The Strong CP problem and instantonic symmetries

Eduardo García-Valdecasas

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Based on 2402.00117 with Daniel Aloni, Matt Reece and Motoo Suzuki.

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1. Introduction
    - Higher form global symmetries.
    - Definition of (-1)-form U(1) symmetry and properties.

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    - A necessary condition for the Strong CP Problem.

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    - A necessary condition for the Strong CP Problem.
4. A different look at Strong CP problem solutions.
    - Known solutions by gauging (-1)-form U(1) symmetry.
    - (Failed) attempts of a new solution using explicit breaking of the (-1)-form U(1).

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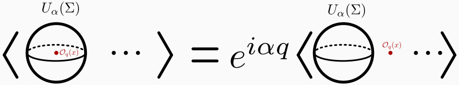

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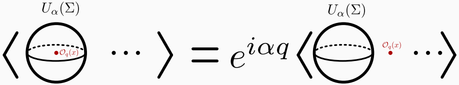

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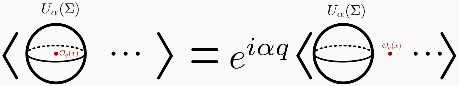

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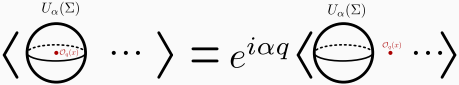

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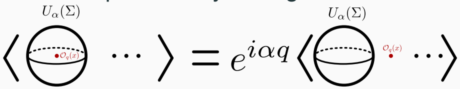

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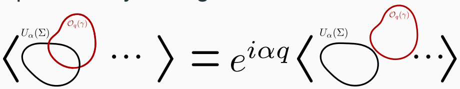

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- The Goldstone boson is the photon,

$$\langle 0 | j^{(2)}_{e,\mu\nu} | \lambda, p\rangle = (\lambda_\mu p_\nu - \lambda_\nu p_\mu) e^{ipx}$$

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- **Our point of view:** (-1)-form U(1) symmetries are features of QFT's that share some properties with higher form symmetries.

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7. **Spontaneous Breaking**. Charged operator takes a vev.

$$\langle W_q[\gamma_p] \rangle \neq 0$$

The conserved current acts on the vacuum creating massless Goldstone,

$$\langle 0 | j_\mu(x) | p \rangle = p_\mu e^{ipx}$$

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Only large gauge transformations.

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$\Longrightarrow$ **In this talk: explore this possibility.**

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In this talk: the Strong CP Problem.

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Spontaneously broken (-1)-form U(1) symmetries

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Is there a sense in which $U(1)_m^{(-1)}$ is spontaneously broken?

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$$\phi(t) = \int_0^{2\pi R} dx A_1(x,t) \quad \rightarrow \quad S = \int dt \left[\frac{1}{4\pi e^2 R} \dot{\phi}^2 + \frac{\theta}{2\pi} \dot{\phi} \right], \qquad \phi \sim \phi + 2\pi$$

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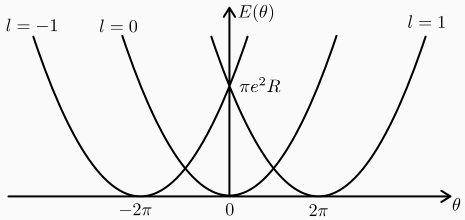
Solved by eigenstates $\phi_l = e^{il\phi}$ with energy,

$$E_l = \pi e^2 R \left(l - \frac{\theta}{2\pi}\right)^2$$

Excited states (not drawn): adding 2 probe particles.

Classically confined.



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- The gauge boson A eats the would-be Goldstone ϕ and becomes **massive.**

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- Dualize to magnetic frame $d\phi = \star d\tilde{\phi}$. We find $\tilde{\phi}$ eaten by A,

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We conclude that the gauged (-1)-form U(1) symmetry is in the Higgs phase. We interpret this to mean that the global (-1)-form U(1) symmetry of $2d$ Maxwell is spontaneously broken.

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$\mathcal{X} \neq 0$ related to “masslessness” of $A_\mu(x)$

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- C_{d-1} is the electromagnetic dual of the "Goldstone" (-1)-form gauge field of the spontaneously broken (-1)-form symmetry.

A different look at the Strong CP Problem

Spontaneous breaking of (-1)-form symmetry in SU(N) Yang Mills

$$S = \int \text{tr}\left(-\frac{1}{g^2} F \wedge \star F + \frac{\theta}{8\pi^2} F \wedge F\right)$$

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$\Longrightarrow$ pole at $q = 0$ in K_1 2-point function.

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- The $U(1)^{(-1)}$ symmetry is spontaneously broken. The Goldstone field is C_3.
- In fact, at large N the IR of SU(N) YM can be described using an effective theory in terms of $F_4 = dC_3$ (Di Vechia, Veneziano, Shifman, Gabadadze, Dvali),

$$\mathcal{L} = -\frac{1}{2\mathcal{X}}F_4\wedge\star F_4 + \frac{1}{2\pi}\theta F_4$$

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Spontaneous breaking of (-1)-form symmetry in SU(N) Yang Mills

Additional check for SSB of $U(1)^{(-1)}$: gauge it by coupling to an axion ϕ. Physics is well known,

- The axion ϕ is massive.
- The magnetic objects, i.e. axionic strings are confined.
- Magnetic confinement can be seen explicitly from the effective action by dualizing $d\phi \sim \star dB_2$,

$$\mathcal{L}' \supset |dB_2 - C_3|^2$$

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Away from large N, or in physical QCD $U(1)^{(-1)}$ is still spontaneously broken but the effective theory in terms of C_3 takes a more complicated form. Generically,

$$\mathcal{L} = -\frac{1}{2}|F_4|^2 + \frac{1}{2\pi}\theta F_4 + \mathbf{K}(F_4)$$

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$\Longrightarrow$ If we prevent this phenomenon by either gauging or explicitly breaking the (-1)-form U(1) symmetry, the Strong CP problem is avoided.

A different look at Strong CP Problem solutions

Solving the problem with an axion

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- Related formulations of this solution exist in QCD but they are more involved.

Exploring new solutions to the Strong CP Problem

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- Open question. Are there other anomalies that can be used to solve the problem?

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- **Are emergent (-1)-form U(1) symmetries exact?**
- If they are not, we could potentially solve the Strong CP problem for free with GUT models.

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We expect this lesson to generalize, implying that the Strong CP problem can't be solved in this way.

Outlook

- A firmer footing for (-1)-form symmetries. Perhaps using the SymTFT. Or Holography?
- Are (-1)-form symmetries matched under dualities?
- Goldstone Theorem?
- Better understanding of explicit breaking.
- Breaking by monopoles in GUT theories. Extensive study in 2408.00067.
- Application to other axion-like fields in particle physics. In particular axion monodromy.

Thanks!

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