Preliminary development of the spin tune model at the hybrid ring for the EDM investigation

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Summary. — Electric Dipole Moments (EDMs) are very sensitive probes of CP violation and could represent the tool to solve some open questions the Standard Model cannot answer. Recent developments in the storage ring technology have allowed to propose a new method for the EDM investigation. One of the main parameters involved in this study is the Spin Coherence Time (SCT), *i.e.*, the time during which the spins of the particles of the beam precess coherently, maintaining a net polarization greater than 1/e. The SCT represents the time available for the experiment, which, therefore, must be as long as possible. To identify the working conditions that maximize the SCT, the single particle spin tune, defined as the number of spin precessions around the vertical axis per turn of the particle around the ring, must be tracked with very high accuracy. A model to precisely track the spin tune has been developed and tested on the hybrid ring. The model allows to estimate the spin tune, finds its direct application on the SCT optimization and is extendable to different storage rings, both existing and future.

1. – Introduction

The Standard Model (SM) is the modern quantum field theory of particle physics, but it cannot be considered the ultimate theory since it leaves several open questions. One of them is the observed matter-antimatter imbalance in the Universe, known as baryon asymmetry. The responsible process, called baryogenesis, is an asymmetric process that requires CP violation [1]. The baryon asymmetry is quantified by the asymmetry parameter $\eta = (N_B - N_{\bar{B}})/N_\gamma$, where N_B ($N_{\bar{B}}$) and N_γ are, respectively, the total number of baryons (antibaryons) and photons in the Universe. While the SM predictions are consistent with $\eta \sim 10^{-18}$ [2], the measured value is of the order of 10^{-10} [3]: this means that the CP violation incorporated in the SM [4] is insufficient to account for the observed baryon asymmetry. Therefore, new sources of CP violation beyond the SM are needed.

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A very sensitive probe of CP violation is the Electric Dipole Moment (EDM) [5]. It is an intrinsic fundamental property of particles, defined as a permanent asymmetry in the distribution of electric charges within the particle, with respect to the spin orientation. In the SM the EDMs are predicted to be extremely small, but many SM extensions predict non-zero and significantly larger EDMs, which can be experimentally accessed [6].
Since direct measurements of the EDM are experimentally challenging because of its tiny magnitude, very high precision techniques are required. One of them involves the use of storage rings [7, 8]. By injecting in a storage ring a polarized particle beam under the frozen spin condition, *i.e.*, with all the spins remaining aligned with their momenta, the existence of a non-vanishing EDM can be revealed as the rotation of the polarization vector from the ring plane to the vertical axis, due to its interaction with a radial electric field, and detected using a polarimeter, as depicted in fig.1.

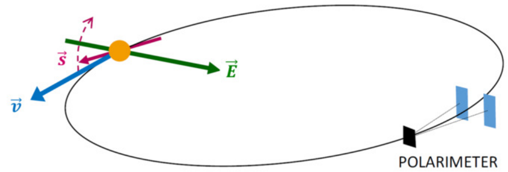

Fig. 1. – Schematic of the principle of the EDM measurement. For simplicity, only one particle is depicted. Legend: $\vec{v}$ is the particle velocity; $\vec{s}$ is the particle spin; $\vec{E}$ is the electric field.

The spin tune and the Spin Coherence Time (SCT) are two connected quantities to consider when performing an EDM experiment in a storage ring. The single particle spin tune is defined as the number of spin precessions around the vertical axis per turn of the particle around the ring, and to first order is given by [9]:

$$\nu_s = G\gamma - \frac{r(G+1)}{\gamma(\beta+r)}, \tag{1}$$

where G is the gyro-magnetic anomaly, γ is the relativistic factor, $r = E/(Bc)$ is the normalized field ratio and $\beta = v/c$ is the particle velocity. The SCT is the time during which the spins of the particles of the beam precess coherently, maintaining a net polarization greater than 1/e. Since this condition is necessary for the EDM measurement, it also represents the time available for the experiment, which, therefore, must be as long as possible. However, since the spin tune depends on the particle velocity through γ, any mechanism which changes the particle velocity directly affects the spin tune; as a result, the spins spread out and the polarization is lost. Therefore, to obtain a long SCT, it is necessary to minimize the spin tune.
A model for precisely tracking the spin tune has been developed and tested on the hybrid storage ring. Previous results have already been obtained through simulations at the Prototype Storage Ring (PSR) [10]. The single particle spin tune formula is expressed by a sum of contributions taking into account the dependencies of the spin tune on the synchrotron and betatron motions together with chromaticities. The software used to build the hybrid ring lattice and to perform the simulations is Bmad [11]. The model allows to estimate the spin tune and to directly determine the hybrid lattice optical properties which maximize the SCT. Furthermore, it is lattice-independent because it is extendable to a variety of lattices representing existing storage rings as well as new generation dedicated devices, *e.g.*, the PSR.

2. – Hybrid storage ring

Hybrid [12, 13] means that it features electric bending elements and magnetic focusing elements. It is a 24-fold symmetric ring in which each FODO cell alternates two straight sections and two bending sections: in each straight section there is a quadrupole magnet for particle focusing, while each bending section consists of two electric deflectors for particle confinement. Superimposed on each quadrupole magnet there is a sextupole magnet. Two straight sections house the injection points for the clockwise (CW) and counter-clockwise (CCW) proton beams, while another one contains the radio-frequency (RF) cavity. The layout of the hybrid ring lattice is shown in fig.2.

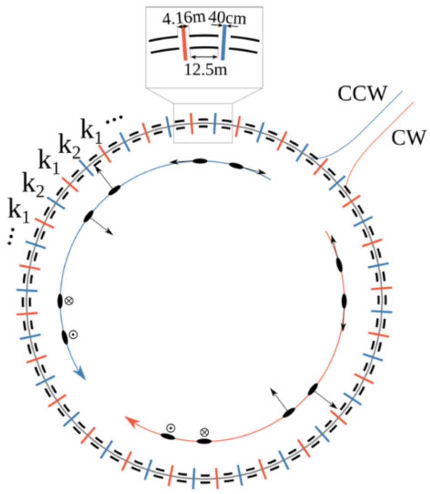

Fig. 2. – Layout of the hybrid ring lattice without the sextupole magnets and the RF cavity. Legend: k_1 and k_2 are the quadrupole magnets strengths.

The main advantage in using the hybrid ring is the simultaneous circulation of two counter-rotating beams that allows to control different systematic errors. Moreover, the electric field provided by the confinement system totally contributes to the EDM signal.

3. – Travel time model: RF cavity OFF

The starting point for the development of the spin tune model was the study of the dependences of the average path length in a storage ring [14]. The path lengthening formula is given by the sum of the longitudinal (L) and transverse (T) contributions:

$$\frac{\Delta L}{L} = \left(\frac{\Delta L}{L}\right)_L + \left(\frac{\Delta L}{L}\right)_T = \alpha_0\delta + \alpha_1\delta^2 - \frac{\pi}{L}\left(\epsilon_x\xi_x + \epsilon_y\xi_y\right), \quad (2)$$

where $\delta = \Delta p/p$ is the longitudinal momentum offset, ϵ_x and ϵ_y are the horizontal and vertical emittances, ξ_x and ξ_y are the horizontal and vertical chromaticities, α_0 and α_1 are the 1st and 2nd order compaction factors and L is the ring circumference length.
For convenience, the path lengthening has been studied in terms of the travel time [15]:

$$\frac{\Delta t}{t} = \eta_0\delta + \vec{\epsilon}\cdot\vec{s} + \vec{\epsilon}\cdot(T\vec{\xi}), \quad (3)$$

where η_0 is the 1st order slip factor, $\vec{\epsilon}$ is vector of the beam properties, $\vec{\xi}$ is the vector of the optical properties, $\vec{s}$ is the vector of the coefficients and T is the 3×2 matrix of the coefficients. The coefficients encoded in $\vec{s}$ and T are free parameters of the model and set the strengths of the contributions.
If the travel time model is accurate, T_{11} and T_{22} should be equal to $-\pi/L$.

4. – Spin tune model: RF cavity OFF

Considering the travel time model results, a formula with the same structure has been developed for the spin tune:

$$\nu_s = \sigma_0 \delta + \vec{\epsilon} \cdot \vec{v} + \vec{\epsilon} \cdot (W\vec{\xi}), \tag{4}$$

where σ_0 is the 1st order spin tune factor, while $\vec{\epsilon}$, $\vec{\xi}$, $\vec{v}$ and W represent the same quantities that appear also in eq.(3).
The goal is the comparison between the spin tune values obtained through the formula given by eq.(4) and the spin tune values evaluated using the raw data provided by Bmad; the former are referred to as calculated values, while the latter as simulated values. The procedure is divided into three steps: the first step consists in the derivation of the coefficients of the formula, the second one is dedicated to the calculated values and the last one to the simulated values. To determine the free parameters, the terms have been considered separately in order to independently study the dependence of the spin tune on the synchrotron and betatron motions together with chromaticities. The simulated values have been directly evaluated from the plot of the spin rotation angle as a function of the number of turns.

5. – Spin tune model: RF cavity ON

Since during the EDM measurement the RF cavity will have to be switched ON, the spin tune model must be developed with the RF cavity ON.
The starting point is the time-averaged spin tune:

$$\langle \nu_s \rangle = \sigma_0 \langle \delta \rangle + \langle \vec{\epsilon} \rangle \cdot \vec{v} + \langle \vec{\epsilon} \rangle \cdot (W\vec{\xi}), \tag{5}$$

where $\langle \delta \rangle$ is the mean longitudinal momentum offset and $\langle \vec{\epsilon} \rangle$ is the time-averaged vector of the beam properties.
Modelling appropriately the longitudinal momentum and using the boundary condition of the longitudinal focusing, *i.e.*, $\langle \Delta t/t \rangle = 0$, the spin tune model is:

$$\langle \nu_s \rangle = \langle \vec{\epsilon} \rangle \cdot \vec{m} + \langle \vec{\epsilon} \rangle \cdot (N\vec{\xi}), \tag{6}$$

where:

$$\vec{m} = \vec{v} - \frac{\sigma_0}{\eta_0} \vec{s}, \qquad N = W - \frac{\sigma_0}{\eta_0} T. \tag{7}$$

To improve the precision of the model, the coefficients can be directly evaluated from RF cavity ON simulations.
Also in this case, the three-step procedure has been used to make a comparison between the calculated and simulated spin tune values.

6. – SCT optimization

To obtain a long SCT, it is necessary to minimize the spin tune. This translates into equating eq.(6) to zero and leads to:

$$\vec{m} + N\vec{\xi} = 0. \tag{8}$$

Inverting eq.(8), it is possible to derive the setting of optimized chromaticities that maximize the SCT:

$$\vec{\xi}_{opt} = -(N)^{+}\vec{m} = -[(N^T N)^{-1} N^T]\vec{m}, \tag{9}$$

where N^+ is the pseudo-inverse matrix since N is a 3×2 matrix. Since eq.(8) is an overdetermined system and a unique solution is not guaranteed, eq.(9) represents the condition for the partial SCT optimization. A full SCT optimization can be obtained for lattices with a third family of sextupole magnets, *e.g.*, the PSR.

7. – Results

To test the travel time model, the values of the coefficients T_{11} and T_{22} have been derived from single particle simulations obtaining $T_{11,mod} = (-3.91919 \times 10^{-3}) \pm (1.6 \times 10^{-8})$ and $T_{22,mod} = (-3.91919 \times 10^{-3}) \pm (1.1 \times 10^{-8})$. They are compatible with those predicted by theory ($T_{11,th} = T_{22,th} = -\pi/L = -3.91915 \times 10^{-3}$).
The spin tune models have been tested using about 10^4 sets of single particle tracking simulations, each characterized by unique randomly assigned momentum offsets, transverse emittances and chromaticities. The comparison between simulated and calculated values has been evaluated in terms of the residuals, *i.e.*, considering their difference. The histograms of the residuals are shown in fig.3: they both peak at zero and this means that the discrepancy between the simulated and calculated values is negligible for the majority of the sets of simulations considered and suggests that the models correctly predict the values provided by the simulator. The goodness of the results are aligned with those obtained for the PSR [10].

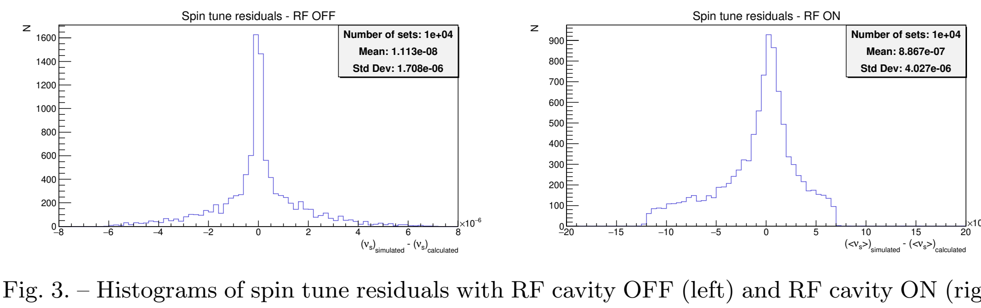

Fig. 3. – Histograms of spin tune residuals with RF cavity OFF (left) and RF cavity ON (right).

The SCT optimization condition has been tested by simulating a particle beam of about 10^3 particles performing 10^6 turns around the ring, each characterized by the optimized chromaticities and unique randomly assigned momentum offsets and transverse emittances. The SCT has been measured from the plot of the polarization vector as a function of time obtaining $\tau = (132 \pm 2)$ s, as shown in fig.4. This value is one order of magnitude smaller than the one measured for the PSR [16] due to the partial SCT optimization.

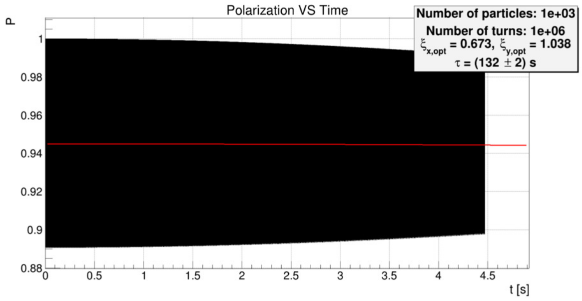

Fig. 4. – Plot of the polarization vector as a function of time, to measure the SCT.

8. – Conclusions

A model for accurately tracking the spin tune has been developed and tested on the hybrid storage ring. A staged approach has been used, beginning with the path lengthening formula, followed by the travel time model and finishing with the innovative spin tune model along with its direct application on the SCT optimization. The result is a lattice-independent model capable of estimating the spin tune and of directly determining the lattice optical properties that maximize the SCT. Looking ahead, this model represents a significant advancement in EDM experimental techniques and contributes to extract information about the EDM with unprecedented sensitivity and to have a deeper mathematical understanding of the phenomenon.

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