The Gallium Anomaly: An Unsolvable Puzzle?

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Summary. — The Gallium Anomaly, a persistent discrepancy between observed and predicted rates of neutrino-induced transitions on ^{71}Ga, remains a significant open question in neutrino physics. In this work, we revisit the theoretical calculation of the neutrino cross section for the reaction $\nu_e + ^{71}\mathrm{Ga} \rightarrow ^{71}\mathrm{Ge} + e^-$ by incorporating recent advancements, providing a refined and self-consistent theoretical framework. The goal of this study is to quantify the impact of these updates on the predicted cross section and re-assess the significance of the gallium anomaly in light of these findings.

The Gallium Anomaly first emerged in the 1990s through the SAGE [1,2] and GALLEX [3-6] experiments, which were designed to study solar neutrinos via the neutrino capture reaction on a gallium target, $^{71}\mathrm{Ga}(\nu_e, e^-)^{71}\mathrm{Ge}$. During dedicated calibration campaigns employing intense artificial neutrino sources, these experiments observed a neutrino capture rate significantly lower than that predicted by theoretical models. This deficit, amounting to approximately 20% relative to expectations, raised fundamental questions regarding the accuracy of neutrino interaction modeling and suggested the potential existence of new, yet unexplained, physical phenomena.

The Gallium Anomaly is part of a wider landscape of unresolved issues in neutrino physics. Similar deficits observed in other contexts, such as the reactor antineutrino anomaly [7-11] and the so-called LSND anomaly [12] , have collectively hinted at the existence of sterile neutrinos, a hypothetical fourth type of neutrino that does not interact via the known fundamental forces except gravity. These anomalies have driven renewed interest in precision neutrino measurements and motivated new experimental efforts. Notably, the recent BEST experiment [13, 14], has confirmed the persistence of the Gallium Anomaly, achieving a statistical significance of up to 5σ based on its own data. However, despite this high level of significance, the results do not yet constitute definitive evidence for the existence of sterile neutrinos.

A central quantity in the analysis of neutrino interactions with gallium is the ground-state inverse beta decay (IBD) cross section for the process $\nu_e + ^{71}\mathrm{Ga} \rightarrow ^{71}\mathrm{Ge} + e^-$, which can be expressed as [15]:

$$\sigma_{\text{gs}} = \frac{G_F^2\,|V_{ud}|^2\,g_A^2}{\pi(2J_{\text{Ga}}+1)} \sum_j p_e^j E_e^j\,\mathcal{F}(E_e^j, Z)\,\left|\mathcal{M}_{\text{nuc}}^{\text{IBD}}\right|^2\,\mathcal{B}(E_e^j)\,, \tag{1}$$

where G_F is the Fermi constant, $|V_{ud}|$ is the relevant CKM matrix element [16], and $g_A = 1.2766$ is the axial-vector coupling constant [17]. The nuclear spin of gallium is $J_{\text{Ga}} = 3/2$.
The quantities p_e and E_e denote the momentum and energy of the emitted electron, related to the incoming neutrino energy by $E_e = E_\nu - Q_{\text{EC}} + m_e - 0.09\,\text{keV}$. The function $\mathcal{F}(E_e, Z)$ is the generalized Fermi function, which accounts for the Coulomb distortion of the outgoing electron wave function due to the daughter nucleus with atomic number Z. The sum runs over all allowed discrete electron energies E_e^j, weighted by the corresponding branching ratios $\mathcal{B}(E_e^j)$ specific to the neutrino source employed in gallium-based experiments.

The nuclear matrix element $\mathcal{M}_{\text{nuc}}^{\text{IBD}}$ represents the transition amplitude associated with the Gamow-Teller operator acting between the initial and final nuclear states. One of the main theoretical challenges in calculating the cross section is the determination of this matrix element. The typical strategy relies on the use of the inverse process, the electron capture (EC) in ^{71}Ge, whose half-life $t_{1/2}$ is experimentally well measured [18-21]. According to the principle of detailed balance, the nuclear matrix elements governing the IBD and EC processes are equivalent, such that $|\mathcal{M}_{\text{nuc}}^{\text{IBD}}|^2 \overset{\text{db}}{=} |\mathcal{M}_{\text{nuc}}^{\text{EC}}|^2$. This equivalence allows the IBD cross section to be recast in a more convenient form [15]:

$$\sigma_{\text{gs}}^{\text{db}} = \frac{2\pi^2 \ln 2}{f_{\text{EC}}\, t_{1/2}} \left(\frac{2J_{\text{Ge}}+1}{2J_{\text{Ga}}+1}\right) \sum_j p_e^j E_e^j\,\mathcal{F}(E_e, Z)\,\mathcal{B}(E_e^j)\,, \tag{2}$$

where $J_{\text{Ge}} = 1/2$ is the nuclear spin of gallium and f_{EC} is the phase-space factor for allowed electron capture transitions. Given the dominance of the K-shell capture, this latter factor can be approximated as [22, 23]:

$$f_{\text{EC}} \simeq 2\pi^2\,|\psi_{e,1s}^{\text{b}}(r_0)|^2\,(E_\nu^{1s})^2\,(1+\epsilon_{\text{o}}^{1s})\left[1 + \frac{P_{\text{L}}+P_{\text{M}}}{P_{\text{K}}}\right], \tag{3}$$

where P_{K}, P_{L}, and P_{M} are the experimentally determined capture probabilities for the corresponding atomic shells [18, 24, 25], and $|\psi_{e,1s}^{\text{b}}(r_0)|^2$ denotes the bound-state electron density at the nuclear surface.
Both the Fermi function and the bound-state electron density are computed by developing a dedicated numerical code based on the `RADIAL` package [26], which allows us to solve the Dirac-Hartree-Fock-Slater (DHFS) equations using a realistic atomic potential. The DHFS potential is given by

$$V_{\text{DHFS}}(r) = V_{\text{nuc}}(r) + V_{\text{el}}(r) + V_{\text{ex}}(r)\,, \tag{4}$$

where $V_{\text{nuc}}(r)$ is the nuclear potential, modeled using a two-parameter Fermi (2pF) distribution [27] with a measured root-mean-square charge radius of $R_{\text{ch}} = 4.032(2)$ fm [28] and skin thickness $t = 2.3\,\text{fm}$. The term $V_{\text{el}}(r)$ represents the electrostatic interaction between the electron and the surrounding atomic electron cloud, while $V_{\text{ex}}(r)$ accounts

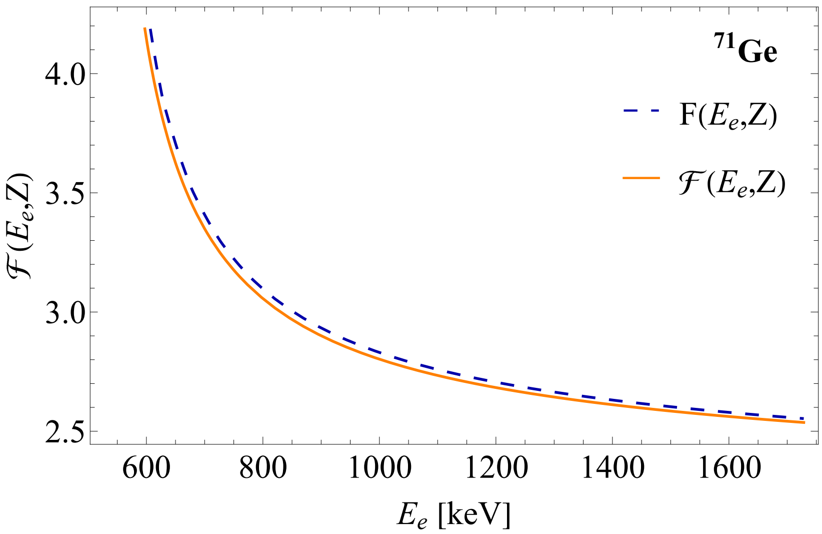

Fig. 1.: Comparison between the numerical Fermi function $\mathcal{F}(E_e, Z)$, obtained through the direct solution of the Dirac equation, and the analytical Fermi function derived under the point-like nucleus approximation with additional correction terms in Eq. (5). The comparison is shown over the electron energy range corresponding to the neutrino spectrum of the source.

for the exchange interaction. In Fig. 1 we compare our numerical Fermi function with the analytical expression widely used in the literature (see e.g. Ref. [29]), which is expressed as the product of various multiplicative corrections

$$\mathrm{F}(E_e, Z) = F_0(E_e, Z) L_0(E_e, Z) U(E_e, Z) S(E_e, Z)\,, \tag{5}$$

where $F_0(E_e, Z)$ is derived from the solution of the Dirac equation for a point-like nucleus and the terms L_0, U, and S introduce corrections for finite nuclear size, screening, and electron exchange effects, respectively.

The ultimate goal of this study is to provide an updated and self-consistent theoretical evaluation of the neutrino capture cross section on ^{71}Ga, a key quantity in interpreting the results of the SAGE [1,2], GALLEX [3-6], and BEST [13,14] experiments. Our approach incorporates a fully numerical treatment of both the outgoing electron's Fermi function and the bound-state electron wave function, accounting for finite nuclear size effects, electronic screening, and exchange corrections through the DHFS potential framework. We focus in particular on quantifying the impact of these theoretical improvements on the predicted cross section, re-evaluating the statistical significance of the Gallium Anomaly, and providing a revised interpretation in terms of oscillations into sterile neutrinos.

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