Directions of development in neutrino physics

Francesco Vissani([1])
([1]) *INFN, Laboratori Nazionali del Gran Sasso*

Summary. — A number of selected topics in neutrino physics are examined, with a focus on the theoretical framework. The salient results and evolution of these topics are presented, along with their synergies and lines of development. For ease of presentation, the discussion is organised into three distinct sections: neutrinos in particle physics, neutrinos in cosmology, and neutrinos in astrophysics.

1. – Ten years ago - introduction

The discovery of neutrino oscillations with atmospheric and solar neutrinos - at Super-Kamiokande and SNO respectively - was announced in Stockholm as follows [1]

> For particle physics this was a historic discovery. Its Standard Model of the innermost workings of matter had been incredibly successful, having resisted all experimental challenges for more than twenty years. However, as it requires neutrinos to be massless, the new observations had clearly showed that the Standard Model cannot be the complete theory of the fundamental constituents of the universe.

Although this is the only observational evidence of this type, moreover tested in many laboratory experiments, there are colleagues interested in "physics beyond the standard model" who tend to downplay the meaning of these discoveries, or sometimes the entire field of neutrino research, see e.g., [2, 3]. Thus, it might be useful to summarise the current state of neutrino physics and to examine some of its most exciting progresses.

I do not even try to be exhaustive; such a broad field would not allow this in the space assigned. Among the most important omissions, fortunately covered elsewhere at IFAE 2025, I mention the lively study of low-energy neutral current interactions, the fruitful connections with nuclear physics, the future oscillation experiments, the direct search for neutrino masses and the development of new detectors. Here I prefer to outline with a minimum of completeness and historical accuracy certain lines of development of neutrino physics that, as those at accelerators [4], deserve a special attention.

This is important as several neutrino experiments require long evolution times, on the order of (several) decades. Concerning time scales, it is interesting to recall that twelve years ago the INFN community was engaged in intense reflection on its future. The lines that led to the greatest successes received comparably little attention, but one of the discussion group [5] highlighted the scientific opportunities offered to neutrino astrophysics by Borexino, which was finally working well, and by KM3NeT, more motivated than ever after the initial IceCUBE discoveries. We will talk about them later.

2. – Neutrinos in particle physics

2˙1. *What do we know on baryon and lepton numbers?*. – The current theory of quarks, leptons and of their non-gravitational interactions based on $\mathrm{SU(3)_c{\times}SU(2)_L{\times}U(1)_Y}$, the standard model, has 4 accidental, global symmetries: the baryon number $\mathbf{B}$ and three lepton numbers $\mathbf{L}_e, \mathbf{L}_\mu, \mathbf{L}_\tau$. Therefore, even before any interpretation, it is utmost interest to notice that the appearance of neutrino types different from those initially produced has been observed in OPERA [6], T2K [7], NOνA [8] (and indirectly by Super-Kamiokande and DeepCore). This implies that only the total lepton number

$$\mathbf{L} = \mathbf{L}_e + \mathbf{L}_\mu + \mathbf{L}_\tau \tag{1}$$

survives the experimental tests. But only certain combinations are non-anomalous symmetries, as proved by 'T HOOFT [9]; thus, there is only one *exact* number that is untested

$$\mathbf{B} - \mathbf{L} \tag{2}$$

Concerning expectations, here is WEINBERG on the point [10]

> If effects of a tiny nonconservation of baryon or lepton number such as proton decay or neutrino masses are discovered experimentally, we will then be left with gauge symmetries as the only true internal symmetries of nature, a conclusion that I would regard as most satisfactory.

Higher dimensional operators that respect the gauge invariance offer us a convenient way to contemplate plausible deviations from the standard model [11]. Table I resumes schematically some of the most interesting ones, indicating their selection rules and the new signal that they induce. They are ordered in powers of inverse mass, starting from the one of dimension 5 (first entry) which is of particular relevance in the following.

2˙2. *On the nature of neutrinos*. – To understand how to extend the standard model, it is useful to recall a very popular infographic of its matter particles, the one that shows quarks and leptons on one side and their antiparticles on the other, as if they were reflected in a mirror. Although such a graphical presentation could seem innocent and even convincing, it hides two flaws concerning neutrinos:

- we know experimentally that the states ν_e, ν_μ and ν_τ are not the mass eigenstates;
- what should distinguish neutrinos from antineutrinos, as their charge is zero?

The first point refers to the phenomenon of neutrino oscillations. The second is more subtle, since neutrinos and antineutrinos differ in principle due to their hyper-charge. At first sight, neutrino could be massive as the other particles - an option called "DIRAC neutrinos", see here [12] for the terminology. But this requires a significant modification of the standard model, namely the addition of right-handed neutrinos, which have no gauge interactions. Everything would suggest that their mass has nothing to do with the

TABLE I. – *Representative gauge invariant effective operators, of canonical dimension 5,6,7 and 9 respectively, that describe phenomena absent in the standard model.*

operator	$\|\mathbf{\Delta B}\|$	$\|\mathbf{\Delta L}\|$	$\|\mathbf{\Delta (B-L)}\|$	process induced	name
$(\ell H)^2/M$	0	2	2	$2n \to 2p + 2e^-$	neutrinoless $\beta\beta$ decay
$qqq\ell/M'^2$	1	1	0	$p \to \pi^0 + e^+$	proton decay
$ddd\,\overline{\ell H}/M''^3$	1	1	2	$n \to K^+ + e^-$	nucleon decay
$(udd)^2/M'''^5$	2	0	2	$n \leftrightarrow \bar{n}$	$n - \bar{n}$ oscillations

standard model, which leads us to look with some suspicion at this theoretical option. But there is a way to avoid such doubts. Let's take the lowest-dimensional non-renormalisable operator of table I. After spontaneous gauge symmetry breaking, it describes a bilinear term in neutrino fields, which violates the lepton number **L** and thus corresponds to a type of mass that differs from those in the standard model.

We can understand its nature with a simple thought experiment. The mass of the known neutrinos is very small, and therefore when they are ultra-relativistic, neutrino states have negative helicity and antineutrino states have positive helicity in very good approximation. Therefore they are well distinct, as a consequence of the $V - A$ structure of the standard model. However, neutrinos do have mass; thus, there is a reference frame in which they are at rest, and where it no longer makes sense to speak of helicity, but only of spin. Therefore, the minimum option is that the neutrino and the antineutrino are simply two states oriented in different directions, or in plain words, *they are the same particle*, a possibility imagined in 1937 [13].(1) Summarising: neutrinos masses are likely to be due to the dimension 5 operator [11] of table I; this implies that they are MAJORANA neutrinos, matter and antimatter at once.

This scenario admits a crucial experimental test. The e-e element of this mass matrix violates $\mathbf{L}_e$ by two units and can be written as the sum of neutrino masses weighted with the leptonic mixing $m_{\beta\beta} = \left|\sum_i U_{ei}^2 m_{\nu_i}\right|$. By exchanging a virtual neutrino, a nucleus can emit two electrons, increasing its charge by two units [14]. This is called "neutrinoless double beta decay". Neutrinoless double beta decay attracts a lot of experimental efforts and its importance is justly emphasised in textbooks, such as e.g., MARK THOMSON's [15]. Let's emphasise that the observation of this process would have an importance as great as that of the proton decay and possibly even more, as it will test $\mathbf{B} - \mathbf{L}$.

2·3. *On the meaning of neutrino masses and their connections.* – The leptonic mixing matrix (PMNS) is strongly non diagonal, unlike the hadronic mixing matrix (CKM). The smallest leptonic mixing angle $\theta_{13}^l \sim 8.7°$ is comparable to the largest hadronic mixing angle, the CABIBBO angle $\theta_{12}^h \sim 13°$. The mass differences between neutrinos are modest compared to those between charged leptons; furthermore, we do not know the value of the lightest mass, nor the type of mass spectrum, which according to the data available today could vaguely resemble that of quarks (normal spectrum) or be very different (inverse spectrum). For an illustration, see [16].

(1) In the case of the "DIRAC mass" and in the rest system, there would be 4 states per particle type. E.g., the electron/positron system has 2 spin states and 2 charge states. Even if neutrino could also have a charge, like the lepton number **L**, this cannot be considered equally necessary, since it is not a gauge symmetry. So 2 states could suffice to describe it fully.

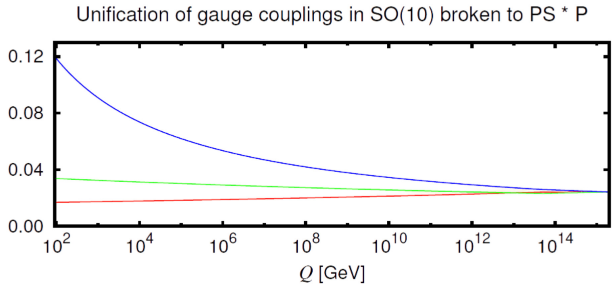

Fig. 1. – *Gauge couplings in a SO(10) model with an intermediate scale* $M = 5 \times 10^{13}$ GeV*.*

Unfortunately, the global fits of the most recent oscillation data [17, 18] have lost quality, as T2K and NOνA are not in good agreement. The χ^2 tables, obtained by Super-Kamiokande analysing atmospheric neutrinos, continue to have an important weight and are the reason for the mild indication in favor of the normal spectrum, which amounts to 2.2σ. The CP violation parameter and the departure of the angle θ^l_{23} from $\pi/4$ are poorly determined. In short, there is much room for observational progress.

Conversely, there would also be good opportunities for the theory of fermion masses, even if history suggests taking with a bit of detachment the indications (especially those of uneconomic models). As discussed above, the point of view of non-renormalisable operators has consolidated over time; the lowest dimension operator plays the very same role that the Fermi operator plays for the weak interactions and provides Majorana mass. The observed neutrino mass scale $m_\nu^{\rm exp} \sim \sqrt{\Delta m^2_{atm}} \sim 50$ meV compares well with

$$m_\nu^{\rm th} \sim \frac{M_W^2}{M} \sim 65 \text{ meV} \times \frac{10^{14} \text{ GeV}}{M} \tag{3}$$

M plausibly suggests some mass scale much larger than the one of the standard model.

This interpretation indicates a scale of violation of the lepton number $\mathbf{B} - \mathbf{L}$ much larger than the standard model one, of the kind expected from Grand Unified models. A SO(10) model broken at Pati-Salam symmetry at a similar mass scale allows successful gauge coupling unification, see Fig. 1. Very interesting features of such models are *(1)* the presence, in the 16-spinorial representation, of the 15 Weyl fields of a family + 1 right-handed neutrino, whose mass is related to the gauge breaking of an intermediate symmetry; *(2)* the possibility of predicting the decay of the proton in a complete model. In fact, the search for matter stability in lab still attracts interest:

- The research continues with Super- & Hyper-Kamiokande, JUNO and DUNE, the latter optimised for the $p \to K^+ \bar\nu$ mode, especially motivated in certain theories;
- A good news for the water Cherenkov detectors is that the background for 'vanilla' mode $p \to \pi^0 e^+$ search is small, i.e., 1.3 /(Mton yr) [19];
- Nucleon decay effective operators as a rule do not probe $\mathbf{B} - \mathbf{L}$; the (unexpected but testable) exception mentioned in table I is $n \to K^+ \ell^-$ [20].

Well defined SO(10) models give useful constraints/indications, that typically do not contradict unsophisticated expectations such those *à la* Froggatt-Nielsen [21, 22]:

1. the mass spectrum is normal;
2. the lightest neutrino mass $m_1 \lesssim \sqrt{\Delta m^2_{sol}} \sim 10$ meV;
3. $m_{\beta\beta}$ has values of a few meV.

Moreover, we can learn on cosmic baryonic unbalance. However, there are no particular indications on the nature of dark matter.

Different points of view are admissible, e.g. that defended by Shaposhnikov and collaborators whose argument (of semi-phenomenological nature) goes more or less as follows: If, in addition to standard model particles, we can dispose of right neutrinos, we can use them to explain not only the light neutrino masses, but also the origin of the baryonic excess and the presence of dark matter. This works if there is a new neutrino with a mass around 1-10 keV and two others with a mass around GeV, which are interesting for accelerator research with the SHiP experiment [23]. The coherence of the scheme is indisputable but from a theoretical point of view one gives up *a priori* to relate neutrinos to other particles; lowering the chances to understand their masses.

3. – Neutrinos in cosmology

3˙1. *Other neutrinos?*. – The standard model predicts three neutrinos, one per family. They are light and oscillate among them. If additional light neutrinos with little or no (gauge) interactions existed, by mixing with the three known ones could cause new observable manifestations. There are some hints for this possibility; a relatively recent one dates back to 1995, when the LSND experiment (originally designed to investigate other aspects) observed a certain excess of events interpretable in terms of $\bar{\nu}_\mu \to \bar{\nu}_e$. However, already in 1996, Bilenky and collaborators [24] noted that, together with this phenomenon of neutrino appearance, one expects disappearance of electronic and muon neutrinos, which is not observed. All global analyses of the data (that is, any serious consideration of the 4-neutrino hypothesis) from 2005 [25] to 2018 [26], have supported this conclusion with increasing levels of confidence. But recently, further clues have been collected, including those on gallium anomaly, the results of Neutrino-4 and the Best experiment (supporting the disappearance of $\bar{\nu}_e$), and certain results of IceCUBE 2024 (compatible with the disappearance of $\bar{\nu}_\mu$).

3˙2. *Cosmology and the number of neutrinos*. – At the same time, Gamow's big-bang nucleosynthesis theory has become increasingly precise, also thanks to the measurements of the relevant nuclear cross sections. On these bases, the LUNA collaboration [27] has argued that, at the time of nucleosynthesis, there were

$$N_{\nu,\text{eff}}^{\text{BBN}} = 2.8 \pm 0.3 \tag{4}$$

that applies to any type of neutrino, including those that have been invoked to explain the anomalies discussed just above; let us note that the observed value of the primordial ^{4}He abundance is still crucial for the inference, as remarked by Pisanti 2021 [28]. In addition to that, modeling the effects of neutrinos on the cosmic microwave background radiation and on mass distributions leads to independent limits, that currently read

$$N_{\nu,\text{eff}}^{\text{CMB}} = 3.0 \pm 0.2 \tag{5}$$

nicely consistent with the previous one. It is not impossible but not easy to reconcile these relationships with the existence of a fourth neutrino with a mass around 1 eV and significantly mixed with the others. The minimal interpretation of these results is that no new neutrinos of this type exist.

3˙3. *Cosmology and the mass of neutrinos.* – Regardless of this, it is quite exciting that, since ten years cosmological data provide valuable information also on the sum of neutrino masses $\Sigma = \sum_i m_{\nu_i}$. Again, neutrinos with masses around eV are not supported, but well beyond this, the analyses agree better with the "normal mass spectrum" case. E.g, the DESI 2025 [29] results based on the ΛCDM model imply $\Sigma < 64$ meV at 95% CL, a value quite close to the minimum allowed. Indeed, the key role of the a priori $\Sigma \geq 0$ has put some cosmologists on guard and suggested replacing the cosmological constant hypothesis with a component obeying a variable equation of state $p/\rho = w_0 + w_a(1-a)$, with an evidence of $2.5 - 3.9\,\sigma$ for $w_a < 0$. In this case, the neutrino limit weakens to $\Sigma < 163$ meV at 95% CL and the overall agreement of the data improves. We should probably accept the exploratory nature of these studies and avoid expecting too much from them now. But we should not ignore that these impressive bounds already have an important impact on laboratory expectations for neutrinos, and in particular on those relating to neutrinoless double beta decay, as pointed out, e.g., here [30].

Cosmology has led particle physics to critical situations more than once in the past; think e.g., to the 30 eV neutrino or the ones of HCDM model. The question as to how much we can rely on its results remains open. Perhaps someone might be tempted to bring to an end the discussion by invoking the maxim attributed to Landau

> Cosmologists are often wrong, but never in doubt

of which I have never been able to find the source, if there ever was one.

4. – Neutrinos in astrophysics

We illustrate a rapidly expanding field by touching upon three specific examples, concerning very promising areas of development.

4˙1. *Solar neutrinos odyssey.* – The story of neutrinos from the Sun is very interesting, but not usually remembered with sufficient completeness. We can derive a proper appreciation of the significance of these results from John Bahcall's book [31], a monument to the knowledge (see also [32]).

The Homestake experiment came about in spite of the doubts of the scientific community and of technical difficulties; astronomers and nuclear physicists believed that even if it succeeded, it would still not provide useful information. When results arrived, deviating from expectations, the community of critics increased. In fact particle physicists had meanwhile started to believe that neutrinos were massless and did not accepted the explanation proposed by Pontecorvo and co-workers [33, 34]. These circumstances delayed for 20 years the carrying out of verifications that later - thanks to (Super-)Kamiokande, Gallex/GNO, SAGE,SNO, KamLAND, etc - confirmed the results.

However, Homestake, Gallex/GNO and SAGE only probed the neutrino signals above their observation threshold; (Super-)Kamiokande and SNO measured boron neutrinos only. Great progress came thanks to the Borexino experiment [35, 36], capable to observe and isolate almost all the reactions that fuel the sun (except for the hep branch, insignificant for energy production). These results have had the effect of a game-changer, showing how much more we can learn from solar neutrinos. The impression is that little by little the majority of the scientific community has realised that Borexino has obtained epochal results, that opens up very promising directions of research for the future.

It is worth remarking that the cross section of neutrinos interacting with gallium is not so well known *a priori* and the calibrations of the experiments with gallium are creating a certain perplexity for interpretations. By contrast the elastic scattering with electrons $\nu + e \to \nu + e$ (used by Super-Kamiokande and Borexino) is known very precisely.

4.2. *Supernova neutrinos.* – The chance to observe a core collapse supernova in the Milky Way is estimated to be 1.63±0.43 supernovæ per century [37]. The typical distance of a galactic supernova is 8.5 kpc, the distance of the Galactic center. SN1987A was 50 kpc away from us.(2) The local group comprises ∼50 galaxies (most of which are small) within ∼1 Mpc from us; this should double the chance.

To have an idea of the expected number of events in detectors composed of ordinary targets (water or hydrocarbon), recall that is largely dominated by the IBD reaction, $\bar{\nu}_e + p \to e^+ + n$, very precisely known [38]. A rough but useful estimation is given by

$$\text{number of events} \sim 1 \times \left(\frac{\text{detector mass}}{50\ \text{kton}}\right) \times \left(\frac{1\ \text{Mpc}}{\text{distance}}\right)^2 \tag{6}$$

Super-Kamiokande (SK) has a mass of 50 kton and ∼ 1/2 fiducial volume; Hyper-Kamiokande will be 5 times larger. From Eq. 6, we see that in the Milky Way we expect high statistics and many opportunities for synergy with other detectors. In the local group, it depends, as the largest galaxy (Andromeda) is quite distant, 0.8 Mpc. If the supernova explodes further away, the correspondence with light is less clear; we just see the average signal of many supernova events, some kind of integrated spectrum. In principle, there are interesting intermediate cases where only a few or only one neutrino event is received, and in this case it is possible to exploit the synergy with other detectors.

There is a special opportunity for water-based CHERENKOV detectors in the case of a galactic event: they can observe the elastic neutrino scattering [39, 40]. This contributes only ∼ 3% of the events in SK and the efficiency for online detection is not high, about 30% [41], but this is enough to see the supernova with a precision of

$$\delta\theta \sim 3^\circ - 7^\circ \tag{7}$$

We will get more information, valuable for (or in conjunction with) the gravitational wave signal: we will have a temporal trigger (SNEWS) *and also a directional one.* Note that this identification will precede the light signal, i.e., the supernova of astronomers. This could be even more dramatic if the supernova "fails" to explode, a possibility remarked by NOMOTO ET AL. back in 1992 [42], that should occur in about 10% of cases.

4.3. *The super neutrino of KM3NeT.* – Perhaps the most impressive recent neutrino-related news is the detection of a near-horizontal muon event in KM3NeT, attributed to a neutrino of huge energy [43]. Its range is [0.11,0.79] EeV at 1σ and [0.07,2.6] EeV at 2σ. While it is fair to say that this was not expected, in part because of the constraints

(2) Note that the scaling law $1/r^2$ is more severe than the one $1/r$ for a gravity wave signal, as in that case what counts is the amplitude of the wave and not its intensity.

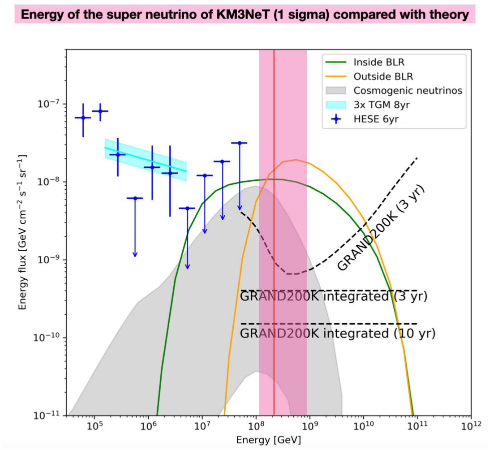

Fig. 2. – *Estimated energy of the event observed by KM3NeT compared with expectations. In grey, cosmogenic neutrinos; coloured curves, neutrinos from a class of active galactic nuclei. Adapted from [46].*

imposed by IceCUBE and Auger, in no way this contradicts current knowledge, that it is still limited by the small sample of events collected and, most of all, observationally driven. Note *en passant* that the oft-used hypothesis that cosmic neutrinos are power law distributed is not a theory, but only a cursory description.([3])

There are two major competing astrophysical models that could explain this event. The first ("cosmogenic neutrinos") was proposed by Berezinsky and Zatsepin [45]. Postulating that the ultra-high energy cosmic rays (UHECR) are mainly composed of protons, a fraction of them will interact with IR photons during their propagation in the cosmos, thereby producing neutrinos. This is somewhat challenged by current evidence, obtained by AUGER experiment, that the UHECR are composed of heavy nuclei. The other explanation instead suggests that the UHECR are produced in active galactic nuclei, and the target that allows their conversion into neutrinos are the X-rays around the central black hole [46, 47]. Few remarks are in order:

- in both cases the theoretical uncertainties are rather significant, but the neutrinos are not power law distributed;
- in the latter case the energies are larger due to the energy of the target photons and interestingly it should be possible to identify the astrophysical sources.

See Fig. 2 for a visual summary, where we overlapped to the theoretical prediction the estimated energy of the super neutrino event just observed by KM3NeT (vertical band).

It should be emphasized that now that KM3NeT is operational, we will be able to study neutrino emission much better than we have been able to do so far, for its superior angular resolution and different location, more advantageous with respect to the Galaxy.

([3]) However, once adopted it should be taken seriously: it is not reasonable to declare different power laws to analyse neutrinos of different flavors, as the oscillations correlate them tightly; if it is very steep at low energies, one needs to explain why there is no overproduction of γ rays; the prompt component should be fixed to the predicted value and not set to zero; *etc.* See [44].

5. – On the development of science - epilogue

Here are a few more quotations from LANDAU, this time with references. The first, appeared just before the observation of the neutron, concerns an outdated model [48]

> we have always protons and electrons in atomic nuclei very close together

The second (two years later) describes a calculation of e^-e^+ production, based on the assumption that negative energy states exist

> the production of electronic pairs by collisions [...] may then be described as a transition of a negative-energy electron into a positive-energy state

The last is from the famous paper on neutrinos [50], that follows LEE & YANG hypothesis of parity violation in weak interactions:

> The mass of the longitudinal neutrino [...] vanishes automatically

The possibility that neutrinos have mass is ruled out, contrary to what we know today (see [12, 51] for historical insights). These remarks are not intended to disrespect one of the greatest physicists of all time, from whose books many of us learned and whose ideas continue to guide us. However, they lead us to believe that perhaps *the condition of being often in error, but rarely in doubt, does not only concern cosmologists.* This remark is a note of caution for scientists that subtends useful epistemological considerations. It is imagination, free of constraints, that allows us to investigate new situations; only then do we refine our theories, separating what is valid and what is not by systematic investigation. But this as not an excuse to avoid trying to understand what direction we are moving in; to reason about it; to do our best and keep striving forward.

Returning to the specific points of this discussion, it is important to emphasise two practical considerations: Firstly, progress in neutrino physics requires long and demanding programmes of theoretical research, and secondly, they are based on large and difficult experiments. This requires theoretical and experimental physicists, as well as other scientists and administrators – all – to engage in sincere and profound dialogue to achieve the greatest possible chance of success with projects that are realistic, coherent and informed by science. See [52] for a similar appeal formulated at IFAE in more challenging circumstances than the current ones.

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I thank GIUSEPPE BOZZI and the Organisers for the invitation and CARLO MASCARETTI, ALEANDRO NISATI, NICOLA ROSSI and PIERA SAPIENZA for very precious discussions. With partial support of the PRIN 2022 grant 2022E2J4RK *(PANTHEON),* funded by the Italian Ministero dell'Università e Ricerca (MUR) & European Union – Next Generation EU.

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