Thermal Noise in Advanced Gravitational Wave Detectors Workshop

Material Loss Angles from TNI

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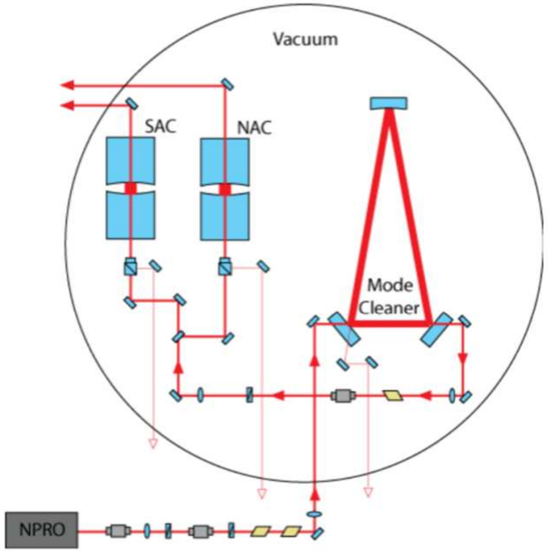
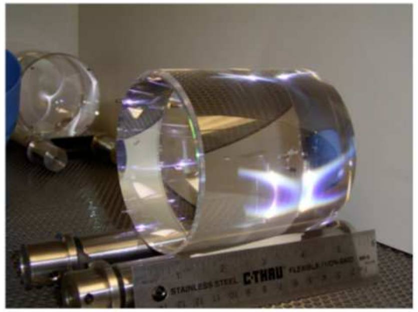
TNI Measurements

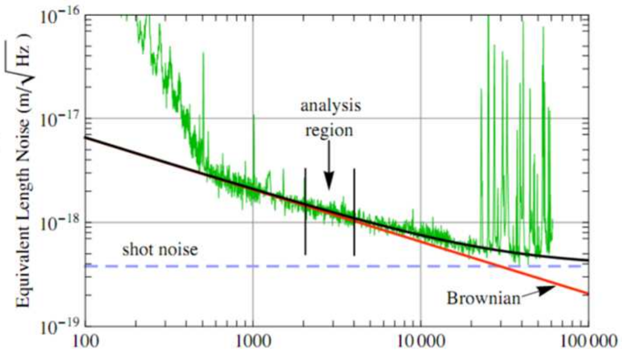




fit data to model

[G. Harry et al., CQG 19 (2002) 897]

Spectrum Analyzer → calibration →

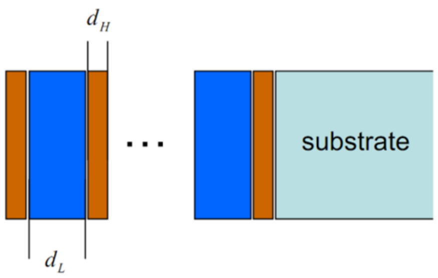


$$S_B(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \phi_c$$

→ ϕ_c

TNI Prototype Coatings Tested

Coating	Type	Low Index	High Index	Mftr.	$\phi_c \times 10^6$
1	QWL	Silica	Tantala	REO	8.25 ± 0.3
2	Optimized	Silica	Tantala	LMA	6.85 ± 0.2
3	QWL	Silica	Doped Tantala	LMA	6.0 ± 0.25
4	Dichroic	Silica	Doped Tantala	LMA	5.5 ± 0.5



$$\phi_C = A\phi_{SiO_2} + B\phi_{Ta_2O_5}$$

$$A = b_{SiO_2} d_{SiO_2}$$

$$B = b_{Ta_2O_5} d_{Ta_2O_5}$$

$$b_X = \frac{1}{\sqrt{\pi}}\left(\frac{Y_X}{Y_S} + \frac{Y_S}{Y_X}\right)$$

Rationale

Measurements on 2 coatings using the same materials yield 2 equations in 2 unknowns (the loss angles)

$$\phi_C^{(QWL)} = A\phi_{SiO_2} + B\phi_{Ta_2O_5}$$

$$\phi_C^{(OPT)} = C\phi_{SiO_2} + D\phi_{Ta_2O_5}$$

Measurement uncertainties can be assumed as Gaussian distributed (consistent with fitting residual distribution).
This yields, in turn a bi-variate Gaussian distribution for the material loss angles, via std. linear algebra...

Matrix Notation

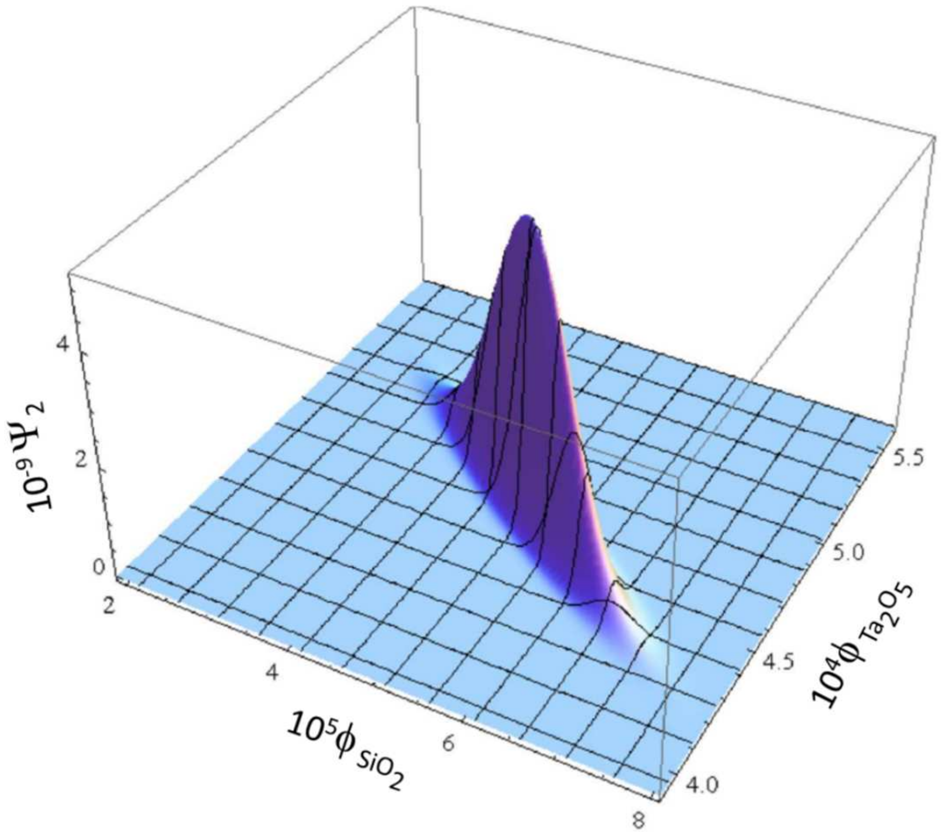
$$\mathrm{M}\cdot\boldsymbol{\phi}=\boldsymbol{\Phi_c}$$, with

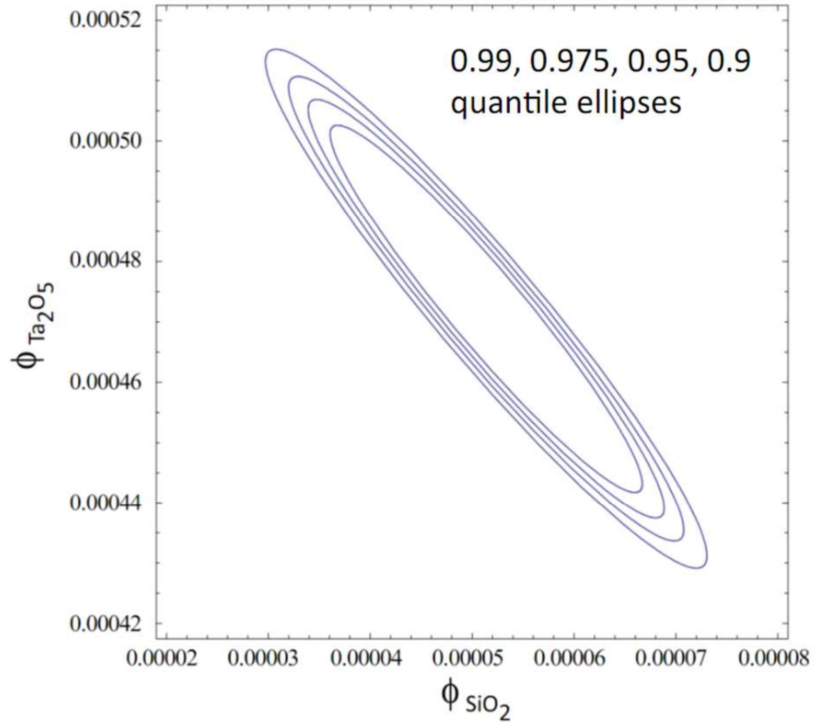
$$\begin{cases}\boldsymbol{\Phi_c}=\left\{\Phi_c^{(QWL)},\Phi_c^{(OPT)}\right\}\\ \boldsymbol{\phi}=\left\{\phi_{SiO_2},\phi_{Ta_2O_5}\right\}\\ \mathrm{M}=\begin{bmatrix}b_{SiO_2}d_{SiO_2}^{(QWL)} & b_{Ta_2O_5}d_{Ta_2O_5}^{(QWL)}\\ b_{SiO_2}d_{SiO_2}^{(OPT)} & b_{Ta_2O_5}d_{Ta_2O_5}^{(OPT)}\end{bmatrix}\end{cases}$$

⇓

$$E(\boldsymbol{\phi})=\mathrm{M}^{-1}E(\boldsymbol{\Phi_c}),\quad cov(\boldsymbol{\phi})=\mathrm{M}^{-1}\cdot\begin{bmatrix}\sigma^2_{\Phi_c^{(QWL)}} & 0\\ 0 & \sigma^2_{\Phi_c^{(OPT)}}\end{bmatrix}\cdot\left[\mathrm{M}^{-1}\right]^T$$

Silica/Plain-Tantala Coatings

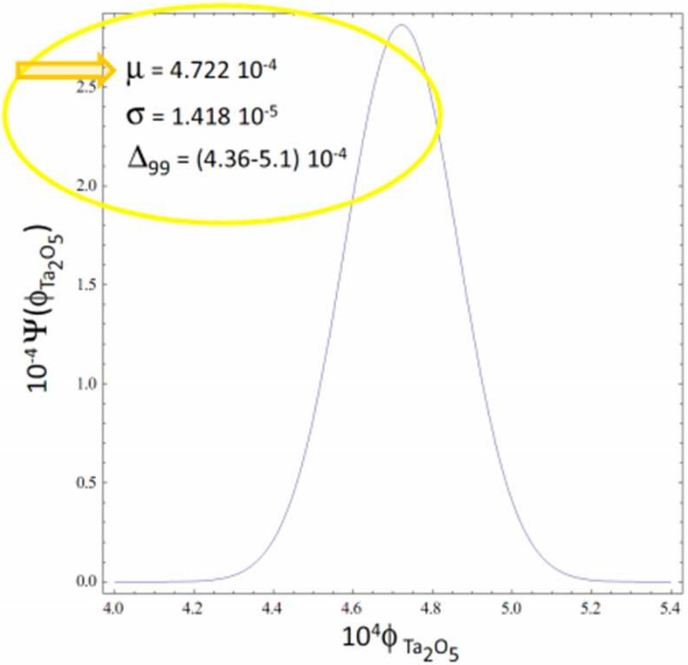


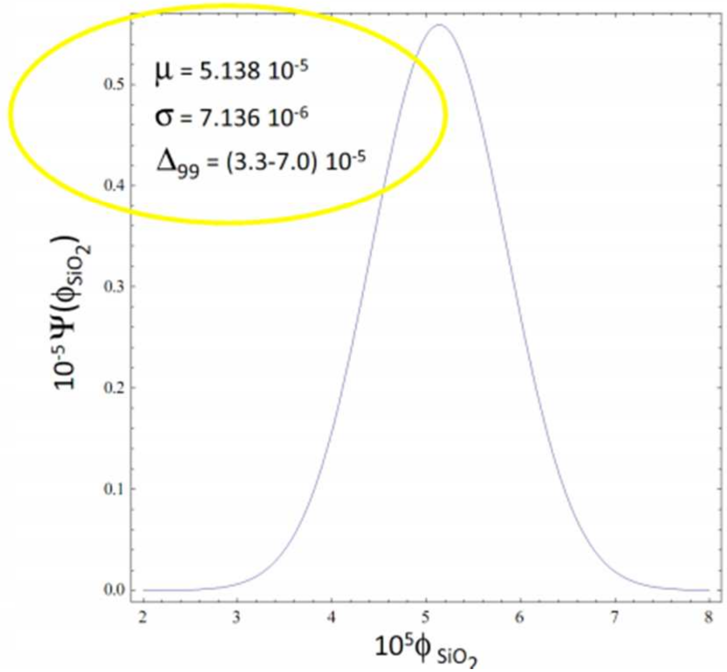


Silica/Plain-Tantala Coatings, contd.

The *marginal* distributions (which take into account uncertainties in *both* numbers) of Silica and plain Tantala loss angles are easily computed

$$\Psi(\phi_{SiO_2}) = \int_{-\infty}^{\infty} d\phi_{Ta_2O_5} \Psi_2(\phi_{SiO_2}, \phi_{Ta_2O_5}), \quad \Psi(\phi_{Ta_2O_5}) = \int_{-\infty}^{\infty} d\phi_{SiO_2} \Psi_2(\phi_{SiO_2}, \phi_{Ta_2O_5})$$





Silica/Doped-Tantala Coatings

Doped Tantala is harder because the system is *ill-conditioned*, or

$$\det M \approx 0$$

However, the low-index layers here are made from fused silica, i.e., the same material used in the undoped coatings. This means we can use the previous value for the fused-silica loss,

$$\phi_{SiO_2} = (5.138 \pm 0.714) \times 10^{-5}$$

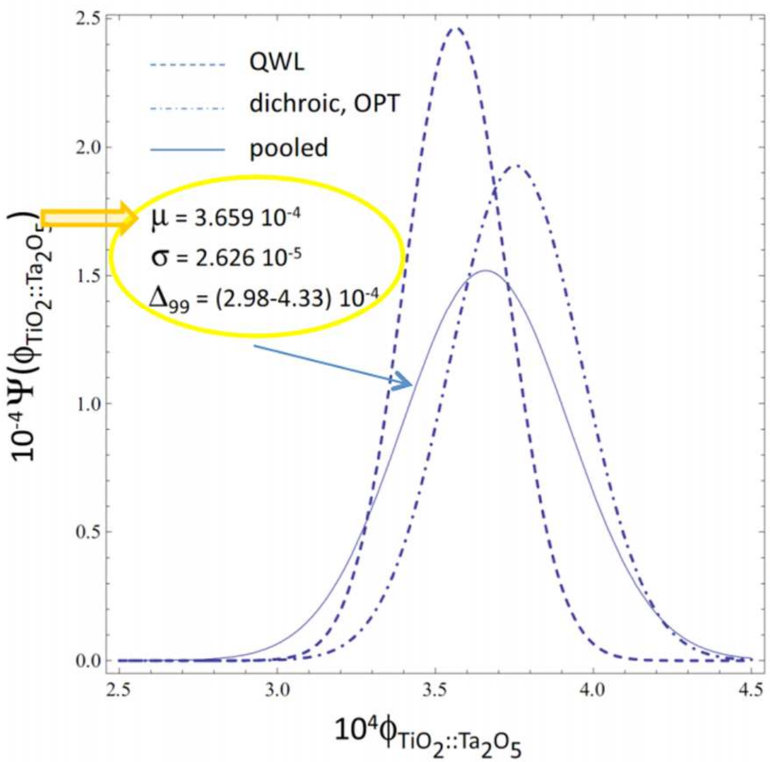
leaving us with only one unknown. We still have two equations, and we can use this previous value for the silica loss to get two (Gaussian) distributions for the doped-tantala loss. We expect them to agree, and they do, yielding

$$\phi_{Ta_2O_5:TiO_2} = (3.56 \pm 0.16) \times 10^{-4}$$

$$\phi_{Ta_2O_5:TiO_2} = (3.75 \pm 0.21) \times 10^{-4}$$

Silica/Doped-Tantala Coatings, contd.

We may pool (combine) the two distributions of the doped-Tantala loss. Average and uncertainties propagate in the usual way.



$$\langle \phi_{TiO_2::Ta_2O_5} \rangle = \frac{\langle \phi_{TiO_2::Ta_2O_5} \rangle_1 + \langle \phi_{TiO_2::Ta_2O_5} \rangle_2}{2}$$

$$\sigma_{\phi_{TiO_2::Ta_2O_5}} = \left(\sigma^2_{\phi_{TiO_2::Ta_2O_5},1} + \sigma^2_{\phi_{TiO_2::Ta_2O_5},2} \right)^{1/2}$$

$$\phi_{Ta_2O_5:TiO_2} = (3.66 \pm 0.26) \times 10^{-4}$$

Conclusions

1. First extraction of individual-material loss angles from direct observations of thermal noise.
2. Simple predictive model for composite material properties including elastic moduli, loss angle.
3. Loss angles derived from direct noise measurements not entirely in agreement with Qs.
4. Loss-angle ratios between doped and undoped are consistent.

5. Using *all* (4) prototype measurements as inputs of a single inverse problem, to retrieve the loss angles of Silica, Tantala and doped Tantala yields *consistent* results

Doped Tantala: TNI vs Mixture Approach

We can compare the doped Tantala loss angle distribution obtained from TNI measurements to the prediction of EMT, using Scott-MacCrone loss angle for Titania, and the TNI result for plain Tantala

TNI : distribution deduced from doped coating measurement, using the marginal distribution of Silica loss angle from the undoped coating measurements.

Bruggeman-Barta : distribution deduced using Scott-MacCrone value for Titania loss angle, with plain Tantala loss-angle distribution from un-doped coating measurements.