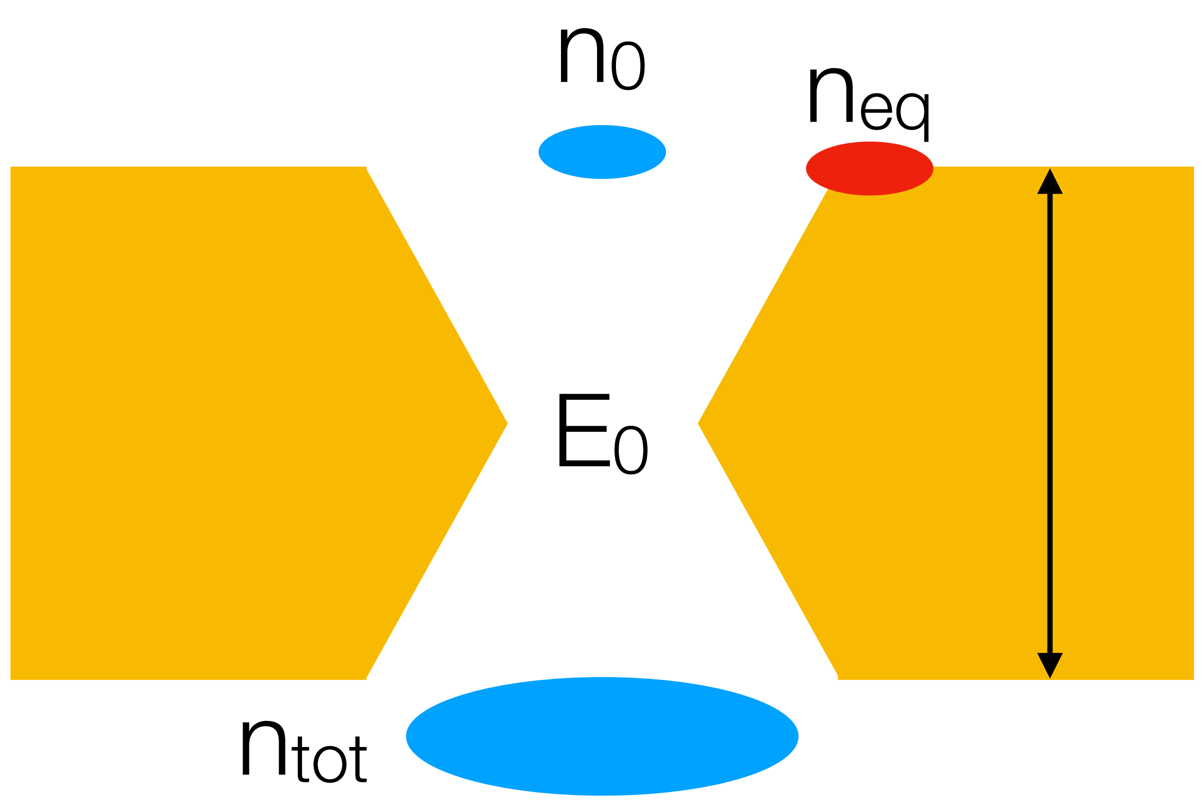
# Saturation of the GEM gain

# A simple model

Let's suppose that during the development of the avalanche within the gem multiplication channels a significant amount of electrons and positive ions are produced.

Under the effect of the electric field present in the channel, these slowly migrate toward the lower potential plane of the GEM, tending to partially shield the field itself.

If $n_0$ is the number of electrons entering a GEM channel and $E_0 = V_{GEM}/d$ the electric field in it:




d is the GEM thickness

$V_{GEM}$ is the voltage drop between GEM sides

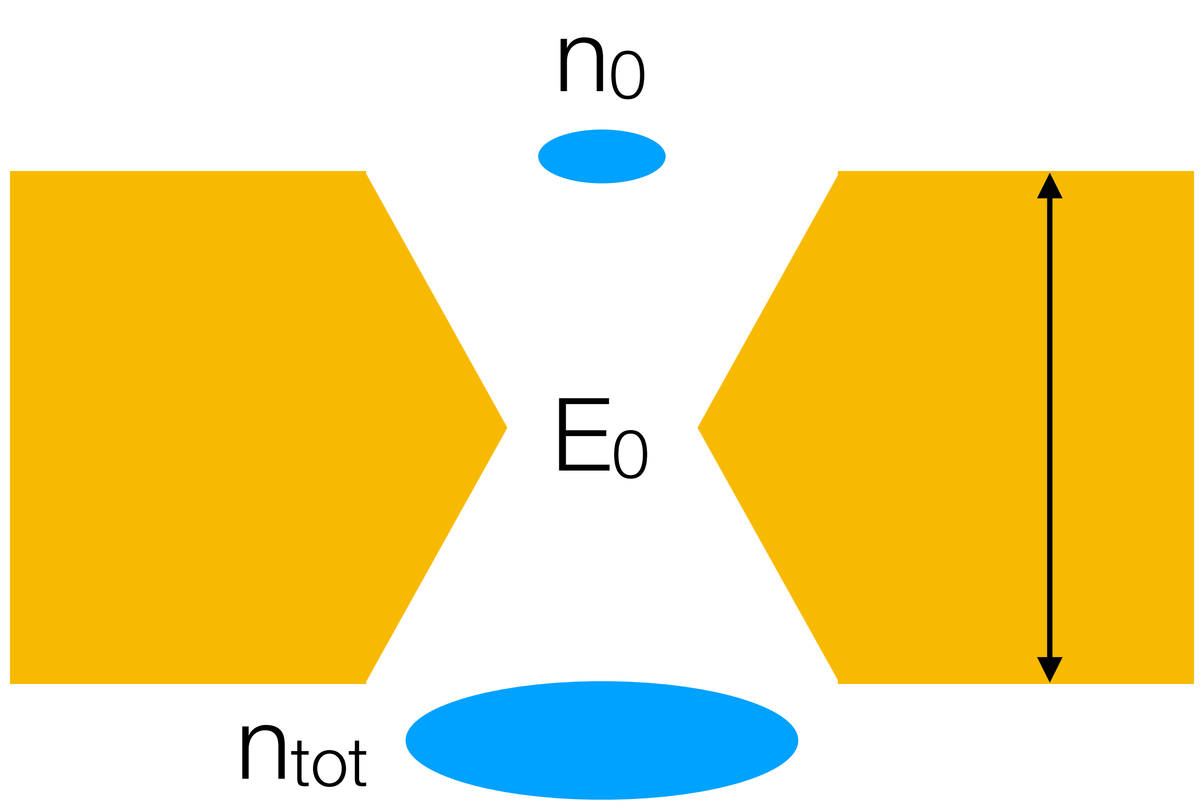
Multiplication is described by a modified Townsend equation

$$\frac{dn}{ds} = \alpha E_0(1 - \beta n)n$$

where $\beta$ is can be interpreted as the inverse of the number of charges $\beta = 1/n_{eq}$ present on the GEM border of the channel and needed to produce $E_0$ in it ($\beta \propto 1/V_{GEM}$);

# A simple model

$$\frac{dn}{ds} = \alpha E_0(1 - \beta n)n$$




d is the GEM thickness

$V_{GEM}$ is the voltage drop between GEM sides

$$\int_{n_0}^{n_{tot}} \frac{dn}{(1-\beta n)n} = \int_0^d \alpha E_0 ds$$

$$G = \frac{e^{\alpha V}}{1 + \beta n_0 (e^{\alpha V} - 1)}$$

where $G = n_{tot}/n_0$ is the average gain of the single channel.

It should be noticed that it depends on the amount of primary electrons entering the channel. In particular it decreases with $n_0$ and:

- if $\beta n_0 \simeq 0$ (i.e. negligible screen effect), $G = e^{\alpha} V$
- if $\beta n_0 \simeq 1$ (i.e. total screen effect), $G = 1$

# A simple model

Let's suppose that only in GEM#3 we have non linear gain because of the larger amount of charges.

$$G_{tot} = \frac{G_1 G_2 e^{\alpha V}}{1 + \beta n_0 (e^{\alpha V} - 1)} = \frac{G_1 G_2 G_3}{1 + (p_1/V_{GEM}) G_1 G_2/\sigma^3 \ (G_3 - 1)}$$

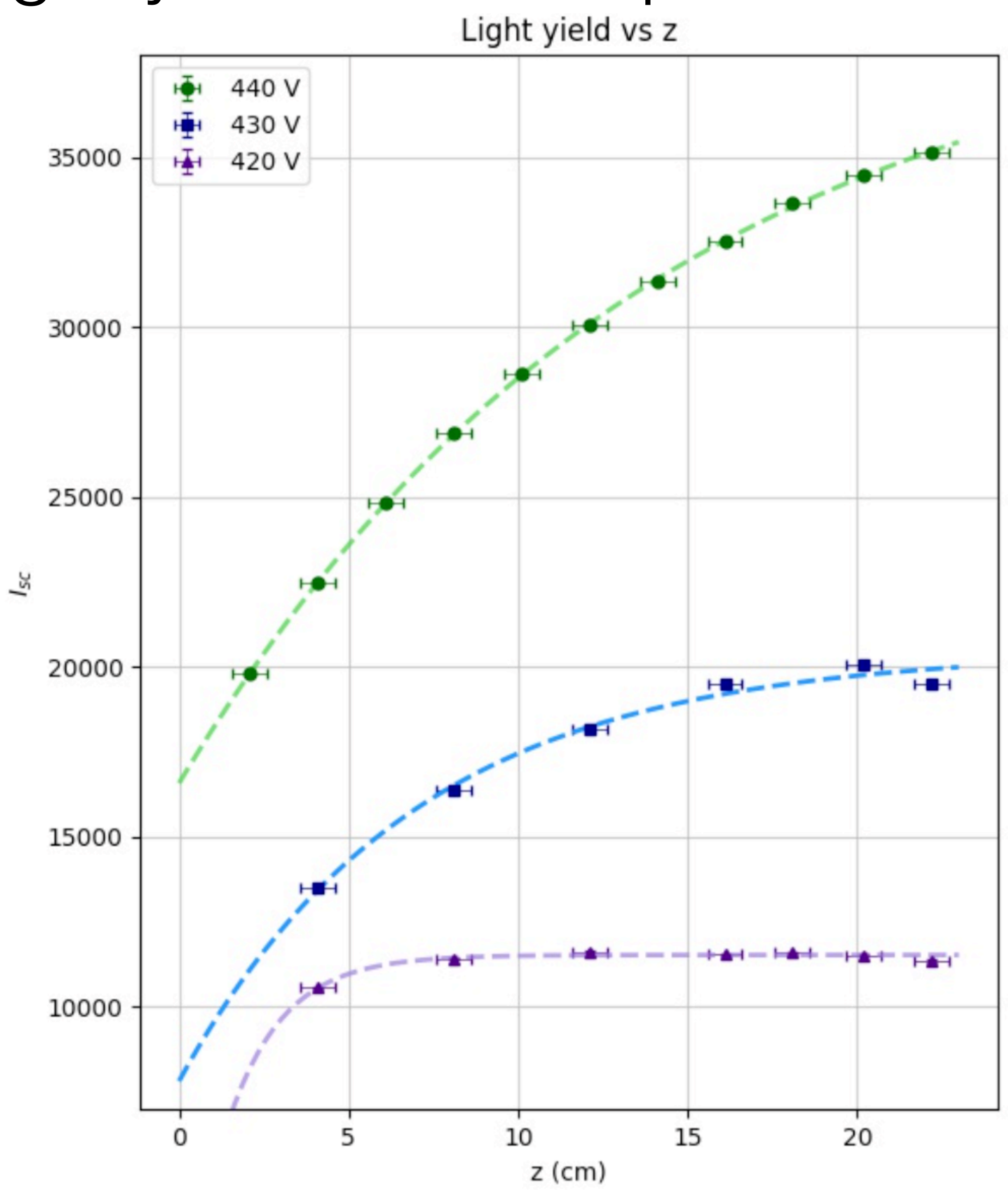
If $G_1 = G_2 = G_3 = e^{\alpha V} = p_0$

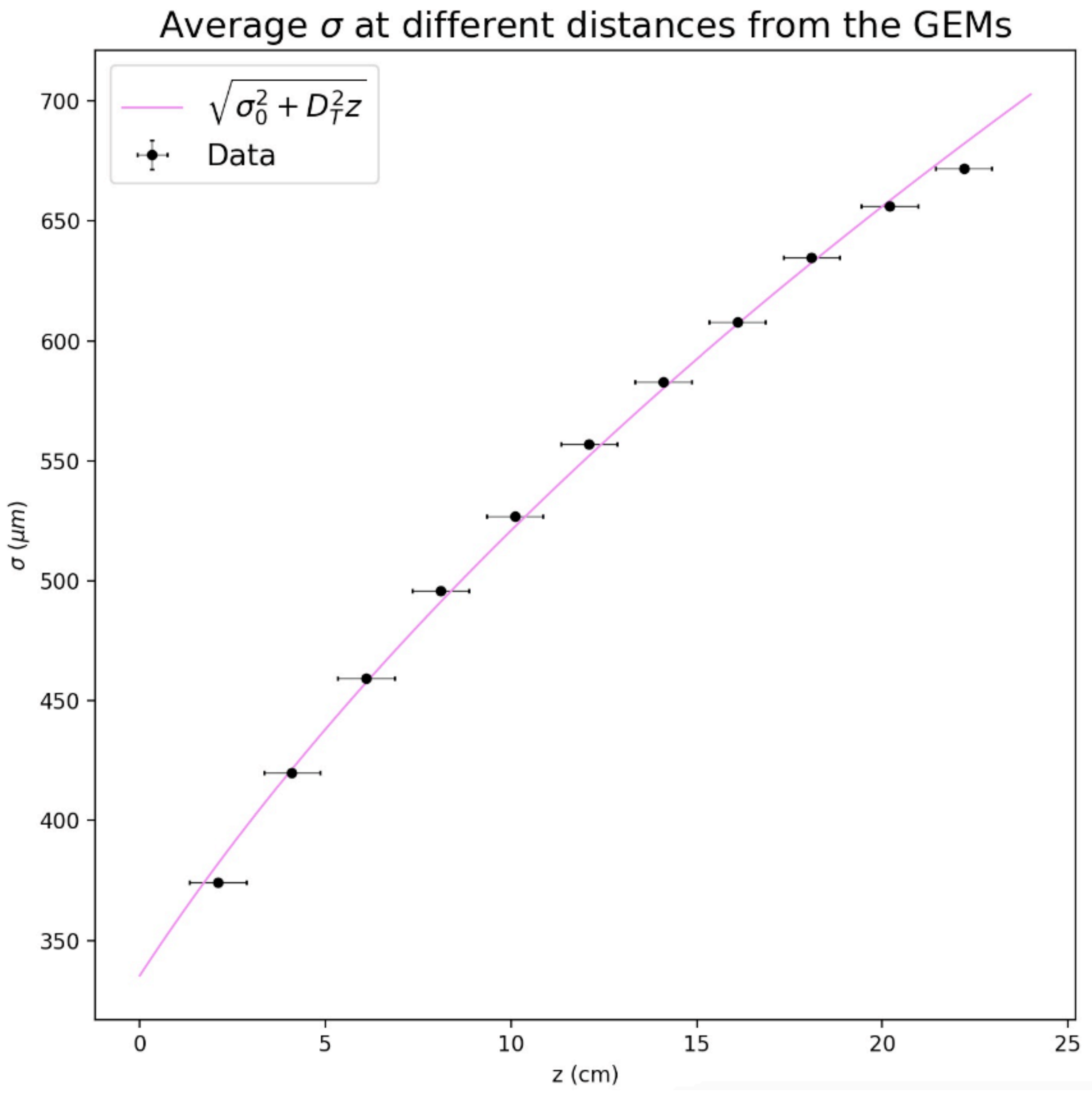
$$G_{tot} = \frac{p_0^3}{1 + (p_1/V_{GEM}) p_0^2/\sigma_3 (p_0 - 1)} = \frac{p_0^3 \sigma^3}{\sigma^3 + (p_1/V_{GEM}) p_0^2 (p_0 - 1)}$$

- We can try to fit this last function on the data expecting:
  - $p_0$ to be the not-saturated gain of the three GEMS;
  - $p_1$ is constant

# A simple model

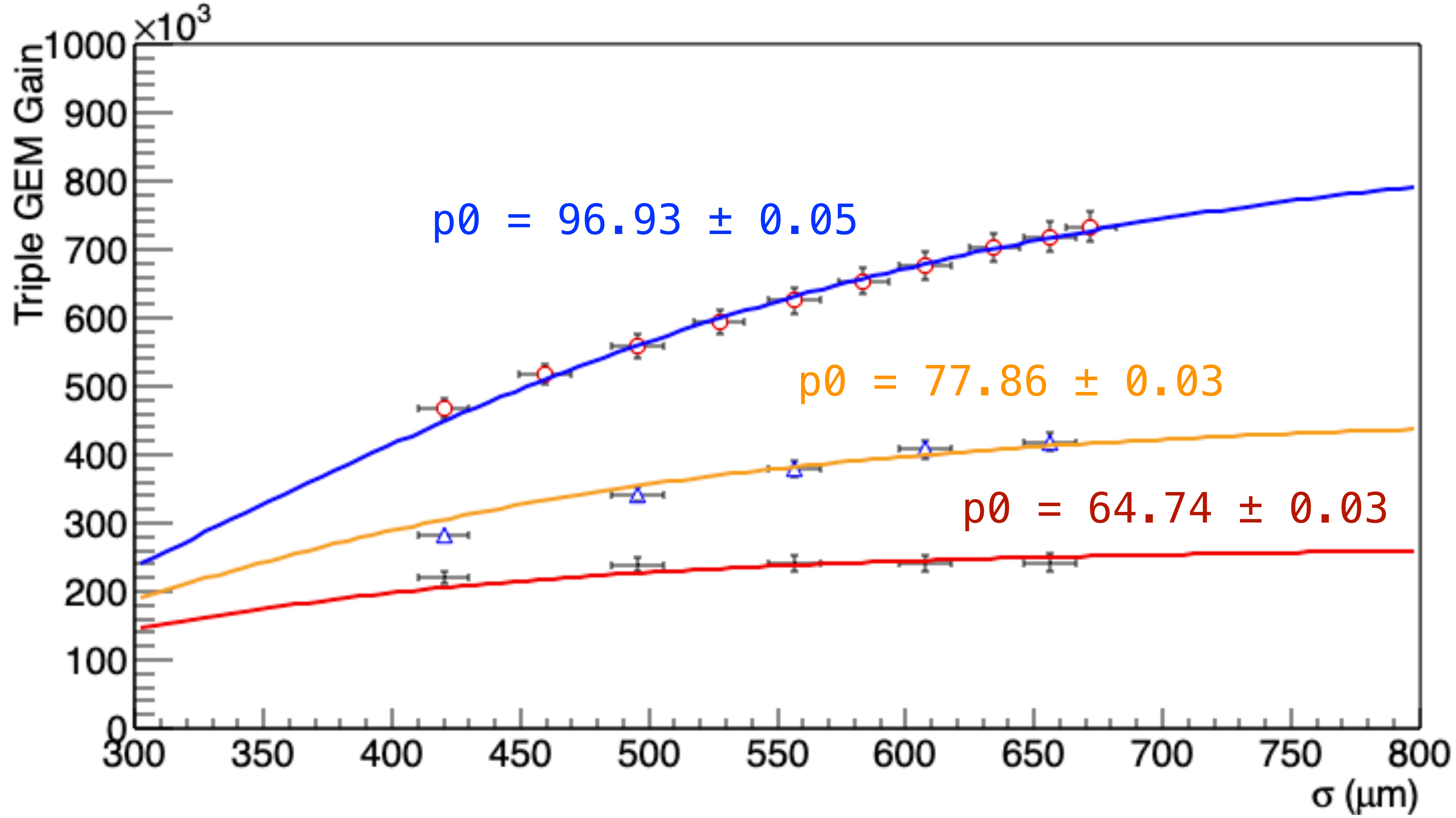
- From the GIN data we can evaluate the electron gain in 3 different $V_{GEM}$ setup (440, 430 and 420) and the behavior of $\sigma$
- We can start from the light yield for $^{55}Fe$ spots




- The electron gain is evaluated by taking into account 0.07 $\gamma/e$, 150 $n_e$ and $\Omega = 9.2 \times 10^{-4}$

# A simple model




We can fit the behavior

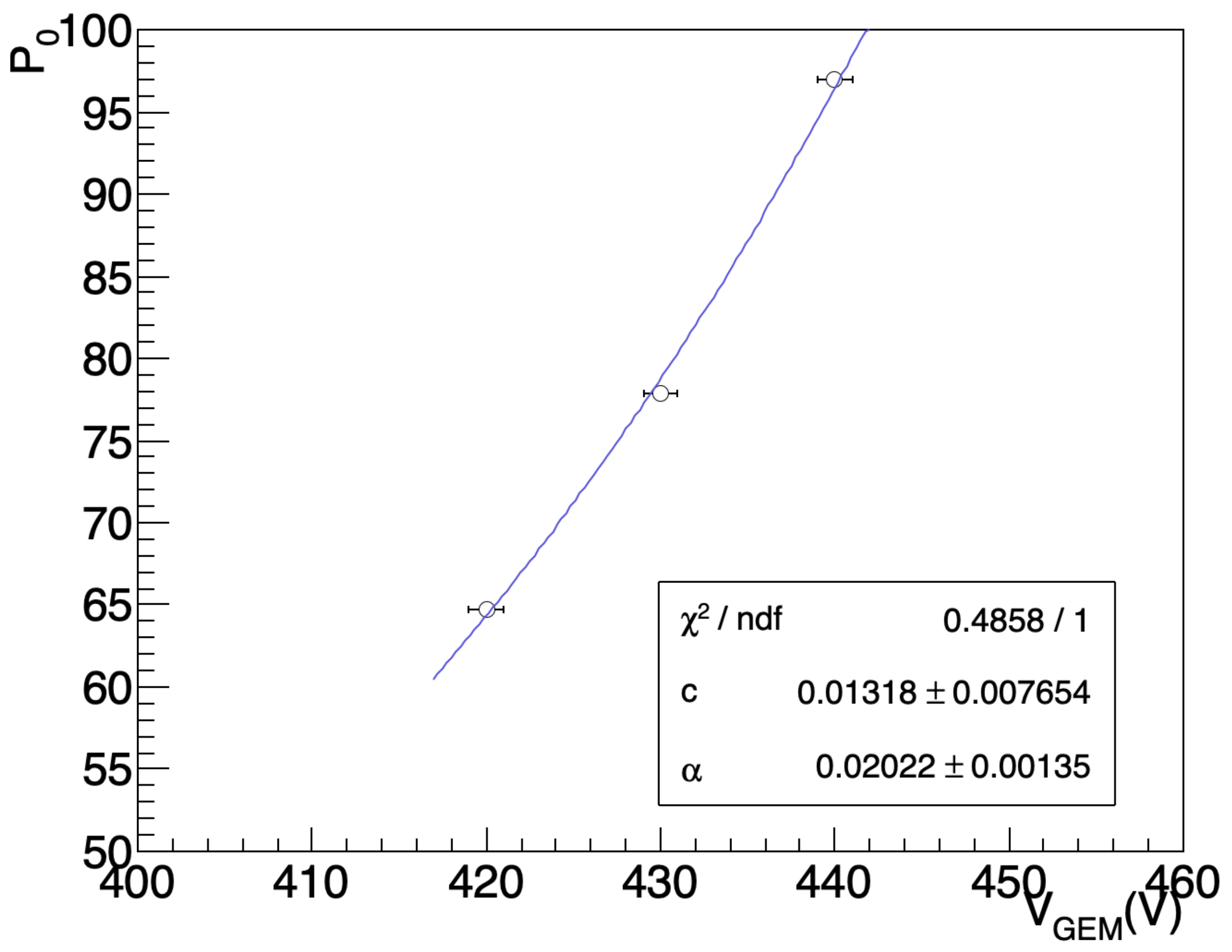
$$G_{tot} = \frac{p_0^3 \sigma^3}{\sigma^3 + (p_1 / V_{GEM}) p_0^2 (p_0 - 1)}$$

Where $p_0$ is the single GEM non-saturated gain;

A Minuit simultaneous fit was performed with 4 parameters: the three not saturated gains and a common $p_1$;

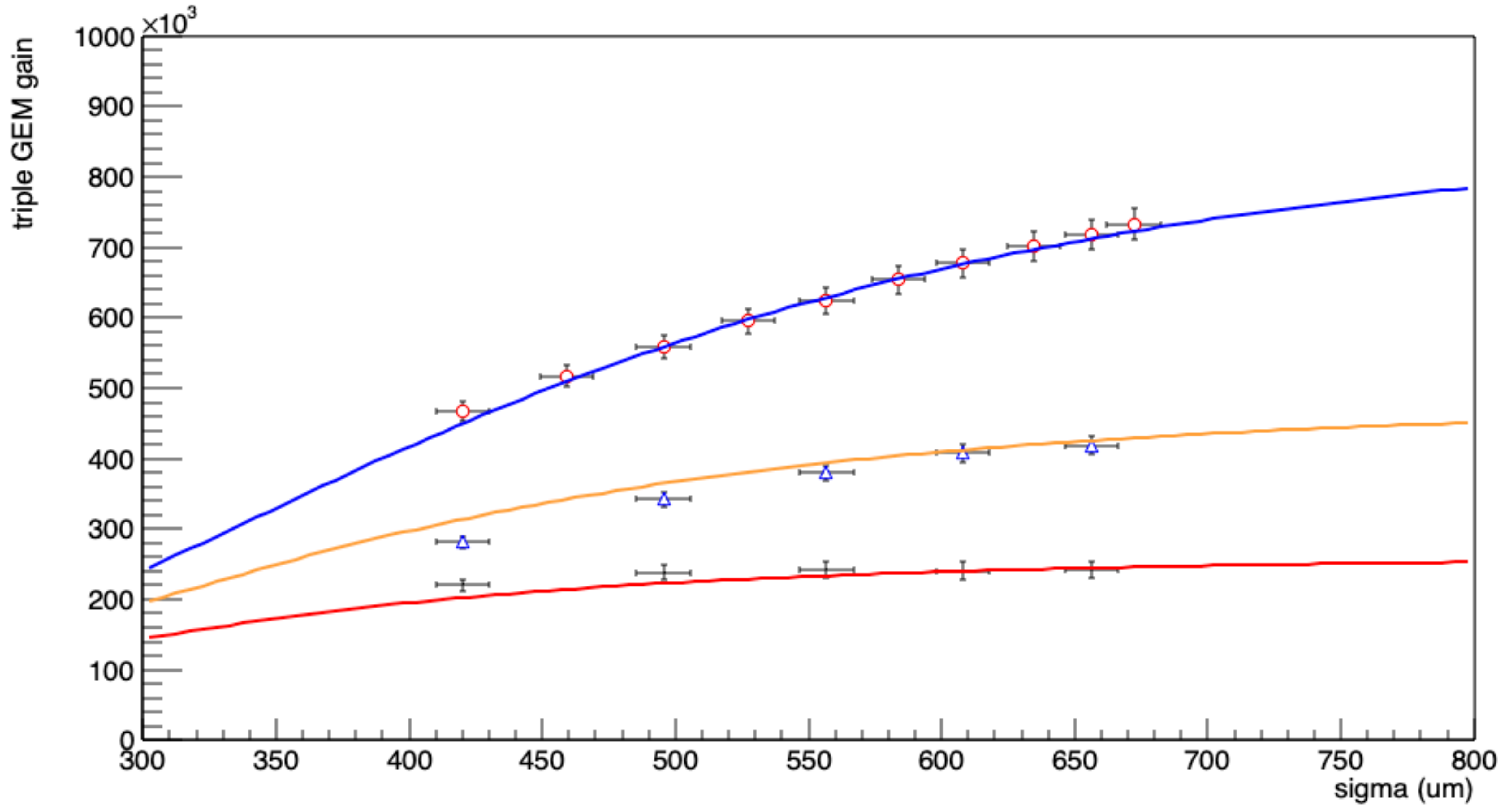
$p_1$ is $(3.730 \pm 0.014)\ 10^4$ in all the three fits

Identical results obtained with Minuit2




By fitting $\boldsymbol{p_0}$ vs $V_{GEM}$ ($p_0 = ce^{\alpha V_{GEM}}$) we evaluate a negligible constant term and $\alpha = 0.020 \pm 0.001$

# A simple model




We can fit the behavior

$$G_{tot} = \frac{p_0^3 \sigma^3}{\sigma^3 + (p_1/V_{GEM}) p_0^2 (p_0 - 1)}$$

Where $p_0$ is the single GEM non-saturated gain and can be expressed as:

$$p_0 = c e^{\alpha V_{GEM}}$$

A Minuit simultaneous fit was performed with 3 parameters:

- a normalisation $c = 0.012 \ \pm \ 0.001$

- $\alpha = 0.020 \ \pm \ 0.001$

- $p_1 = (3.66 \pm 0.014)\ 10^4$