Minimal SU(5): *the importance of being effective*

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Abetone 2024

In collaboration with Goran Senjanović

Why Grand Unification

Grand unification is one of the most appealing candidates for physics beyond Standard Model

↓

Charge quantization
Existence of magnetic monopoles

Proton decay

Unification of gauge forces

Typical scales at around $10^{14} - 10^{16}$ GeV. Is there any hope to probe it?

Magnetic Monopoles

Existence of magnetic monopoles implies quantization of charge

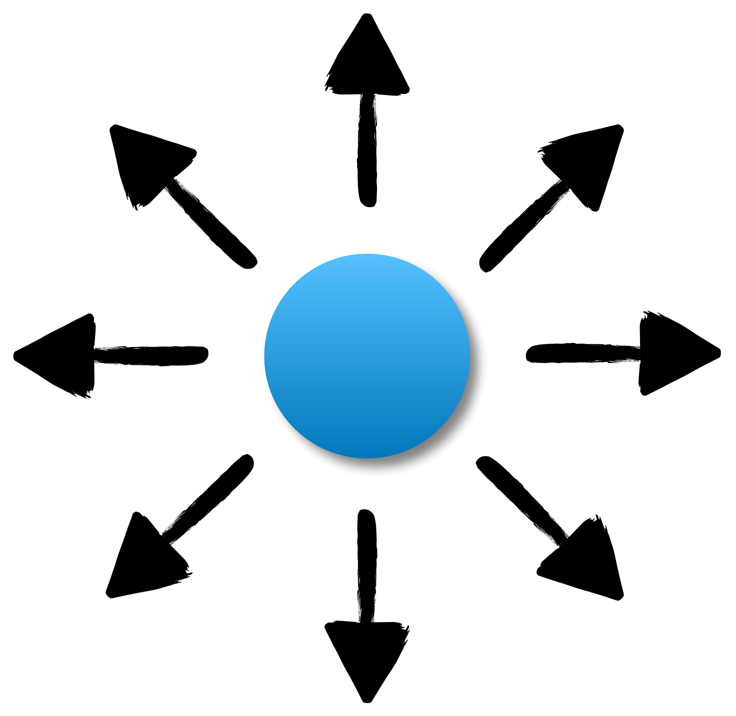
Dirac '31

$$g_m e_q = 2\pi n$$

Wu, Yang '68

Grand Unified theories charge is quantized and leads to the existence of magnetic monopoles

First explicit solution in $SU(2)$:
't Hooft '74, Polyakov '74 monopoles



No monopole has been observed so far. However, one would expect them to be produced in the early Universe.

Kibble '76

→ Monopole problem

Zel'dovich, Khlopov '78
Preskill '79

Minimal $SU(5)$ model: an instructive predictive failure

Georgi Glashow model '74 is the minimal theory

Gauge Sector

$$24_{\text{gauge}} = 8_{\text{C}} + 3_{\text{L}} + 1 + (3_{\text{C}}, 2_{\text{L}}) + (\overline{3}_{\text{C}}, 2_{\text{L}})$$

$$= \text{gluons} + W's + B + (X, Y) + (\overline{X}, \overline{Y})$$

Fermion Sector

$$\overline{5}_{\text{F}} = (d^{r,g,b}, \nu, e)_{\text{L}} \,,$$

$$10_{\text{F}} = (u^c, u, d, e^c)_{\text{L}}$$

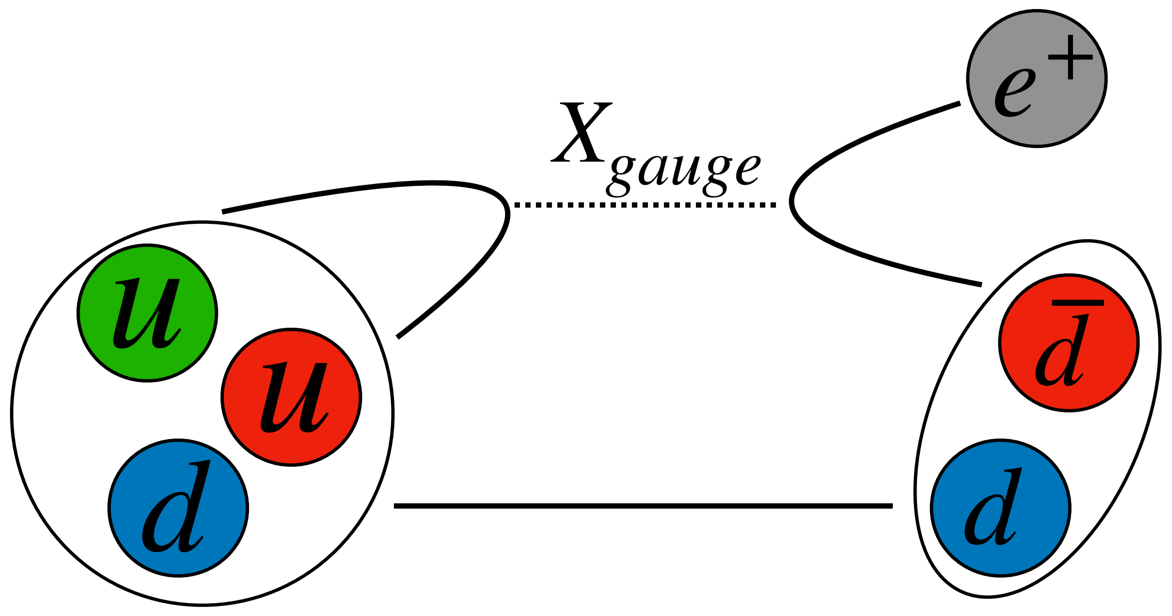
Baryon number violating interaction

$$\mathcal{L}_{\not{B}} = \frac{\alpha_{\text{GUT}}}{M_X^2} \left[(\overline{u}^c u)\,(\overline{e}^c d + \overline{d}^c e) + (\overline{u}^c d)\,(\overline{e}^c u + \overline{d}^c \nu) \right]$$

↓

Leads to p-decay

Minimal $SU(5)$ model: Proton decay



Channel	Lifetime (10^{30}yrs)
$N \to e^+ \pi$	5300 (n), 16000 (p)
$N \to \mu^+ \pi$	3500 (n), 7700 (p)
$N \to \nu \pi$	1100 (n), 390 (p)
$N \to e^+ K$	17 (n), 1000 (p)
$N \to \mu^+ K$	26 (n), 1600 (p)
$N \to \nu K$	86 (n), 5900 (p)

Proton lifetime from Super-Kamiokande requires

$$\tau_p = \simeq C \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} \left(m_{\text{proton}}\right)^{-5} \gtrsim 10^{34}\text{yrs} \longrightarrow M_{\text{GUT}} \gtrsim 4 \cdot 10^{15}\text{GeV} \left(\frac{\alpha_{\text{GUT}}}{40^{-1}}\right)^{1/2}$$

C is of order one and branching ratio are predicted *Mohapatra '79*. The same is not true in extensions.
Crucial: it can spoil relation between τ_p and M_{GUT}

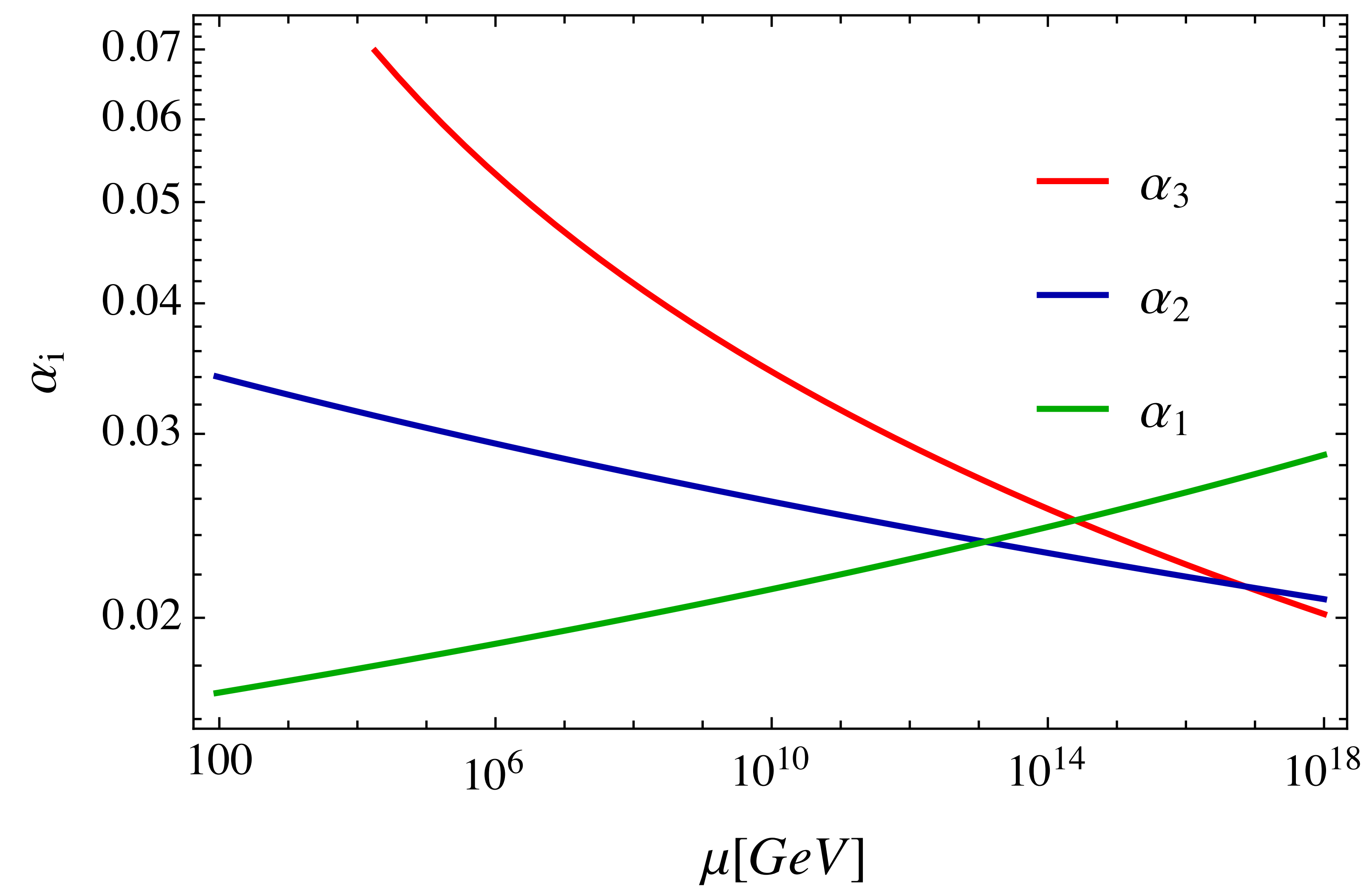
Scales in minimal $SU(5)$

Georgi, Quinn, Weinberg '74: Unification of gauge couplings in SM

Values found by GQW:

$g_{3o}^2/4\pi$	M (GeV)	$\sin^2\theta$
0.5	2×10^{17}	0.175
0.2	2×10^{16}	0.187
0.1	5×10^{14}	0.207
0.05	2×10^{11}	0.248

Standard Model failure



However, new particle states can aid unification:

GUT symmetry breaking realized via $24_H = 8_C + 3_L + 1 + (3_C,2_L) + (\bar{3}_C,2_L)$

Eaten by X, Y

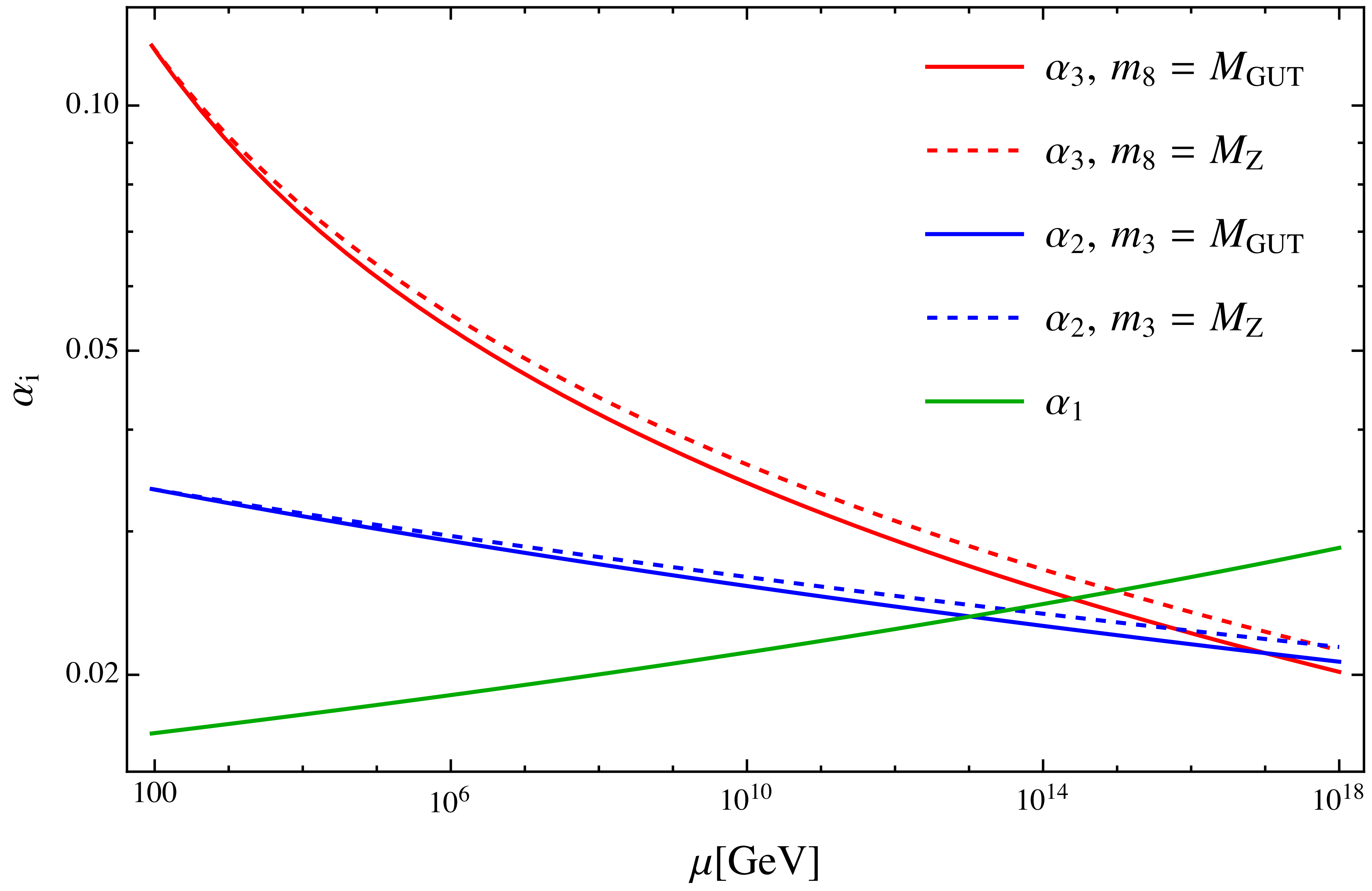
Scales in minimal $SU(5)$

GUT symmetry breaking realized via $24_H = 8_C + 3_L + 1 + \cancel{(3_C, 2_L) + (\bar{3}_C, 2_L)}$

Eaten by X, Y

EW symmetry breaking realized via $5_H = 3_C + 2_L = 3_C + H_{SM}$

$m_T = M_{GUT}$



- Color octet 8_C of mass m_8
- Weak triplet 3_L of mass m_3
- Color triplet 3_C of mass m_T
- Singlet 1 irrelevant

More is needed

3_C normally required heavy due to proton decay. See *Dvali '92* for the light case scenario.

Failures of minimal $SU(5)$

No gauge coupling unification

Neutrino massless (Right-handed neutrino missing)

Just as in the Standard Model, one could add a sterile neutrino.

Predicts wrong Yukawa relations

$$m_d = m_e$$

This is due to an accidental $SU(4)$ symmetry

What to do?

What to do?

1) Give up on grand unification.

Aka give up on: p-decay, existence of monopoles, charge quantization, unification of forces

2) Add new representations.

For example, addition of 45_H can address Yukawa and unification.
Yet, this implies lots of new degrees of freedom → simplicity and predictivity lost?
Change symmetry group? ($SO(10)$? More later.)

Babu, Ma, '84
See also Murayama, Yanagida '92, Doršner, Fileviez Perez '05, Haba et al '24 …

3) Go effective.

Treat Georgi-Glashow model as an effective theory and see if higher-dimensional operators can salvage it. In this approach, minimal model believed to be ruled out. (See *Doršner, Fileviez Perez '05, Bajc, Senjanović '07* adding new representations)

Georgi-Glashow as effective theory works. *Senjanović, MZ, '24*

Minimal $SU(5)$ on the edge: *the importance of being effective*

Idea: Keep the particle content of minimal $SU(5)$ model, but allow for $d > 4$ operators.

Perturbativity requirement:
All higher-dimensional operators suppressed by high scale Λ with

$$\Lambda \gtrsim 10\, M_{\mathrm{GUT}}$$

Contra:
$SU(5)$ features are altered by unknown physics (gravity?). Not ideal.

Pro:
Small number of degrees of freedom → simplified - and possible - analysis of scales
Simplicity → correlations between τ_p and the scale of the new scalar particle (light)

Minimal $SU(5)$ on the edge: *the importance of being effective*

Predicts wrong Yukawa relations

$$m_d = m_e$$

$$\mathcal{L}_{\mathrm{y}}^{d=4} = \overline{5}_{\mathrm{F}}\, Y_{\mathrm{d}}\, 5^*_{\mathrm{H}}\, 10_{\mathrm{F}} + 10_{\mathrm{F}}\, Y_{\mathrm{u}}\, 5_{\mathrm{H}}\, 10_{\mathrm{F}}$$

This can be easily corrected by the addition of $d = 5$ Yukawa couplings

Ellis, Gaillard, '79

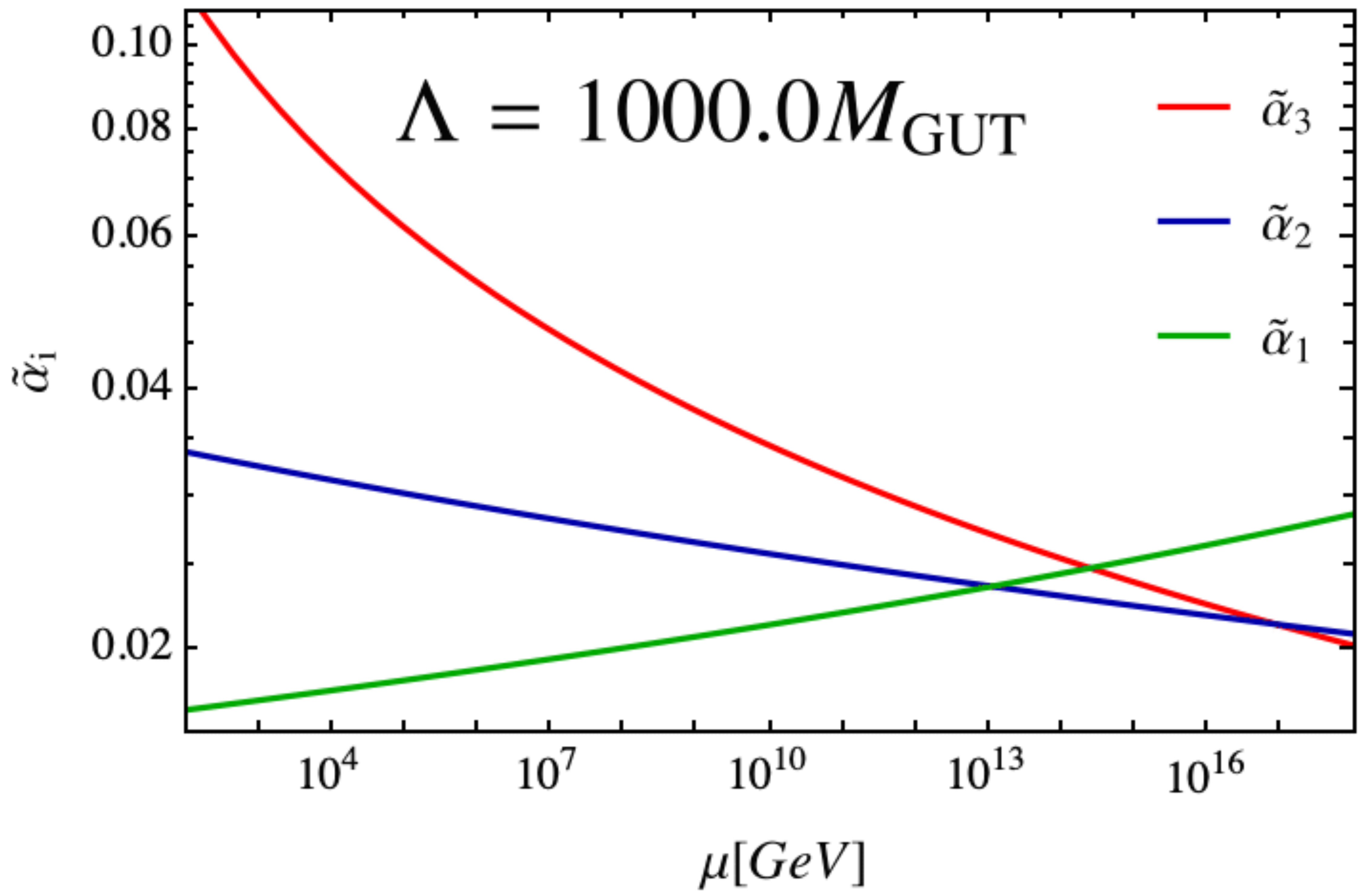
$$\mathcal{L}_{\mathrm{y}}^{d=5} = \frac{1}{\Lambda} \overline{5}_{\mathrm{F}}\, 24_{\mathrm{H}}\, 5^*_{\mathrm{H}}\, 10_{\mathrm{F}} + \ldots$$

$\langle 24_{\mathrm{H}} \rangle \propto \mathrm{diag}\,(\underbrace{2,2,2,}_{SU(3)_{\mathrm{C}}} \underbrace{-3-3}_{SU(2)_{\mathrm{L}}})$ breaks the accidental symmetry

This makes the p-decay branching ratio unpredictable, as opposed to the minimal case.

Minimal $SU(5)$ on the edge: *the importance of being effective*

Particle thresholds insufficient ⟶ include $d = 5$ gauge boson kinetic energy



$$\Delta \mathcal{L}_{\rm kin} = \frac{1}{\Lambda} \mathrm{Tr}\, F_{\mu\nu} \langle 24_{\rm H} \rangle F^{\mu\nu}$$

Shafi, Wetterich '84

$$\langle 24_{\rm H} \rangle = M_{\rm GUT}\, \mathrm{diag}\,(2,2,2,-3-3)$$

$$\alpha_3 \to \tilde{\alpha}_3 = \left(1 - \frac{M_{\rm GUT}}{\Lambda}\right) \alpha_3\,,$$

$$\alpha_2 \to \tilde{\alpha}_2 = \left(1 + \frac{3\, M_{\rm GUT}}{2\,\Lambda}\right) \alpha_2\,,$$

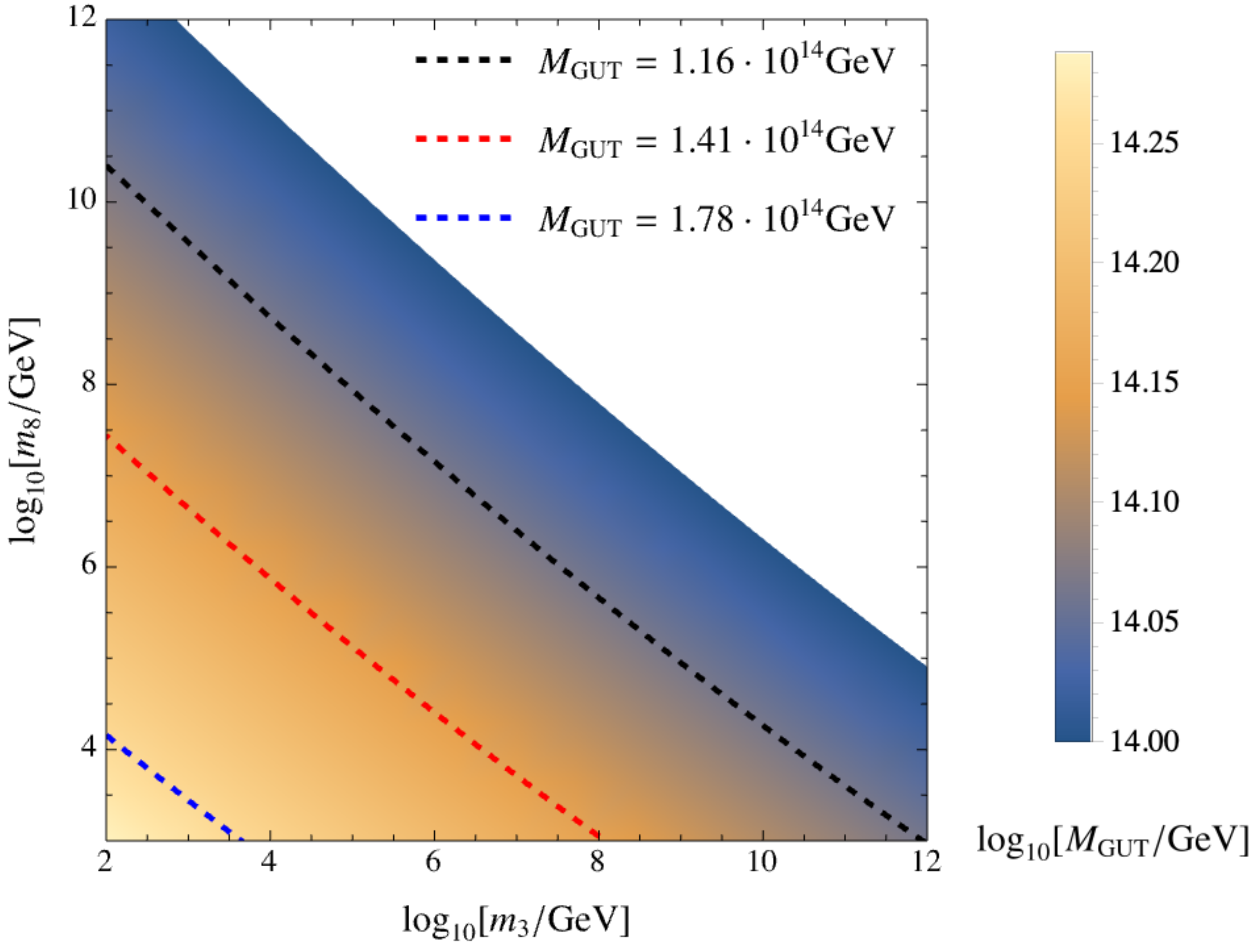
$$\alpha_1 \to \tilde{\alpha}_1 = \left(1 + \frac{M_{\rm GUT}}{2\,\Lambda}\right) \alpha_1\,,$$

Unification condition: $\tilde{\alpha}_3\left(M_{\rm GUT}\right) = \tilde{\alpha}_2\left(M_{\rm GUT}\right) = \tilde{\alpha}_1\left(M_{\rm GUT}\right)$

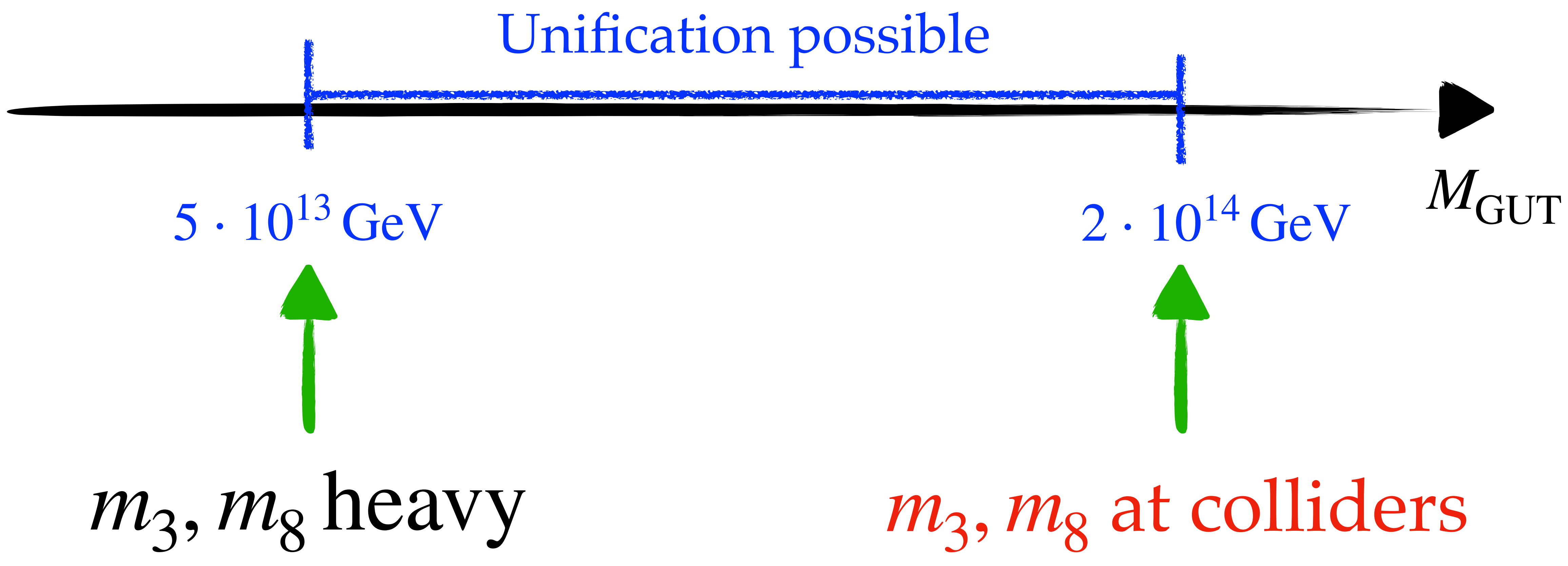
$\Lambda \gtrsim 20$ compatible with perturbativity. But $M_{\rm GUT} \simeq 10^{14}$GeV leads to too short τ_p?

Minimal $SU(5)$ on the edge: *the importance of being effective*

Senjanović, MZ, '24



Theory unifies in a small energy range



$$\frac{M_{GUT}}{M_Z} = \exp\left\{\frac{\pi}{21}\left[5\left(1+\frac{\epsilon}{4}\right)\alpha_1^{-1} - 3\left(1+\frac{5\,\epsilon}{4}\right)\alpha_2^{-1} - 2\left(1-\frac{5\,\epsilon}{4}\right)\alpha_3^{-1}\right]\right\}\left(\frac{M_Z^2}{m_3\, m_8}\right)^{\frac{1}{42}}$$

$$\tau_p = \simeq C\,\frac{M_{GUT}^4}{\alpha_{GUT}^2}\left(m_{\text{proton}}\right)^{-5} \gtrsim 10^{34}\text{yrs} \quad \not\leftrightarrow \quad M_{GUT} \gtrsim 4 \cdot 10^{15}\text{GeV}\left(\frac{\alpha_{GUT}}{40^{-1}}\right)^{1/2} \gg 10^{14}\,\text{GeV}$$

C is not fixed, and this spoils the relation between M_{GUT} and τ_p

Nandi, Stern, Sudarschan '82

Minimal $SU(5)$ on the edge: *the importance of being effective*

$$\mathscr{L}_{\not B} = \left(\bar{u}^c u\right)\left(\bar{e}^c d + \bar{d}^c e\right) + \left(\bar{u}^c d\right)\left(\bar{e}^c u + \bar{d}^c \nu\right)$$

$\downarrow$ Fermion rotation to mass basis

$$f \to F f; \quad f^c \to F_c f^c; \quad F F^\dagger = F_c {F_c}^\dagger = 1$$

$$\mathscr{L}_{\not B} = \left(\bar{u}^c\, U_c^\dagger U\, u\right)\left(\bar{e}^c\, E_c^\dagger D\, d + \bar{d}^c\, D_c^\dagger E\, e\right) + \left(\bar{u}^c\, U_c^\dagger D\, d\right)\left(\bar{e}^c\, E_c^\dagger U\, u + \bar{d}^c\, D_c^\dagger N\, \nu\right)$$

The unitary matrices determine C . Notice, $V_{\rm CKM} = U^\dagger D$

$$\tau_p \simeq C\, \frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2} \left(m_{\rm proton}\right)^{-5}$$

$C = \infty \implies \theta_{13} = 0$ → No p-decay! *Nandi, Stern, Sudarschan '82*

$\tau_p = \infty$

$$\tau_p \simeq C\, \frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2} \left(m_{\rm proton}\right)^{-5} \lesssim \left(\theta_{13}\right)^{-2} \frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2} \left(m_{\rm proton}\right)^{-5} \to M_{\rm GUT} \text{ can be way smaller}$$

Minimal $SU(5)$ on the edge: *the importance of being effective*

$$\Gamma(p \to K^+\bar{\nu}) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\sum_{i=1}^{3}\left|\frac{2m_p}{3m_B}D\ c(\nu_i, d, s^C)+[1+\frac{m_p}{3m_B}(D+3F)]c(\nu_i, s, d^C)\right|^2$$

$$\Gamma(p \to \pi^+\bar{\nu}) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\sum_{i=1}^{3}\left|c(\nu_i, d, d^C)\right|^2$$

$$\Gamma(p \to \eta e^+_\beta) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\left\{\left|c(e_\beta, d^C)\right|^2+\left|c(e^C_\beta, d)\right|^2\right\}$$

$$\Gamma(p \to K^0 e^+_\beta) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\left\{\left|c(e_\beta, s^C)\right|^2+\left|c(e^C_\beta, s)\right|^2\right\}$$

$$\Gamma(p \to \pi^0 e^+_\beta) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\left\{\left|c(e_\beta, d^C)\right|^2+\left|c(e^C_\beta, d)\right|^2\right\}$$

$$\Gamma(n \to \pi^- e^+_\beta) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\left\{\left|c(e_\beta, d^C)\right|^2+\left|c(e^C_\beta, d)\right|^2\right\}$$

$$\Gamma(n \to \pi^0\bar{\nu}) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\sum_{i=1}^{3}\left|c(\nu_i, d, d^C)\right|^2$$

$$\Gamma(n \to \eta\bar{\nu}) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\sum_{i=1}^{3}\left|c(\nu_i, d, d^C)\right|^2$$

$$\Gamma(n \to K^0\bar{\nu}) \propto \frac{\alpha^2_{\rm GUT}}{M_X^4}\sum_{i=1}^{3}\left|c(\nu_i, d, s^C)[1+\frac{m_n}{3m_B}(D-3F)]-c(\nu_i, s, d^C)[1+\frac{m_n}{3m_B}(D+3F)]\right|^2$$

$$c(e^C_\alpha, d_\beta) = \left(U_c^\dagger U\right)_{11}\left(E_c^\dagger D\right)_{\alpha\beta} + (U_c^\dagger D)_{1\beta}(E_c^\dagger U)_{\alpha 1}$$

$$c(e_\alpha, d^C_\beta) = \left(U_c^\dagger U\right)_{11}\left(D_c^\dagger E\right)_{\beta\alpha}$$

$$c(\nu_l, d_\alpha, d^C_\beta) = \left(U_c^\dagger D\right)_{1\alpha}(D_c^\dagger N)_{\beta l}$$

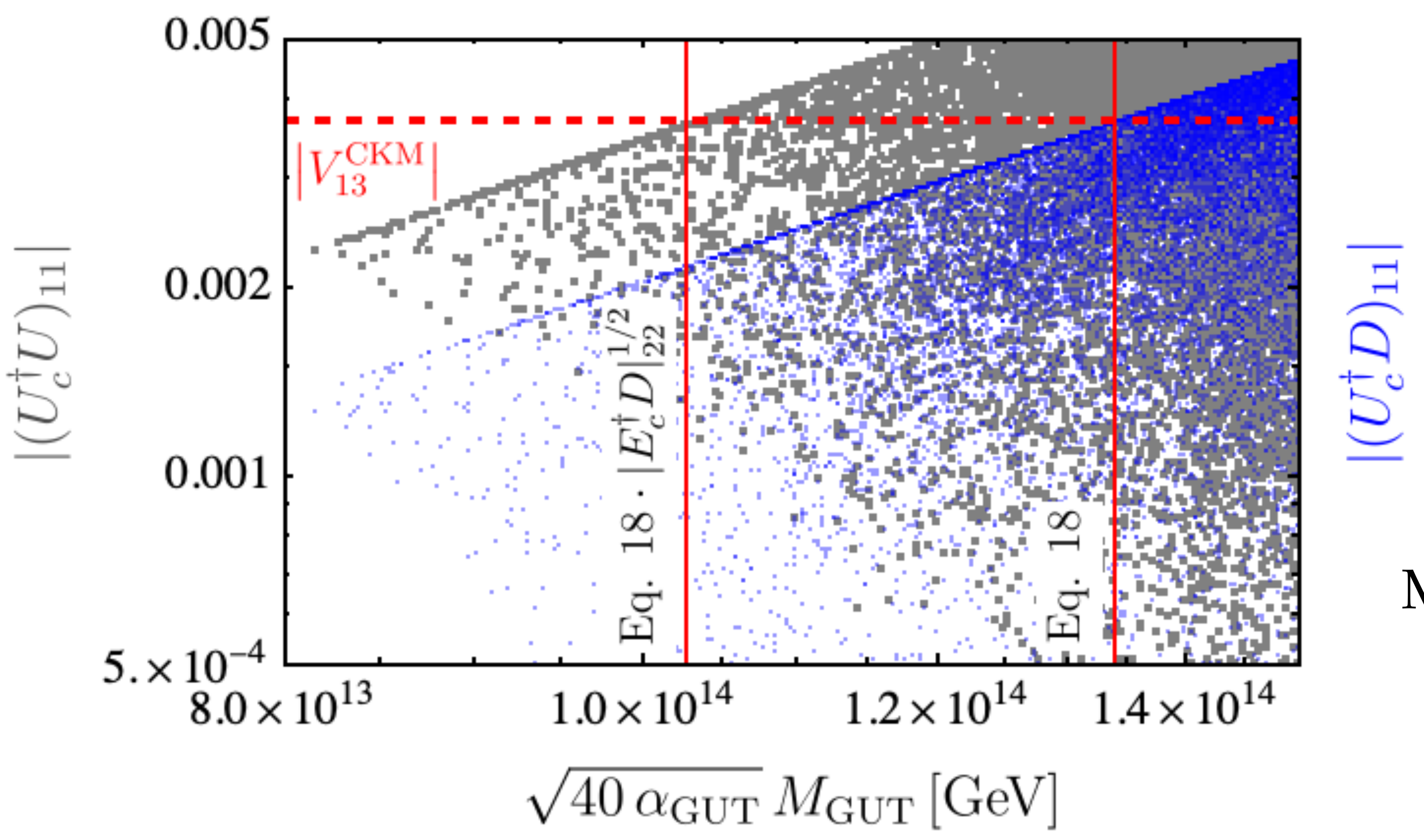
$$\left(U_c^\dagger U\right)_{11} = 0\,, \quad \left(E_c^\dagger U\right)_{a1} = 0\,, \quad \left(U_c^\dagger D\right)_{1a} = 0\,, \quad a = 1{,}2 \implies (V_{\rm CKM})_{13} = 0$$

1) $\left(U_c^\dagger U\right)_{11} = 0\,, \quad \left(E_c^\dagger U\right)_{a1} = 0\,, \quad \left(U_c^\dagger D\right)_{11} = 0 \implies M_{\rm GUT} \simeq 3 \cdot 10^{14}\,{\rm GeV}$

2) $\left(U_c^\dagger D\right)_{1a} = \left(E_c^\dagger D\right)_{1a} = \left(E_c^\dagger D\right)_{a1} = 0\,, \quad \left(D_c^\dagger E\right)_{1a} = \left(D_c^\dagger E\right)_{a1} = 0 \implies M_{\rm GUT} \simeq 1.3 \cdot 10^{14}\,{\rm GeV}$

Minimal $SU(5)$ on the edge: *the importance of being effective*

$$\tau_p \simeq C\,\frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2}\left(m_{\rm proton}\right)^{-5} \lesssim (\theta_{13})^{-2}\frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2}\left(m_{\rm proton}\right)^{-5} \rightarrow M_{\rm GUT}\text{ can be way smaller}$$



Channel	Lifetime $(10^{30}$yrs)
$N\to e^+\,\pi$	5300 (n), 16000 (p)
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Monte Carlo analysis to find the lowest possible leads to

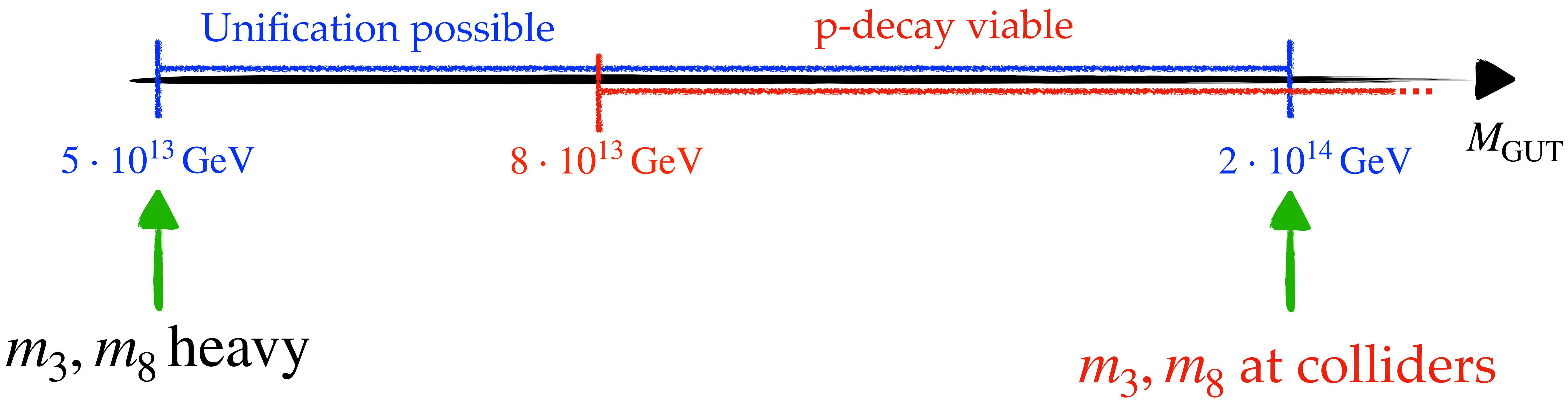
$$M_{\rm GUT} \gtrsim 8\cdot 10^{13}\,\text{GeV}$$

Senjanović, MZ, '24

Appreciably larger bound in Doršner, Fileviez Perez '05

Minimal $SU(5)$ on the edge: *the importance of being effective*

Theory unifies in a small energy range, near the lower bound from p-decay



Improvement of p-lifetime by a factor 10 (20) $\Longrightarrow m_3$ or $m_8 \lesssim$ 100 (10) TeV

Improvement by factor ~50 would rule out the model.

Senjanović, MZ, '24

* improvement by factor 10 will be achieved in the near future.

Minimal $SU(5)$ on the edge: *the importance of being effective*

Gauge gauge coupling unification compatible with p-decay

Neutrino massless (Right-handed neutrino missing)

Just as in the Standard Model, one could add a sterile neutrino

Yukawa relations can be corrected by higher-dimensional operators

$$m_d \neq m_e$$

Minimal $SU(5)$ on the edge: *the importance of being effective*

Neutrino massless (Right-handed neutrino missing)

The leading contribution comes from higher-dimensional operators, via *Weinberg operator '79*

$$\mathscr{L}_{\not{L}} = \frac{c_\nu}{\Lambda}\,\overline{5}_F\,5_H\,5_H\,\overline{5}_F \longrightarrow m_\nu \simeq \frac{c_\nu}{2}\frac{\left(v_{SM}\right)^2}{\Lambda}$$

$$m_\nu \gtrsim 0.1\,\text{eV} \implies \Lambda \lesssim c_\nu\, 3\cdot 10^{14}\,\text{GeV}$$

Borderline with perturbativity requirement: $c_\nu \sim 3$

Just as in the Standard Model, one could add a sterile neutrino, and realize the seesaw mechanism.

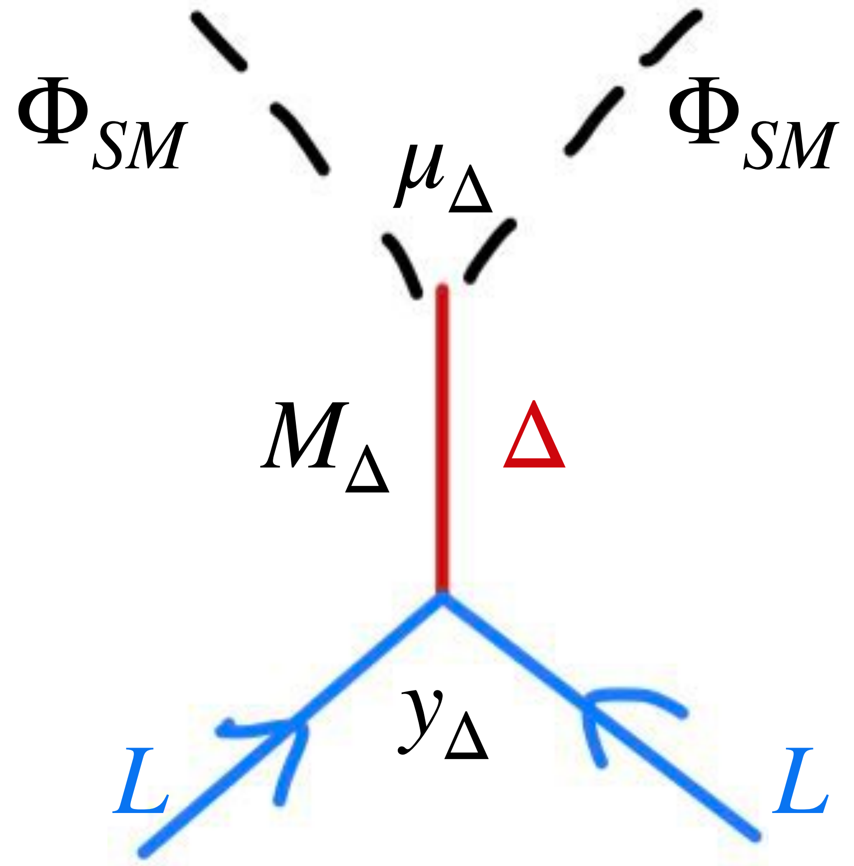
Minkowski '77
Mohapatra, Senjanović '79
Yanagida '79

Implementation of neutrino mass

Another possibility is adding new representations

Minimal SU(5) + 15_H *Doršner, Fileviez Perez '05*

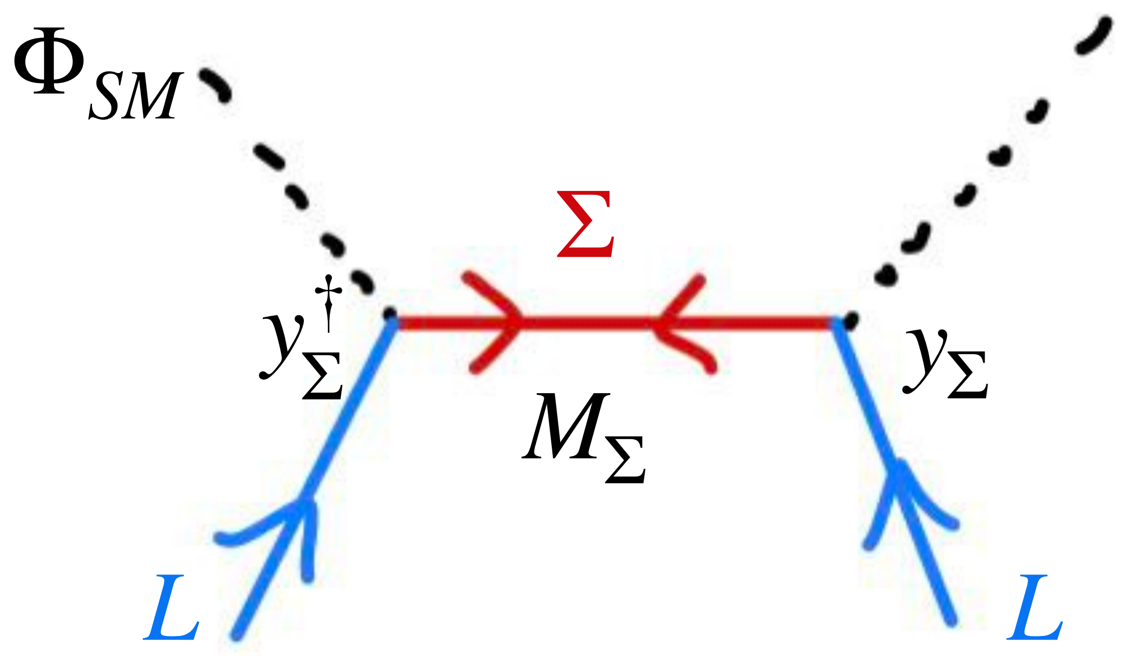
15_H contains a scalar weak triplet Δ with Y=2



$$m_\nu \simeq y_\Delta \mu_\Delta \frac{v^2}{M_\Delta^2}$$

Minimal SU(5) + 24_F *Bajc, Senjanović '07*

24_F contains a fermionic weak triplet Σ with Y=0



$$m_\nu \simeq v^2 \left(y_\Sigma^T \frac{1}{M_\Sigma} y_\Sigma \right)$$

These theories still require non-renormalizable operators. However, the tight correlations on the scales are relaxed due to the higher number of d.o.fs

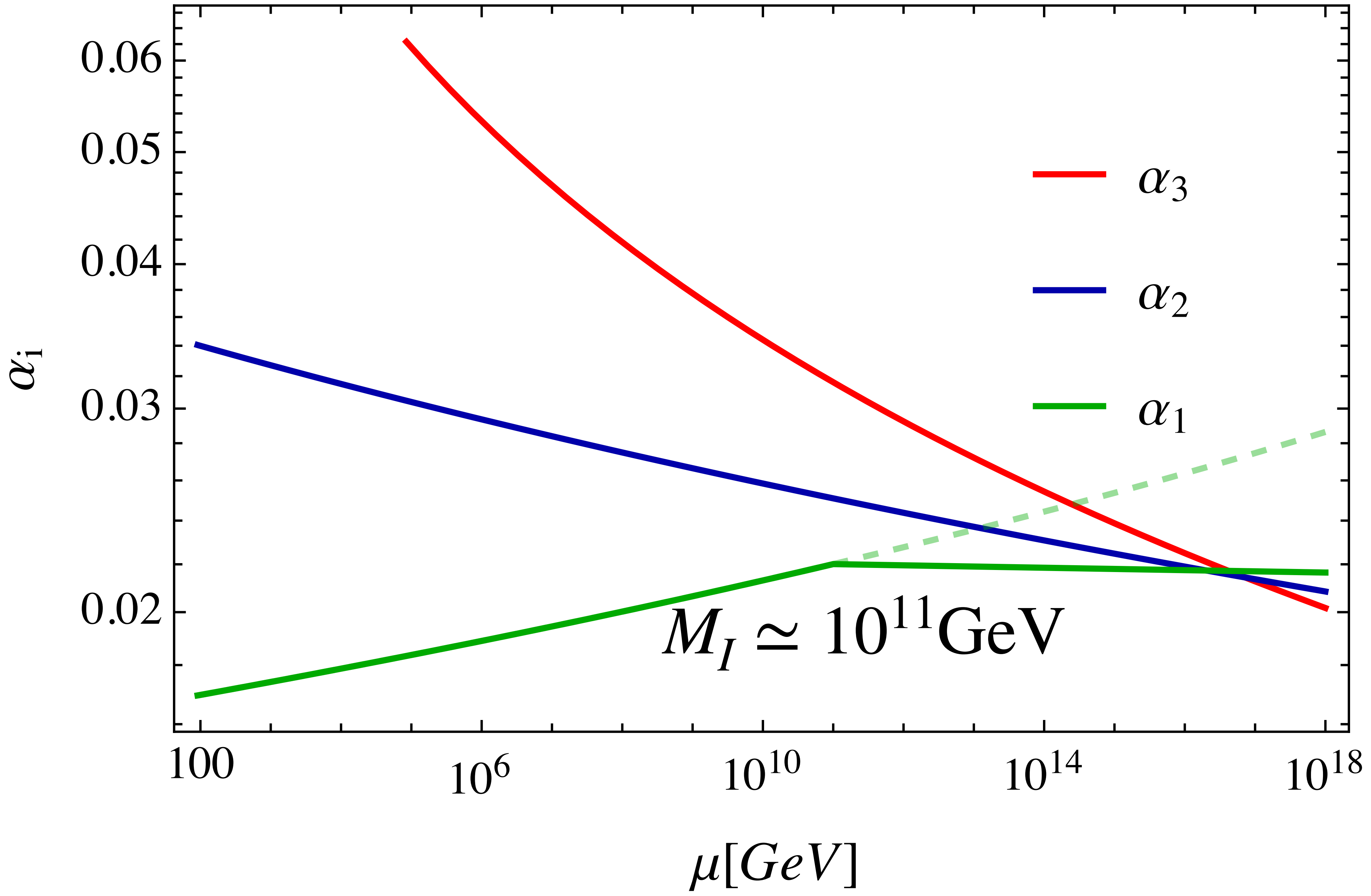
Thank you!

Backup

$SO(10)$: a true GUT for neutrino mass

Georgi '74
Fritzsch, Minkowski, '74

Del Aguila, Ibanez '80
Adding W_R *Rizzo, Senjanović '80*



$$16_F = \underbrace{10 + 5 + 1}_{\text{Just as in } SU(5)} = \begin{pmatrix} u_\alpha \\ \nu \\ d_\alpha \\ e \\ e^c \\ d^c_\alpha \\ \nu^c \\ u^c_\alpha \end{pmatrix}$$

Unified generation in same multiplet

↓

→ Right handed neutrino already there
Charge conjugation is gauge symmetry

In generic $SO(10)$ model, intermediate scale can always be chosen to ensure unification.
No need for new light particle states due to gauge coupling unification?

$SO(10)$: model building approach

At the renormalizable lever, the theory requires large representations such as 120_H, 126_H which introduce lots of new particle thresholds. Hard to make predictions. Moreover, even the minimal model seems to be ruled out *See Susić, Jarkovska, Malinsky '24*

What happens if, just as in the $SU(5)$, we go effective and choose the smallest possible representations allowing for a realistic model?

A. Preda, G. Senjanović, MZ, '22

$SO(10)$: still important to be effective?

$SO(10)$

$\downarrow \langle 45_H \rangle \simeq M_{GUT}$

$\mathcal{G}_I$

$\downarrow \langle 16_H \rangle \simeq M_I$

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$\downarrow \langle 10_H \rangle \simeq M_W$

$SU(3)_C \times U(1)_{em}$

Scalar sector with small representations:

Adjoint 45_H, Spinor 16_H, Fundamental 10_H

At renormalizable level, only 10_H couples to fermions

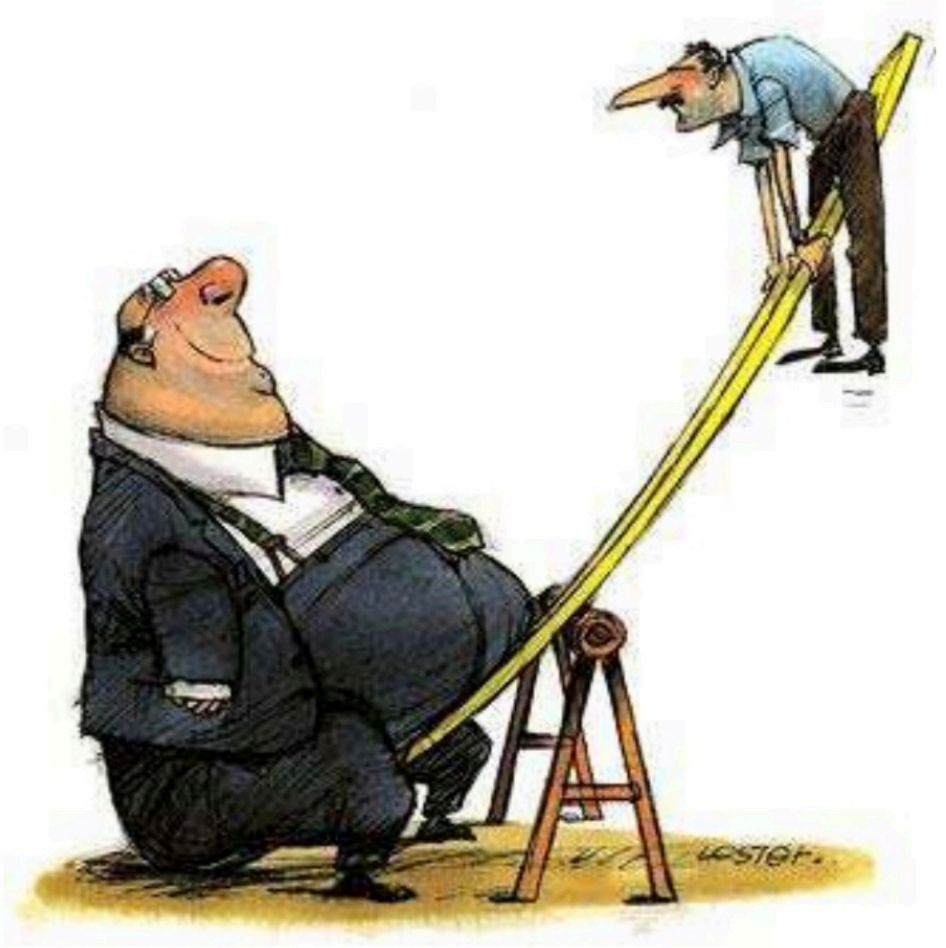
$$16_F\, 16_F \langle 10_H \rangle \implies m_d = m_e, \quad m_u = m_D$$ *Neutrino Dirac mass*

$\downarrow$

$$\boxed{m_{3D} = m_t}$$

This is crux of it all!

$SO(10)$: still important to be effective?



The right handed neutrino N obtains mass from $d = 5$ operator

$$16_F 16_F 16^*_H 16^*_H / \Lambda$$

$$m_{\mathrm{N}} \simeq \frac{M_{\mathrm{I}}^2}{\Lambda}$$

$$\Big\downarrow \quad \begin{pmatrix} 0 & m_D^T \\ m_D & m_N \end{pmatrix} \begin{matrix} \nu \\ N \end{matrix}$$

$$m_\nu \simeq \frac{(m_{3D})^2}{m_N} \simeq \frac{m_t^2 \Lambda}{M_I^2} \simeq \mathrm{eV} \left(\frac{m_t}{100\mathrm{GeV}} \frac{4 \cdot 10^{14}\mathrm{Gev}}{M_I} \right)^2 \left(\frac{\Lambda}{4 \cdot 10^{16}\mathrm{GeV}} \right) \lesssim \mathrm{eV}$$

$$\Longrightarrow \; M_I \sim M_{GUT}$$

$SO(10)$: still important to be effective?

Due to the choice of scalar sector, we are pushed into single step breaking, just as in $SU(5)$.

However, more thresholds states i.e., more freedom in unification?

$45_H, 16_H, 10_H$ as opposed to $24_H, 5_H$

↑ $SO(10)$ ↑ $SU(5)$

Is the $SO(10)$ model still capable of informing us about scales?

$SO(10)$: still important to be effective?

Key-point: Different gauge kinetic operators are at play

$$\Delta\mathscr{L}_{\text{kin}}: \quad \frac{1}{\Lambda^2}\,\text{Tr}\,F_{\mu\nu}\langle 45_{\text{H}}^2\rangle F^{\mu\nu} \propto \frac{M_{\text{GUT}}^2}{\Lambda^2} \lesssim 10^{-2} \qquad \frac{1}{\Lambda}\,\text{Tr}\,F_{\mu\nu}\langle 24_{\text{H}}\rangle F^{\mu\nu} \propto \frac{M_{\text{GUT}}}{\Lambda} \lesssim 10^{-1}$$

$SO(10)$

Here the first operator is $d = 6$. Negligible! **Unification can fail!**

$SU(5)$

This operator was **always ensuring unification**, regardless of particle scales

$SO(10)$ has more particle states. However, they need to need to ensure unification as opposed to $SU(5)$. Therefore, more is demanded from them.

$SO(10)$: still important to be effective

Bottom line: Surprisingly, something can still be said about scales.

Under the assumption of no rotation in p-decay ($C = 1$):
Scalar weak triplet 3_L, a squark $(2_L, 3_C)$ and an octet scalar gluon 8_C, need to lie below 10 TeV.
Moreover, p-decay has to be found before 2030.

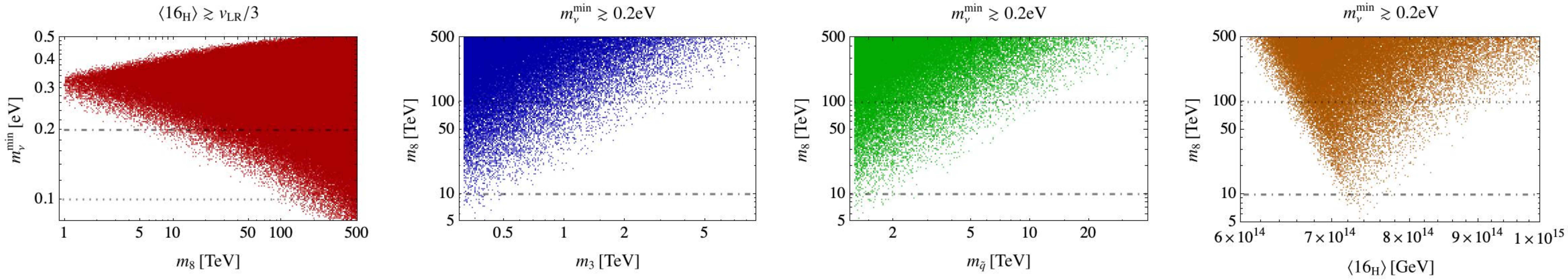
A. Preda, G. Senjanović, MZ, '22

The assumption of no p-decay cancellation is commonly accepted by the community.

However, this would not exhaust all parameter space of the theory. Theories live on a point. Nevertheless, result is still surprising.

$SO(10)$: still important to be effective

The full parameter space allows still for a relaxed statement. If we allow for cancellations in p-decay amplitudes:



If the colour octet 8_C lies below 10 TeV $\Longrightarrow$

- $m_\nu \gtrsim 2 \cdot 10^{-1}$eV
- Weak triplet below 500 GeV
- Squark below 2 TeV

Potentially within experimental reach

At collider energies

A. Preda, G. Senjanović, MZ, to appear

Example of what happens when we analyze a GUT theory in its full parameter space.

Conclusions

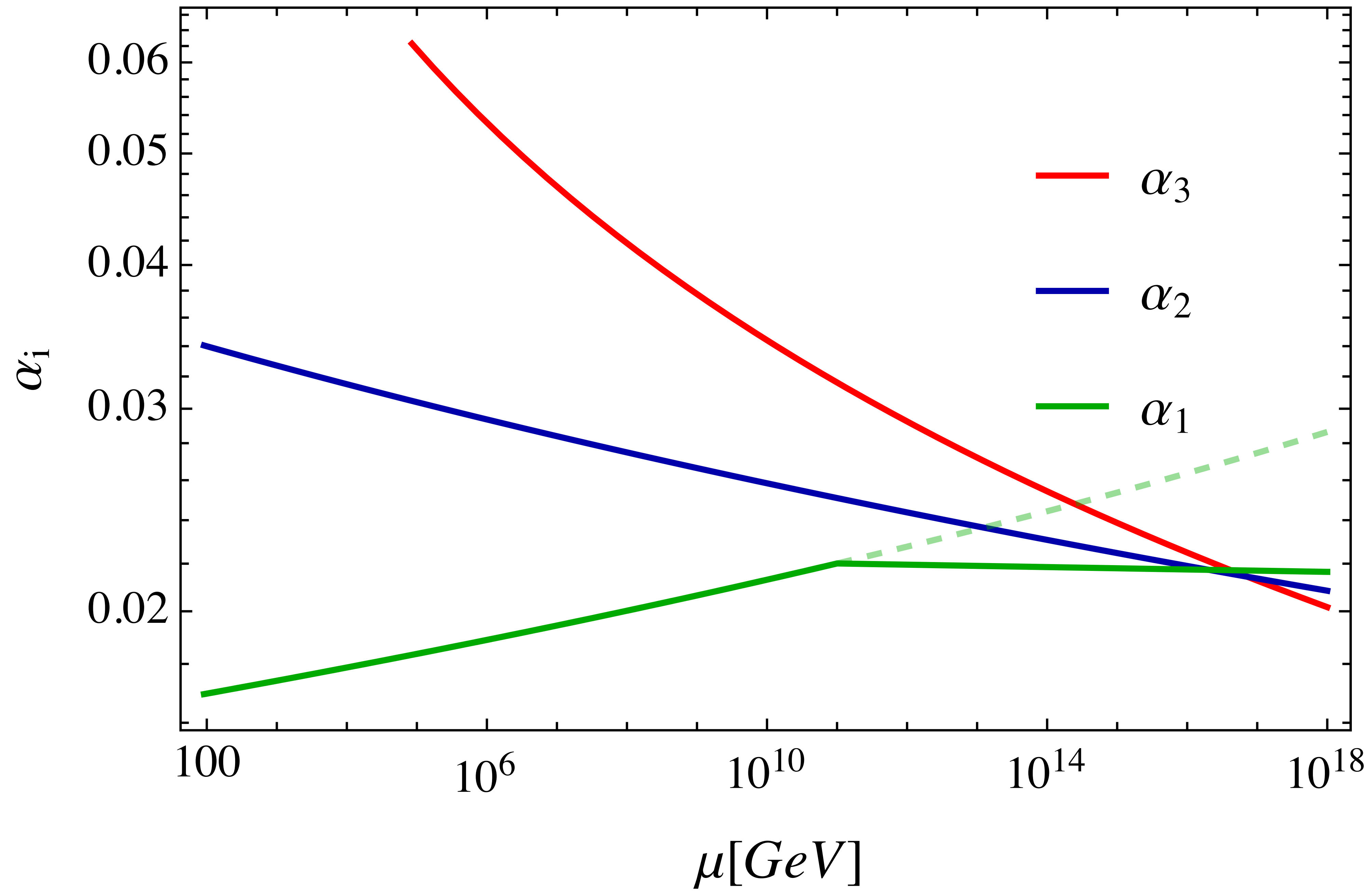
- We showed that the original $SU(5)$ model by Georgi-Glashow, known to be ruled out, is viable as an effective theory
- Its advantage lies in the **small number of new degrees of freedom** introduced, which leads to constraint in the particle spectrum
- New particle states, **a weak triplet and a color octet need to be light** for the theory to be viable
- **Their lightness is correlated with bounds on proton lifetime**. An improvement in current bounds by a factor of order 10 would imply these particle are at collider energies

- **A similar situation** emerges in minimal $SO(10)$ based on small representation if treated as an effective theory.
- Because $SO(10)$ naturally predicts neutrino mass, the **potentially light particle states imply a lower bound on neutrino mass**, within the reach of current experiments.

- Both theories, however, cannot be fully ruled out within the near future. What to do?

More on $SO(10)$?

Adding W_R



- Unification ensured by intermediate scale M_I
- Right-handed neutrino gets a mass — fermions unified in 16_F
- Can realize seesaw mechanism

Higgs sector:

$$45_H; \quad 16_H; \quad 10_H$$

SM undergoes usual breaking with 10_H

- 10_H real does not work since $m_u = m_d$ hence the need for complex 10_H
- At renormalizable level still $m_d = m_e$, so higher dimensional operators are needed for Yukawa

In what follows, for validity of perturbativity, we will require cutoff

$$\Lambda \gtrsim 10\, M_{GUT}$$

Crux: $m_{3D} = m_t$

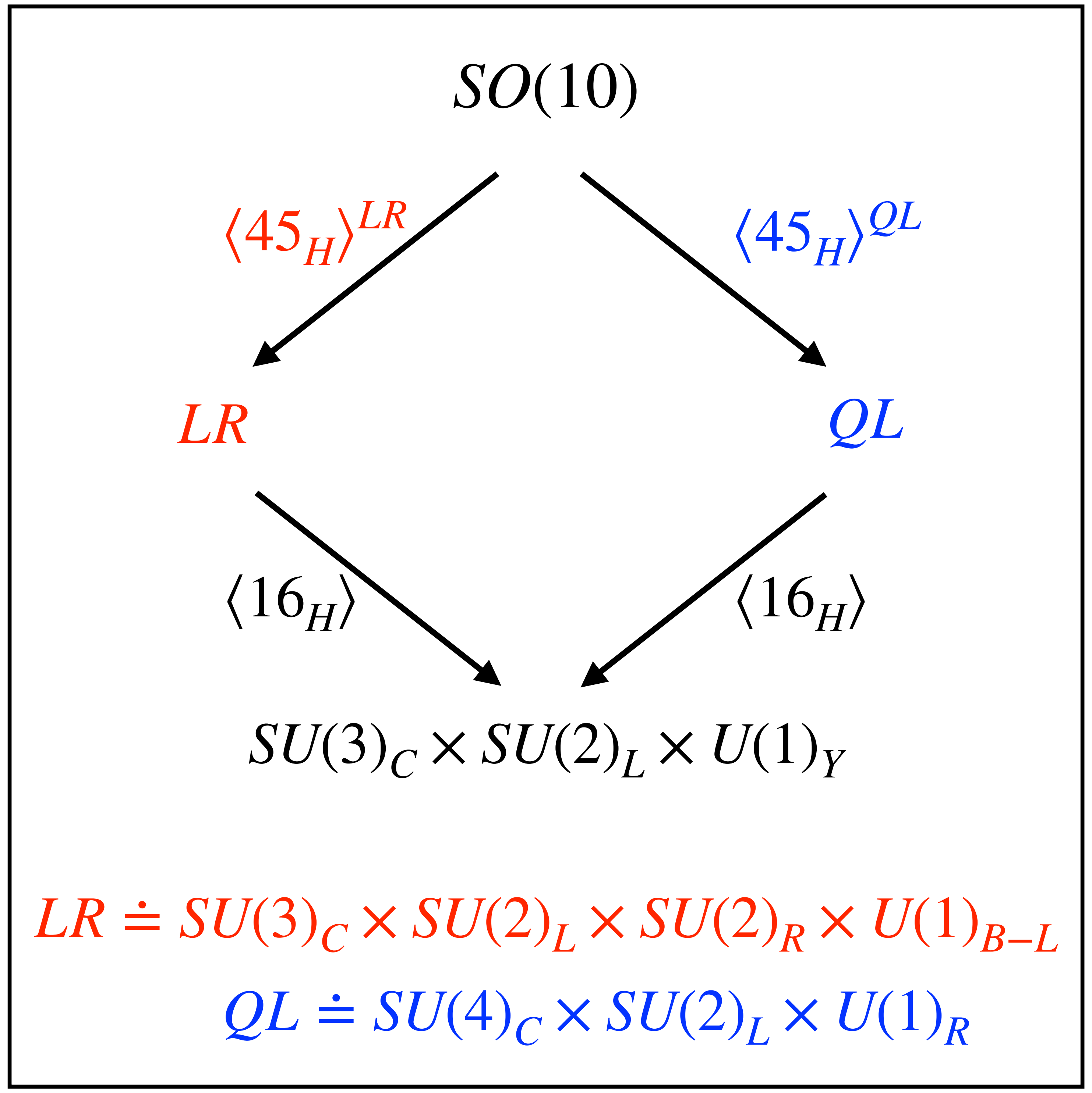
16_H

$4_C\,2_L\,2_R$	$4_C\,2_L\,1_R$	$3_c\,2_L\,2_R\,1_X$	$3_c\,2_L\,1_R\,1_X$	$3_c\,2_L\,1_Y$	5	$5'\,1_{Z'}$	$1_{Y'}$
$(4,2,1)$	$(4,2,0)$	$\left(3,2,1,+\frac{1}{6}\right)$	$\left(3,2,0,+\frac{1}{6}\right)$	$\left(3,2,+\frac{1}{6}\right)$	10	$(10,+1)$	$+\frac{1}{6}$
		$\left(1,2,1,-\frac{1}{2}\right)$	$\left(1,2,0,-\frac{1}{2}\right)$	$\left(1,2,-\frac{1}{2}\right)$	$\overline{5}$	$(\overline{5},-3)$	$-\frac{1}{2}$
$(\overline{4},1,2)$	$\left(\overline{4},1,+\frac{1}{2}\right)$	$\left(\overline{3},1,2,-\frac{1}{6}\right)$	$\left(\overline{3},1,+\frac{1}{2},-\frac{1}{6}\right)$	$\left(\overline{3},1,+\frac{1}{3}\right)$	$\overline{5}$	$(10,+1)$	$-\frac{2}{3}$
	$\left(\overline{4},1,-\frac{1}{2}\right)$		$\left(\overline{3},1,-\frac{1}{2},-\frac{1}{6}\right)$	$\left(\overline{3},1,-\frac{2}{3}\right)$	10	$(\overline{5},-3)$	$+\frac{1}{3}$
		$\left(1,1,2,+\frac{1}{2}\right)$	$\left(1,1,+\frac{1}{2},+\frac{1}{2}\right)$	$(1,1,+1)$	10	$(1,+5)$	0
			$\left(1,1,-\frac{1}{2},+\frac{1}{2}\right)$	$(1,1,0)$	1	$(10,+1)$	$+1$

45_H

$4_C\,2_L\,2_R$	$4_C\,2_L\,1_R$	$3_c\,2_L\,2_R\,1_X$	$3_c\,2_L\,1_R\,1_X$	$3_c\,2_L\,1_Y$	5	$5'\,1_{Z'}$	$1_{Y'}$
$(1,1,3)$	$(1,1,+1)$	$(1,1,3,0)$	$(1,1,+1,0)$	$(1,1,+1)$	10	$(10,-4)$	$+1$
	$(1,1,0)$		$(1,1,0,0)$	$(1,1,0)$	1	$(1,0)$	0
	$(1,1,-1)$		$(1,1,-1,0)$	$(1,1,-1)$	$\overline{10}$	$(\overline{10},+4)$	-1
$(1,3,1)$	$(1,3,0)$	$(1,3,1,0)$	$(1,3,0,0)$	$(1,3,0)$	24	$(24,0)$	0
$(6,2,2)$	$\left(6,2,+\frac{1}{2}\right)$	$\left(3,2,2,-\frac{1}{3}\right)$	$\left(3,2,+\frac{1}{2},-\frac{1}{3}\right)$	$\left(3,2,\frac{1}{6}\right)$	10	$(24,0)$	$-\frac{5}{6}$
	$\left(6,2,-\frac{1}{2}\right)$		$\left(3,2,-\frac{1}{2},-\frac{1}{3}\right)$	$\left(3,2,-\frac{5}{6}\right)$	24	$(10,-4)$	$+\frac{1}{6}$
		$\left(\overline{3},2,2,+\frac{1}{3}\right)$	$\left(\overline{3},2,+\frac{1}{2},+\frac{1}{3}\right)$	$\left(\overline{3},2,+\frac{5}{6}\right)$	24	$(\overline{10},+4)$	$-\frac{1}{6}$
			$\left(\overline{3},2,-\frac{1}{2},+\frac{1}{3}\right)$	$\left(\overline{3},2,-\frac{1}{6}\right)$	$\overline{10}$	$(24,0)$	$+\frac{5}{6}$
$(15,1,1)$	$(15,1,0)$	$(1,1,1,0)$	$(1,1,0,0)$	$(1,1,0)$	24	$(24,0)$	0
		$\left(3,1,1,+\frac{2}{3}\right)$	$\left(3,1,0,+\frac{2}{3}\right)$	$\left(3,1,+\frac{2}{3}\right)$	$\overline{10}$	$(\overline{10},+4)$	$+\frac{2}{3}$
		$\left(\overline{3},1,1,-\frac{2}{3}\right)$	$\left(\overline{3},1,0,-\frac{2}{3}\right)$	$\left(\overline{3},1,-\frac{2}{3}\right)$	10	$(10,-4)$	$-\frac{2}{3}$
		$(8,1,1,0)$	$(8,1,0,0)$	$(8,1,0)$	24	$(24,0)$	0

More on the breaking Pattern



Content

At tree level only one VEV allowed for

$$\langle 45_H \rangle^{SU(5)} = v_{GUT}\, \sigma_2 \otimes \mathrm{diag}(1,1,1,\pm 1,\pm 1),$$

Non viable for unification

Including higher dimensional operators, new directions are possible

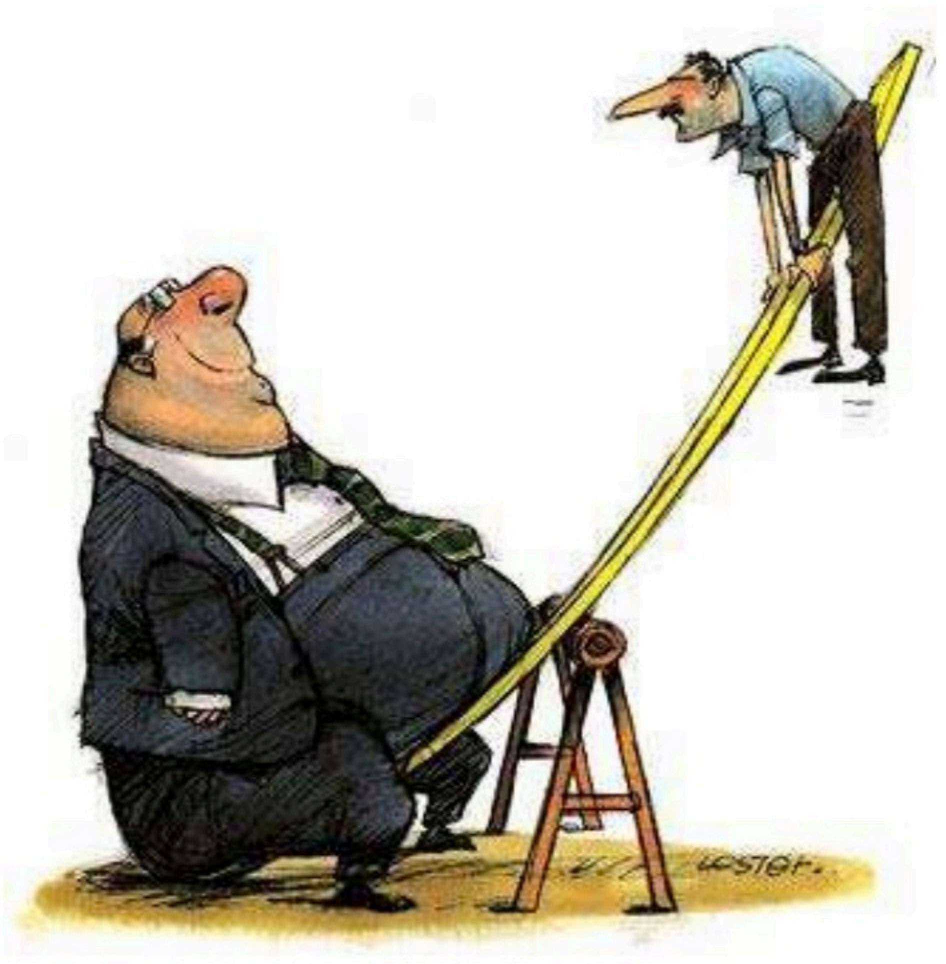
$$\langle 45_H \rangle^{LR} = v_{GUT}\, \sigma_2 \otimes \mathrm{diag}(1,1,1,0,0),$$
$$\langle 45_H \rangle^{QL} = v_{GUT}\, \sigma_2 \otimes \mathrm{diag}(0,0,0,1,1)$$

→ Breaking pattern possible also at tree-level via inclusion of radiative corrections (see Bertolini, Di Luzio, Malinsky 0912.1796)

→ Higher-dimensional operators give more freedom in the spectrum

$\langle 16_H \rangle \simeq M_I$ realizes tadpole for vanishing component when acquiring VEV via $16_H\, 45_H\, 16^*_H$

Seesaw



The right handed neutrino N obtains mass from $d = 5$ operator

$$16_F 16_F 16^*_H 16^*_H / \Lambda$$

$$m_{\rm N} \simeq \frac{M_{\rm I}^2}{\Lambda}$$

$$\downarrow \qquad \begin{pmatrix} 0 & m_D^T \\ m_D & m_N \end{pmatrix} \begin{matrix} \nu \\ N \end{matrix}$$

$$m_\nu \simeq \frac{(m_{3D})^2}{m_N} \simeq \frac{m_t^2 \Lambda}{M_I^2} \simeq \text{eV} \left(\frac{m_t}{100\text{GeV}} \frac{4 \cdot 10^{14}\text{Gev}}{M_I} \right)^2 \left(\frac{\Lambda}{4 \cdot 10^{16}\text{GeV}} \right) \lesssim \text{eV}$$

$$\Longrightarrow \; M_I \gtrsim 0.1\, M_{GUT}$$

But in the absence of an intermediate scale, we learnt from $SU(5)$ that new light states are necessary

Scalar weak triplet 3_W, a squark $(2_W, 3_C)$ and an octet scalar gluon 8_C

	$4_C 2_L 2_R$	$3_c\, 2_L\, 1_Y$	m_i
16_H	$(4,2,1)$	$(3,2,+\frac{1}{6})$	m_{sq}
		$(1,2,-\frac{1}{2})$	
	$(\overline{4},1,2)$	$(\overline{3},1,+\frac{1}{3})$	m_{sd}
		$(\overline{3},1,-\frac{2}{3})$	m_{sup}
		$(1,1,+1)$	m_{sel}
		$(1,1,0)$	

	$4_C 2_L 2_R$	$3_C 2_L 1_Y$	m_i
45_H	$(1,1,3)$	$(1,1,+1)$	m_{sel}
		$(1,1,0)$	
		$(1,1,-1)$	
	$(1,3,1)$	$(1,3,0)$	m_3
	$(6,2,2)$	$(3,2,\frac{1}{6})$	m_{sq}
		$(3,2,-\frac{5}{6})$	
		$(\overline{3},2,+\frac{5}{6})$	
		$(\overline{3},2,-\frac{1}{6})$	
	$(15,1,1)$	$(1,1,0)$	
		$(3,1,+\frac{2}{3})$	
		$(\overline{3},1,-\frac{2}{3})$	m_{sup}
		$(8,1,0)$	m_8

10_H contains 2 doublets (one of which is SM one). Also, it contains 2 coloured triplets mediating p-decay → heavy or decoupled *Dvali '92*

One-loop RG leads to

$$\frac{M_{GUT}}{M_Z} \simeq \exp\left\{\frac{\pi}{10}\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)_{M_Z}\right\}\left[\left(\frac{M_Z}{M_I}\right)^{22}\left(\frac{M_Z^2 m_{sel} m_{sup}}{m_3 m_8 m_{sq}^2}\right)\right]^{\frac{1}{20}}$$

- p-decay → $M_{GUT} \gtrsim 4 \cdot 10^{15}$GeV
- ν mass → $M_I \gtrsim 10^{14}$GeV and $\Lambda \sim 10\, M_{GUT}$

Typically $M_I \sim 10^{12}$GeV → scalars must be light

Cutoff not so far from M_{GUT}

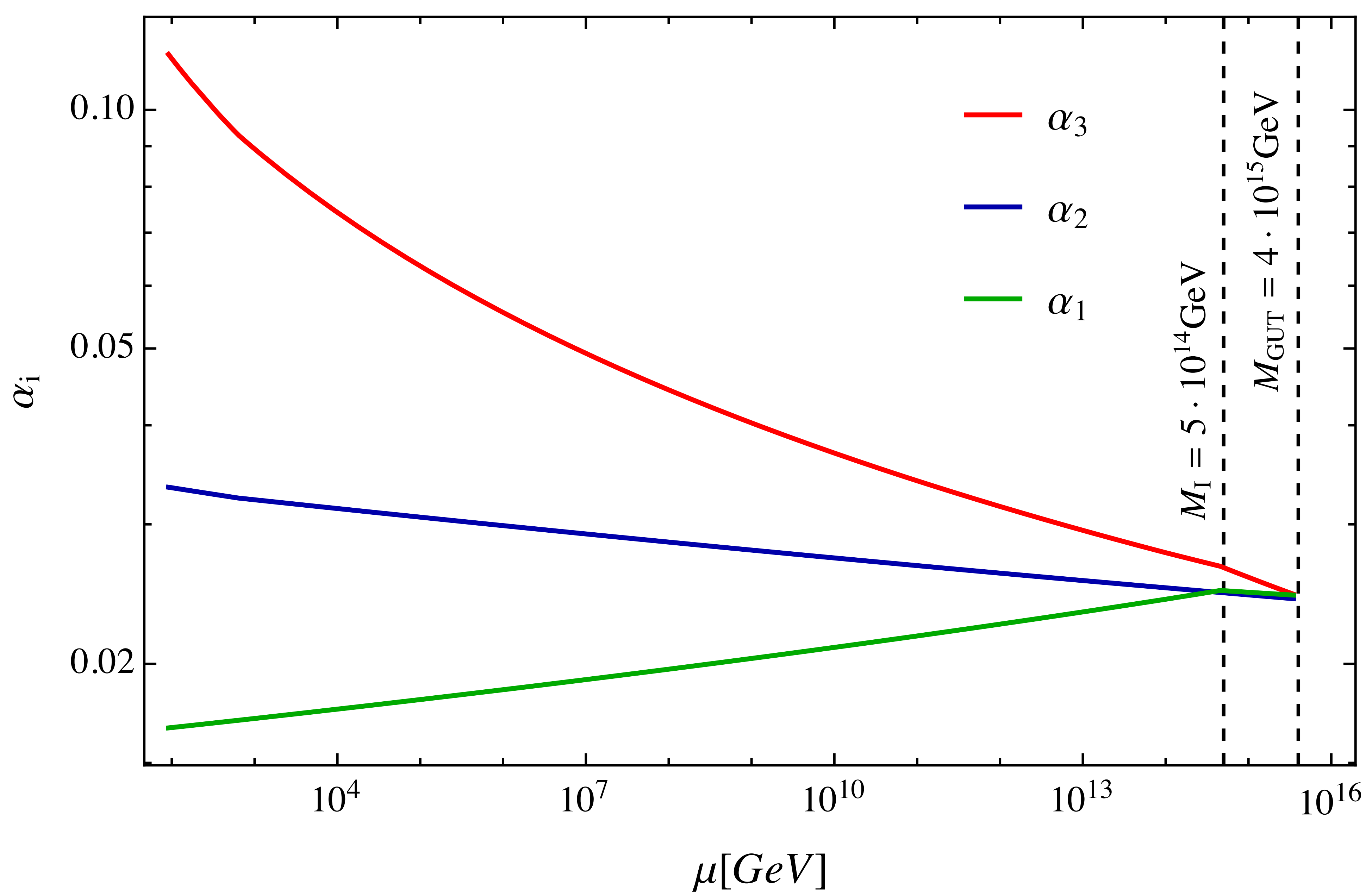
$$\frac{1}{\Lambda^2} F F\, 45_H^2$$

$$\delta\alpha_1^{LR} = \delta\alpha_1^{QL} = \left(\frac{M_{GUT}}{\Lambda}\right)^2 ; \quad \delta\alpha_3^{LR} = \frac{5}{2}\left(\frac{M_{GUT}}{\Lambda}\right)^2 ; \quad \delta\alpha_2^{QL} = \frac{5}{3}\left(\frac{M_{GUT}}{\Lambda}\right)^2$$

2-loops RG analysis

We varied all particle masses and took into account the effects of higher dimensional operators

QL case



Example of realization:
$m_3 = m_8 = m_{sq} = \text{TeV}$

Spectrum

Particle	Mass range
scalar quark doublet	$m_{sq} \lesssim 10\text{TeV}$
weak triplet	$m_3 \lesssim 10\text{TeV}$
color octet	$m_8 \lesssim 10\text{TeV}$
scalar lepton doublet	$10^3\text{GeV} - M_I$
second Higgs doublet	$10^3\text{GeV} - M_{GUT}$
scalar down quark	$10^{12}\text{GeV} - M_{GUT}$
color triplet Higgs partners	$10^{12}\text{GeV} - M_{GUT}$
scalar up quark	$10^{14}\text{GeV} - M_{GUT}$
scalar electron	$10^{14}\text{GeV} - M_{GUT}$

- m_3, m_{sq}, m_8 **always lie below** 10TeV **to ensure unification, p-lifetime and neutrino mass**
- $M_{GUT} < 10^{16}\text{GeV}$ **always implying** $\tau_p < 10^{35}\text{yrs}$

The problem of scale in supersymmetry

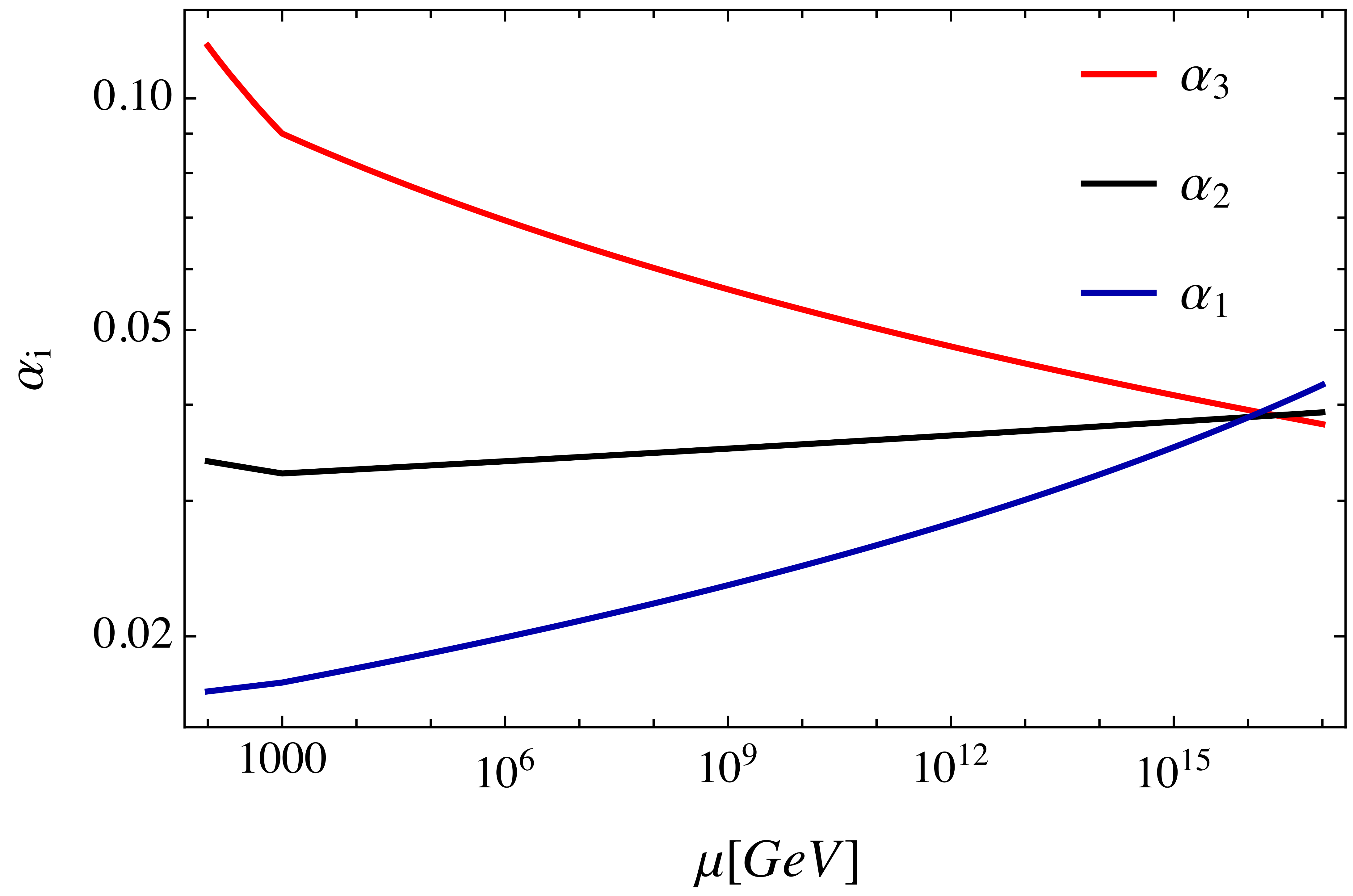
Supersymmetry: particle p $\longrightarrow$ sparticle $\tilde{p}$

$$m_h^2 = m_0^2 + \frac{y_t}{16\pi^2}\Lambda^2 + m_t^2 + \ldots \quad \text{SM Higgs correction}$$

$$-\frac{y_t}{16\pi^2}\Lambda^2 - m_{\tilde{t}}^2 + \ldots \quad \text{MSSM addition}$$

$$m_{\tilde{p}} \simeq TeV$$

$\Lambda^{\mathrm{MSSM}} \sim \mathrm{TeV}, \quad \mathrm{M_{GUT}} \simeq 10^{16}\mathrm{GeV}$



The problem of scale in supersymmetry

$$\Lambda = \Lambda^{\rm MSSM} \left(\frac{M_{\rm GUT}^2}{m_3 m_8}\right)^{3/4} .$$

For more, tune in to G. Senjanović closing talk on Friday

Senjanović, MZ '23

$\Lambda_s \sim m_8 \sim 10^{11}\text{GeV}, \quad m_3 \sim 10^9\text{GeV}, \quad M_G \simeq 10^{16}\text{GeV}$