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Stochastic DM production

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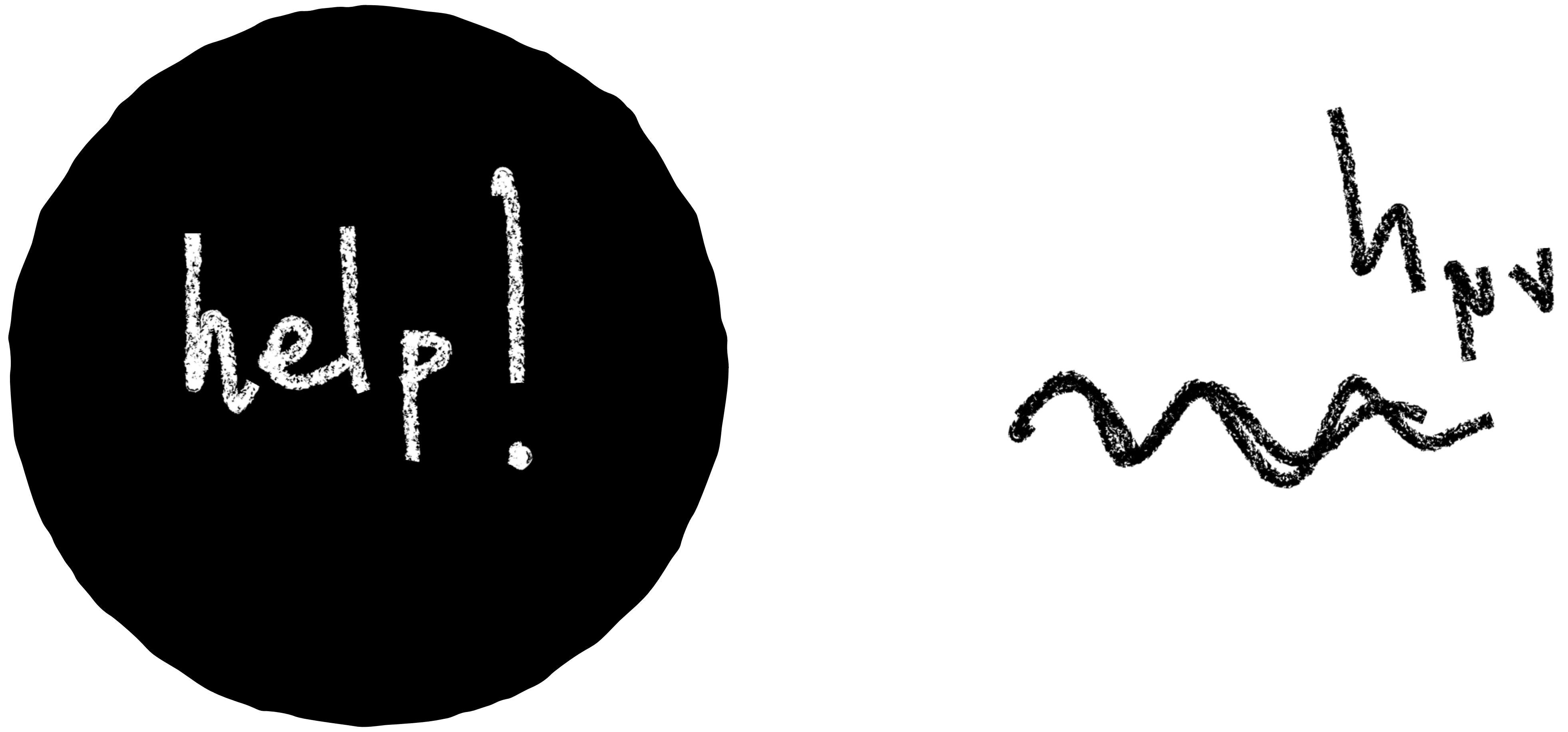
with Michele & Raghuveer

How dark can be the dark matter?*



I will assume only gravitational coupling

* recurrent question in conversations with <u>Michele</u>

Real challenge: jump start the empty dark sector

two possibilities (so far....)

* Gravitational Freeze-in [GFI]

$$\rho_{SM} \sim T_R^4$$

* Gravitational Particle Production [GPP]

$a(t)$ time dependence of background

Prototype of dark-dark-matter

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{M}{2}\left(\Psi\Psi + \bar{\Psi}\bar{\Psi}\right)$$

* M, mass
* Free theory coupled to gravity
* Shares feature of Weyl/conformal sectors
  ↳ obstruction to GPP [nightmare]

Outline

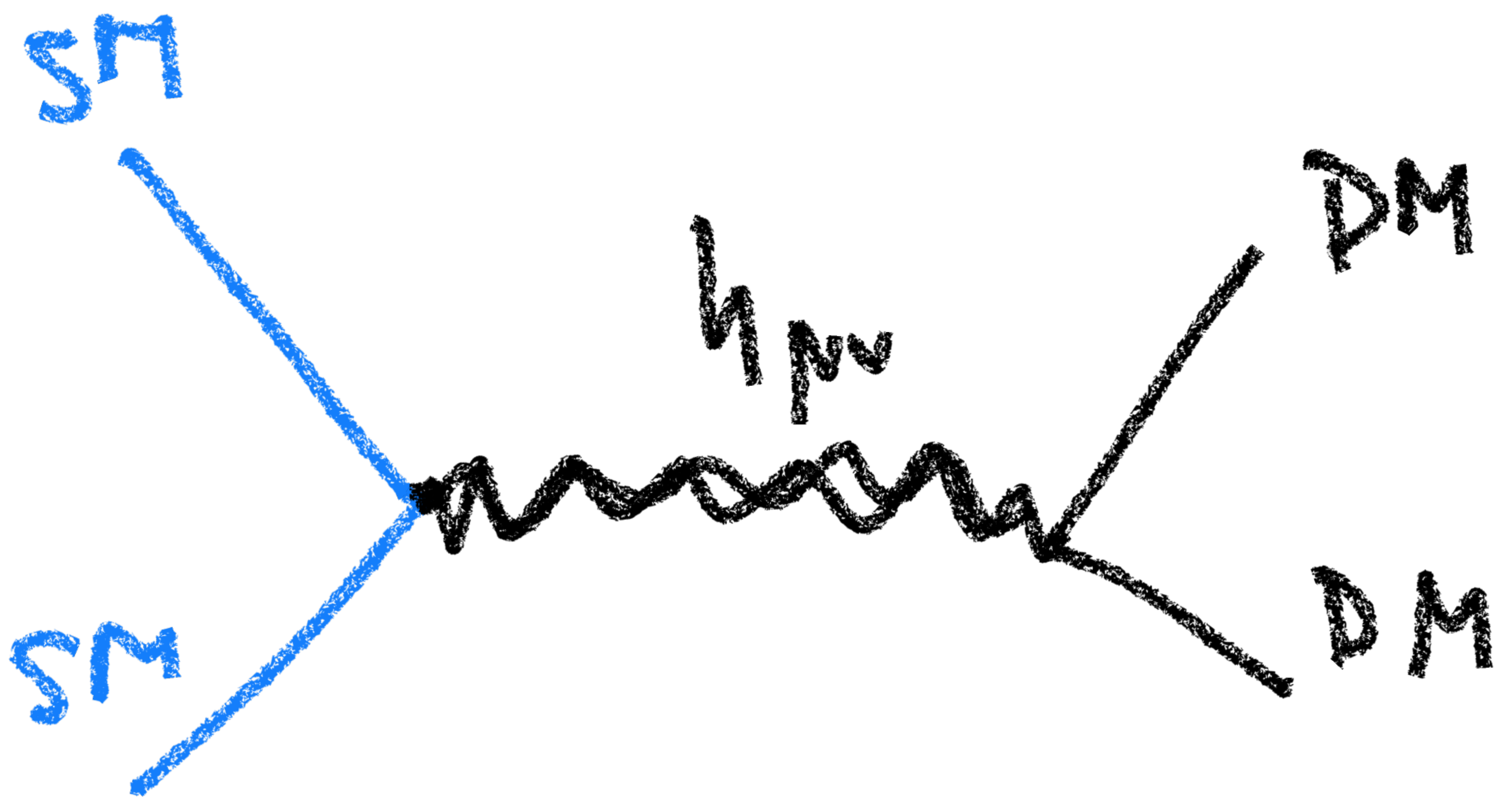
$\rightarrow$ Compute $\quad M \left.\frac{n}{s}\right|_{\text{today}} = 0.4\,\text{eV}$

1) Gravitational Freeze-In
2) Gravitational Particle Production

   and

3) Stochastic Gravitational Particle Production

Gravitational Freeze-In



SM thermal states annihilate into DM via graviton-exchange

$$\frac{1}{M_{Pl}} h_{\mu\nu} T^{\mu\nu}$$

* From direct computation:

$$\Omega_{DM}\Big|_{GFI} \sim 10^{-5} \frac{M K_R^3}{3H_0^2 M_{Pl}^2}$$

- $K_R \sim T_R T_0 / M_{Pl}$
- $T_R > M$
- CFT computation [c-charge dependence]

Gravitational Particle Production

QM Hamiltonian with time dependence → initial vacuum not final vacuum

In QFT time dependence of background is not enough:

* effect proportional to $M \cdot a(t)$
* physical effect modulo Weyl rescaling

$$\boxed{i \bar{\sigma}^\mu \partial_\mu \psi = M a(t) \bar{\psi}}$$

Bogoliubov transformations: free fermion

At early times, Bunch-Davies vacuum, $a(t) \to 0$

* when mass is negligible $k > aH > aM$

→ only positive frequency solutions [as in flat space]

$$\psi = \sum_h \int \frac{d^3k}{(2\pi)^3} \left[v_{k,h}(\tau)\, \xi_{k,h}\, e^{i\vec{k}\cdot\vec{x}}\, a_{\vec{k},h} + \dots \right]$$

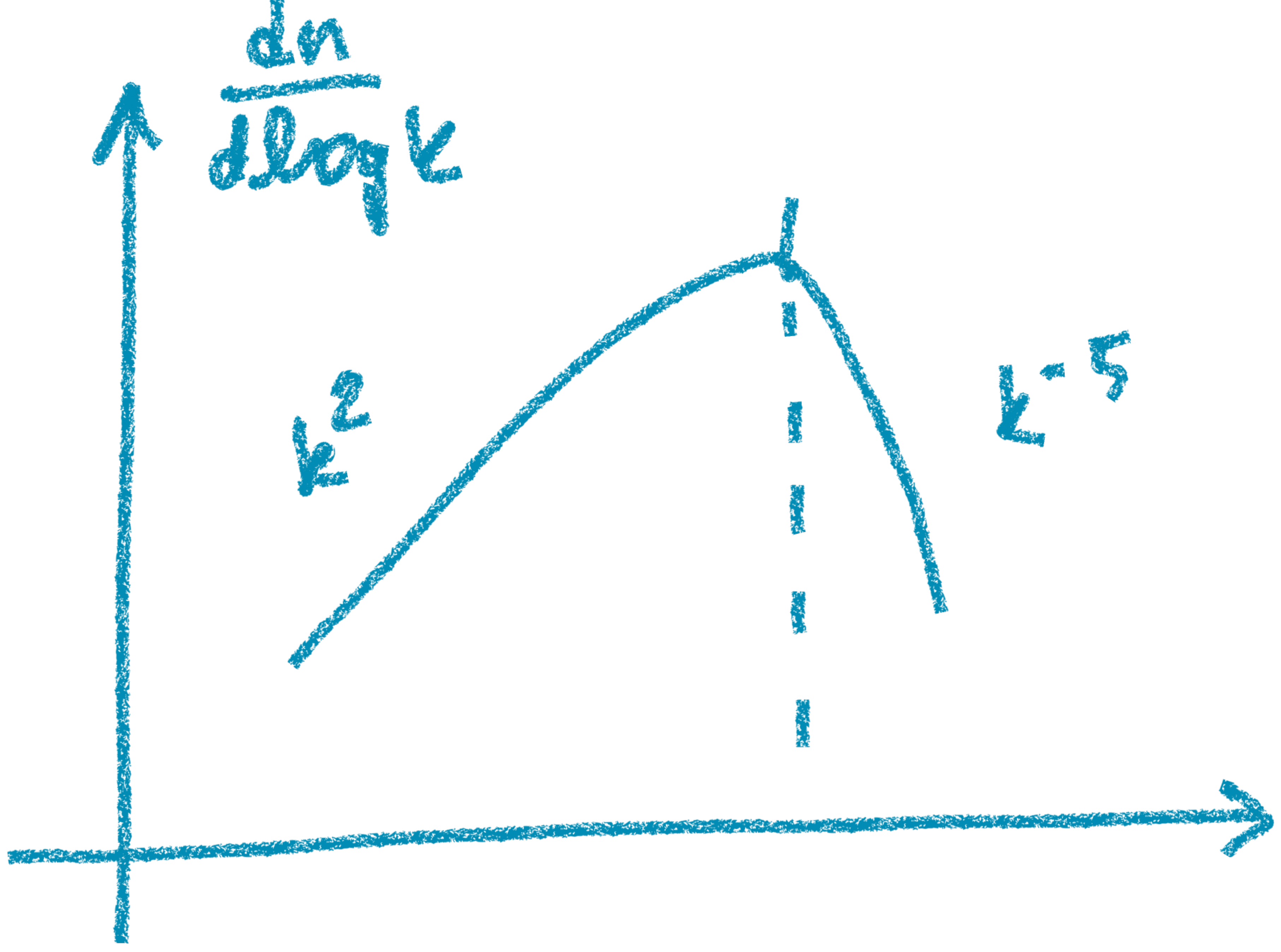
$$v_{k,h}(-\infty) = e^{-ik\tau}\,\delta_{h,-}\ ; \quad v_{k,h}(+\infty) \sim \alpha_k\, e^{-i\int Ma\,d\tau} + h\,\beta_k\, e^{+i\int Ma\,d\tau}$$

Number density from Bogolivbov coefficients

$$n = \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2$$

$$\Omega_{DM}\big|_{GPP} \sim 10^{-2} \frac{M k_M^3}{3 H_0^2 M_{Pl}^2}$$

- $k_M \sim a_M M$; $a_M = a(H \sim M)$
- strong suppression with M
- $M \sim 2.6 \times 10^8$ GeV



w/ Michele & Raghuveer
2408 + in progress

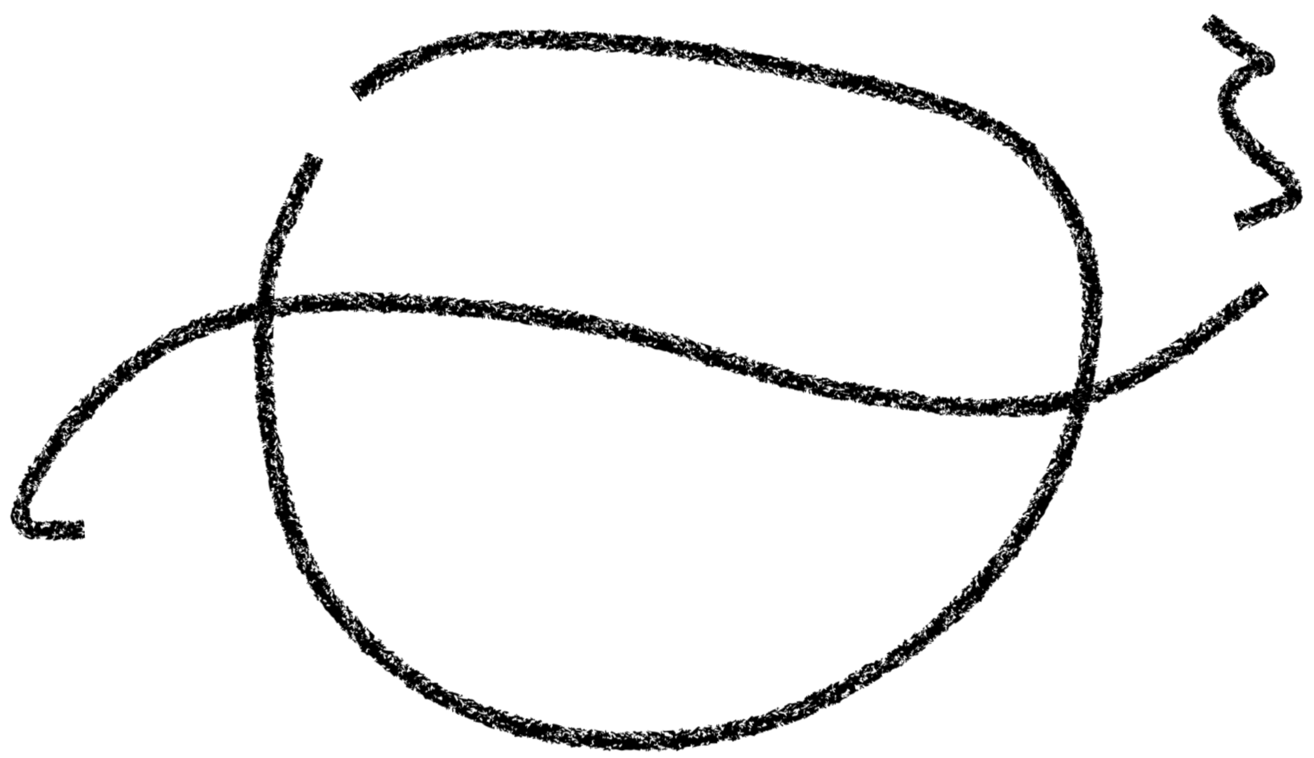
Stochastic Gravitational Particle Production

$$M_a(t) \longleftrightarrow \Phi(t, \vec{x})$$

cosmological perturbations

Curvature perturbations as source of DM

* they exist (are time dependent)
* in conformal gauge: $ds^2 = a^2 d\tau^2 [1 + 2\Phi] - a^2 dx^2 [1 - 2\Psi]$



$$\langle \zeta_q \zeta^*_{q'} \rangle \sim \delta^3(q - q')\, \Delta_\zeta(q)$$

$$\Delta_\zeta \sim 10^{-9} \text{ @ CMB}$$

Φ, Ψ matched to super-horizon value of ζ

$\rightarrow \Phi, \psi$ "active" in radiation domination

Strong analogy with Bogoliubov

$$\boxed{i\bar{\sigma}^\mu \partial_\mu \psi = i\left(2\Phi\dot{\psi} - \frac{1}{2}\nabla\Phi\cdot\vec{\sigma}\psi + \frac{3}{2}\dot{\Phi}\psi\right)}$$

* Linear in fermion field
* time dependent source
* interactions with fluctuations $\longrightarrow$ negative frequency β_k

$\rightarrow$ find solution perturbatively with $G^{ret.}$
starting from Bunch-Davies ... $n \sim |\beta|^2$

After some manipulations [stochastic ζ]

$$\frac{dn}{d\log k} = \frac{k^3}{4\pi^2}\int \frac{d\cos\theta}{q}\, dq\, \Delta_\zeta(q)\, |\mathcal{I}(q, k+\omega)|^2\, \mathcal{K}[k, q, \cos\theta]$$

* $\mathcal{K}$, spin-dependent kernel
* $\mathcal{I}$, transfer function of gravitational potential
* $\Delta_\zeta(q)$, primordial power spectrum of ζ

$\longrightarrow$ assume peak @ q_* for $\Delta_\zeta(q)$ ...

formula appears complicated: universal properties

Stochastic Dark Matter

$$\boxed{n = 0.015 \int \frac{dq}{q}\, q^3 \Delta_s(q)}$$

Free Fermion

$$q_* \sim 10^{-7}\,\mathrm{eV} \left(\frac{10^6\,\mathrm{GeV}}{M}\right)^{1/3} \left(\frac{0.001}{\Delta(q_*)}\right)^{1/3}$$

* related to last e-folds of inflation
* <u>valid</u>: $k \sim q_* \gg aM$

  Φ, evolves after T_R (not diluted!)

New contribution to DM abundance

	GFI	GPP	Stochastic
$\Omega_{DM} \propto$	$10^{-5} M k_R^3$	$10^{-2} M k_m^3$	$10^{-4} M q_*^3 \Delta_\xi(q_*)$

dominance of stochastic for $q_* \gtrsim k_R$ [reheating]

$\longrightarrow$ Computations simplified & Scope broadened exploiting "CFT-ideology"

see Michele's talk!

ENJOY
THE
SLOPES!

* if you want to join a newbie...