: $\bar{\gamma} \to l\bar{l}$ $(l: e, \mu)$ DP's contribution

$$\Gamma_{\bar{\gamma}l\bar{l}} = \frac{1}{3}\alpha m\varepsilon^2 \left[1 - \left(\frac{2m_l}{m}\right)^2\right]^{1/2} \left[1 + 2\left(\frac{m_l}{m}\right)^2\right]$$

TH. $m > 2m_l$, $\varepsilon < 3 \cdot 10^{-2}$ $GK2016$ $\left(\varepsilon^{exp} \leq 10^{-3}\right)$,

TH. $m < 3.3\ GeV$ … $(m^{exp} \leq 8\ GeV)$

❖Dark Photon. Registration II

$$\Gamma_{tot} \cong \frac{1}{3}\alpha m \varepsilon^2 \left\{1 + A_{(\mu^+\mu^-)}\left[1 + R_{(had/\mu^+\mu^-)}\right]\right\}$$

$$A_{(\mu^+\mu^-)} = \left[1 - \left(\frac{2m_\mu}{m}\right)^2\right]^{1/2}\left[1 + 2\left(\frac{m_\mu}{m}\right)^2\right]$$

- $R_{(had/\mu^+\mu^-)} = \frac{\sigma(e^+e^-\to hadrons)}{\sigma(e^+e^-\to\mu^+\mu^-)} = \frac{6\pi}{\alpha} g_{\phi\gamma\gamma} f, \qquad m > 2m_\mu$

Bazar, Kharzeev, Skokov, 2012

$$g_{\phi\gamma\gamma} = \left[-\alpha b_{EM}/(8\pi f)\right] \leq 10^{-6}\, GeV^{-1}$$

G.K. 2021

$R_{(had/\mu^+\mu^-)}$: $|(3/4)b_{EM}| \sim 7$ *increasing in hh – channel compared to that of* $\mu^+\mu^-$

❑ If NO excess found → *bounds on* $\boldsymbol{\varepsilon}$ *–* ***mixing***, $\quad \boldsymbol{\varepsilon}(m, m_\phi)$

- **Bright application: KOTO exp. anomaly:**

✓ $K_L \to \pi^0 \nu\bar{\nu}$ ***observed excess***:

due to scalar (LLP) missing energy, $K_L \to \pi^0(LLP)$.

Shinohara, Egana – Ugrinovich, 2020

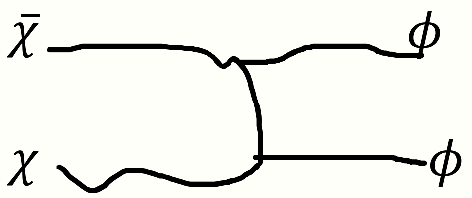
BS. Early Universe.

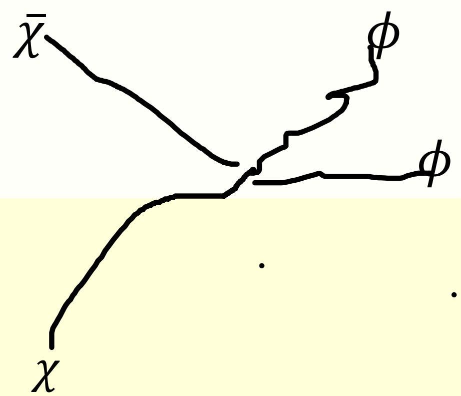
- *Early Universe.* $\boldsymbol{Thermal\ \chi\ - clouds\ form\ BS's\ (\mathcal{R} \to \infty)}$
- *Compound composite stars:*
- *Proca Star* + *Axion Star* $\Rightarrow \boldsymbol{X_*^2 = V_\mu V^\mu + A^2}$ $\qquad V.Fock, 1929$
- $\boldsymbol{V_\mu = \bar{\chi}\gamma_\mu\chi}$ $\qquad \boldsymbol{A}$ $\qquad$ *M. Fierz, 1937*

- $\boldsymbol{T < T_c}$: $\boldsymbol{Confinement\ of\ SDM, Abelian\ Higgs\ effect\ (SDM \backslash \gamma, \bar{\gamma}, \bar{g})}$
- *Evolving star, SSB by* $\boldsymbol{\bar{\chi}\chi\ condensate, m, conformal\ anomaly}$

- *SSB:* $\boldsymbol{\langle 0|D^\mu(0)|X_*(q)\rangle = -i\Lambda_* q^\mu,}$ $\qquad \boldsymbol{\langle 0|\,\partial_\mu D^\mu(0)\,|X_*(q)\rangle = -\Lambda_* M_*^2}$
- $\downarrow$

  $\boldsymbol{m_h \sim M_{Pl} e^{-a}, a \sim 40,}$ $\qquad \boldsymbol{\sim O(M_{Pl})}$





❑ BS. Thermo-Dynamics

➢ *Cosmological inputs:*

- *Cosmo CP; Bgr.* $\rho_{\gamma l} \sim T^4 \sim O(10^{39}\ GeV\,cm^{-3}) \gg \rho_{\odot} \approx 0.43\ GeV\,cm^{-3}$

- $\boldsymbol{DM\ \chi\ \ m \sim 0.1 - 10\ TeV\ based\ on\ \Omega_{DM}h^2 \to \Omega_{Pl}h^2 \sim 0.2\ \ Planck\ Coll.,2016}$

- ***BS Cosmo barrier (high to low*** $\boldsymbol{\rho}$***)*** $\boldsymbol{\rho_* \ll 10^{-10}\rho_{\odot}}$ ***G.K. 2023***

○ *Low* ρ: free gas SDM + SM

- *Th. Dynamics* $\boldsymbol{E_* = -PV + sT + \mu N,\quad P > 0\ if\ \ T, |\mu|\ large}$

- *BS* $(R \to \infty)$: $T, |\mu|$ *low,* ***P < 0 binding the SDM inside***

- *CP:* $\boldsymbol{P(T_c, \mu_c) = 0,\ \ energy\ fed\ into\ BS \sim e^{(E_* - \mu_c N)/T}}$
- ***BS*** *unstable, SDM* $\phi \to \overline{\gamma}' s, \bar{\nu}\nu$ $\boldsymbol{LHC\ data\ need}$!

❑ Interactions. Spinor DM. Higgs Portal

$$L \supset y_\chi \hat{m} \bar{\chi}\chi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \ - \Omega, \qquad \hat{m} = \frac{m\phi}{\phi_0} = m\varphi$$

❖ ***DM χ enjoys a global $U(1)$ symmetry $\rightarrow$ ensures stability***

- ***Below PT: Yukawa int's $m \sim y_\chi \phi_0 > T_c \sim m_\phi$, $\phi_0 = \langle\phi\rangle$***
- ***ThDP: $\Omega = V(\phi, |H|^2) + \Omega_{DM} + \Omega_{T=0}$***
- ***Higgs Portal allows SDM to decay to SM, BS instability***
- $V(\phi, |H|^2) = \frac{1}{4}\kappa_D \phi^2 \ |H|^2 + \frac{\lambda}{4}\phi^4 \left[ln\left(\frac{\phi}{\phi_0}\right) - \frac{1}{4}\right],$

$\downarrow$ $\downarrow$

subjects to LHC

- $\kappa_D \leq O(1), \quad H = \frac{v+h}{\sqrt{2}}, \quad \langle|H|^2\rangle = \frac{v^2}{2}$
- $m_\phi^2 = \frac{d^2\phi}{d\phi^2}\Big|_{\langle\phi\rangle=\phi_0}, \quad \lambda = \left(\frac{m_\phi}{\phi_0}\right)^2 (1-\delta_D), \quad \delta_D = \kappa_D \left(\frac{v}{2m_\phi}\right)^2$

❑ ThDP

$$\Omega = V(\phi, |H|^2) + \Omega_{DM} + \Omega_{T=0}$$

- *Open cosmological system* $\subset\ \chi\, DM + SDM$
- $\Omega_D = -\frac{1}{\beta\, V} ln(Z_0 \cdot Z[\varphi, \mu])$
- $Z[\varphi,\mu] = \int D\bar{\chi}\, D\chi\, exp\left[-\int_0^\beta d\tau \int_V\ d^3\vec{x}\,\bar{\chi}\,(p_\mu\gamma^\mu - \widehat{m})\chi + \mu\bar{\chi}\chi\right]$

✓ **SDM** ← *decoupled thermally* → **SM;** $T_{DM} \neq T_{SM},\ T_{DM} < T_{SM}$ *freeze − out*

Min. ThDP: $\widetilde{\varphi^2}\, ln\widetilde{\varphi} = \frac{1}{\lambda\phi_0^2}\left\{\left(\frac{2m}{\phi_0}\right)^2 [F(\beta,\mu) + \mu \to -\mu] - \frac{\kappa_D}{4}v^2\right\}, F(\beta,\mu)=\int d^3p \frac{n_p}{\sqrt{p^2+\widetilde{m^2}}}$

$n_p = (1 + e^{E\beta})^{-1}$ *occupation number*; $\quad E = \sqrt{p^2 + \widetilde{m^2}},\ \ \widetilde{m} = m\widetilde{\varphi}$

SDM field fluctuation with *T around its equilibrium state*

➢ $\widetilde{\varphi}(\beta) \approx 1 + \frac{1}{\lambda\phi_0^2}\left[\left(\frac{2m}{\phi_0}\right)^{5/2} \frac{2}{\phi_0^2\,\beta^{3/2}} e^{-m\beta} - \frac{1}{4}\kappa_D v^2\right]$

✓ **DM can exist at** $T = 0$, *total condensate, no Higgs portal* $\kappa_D = 0$

Higgs Portal. Details. Couplings

- $DM\ inter's\ Higgs\ portal\ m_\phi \sim [0.1\ GeV, m_h/2]$ *CMS 2023*

  $DM\ Relic\ density\ \Omega_D h^2 = 0.1199 \pm 0.0027$ *Planc Coll. 2014*

$$\Downarrow$$

- $m_\phi > 1\ TeV\ \ \sim O(1) \leftarrow \kappa_D \rightarrow \sim O(10^{-4}),\qquad m_\phi \sim m_h/2$
- *CMS:* $BR(h \rightarrow inv) < 0.15,\ @95\%\ C.L.$ *CMS, 2023*
- $\downarrow$
- $\kappa_D < 0.034,\qquad m_\phi \sim 62\ GeV$
- *CMS:* $\kappa_D < 0.028 \left[1 - \left(2m_\phi/m_h\right)^2\right]^{-1/4},\quad m_\phi < \frac{m_h}{2}$ *CMS, 2019*

- $EW\ \phi_0 \sim O\left(10^3 GeV\right)$ *LHC, 2020* $\Rightarrow \lambda \sim O(10^{-4})$
- **Vacuum energy density:**

$$|\rho| \sim \frac{m_\phi^4}{16\lambda}(1 - 6\delta_D) \sim O\left(10^{50} GeV\ cm^{-3}\right)$$

$\lambda = 0\ configuratin$ $\qquad \rho \sim \kappa_D \phi_0^2 v^2 \le 3 \cdot 10^{50} GeV\ cm^{-3}$

❖ *Inside a star. Binding*

- $PT\ binding\ SDM + SM = scalar\ mixed\ state$

$\downarrow$

Expand and merge into BS under $GR + \lambda\ self - couplings$

$until\ Universe\ evolved\ and\ transitioned$

Critical: inter'n energy inside? Relativistic? typical $mom.\ p \sim R_*^{-1} \sim m_\phi$

- **Heisenberg:** $\lambda \to 0, \kappa_D \to 0.\ Equilibrium\ M_*^{eq} = M_{Pl}^2/m_\phi$ "relat. equil. mass"

> **Interaction effect:** $\left(\frac{int}{non-int}\right) \sim Q_{GPE} \sim V(\phi, |H|^2)/(m_\phi^2\, \phi^2)$

❑ **Families of equilibria between the fields and GR**

$$Q_{GPE} = \frac{1}{2}\delta_D + \lambda_{eff}\left(\frac{M_{Pl}}{2m_\phi}\right)^2 \qquad \delta_D = \kappa_D\left(\frac{v}{2m_\phi}\right)^2,\ \lambda_{eff} = \lambda\left[\ln\left(\frac{M_{Pl}}{\phi_0}\right) - \frac{1}{4}\right]$$

The λ_{eff} may only be ignored if $\lambda < O(10^{-35})\ at\ Q_{GPE} \ll 1$

$\kappa_D < 0.034, \quad m_\phi \cong \frac{m_h}{2}\ CMS\ (2023), \quad \phi_0 \cong 3 \cdot 10^3\ GeV\ \ LHC$ **di-photon, di-jets, 60-160 GeV**

❖ Gravity-potential equilibrium. Max BS mass

❑ **Families of equilibria between the fields and GR**

$$Q_{GPE} = \frac{1}{2}\delta_D + \lambda_{eff}\left(\frac{M_{Pl}}{2m_\phi}\right)^2 \qquad \delta_D = \kappa_D\left(\frac{v}{2m_\phi}\right)^2, \ \lambda_{eff} = \lambda\left[\ln\left(\frac{M_{Pl}}{\phi_0}\right) - \frac{1}{4}\right]$$

Characteristic of marginally relativistic BS with $Q_{GPE} \sim O\left(10^{32}\right)$ *and* $m_\phi \sim m_h/2$

$$M_*^{max,eq} \sim \sqrt{Q_{GPE}}M_*^{eq} \sim \sqrt{\lambda_{eff}}\frac{M_{Pl}^3}{m_\phi^2} \sim O\left(10^{52}\ GeV\right) < M_\odot \sim O\left(10^{57}\ GeV\right), \ Q_{GPE} \gg 1$$

✓ $M_*^{max,eq} \sim M_{PBH} \sim 10^{-5} M_\odot$ *comprising* 1% *of total DM*

U-short timescale microlensing event, optical GR lensing OGLE, Niikura et al., '19

$$M_*^{max} < \left(\frac{1-\delta_D}{2}\right)^{1/2}\left(\frac{\pi}{2m}\right)^{5/4}\frac{M_{Pl}^2\,\phi_0}{T^{3/4}}e^{m/2T} \sim O\left(10^{70} GeV\right)$$

Astrophysical relevant for therm. produced $DM\ with\ m \sim 10\ TeV, T_c \sim m_\phi$

✓ **Thermal Dynamical indicator for stable and unstable configurations of the BS**

❖BS size. Critical temperature

$$M_*^{max} < \left(\frac{1-\delta_D}{2}\right)^{1/2} \left(\frac{\pi}{2m}\right)^{5/4} \frac{M_{Pl}^2\,\phi_0}{T^{3/4}}\, e^{m/2T} \sim O\left(10^{70} GeV\right)$$

➢ **BS nucleates wide @ T_c: $R_* < M_*^{max} M_{Pl}^{-2} \sim O\left(10^{13} km\right),\ T_c < m$**

Then shrinks, dense, accumulates SDM + Higgs as T < T_c

BEC/SDM condensate $\sum_f n_f = \sum_f \frac{1}{\rho_0^{-1} e^{F(f)\beta} - 1} = N,\ F(f) = E(f) - \mu Q(f)$

$\bar{\rho} = \rho_*/\rho$, **stable** $\bar{\rho} = \overline{\rho_0}$ **BS formed,** $\overline{\rho_0} \to 1, high\, T,\ N\, large$

$$T_c = 2\pi \left[\frac{N}{\zeta(3/2)}\right]^{2/3} \left(\frac{M_{Pl}^2}{M_*^{max}}\right)^2 \frac{1}{m_\phi} \sim 10^{-65}\ N^{2/3}\ \text{GeV}$$

- $N \sim O\left(10^{100}\right)\ for\ T_c > m_\phi$

If $m_\phi > T_c$ *condensate is weak, ECO of DM with R and M is neither NS, nor the BH*

❖Lower bound on the BS mass

❑ Relation between energy densities of DM in *galactic halos* and *the ⊙ system*

$$\rho_{DM}^{MW} = M_{DM}^{MW}\Omega_{halo} = \rho_* + \rho_\chi + \rho_{\phi/h}$$

$MW\ halo\quad \Omega_{halo} = \left(4\pi R_{halo}^3/3\right)^{-1};$

$BS\quad \rho_* = M_* n_*,\quad n_* \ll (1.5-1.8)\times 10^{-37} cm^{-3}\quad GK\ (2023)$

$DM\quad \rho_\chi = m\, n_\chi, n_\chi \sim O\left(10^{-22} cm^{-3}\right),\qquad Baker\ et\ al., (2020)$

SDM-Higgs $\rho_{\phi/h} = m_\phi n_\phi (1+\delta_h), n_\phi < \pi \cdot 10^{-11} cm^{-3}, \delta_h < 10^{-14}$, $GK\ (2023)$

$Result:\quad M_* \gg (3.9-4.7)\times 10^{35}\ GeV$

$M_{DM}^{MW} \sim 0.95 M^{MW} \sim 10^{12} M_\odot,\qquad M^{MW} = (0.8-1.5)\times 10^{12} M_\odot$


DM (+SM) is the Fundamental *Entity* in the Universe
SYMMETRIES
Church and the Empire were fused in a single *Entity*
Thank you!