Isospin-violating decays of charmonium in $\Lambda\bar{\Sigma}^0$

F. Rosini([1])([2])

([1]) *INFN, Sezione di Perugia - Perugia, Italy*
([2]) *Dipartimento di Fisica e Geologia, Università degli Studi di Perugia - Perugia, Italy*

Summary. — Assuming isospin conservation, the decay of a $c\bar{c}$ vector meson into the $\Lambda\bar{\Sigma}^0$+c.c. final state is purely electromagnetic. At the leading order, the $c\bar{c}$ vector meson first converts into a virtual photon that, then produces the $\Lambda\bar{\Sigma}^0$ + c.c. final state. Moreover, such a mechanism, i.e., the virtual photon coupling to $\Lambda\bar{\Sigma}^0$+c.c., is the solely intermediate process through which, in Born approximation, the reaction $e^+e^- \to \Lambda\bar{\Sigma}^0$ + c.c. does proceed. It follows that any significant difference between the amplitudes of the processes $c\bar{c} \to \Lambda\bar{\Sigma}^0$ + c.c. and $e^+e^- \to \Lambda\bar{\Sigma}^0$ + c.c. at the $c\bar{c}$ mass must be ascribed to an isospin-violating contribution in the $c\bar{c}$ decay.

1. – Parameterizations of the amplitudes

The processes under consideration are the decay of a $c\bar{c}$ vector meson, called ψ, into a baryon-antibaryon $B_1\bar{B}_2$ + c.c. final state, and the e^+e^- annihilation which produces the same final state. The reactions are

$$\psi \to B_1\bar{B}_2 + \text{c.c.}\,, \quad e^+e^- \to B_1\bar{B}_2 + \text{c.c.}$$

In general, at the leading-order, the ψ and γ-hyperons currents are

$$\begin{aligned}
J^{\psi}_{\mu} &= \bar{u}(k_1)G_{\mu}[g_E^{B_1B_2}, g_M^{B_1B_2}]v(k_2)\,,\\
J^{\gamma}_{\mu} &= \bar{u}(k_1)G_{\mu}[G_E^{B_1B_2}(q^2), G_M^{B_1B_2}(q^2)]v(k_2)\,,
\end{aligned}$$

where u and v are the Dirac spinors, k_1 and k_2 are the four-momenta of the hyperons, $q = k_1+k_2$, $g_E^{B_1B_2}$ and $g_M^{B_1B_2}$ are the so-called psionic coupling constants, while $G_E^{B_1B_2}(q^2)$ and $G_M^{B_1B_2}(q^2)$ are Sachs form factors. The Lorentz four-vector G_μ in both the currents has the same expression, in that, the second can be obtained from the first by the substitutions: $g_{E,M}^{B_1B_2} \to G_{E,M}^{B_1B_2}(q^2)$ and vice versa.

We define the effective unique psionic coupling constant $A^{\psi}_{B_1B_2}$ and the effective form factor $A_{B_1B_2}(q^2)$ as

$$
\begin{aligned}
(1) \qquad |A^{\psi}_{B_1B_2}| &= \sqrt{\frac{2M^2_{B_1B_2}}{M^2_{\psi}}\left|g^{B_1B_2}_E\right|^2 + \left|g^{B_1B_2}_M\right|^2}\,, \\
|A_{B_1B_2}(q^2)| &= \sqrt{\frac{2M^2_{B_1B_2}}{q^2}\left|G^{B_1B_2}_E(q^2)\right|^2 + \left|G^{B_1B_2}_M(q^2)\right|^2}\,,
\end{aligned}
$$

in terms of which the decay $\psi \to B_1\bar{B}_2 + \text{c.c.}$ and the cross section of the annihilation $e^+e^- \to B_1\bar{B}_2 + \text{c.c.}$ are defined as

$$
\begin{aligned}
(2) \qquad \Gamma^{\psi}_{B_1B_2} &= \beta_{B_1B_2}(M^2_{\psi})\Gamma^{\psi}_{\mu\mu}\left|A^{\psi}_{B_1B_2}\right|^2\,, \\
\sigma_{B_1B_2}(q^2) &= \frac{4\pi\alpha^2\beta_{B_1B_2}(q^2)}{3q^2}\left|A_{B_1B_2}(q^2)\right|^2\,,
\end{aligned}
$$

where $\beta_{B_1B_2}(q^2) = \sqrt{1 - 4M^2_{B_1B_2}/q^2}$. In the case of a pure isovector final state, as for $(B_1, B_2) = (\Lambda, \Sigma^0)$, and assuming isospin conservation, the psionic coupling constants coincide with the Sachs form factors at the vector meson mass, i.e.,

$$
(3) \qquad g^{B_1B_2}_{E,M} = G^{B_1B_2}_{E,M}(M^2_{\psi})\,.
$$

An isospin-violating contribution can be parametrized by the pair of coupling constants $g_{E,\not{I}}$ and $g_{M,\not{I}}$ that sum up to the isospin-conserving ones to give the total psionic coupling constants as

$$
g^{B_1B_2}_{E,M,\text{tot}} = g^{B_1B_2}_{E,M} + g^{B_1B_2}_{E,M,\not{I}}\,.
$$

In terms of the effective coupling constant and the effective form factor defined in Eq. 1, the total coupling constant is defined as

$$
A^{\psi}_{B_1B_2,\text{tot}} = A^{\psi}_{B_1B_2} + A^{\not{I}}_{B_1B_2} = A_{B_1B_2}(M^2_{\psi}) + A^{\not{I}}_{B_1B_2}\,,
$$

where, as a consequence of Eq. 3, the effective isospin-conserving psionic coupling constant coincides with the effective form factor at the ψ mass.
It follows that, by allowing the isospin violation, which can occur only in the decay process $\psi \to \Lambda\bar{\Sigma}^0 + \text{c.c.}$, since the reaction $e^+e^- \to \Lambda\bar{\Sigma}^0 + \text{c.c.}$ is purely electromagnetic, the expression of the decay rate of Eq. 2 becomes

$$
\begin{aligned}
\Gamma^{\psi}_{B_1B_2} &= \beta_{B_1B_2}(M^2_{\psi})\Gamma^{\psi}_{\mu\mu}\left|A^{\psi}_{B_1B_2,\text{tot}}\right|^2 \\
(4) \qquad &= \beta_{B_1B_2}(M^2_{\psi})\Gamma^{\psi}_{\mu\mu}\left|A_{B_1B_2}(M^2_{\psi}) + A^{\not{I}}_{B_1B_2}\right|^2\,.
\end{aligned}
$$

Using the cross section formula of Eq. 2 at $q^2 = M^2_{\psi}$ and that for the decay rate of Eq. 4, the ratio reads

$$
R^{\psi}_{B_1B_2} \equiv \left|A_{B_1B_2}(M^2_{\psi}) + A^{\not{I}}_{B_1B_2}\right| / \left|A_{B_1B_2}(M^2_{\psi})\right|
$$

$$\equiv \sqrt{\frac{4\pi\alpha^2}{3M_\psi^2\Gamma_{\mu\mu}^\psi}\frac{\Gamma_{B_1B_2}^\psi}{\sigma_{B_1B_2}(M_\psi^2)}}\,. \tag{5}$$

The intensity and hence the modulus of the isospin-violating coupling constant can be extracted from the experimental value $R^\psi_{B_1B_2}$ as a function of its relative phase to the electromagnetic coupling constant $A_{B_1B_2}(M_\psi^2)$. The relative isospin-violating-to-pure-electromagnetic intensity is expressed as

$$\begin{aligned}\left|A^{\not I}_{B_1B_2}\right|/\left|A_{B_1B_2}(M_\psi^2)\right| = &\sqrt{\left(R^\psi_{B_1B_2}\right)^2 - \sin^2\left(\phi^\psi_{B_1B_2}\right)}\\ &- \cos\left(\phi^\psi_{B_1B_2}\right)\,.\end{aligned} \tag{6}$$

The behavior of the ratio $\left|A^{\not I}_{B_1B_2}\right| / \left|A_{B_1B_2}(M_\psi^2)\right|$ as a function of the relative phase ϕ_ψ is symmetric with respect to $\phi^\psi_{B_1B_2} = \pi$, and it is minimum and equal to $R^\psi_{B_1B_2} - 1$ at $\phi^\psi_{B_1B_2} = 0$ when the interference between the two coupling constants is constructive and reaches the maximum value of $R^\psi_{B_1B_2} + 1$ at $\phi^\psi_{B_1B_2} = \pi$, when, instead, the interference is destructive.

2. – The scaled cross section

To collect as much as possible information on the modulus of the effective form factor $A_{B_1B_2}(q^2)$, for a specific final state $B_1\bar{B}_2$ + c.c., we use cross section data of all the B_1B_2 pairs of neutral baryons belonging to the spin-1/2 SU(3) baryon octet [11]. In particular, the amplitudes of reactions producing pairs of neutral baryon-anti-baryon are proportional to the same coupling constant by known coefficients.
As a consequence, the effective form factors of these neutral baryon-anti-baryon pairs and hence the cross sections can be expressed in terms of the effective form factor of an only pair scaled by the corresponding coefficient:

$$A_{B_1B_2}(q^2) = N_{B_1B_2}A_{\Lambda\Sigma^0}(q^2)\,, \tag{7}$$

with $(N_{nn}, N_{\Lambda\Lambda}, N_{\Sigma^0\Sigma^0}, N_{\Xi^0\Xi^0}) = (-2,-1,1,-2)/\sqrt{3}$. The scaled cross section is defined as [14]

$$\tilde{\sigma}(q^2) = \frac{\sigma_{B_1B_2}(q^2)}{N^2_{B_1B_2}\beta_{B_1B_2}(q^2)} = \frac{4\pi\alpha^2}{3q^2}\left|A_{\Lambda\Sigma_0}(q^2)\right|^2\,.$$

The data sets used to extract the scaled cross section values are on all four neutral baryon-anti-baryon channels, besides that under study, namely: $(B_1, B_2) = (\Lambda, \Sigma^0)$. The measurements were performed by the following experiments: BaBar on the reactions: $e^+e^- \to \Lambda\bar{\Lambda}$, $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ and $e^+e^- \to \Lambda\bar{\Sigma}^0$ + c.c. [10]; Belle on the reaction: $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ [9]; BESIII on the reactions: $e^+e^- \to \Lambda\bar{\Sigma}^0$+c.c. [8], $e^+e^- \to \Lambda\bar{\Lambda}$ [7], $e^+e^- \to n\bar{n}$ [6], and $e^+e^- \to \Xi^0\bar{\Xi}^0$ [5]. To extract the values of the modulus of the effective form factor $A_{\Lambda\Sigma^0}(q^2)$ at the J/ψ and $\psi(2S)$ masses, we assume the high-q^2-power law behavior

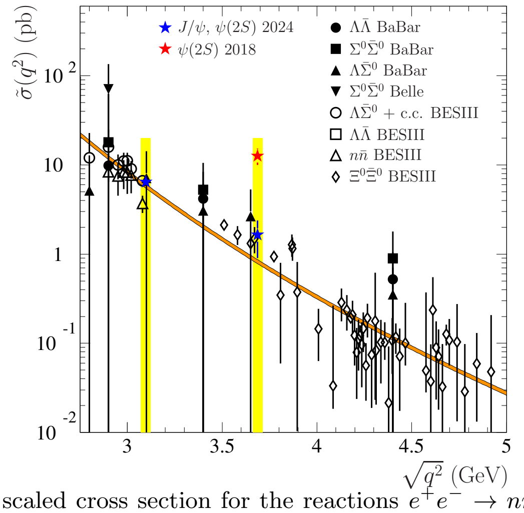

Fig. 1. – Data on the scaled cross section for the reactions $e^+e^- \to n\bar n$, $\Lambda\bar\Lambda$, $\Sigma^0\bar\Sigma^0$, $\Xi^0\bar\Xi^0$ and $\Lambda\bar\Sigma^0$ + c.c., collected by the experiments: BaBar [10], Belle [9] and BESIII [8, 7, 6, 5]. Energy regions 50 MeV-wide around the J/ψ and $\psi(2S)$ masses are highlighted in yellow [15].

predicted by the perturbative QCD [4, 3] and use the fitting function

$$\tilde\sigma_{\rm fit}(q^2) = \frac{A}{(q^2)^5\left(\pi^2 + \ln^2\left(q^2/\Lambda^2_{\rm QCD}\right)\right)^2}\,. \tag{8}$$

This function depends on only one free parameter A, the QCD scale is fixed at $\Lambda_{\rm QCD} =$ 0.35 GeV. The best value of the parameter A is obtained by minimizing the χ^2

$$\chi^2 = \sum_{j=1}^{N}\left(\frac{\tilde\sigma_{\rm fit}(q_j^2) - \tilde\sigma_j}{\delta\tilde\sigma_j}\right)^2,$$

where $\sigma_{B_1B_2,j} \pm \delta\sigma_{B_1B_2,j}$ is the experimental value of the cross section of the annihilation $e^+e^- \to B_1\bar B_2$ + c.c. measured at $q^2 = q_j^2$, for all data sets with the cut-off for the four-momentum transfer squared $q_j^2 \geq (2.8\,\text{GeV})^2$ has been adopted to exclude the production threshold region. The total number of data points considered for the minimization is $D = 70$ and the obtained best value for the parameter A is

$$A = (3.92 \pm 0.21) \times 10^8\ \text{GeV}^{10}\,\text{pb}\,. \tag{9}$$

The black symbols of Fig. **2** represent the data of all the processes considered. The blue stars are the values of the scaled cross section at the J/ψ and $\psi(2S)$ masses extracted from the 2024 [12] decay rates $\Gamma\left(J/\psi \to \Lambda\bar\Sigma^0 + \text{c.c.}\right)$ and $\Gamma\left(\psi(2S) \to \Lambda\bar\Sigma^0 + \text{c.c.}\right)$, while the red star is the scaled cross section at the $\psi(2S)$ mass obtained by the 2018 [13] decay rate $\Gamma(\psi(2S) \to \Lambda\bar\Sigma^0 + \text{c.c.})$. The orange band is the fit function of Eq. 8 with the parameter value given in Eq. 9.

The predictions for the scaled cross section at the J/ψ and $\psi(2S)$ masses are

$$\begin{aligned} \tilde{\sigma}_{\rm fit}(M^2_{J/\psi}) &= (5.79 \pm 0.30)\ \mathrm{pb}\,, \\ \tilde{\sigma}_{\rm fit}(M^2_{\psi(2S)}) &= (0.825 \pm 0.043)\ \mathrm{pb}\,. \end{aligned} \tag{10}$$

On the other hand, using the branching fractions [12]

$$\begin{aligned} \mathrm{BR}(J/\psi \to \Lambda\bar{\Sigma}^0 + \mathrm{c.c.}) &= (2.83 \pm 0.23) \times 10^{-5}\,, \\ \mathrm{BR}(\psi(2S) \to \Lambda\bar{\Sigma}^0 + \mathrm{c.c.}) &= (1.6 \pm 0.7) \times 10^{-6}\,, \end{aligned} \tag{11}$$

the corresponding scaled cross sections, represented by blue stars in Fig. **2**, are

$$\begin{aligned} \tilde{\sigma}_\star^{J/\psi} &= (6.45 \pm 0.53)\ \mathrm{pb}\,, \\ \tilde{\sigma}_\star^{\psi(2S)} &= (1.64 \pm 0.74)\ \mathrm{pb}\,. \end{aligned} \tag{12}$$

It is interesting to notice how much the last value, which refers to the $\psi(2S)$ decay, has changed since 2018 when the branching fraction was [14]

$$\mathrm{BR}_{18}(\psi(2S) \to \Lambda\bar{\Sigma}^0 + \mathrm{c.c.}) = (1.23 \pm 0.24) \times 10^{-5}\,, \tag{13}$$

corresponding to the scaled cross section

$$\tilde{\sigma}_{18\star}^{\psi(2S)} = (12.6 \pm 2.6)\ \mathrm{pb}\,, \tag{14}$$

indicated by a red star in Fig. **2**.

3. – Interpretation

Assuming a dominant electromagnetic contribution and the power scaling predicted by perturbative QCD, the expected ratio between J/ψ and $\psi(2S)$ decay rates divided by their electric widths should be of the order of the eight power of the inverse mass ratio, i.e., $\simeq 4.0$. The 2018 experimental values, considering the 2018 and 2024 data for the width $\Gamma_{\Lambda\Sigma^0}^{\psi(2S)}$ are

$$\frac{\Gamma_{\Lambda\Sigma^0}^{J/\psi}/\Gamma_{\mu\mu}^{J/\psi}}{\Gamma_{\Lambda\Sigma^0}^{\psi(2S)}/\Gamma_{\mu\mu}^{\psi(2S)}} = \left\{ \begin{array}{ll} 0.31 \pm 0.07 & \quad 2018\ [13] \\ 2.4 \pm 1.0 & \quad 2024\ [12] \end{array} \right. .$$

With the results of Eq. 10, the squared values of the ratios defined in Eq. 5 for J/ψ and $\psi(2S)$ are

$$\left(R_{\Lambda\Sigma^0}^{\psi}\right)^2 = \left\{ \begin{array}{ll} 1.15 \pm 0.11 & \quad \psi = J/\psi \\ 2.00 \pm 0.88 & \quad \psi = \psi(2S) \end{array} \right. , \tag{15}$$

where we have used the definition of the complete cross section $\sigma_{\Lambda\Sigma^0}(M_\psi^2) = \beta_{\Lambda\Sigma^0}(M_\psi^2)\tilde{\sigma}_{\rm fit}(M_\psi^2)$. The relative isospin-violating-to-pure-electromagnetic intensities for the J/ψ and $\psi(2S)$ as functions of the relative phase ϕ_ψ, as defined in Eq. 6, are shown in Fig. **3**. The possibility of significative isospin-violating effects for the $\psi(2S)$, which was the case of study

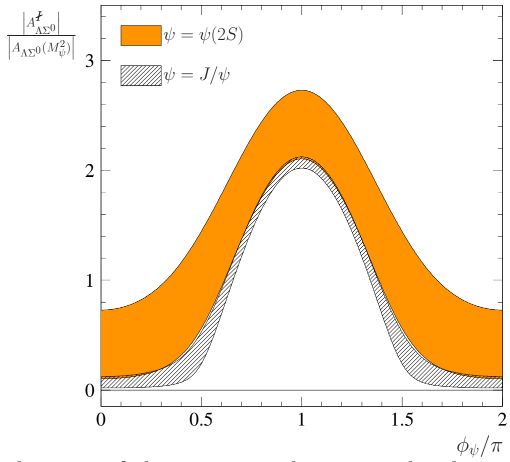

Fig. 2. – Relative contributions of the isospin-violating amplitude with respect to the electromagnetic one as a function of the relative phase ϕ_ψ in units of π radiants in the cases of $\psi(2S)$, orange band, and J/ψ, lined band [15].

of Ref. [14] is drastically reduced considering the new value of the $\psi(2S) \to \Lambda\bar{\Sigma}^0 + \text{c.c.}$ branching fraction. The entity of such an effect can be evaluated by the distance of the ratio $R^{\psi(2S)}_{\Lambda\Sigma^0}$ from unity, i.e., by the compatibility with zero of the minima of the curve of Fig. **3**. Using the datum of Eq. 15 for the $\psi(2S)$ case the minimum is

$$\left.\frac{\left|A^{\not I}_{\Lambda\Sigma^0}\right|}{\left|A_{\Lambda\Sigma^0}\left(M^2_{\psi(2S)}\right)\right|}\right|_{\phi_{\psi(2S)}=0} = R^{\psi(2S)}_{\Lambda\Sigma^0} - 1 = 0.41 \pm 0.31\,,$$

which is compatible with zero within less than two standard deviations. The situation for the J/ψ is unchanged since the previous study [14], in this case the minimum is

$$\left.\frac{\left|A^{\not I}_{\Lambda\Sigma^0}\right|}{\left|A_{\Lambda\Sigma^0}\left(M^2_{J/\psi}\right)\right|}\right|_{\phi_{J/\psi}=0} = R^{J/\psi}_{\Lambda\Sigma^0} - 1 = 0.07 \pm 0.06\,,$$

definitely compatible with zero.
In both cases, the possibility of few % isospin-violating effects remains, even though, as is theoretically expected. However, with the recent measurement of the BESIII Experiment [1] of the $\psi(2S) \to \Lambda\bar{\Sigma}^0 + \text{c.c.}$ branching fraction, spectacular isospin violation phenomena are excluded.

REFERENCES

[1] Ablikim, M. and others, *Phys. Rev. D*, **103** (2021) 112004.
[2] Dobbs, S. and Seth, Kamal K. and Tomaradze, A. and Xiao, T. and Bonvicini, G., *Phys. Rev. D*, **96** (2017) 092004.

[3] Brodsky, Stanley J. and Farrar, Glennys R., *Phys. Rev. Lett.*, **31** (1973) 1153.
[4] Matveev, V. A. and Muradyan, R. M. and Tavkhelidze, A. N., *Phys. Rev. Lett*, **31** (1973) 1153–1156.
[5] Ablikim, M. and others, *JHEP*, **11** (2024) 062.
[6] Ablikim, M. and others, *Nature Phys.*, **17** (2021) 11.
[7] Ablikim, M. and others, *Phys. Rev. Lett.*, **123** (2019) 12.
[8] Ablikim, M. and others, *Phys. Rev. Lett.*, **109** (2024) 012002.
[9] Gong G. and others, *Phys. Rev. D*, **107** (2023) 072008.
[10] Aubert, B. and others, *Phys. Rev. D*, **76** (2007) 092006.
[11] Baldini Ferroli, R. and Mangoni, A. and Pacetti, S., *Phys. Lett. B*, **799** (2019) 135041.
[12] Navas, S. and others, *Phys. Rev. D*, **110** (2024) 030001.
[13] Tanabashi, M. and others, *Phys. Rev. D*, **98** (2018) 030001.
[14] Baldini Ferroli, R. and Mangoni, A. and Pacetti, S., *Eur. Phys. J. C*, **80** (2020) 903.
[15] Rosini, F. and Pacetti, S., *Eur. Phys. J. C*, **85** (2025) 236.