

ECOS2012, Villa Vigoni, Como Lake
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Nuclear Astrophysics With Indirect Methods



The Cat's Eye Nebula — NGC 6543  HUBBLESITE.org



Charged particle cross section measurements at astrophysical energies

$\sigma \sim \text{picobarn} \Rightarrow$ Low signal-to-noise ratio due to the **Coulomb barrier** between the interacting nuclei



Extrapolation from the higher energies by using the

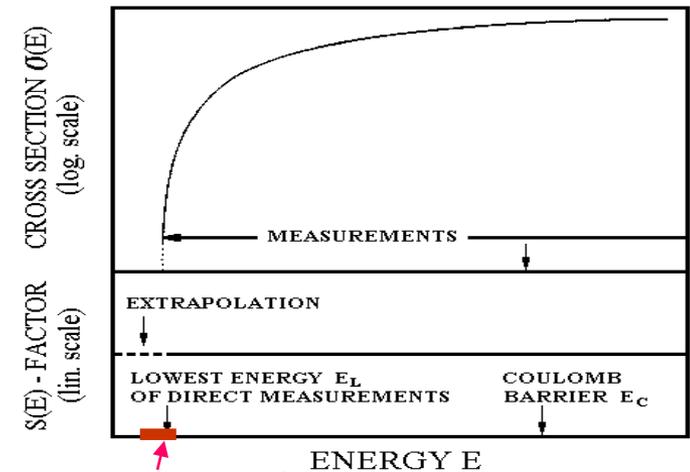
ASTROPHYSICAL FACTOR

$$S(E) = \sigma(E) E \exp(2\pi\eta)$$

$S(E)$ is a smoothly varying function of the energy than the cross section $\sigma(E)$

...but large uncertainties in the extrapolation

\rightarrow EXPERIMENTAL IMPROVEMENTS/SOLUTIONS



Astrophysical energies

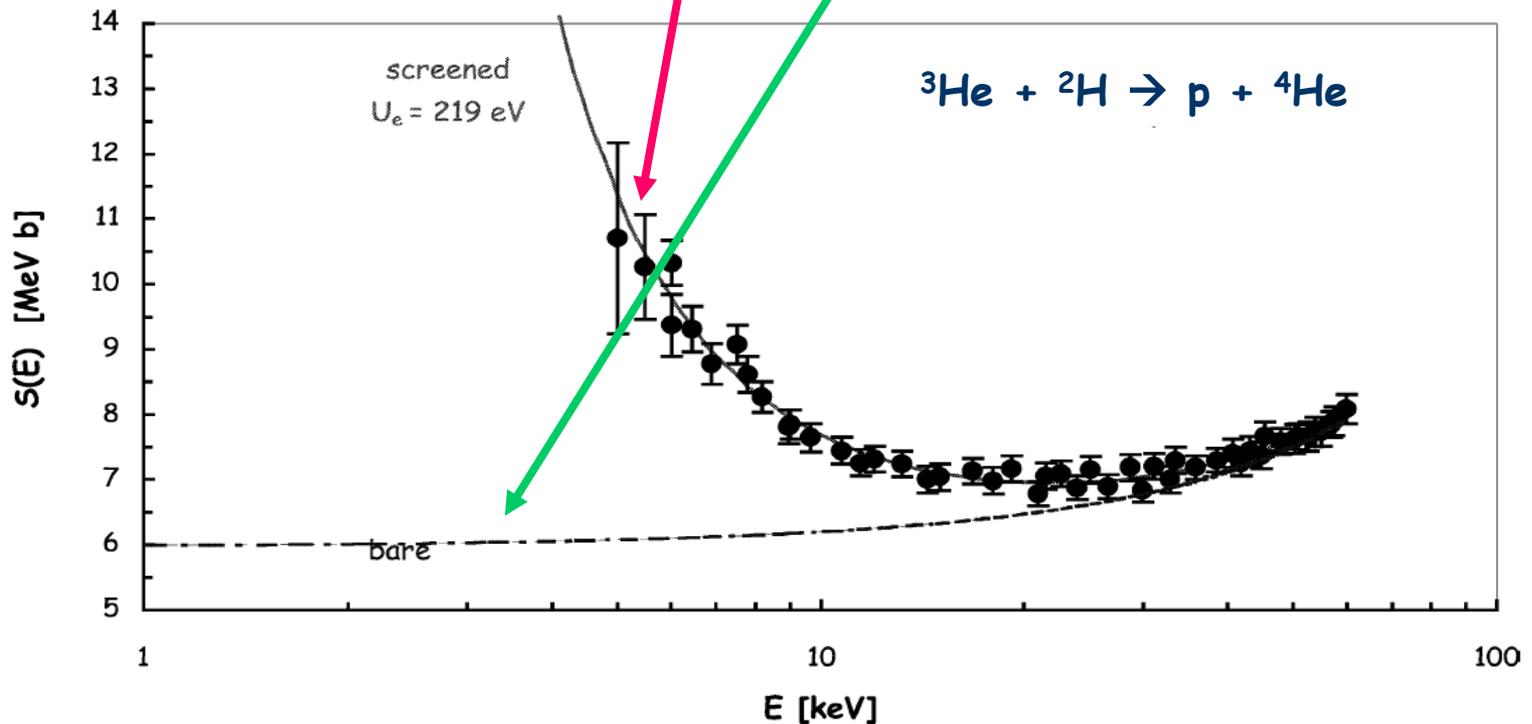
- > to increase the number of detected particles
- > to reduce the background

...but... further problem at astrophysical energies
→ → → →

Electron Screening Electron Screening

$S(E)$ enhancement experimentally found due to the Electron Screening

$$S(E)_s = S(E)_b \exp(\pi\eta U_e / \dots)$$



Electron Screening

In astrophysical plasma:

- the screening, due to free electrons in plasma, can be different \rightarrow we need $S(E)_b$ to evaluate reaction rates



A theoretical approach to extract the electron screening potential U_e in the laboratory is needed

Experimental studies of reactions involving light nuclides have shown



that the **observed exponential enhancement** of the cross section at low energies were in all cases significantly larger

(about a factor of 2)

than it could be accounted for from available atomic-physics model, i.e. the adiabatic limit $(U_e)_{ad}$

Although we try to improve experimental techniques to measure at very low energy $\rightarrow \rightarrow$

$S_b(E)$ -factor extracted from extrapolation of higher energy data

... new methods are necessary

- to measure cross sections at never reached energies
- to get independent information on U_e
- to overcome difficulties in producing the beam or the target (Radioactive ions, neutrons..)

-> -> -> **INDIRECT METHODS**

❖ **Coulomb dissociation**

...to determine the absolute $S(E)$ factor of a radiative capture reaction $A+x \rightarrow B+\gamma$ studying the reversing photodisintegration process $B+\gamma \rightarrow A+x$

❖ **Asymptotic Normalization Coefficients (ANC)**

... to determine the $S(0)$ factor of the radiative capture reaction, $A+x \rightarrow B+\gamma$ studying a peripheral transfer reaction into a bound state of the B nucleus

❖ **Trojan Horse Method (THM)**

...to determine the $S(E)$ factor of a charged particle reaction $A+x \rightarrow c+C$ selecting the Quasi Free contribution of an appropriate $A+a(x+s) \rightarrow c+C+s$ reaction

From Coulomb dissociation to photoabsorption

The cross-section for Coulomb dissociation (C.D.) can be linked to the photoabsorption one by the following expression:

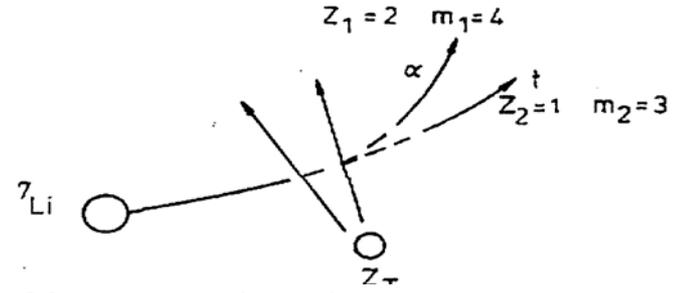
$$\frac{d^2 \sigma}{d\Omega dE_\gamma} = \frac{1}{E_\gamma} \frac{dn_{\pi,\lambda}}{d\Omega} \sigma_{\pi,\lambda}^{\text{photo}}$$

Some important considerations:

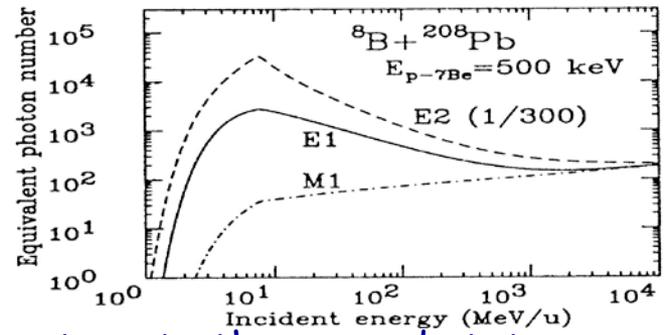
-  C.D. enhances the number of events with respect to the ones for the original capture process, by a large factor. This is due to:
 - large virtual photon number,
 - possibility to use thick targets,
 - phase space factor $(k_x/k_\gamma)^2$ from detailed balance linking σ_{photo} with σ_{capt} .
-  Contrary to a photodissociation reaction, fragments emerge with high velocity making their detection easier.

☹️ The nuclear contribution to the break-up must be negligible
 → large impact parameters → small fragment detection angles needed. Otherwise
 quantal calculations (DWBA/Eikonal), optical potentials needed

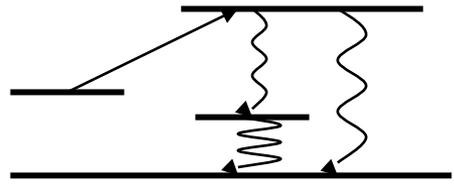
☹️ Post acceleration effects must be negligible → high projectile velocities decrease their
 effects. Otherwise higher-order effect of Coulomb interaction accounted for with
 various theoretical approaches such as:
 Time-dependent dynamical calculations



☹️ One has to take properly into account that different multipolarities can contribute with
 different weights in the dissociation processes and radiative capture processes.
 Effects on angular distributions,
 on the slope of the extracted S factor.



☹️ C.D. provides only information on the radiative capture to the ground state.



Some experimental prescriptions

Some examples taken from the study of
 ${}^6\text{Li} + {}^{208}\text{Pb} \rightarrow \alpha + d + {}^{208}\text{Pb}$ (J.Kiener et al.: PRC 44,2195,(1991))

We have to study the 3-body reaction:

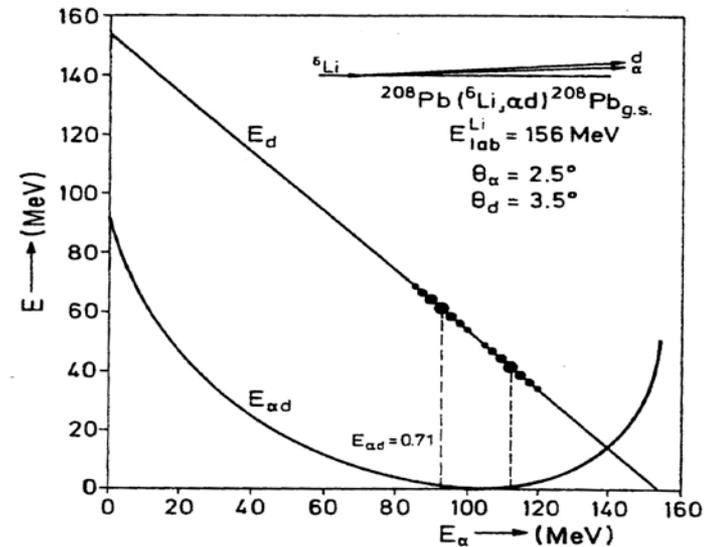


In a way which allows us to completely reconstruct the kinematics.



1) We have to identify and detect in coincidence the fragments A and x, measuring their energies and emission angles. Fragments have to be detected at small angles (large impact parameters)

2) Elastic B.U. events must be selected



$$E_{\alpha} + E_d = E_{\text{proj}} - Q_{\text{th}} - E^*(\text{Pb}) - E_k(\text{Pb}) \approx E_{\text{proj}} - Q_{\text{th}}$$

Elastic B.U. events form a straight line in a $E_A - E_x$ plot

3) The relative energy E_{A-x} must be reconstructed event by event.

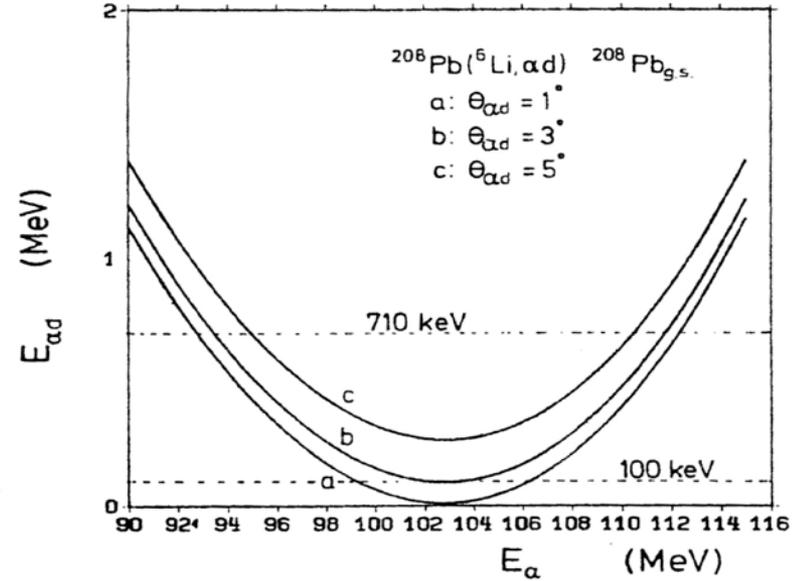
• **Magnifying glass effect:**

There is a weak dependence of the relative energy E_{A-x} around its minimum on the energies of the two fragments.

Example: $D(E_{\alpha-d})/D(E_{\alpha}) \approx 5\%$

• Small relative angles q_{A-x} must be measurable to explore low relative energies.

• The relative angle between the fragments must be measured with good resolution to obtain a good relative energy resolution.



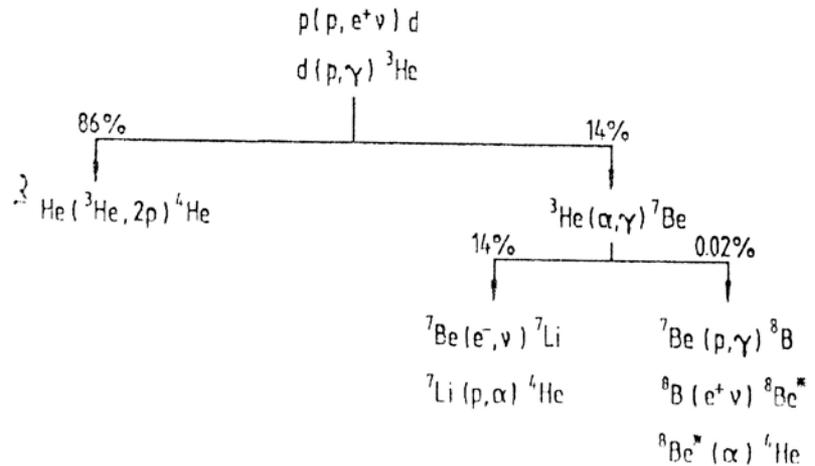
4) For different bins of relative energy E_{A-x} angular distributions can be extracted

5) Reproducing angular distribution one can extract the value of $\sigma_{\text{photo}}(B+\gamma \rightarrow A+x)$

6) Applying the detailed balance principle, one get back to
$$\sigma_{\text{capt}} = \sigma_{\text{photo}} \frac{2(2J_B+1)}{(2J_A+1)(2J_x+1)} \frac{(k_\gamma)^2}{(k_x)^2}$$

The ${}^7\text{Be}(p,\gamma){}^8\text{B}$ case

The ${}^7\text{Be}(p,\gamma){}^8\text{B}$ reaction is connected with the Solar Neutrino problem and has been studied by different groups.



Results of Riken C.D. experiments

Refs.: T. Motobayashi et al.: PRL 73,2680,(1995)

T. Tikuchi et al: PLB 391,261,(1997)

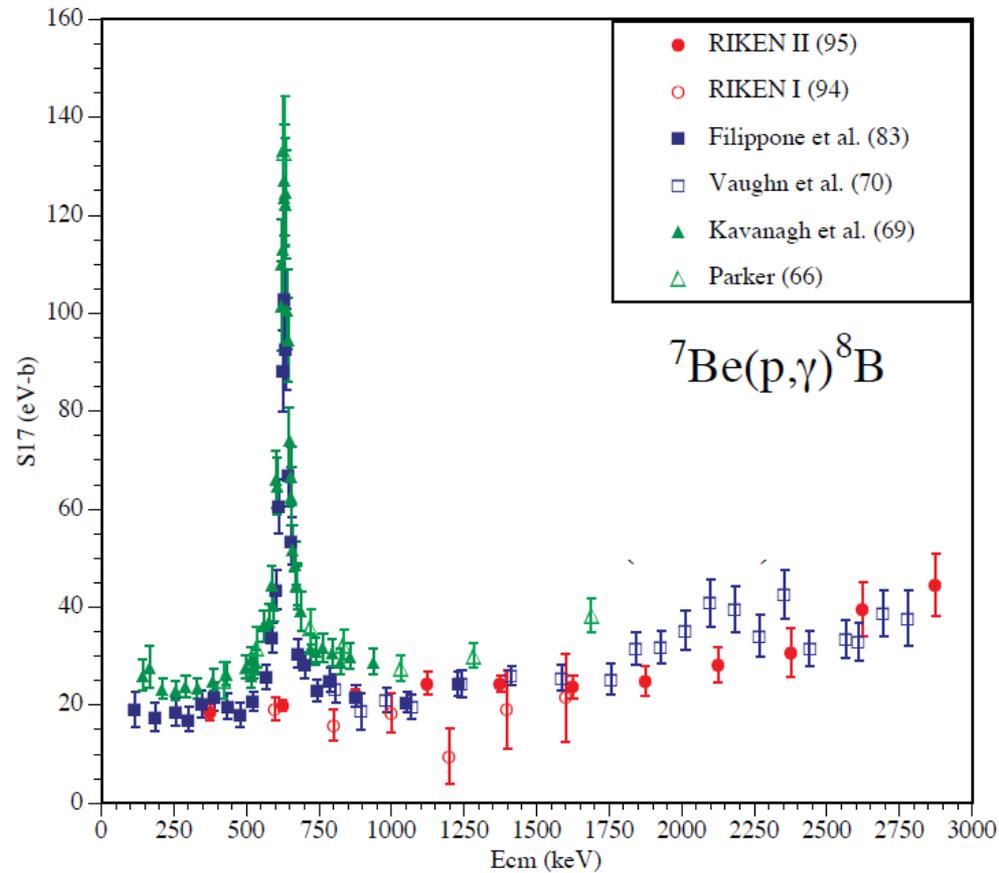
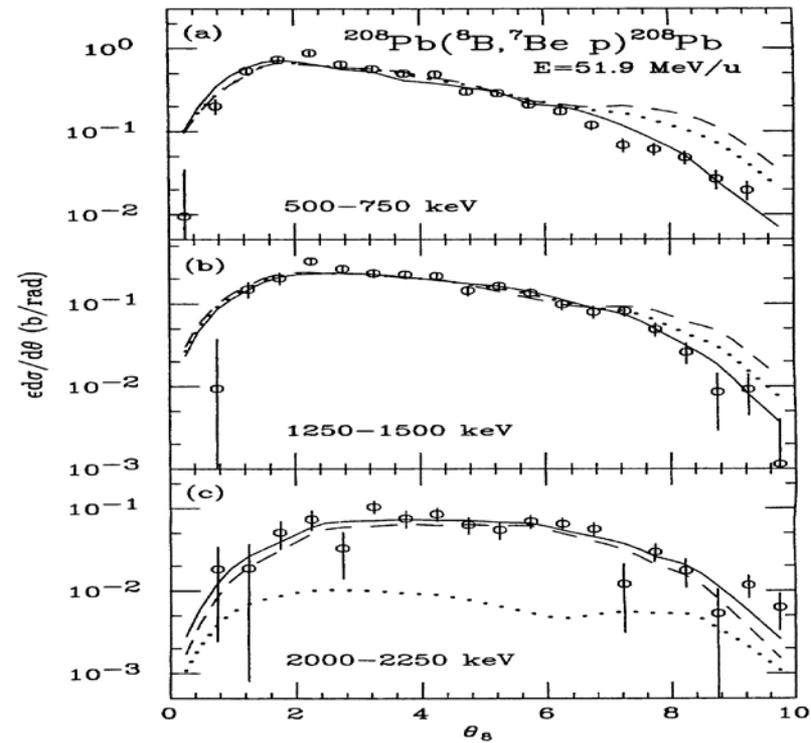
T. Tikuchi et al: EPJ A3,213,(1998)

${}^8\text{B} + {}^{208}\text{Pb} \rightarrow p + {}^7\text{Be} + {}^{208}\text{Pb}$ @ 51.9 MeV/nucleon

$i({}^8\text{B}) = 10^4$ pps

- Particles were detected using a telescope system based on plastic scintillators.
- Elastic B.U. events were selected and relative energy spectra reconstructed.

Angular distributions for different bins in relative energy reconstructed and fitted and astrophysical $S(E)$ factor extracted.



GSI experiments 2003: Significant contribution from E2 multipolarity excluded. $S_{17}(0)=18.1\pm 0.3$ eV·b

However, two major differences between CD and direct S(E) factors:

- . $S_{17}(0)$ from CD measurements about 10% lower than the mean of direct measurements
- . $S_{17}(E)$ slope from CD steeper than from direct measurements

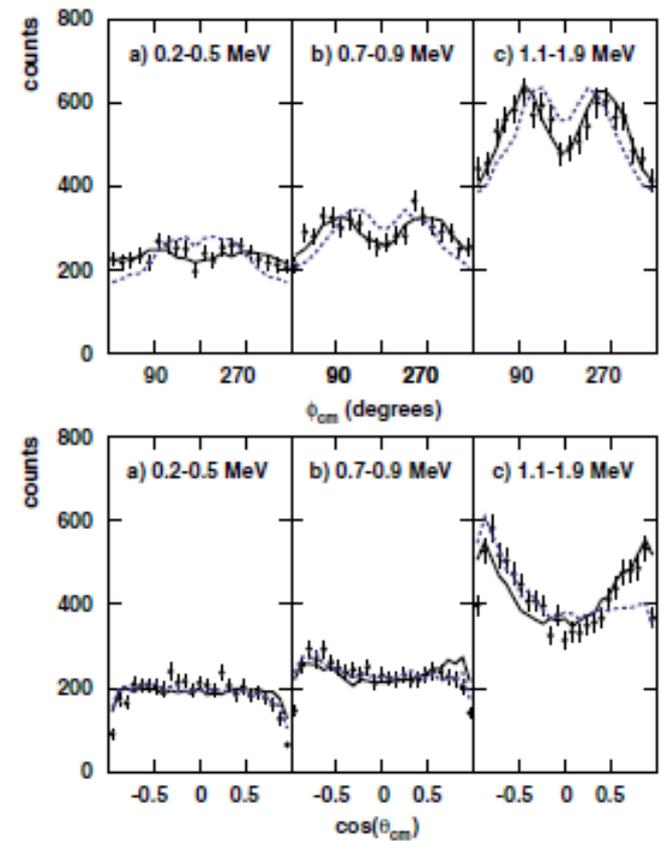
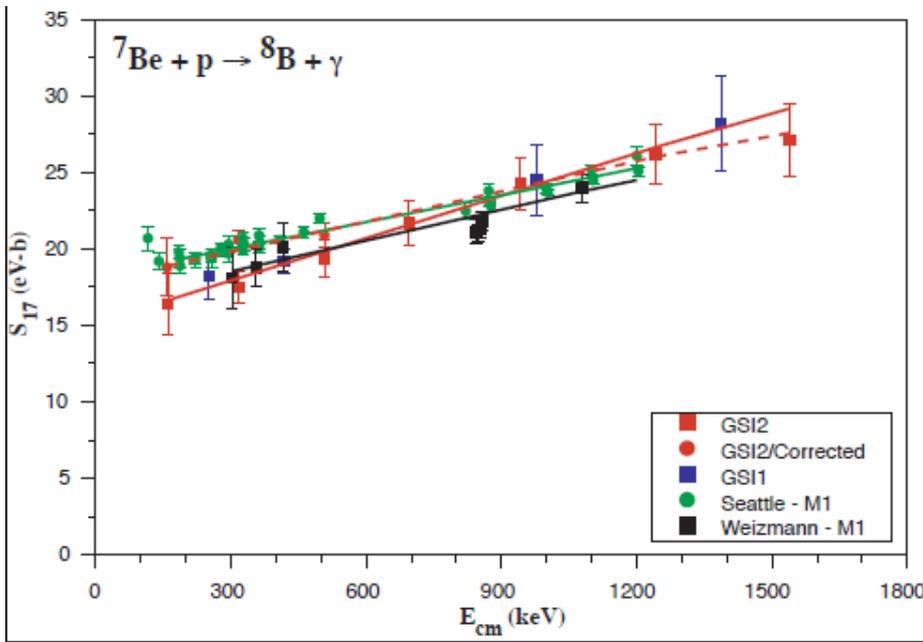


FIG. 3 (color online). Top: Experimental distributions of the proton azimuthal angular (ϕ_{cm}) distributions for three different bins of the p - ${}^7\text{Be}$ relative energy, E_{rel} . The full histograms denote a first-order perturbation-theory calculation for $E1$ multipolarity, and the dashed ones for $E1 + E2$. All theoretical curves were individually normalized to the data points in each frame. Bottom: the same for the polar breakup angles, θ_{cm} .

GSI corrected: more accurate Coloumb break-up theory brings to the agreement
 ...alternative analysis via ANC

Some References on C.D.

- 1) G. Baur et al. J.Phys. G 20,1, (1994)
 - 2) G. Baur et al. Annu. Rev. Nucl. Part. Sci. 46, 321,(1996)
 - 3) T. Motobayashi et al.:NPA 719,65c,2003)
 - 4) J. Kiener et al. PRC 44,2195,(1991)
- ${}^7\text{Be}(p,\gamma){}^8\text{B}$**
- 5) T.Motobayashi et al.: PRL 73,2680,(1994).
NPA 693,258,(2001)
 - 6) T.Tikuchi et al: PLB 391,261,(1997)
 - 7) T.Tikuchi et al: EPJ A3,213,(1998)
 - 8) B.Davids et al.: PRL 86,2750,(2001)
EPJ A15,65,(2002)
 - 9) F. Schuemann et al., PRL 90 (2003) 232501
 - 10) H. Esbensen et al., PRL 94 (2005) 42502
 - 10) M. Gai et al., PRC 74 (2006) 025810

${}^{13}\text{N}(p,\gamma){}^{14}\text{O}$

- 11) J. Kiener et al.: NPA 552, 66, (1993)
- 12) Motobayashi et al. PLB 264, 259, (1991)

${}^2\text{H}(\alpha,\gamma){}^6\text{Li}$

- 13) F. Hammache et al., Phys. Rev. C 82 (2010) 065803
- 14) J.Kiener et al.: PRC 44,2195,(1991)

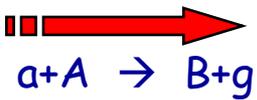
${}^{14}\text{C}(n,\gamma){}^{15}\text{C}$

- H. Esbensen, Phys. Rev. C 80, 024608 (2009)

Table 2 Radiative capture reactions of interest for light-element synthesis accessible by of fast projectiles

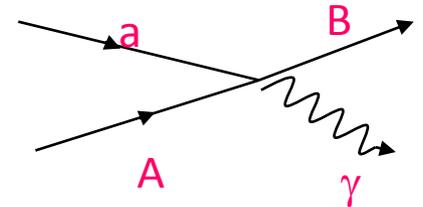
Reaction	$T_{1/2}$ (projectile)	Astrophysical application	
${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$	53.3 days	Solar-neutrino problem ${}^3\text{He}$ abundancy	
${}^7\text{Be}(p,\gamma){}^8\text{B}$	770 ms		
${}^7\text{Be}(\alpha,\gamma){}^{11}\text{C}$	20.4 min		
${}^4\text{He}(d,\gamma){}^6\text{Li}$	Stable	Primordial nucleosynthesis of Li Be B-isotopes	
${}^6\text{Li}(p,\gamma){}^7\text{Be}$	53.3 days		
${}^6\text{Li}(\alpha,\gamma){}^{10}\text{B}$	Stable		
${}^4\text{He}(t,\gamma){}^7\text{Li}$	Stable		
${}^7\text{Li}(\alpha,\gamma){}^{11}\text{B}$	Stable		
${}^{11}\text{B}(p,\gamma){}^{12}\text{C}$	Stable		
${}^9\text{Be}(p,\gamma){}^{10}\text{B}$	Stable		
${}^{10}\text{B}(p,\gamma){}^{11}\text{C}$	20.4 min		
${}^7\text{Li}(n,\gamma){}^8\text{Li}$	842 ms		Primordial nucleosynthesis in inhomogeneous B
${}^8\text{Li}(n,\gamma){}^9\text{Li}$	178 ms		
${}^{12}\text{C}(n,\gamma){}^{13}\text{C}$	Stable		
${}^{14}\text{C}(n,\gamma){}^{15}\text{C}$	2.45 s		
${}^{14}\text{C}(\alpha,\gamma){}^{18}\text{O}$	Stable		
${}^{12}\text{C}(p,\gamma){}^{13}\text{N}$	10 min	CNO cycles	
${}^{16}\text{O}(p,\gamma){}^{17}\text{F}$	65 s		
${}^{13}\text{N}(p,\gamma){}^{14}\text{O}$	70.6 s		
${}^{20}\text{Ne}(p,\gamma){}^{21}\text{Na}$	22.5 s	Hot p - p chain	
${}^{11}\text{C}(p,\gamma){}^{12}\text{N}$	11 ms		
${}^{15}\text{O}(\alpha,\gamma){}^{19}\text{Ne}$	17.2 s		rp process
${}^{31}\text{S}(p,\gamma){}^{32}\text{Cl}$	291 ms		
${}^{12}\text{C}(\alpha,\gamma){}^{16}\text{O}$	Stable	Helium burning	
${}^{16}\text{O}(\alpha,\gamma){}^{20}\text{Ne}$	Stable		
${}^{14}\text{N}(\alpha,\gamma){}^{18}\text{F}$	109.7 min		

Asymptotic Normalization Coefficients



At low relative energies the $S(0)$ for a (peripheral) direct capture reaction

$$S(0)_{DC} \propto (C_{aA}^B)^2$$

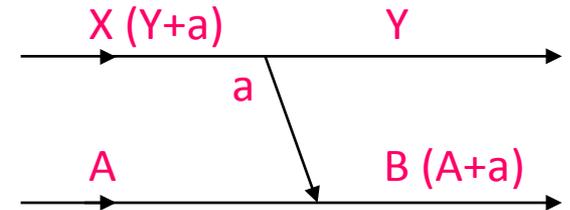


C_{aA}^B is the so called ANC that specifies the tail of the B overlap function in the a+A channel



For a peripheral transfer reaction $X+A \rightarrow Y+B$ into a bound state of B,

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}} \propto (C_{aA}^B)^2 \cdot (C_{Ya}^X)^2 \left(\frac{d\tilde{\sigma}}{d\Omega} \right)_{DW}$$



reduced DWBA cross section
insensitive to the bound
state potential parameters

The ANC C_{aA}^B can be obtained normalising the experimental angular distribution to the calculated one. What we need: precise optical potentials and one additional ANC (from elastic scattering angular distributions)

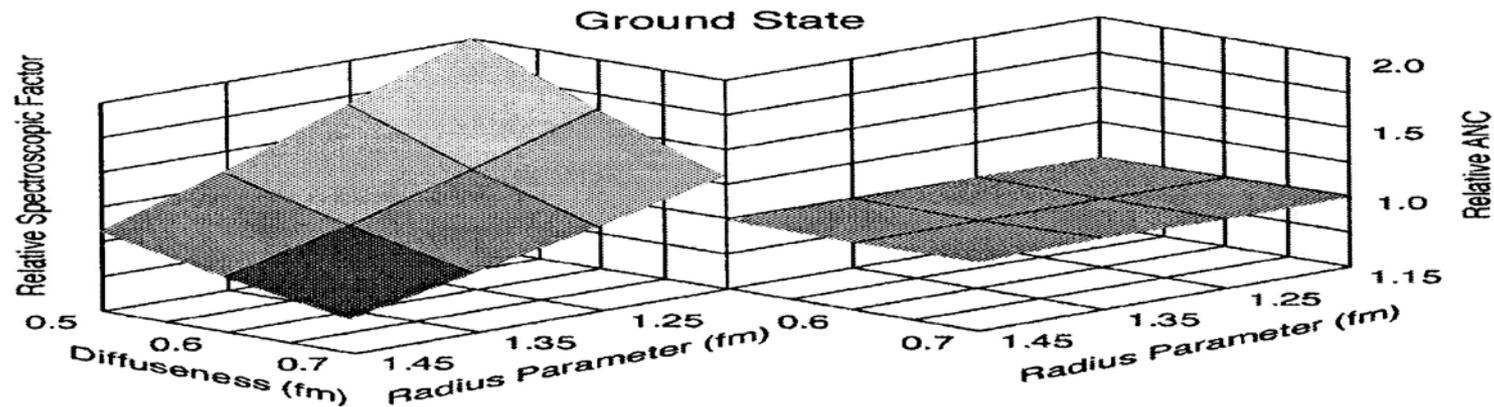
Uncertainties on Spectroscopic factors and ANC

✦ The spectroscopic factor in conventional DWBA analysis is linked to the properties of nuclear interior and its value depends upon the parameters chosen for the bound state potential in the calculations.

✦ The ANC in DWBA analysis of peripheral transfer reactions is less sensitive upon the parameters used for the bound state wave function.

Example

Relative variation of spectroscopic factor and ANC for the g.s. of ^{15}O as obtained from DWBA analysis of $^{14}\text{N}(^3\text{He},d)^{15}\text{O}_{\text{g.s.}}$
(F.P.Bertone et al.:PRC66,055804,(2002))



The ${}^7\text{Be} (p,\gamma){}^8\text{B}$ case

Ref: A. Azahari et al. Phys.Rev.C63, 055803(2001)

ANC for ${}^8\text{B}$, $C_{p{}^7\text{Be}}^{8\text{B}}$ were extracted for two transfer reactions:

${}^{10}\text{B} ({}^7\text{Be}, {}^8\text{B}) {}^9\text{Be}$ and ${}^{14}\text{N} ({}^7\text{Be}, {}^8\text{B}) {}^{13}\text{C}$

Experiment

$E({}^7\text{Be})=85\text{MeV}$ $\Delta E/E=1.9\%$ $i=5\cdot 10^4$ pps

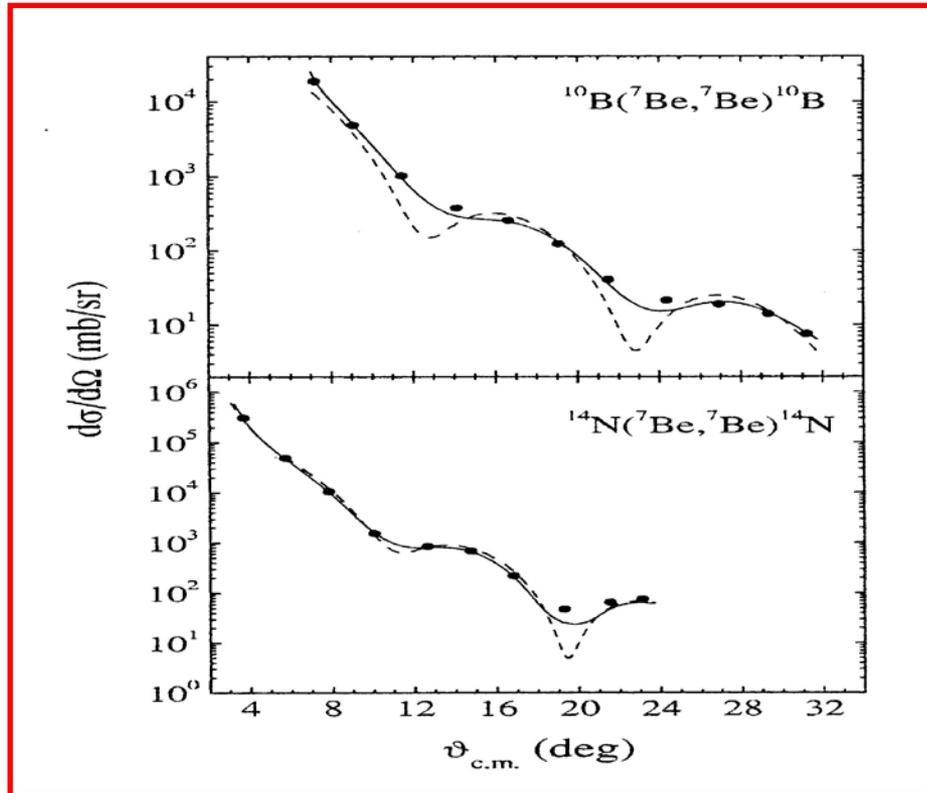
1.7 mg/cm²
 ${}^{10}\text{B}$ Target

Reaction
Telescopes

Reaction products detected and identified by two DE(100mm) E(1000mm) position sensitive Si telescopes on both sides of the beam.

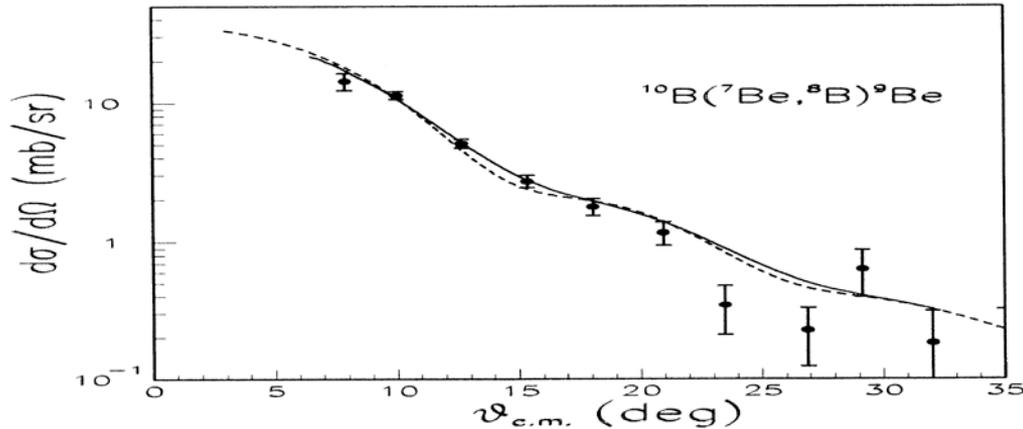
Elastic angular distributions

Appropriate optical model potentials to reproduce elastic scattering.



- - - - Calculated angular distributions
- Same angular distributions corrected for finite angular resolution

Transfer angular distribution and ANC for $^{10}\text{B}(^7\text{Be}, ^8\text{B})^9\text{Be}$



$d\sigma/d\Omega$ transfer

----- dominant contribution

———— smoothed for angular resolution

Attention! The $p_{3/2}$ proton in the g.s. of ^{10}B transfers to either the $p_{1/2}$ or $p_{3/2}$ (dominant contribution) orbitals forming the g.s. of ^8B . Therefore we should add:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = (C^{^{10}\text{B}})^2 \cdot (C_{p_{3/2}}^{^8\text{B}})^2 \left[\left(\frac{d\sigma}{d\Omega}\right)_{p_{3/2}DW} \frac{1}{(b_{^{10}\text{B}})^2 (b_{^8\text{B}_{p_{3/2}}})^2} \right] +$$

$$+ (C^{^{10}\text{B}})^2 \cdot (C_{p_{1/2}}^{^8\text{B}})^2 \left[\left(\frac{d\sigma}{d\Omega}\right)_{p_{1/2}DW} \frac{1}{(b_{^{10}\text{B}})^2 (b_{^8\text{B}_{p_{1/2}}})^2} \right]$$

$$(C_{p_{3/2}}^{^8\text{B}})^2 = 0.410 \pm 0.055 \text{ fm}^{-1} \quad \text{extracted from } ^{10}\text{B}(^7\text{Be}, ^8\text{B})^9\text{Be}$$

$$(C_{p_{3/2}}^{^8\text{B}})^2 = 0.379 \pm 0.055 \text{ fm}^{-1} \quad \text{extracted from } ^{14}\text{N}(^7\text{Be}, ^8\text{B})^{13}\text{C}$$

Stability of the results

- 1) ANC dependence on the single particle Wood-Saxon potential wells.
- 2) ANC dependence on the Optical Model potentials: the authors quote an uncertainty <10% due to Optical Model potentials.

The ANC were used to calculate $S_{17}(0)$ obtaining:

$$S_{17}(0)=18.4\pm 2.5 \text{ eV}\cdot\text{b} \text{ from } {}^{10}\text{B} ({}^7\text{Be}, {}^8\text{B}){}^9\text{Be}$$

$$S_{17}(0)=16.9\pm 1.9 \text{ eV}\cdot\text{b} \text{ from } {}^{14}\text{N} ({}^7\text{Be}, {}^8\text{B}){}^{13}\text{C}$$

Averaging the $C^{8\text{B}}$ values obtained in the two transfer reactions one obtains:

$$S_{17}(0)=17.3\pm 1.8 \text{ eV}\cdot\text{b}$$

Some References on ANC

1) A.M. Mukhamedzhanov et al.: PRC 56,1302,(1997)

2) H.M.Xu et al :PRL 73,2027,(1994)

3) C.A.Gagliardi et al: EPJ A15,69,(2002)

• ${}^7\text{Be}(p, \gamma){}^8\text{B}$ via ${}^{10}\text{B}({}^7\text{Be}, {}^8\text{B}){}^9\text{Be}$ and ${}^{14}\text{N}({}^7\text{Be}, {}^8\text{B}){}^{13}\text{C}$

4) A.Azhari et al: PRC 63,055803,(2001)

• ${}^{16}\text{O}(p, \gamma){}^{17}\text{F}$ via the ${}^{16}\text{O}({}^3\text{He}, d){}^{17}\text{F}$ transfer reaction

5) C.A.Gagliardi et al.: PRC 59,1149,(1999)

• ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}$ via the ${}^{14}\text{N}({}^3\text{He}, d){}^{15}\text{O}$ transfer reaction

5) F.P.Bertone et al.:PRC66,055804,(2002)

• ${}^{12}\text{C}(n, \gamma){}^{13}\text{C}$ via the ${}^{12}\text{C}(d, p){}^{13}\text{C}$ transfer reaction

6) N.Imai et al.:NPA, 688,281,(2001)

• ${}^{15}\text{N}(p, \gamma){}^{16}\text{O}$ via the ${}^{15}\text{N}({}^3\text{He}, d){}^{16}\text{O}$ transfer reaction

7) A.M. Mukhamedzhanov et al., J. Phys.: Conf. Ser. 202 012017 (2010)

Trojan Horse Method

Basic principle: astrophysically relevant two-body σ from quasi-free contribution of an appropriate three-body reaction



a: $x \oplus s$ clusters

Quasi-free mechanism

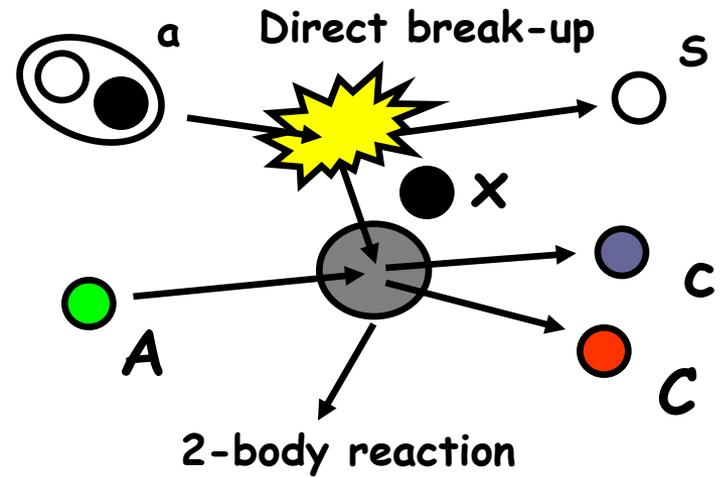
- ✓ only $x - A$ interaction
- ✓ $s = \text{spectator}$ ($p_s \sim 0$)

$E_A > E_{\text{Coul}} \Rightarrow$ NO Coulomb suppression

NO electron screening

$$E_{\text{q.f.}} = E_{Ax} - B_{x-s} \pm \text{intercluster motion}$$

plays a key role in compensating for the beam energy



→ $E_{\text{q.f.}} \approx 0 \quad !!!$

Theoretical approaches to the THM



PWIA hypotheses:

- A does not interact simultaneously with x and s
- The presence of s does not influence the A-x interaction

$$\frac{d^3 b}{d\Omega_c d\Omega_C dE_c} = C \cdot KF \cdot \left| \langle \Theta | \mathbf{p}_s \rangle \right|^2 \frac{d\sigma^N}{d\Omega}$$

MPWBA formalism

(S. Typel and H. Wolter, *Few-Body Syst.* 29 (2000) 75)

- distortions introduced in the c+C channel, but plane waves for the three-body entrance/exit channel
- off-energy-shell effects corresponding to the suppression of the Coulomb barrier are included

KF kinematical factors

$|\phi|^2$ momentum distribution of s inside a

$d\sigma^N/d\Omega$ Nuclear cross section for the $A+x \rightarrow C+c$ reaction

A. Tumino et al., PRL 98, 252502 (2007)

but No absolute value of the cross section

What has to be done practically?

Before data taking

- 1) Suitable Trojan Horse nucleus must be found e.g. ${}^6\text{Li}$ (α -d structure with $E_{\text{binding}}=1.47\text{MeV}$), d (p-n structure with $E_{\text{binding}}=2.22\text{MeV}$)
- 2) Suitable kinematical conditions which correspond to the expected quasi free contribution must be found

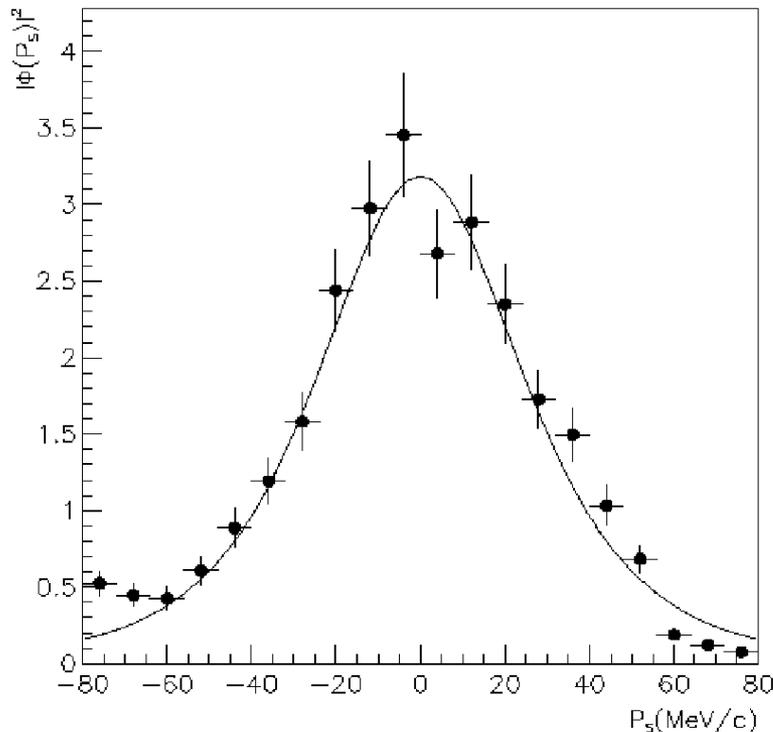
After data taking

- 3) Selection of the three body reaction of interest.
- 4) Check if the quasi free reaction mechanism is present and can be discriminated from others.
- 5) Reconstruct $\sigma_{\text{bare}}^{2b}$ and multiply it by the penetration factor.
- 6) Normalise σ_{THM}^{2b} to $\sigma_{\text{Direct}}^{2b}$ above barrier.
- 7) Verify that all direct data are reproduced
 - ✦ excitation functions including resonances
 - ✦ angular distributions
- 8) If points 1-7 are true, we believe that THM data are reliable where direct data are not available.

Selection of quasi-free contribution

Momentum Distribution

An observable which turns out to be very sensitive to the reaction mechanism is the shape of the experimental momentum distribution



The extracted experimental momentum distribution is compared with the theoretical one. For p-n system it is given by the Hulthén wave function in momentum space:

$$G^2(p_s) = N \left[\frac{1}{a^2 + p_s^2} - \frac{1}{b^2 + p_s^2} \right]^2$$

N: normalization parameter

$$a = 0.2317 \text{ fm}^{-1}$$

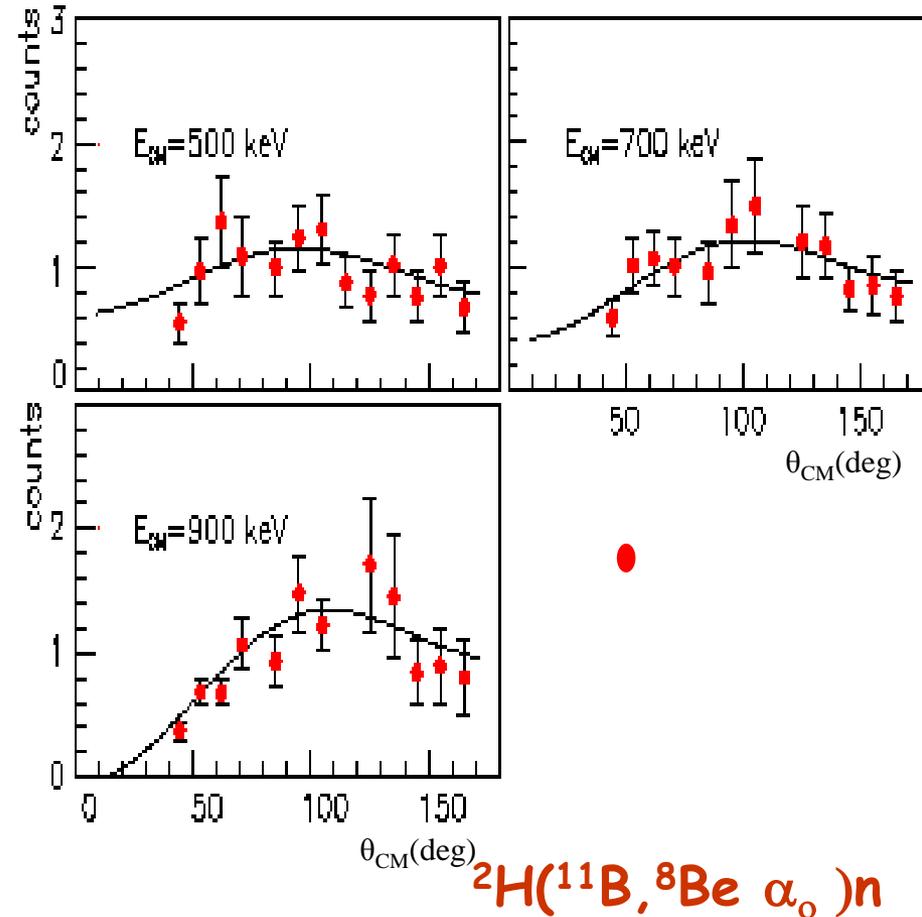
$$b = 1.202 \text{ fm}^{-1}$$

Extraction of the 2-body cross section

Monte Carlo simulation of the three-body cross section under the assumptions:

- PWIA/DWBA approach
- Quasi-free contribution is the only reaction mechanism
- a p_s window of 20 MeV/c is considered

$$\sigma_{\text{bare}}(E) = \frac{\text{Coincidence yield}}{KF |\phi(p_s)|^2 P_0^{-1}}$$



Spitaleri et al, PRC 69, 55806 (2004)

The indirect THM cross section $\sigma_{\text{bare}}(E)$ is normalized to the direct data at high energies, where the electron screening is negligible

Table XI.1. Two-Body reactions studied via Trojan Horse Method.

	Reaction	Indirect reaction	E_{inc} (MeV)	Q_2 (MeV)	THM-Nucl. Cluster-x
1	${}^7\text{Li}(p, \alpha){}^4\text{He}$	${}^2\text{H}({}^7\text{Li}, \alpha \alpha)n$	19-22	15.122	${}^2\text{H}$ (p)
2	${}^7\text{Li}(p, \alpha){}^4\text{He}$	${}^7\text{Li}({}^3\text{He}, \alpha \alpha){}^2\text{H}$	33	11.853	${}^3\text{He}$ (p)
3	${}^6\text{Li}(p, \alpha){}^3\text{He}$	${}^2\text{H}({}^6\text{Li}, \alpha {}^3\text{He})n$	14,25	1.795	${}^2\text{H}$ (p)
4	${}^6\text{Li}(d, \alpha){}^4\text{He}$	${}^6\text{Li}({}^3\text{He}, \alpha \alpha){}^1\text{H}$	17.5	16.879	${}^3\text{He}$ (d)
5	${}^6\text{Li}(d, \alpha){}^4\text{He}$	${}^6\text{Li}({}^6\text{Li}, \alpha \alpha){}^4\text{He}$	5	22.372	${}^6\text{Li}$ (d)
6	${}^9\text{Be}(p, \alpha){}^6\text{Li}$	${}^2\text{H}({}^{10}\text{Be}, \alpha {}^6\text{Li})n$	22.35	-0.099	${}^2\text{H}$ (p)
7	${}^{10}\text{B}(p, \alpha){}^7\text{Be}$	${}^2\text{H}({}^{10}\text{B}, \alpha {}^7\text{Be})n$	24.4	-1.079	${}^2\text{H}$ (p)
7	${}^{11}\text{B}(p, \alpha){}^8\text{Be}$	${}^2\text{H}({}^{11}\text{B}, \alpha {}^8\text{Be})n$	27	6.36	${}^2\text{H}$ (p)
8	${}^{15}\text{N}(p, \alpha){}^{12}\text{C}$	${}^2\text{H}({}^{15}\text{N}, \alpha {}^{12}\text{C})n$	60	2.74	${}^2\text{H}$ (p)
9	${}^{17}\text{O}(p, \alpha){}^{14}\text{N}$	${}^2\text{H}({}^{17}\text{O}, \alpha {}^{14}\text{N})n$	45	-1.032	${}^2\text{H}$ (p)
10	${}^{18}\text{O}(p, \alpha){}^{15}\text{N}$	${}^2\text{H}({}^{18}\text{O}, \alpha {}^{15}\text{N})n$	54	1.76	${}^2\text{H}$ (p)
11	${}^3\text{He}(d, p){}^4\text{He}$	${}^6\text{Li}({}^3\text{He}, p {}^4\text{He}){}^4\text{He}$	5,6	16.879	${}^6\text{Li}$ (d)
12	${}^2\text{H}(d, p){}^3\text{H}$	${}^2\text{H}({}^6\text{Li}, p {}^3\text{He}){}^4\text{He}$	14	2.59	${}^6\text{Li}$ (d)
13	${}^2\text{H}(d, p){}^3\text{H}$	${}^2\text{He}(d, p {}^3\text{H}){}^1\text{H}$	18	-1.46	${}^3\text{He}$ (d)
14	${}^2\text{H}(d, n){}^3\text{He}$	${}^2\text{H}(d, n {}^3\text{He}){}^1\text{H}$	18	-2.224	${}^3\text{He}$ (d)
15	${}^{12}\text{C}(\alpha, \alpha){}^{12}\text{C}$	${}^6\text{Li}({}^{12}\text{C}, \alpha {}^{12}\text{C}){}^2\text{H}$	20,16	0	${}^6\text{Li}$ (α)
16	${}^6\text{Li}(n, t){}^4\text{He}$	${}^2\text{H}({}^6\text{Li}, t \alpha){}^1\text{H}$	14	2.224	${}^2\text{H}$ (n)
17	${}^1\text{H}(p, p){}^1\text{H}$	${}^2\text{H}(p, p p)n$	5,6	2.224	${}^2\text{H}$ (p)
18	${}^{19}\text{F}(p, \alpha){}^{16}\text{O}$	${}^{19}\text{F}(p, \alpha {}^{16}\text{O})n$	50	8.11	${}^2\text{H}$ (p)

$d+d \rightarrow 3He+n$ two-body cross section

Primordial Nucleosynthesis + inertial fusion

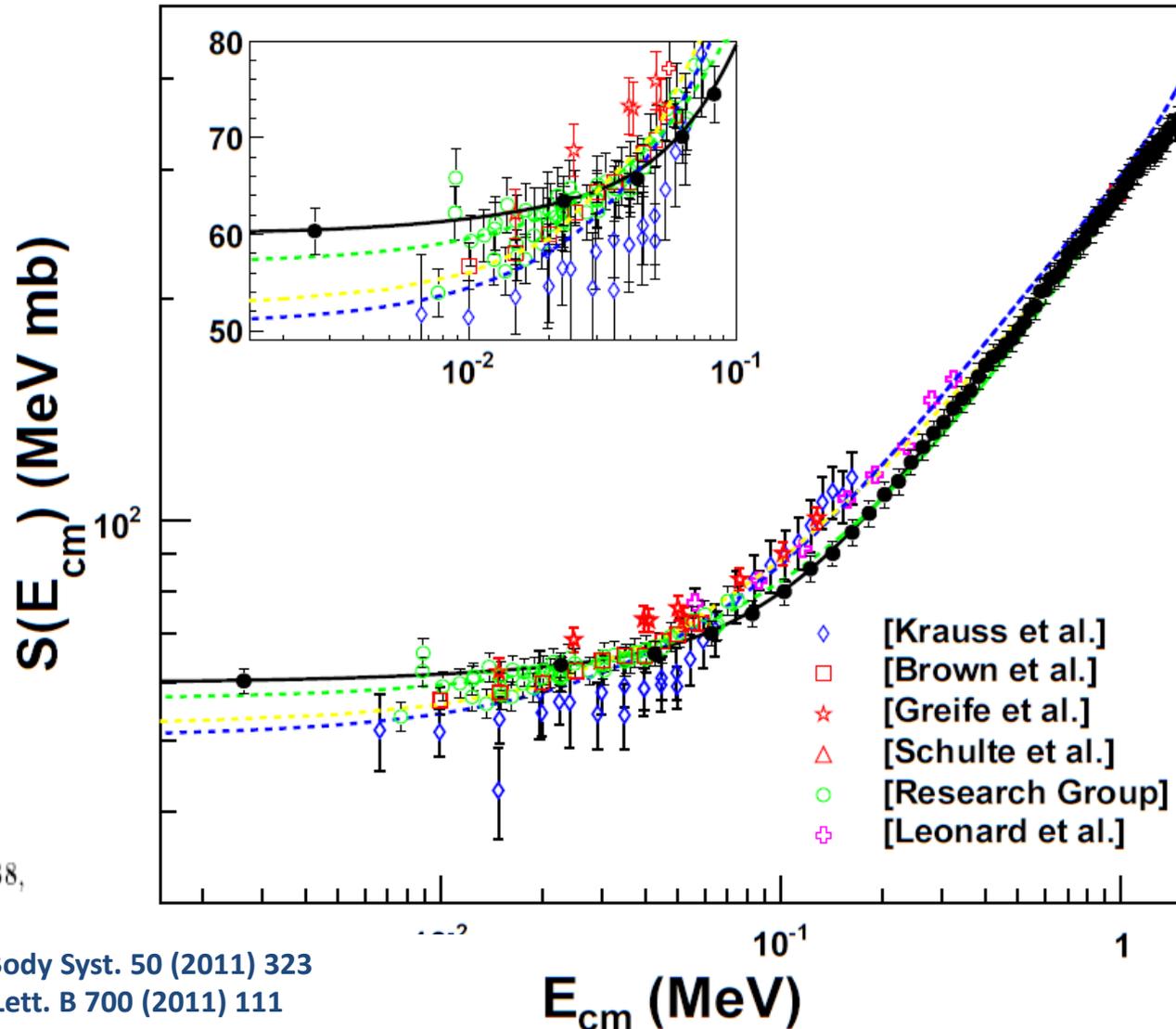


Comparison between THM data (black dots) and direct data (colored symbols)

Yellow line: polynomial expansion reported in the NACRE compilation

Blue line: calculation from the Cyburt compilation

Green line: calculation by P. Descouvemont et al.



C. Angulo *et al.*, Nucl. Phys. A656, 3 (1999)

R.H. Cyburt, Phys. Rev. D70, 023505 (2004)

P. Descouvemont *et al.*, At. Data Nucl. Data Tables 88, 203 (2004)

A. Tumino *et al.*, Few Body Syst. 50 (2011) 323

A. Tumino *et al.*, Phys. Lett. B 700 (2011) 111

$d+d \rightarrow 3H+p$ two-body cross section



Symbols and lines with same meaning as in the previous figure

Screening potential estimate

$$f_{\text{lab}}(E) = \exp(U_e/E)$$

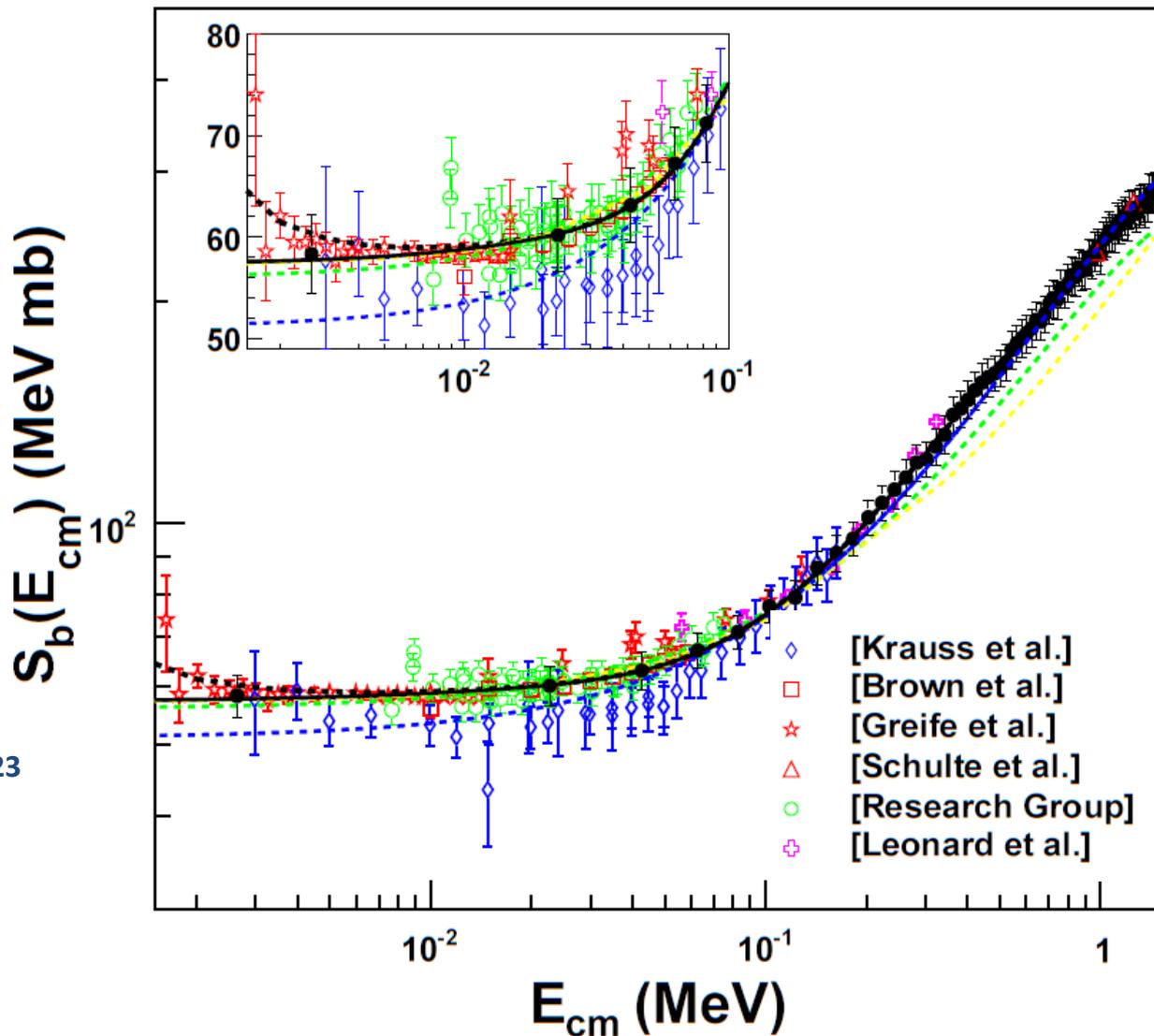
(Assenbaum, H.J. et al., 1987, Z. Phys. A, 327, 461)

$$\rightarrow U_e = 13.2 \pm 1.8 \text{ eV}$$

In agreement with the adiabatic limit

A. Tumino et al., Few Body Syst. 50 (2011) 323

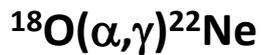
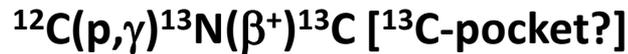
A. Tumino et al., Phys. Lett. B 700 (2011) 111



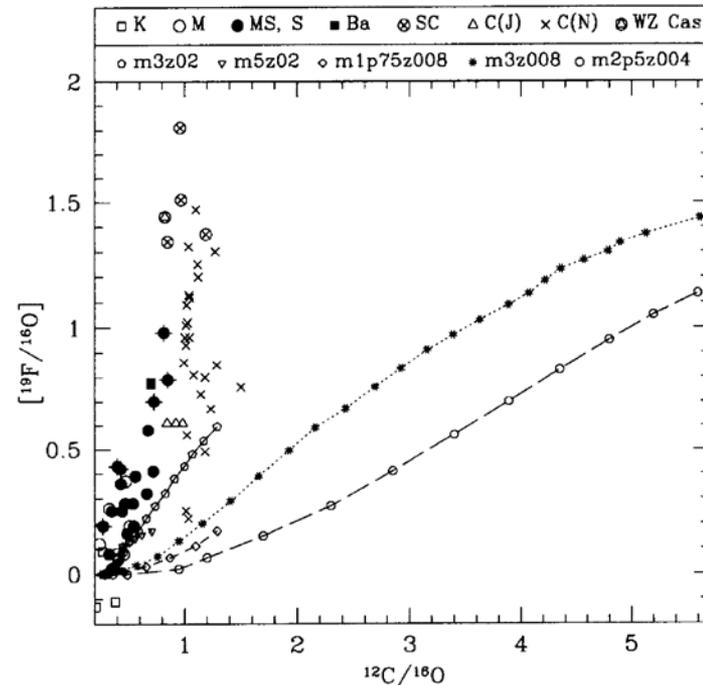
Recent results for resonant reactions



...reactions belonging to the ${}^{19}\text{F}$ production/destruction path



} ${}^{19}\text{F}$ depleting reactions



The importance of ${}^{19}\text{F}$ in astrophysics:

◆ its abundance observed in red giants can constrain AGB star models

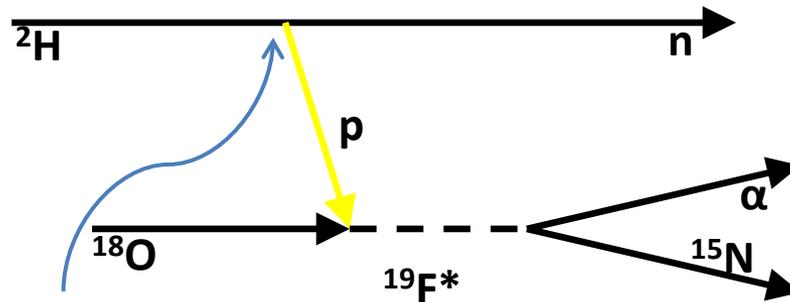
Open problem:

◆ fluorine abundance in red giants is enhanced by large factors with respect to the solar one

This would imply C/O values much larger than what experimental data suggest

The Trojan horse method for resonant reactions

In the THM the astrophysically relevant reaction, in particular $^{17,18}\text{O}(p, \alpha)^{14,15}\text{N}$, studied through an appropriate three-body process $^2\text{H}(^{17,18}\text{O}, \alpha)^{14,15}\text{N}n$:



The process is a transfer to the continuum where proton (p) is the transferred particle

Upper vertex: direct deuteron breakup

When narrow resonances dominate the S-factor the reaction rate can be calculated by means of the resonance strength:

From Modified R-Matrix strength of narrow resonances:

$$(\omega\gamma)_i = \frac{1}{2\pi} \omega_i N_i \frac{\Gamma_{(p^{18}\text{O})_i}}{|M_i|^2}$$

Advantages:

- possibility to measure down to zero energy
- No electron screening
- No spectroscopic factors in the $\Gamma_{(p^{18}\text{O})} / |M_i|^2$ ratio

- $M_i(E)$ is the amplitude of the transfer reaction (upper vertex) that can be easily calculated
- $\Gamma_{(p^{18}\text{O})}$ is the partial width for the p+ ^{18}O channel

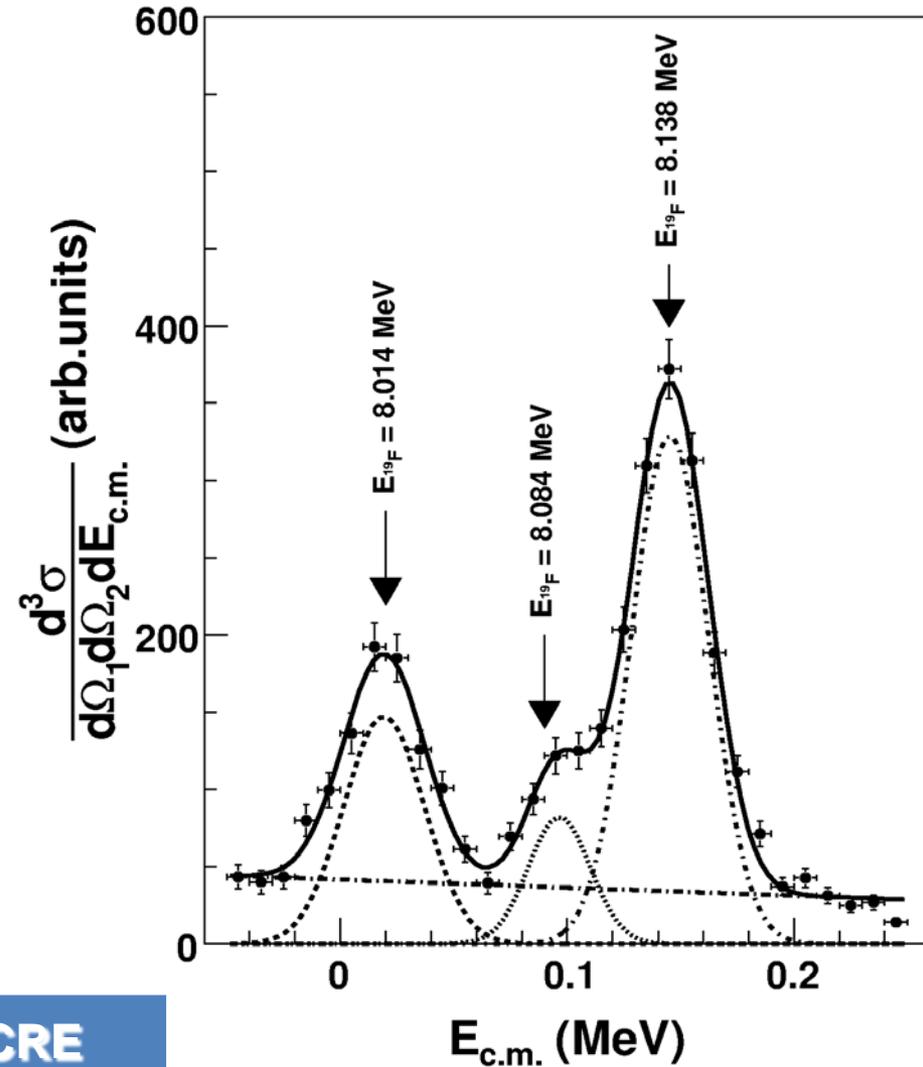
$^{18}\text{O} + p \rightarrow \alpha + ^{15}\text{N}$ THM Results

In case of narrow resonances reaction rate depending on resonance strength:

$$(\omega\gamma)_i = \frac{\omega_i}{\omega_3} \frac{\Gamma_{p_i}(E_{R_i})}{|M_i(E_{R_i})|^2} \frac{|M_3(E_{R_3})|^2}{\Gamma_{p_3}(E_{R_3})} \frac{N_i}{N_3} (\omega\gamma)_3,$$

Advantages:

- possibility to measure down to zero energy
- No electron screening
- No spectroscopic factors in the $\Gamma_{(p^{18}\text{O})} / |M_i|^2$ ratio
- no need to know the absolute cross section



M. La Cognata et al. PRL 101, 152501 (2008)
M. La Cognata et al. Ap. J. 708, 796 (2010)

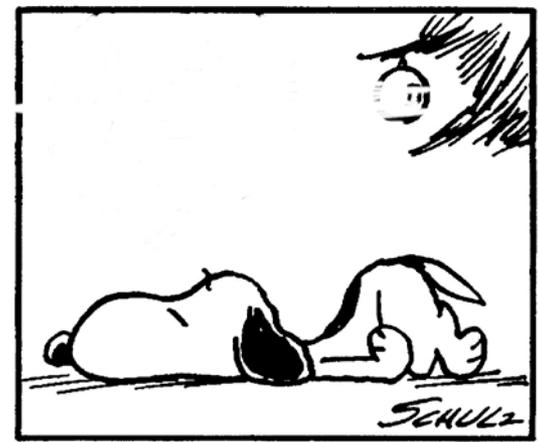
$\omega\gamma$ (eV)	Present work	NACRE
20 keV	$8.3^{+3.8}_{-2.6} 10^{-19}$	$6^{+17}_{-5} 10^{-19}$
90 keV	$1.8 \pm 0.3 10^{-7}$	$1.6 \pm 0.5 10^{-7}$

Some References on THM

- 1) C.Spitaleri et al.: NPA719, 99c, (2003)
- 2) S. Typel et al.: Few Body System 29,75,(2000)
 - ${}^7\text{Li}(p,\alpha)\alpha$
- 3) Lattuada M. et al.: 2001 Ap. J., 562, 1076
- 4) Spitaleri C. et al.: 1999, P.R.C, 60, 55802
 - ${}^6\text{Li}(p,\alpha){}^3\text{He}$
- 5) A. Tumino et al: PRC 67,065803,(2003)
 - ${}^6\text{Li}(d,\alpha){}^4\text{He}$
- 6) C.Spitaleri et al PRC 63,055801,(2001)
- 7) A.Musumarra et al. PRC 64,068801 (2001)
 - ${}^{12}\text{C}(\alpha,\alpha){}^{12}\text{C}$
- 8) C.Spitaleri et al: EPJ A7,181,(2000)
- 9) M.G.Pellegriti et al.:NPA 688,543,(2001)
 - ${}^{11}\text{B}(p,\alpha){}^8\text{Be}$, • ${}^9\text{Be}(p,\alpha){}^6\text{Li}$
- 10) L. Lamia et al JPG (2012)
 - ${}^1\text{H}(p,p){}^1\text{H}$
- 11) Tumino, A. et al., 2007, Phys. Rev. Lett., 98, 252502
- 12) Tumino, A. et al., 2008, Few-Body Systems, 43, 219
- 13) A. Tumino et al., PRC 68 (2008) 064001
 - ${}^{18}\text{O}(p,\alpha){}^{15}\text{N}$
- 14) M. La Cognata et al. PRL 101, 152501 (2008)
- 15) M. La Cognata et al. Ap. J. 708, 796 (2010)
 - ${}^{17}\text{O}(p,\alpha){}^{14}\text{N}$
- 16) M.L. Sergi et al, PRC(R) 82 032801 (2010)
 - ${}^2\text{H}(d,p){}^3\text{H}$, • ${}^2\text{H}(d,n){}^3\text{He}$
- 17) A. Tumino et al., PLB 700 (2011) 111

Conclusions

Short !!!



- Indirect methods

- To extract cross-sections of astrophysical relevance in an energy range that cannot be reached with direct reactions.

- To obtain complementary information that cannot be extracted with direct experiments

- To confirm in another independent way already existing results of important reactions

- similar characteristics and theoretical concepts

- importance of nuclear reaction theory

- direct reaction theory with certain kinematical conditions

- peripheral reactions, asymptotics of wave functions

- approximations → range of validity, accuracy

- still great potential for future applications (also beyond astrophysical applications)!

Suggestion on Key physics issues

Helium burning of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ at thermonuclear evolution of massive stars and for the nucleosynthesis up to Fe.

Cross section at the Gamow energy ($E_0 \sim 0.1$ MeV) is not based on direct measurements. The present low energy uncertainty at E_0 exceeds 50%. Theoretical models based on superposition of E1 and E2 capture procedures.

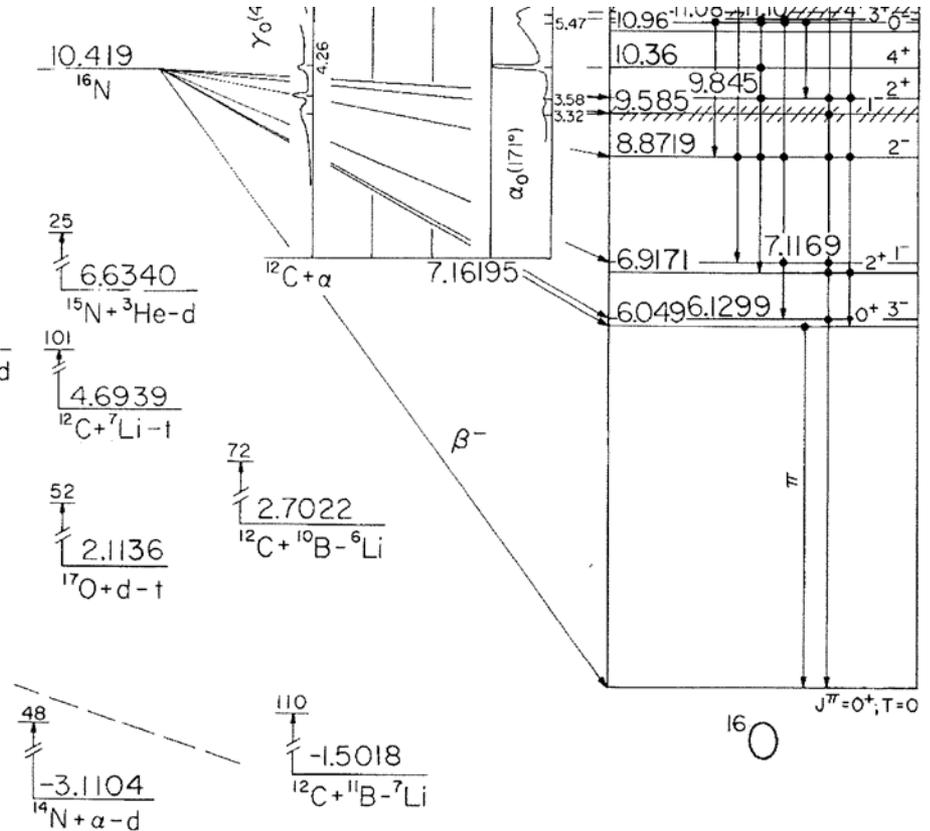
In addition, below 1 MeV interference between 1^- at 7.117 MeV and 2^+ at 6.917 MeV) of unknown energy resonance (1^- at 9.552 MeV) with

Additional efforts with indirect techniques in the future.

- **Coulomb Dissociation:** challenging case at high energy > 100 MeV/amu; $E2 \gg E1$. However, fragmentation \rightarrow disentangle the contribution of different multipolarities

Interesting angular region for detecting the fragments is $\Theta < 5^\circ \rightarrow$ careful attention to angular accuracy and angular resolution.

- **THM:** $^{12}\text{C}(\alpha, \alpha)^{12}\text{C}$ elastic scattering via $^{12}\text{C}(^6\text{Li}, \alpha)^{12}\text{C}^2\text{H}$ to perform the spectroscopy of the unknown states.



The $^{12}\text{C} + ^{12}\text{C}$ experiment

Currently a great interest in the fusion channel in the low energy region because of its critical role in studying a wide range of stellar burning scenarios in carbon-rich environments \rightarrow constraints on the models



Carbon burning temperature from 0.8 to 1.2 GK, corresponding to center-of-mass energies E_{cm} from 1 to 3 MeV

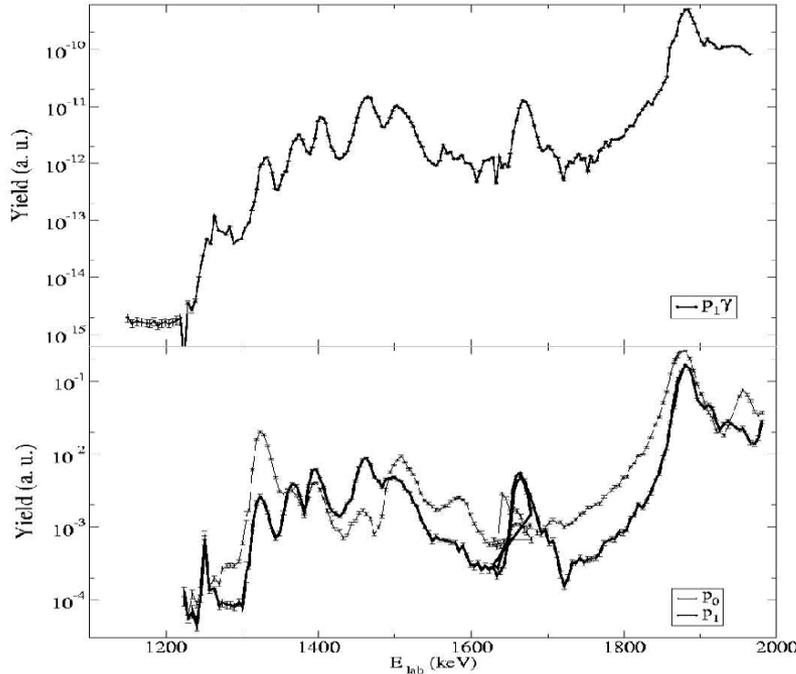
Measured down to $E_{\text{cm}} = 2.14$ MeV, still at the beginning of the region of astrophysical interest.

Extrapolation from current data to the ultra-low energies is complicated by the presence of resonant structures even in the low-energy part of the excitation function
Further measurements extending down to 1 MeV would be extremely important!

THM: $^{12}\text{C} + ^{12}\text{C}$ burning by means of $^{16}\text{O} (\alpha + ^{12}\text{C}) + ^{12}\text{C}$ and $^{14}\text{N} (\text{d} + ^{12}\text{C}) + ^{12}\text{C}$ processes in the quasi-free (QF) kinematics regime, where α from ^{16}O or ^2H from the ^{14}N TH nuclei are spectators to the $^{12}\text{C} + ^{12}\text{C}$ two-body processes. There is a number of works providing evidence of direct ^{12}C transfer in the $^{12}\text{C}(^{14}\text{N}, \text{d})^{24}\text{Mg}^*$ reaction at 30 MeV of beam energy and up (R.W. Zurnühle et al., Phys. Rev. C 49 , 2549 (1994))

Neutrino

The $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$ reaction



$^{19}\text{F}(\alpha, p)^{22}\text{Ne}$: main ^{19}F destruction channel AGB stars with $M > 2 M_{\odot}$ and WR stars ($\sim 30 M_{\odot}$)

$T \rightarrow 2 \cdot 10^8 \text{ K}$

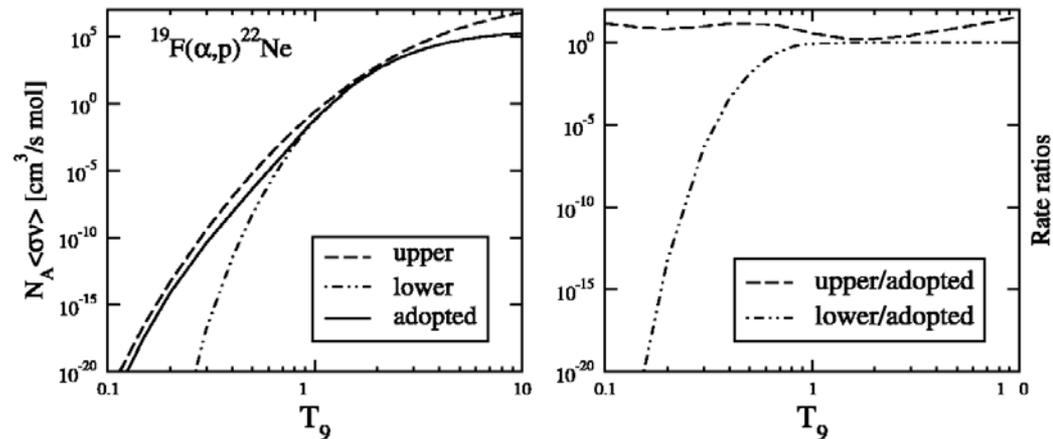
\Rightarrow Energies of interest 300-800 keV

Most recent measurement (2006) down to 800 keV

\Rightarrow Extrapolation impossible because of the many resonances

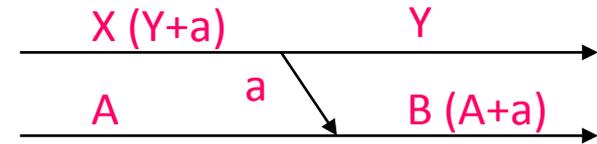
The rate is calculated by using simplified models

Reaction rate: uncertainty of about 14 orders of magnitude



Asymptotic Normalisation Coefficients and Radiative Capture studies

The DWBA cross-section for a transfer reaction



can be written as:

$$\sigma_{tra} \propto \left| \left\langle \chi_f I_{aA}^B \left| \hat{V} \right| I_{Ya}^X \chi_i \right\rangle \right|^2$$

Where:

$\chi_{i,f}$ distorted waves in the initial and final channels

\hat{V} transition operator

$I_{\beta\gamma}^\alpha(\underline{r}_{\beta\gamma})$ overlap function of the bound state α formed by β and γ

The radial part of $I_{\beta\gamma}^\alpha$ is:

$$I_{\beta\gamma}^\alpha(r_{\beta\gamma}) = S^{1/2} \varphi_{\beta\gamma}(r_{\beta\gamma})$$

Where:

$\varphi_{\beta\gamma}$ is the bound state wave function for the relative motion of β and γ forming α

S is the spectroscopic factor of the configuration ($\beta\gamma$) in nucleus α

After substitution:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}} \propto S_{aA}^B S_{Ya}^X \left(\frac{d\sigma}{d\Omega} \right)_{DW}$$

If we deal with peripheral transfer reactions

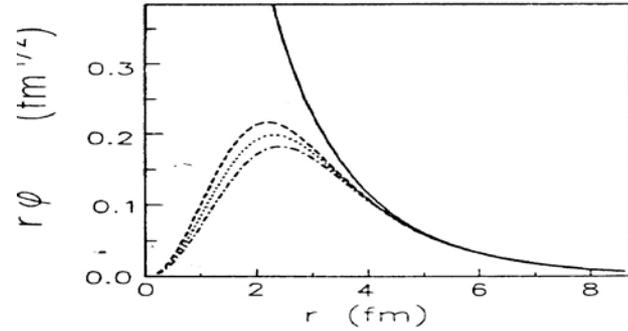
the radial part of the bound state wave function can be approximated by:

$$\varphi_{\beta\gamma}(r_{\beta\gamma}) \stackrel{r_{\beta\gamma} > R_N}{\approx} b_{\beta\gamma} \frac{W(2k_{\beta\gamma} r_{\beta\gamma})}{r_{\beta\gamma}}$$

$b_{\beta\gamma}$ single particle ANC

R_N nuclear interaction radius between β and γ

Ex: Radial behaviour of different
calculated proton wave functions in
 ^{10}B



Therefore:

$$I_{\beta\gamma}^{\alpha}(r_{\beta\gamma}) = S_{\beta\gamma}^{1/2} \varphi_{\beta\gamma} \stackrel{r_{\beta\gamma} > R_N}{\approx} S_{\beta\gamma}^{1/2} b_{\beta\gamma} \frac{W(2k_{\beta\gamma} r_{\beta\gamma})}{r_{\beta\gamma}} = C_{\beta\gamma}^{\alpha} \frac{W(2k_{\beta\gamma} r_{\beta\gamma})}{r_{\beta\gamma}}$$

Here $C_{\beta\gamma}^{\alpha} = S_{\beta\gamma}^{1/2} b_{\beta\gamma}$ is the asymptotic normalisation coefficient we need

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}} \propto (C_{aA}^B)^2 \cdot (C_{Ya}^X)^2 \left[\left(\frac{d\sigma}{d\Omega} \right)_{\text{DW}} \frac{1}{(b_{aA})^2 (b_{Ya})^2} \right]$$

Direct radiative capture

The cross-section for direct capture reaction $a+A \rightarrow B+\gamma$ can be written as:

$$\sigma_{DC} \propto \left| \left\langle I_{aA}^B(\underline{r}_{aA}) \left| \hat{O} \right| \Psi_i(\underline{r}_{aA}) \right\rangle \right|^2$$

Where:

Ψ_i Scattering wave function describing the relative motion in the entrance channel

\hat{O} electromagnetic transition operator

I_{aA}^B Overlap function for the bound state B formed by a and A

For low relative energies $E_{aA} \leftrightarrow$ periferal processes



The radial part of I_{aA}^B for $r > R_N$ is



$$I_{aA}^B(r_{aA}) = C_{aA}^B \frac{W(2k_{aA}r_{aA})}{r_{aA}}$$



$$\sigma_{DC} \propto (C_{aA}^B)^2 \text{ can be calculated}$$

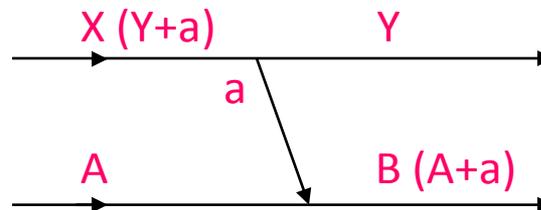
Summarising the idea...

➔ At low relative energies the cross-section for a (peripheral) direct capture reaction $a+A \rightarrow B+\gamma$ can be calculated if C_{aA}^B is known.

➔ Studying a peripheral transfer reaction $X+A \rightarrow Y+B$ the ANC C_{aA}^B can be obtained normalising the experimental angular distribution to the calculated one.

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} \propto (C_{aA}^B)^2 \cdot (C_{Ya}^X)^2 \left[\left(\frac{d\sigma}{d\Omega}\right)_{DW} \frac{1}{(b_{aA})^2 (b_{Ya})^2} \right]$$

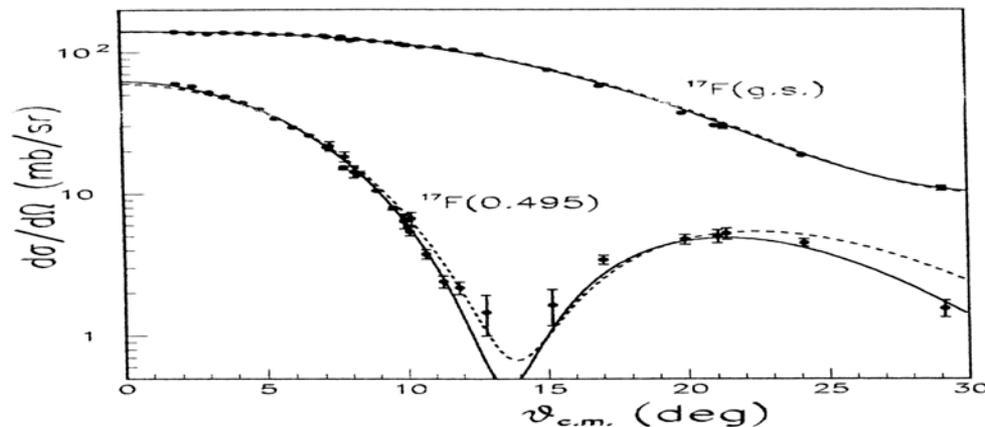
precise optical potentials and one additional ANC needed



The $^{16}\text{O}+p \Rightarrow ^{17}\text{F}+\gamma$ case

Ref: C.A. Gagliardi et al. Phys.Rev.C 59, 1149(1999)

Studied via the $^{16}\text{O}(^3\text{He},d)^{17}\text{F}$ transfer reaction.



Extracted ANC:

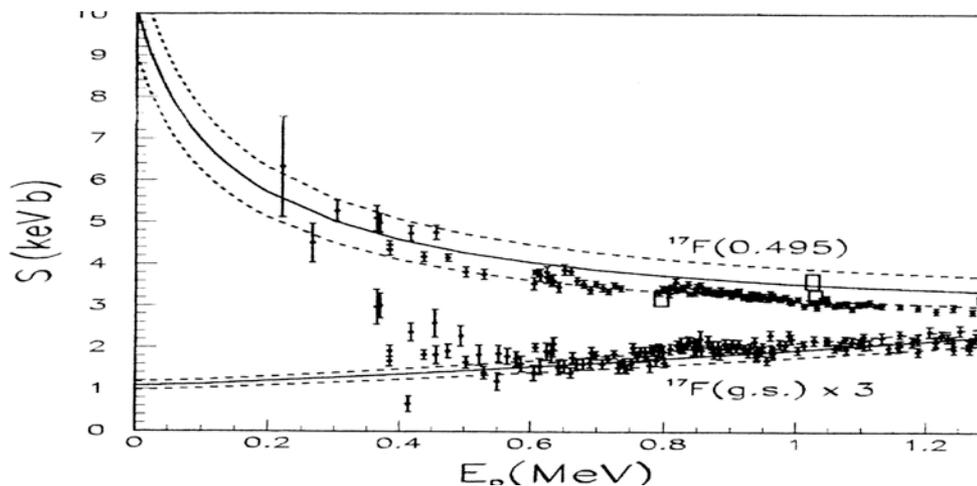
$$C_{g.s.}^{17F}$$

$$= 1.08 \pm 0.10 \text{ fm}^{-1}$$

$$C_{1^{st} \text{ excited}}^{17F}$$

$$= 6490 \pm 680 \text{ fm}^{-1}$$

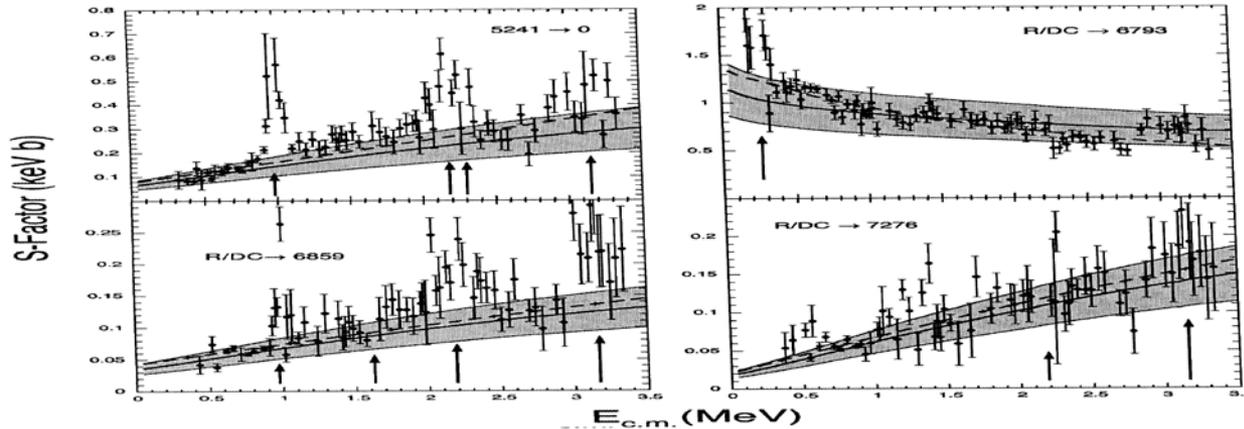
Calculated σ_{DC} for $^{16}\text{O}+p \rightarrow ^{17}\text{F}+\gamma$ are in good agreement with direct data.



The $^{14}\text{N}(p,\gamma)^{15}\text{O}$ case

Ref: P.F. Bertone et al. Phys.Rev.C 66,055804(2002)

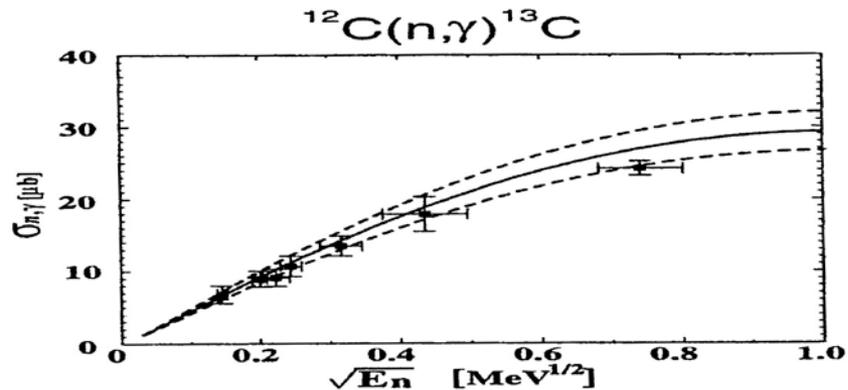
Studied via the $^{14}\text{N}(^3\text{He,d})^{15}\text{O}$ transfer reaction.



The $^{12}\text{C}(n,\gamma)^{13}\text{C}$ case

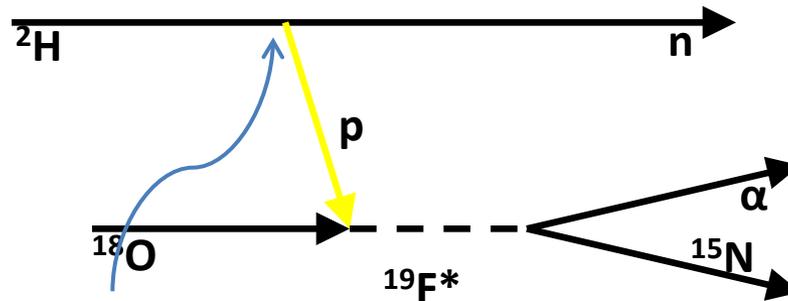
N. Imei et al. Nucl.Phys.A 688,218c(2001)

Studied via the $^{12}\text{C}(d,p)^{13}\text{C}$ transfer reaction



The Trojan horse method for resonant reactions

In the THM the astrophysically relevant reaction, in particular $^{17,18}\text{O}(p,\alpha)^{14,15}\text{N}$, studied through an appropriate three-body process $^2\text{H}(^{17,18}\text{O},\alpha^{14,15}\text{N})n$:



The process is a transfer to the continuum where proton (p) is the transferred particle

Upper vertex: direct deuteron breakup

Standard R-Matrix approach cannot be applied to extract the resonance parameters of the $^{18}\text{O}(p,\alpha)^{15}\text{N}$ → Modified R-Matrix is introduced instead

In the case of a **resonant** THM reaction the cross section takes the form

$$\frac{d^2\sigma}{dE_{C_c} d\Omega_s} \propto \frac{\Gamma_{(C_c)_i}(E) |M_i(E)|^2}{(E - E_{R_i})^2 + \Gamma_i^2(E)/4}$$

$M_i(E)$ is the amplitude of the transfer reaction (upper vertex) that can be easily calculated
→ The resonance parameters can be extracted and in particular the strength

How to extract the resonant strength?

When narrow resonances dominate the S-factor the reaction rate can be calculated by means of the resonance strength:

$$(\omega\gamma)_i = \frac{\hat{J}_i}{\hat{J}_p \hat{J}_{^{18}\text{O}}} \frac{\Gamma_{(p^{18}\text{O})_i}(E_{R_i}) \Gamma_{(\alpha^{15}\text{N})_i}(E_{R_i})}{\Gamma_i(E_{R_i})} \quad ({}^{18}\text{O}(p,\alpha){}^{15}\text{N} \text{ case})$$

Where:

- $\hat{J}=2J+1$ → Area of the Breit-Wigner describing the resonance
- $\Gamma_{(AB)}$ is the partial width for the A+B channel
- Γ_i is the total width of the i-th resonance → no need to know the resonance shape
- E_{R_i} is the resonance energy

$$(\omega\gamma)_i = \frac{1}{2\pi} \omega_i N_i \frac{\Gamma_{(p^{18}\text{O})_i}}{|M_i|^2}$$

Where:

- $\omega_i = \hat{J}_i / \hat{J}_p \hat{J}_{^{18}\text{O}}$ statistical factor
- $N_i = \text{THM resonance strength}$
- $M_i = \text{transfer amplitude}$

Advantages:

- possibility to measure down to zero energy
- No electron screening
- No spectroscopic factors in the $\Gamma_{(p^{18}\text{O})} / |M_i|^2$ ratio