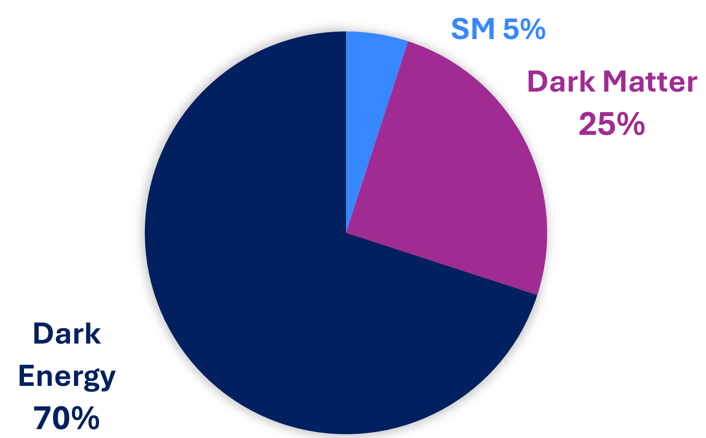
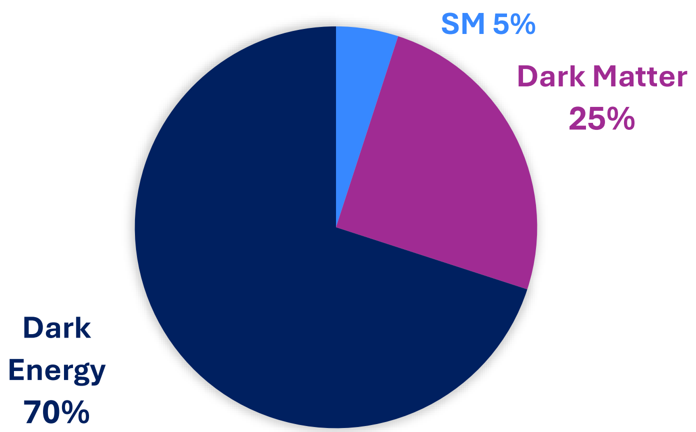
Topological Portal to the Dark Sector

Joe Davighi, CERN

14th March 2025, La Thuile



$$L \stackrel{?}{=} L_{\mathrm{SM}} + \sum \mathcal{O}_{\mathrm{SM}} \mathcal{O}_{\mathrm{DM}} + L_{\mathrm{DM}}$$

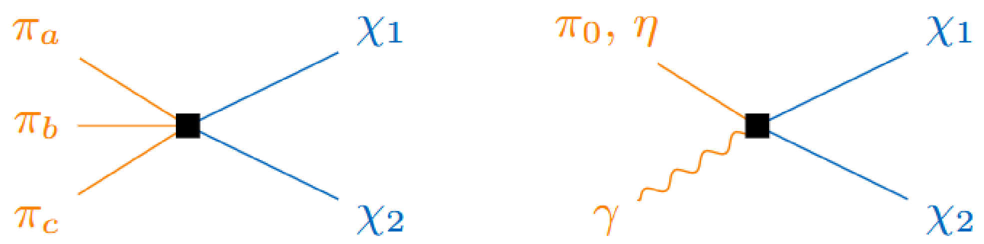


$$L \stackrel{?}{=} L_{\mathrm{SM}} + \sum \mathcal{O}_{\mathrm{SM}} \mathcal{O}_{\mathrm{DM}} + L_{\mathrm{DM}}$$

E.g. Renormalizable Portals:

- Vector ~ $B^{\mu\nu} X_{\mu\nu}$
- Higgs ~ $|H|^2 (\#\Lambda S + \# S^2)$
- Neutrino ~ $\bar{L} H N$

We propose a new non-renormalizable portal that is **topological** – which elegantly delivers some key features of GeV thermal DM, and has distinctive phenomenology at colliders



Outline

1. What are Topological Interactions? Why are they special?
2. Topological Portal: QCD $\overset{S_{\text{top}}}{\longleftrightarrow}$ Dark Sector (DM = light scalars)
3. Phenomenology
4. Generalized symmetries: from IR to UV

1. Topological Interactions

Based on **Davighi**, Gripaios, 1803.07585; **Davighi**, Gripaios, Randal-Williams, 2011.05768

Topology

Topology measures **robust** features that don’t change under **smooth deformations**

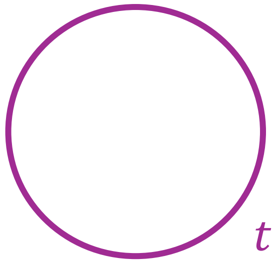
Ex 1:

$N_{\text{holes}} = 1$ $N_{\text{holes}} = 3$

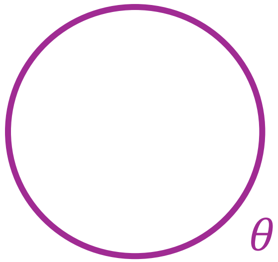
Topology

Topology measures **robust** features that don't change under **smooth deformations**

Ex 2:



$$\theta(t): S^1_t \to S^1_\theta : \theta(2\pi) = \theta(0)$$



Topological: $\frac{1}{2\pi}\int d\theta = W \in \mathbb{Z}$ S_{wind} (Aharonov–Bohm)

Not topological: $\int dt\, \frac{1}{2}\left(\frac{d\theta}{dt}\right)^2$ S_{kinetic}

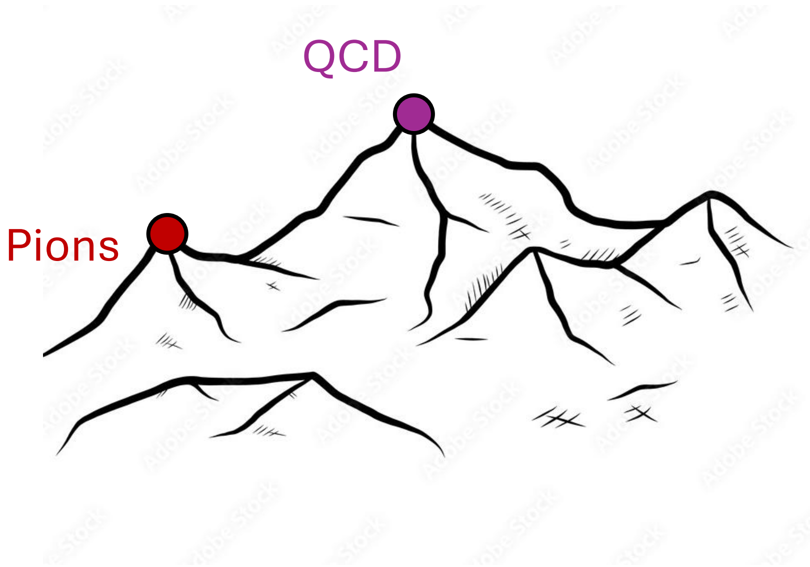
Topology and QFT

QFTs *run*

- RG flow $\frac{dg_i}{d\mu} \neq 0$ is a smooth deformation of QFT

Topological quantities $n \in \mathbb{Z}$ cannot run!

- Sector $\{n_i\}$ of robust "*non-renormalized*" information



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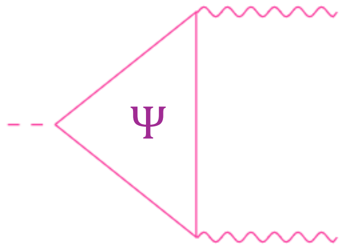
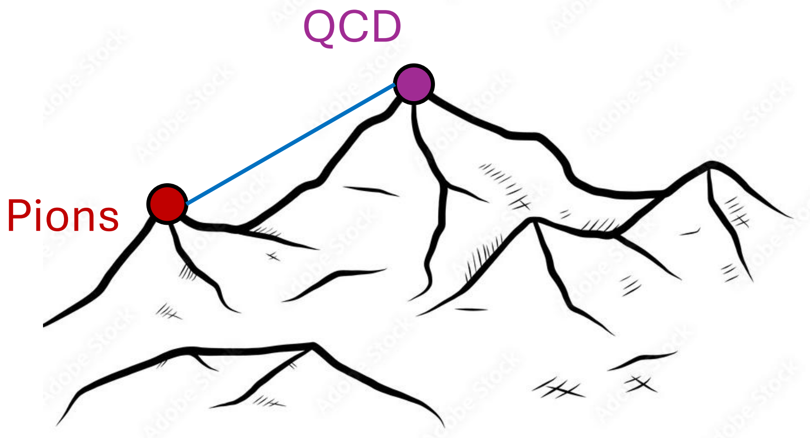
- Sector $\{n_i\}$ of robust "*non-renormalized*" information
- Classic example: **anomaly matching**
- Provide *bridges* from UV to IR
    - ✓ Even across strong coupling transitions

$Z \to e^{i n_c \int \alpha F F} Z$

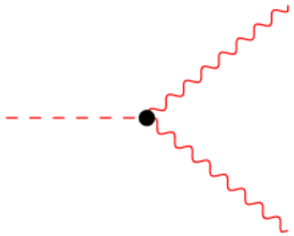
$n_c \in \mathbb{Z}$ because related to a topological index

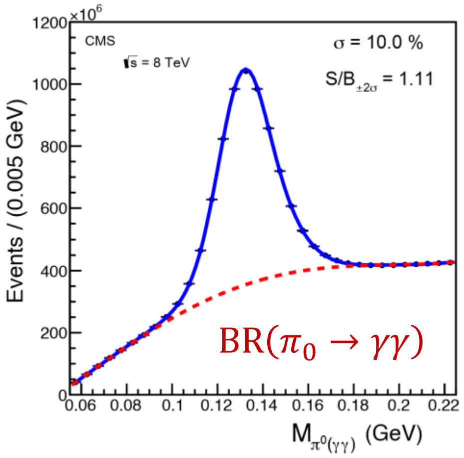




Topological term:

$S \supset \frac{1}{(4\pi)^2} n_c \int \pi_0 F F$





Topological Interactions

- $\frac{B}{2\pi}\int d\theta, \quad n_c\int \pi_0 F\wedge F$ both obtained by directly integrating a **differential form**

Recall: A differential k-form on manifold X is a totally antisymmetric tensor of type $(0,k)$
Can expand in coordinate basis 1-forms dx^μ, as $A = \frac{1}{k!}A_{[\mu_1\ldots\mu_d]}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_d}$

Hence e.g. field strength $F = F_{\mu\nu}dx^\mu\wedge dx^\nu$ defines a 2-form

- These terms are "topological" because they can be written **without a metric**

[Contrast with e.g. kinetic term $S = \int \frac{1}{2}\eta^{\mu\nu}g_{ij}\big(\pi(x)\big)\partial_\mu\pi^i\partial_\nu\pi^j$, or any $\int\sqrt{\det\eta}\, f\big(\pi(x)\big)$]

See geometry talks by T Cohen, T Corbett, M Alminawi

Example: Wess-Zumino-Witten term in QCD

Wess, Zumino Phys. Lett. 1971; Witten Nucl. Phys. B 1983

Low-energy QCD is characterized by **chiral symmetry breaking**:

- $\langle \overline{\psi}_i \psi_j \rangle \sim \Lambda_{\text{QCD}}^3 \delta_{ij} : SU(3)_L \times SU(3)_R \to SU(3)_{L+R}$
- Goldstone theorem $\Longrightarrow$ 8 QCD pions $\{\pi^0, \pi^\pm, K^0, \overline{K}^0, K^\pm, \eta\}$, weakly-interacting

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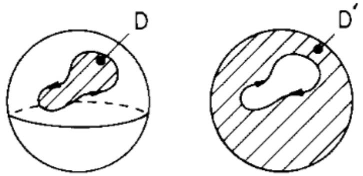
The effective action contains all $SU(3)_L \times SU(3)_R$-invariant interactions

This includes a single topological interaction:



$$S_{\text{WZW}} = n \int_{D_5 \subset SU(3)} \frac{-i}{480\pi^3} \text{Tr}\,(g^{-1} dg)^5, \qquad g \in SU(3), \quad \partial D_5 = \text{Im(Spacetime)}$$

EFT consistency $\Rightarrow$ $n \in \mathbb{Z}$

Here $\frac{-i}{480\pi^3} \text{Tr}\,(g^{-1} dg)^5$ is the (unique) closed, $SU(3)_L \times SU(3)_R$-invariant, 5-form

Anomaly matching $\Rightarrow n = n_c$

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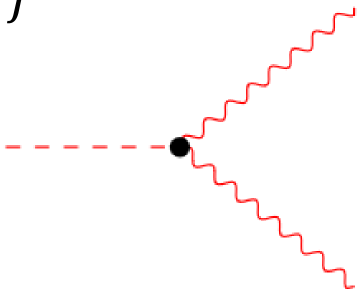
$$S_{\mathrm{WZW}} \sim n \int_{B \subset SU(3)} \mathrm{Tr}\,(g^{-1} dg)^5$$

Expanding locally $g = 1 + \frac{2i}{f_\pi} t_a \pi_a(x) + \cdots$, and gauging QED, can write a local Lagrangian

$$L \sim \frac{1}{f^5} \epsilon^{\mu\nu\rho\sigma} \pi_0 \partial_\mu \pi^+ \partial_\nu \pi^- \partial_\rho K^+ \partial_\sigma K^- + \cdots + \frac{1}{f^3} \epsilon^{\mu\nu\rho\sigma} \pi_0 \partial_\mu \pi^+ \partial_\nu \pi^- F_{\rho\sigma} + \frac{1}{f} \epsilon^{\mu\nu\rho\sigma} \pi_0 F_{\mu\nu} F_{\rho\sigma}$$

n.b. total antisymmetry



2. Topological Portal to Dark Sector

Davighi, Greljo, Selimović 2401.09528

Dark Sectors with Global Symmetry Breaking

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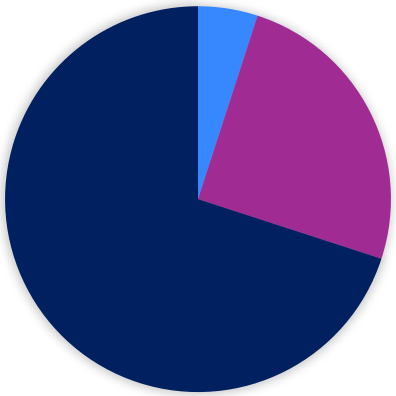
The **dark sector** could also be characterized by a **global symmetry breaking**

- Many QFTs end up this way at low-energy
- e.g. a confining dark QCD-like sector; e.g. a dark Higgs sector

$$G_D \to H_D$$

$\Longrightarrow$ Dark Pions on G_D/H_D



$$L_{\chi\mathrm{PT}} + \sum \mathcal{O}_{\mathrm{SM}}\mathcal{O}_{\mathrm{DM}} + L_{\mathrm{D}\chi\mathrm{PT}}$$

??

Topological Portal to the Dark Sector

Dark sector on G_D/H_D can talk to QCD pions via a **topological interaction**:

Davighi, Greljo, Selimović 2401.09528

$$S_{\text{portal}} \sim n \int_{B_5} \text{Tr}\,(g^{-1}dg)^3 \wedge \Omega_2$$

c.f. $S_{\text{WZW}} \sim \int_{B_5} \text{Tr}\,(g^{-1}dg)^5$

The **only other** invariant form made out of QCD pions!

... so, to make a second topological term we need a **closed, $\boldsymbol{G_D}$-invariant 2-form on $\boldsymbol{G_D/H_D}$**

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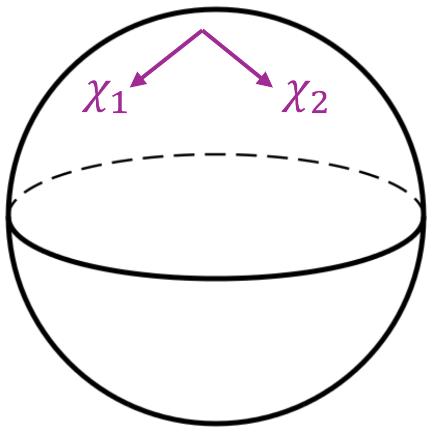
➢ Example (almost unique): $\dfrac{G_D}{H_D} = \dfrac{SU(2)}{SO(2)} = \boldsymbol{S^2}$



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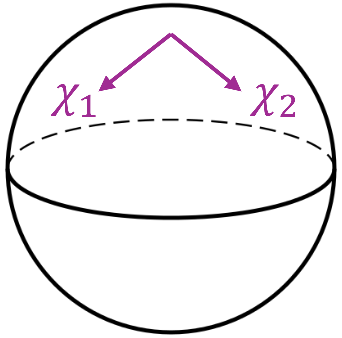

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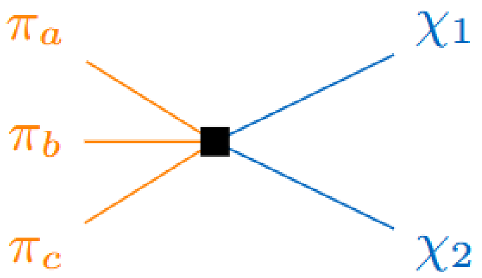
$$S_{\text{portal}} = 2\pi n \int_{X_5} \frac{1}{24\pi^2} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \frac{d\chi_1 \wedge d\chi_2}{4\pi f_D^2}$$

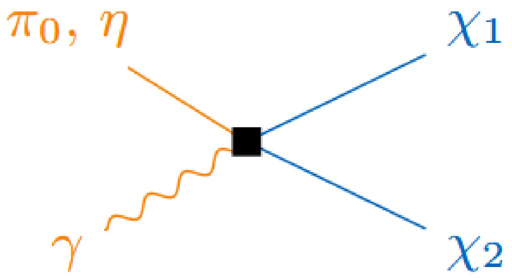
$$= n \int_{M_4} \frac{\epsilon^{\mu\nu\rho\sigma}}{48\pi^2} \left[\frac{1}{f^3} f_{abc}\pi^a \partial_\mu \pi^b \partial_\nu \pi^c + \frac{3e}{f}\left(\pi_0 + \frac{\eta}{\sqrt{3}}\right) F_{\mu\nu} \right] \frac{\partial_\rho \chi_1 \partial_\sigma \chi_2}{f_D^2}$$

Integer! (EFT consistency)

Term from gauging QED

Topological portal requires at least x2 dark pions





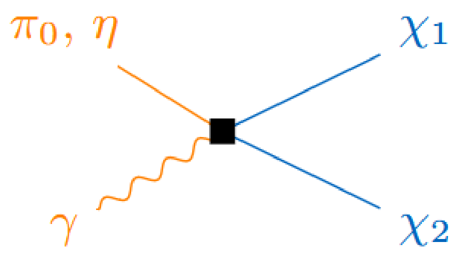
3. Phenomenology

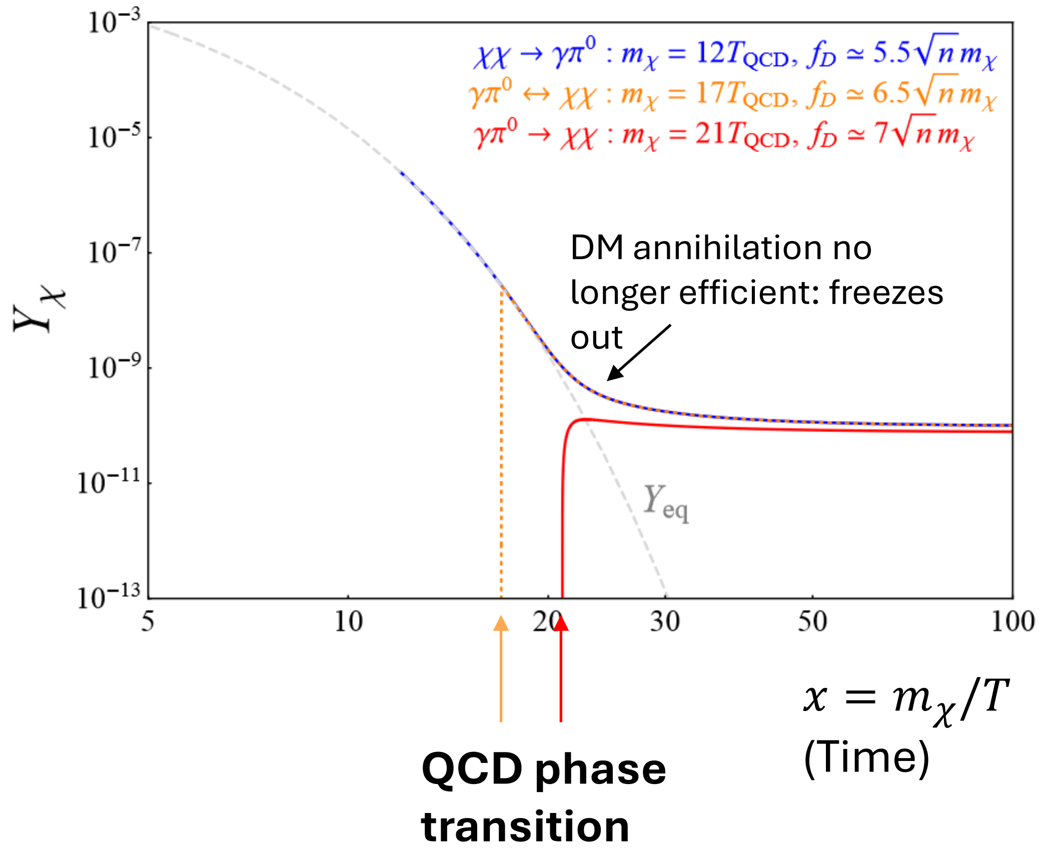
Davighi, Greljo, Selimović 2401.09528 + work in progress

Thermal History & Relic Abundance

Freeze out via $\pi^0\gamma \leftrightarrow \chi_1\chi_2$ after QCD phase transition

- Numerically, need $\frac{m_\chi}{T_{\rm QCD}} \lesssim 23 \Longrightarrow m_\chi \lesssim 3.7\ \mathrm{GeV}$
- Leading order parametric dependence: $\Omega_{\rm DM} \sim \frac{f_D^4}{m_\chi^4}$





Mass splitting (explicit G_D-breaking): $m_{\chi_2} - m_{\chi_1} > m_{\pi^0}$

- $\chi_2 \to \chi_1\gamma\pi^0$ decays shortly after freeze-out
- Leaves χ_1 **as relic DM**

Davighi, Greljo, Selimović [2401.09528]

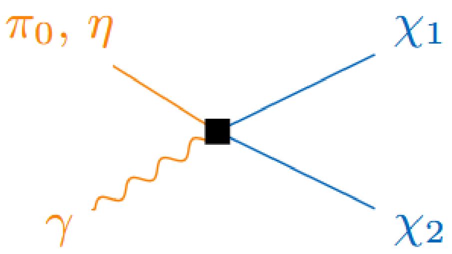
Direct and Indirect Detection? No!

Davighi, Greljo, Selimović 2401.09528

Because the interaction is topological i.e. a **differential form**, it is **perfectly antisymmetric**:

- Total absence of "diagonal" interactions $\chi_1\chi_1 \to \mathrm{SM}$ at leading order
- Natural realisation of **inelastic thermal DM** at **GeV scale**

c.f. Tucker-Smith, Weiner, hep-ph/0101138



$m_1 \approx \text{few GeV}$, the relic DM

$m_2 \approx m_1 + O(100\text{s MeV})$, negligible today

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Davighi, Greljo, Selimović 2401.09528

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Topology can naturally explain why we haven’t seen GeV DM in direct/indirect detection experiments

So how can we test this mechanism?

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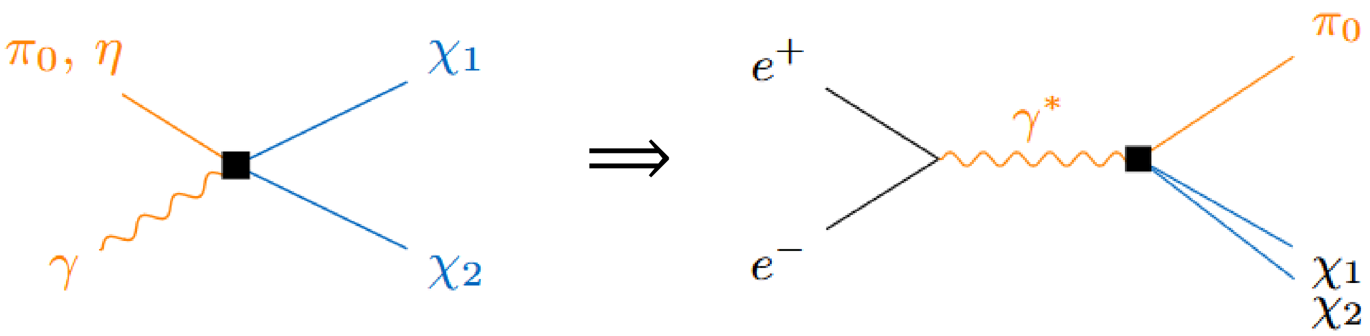

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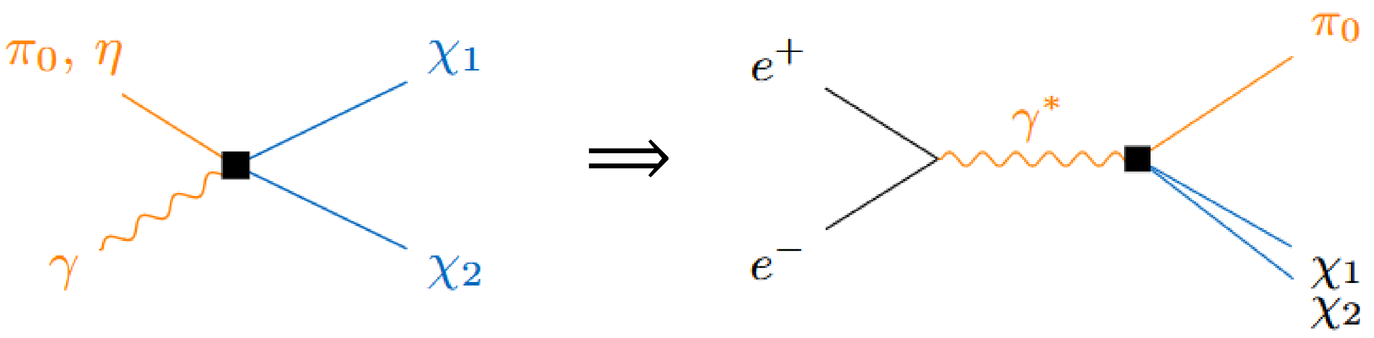

$m_\chi \sim 3$ GeV

Wishlist:

- Clean environment, hermetic (for missing energy)
- Centre of mass energy $\sqrt{s} \sim 10$ GeV to produce $\chi_1 + \chi_2$ w boosted π^0
- High Luminosity for statistics

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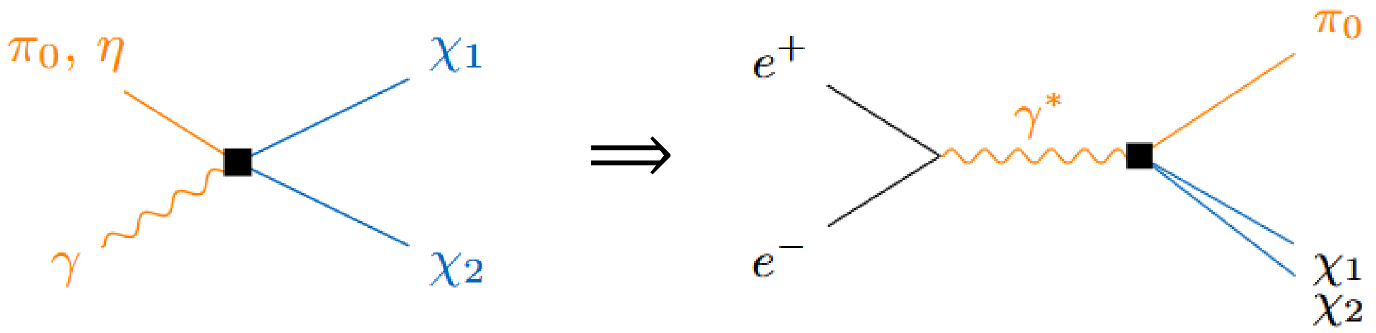

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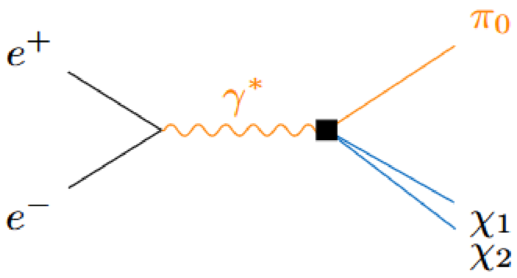
= Belle II

Dark sectors @ Belle II, see talks by F. Trantou, M. Campajola

New Search Strategies

Davighi, Greljo, Selimović 2401.09528



Two regimes (if χ_2 decay prompt, difficult to explain DM):

Δm_χ	$\lesssim 1.7 m_{\pi^0}$	$\gtrsim 1.7 m_{\pi^0}$
Signature	$\pi^0 + \not{E}_T$	$\pi^0 + \not{E}_T + \mathrm{DV}(\pi^0 \gamma \not{E}_T)$

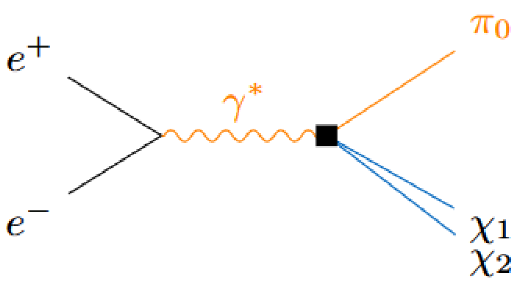
1. Two new signatures to search for! *(work begun with Christopher Hearty and Guorui Lui)*

2. Definite prediction for corresponding channels with η meson instead of π^0!

$$\left[\; L \sim \left(\pi_0 + \frac{\eta}{\sqrt{3}}\right) F_{\mu\nu} \partial_\rho \chi_1 \partial_\sigma \chi_2 \;\right]$$

New Search Strategies

Davighi, Greljo, Selimović 2401.09528



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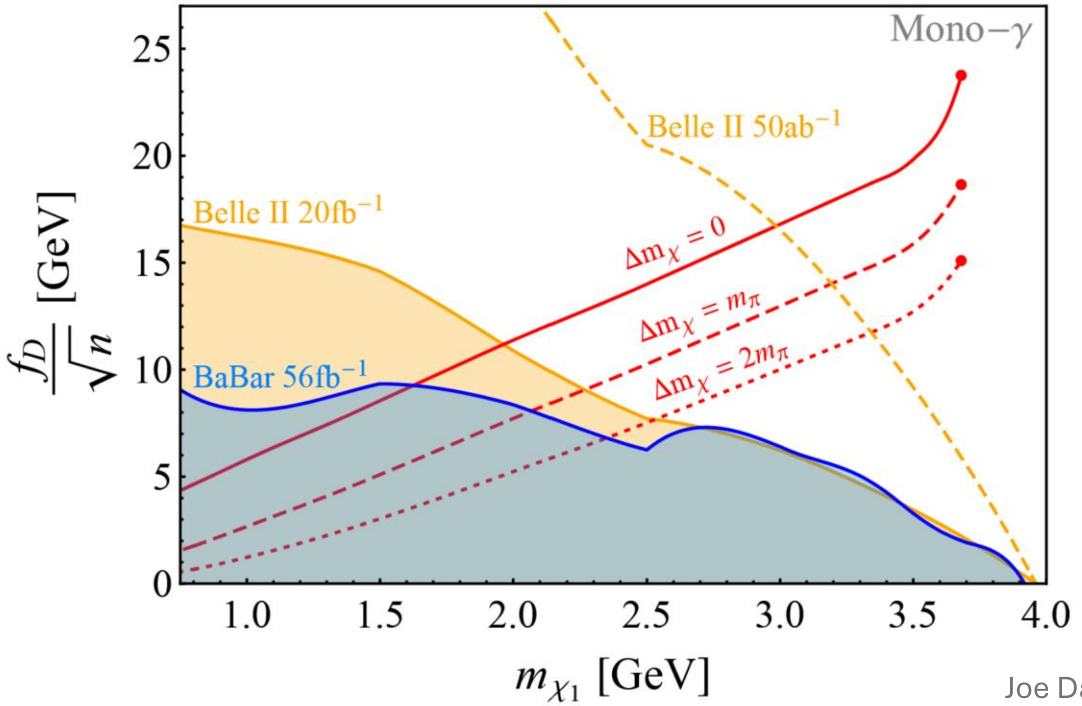
For now: boosted π^0 reconstructed as photon; can recast γ + Inv searches for **signature 1**

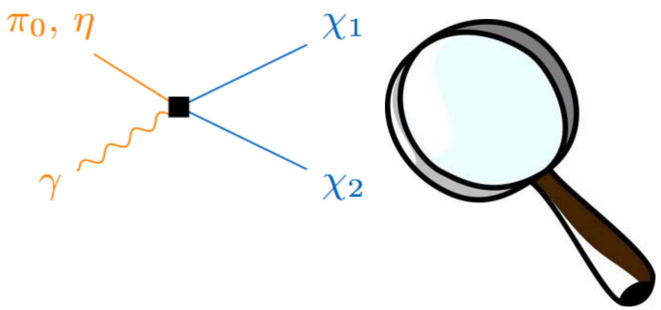


Monophoton recast demonstrates **tremendous prospects at Belle II**

(No data relevant to **signature 2** – DV veto-ed in these mono-photon searches)

Higher Energies?

- Expect further correlated signatures e.g. at beam dump (NA64), and LHC (ATLAS, CMS)
- But **to make predictions**, we need to go above our pion EFT and **find a UV completion**...



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4. Generalized Symmetries: from IR to UV

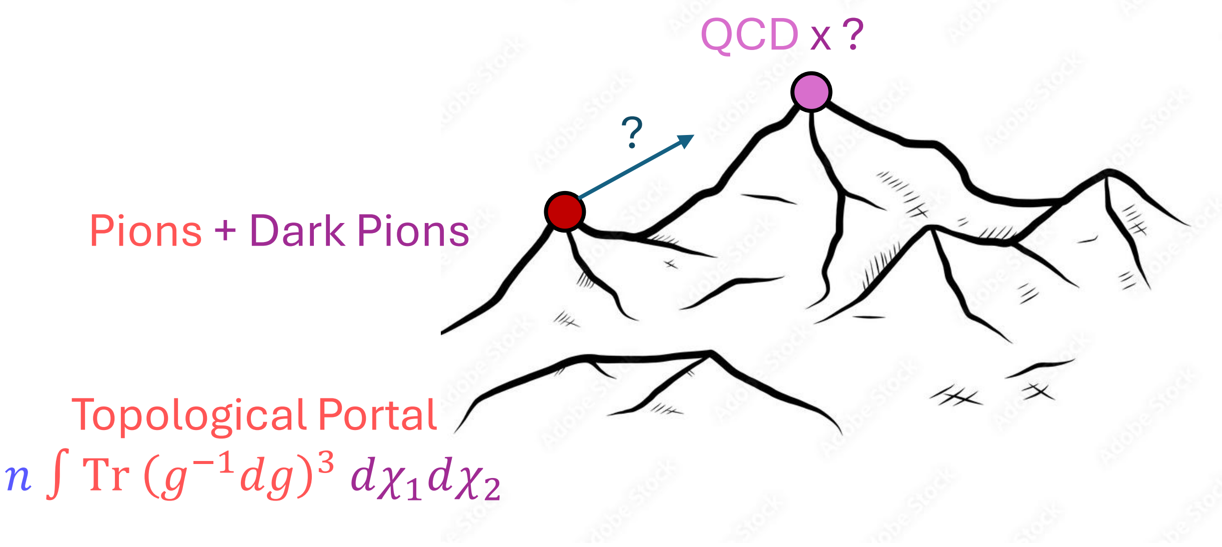
Davighi, Lohitsiri, 2407.20340

WZW terms without anomalies

The topological portal presents a QFT conundrum:

It is an **integer-quantized** WZW term

- BUT, unlike usual WZW, there is no anomaly it matches...
- No known UV completion!



Resolution: **Davighi**, Lohitsiri, 2407.20340

The topological portal **matches** a **generalized symmetry**, not an anomaly!

WZW terms for generalized symmetries!

Davighi, Lohitsiri, 2407.20340

'Ordinary' symmetry: $\partial_\mu j_a^\mu(x) j_b^\nu(y) = i f_{abc} \delta(x-y) j_c^\nu(y)$

$j_{L,a}^\mu = \bar{q}_L \gamma^\mu t_a q_L,\ j_{R,a}^\mu = \cdots$ UV

$j_{L,a}^\mu = -\frac{f_\pi}{2} \partial_\mu \pi^a, \ldots$ IR

WZW terms for generalized symmetries!

Davighi, Lohitsiri, 2407.20340

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$j_{L,a}^\mu = \bar{q}_L \gamma^\mu t_a q_L,\ j_{R,a}^\mu = \cdots$ UV

$j_{L,a}^\mu = -\frac{f_\pi}{2} \partial_\mu \pi^a, \ldots$ IR

Higher "p-form" symmetries are associated with conserved **p+1 forms**, $d \star j^{(p+1)} = 0$

Equivalently, currents have p+1 indices, & totally antisymmetric components

1-form symmetry: $\partial_\mu J^{[\mu\nu]} = 0$

$J^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \partial_\rho \chi_1 \partial_\sigma \chi_2$ IR. UV?

(i.e. $\star$ of volume form on S^2)

WZW terms for generalized symmetries!

Davighi, Lohitsiri, 2407.20340

Flavour symm: $j^{\mu}_{L,a} = -\frac{f_\pi}{2}\partial_\mu \pi^a, \ldots$

1-form symm: $J^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\partial_\rho \chi_1 \partial_\sigma \chi_2$

Couple to background gauge fields, and take variation:

$$S_{\text{portal}}[A_L, A_R] \supset \int \frac{-n}{8\pi^2}[\mathrm{CS}_3(A_L) - \mathrm{CS}(A_R)] \wedge \mathrm{Vol}_{S^2} + \mathrm{Tr} \star j_{L,a} \wedge A_L + (L \to R)$$

$$\boxed{i\partial_\mu j^{\mu}_{L,a}(x) j^{\nu}_{L,b}(y) + f_{abc}\delta(x-y) j^{\nu}_{L,c}(y) = n\frac{1}{8\pi^2}\delta_{ab}\partial_\rho \delta(x-y) J^{\rho\nu}(y)}$$

The topological portal twists the usual QCD current algebra via the dark sector higher-form current, to form a **2-group generalized symmetry**!

Cordova, Dumitrescu, Intriligator 1802.04790; Benini, Cordova, Hsin, 1803.09336
Hsin, Lam, 2007.05915; Lee, Ohmori, Tachikawa, 2108.05369

WZW terms for generalized symmetries!

Davighi, Lohitsiri, 2407.20340

$$i\partial_\mu j^\mu_{L,a}(x) j^\nu_{L,b}(y) + f_{abc}\delta(x-y) j^\nu_{L,c}(y) = n\frac{1}{8\pi^2}\,\delta_{ab}\partial_\rho\delta(x-y)J^{\rho\nu}(y)$$

2-group class $n \in \mathbb{Z}$ is topological ∴ cannot change under RG: matches IR to UV!

Matching this rich symmetry structure guides us to a UV completion:

- Rules out QCD-like dark sector completion! b/c no 1-form current $J^{\rho\nu}$ in non-abelian gauge theory

No go!

WZW terms for generalized symmetries!

Davighi, Lohitsiri, 2407.20340

$$\boxed{i\partial_\mu j^\mu_{L,a}(x) j^\nu_{L,b}(y) + f_{abc}\delta(x-y) j^\nu_{L,c}(y) = n\frac{1}{8\pi^2}\,\delta_{ab}\partial_\rho\delta(x-y) J^{\rho\nu}(y)}$$

2-group class $n \in \mathbb{Z}$ is topological ∴ cannot change under RG: matches IR to UV!

Matching this rich symmetry structure guides us to a UV completion:

- The 1-form symmetry *can* be matched by an *abelian* gauge field

$$J^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\partial_\rho\chi_1\partial_\sigma\chi_2 \qquad \text{IR}$$

$$J^{\mu\nu} = f^{\mu\nu} \qquad \text{UV?}$$

To get the 2-group structure in the UV, f must have a certain mixed anomaly with QCD flavour symmetry $j^\mu_{L,a} = \bar{q}_L\gamma^\mu t_a q_L$…

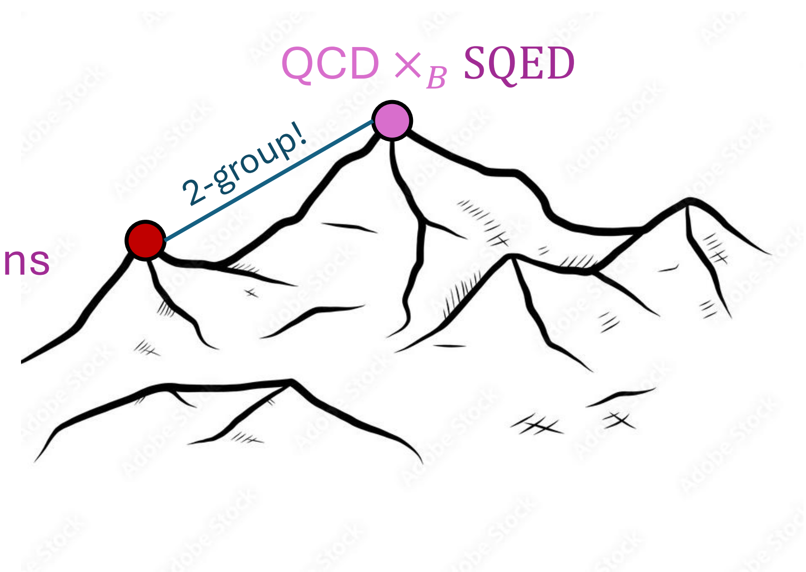
UV completing the topological portal

Davighi, Lohitsiri, 2407.20340

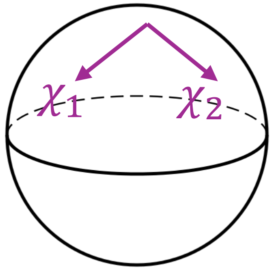
2 group: $i\partial_\mu j^\mu_{L,a}(x) j^\nu_{L,b}(y) + f_{abc}\delta(x-y) j^\nu_{L,c}(y) = n\frac{1}{8\pi^2}\,\delta_{ab}\partial_\rho\delta(x-y)J^{\rho\nu}(y)$

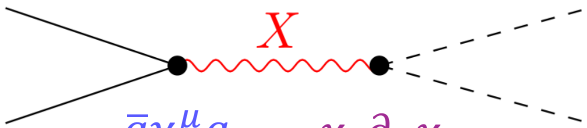
$$L = L_{\rm SM} + \bar{q}X_\mu\gamma^\mu q + \frac{1}{2}\left((\partial_\mu - X_\mu)\phi_i\right)^2 + V(|\phi_i|^2),\ \phi_i \in \mathbb{C}^2$$

Pions + Dark Pions



$\langle\phi\rangle \neq 0 \downarrow$ $\quad \to \chi_{1,2}$ on S^2





$\bar{q}\gamma^\mu q \qquad \chi_1\partial_\mu\chi_2$

$\langle\bar{q}q\rangle \sim \Lambda^3_{\rm QCD} \downarrow$

$$n\int \mathrm{Tr}\,(g^{-1}dg)^3\ \mathrm{Vol}_2$$

***EFT matching onto this operator is tree-level exact!**

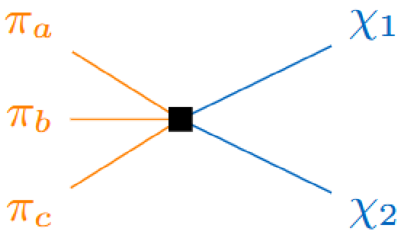
From here we can build a UV completion of the topological portal to dark matter, needed to extend our predictions to high energy:

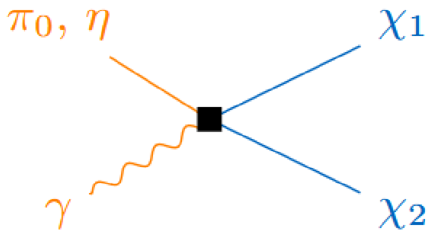
- LHC signatures, constraints — *future work w Greljo, Selimović*
- Opens up option of freeze-out at higher energy — *future work*

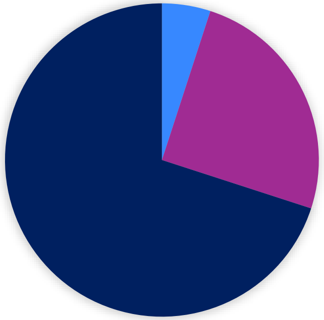
Summary

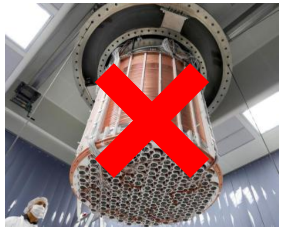
Topological Interactions $\Longrightarrow$

$$S_{\text{portal}} \sim n \int_{B_5} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \Omega_2$$



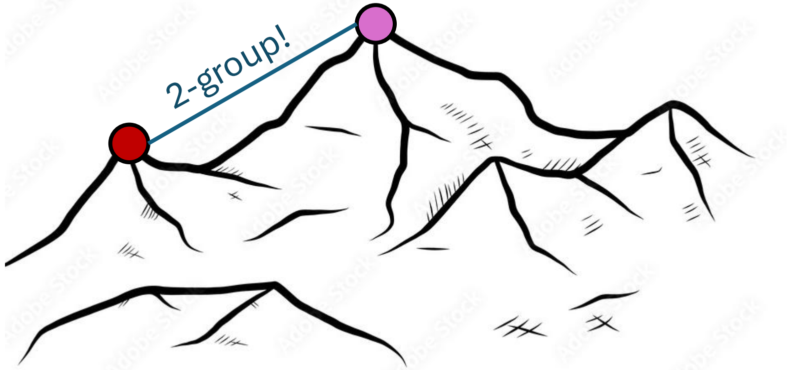


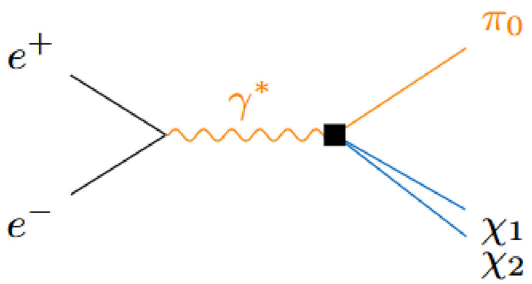




$\Longrightarrow m_\chi \sim 3$ GeV

$\mathrm{QCD} \times_B \mathrm{SQED} \longrightarrow$ LHC phenomenology





Δm_χ	$\lesssim 1.7 m_{\pi^0}$	$\gtrsim 1.7 m_{\pi^0}$
Signature	$\pi^0 + \not{E}_T$	$\pi^0 + \not{E}_T + \mathrm{DV}(\pi^0 \gamma \not{E}_T)$

Backup

Topological Interactions: Two Types

Davighi, Gripaios, 1803.07585; **Davighi**, Gripaios, Randal-Williams, 2011.05768

1. Theta-like

- Integrate a **closed *d*-form** ($d\alpha = 0$)
- No effects perturbatively
- E.g. Aharonov-Bohm
- E.g. 4d gauge theory: $S_\theta = \theta \int \frac{\mathrm{Tr}\,(F \wedge F)}{8\pi^2} = \theta N_{\mathrm{inst}}$

2. WZW-like

- Integrate a *d*-form α that is **NOT closed**.
- New interactions in perturbation theory
- Witten: $\int_{\Sigma_d} \alpha_i = \int_{Y_{d+1}} \omega = d\alpha_i, \partial Y_{d+1} = \Sigma_d$
- E.g. QM on sphere: $S = n \int_{D \subset S^2} \mathrm{Vol}, \partial D = \mathrm{Im}\,(\gamma)$
- E.g. WZW term in QCD: $S_{\mathrm{WZW}} = n \int_{B \subset SU(3)} \omega_5$

Topological Portal to the Dark Sector

Dark sector on G_D/H_D can talk to QCD pions via a **topological interaction**:

Davighi, Greljo, Selimović 2401.09528

$$S_{\text{portal}} \sim n \int_{B_5} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \Omega_2$$

c.f. $S_{\text{WZW}} \sim \int_{B_5} \mathrm{Tr}\,(g^{-1}dg)^5$

Dark sector cosets from global symmetry breaking are typically **symmetric spaces**:

- Invariant forms $\longleftrightarrow$ cohomology classes $\Longrightarrow H^2(G/H) \neq 0$
- Example (almost unique): $\dfrac{G_D}{H_D} = \dfrac{SU(2)}{SO(2)} = \boldsymbol{S^2}$

p	1	Portal 2	3	4	SIMP 5
$H^p(SU(2))$	0	0	$\mathbb{R}$	–	–
$H^p(SU(n)),\ n \geq 3$	0	0	$\mathbb{R}$	0	$\mathbb{R}$
$H^p(SU(2)/SO(2))$	0	$\mathbb{R}$	–	–	–
$H^p(SU(3)/SO(3))$	0	0	0	0	$\mathbb{R}$
$H^p(SU(4)/SO(4))$	0	0	0	$\mathbb{R}$	$\mathbb{R}$
$H^p(SU(n)/SO(n)),\ n \geq 5$	0	0	0	0	$\mathbb{R}$
$H^p(SU(2n)/Sp(2n)),\ n \geq 2$	0	0	0	0	$\mathbb{R}$

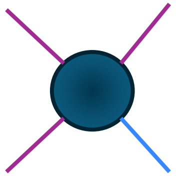
More Topological Portals!

1. From string theory: $S_{\text{top}} = \int_{X_5} (H - n\mathrm{Tr}(g^{-1}dg)^3) \wedge (F - m\, d\chi_1 d\chi_2)$

   From 2-group to sub-zero 3-group symmetry!

JD, Torres

2. For semi-annihilation: $S \sim \int \mathrm{Tr}\,(g^{-1}dg)^3_{\mathrm{D}} \wedge F_{\mathrm{D}} + \int F \wedge \star F_{\mathrm{D}}$



JD, Selimović, Moldovsky, Murayama, Scherbe

3. Weak scale version? In non-minimal CH models e.g. $SO(6)/SO(4)$

JD, Selimović, Zupan

In all cases, topological portals to DM naturally achieve some desirable things:

a) Inelasticity: no $\chi_1\chi_1$ vertices b/c differential forms are antisymmetric

b) Number odd processes: used in e.g. semi annihilation, explosive freeze in etc

Symmetry Defect Operators

Gaiotto, Kapustin, Seiberg, Willett, 1412.5148

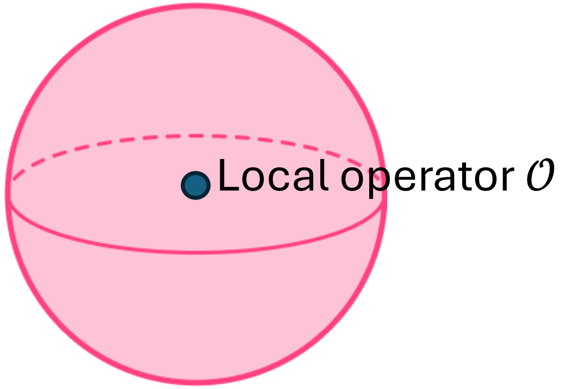
- The charge operator $Q(M_3)$, obtained from infinitesimal Noether procedure, lives in $\mathrm{Lie}\,(G)$
- Exponentiate it to get group elements:

$$U_{g=e^{i\alpha}}(M_3) := \exp i\alpha Q(M_3) = \exp i\alpha \int_{M_3} \star j$$

Key properties

1. $U_g(M_3)$ are all topological ("wiggle-independent") b/c $d \star j = 0$
2. The algebra of these topological operators is a **group**
3. The $U_g(M_3)$ act on local ops: linking between 3-mfd and point.

Action on operators



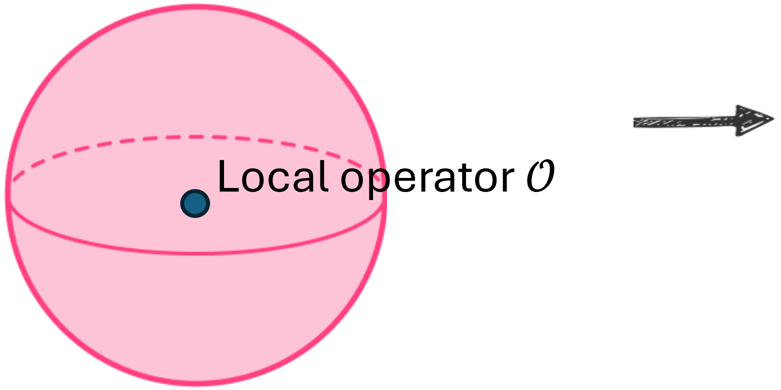
$$U_g(M_3)\, O(x) = e^{i\alpha q_O}\, O(x)$$

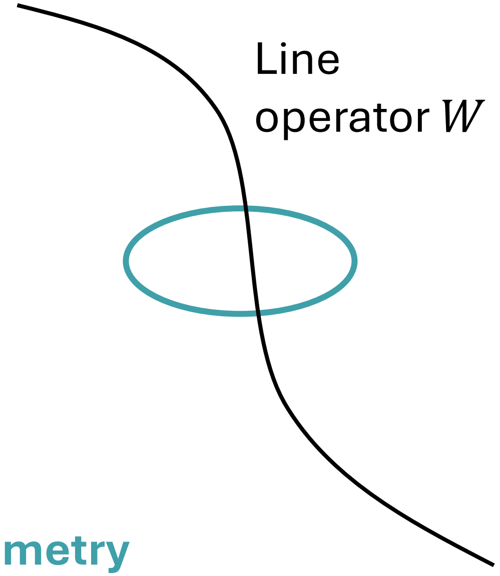
If x inside M_3

From 0-Form to 1-Form Symmetries

Ordinary (henceforth “0-form”) symmetry: charged objects = local operators (0-dimensional)

Generalize to 1-form symmetry: charged objects = line operators (1-dimensional)





Ordinary “0-form” symmetry

- Top ops $U_g(M^{d-1})$ link points (0d)
- Current $J = \star\, j^{(1)}$ is a closed $d-1$ form (if cts.)
- Background g. field is a 1-form $A \mapsto A + d\alpha$
- Minimal coupling $S = \int_{M_4} A^{(1)} \wedge \star\, j^{(1)}$

1-form symmetry

- Top ops $U_g(M^{d-2})$ links with **lines** (1d)
- Current $J = \star\, j^{(2)}$ is a closed $d-2$ form (if cts.)
- Background g. field is a 2-form $B \mapsto B + d\Lambda^{(1)}$
- Minimal coupling $S = \int_{M_4} B^{(2)} \wedge \star\, j^{(2)}$

From 0-Form to Higher-Form Symmetries

- Higher p-form symmetry: charged objects = extended p-dimensional operators
- Current that we integrate is $J = \star\, j$, where $j = j_{\mu_1 \dots \mu_{p+1}}$ is a $d-(p+1)$ form (if cts.)

  [Being a "form" means $j_{\mu_1 \dots \mu_{p+1}}$ is totally antisymmetric in exchanging indices]

- E.g. for a $U(1)$-valued p-form symmetry, defect operator is

$$U_{g=e^{i\alpha}}\left(M_{d-(p+1)}\right) = \exp\left(i\alpha \int_{M_{d-(p+1)}} \star\, j\right)$$

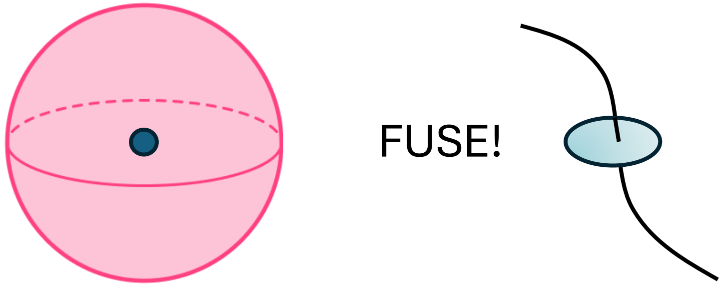
  which acts on p-dimensional extended operators (the objects which can carry charge)

- The defect operator is "topological" i.e. is a symmetry iff $dJ = d \star j = 0$. In components

$$\partial_\mu j^{\mu\mu_1 \dots \mu_p} = 0 \qquad \text{[just } \partial_\mu j^\mu = 0 \text{ in familiar 0-form case]}$$

Higher Group Symmetries

- Higher form symmetries of different degrees can mix to form what is known as a "higher-group" structure in mathematics (described by *higher-bundles* with connection)
- Simplest case is **2-group symmetry**:



0-form symmetry

1-form symmetry

= **2-group** generalized symmetry

- 2-group connection consists of a pair of gauge fields with intertwined g. transformation:

1-form g. field: $A \mapsto A + d\alpha$

2-form g. field: $B \mapsto B + d\Lambda^{(1)} + \frac{n}{2\pi}\alpha dA$

$n \in \mathbb{Z}$, called the "Postnikov class", that classifies the particular 2-group symmetry we have

Higher Group Symmetry is Quantized!

- There is a **2-group current algebra** analogous to the familiar anomalous current algebra:

$$\left\langle \partial^\mu j_\mu^{L,A}(x) j_\nu^{L,B}(y) - \delta(x-y) f^{ABC} j_\nu^C(y) \right\rangle \sim \left\langle n\, \delta^{AB} \partial^\rho \delta(x-y) j_{\rho\nu}^{(2)}(y) \right\rangle$$

From anomaly matching to symmetry matching!

- 2-group class $n \in \mathbb{Z}$ ∴ cannot change continuously under any deformation, including RG
- ... so, like an anomaly, it must match from UV to IR!
- This gives 2-group more power than 1-form and 0-form separately: cannot break one the 1-form symmetry without explicitly breaking the 0-form “flavour” symmetry at the same scale

[like non-abelian current algebra]

This is the “**2-group emergence theorem**” of Cordova, Dumitrescu, Intriligator 1802.04790