The description of shape coexistence and shape mixing in proton-rich nuclei: status and perspectives

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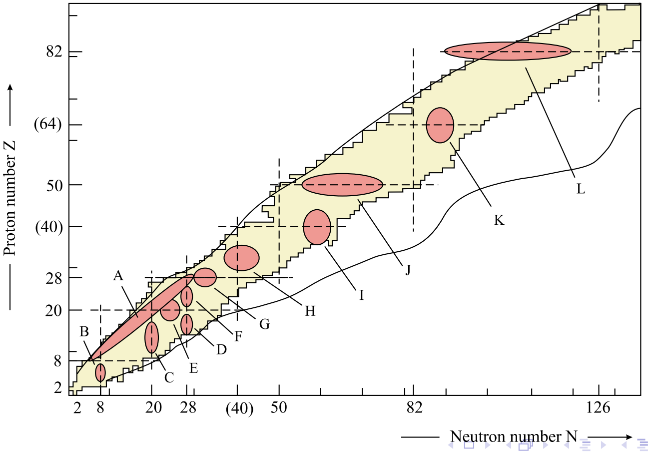
Villa Vigoni, Como lake, Italy, June 18-21, 2012

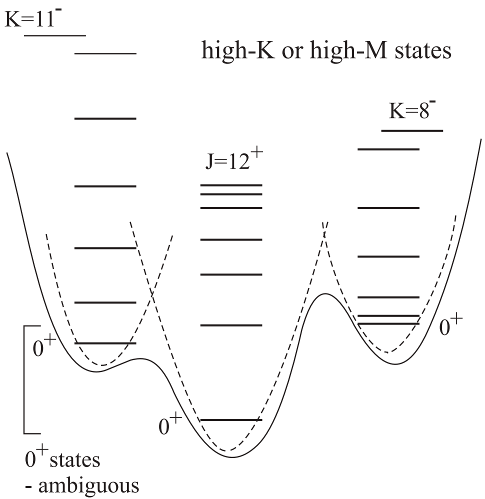
"It would be fair to say that the status of coexistence in nuclei has evolved from an exotic rarity, via the perception that it is a phenomenon which exhibits "islands of occurrence" to the current position in which it occurs in all (but the lightest) nuclei."

K. Heyde and J. L. Wood, RMP 83 (2011) 1467



Shape Coexistence as a global phenomenon

G. D. Dracoulis, Phys. Scr. T88 (2000) 54



In an even-even nucleus, one may find coexistence of:

- low-lying 0^+ levels below 2 times the pairing gap
- spherical "shell model states"
- deformed rotational bands
- high-K isomers that can be associated to distinct deformations through a Nilsson diagram

In odd-A nuclei, verious single-particle states can be coupled to all of these structures

1. paired mean field states of deformations q_1, q_2, $\dots q_n$

$$|q_1\rangle\,,|q_2\rangle\,,\dots|q_n\rangle\,.$$

2. symmetry restoration: projection on particle number and angular momentum

$$|JMq\kappa\rangle = \sum_{K=-J}^{+J} f_{J\kappa}(K)\;\hat{P}^J_{MK}\;\hat{P}^Z\;\hat{P}^N\,|q\rangle = \sum_{K=-J}^{+J} f_J(K)\;|JMKq\rangle$$

3. superposition of symmetry-restored mean-field states

$$|JM\nu\rangle = \sum_q \sum_{K=-J}^{+J} f_{J\nu}(q,K)\,|JMqK\rangle$$

The $f_J(K)$ is the weight of the component K and determined variationally

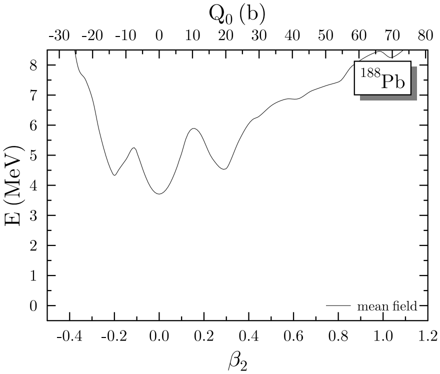
Beyond the self-consistent mean field: why and what for?

- move focus of mean-field methods away from ground-state properties
- description of characteristic collective excited states at low excitation energy
- treat *dynamical* correlations not grasped by the *static* correlations in a symmetry-breaking mean-field calculation, i.e. the fluctuations around the self-consistent minimum.
- treat *dynamical* correlations not easily absorbed into the effective interaction. Usually these are related to the finite size and surface of the system, strongly depend on the structure of the nucleus, and fluctuate rapidly with N, Z, deformation, . . .
- the correlations are not described by a *vertical* expansion in terms of np-nh excitations around the minimum, but by a *horizontal* expansion in terms of occupied states brought to the Fermi energy by the static correlations along a properly chosen collective path
- restore quantum numbers to have selection rules for transitions
- proper description of the transition from vibrational to rotational nuclei
- description of shape coexistence phenomena

Technical details of the results shown here

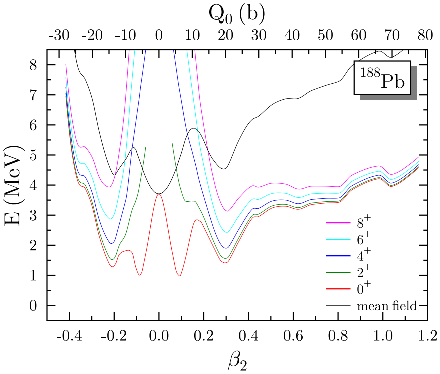
- mostly the parameterizations SLy4 or SLy6 of the Skyrme energy density functional together with a density-dependent local pairing functional
- sometimes Gogny force D1s in results borrowed from Madrid of Bruyères-le-châtel
- the same energy density functional determines the mean field and the configuration mixing
- most calculations limited to axially symmetric mean-field states

Configuration mixing via the projected Generator Coordinate Method



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Configuration mixing via the projected Generator Coordinate Method



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

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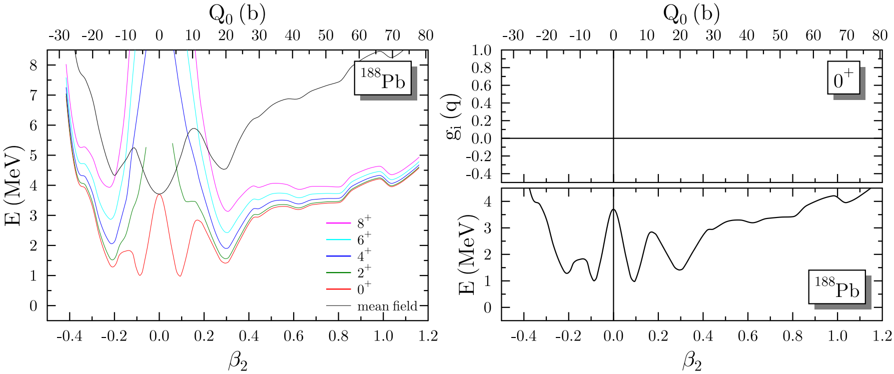
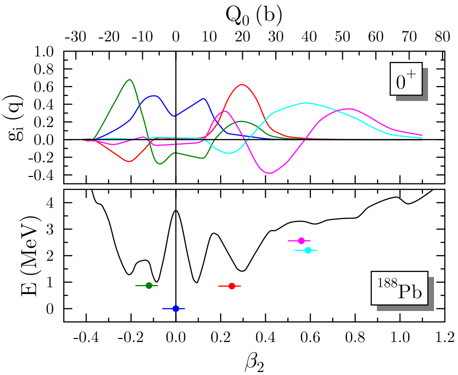


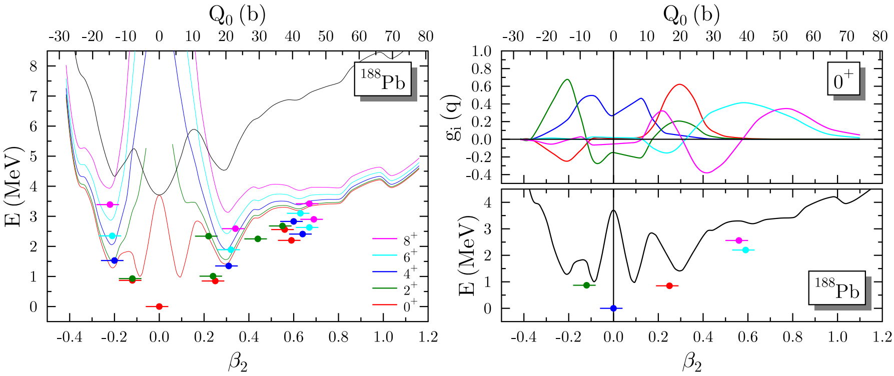
M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Attention: $g_i^2(q)$ is not the probability to find a mean-field state with intrinsic deformation q in the collective state

Configuration mixing via the projected Generator Coordinate Method



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

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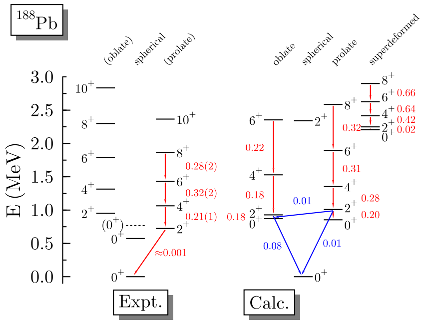
Transition moments

M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Experiment: T. Grahn *et al*, Phys. Rev. Lett. **97** (2006) 062501

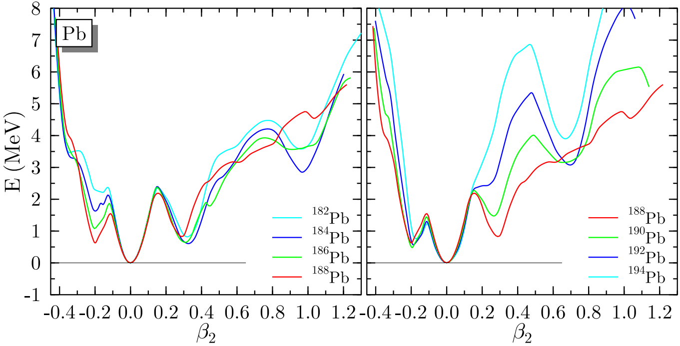


- in-band and out-of-band $E2$ transition moments directly in the laboratory frame with correct selection rules
- full model space of occupied particles
- only occupied single-particle states contribute to the kernels ("horizontal expansion")
- $\Rightarrow$ *no effective charges necessary*
- *no adjustable parameters*

$$B(E2; J'_{\nu'} \to J_\nu) = \frac{e^2}{2J'+1} \sum_{M=-J}^{+J} \sum_{M'=-J'}^{+J'} \sum_{\mu=-2}^{+2} |\langle JM\nu|\hat{Q}_{2\mu}|J'M'\nu'\rangle|^2$$

$$\beta_2^{(t)} = \frac{4\pi}{3R^2A}\sqrt{\frac{B(E2; J \to J-2)}{(J\,0\,2\,0\,|(J-2)\,0)^2 e^2}} \quad \text{with} \quad R = 1.2\,A^{1/3}$$

Mean-field deformation energy: Pb isotopes (SLy6)



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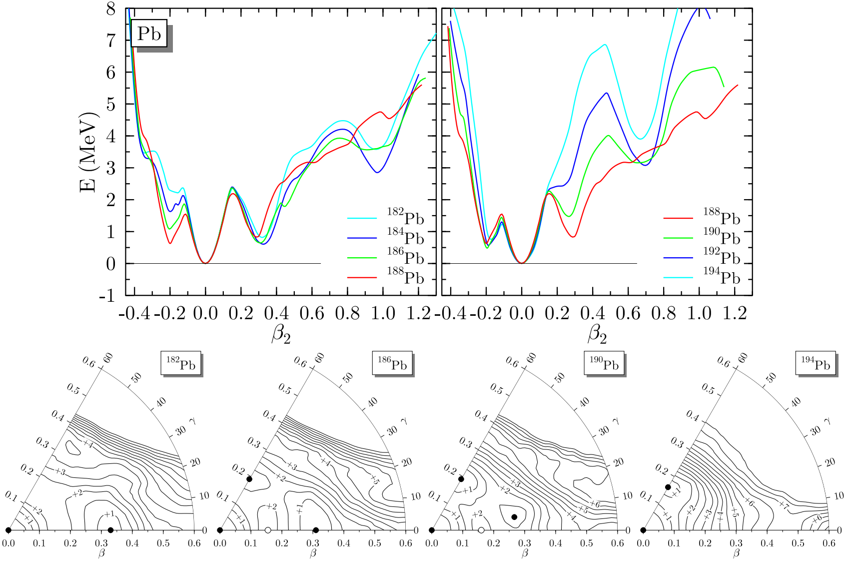
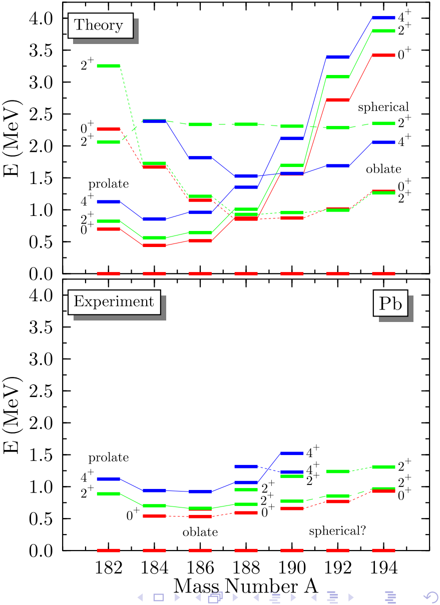


M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

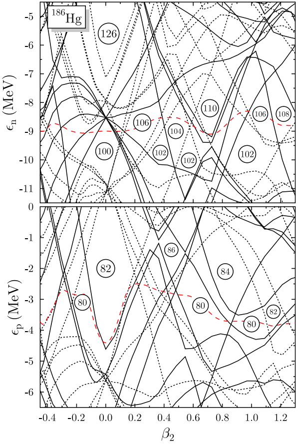
Shape coexistence in the neutron-deficient Pb region

- overall structure of bands and crossing between prolate and oblate bands is well described.
- excitation energy of the projected GCM bandheads is different from that of the mean-field minima.
- projected GCM gives prolate (oblate) bands also in nuclei without prolate (oblate) mean-field minimum
- calculated spectra are too spread out (the variational space used up ti now is too small for fine details of the binding energy that are on the order of < 1 MeV out of 1500 MeV. For experts: we have "Peierls-Yoccoz" instead of "Thouless-Valatin" moments of inertia)

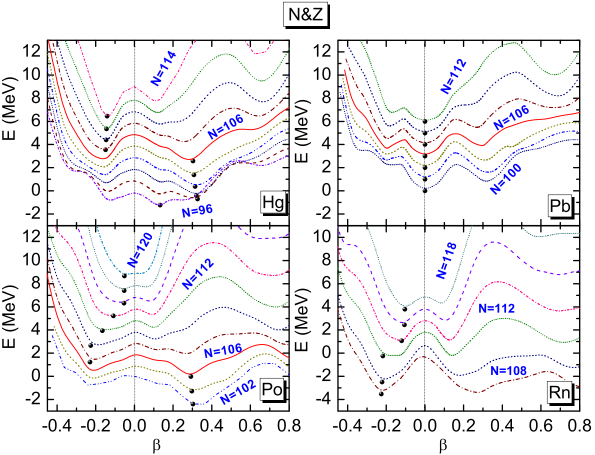


M. B., P. Bonche, T. Duguet, P.-H. Heenen, PRC 69 (2004) 064303.

Coexisting shapes in a mean-field picture: gaps in the Nilsson diagram

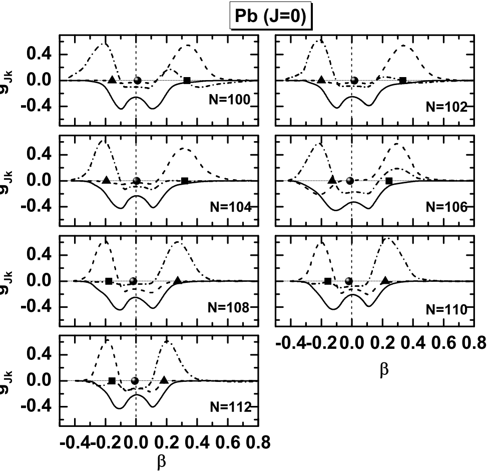


J. M. Yao, M. B., P.-H. Heenen, to be published

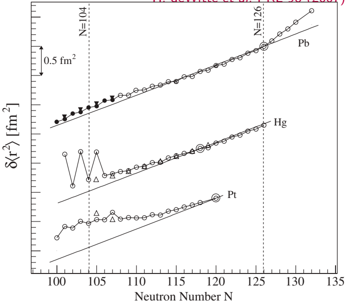


Shape coexistence and shape mixing in the neutron-deficient Pb region

J. M. Yao, M. B., P.-H. Heenen, to be published

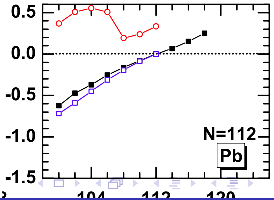


H. deWitte *et al.* PRL 98 (2007) 112502



J. M. Yao, M. B., P.-H. Heenen, to be published

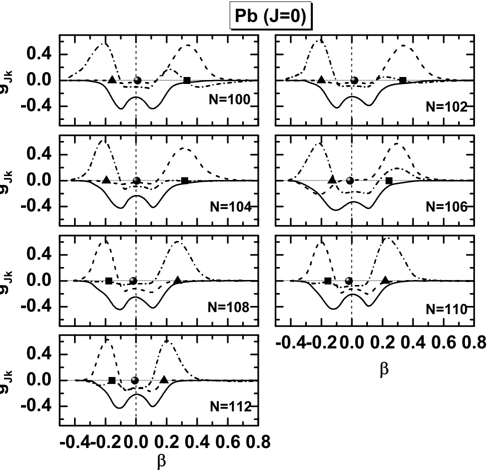
$V_{\text{pair}} = -1250$ MeV fm^3



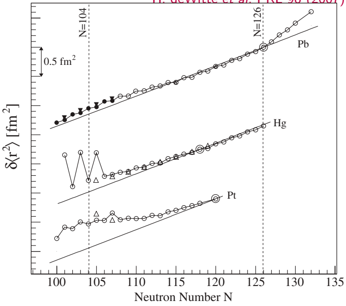
- spread of ground state wave function sensitive to pairing

Shape coexistence and shape mixing in the neutron-deficient Pb region

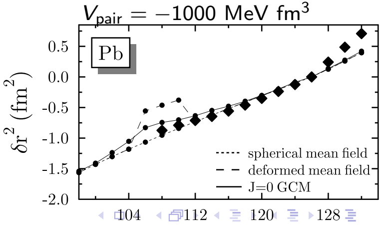
J. M. Yao, M. B., P.-H. Heenen, to be published



H. deWitte *et al.* PRL 98 (2007) 112502

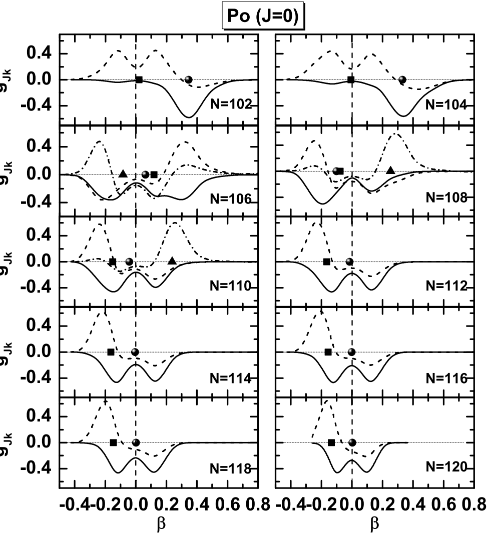


M. B., G. F. Bertsch, P.-H. Heenen, PRC 73 (2006) 034322

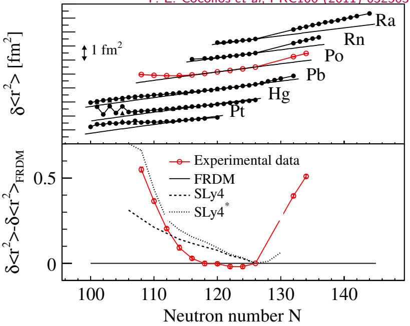


- spread of ground state wave function sensitive to pairing

J. M. Yao, M. B., P.-H. Heenen, to be published



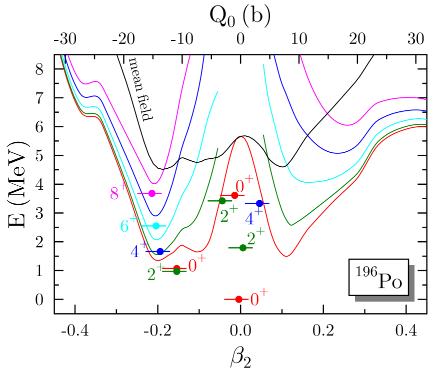
T. E. Cocolios *et al*, PRC106 (2011) 052503



- onset of deformation well reproduced
- reduction of the radius of the reference nucleus would resolve disagreement with experiment in lighter isotopes

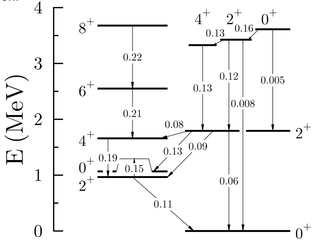
Shape coexistence in ^{196}Po

T. Grahn, *et al.*, PRC 80 (2009) 014323



- Coexistence of a spherical vibrator and an oblate band, which are strongly mixed at low J
- transition strength given in units of β_2 as obtained making the square root of $B(E2)$ dimensionless

calculation:



experiment:

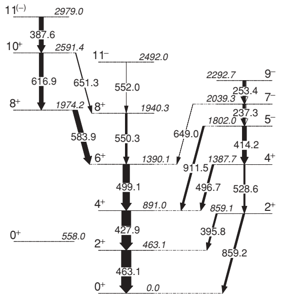
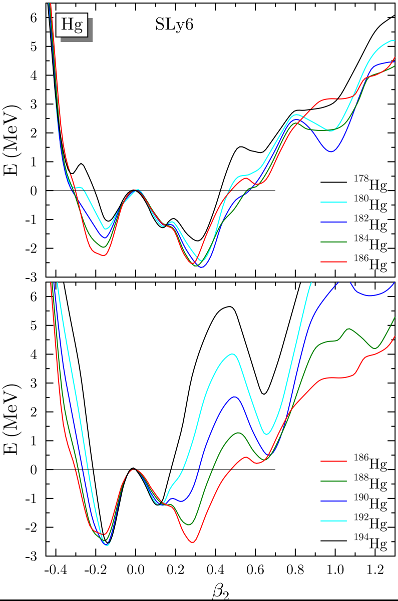
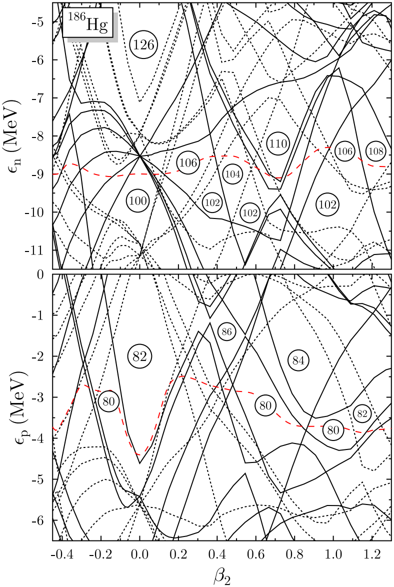


FIG. 1. A partial level scheme of ^{196}Po. Level and transition energies are given in keV and the widths of the arrows are proportional

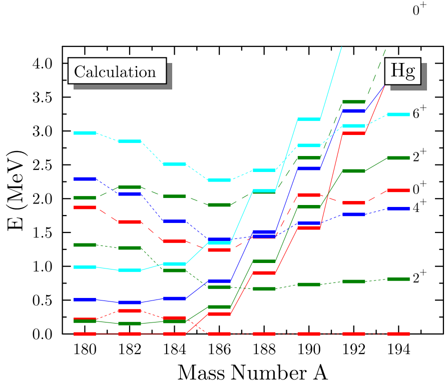
Mean-field deformation energy: Hg isotopes

J. M. Yao, M. B., P.-H. Heenen, unpublished

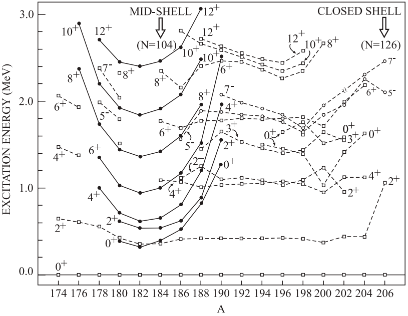




Shape coexistence in the neutron-deficient Hg isotopes

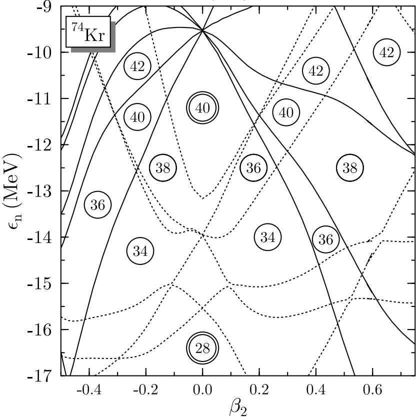


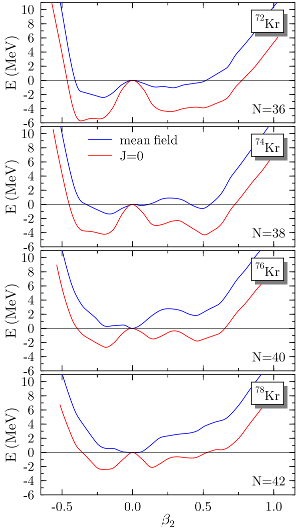
K. Heyde and J. L. Wood, RMP 83 (2011) 1467



Shape coexistence in the neutron-deficient Kr region

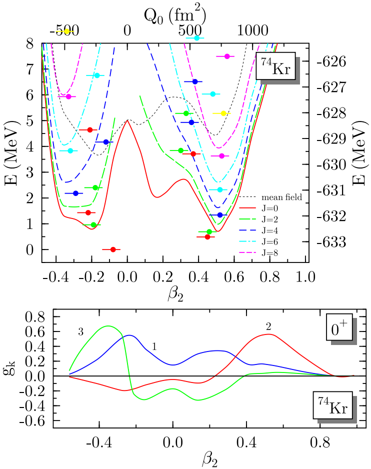
M. B., P. Bonche, P.-H. Heenen, Phys. Rev. C 74 (2006) 024312

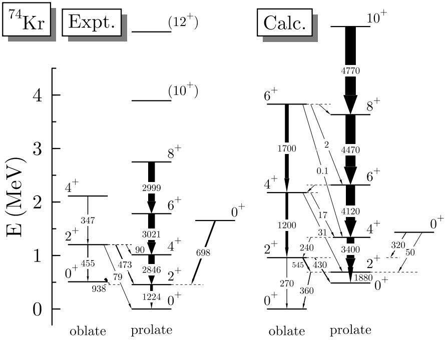




- Nearly each minimum or plateau in the mean-field energy curve can be directly linked to a gap in the Nilsson diagram

Shape coexistence in the neutron-deficient Kr region



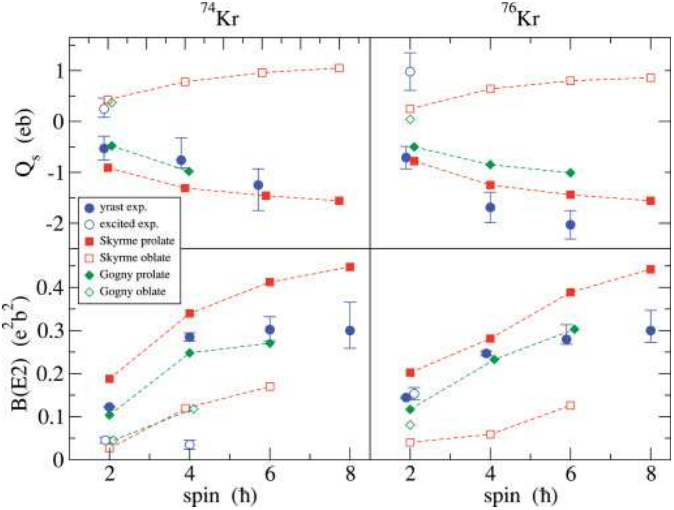


- SLy6+density-dependent pairing
- There are no adjustable parameters. . .

Experiment: E. Clément *et al.* PRC75 (2007) 054313, A. Görgen *et al.* Eur. Phys. J. A26 (2005) 153

M. B., P. Bonche, P.-H. Heenen, Phys. Rev. C 74 (2006) 024312.

E. Clément *et al.* PRC75 (2007) 054313



Shape coexistence in the neutron-deficient Kr region

- Skyrme: axial symmetry-restored GCM using SLy6
- Gogny: 5d Bohr Hamiltonian using D1s

What makes the difference? Triaxiality, the effective interaction, or both?

E. Clément *et al.* PRC75 (2007) 054313

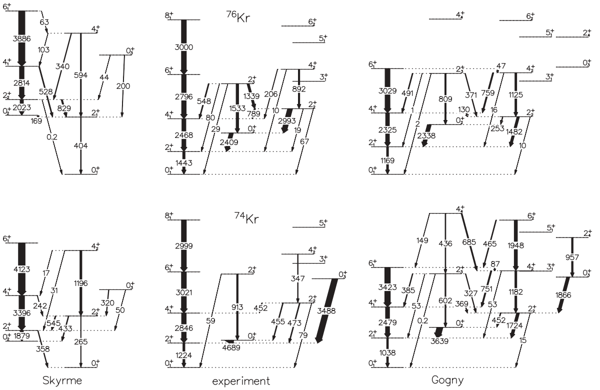
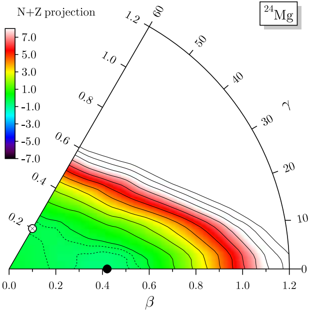


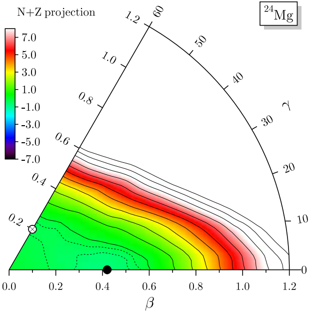
FIG. 16. Comparison between the theoretical and experimental level schemes for the oblate and prolate bands in ^{76}Kr (top) and ^{74}Kr (bottom). The excitation energies of the states are drawn to scale and the widths and labels of the arrows represent the calculated and measured $B(E2)$ values, respectively.

mean-field deformation energy surface

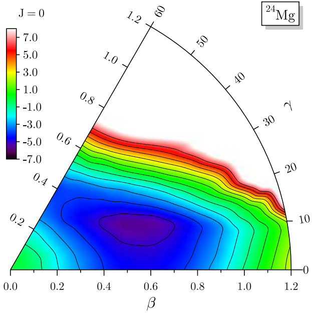


Angular momentum projection of triaxial states

mean-field deformation energy surface



$J = 0$ projected deformation energy surface



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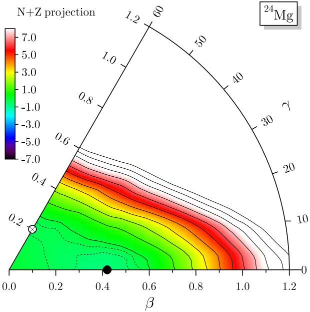

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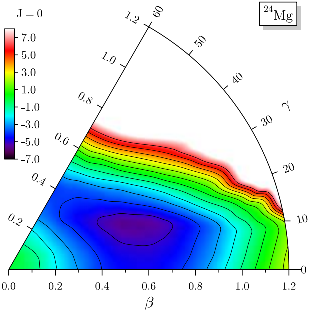
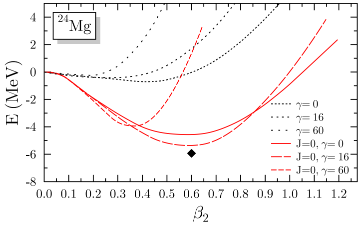
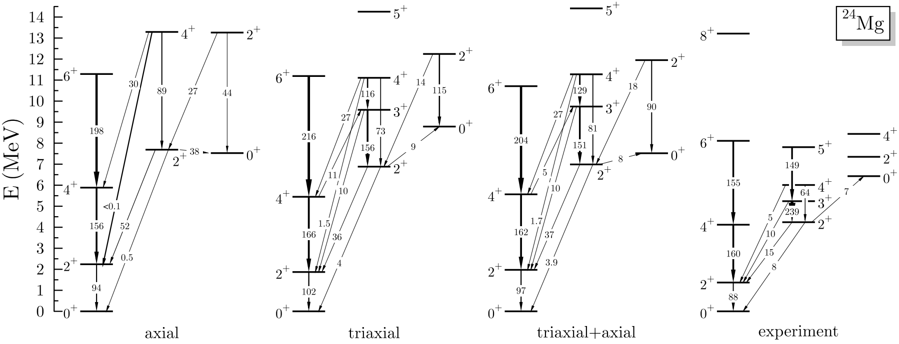




M. B. and P.-H. Heenen, Phys. Rev. C **78** (2008) 024309

Mixing of angular-momentum projected triaxial states of different intrinsic deformation

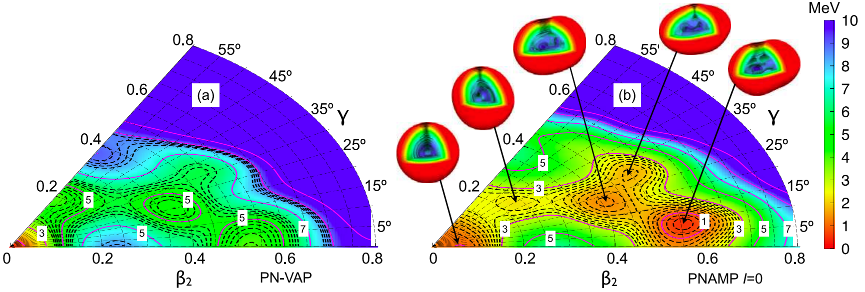


M. B. and P.-H. Heenen, Phys. Rev. C **78** (2008) 024309

Shape coexistence in the neutron-deficient Zr isotopes

⇒ 5 coexisting shapes in the β-γ plane at low spin and excitation energy

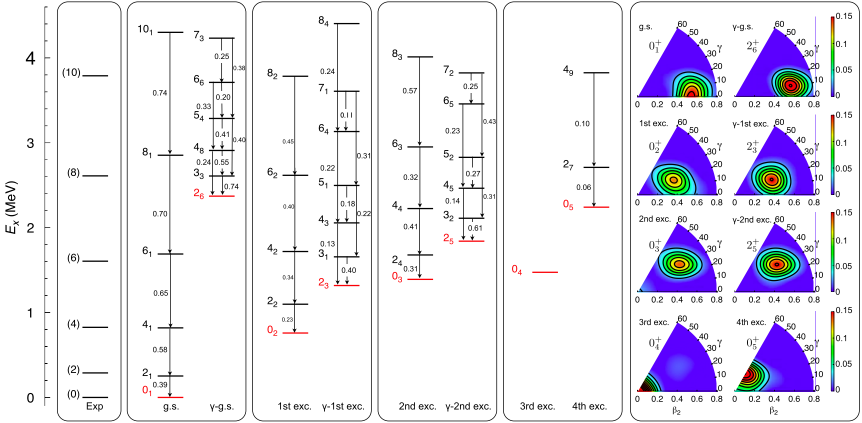
T.R. Rodríguez, J.L. Egido / Physics Letters B 705 (2011) 255–259



Shape coexistence in the neutron-deficient Zr isotopes

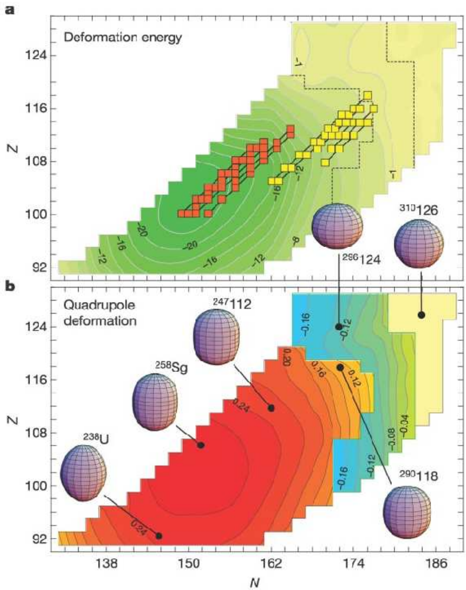
- GCM of symmetry-restored triaxal states

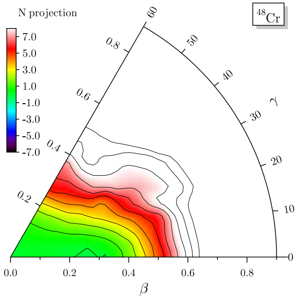
T. Rodriguez-Frutos, J. L. Egido, PLB705 (2011) 255

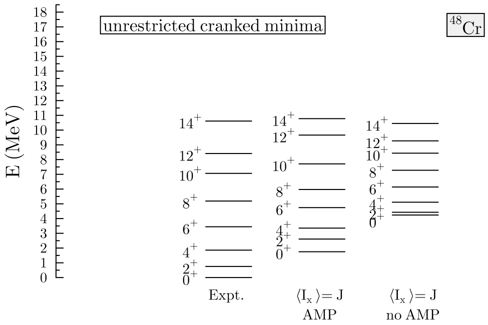


Shape coexistence in superheavy nuclei

S. Ćwiok, P.-H. Heenen, W. Nazarewicz, Nature 433 (2005) 709



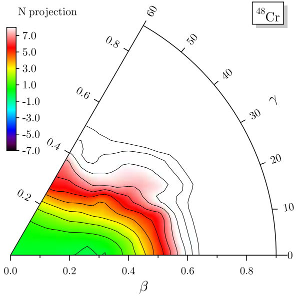


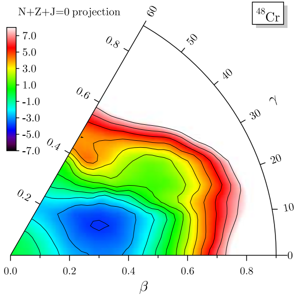


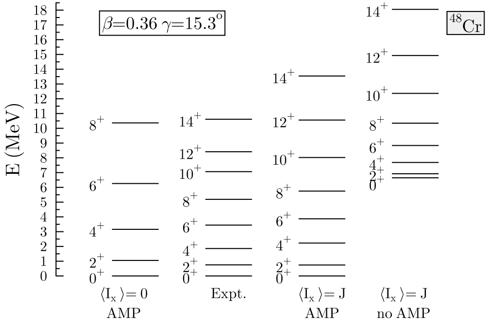
- excitation spectra now too much compressed (at low spin)

B. Avez, B. Bally, M. B., P.-H. Heenen, unpublished

Projection of the rotational band from cranked HFB





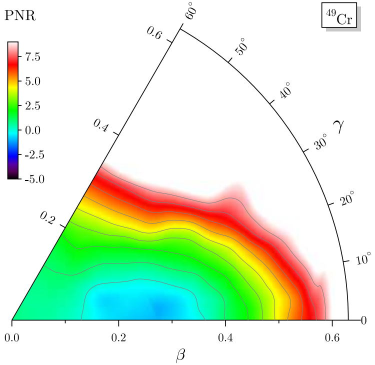


- projection from cranked states improves the moment of inertia at low spin
- for $J > 8$, the projected states from the cranked minima (previous slide) are lower in absolute energy
- backbending cannot be reproduced at fixed deformation
  $\rightarrow$ project the entire surface cranked to each J

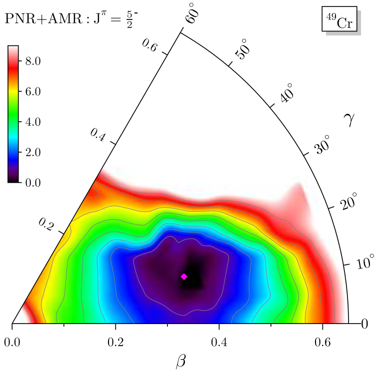
Particle-number and angular-momentum projection of ^{49}Cr

blocked states with $\langle j_x \rangle \approx \frac{5}{2}^-$

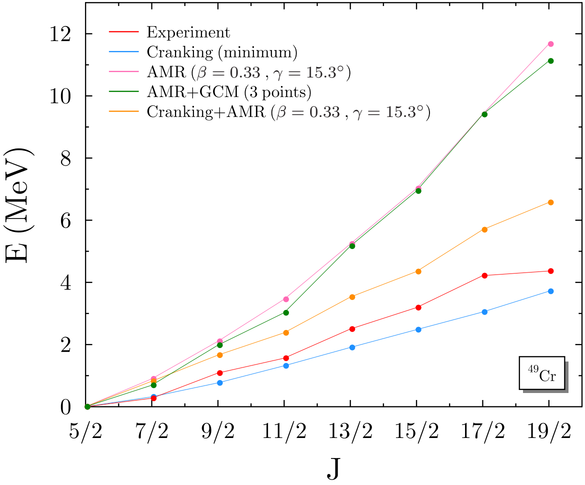
$J^\pi = \frac{5}{2}^-$ from blocked states with $\langle j_x \rangle \approx \frac{5}{2}^-$



B. Bally, B. Avez, M. B., P.-H. Heenen, unpublished



EFfect of cranking on the yrast states of ^{49}Cr



B. Bally, B. Avez, M. B., P.-H. Heenen, unpublished

Summary

Shape coexistence and/or shape mixing

- are crucial to explain the complexity of the excitation spectra spectra of many nuclei
- establishes a coherent picture of masses, radii, excitation spectra, transition moments, spectroscopic moments, ...
- levels corresponding to coexisting shapes might cross; when they do, shapes might be strongly mixed
- the crossing of coexisting shapes often explains deviations from "trivial" trends in the systematics of observables along isotopic/isotonic chains
- provides supplementary information on the evolution of shell structure
- offer insight into fine details of the quantum mechanical A-body problem

Work underway / to do

Necessary developments in the context of mean-field-based models

- explore role of triaxiality
- explore role of other collective degrees of freedom (octupole, . . .)
- explore role of diabatic degrees of freedom (“quasiparticle excitations”) (difficult in the Bohr Hamiltonian framework)
- extension to odd-A nuclei
- construct effective interactions of spectroscopic quality
- clarify open questions about the formalism (treatment of density dependences, general functionals, . . .)

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