

USING AMS DATA TO MODEL COSMIC RAY PROPAGATION IN THE HELIOSPHERE

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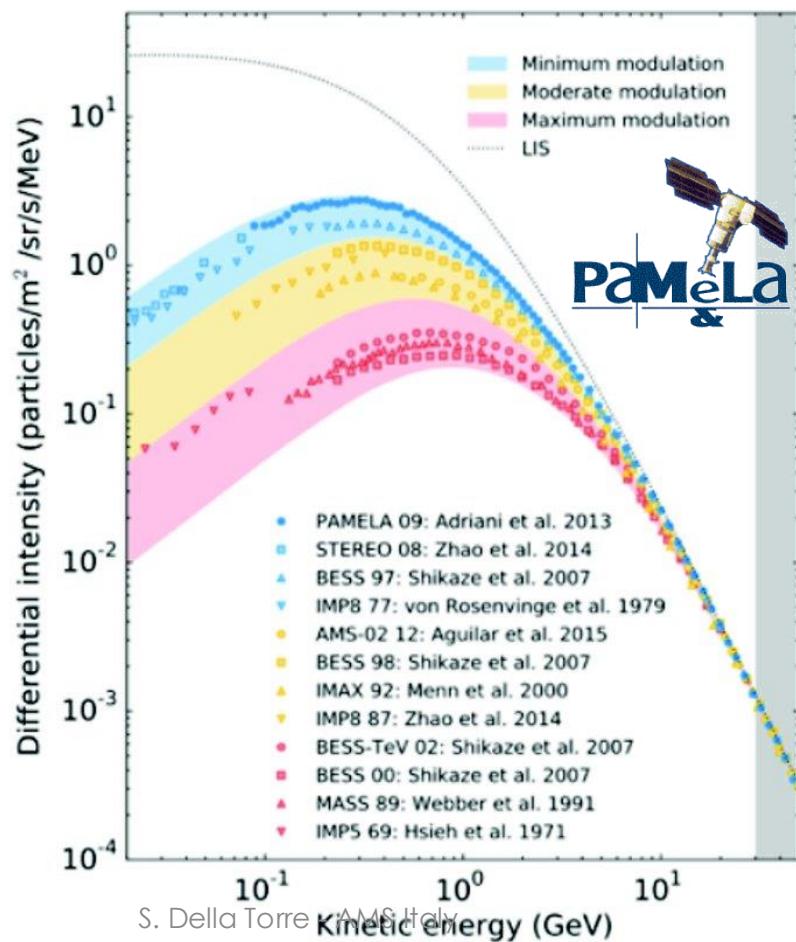


OUTLINE

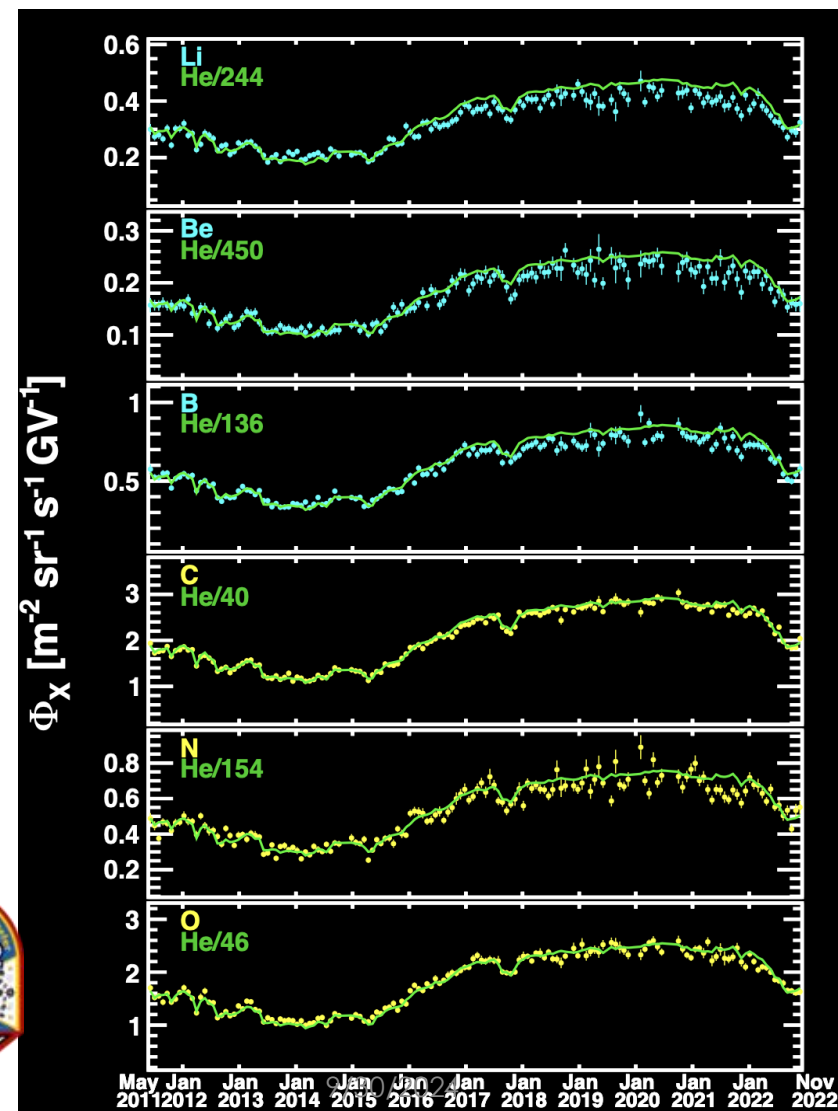
- Brief introduction to Solar Modulation and to Heliosphere
- Cosmic Rays Transport in the Heliosphere
- Applications and implications

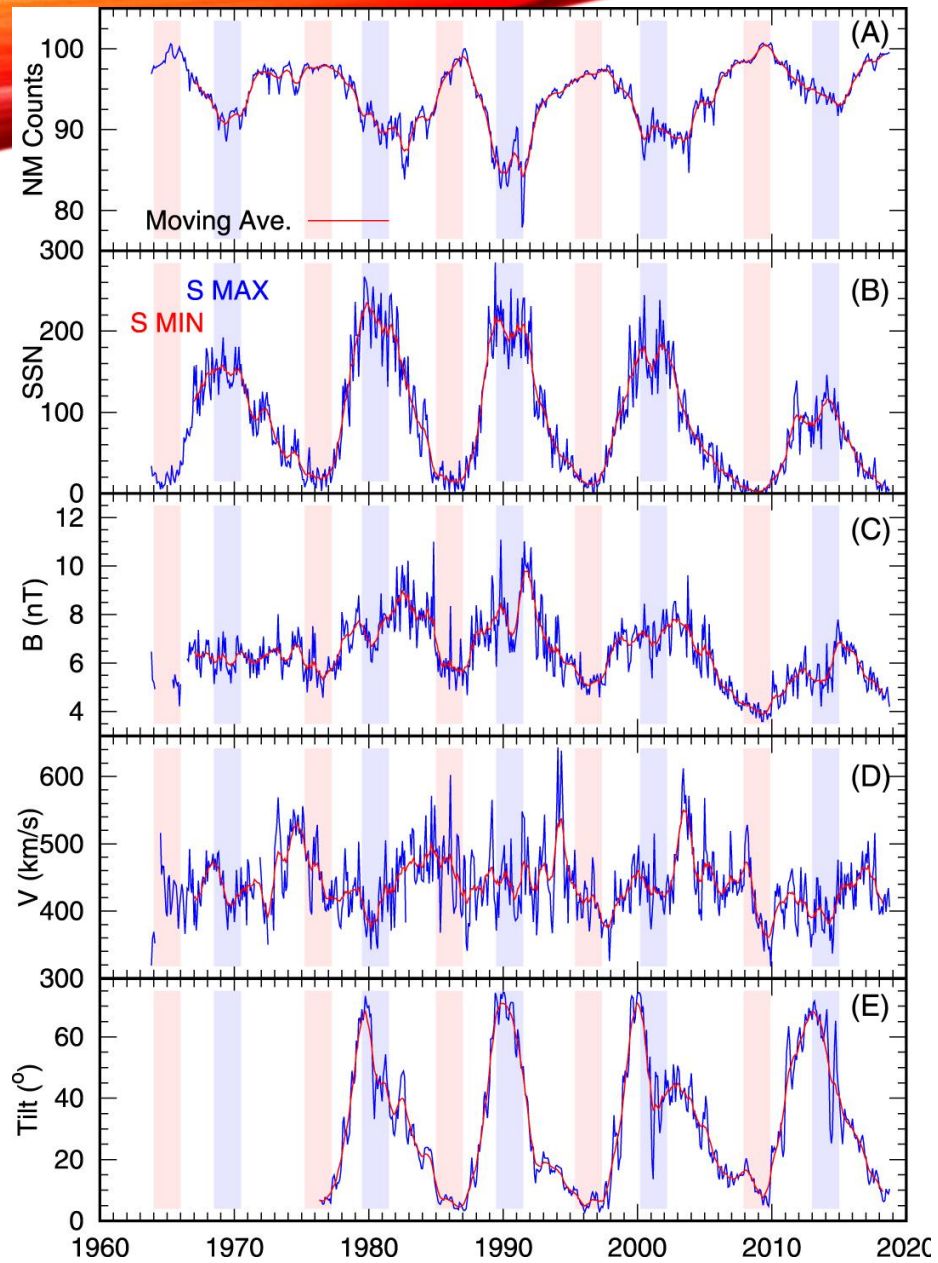
DEFINITION #1: SOLAR MODULATION

Solar Modulation is Energy dependent and is effective <30 GeV/nuc



Solar Modulation affect all Ions in a similar way





The temporal variation is **anti-correlated in time** with parameters inferred from solar observations...

therefore, whatever the **cause** of the variability, it **is linked to our star**.

DEFINITION #2: THE HELIOSPHERE

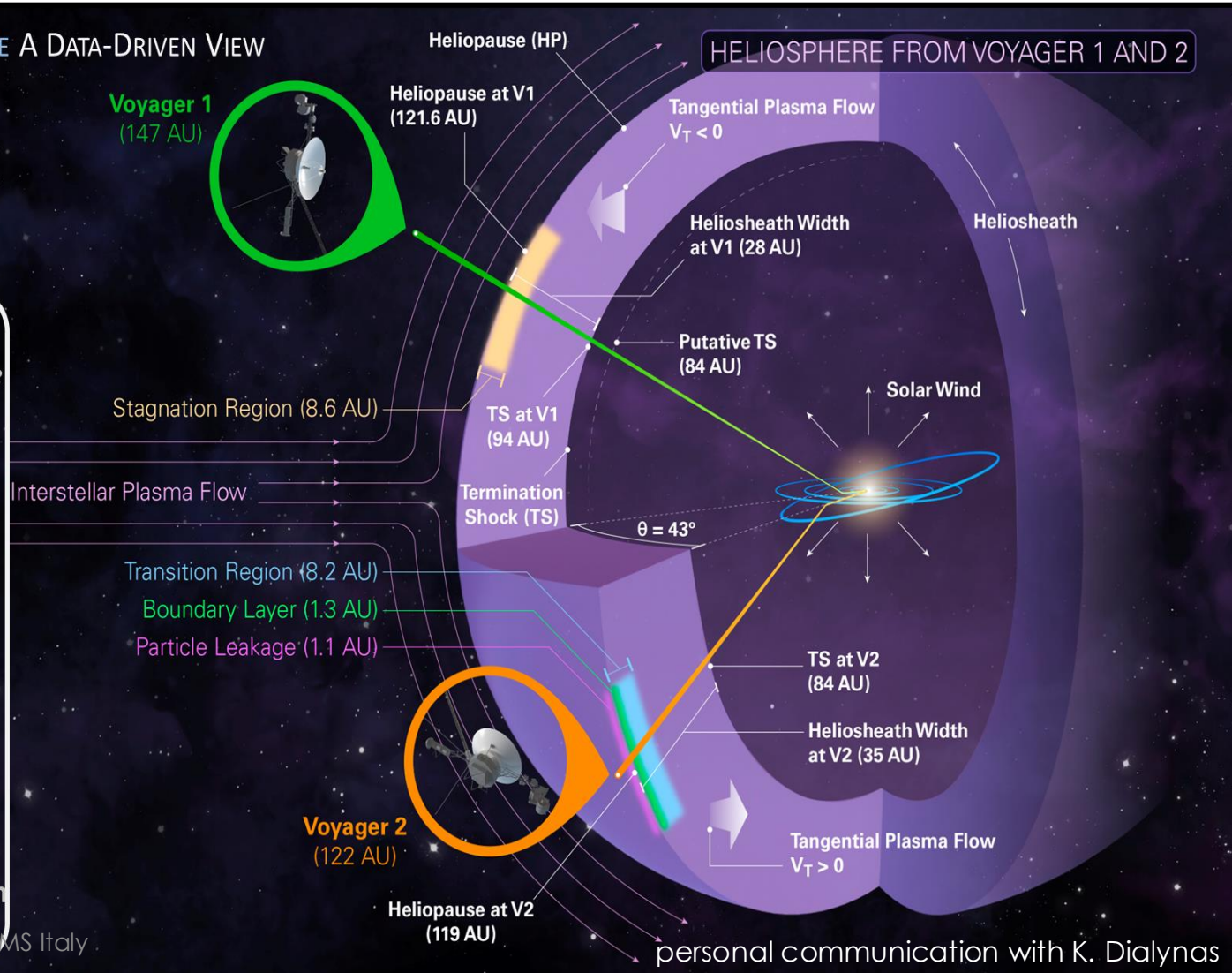
THE GLOBAL HELIOSPHERE A DATA-DRIVEN VIEW

Krimigis+[2019]
 Combined data from V1,
 V2 and Cassini/INCA

Dialynas+[2022]
 The heliosphere from the
 perspective of Cassini,
 HSTOF & Voyager
 measurements!

Kleimann+[2022]
 The heliosphere from the
 perspective of the
 models and
 comparison of models
 with data!

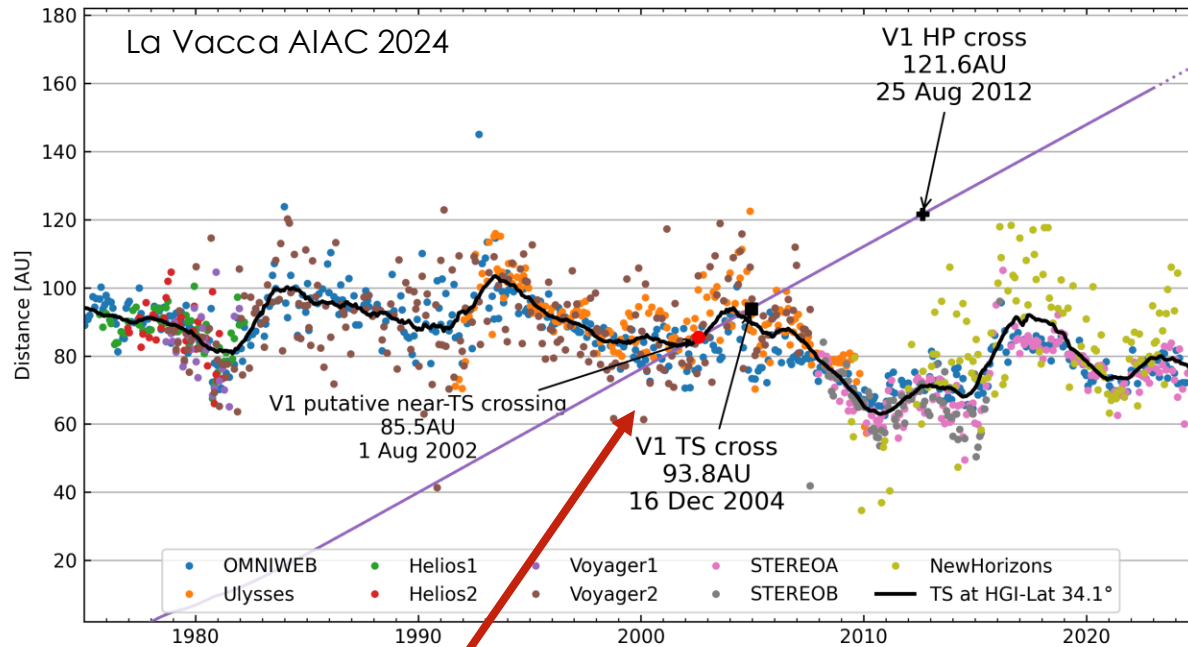
SSRv 2022
 The Heliosphere in the
 Local Interstellar Medium
 Into the Unknown



Simple definition:
 Region of space
 directly influenced
 by Sun dynamics

Plasma emitted by
 the Sun (solar wind)
 transport the
 magnetic field
 defining the shape
 and dimension of
 the heliosphere

HELIOSPHERE BOUNDARIES MOVES AS FUNCTION OF TIME

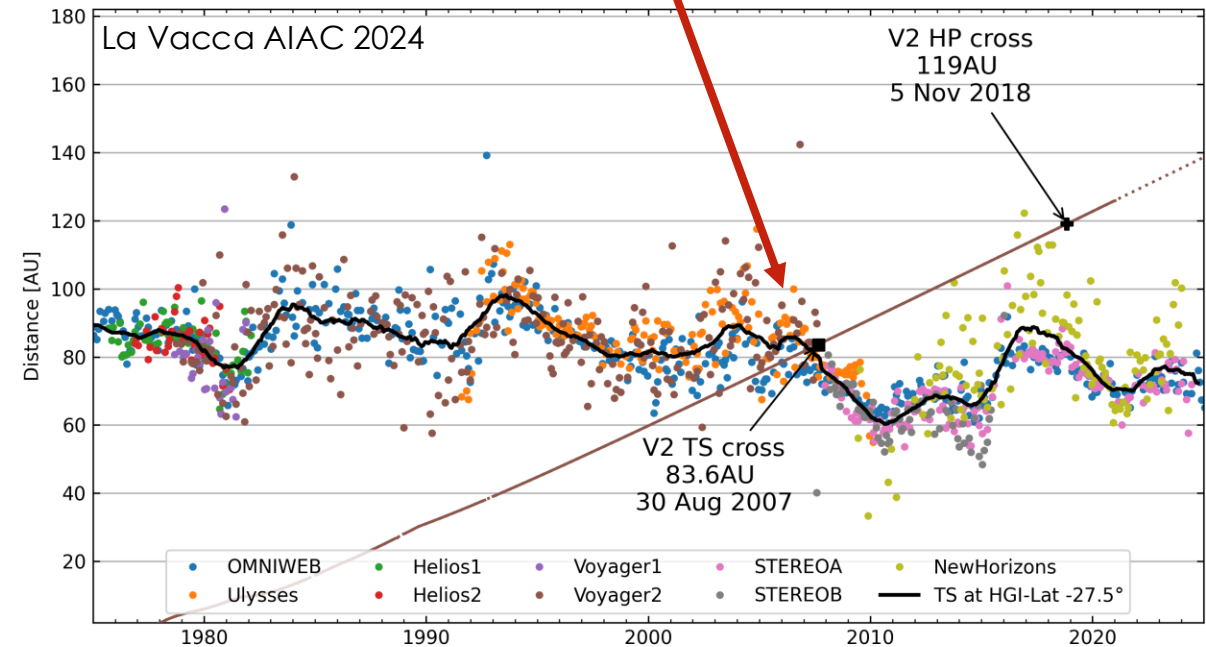


At Voyager 1 latitude:
 - Moving average 2-years amplitude applied to the average

Estimated TS cross
 • 89.5AU on 16 Dec 2004

At Voyager 2 latitude:
 - Moving average 2-years amplitude applied to the average

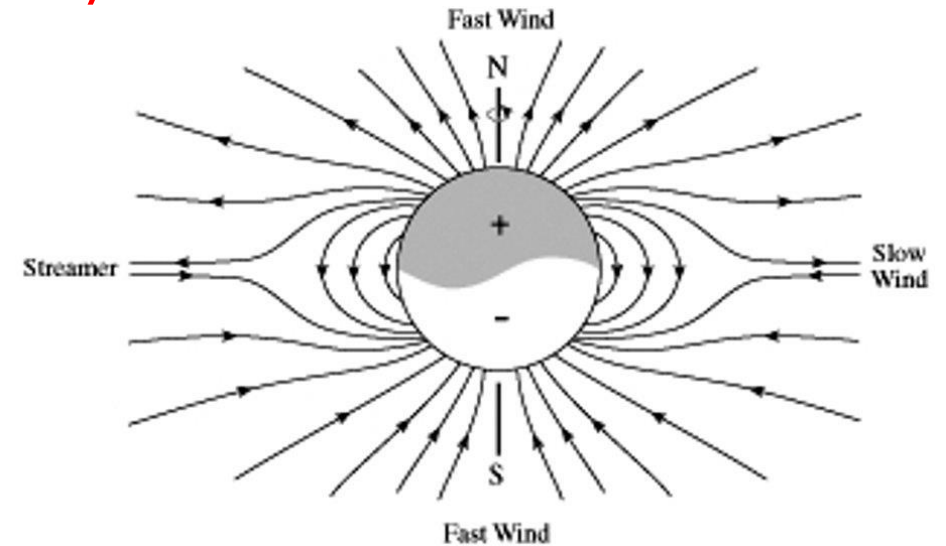
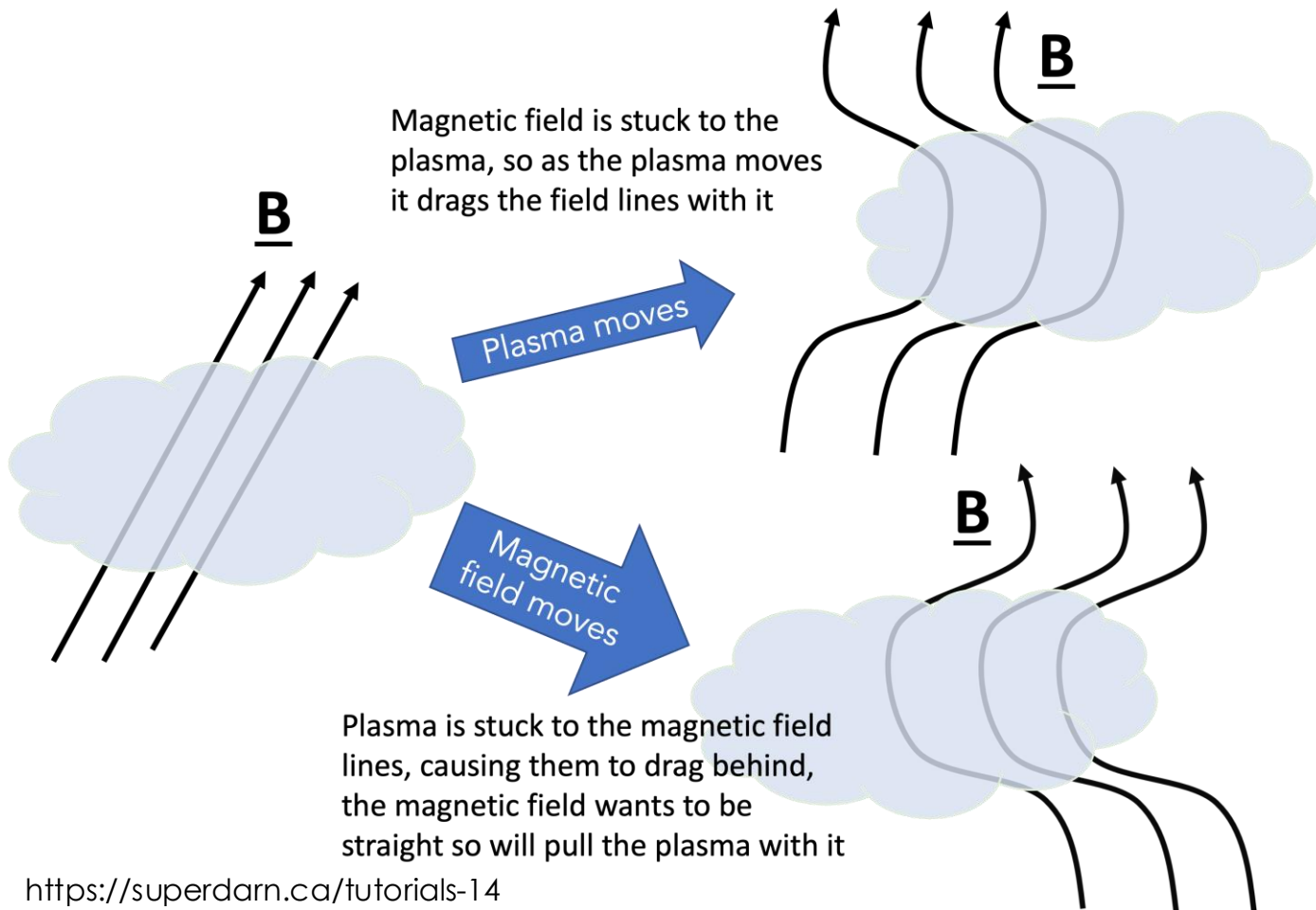
Estimated TS cross
 • 80.0AU on 30 Aug 2007



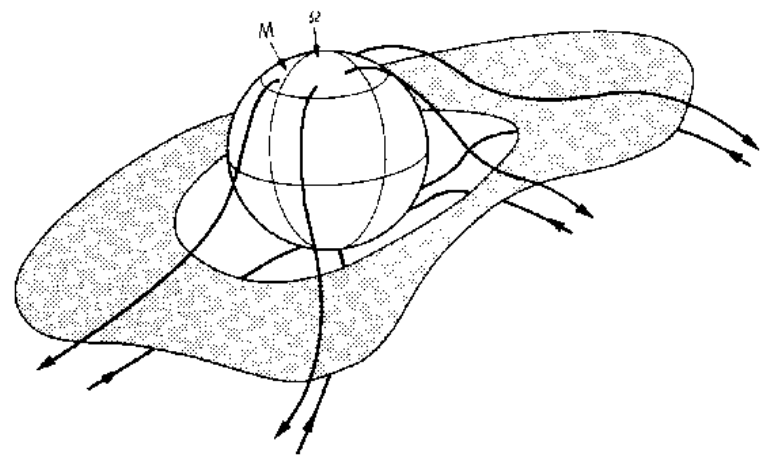
Sun has a magnetic field

Sun emits a plasma flow (solar wind)

Sun Magnetic field permeate the heliosphere by means of solar wind

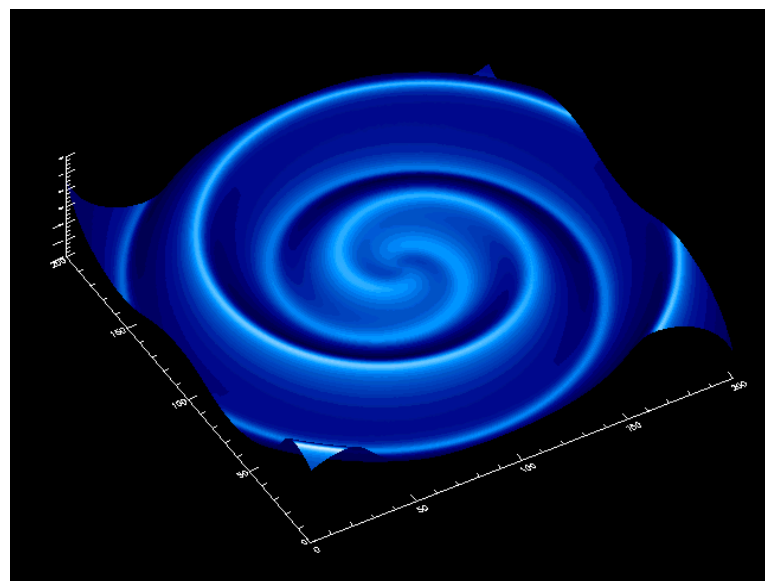


<https://superdarn.ca/tutorials-14>



Solar wind expansion creates the **Neutral Current Sheet** that divide heliosphere in two hemisphere of opposite solar polarity

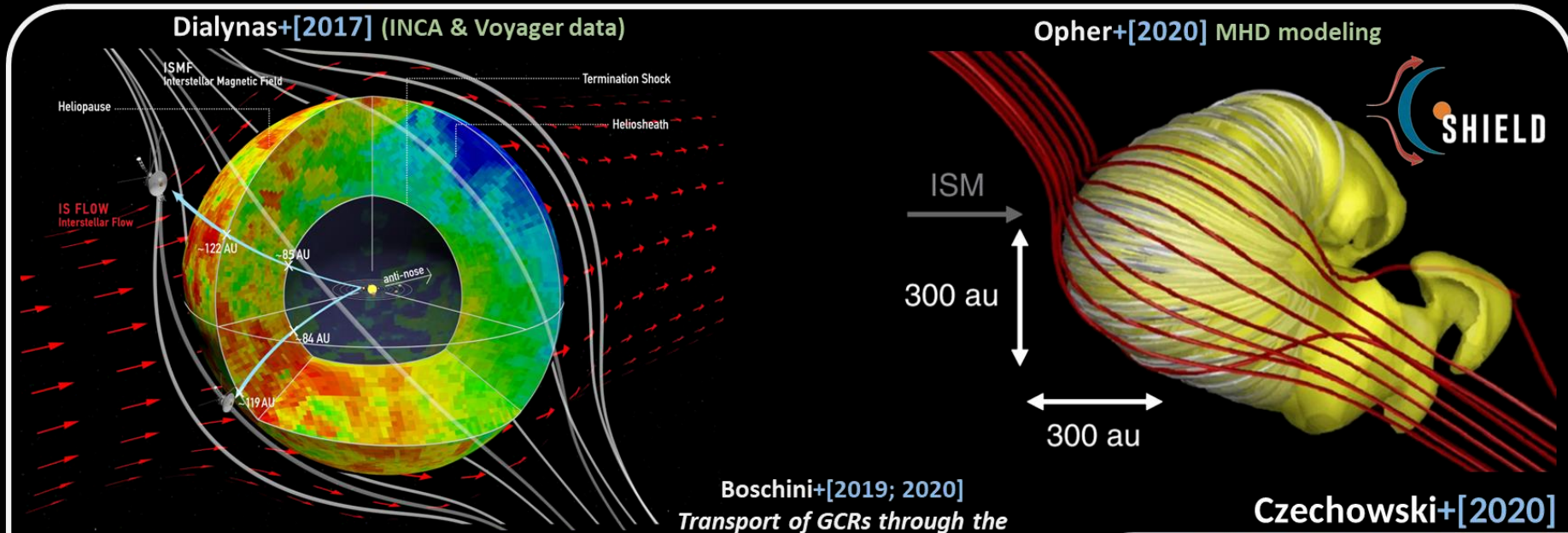
This sheet is more tilted with increasing of solar activity



Due to Solar rotation field line create na Archimedean spiral also known as “Parker Spiral”

The global shape of the heliosphere is still debated.

SHAPE OF THE GLOBAL HELIOSPHERE (1961-2022) // A ROUGH DIAMAGNETIC BUBBLE; A JETS STRUCTURE; A COMET...

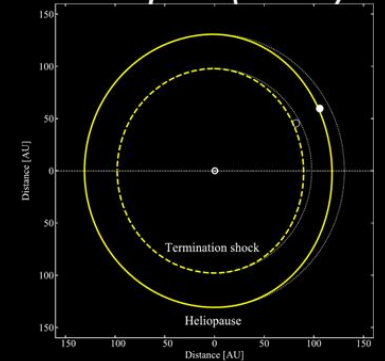


IBEX-Lo Data (Galli+[2016; 2017])
 [...] typical thickness of HS in downwind
 $\sim(220\pm110)$ AU.

IBEX-Hi Data (Reisenfeld+[2016])
 N. pole $L_{HS} \sim 210$ AU || S. pole $L_{HS} \sim 160$ AU

IBEX-Hi Data (Reisenfeld+[2021])
 [...] heliosphere extends at least ~ 350 AU tailwards

Boschini+[2019; 2020]
 Transport of GCRs through the heliosphere (HELMOD).





CR TRANSPORT IN THE HELIOSPHERE

What cause Solar Modulation?

Modulation along horizontal line

Energy loss
e.g. Force-Field Model

Vertical reduction in intensity

Convection-diffusion

Solar modulation consists of both of these processes that act simultaneously

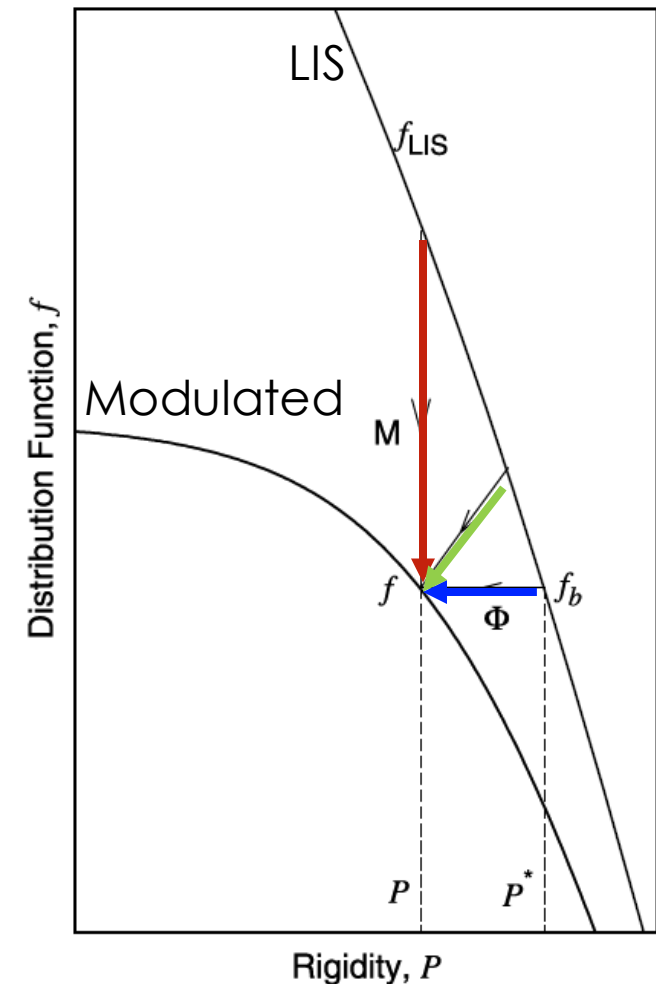


Figure 1. Graphical representation of the description of the modulation with the force field solution (horizontal line) and the convection-diffusion solution (vertical line). The sloped line represents the actual modulation as a combination of intensity reduction and energy loss.

Parker Equation

Parker (1965) proposed a Fokker-Planck like equation to describe the passage of charged particle in interplanetary medium

$$\frac{\partial U}{\partial t} = -\nabla \cdot (U\vec{V}) + \nabla \cdot [\tilde{K} \cdot \nabla U] + \frac{(\nabla \cdot \vec{V})}{3} \frac{\partial}{\partial T} (\alpha_{\text{rel}} TU)$$

U = particle density for unit space and kinetic energy

Convection

Presence of the solar wind moving out from the Sun

Diffusion

Small Scale Magnetic Field irregularity

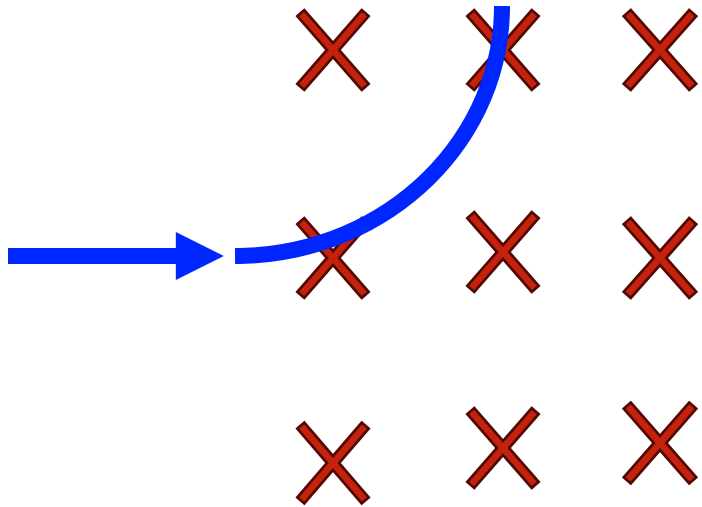
Drift

Large Scale structure of magnetic field (e.g. gradients)

Energetic Loss

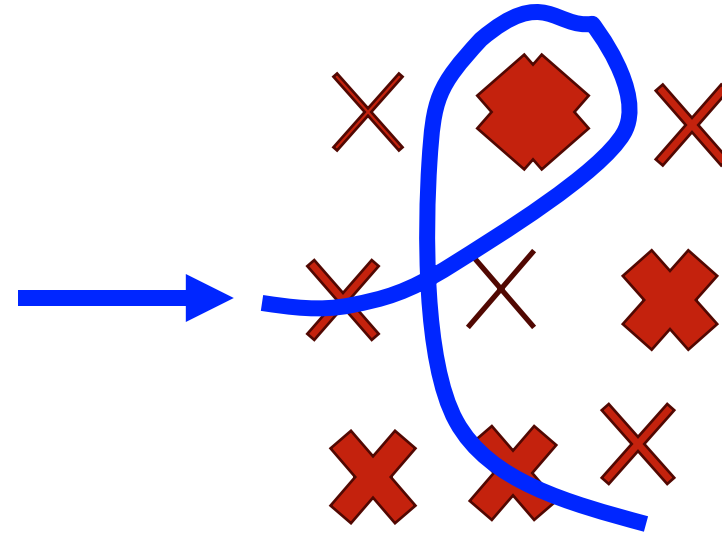
Due to adiabatic expansion of the solar wind

DIFFUSION IN TURBULENT MAGNETIC FIELD



Uniform magnetic field

**Motion defined by
Lorentz's equation**



Nonuniform magnetic field

Motion is not well defined

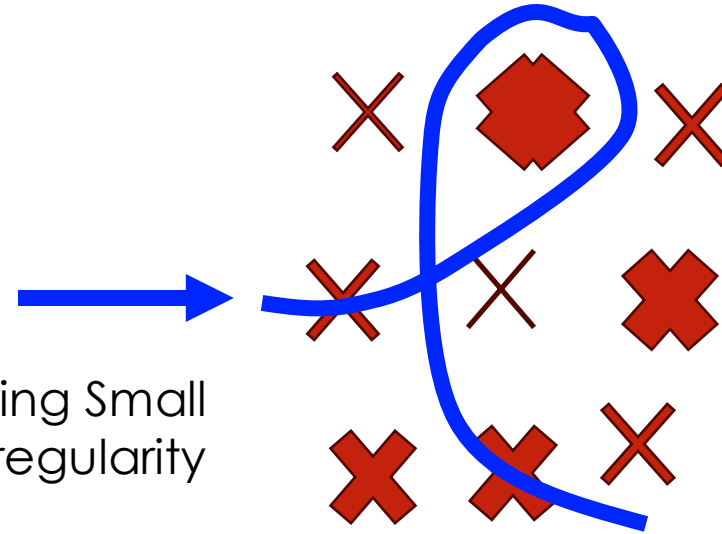
DIFFUSION IN TURBULENT MAGNETIC FIELD

$$\frac{\partial U}{\partial t} = \nabla \cdot [\tilde{\kappa} \cdot \nabla U]$$

$$\hat{\kappa} = \begin{pmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp} \end{pmatrix}$$

Diffusion due to scattering Small Scale Magnetic Field irregularity

Drift motion due to Large Scale structure of magnetic field



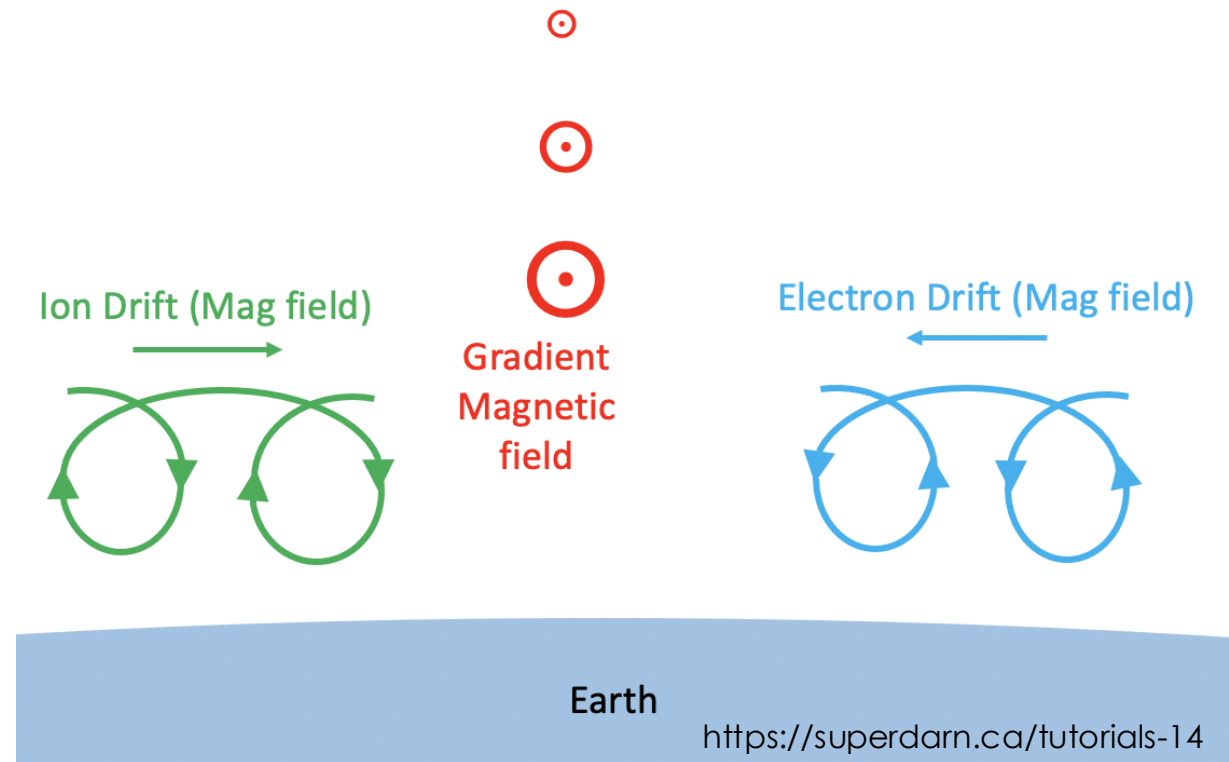
Nonuniform magnetic field

Motion is not well defined

MAGNETIC DRIFT

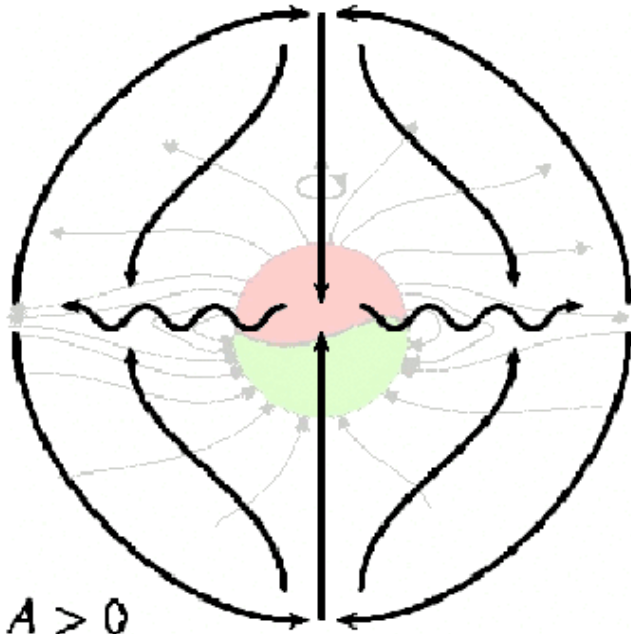
$$\hat{\kappa} = \begin{pmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp} \end{pmatrix}$$

Drift motion due to Large Scale structure of magnetic field

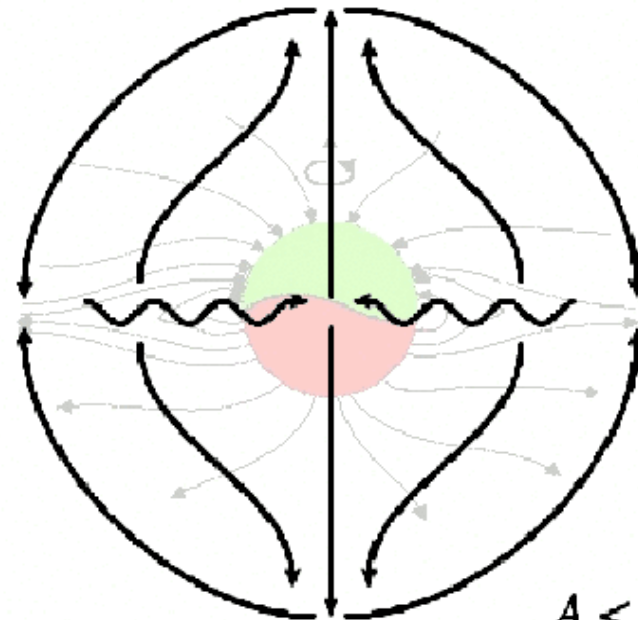


MAGNETIC DRIFT IN THE HELIOSPHERE

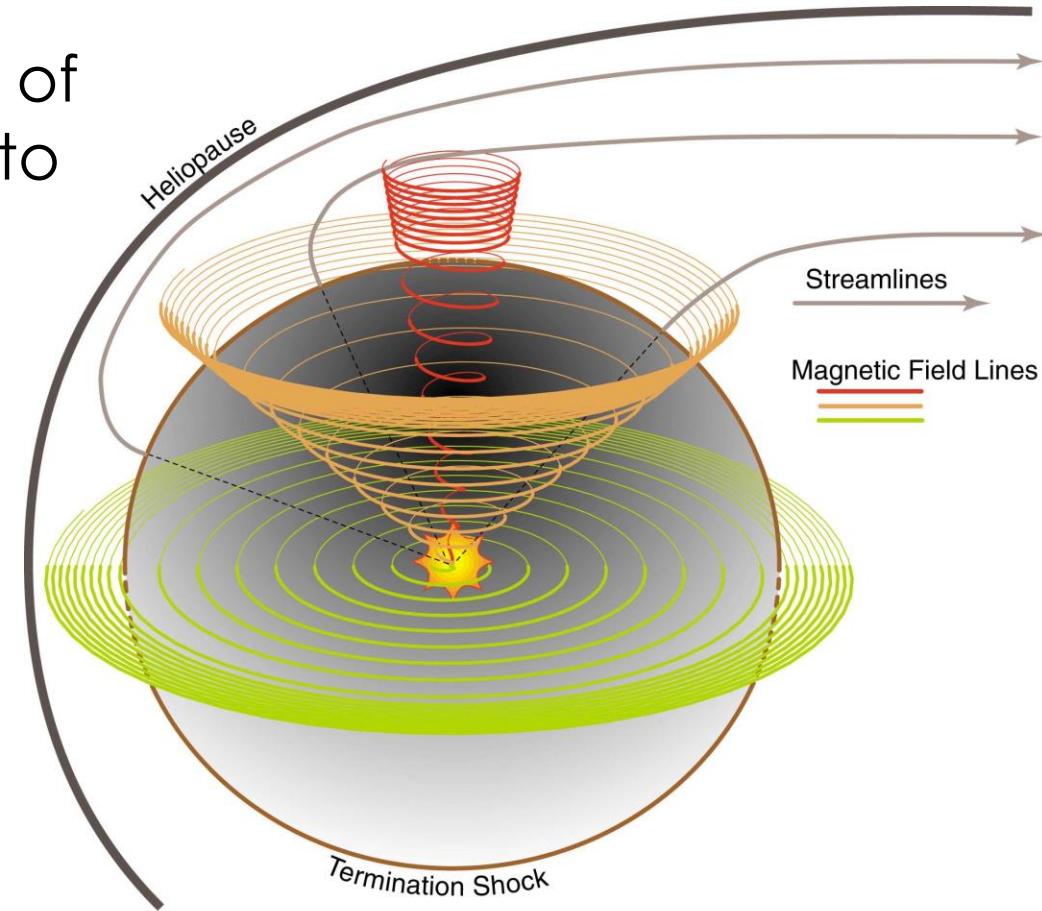
Inside the heliosphere, drift motion create flow of particles from/toward the inner part according to the solar polarity & particle charge



$A > 0$

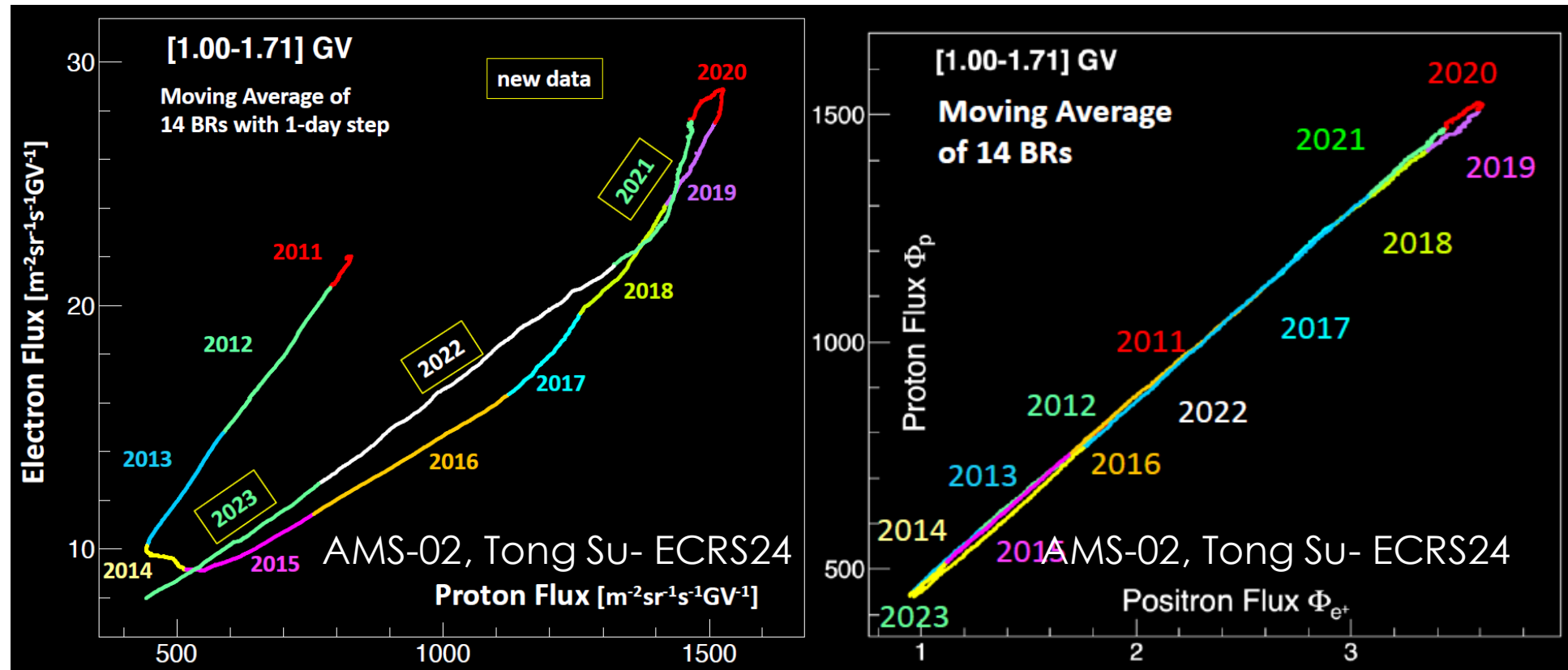


$A < 0$

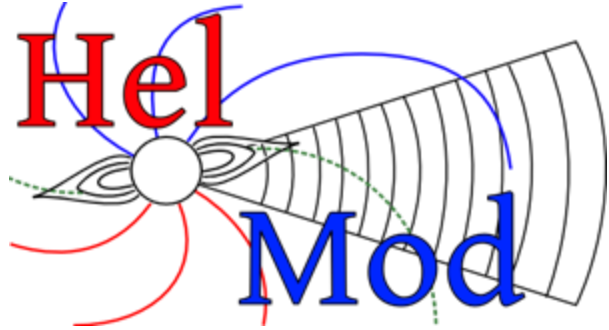


MAGNETIC DRIFT IN THE HELIOSPHERE

This process affect differently particles with opposite charge

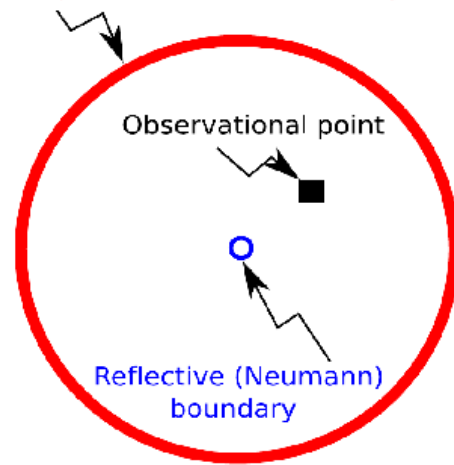


THE **HELIO**SPHERE **MOD**ULATION MODEL

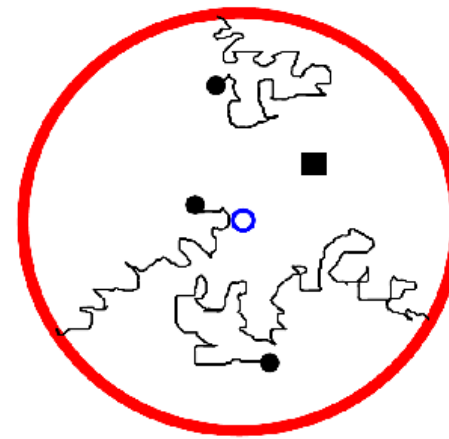


Is a Monte Carlo code that solves numerically the Parker equation in a 2D approximation with the *backward-in-time Stochastic Differential Equation (SDE)* approach.

Heliopause (Dirichlet boundary)

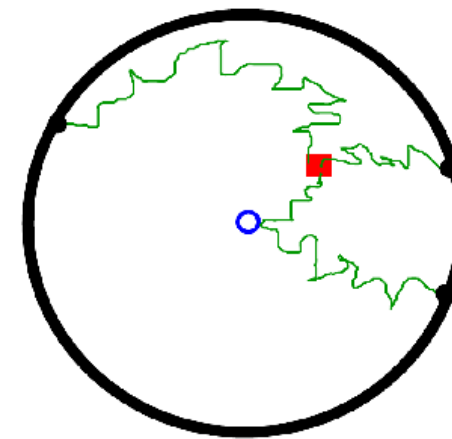


Time-forward integration



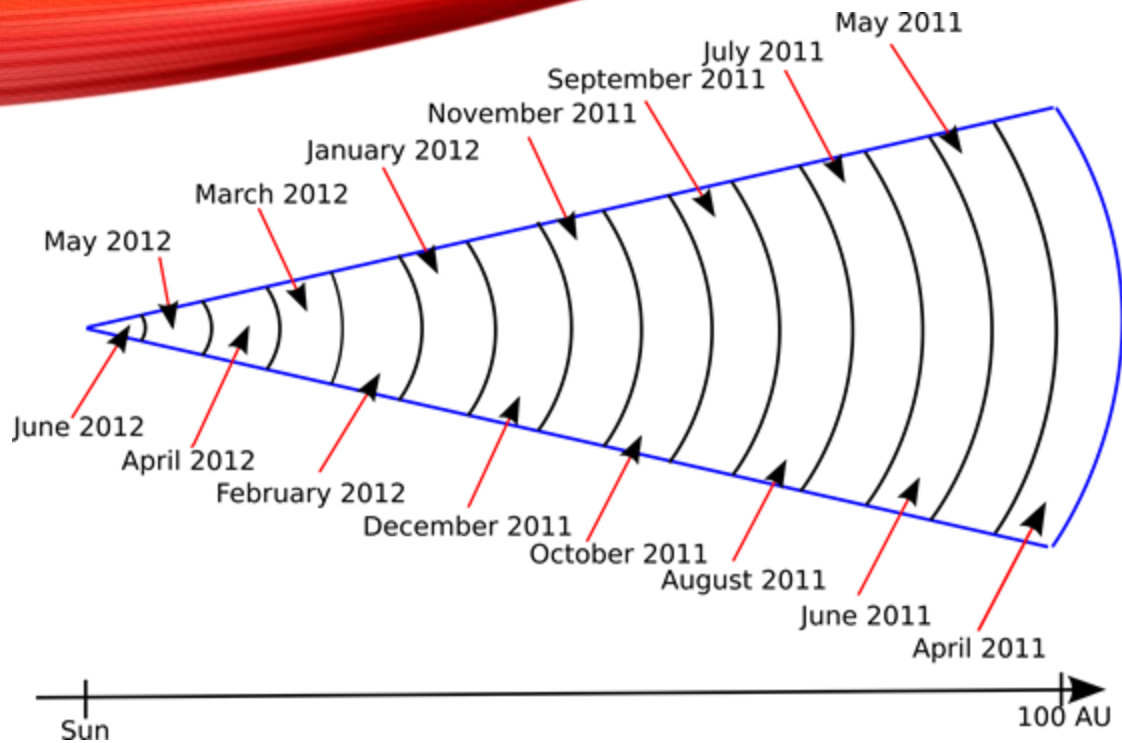
Particles released uniformly from the heliopause

Time-backward integration



Particles released from the observational point

THE COMPUTATIONAL MODEL



**We divide the Heliosphere
in 15 regions,**

each one equivalent to the average of solar activity in the periods before the experiment

The Heliosphere radius and shape varies with time, according with the Sun Activity

Details of the model HelMod-4 (v5.1) can be found in *Boschini et al. Adv. Space Res. (2024)*

Parameters in each region are

Tilt angle
of the Neutral Sheet

Magnetic Field
Magnitude at Earth

Solar Wind Speed

Diffusion
coefficients

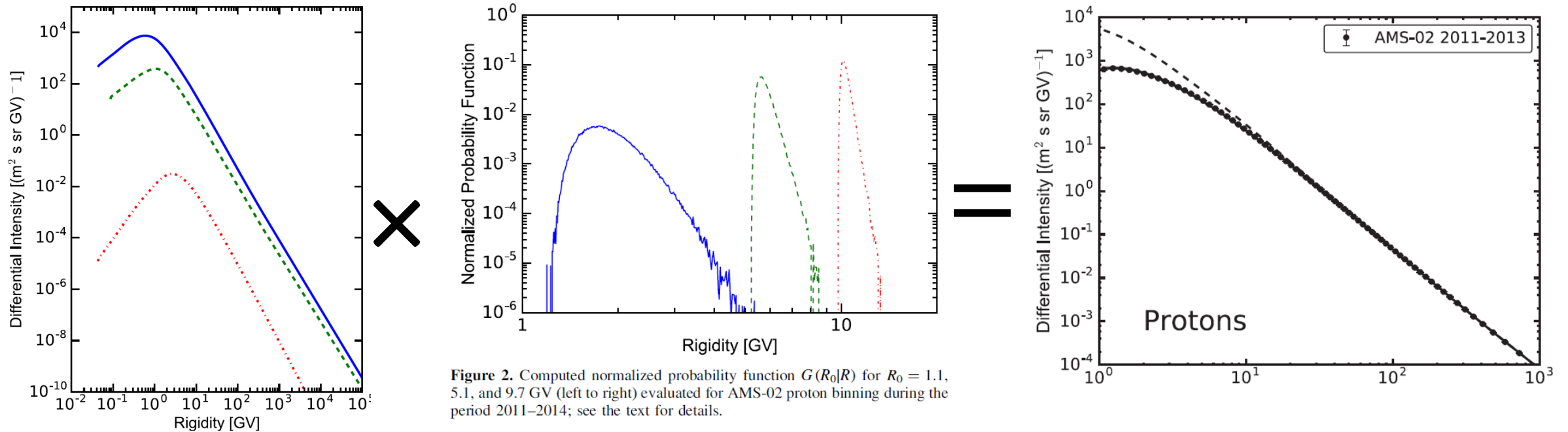
<http://wso.stanford.edu>

Wilcox Solar Observatory

<https://omniweb.gsfc.nasa.gov/>

database

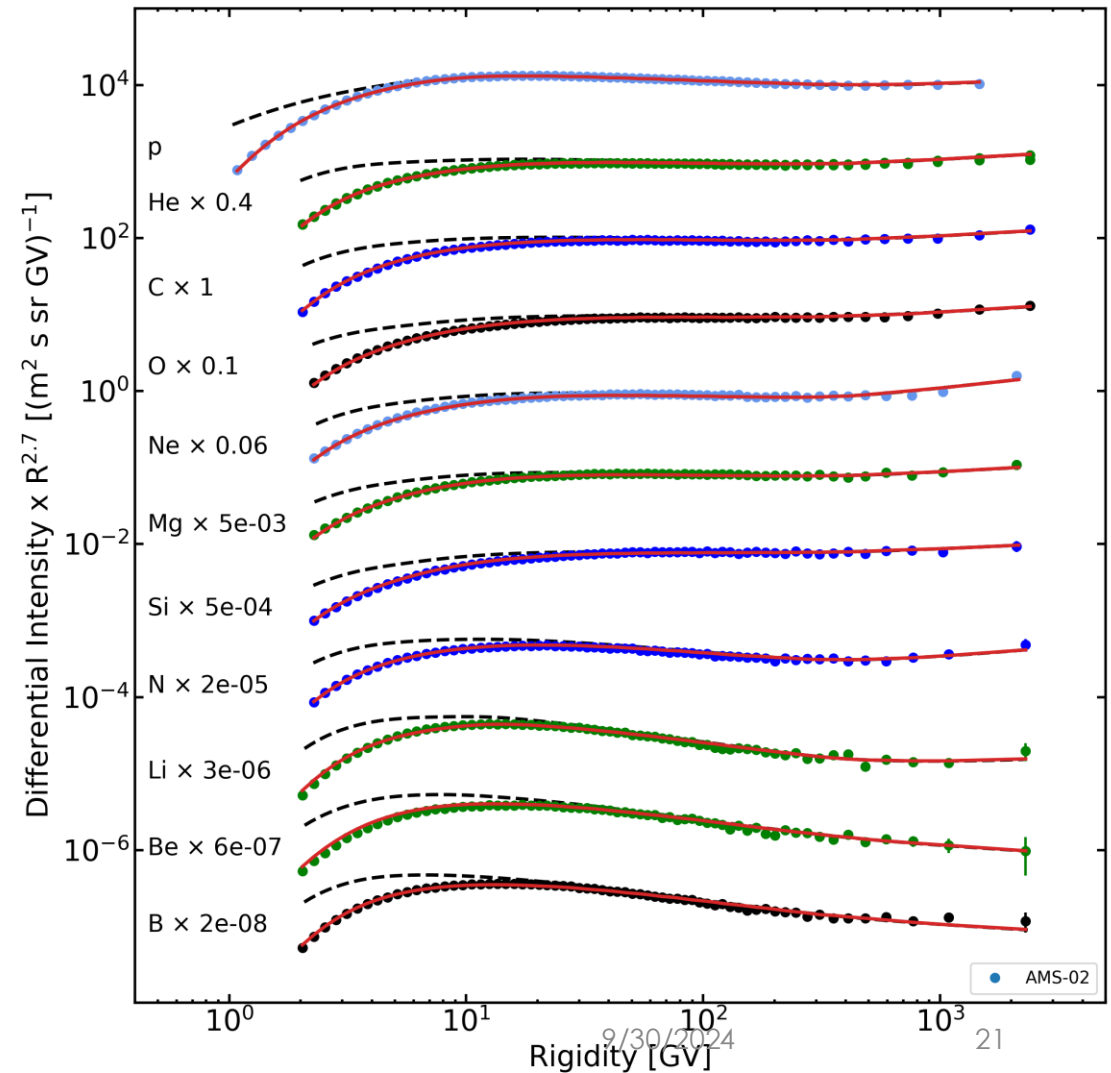
HelMod provide the transformation functions needed to modify the Local Interstellar Spectra (i.e. outside heliosphere) into modulated spectra



LIS is a critical element in evaluating the modulated spectra



The GALPROP-HelMod joint effort allow to intercalibrate the two model in a energy region without direct measurements (outside heliosphere)



- HelMod can reproduce ions:
- along the full 22 years solar cycle
 - At several solar distance
 - Outside the ecliptic plane

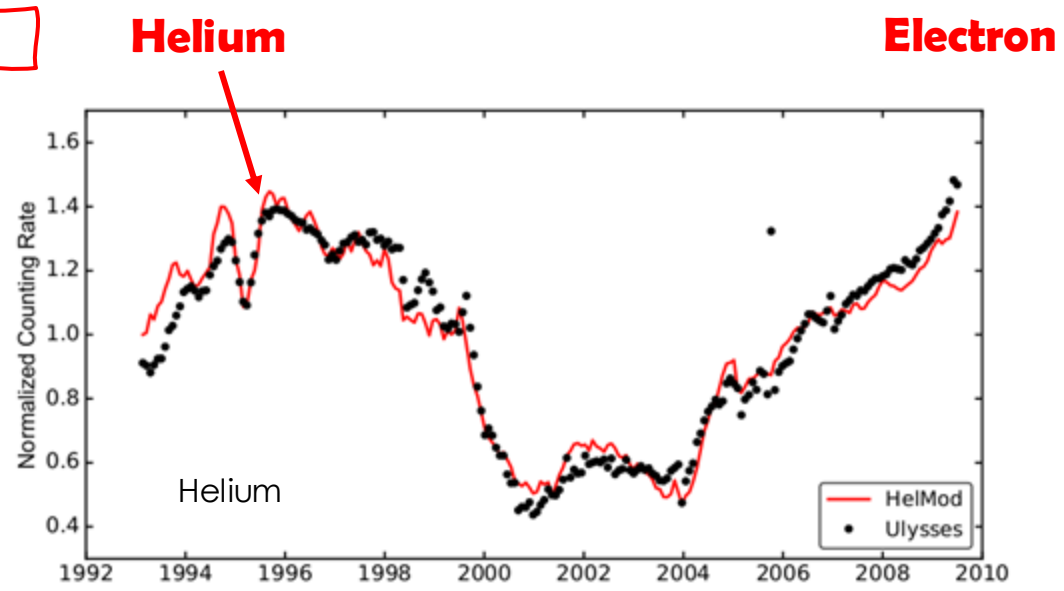
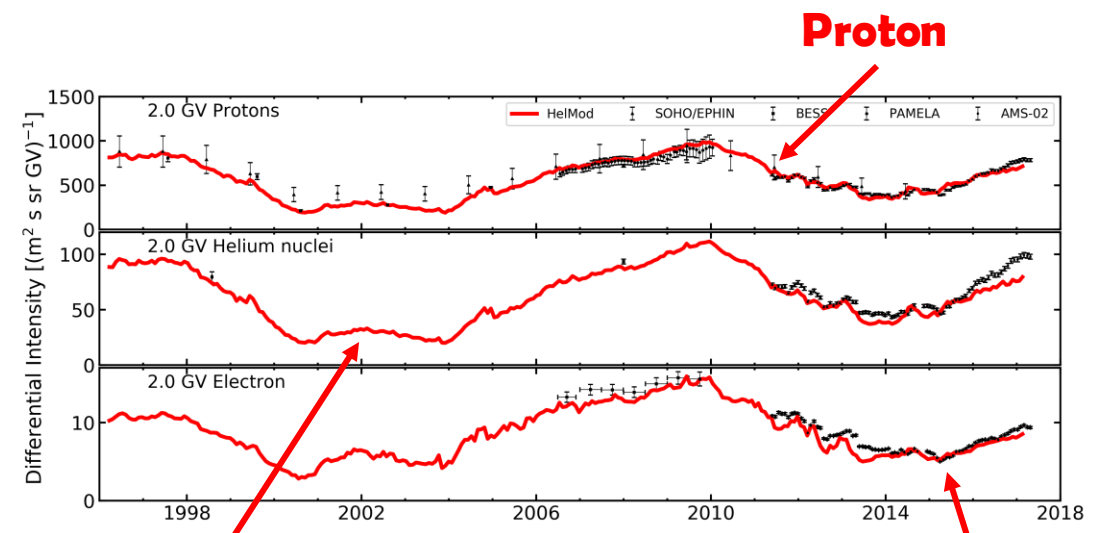
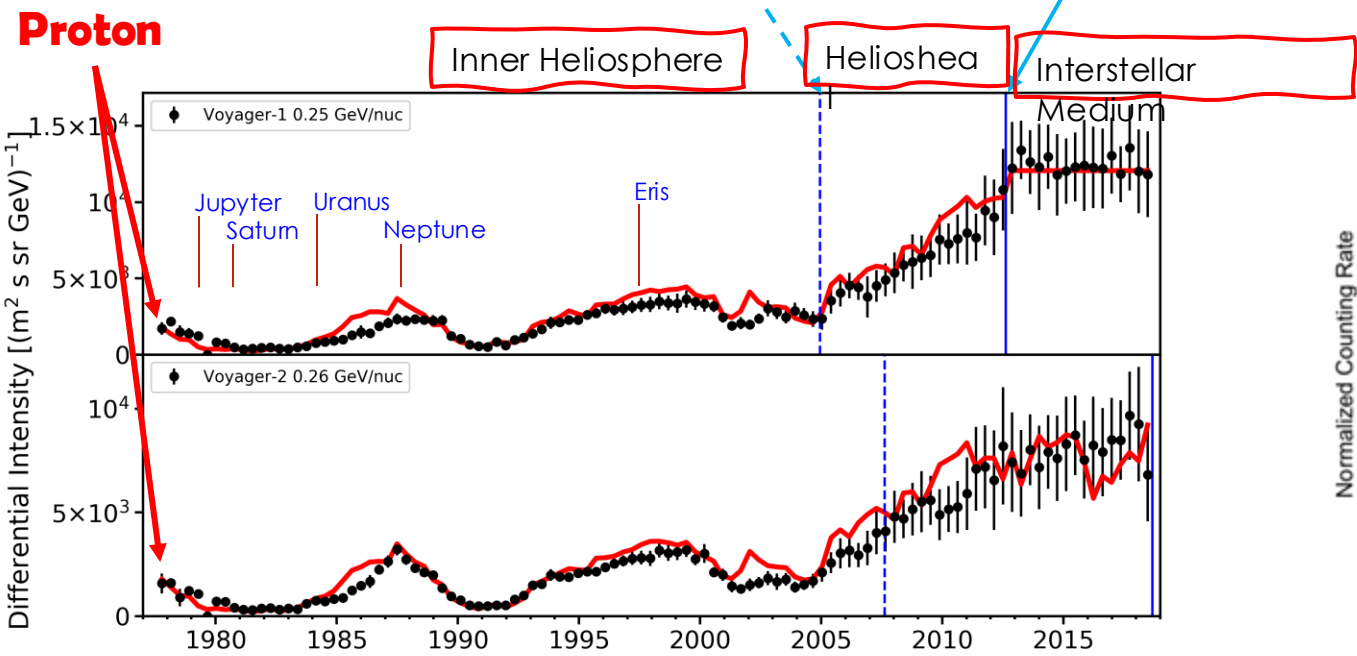
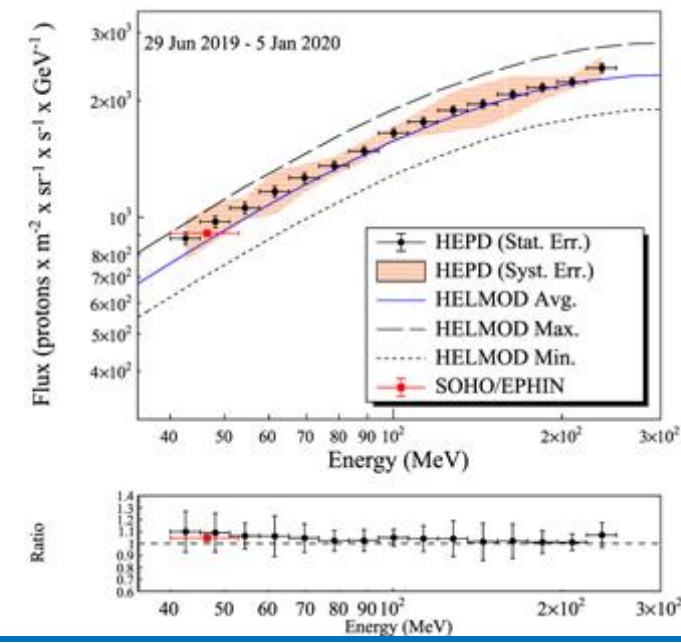
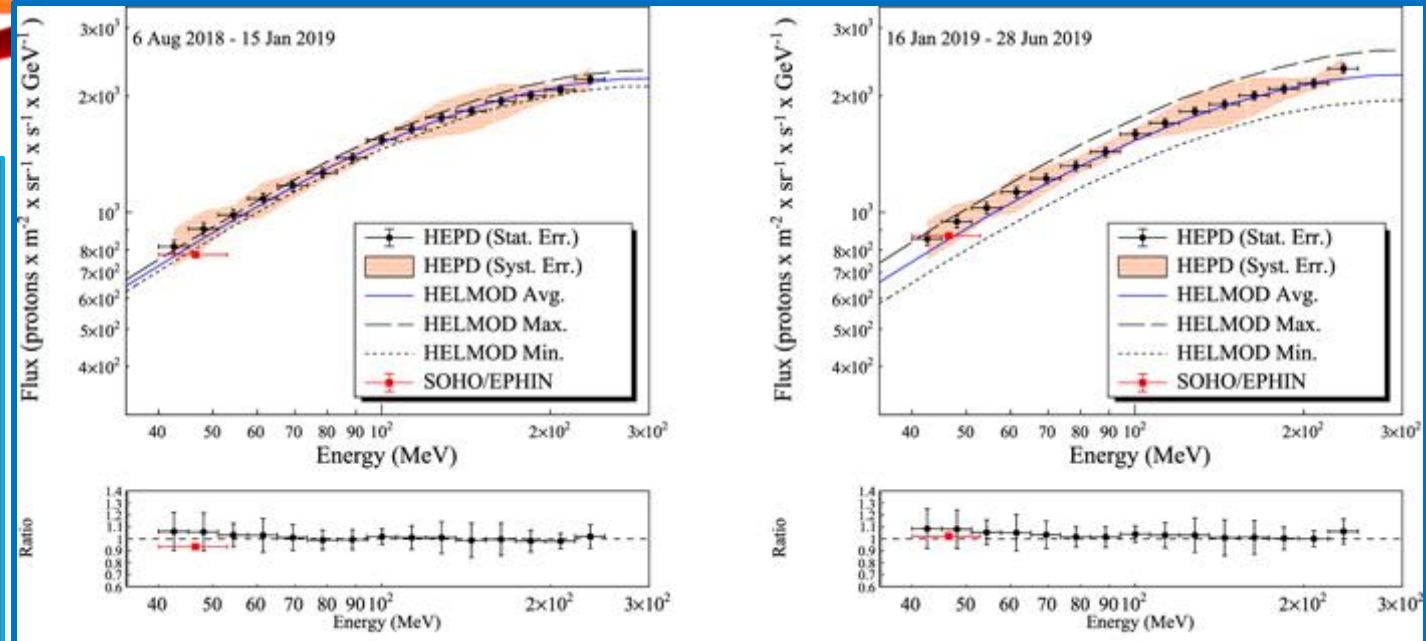
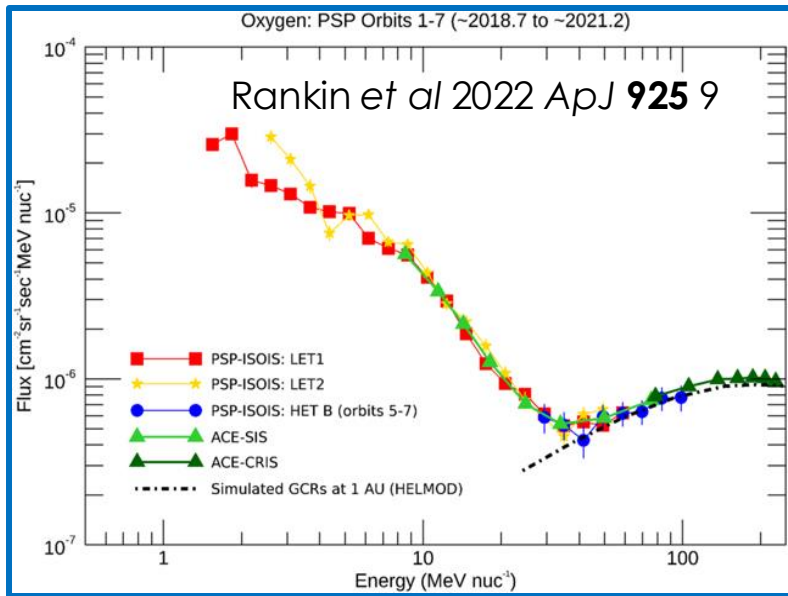
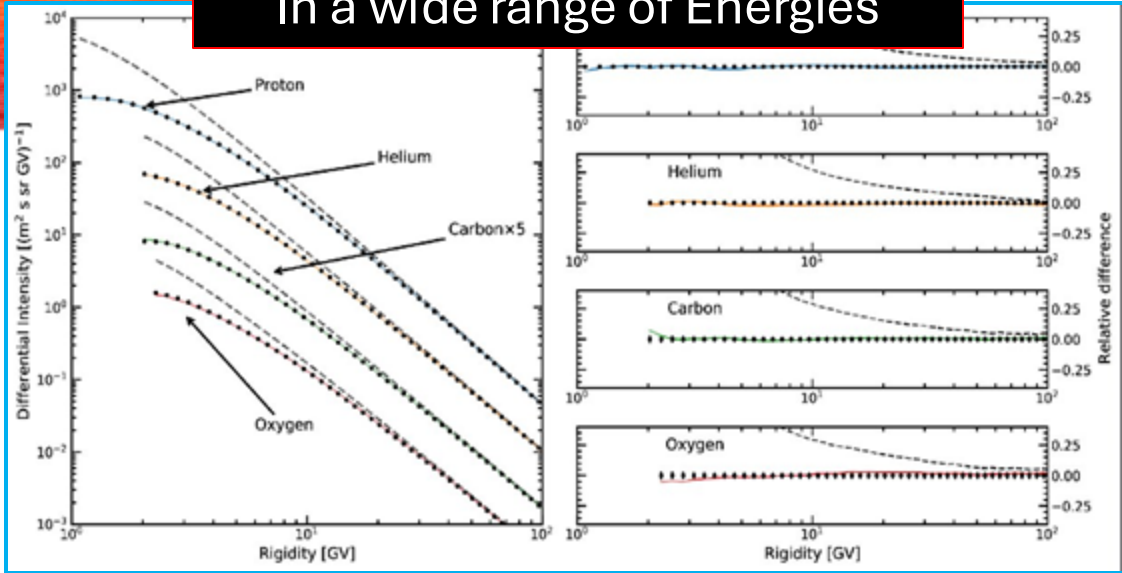


Figure 8: Helium normalized counting rate measured by Ulysses (full black circles) at $\pm 80^\circ$ of solar latitude and 1 to 5 AU compared with the 1 GeV energy modulated spectrum from HELMOD code (red solid line) as function of time.

In a wide range of Energies



Bartocci et al ApJ, 901:8, 2020

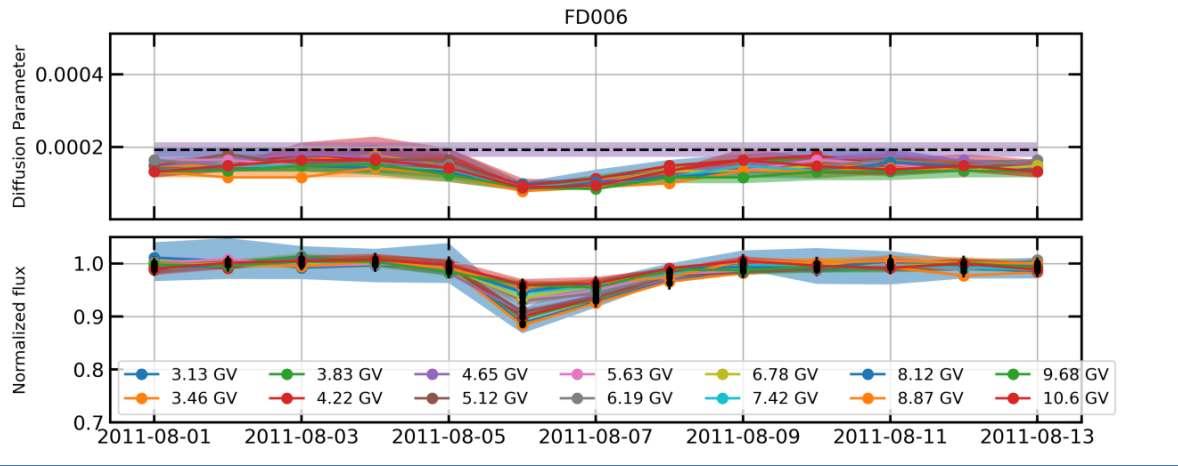


APPLICATIONS AND IMPLICATIONS

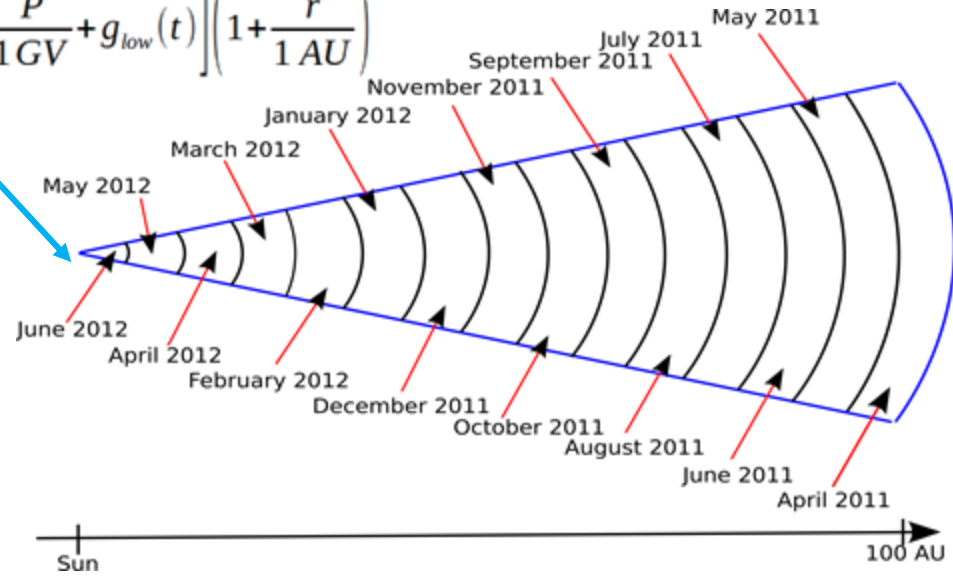
METHODOLOGY

In this work, we modify the value of K_0 parameter in the first shell (i.e. ~ 5 AU) emulating the effect of a local perturbation around Earth and not affecting the rest of the heliosphere

Using a scanning strategy we vary the value of K_0 for each energy and day independently to fit AMS-02 daily spectra



$$\frac{K_{||}}{K_0} = \frac{\beta}{3} \left[\frac{P}{1GV} + g_{low}(t) \right] \left(1 + \frac{r}{1AU} \right)$$



FD	Type	Time	MAR	Ampl
006	ICME	2011/08	16.6	13.6
010	ICME	2011/10	48.5	10.3
017	ICME	2012/03	33.5	35.1
058	ICME	2014/02	33.5	10.0
121	ICME	2017/09	22.8	17.7

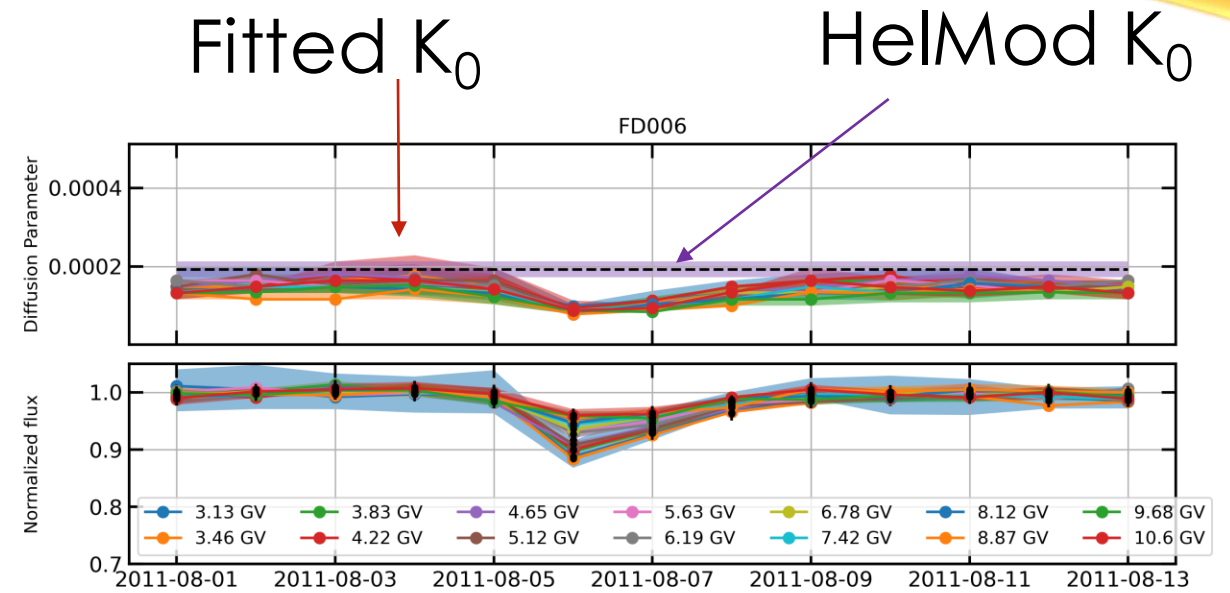
Events from Wang et al ApJ 950:23, 2023.

We apply our study to the five most relevant FD events seen by AMS-02. **For each FD we select a time windows that influence up to 4 quiet days before and after the event.**

PRELIMINARY RESULTS

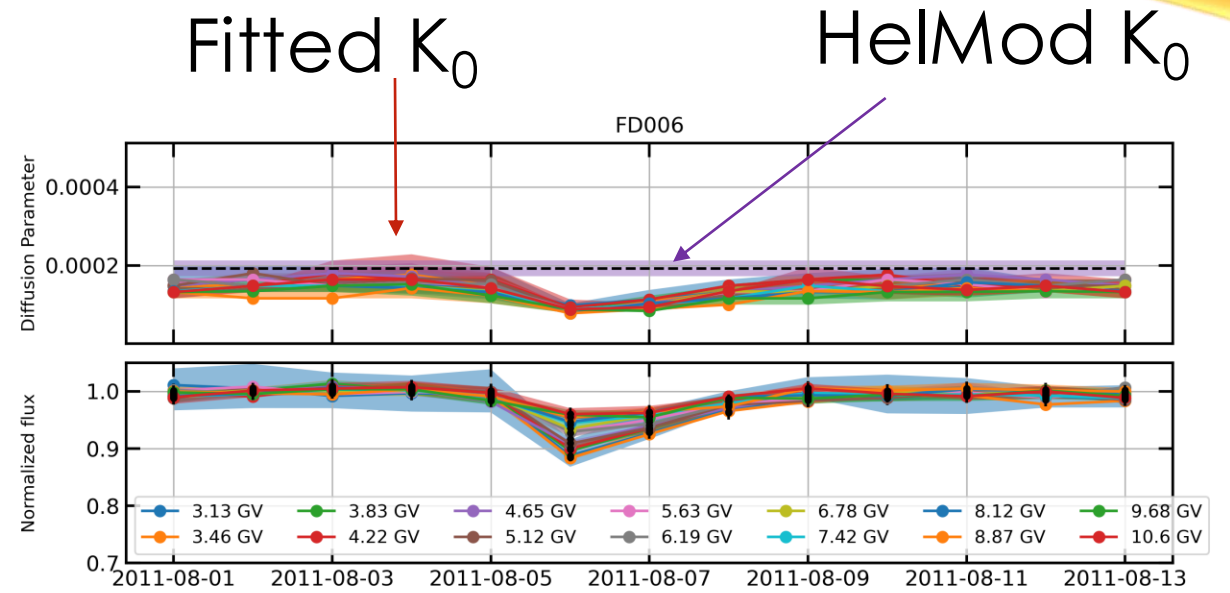
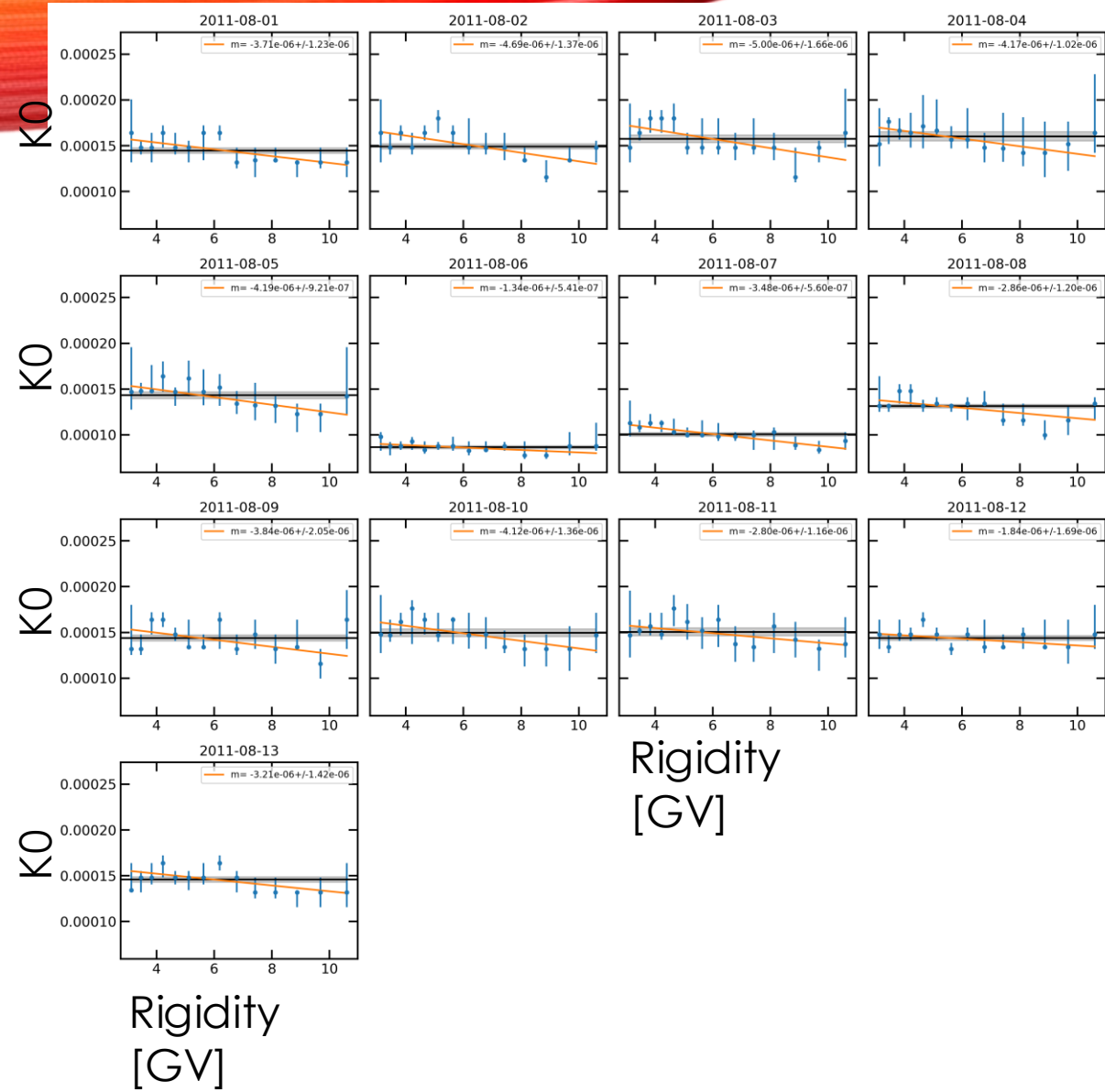
We notice that the daily K_0

1. decrease following the forrush decrease
2. has similar values for all considered rigidities



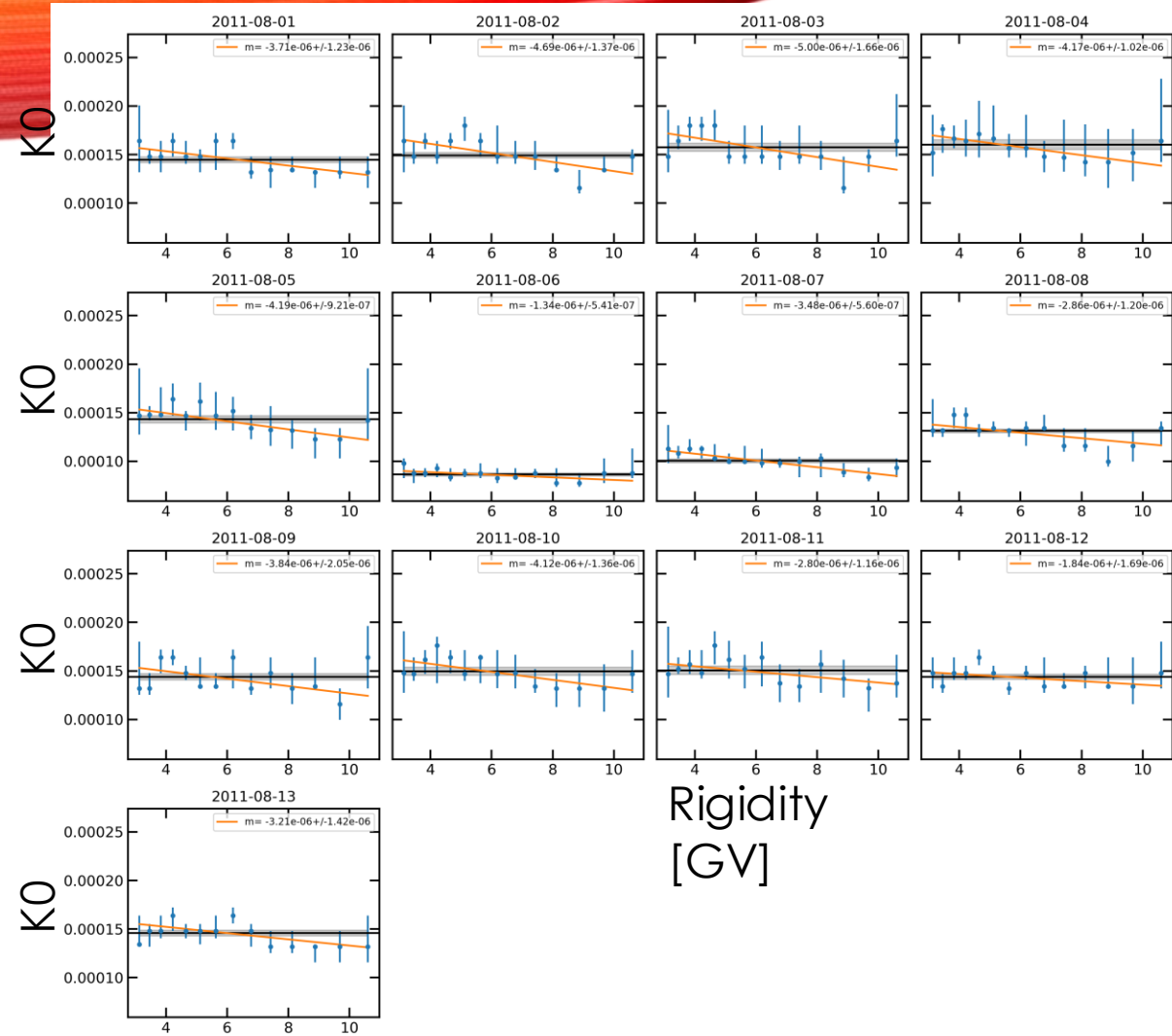
Black point = AMS-02 normalized data
Color band = Simulations best fit

PRELIMINARY RESULTS



We found a linear correlation between K_0 and Rigidity. This may be due to LIS uncertainties or an incorrect rigidity dependence of diffusion tensor.

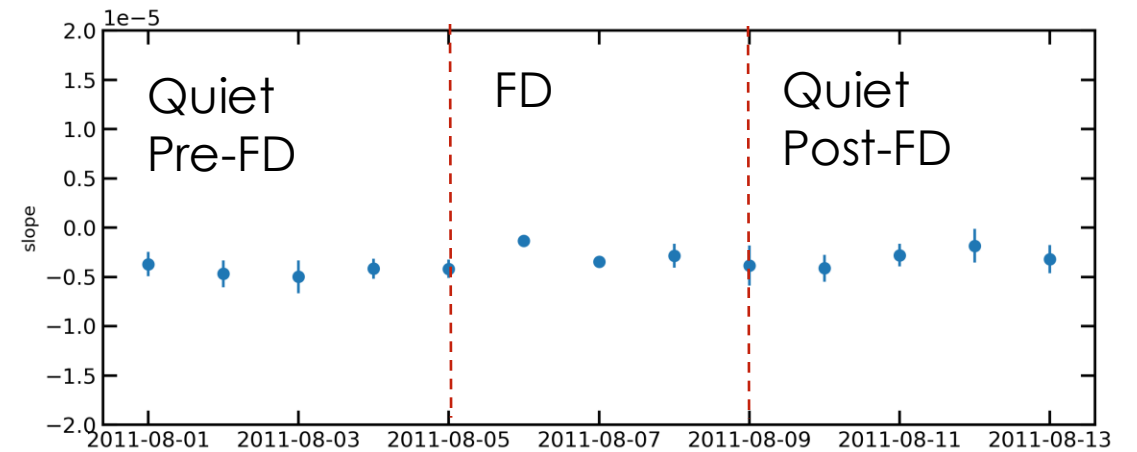
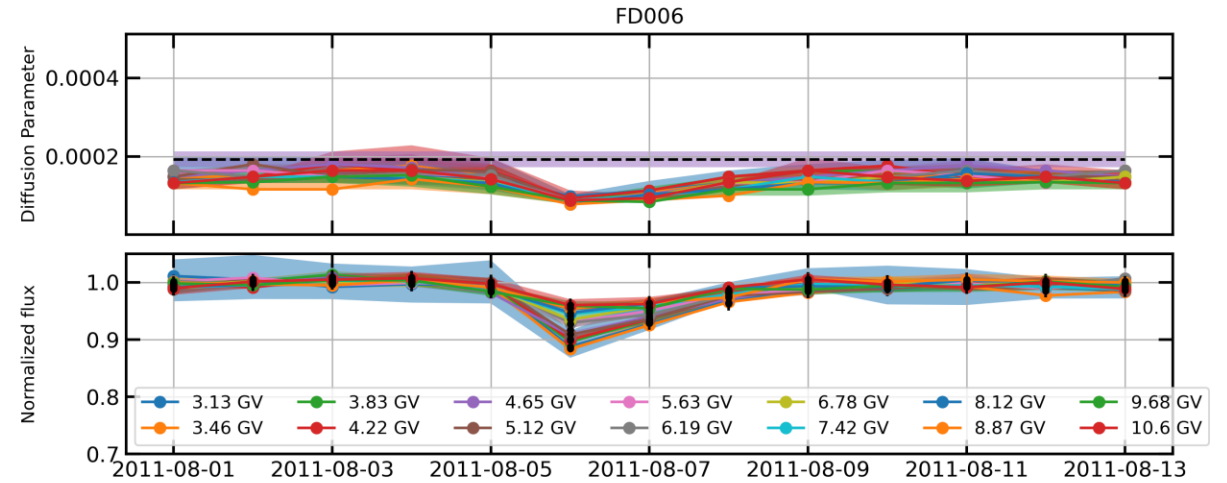
PRELIMINARY RESULTS



Rigidity
[GV]

This Linear Correlation maintains the same slope both quiet and perturbed days.

S. Della Torre - ECRS24

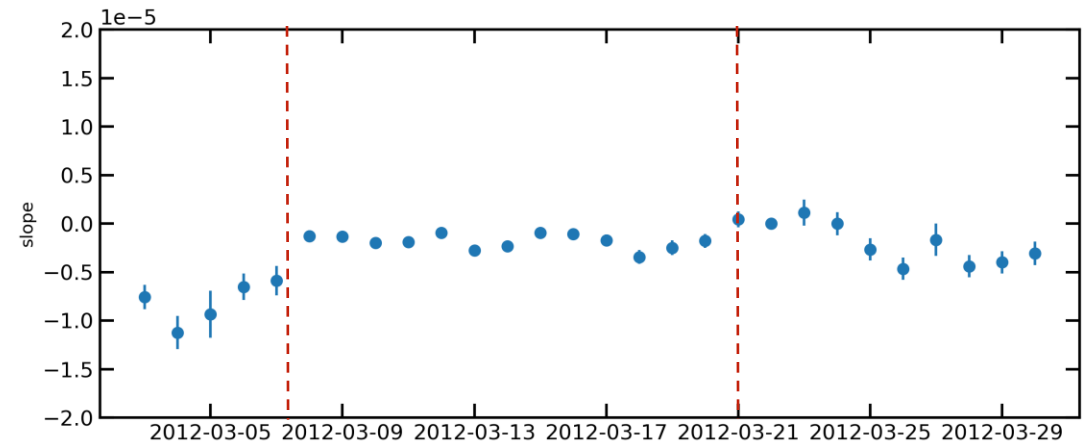
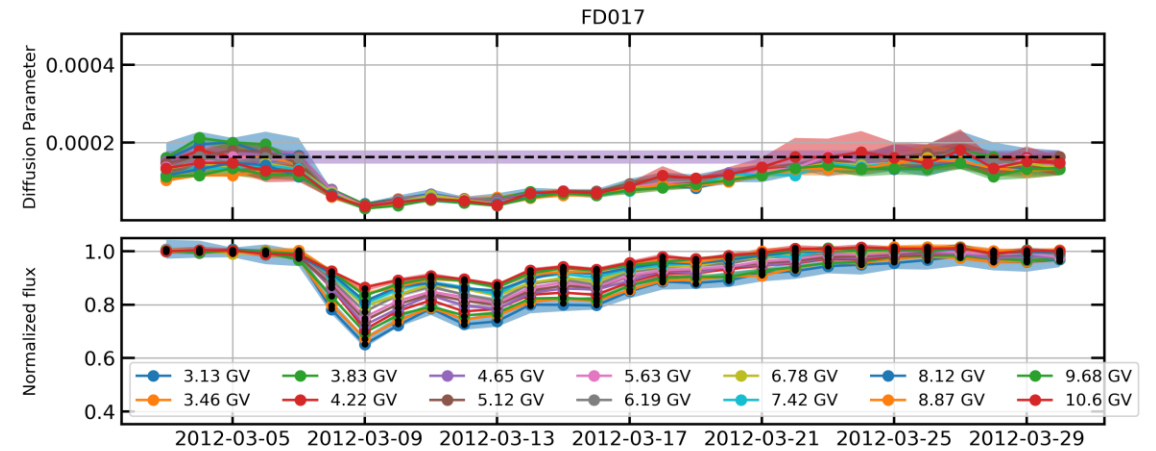
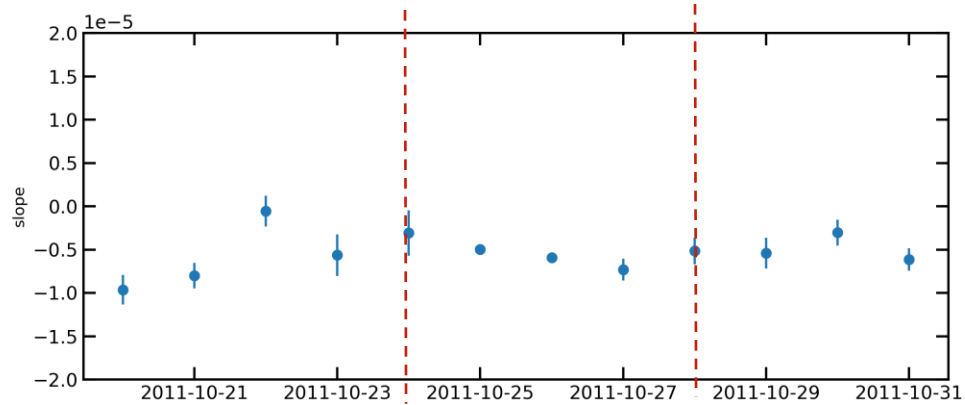
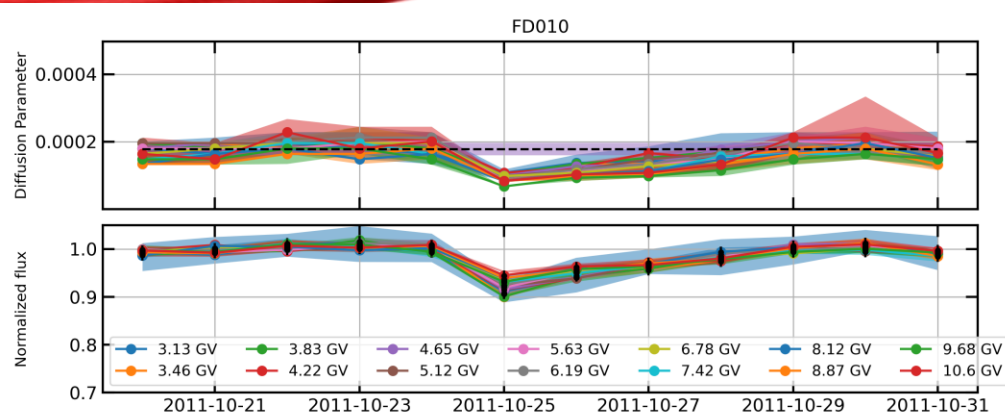


Sept 24th, 2024

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PRELIMINARY RESULTS

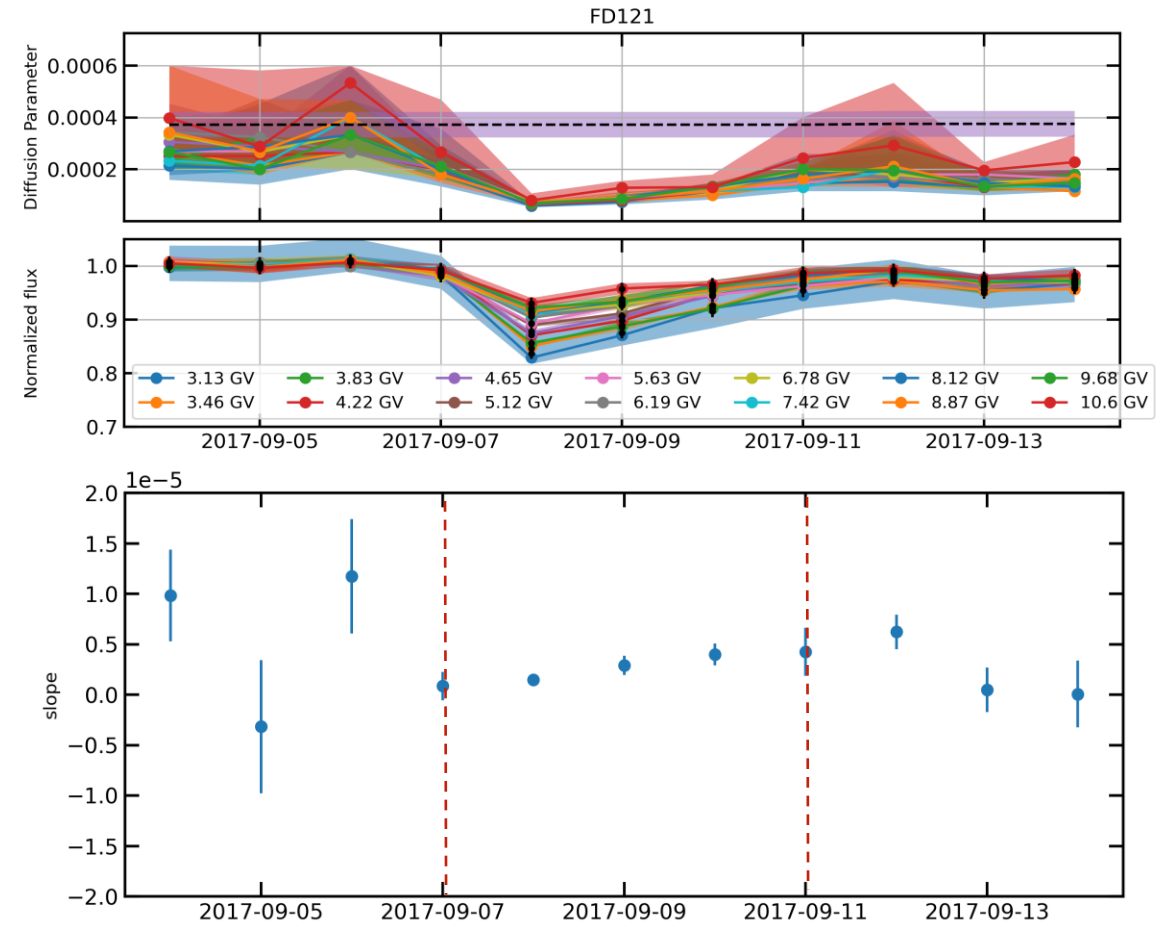
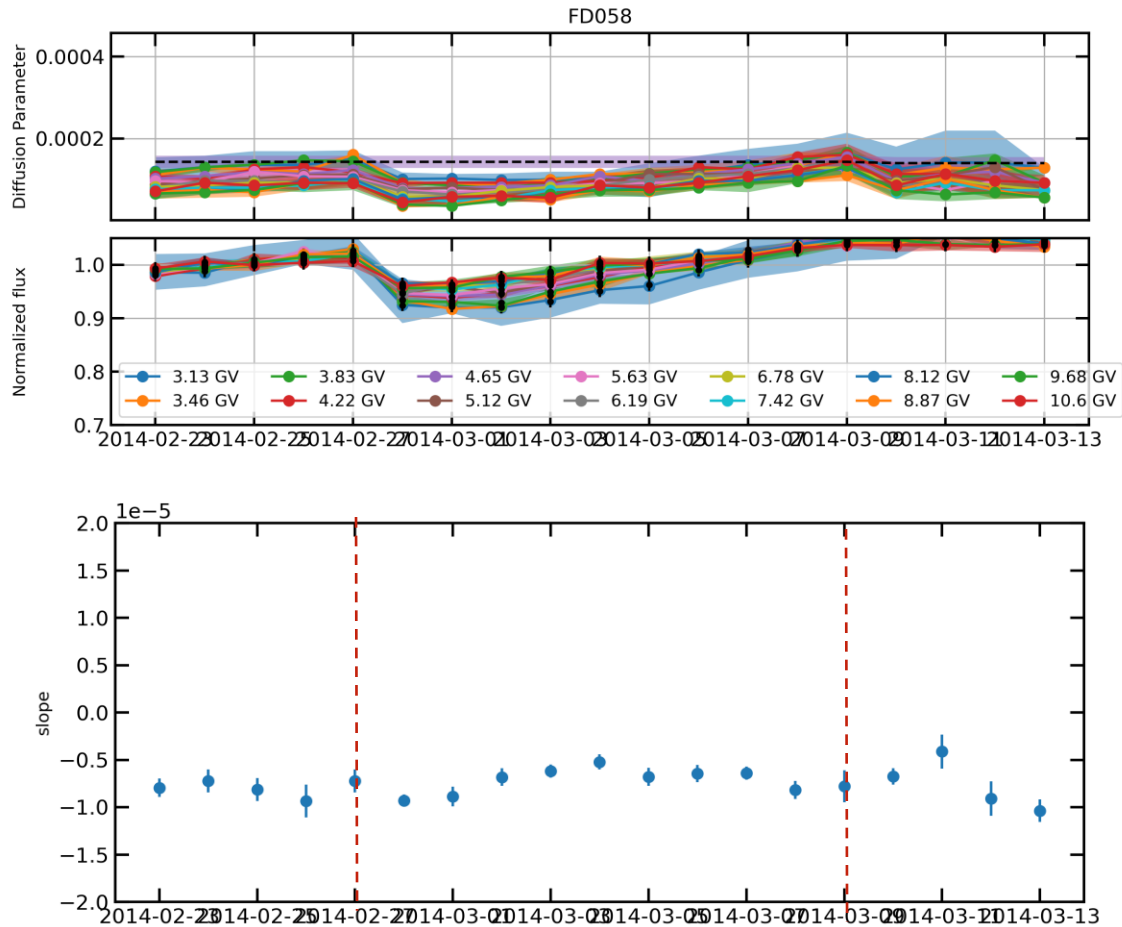
Similar results are observed on others FD of the sample



This Linear Correlation maintains the same slope both quiet and perturbed days.

PRELIMINARY RESULTS

Similar results are observed on others FD of the sample

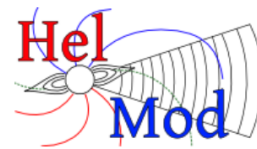


This Linear Correlation maintains the same slope both quiet and perturbed days.

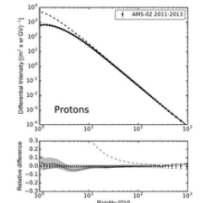


WEBSITE

We developed a website where selected results from HelMod are provided to users



HelMod-4 The Heliospheric Modulation Model Online Calculator (version 5.1)



- Home
- HelMod Calculators
- Local Interstellar Spectra
- Bibliography
- Results
- News
- History and Citation
- Who in HelMod

You are here: Home

Website Search

- HelMod Long Write Up
- The HelMod Model
 - HelMod Heliosphere
 - Heliospheric boundaries in HelMod
 - Heliospheric Magnetic Field
 - Diffusion tensor
 - Monte Carlo Integration
 - Current and Historical Values of default parameters
 - Interpolation Functions for

Propagation of Galactic Cosmic Rays through the Heliosphere with HelMod: the Treatment of Solar Modulation

Website latest update on Apr 11th, 2024

Welcome to HelMod Website supported within the framework of space radiation environment activities of **ASIF**, i.e., ASI (*Italian Space Agency*) Supported Irradiation Facilities. The *helmod website* was initially created in February 2012. In these pages, you can find information about the Solar Modulation Model for the propagation of Galactic Cosmic Rays through the Heliosphere from the Heliopause down to Earth.

As advertised on the *GALPROP website*, *HelMod website* can be used as a service package to seamlessly calculate the effects of the heliospheric modulation for *GALPROP* output files.

HelMod is a 2D Monte Carlo model to simulate the solar modulation of galactic cosmic rays. The model is based on the **Parker transport equation** which contains diffusion, convection, particle drift and energy loss. Following the evolution of the solar activity in time, we are able to modulate the local interstellar spectra (LIS) of cosmic ray species, assuming their isotropy beyond the termination shock, down to the Earth's location inside the heliosphere.

The current HelMod Version is the result of a continuous development since 1998 (*Monte-Carlo approach to Galactic Cosmic Ray propagation in the Heliosphere*, Nucl. Phys B-Proc Sup). The latest review on HelMod was published in 2018 (*Propagation of Cosmic Rays in Heliosphere: the HelMod Model*. Adv. Space Res. 62(10):2859 - 2879, 2018)

In the present website version, a solar modulation calculator is available for Cosmic Rays experiments carried out during solar Cycle 23 and 24. In the 2D-HelMod code version 1 was implemented the standard Parker field with modification in the polar region, the dependence on the particle drift, the Solar Wind description revisited for high and low activity periods, the Heliosphere divided in regions related to spatial propagation of

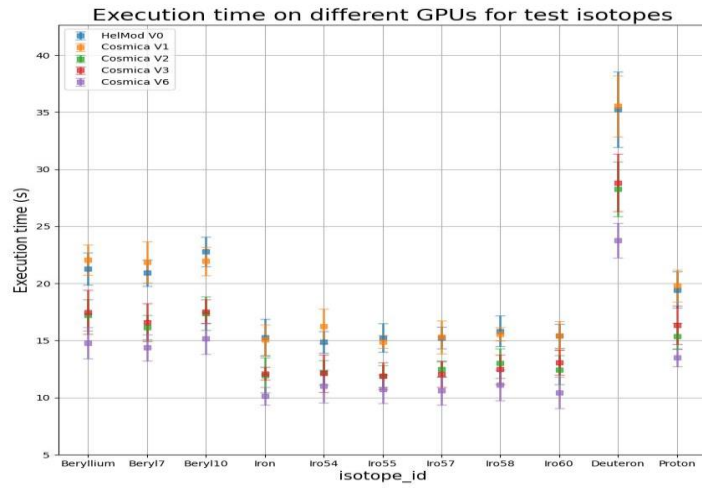
Is under development a new service that allows to run "customized" HelMod Simulation on MiB GPU local Farm

COSMICA AND BEYOND

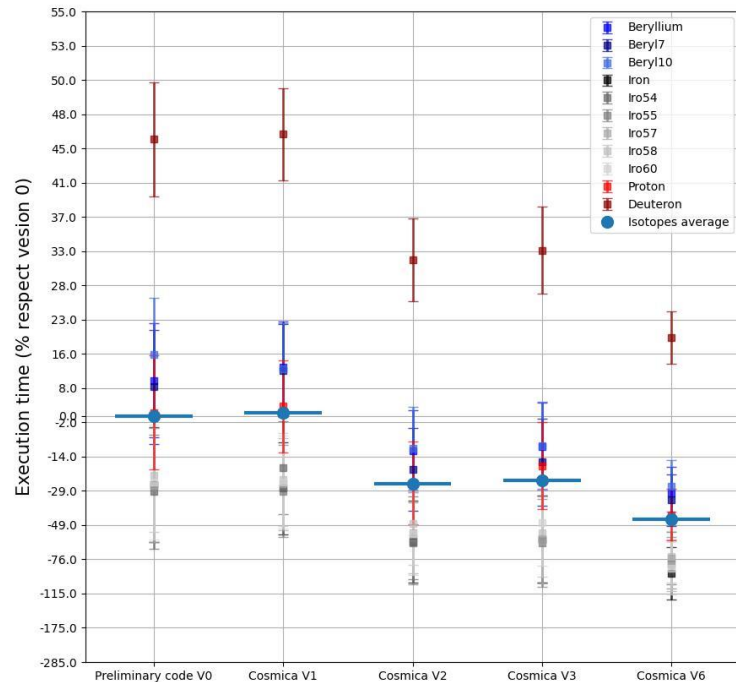
We are working on the next generation of modulation code, that exploit the potential of GPUs

The objectives are

- Made the code to run faster and optimized on GPU (and HPC in general) - *in work*
- Set up and analysis environment to search for modulation parameters with state-of-art algorithm – *started with Ca' Foscari university*
- Review the model descriptions with state-of-art theories – *planned*
- Make the code public for world-wide collaboration toward a common maintained Modulation model – *planned*



Execution time of code versions



-50%



CONCLUSIONS

- Solar modulation is an interesting phenomena that involves several disciplines
- The study of Solar modulation allows to better understand the space close to Earth
- An accurate model for solar modulation is needed in order to study fine structures in actual measurement
- New model and tools are in development, so stay tuned or propose yourself for collaboration



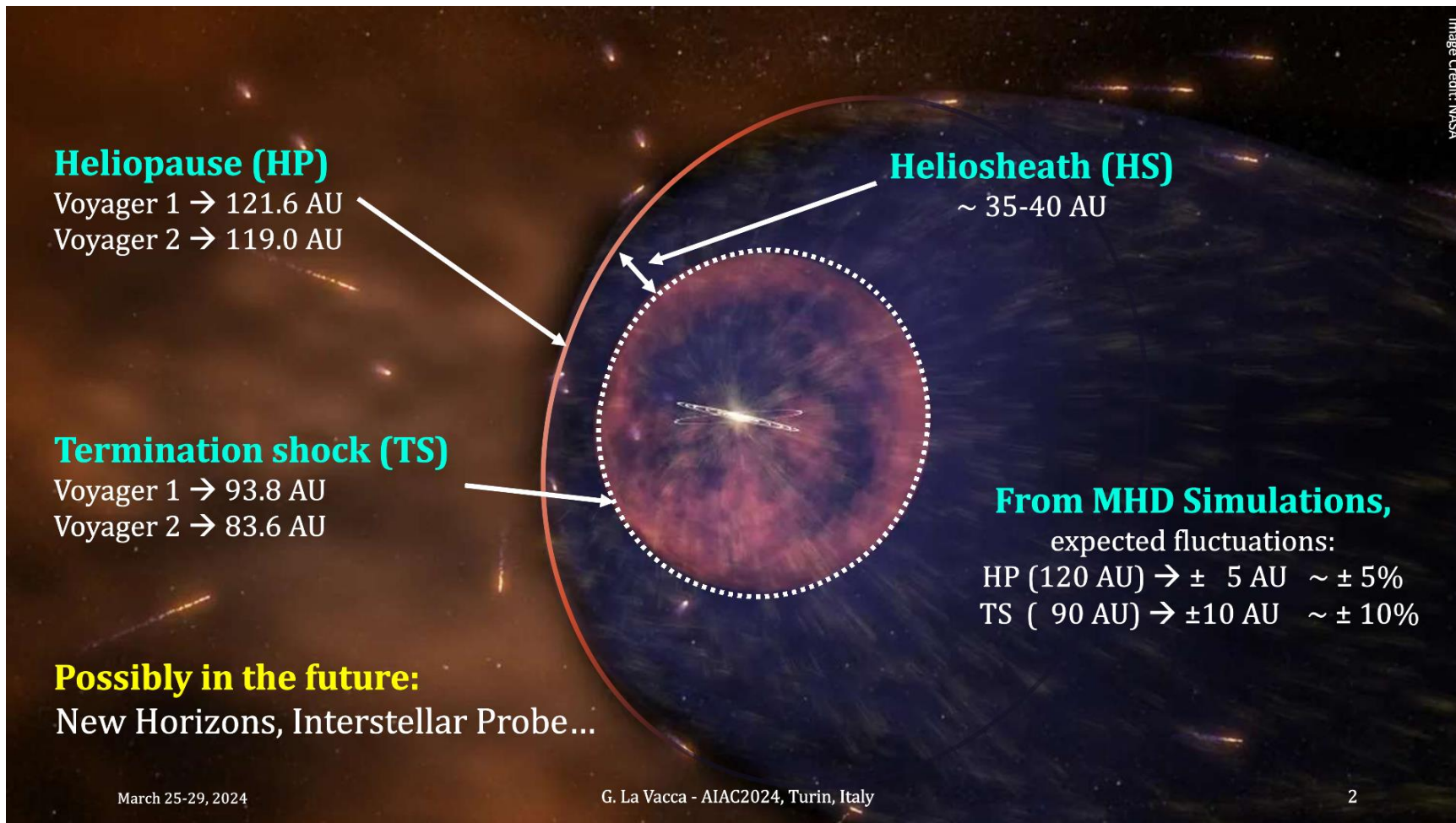
THANKS FOR THE ATTENTION



DEFINITION #2: THE HELIOSPHERE

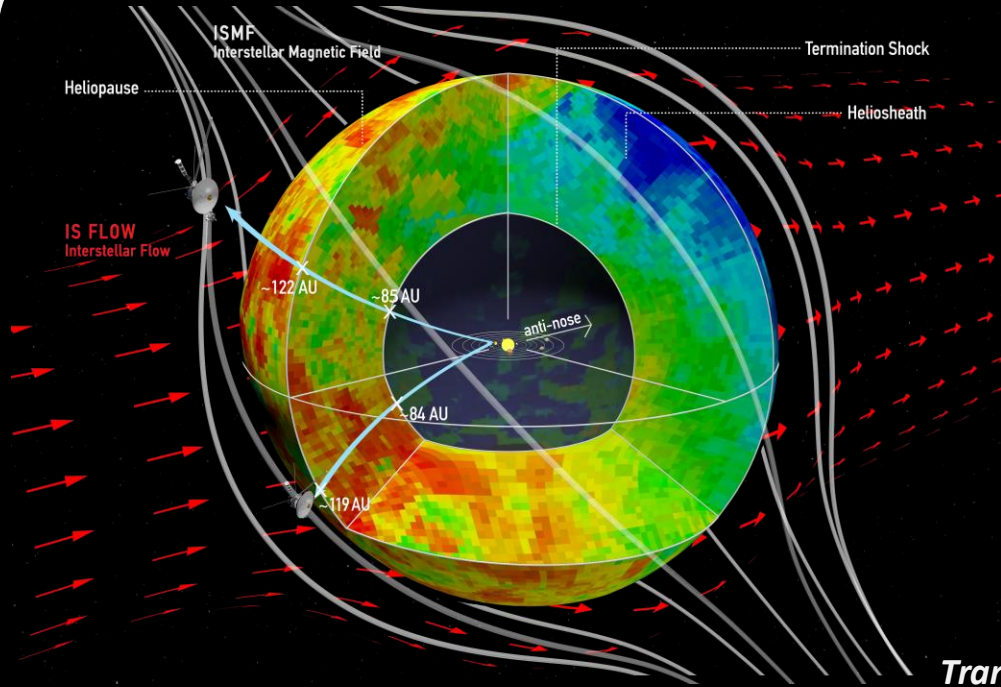
Simple definition:
Region of space directly influenced by Sun dynamics

Plasma emitted by the Sun (solar wind) transport the magnetic field defining the shape and dimension of the heliosphere

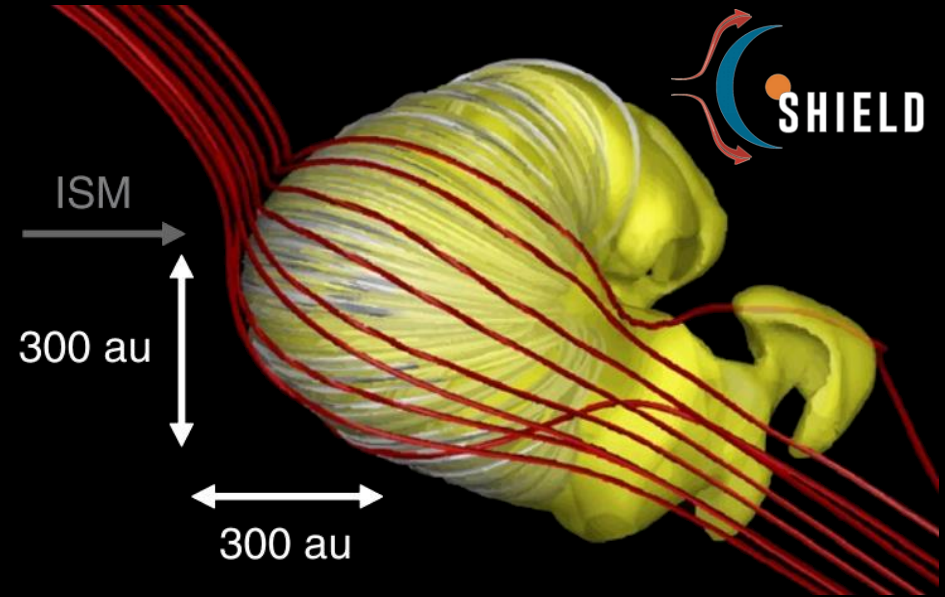


SHAPE OF THE GLOBAL HELIOSPHERE (1961-2022) // A ROUGH DIAMAGNETIC BUBBLE; A JETS STRUCTURE; A COMET...

Dialynas+[2017] (INCA & Voyager data)

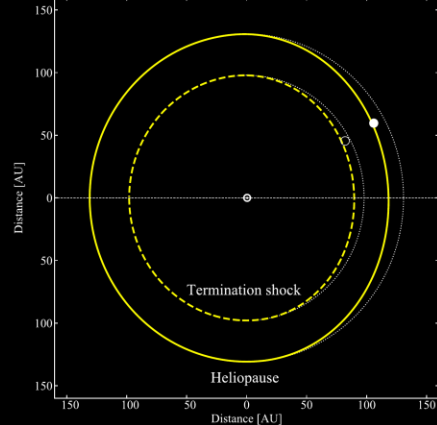


Opher+[2020] MHD modeling



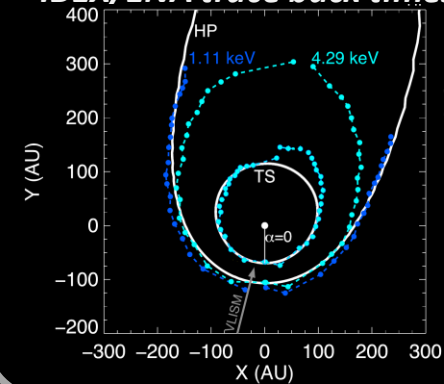
Boschini+[2019; 2020]

Transport of GCRs through the heliosphere (HELMOD).

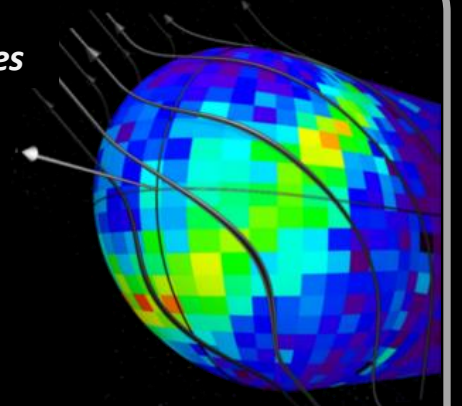


Intermediate Configuration

Zirnstein+[2018]
IBEX/ENA trace back times



McComas+[2013]



IBEX-Lo Data (Galli+[2016; 2017])

[...] typical thickness of HS in downwind
 $\sim(220 \pm 110)$ AU.

IBEX-Hi Data (Reisenfeld+[2016])

N. pole $L_{HS} \sim 210$ AU || S. pole $L_{HS} \sim 160$ AU

IBEX-Hi Data (Reisenfeld+[2021])

[...] heliosphere extends at least ~ 350 AU tailwards

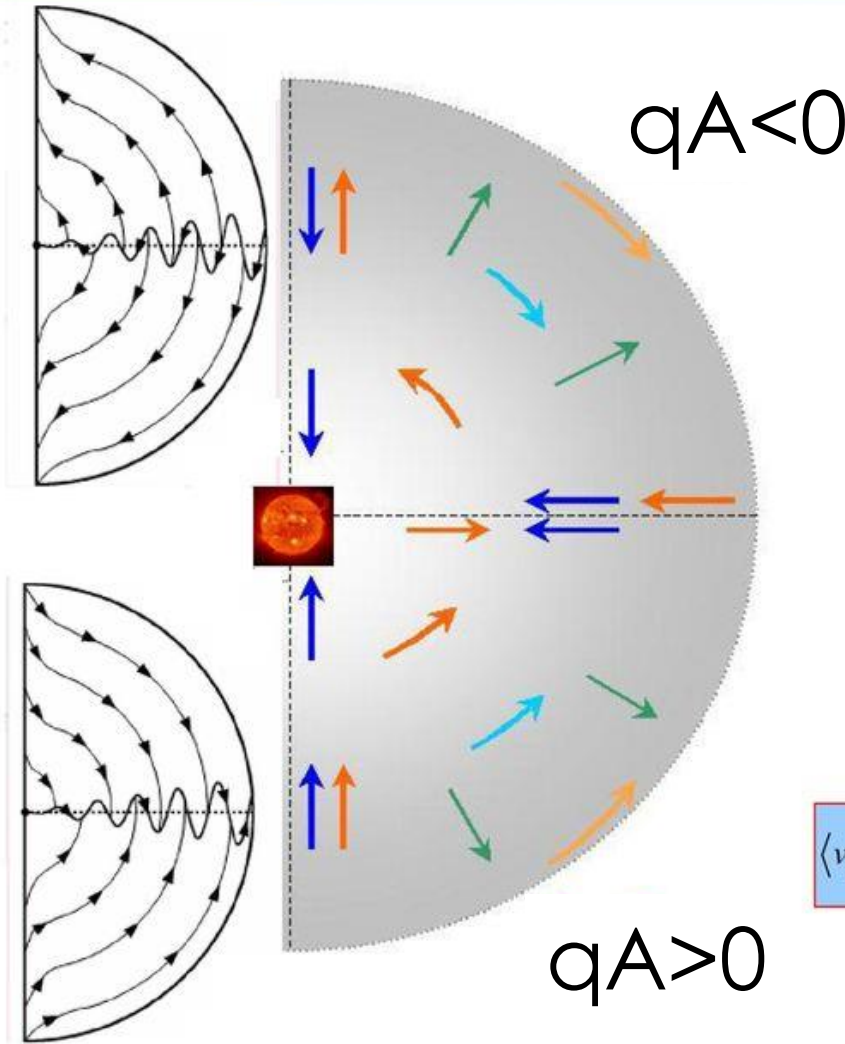
Conceptual illustration of modulation processes

$qA < 0$ Maggiore capacità di penetrazione ai equatore, svuotamento della regione Polare

$qA > 0$ Maggiore capacità di penetrazione ai poli, svuotamento della regione equatoriale

Drift direction of electrons in $A > 0$ cycle

Drift direction of electrons in $A < 0$ cycle



Modulation mechanisms

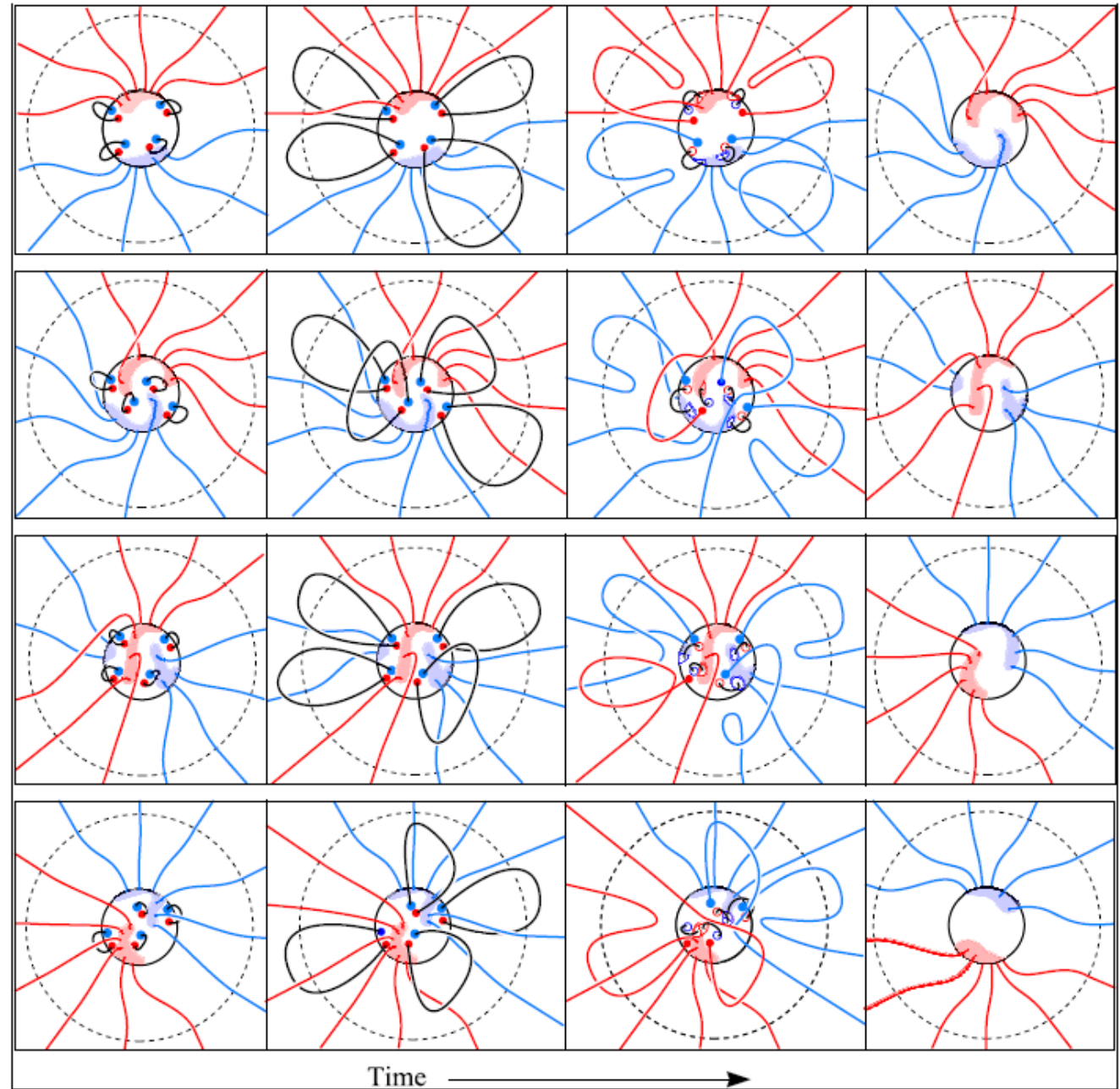
- Convection
- Diffusion
- Perpendicular diffusion
- G, C & NS Drifts
- Shock-drift

$$\langle v_A \rangle_r = -\frac{A}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta K_{\theta r}),$$

$$\langle v_A \rangle_\theta = -\frac{A}{r} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \phi} (K_{\phi \theta}) + \frac{\partial}{\partial r} (r K_{r \theta}) \right],$$

$$\langle v_A \rangle_\phi = -\frac{A}{r} \frac{\partial}{\partial \theta} (K_{\theta \phi}),$$

Solar Activity change
the sun configuration
continuously



Equazione del trasporto – Parker 1965

$$\frac{\partial U}{\partial t} = -\nabla \cdot (U\mathbf{V}) + \nabla \cdot [\vec{K} \cdot \nabla U] + \frac{(\nabla \cdot \mathbf{V})}{3} \frac{\partial}{\partial T} (\alpha_{rel} T U)$$

L'equazione del trasporto fa parte delle equazioni di Fokker-Planck

$$\partial_t F = - \sum_i \partial_i [A_i(\mathbf{x}, t) F] + \frac{1}{2} \sum_{i,j} \partial_i \partial_j \{ [\tilde{\mathbf{D}}(\mathbf{x}, t)]_{ij} F \}$$

*Advettivo (Convettivo,
Deriva)*

Diffusione

STOCHASTIC DIFFERENTIAL EQUATION

Si definiscono SDE equazioni della tipologia:

$$\frac{dx(t)}{dt} = \overset{\text{Drift term}}{a(x, t)} + \overset{\text{Diffusion term}}{b(x, t)\zeta(t)}$$

Funzioni continue

Termine di rumore -
Funzione a variazione

STOCHASTIC DIFFERENTIAL EQUATION

Formalismo di Itô

Drift term

Diffusion term

$$dx(t) = a(x, t)dt + b(x, t)dW(t)$$

Processo di Wiener

$W(t)$ representing the Wiener process; a time stationary stochastic Lévy process where the time increments have a Normal distribution with a mean of zero (i.e. a Gaussian distribution) and a variance of dt ; $dW(t) = W(t + dt) - W(t) \sim N(0, dt)$. In fact, the Wiener process can be understood as the integral of the stochastic Equation, i.e. in its differential form we have $dW(t) = \zeta(t)dt$. See especially the introduction by

STOCHASTIC DIFFERENTIAL EQUATION

In forma integrale

$$x(t) = x_0 + \int_0^t a(x, t') dt' + \int_0^t b(x, t') dW(t')$$

normal (Riemann or Lebesgue) integral

Itô-type stochastic

integral.
Non l'unico modo di calcolare l'integrale, ma certamente quello più facile da computare

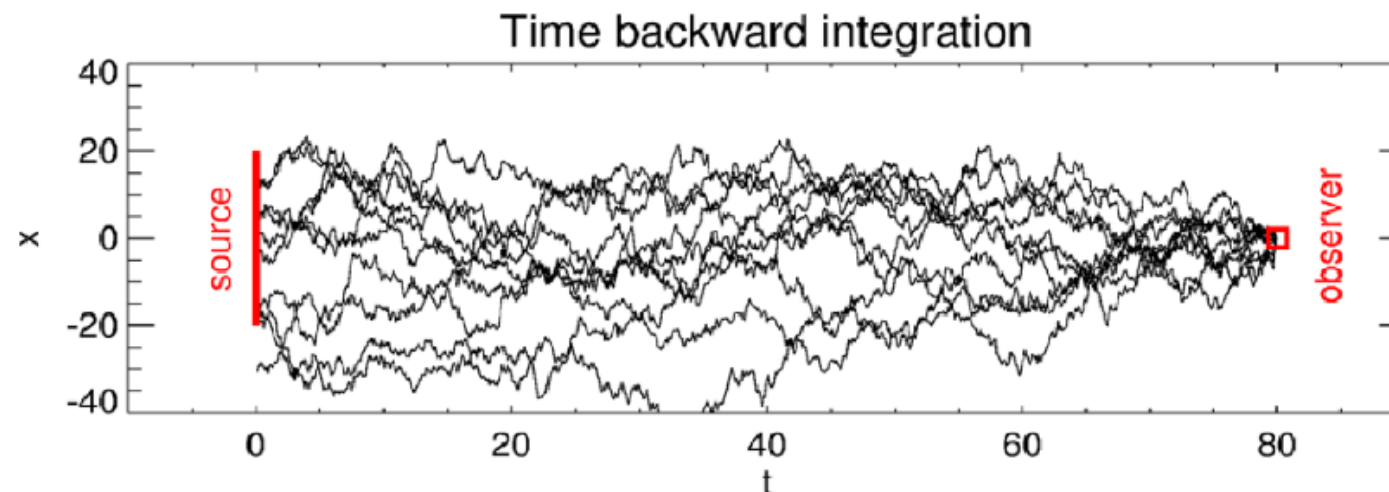
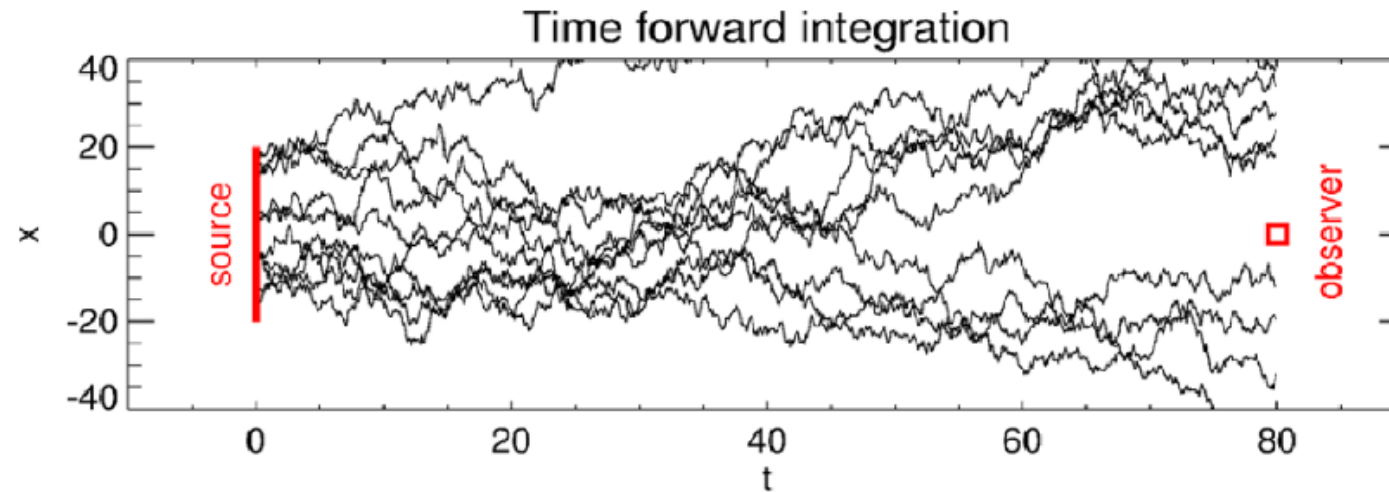
Integrabile numericamente con uno schema Eulero -

Maruyama

$$x(t + \Delta t) = x(t) + a(x, t)\Delta t + b(x, t)\Delta W(\Delta t)$$

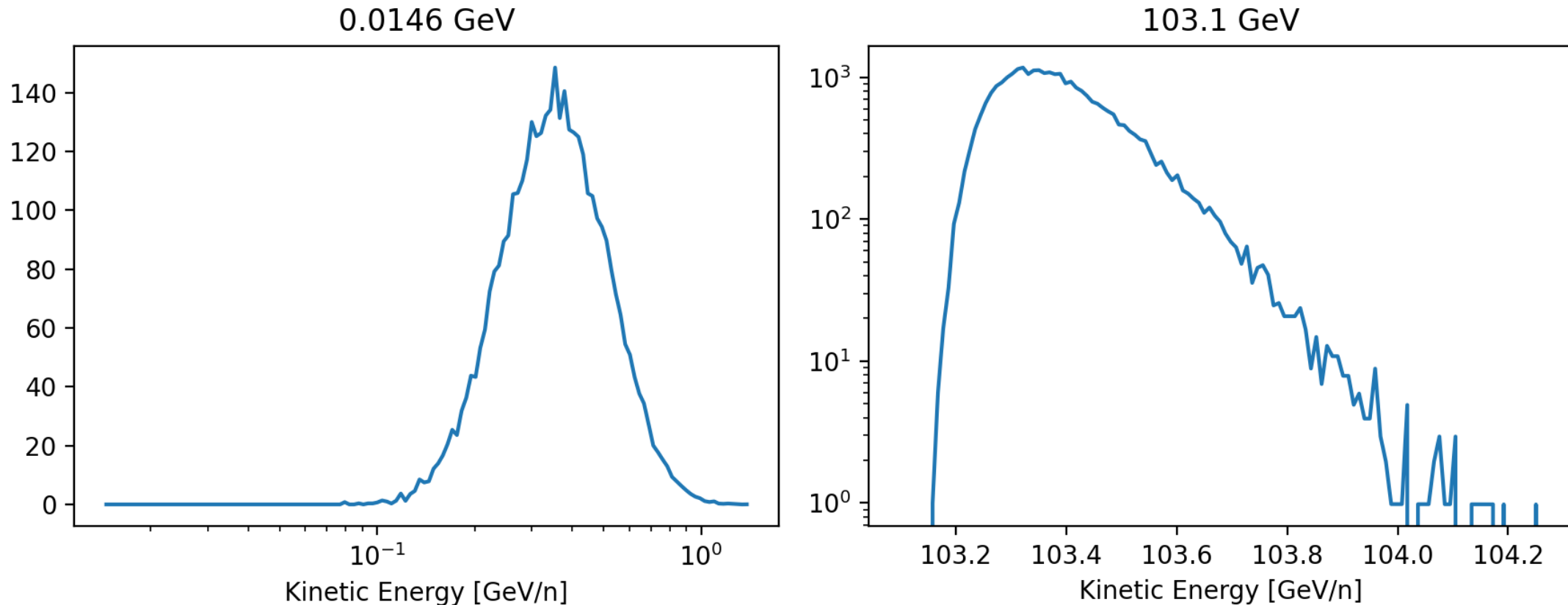
$$\Delta W(\Delta t) = \sqrt{\Delta t} \cdot \Lambda(t),$$

$\Lambda(t)$ is a simulated Gaussian distributed pseudo-random number (PRN).



Il sistema evolve Pseudo-particelle

Evoluzione di un elemento della densità dello spazio delle fasi
L'integrazione di una singola pseudo-particella non è significativa.



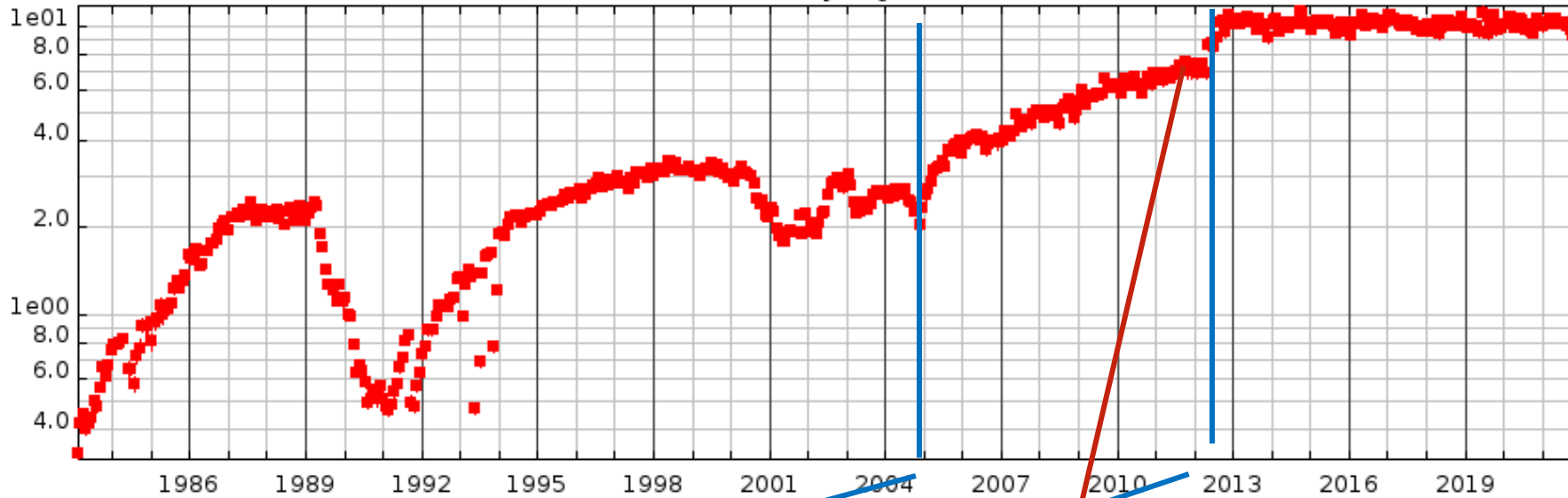
Occorre un gran numero di realizzazioni indipendenti al fine di avere una sorta di distribuzione di probabilità di trasmissione tra sorgente e osservatore (*quasi-funzioni di green*)

I CONFINI DEL MODELLO

Le sonde Voyager sono le uniche sonde uscite dall'Eliosfera e hanno potuto confermare che oltre l'eliopausa NON si osserva più Modulazione Solare

Voyager-1 CRS (27 day average Flux ((m²_ster_sec)⁻¹))

■ 270.050 - 346.034 MeV/n Hydrogen flux derived from IIPH

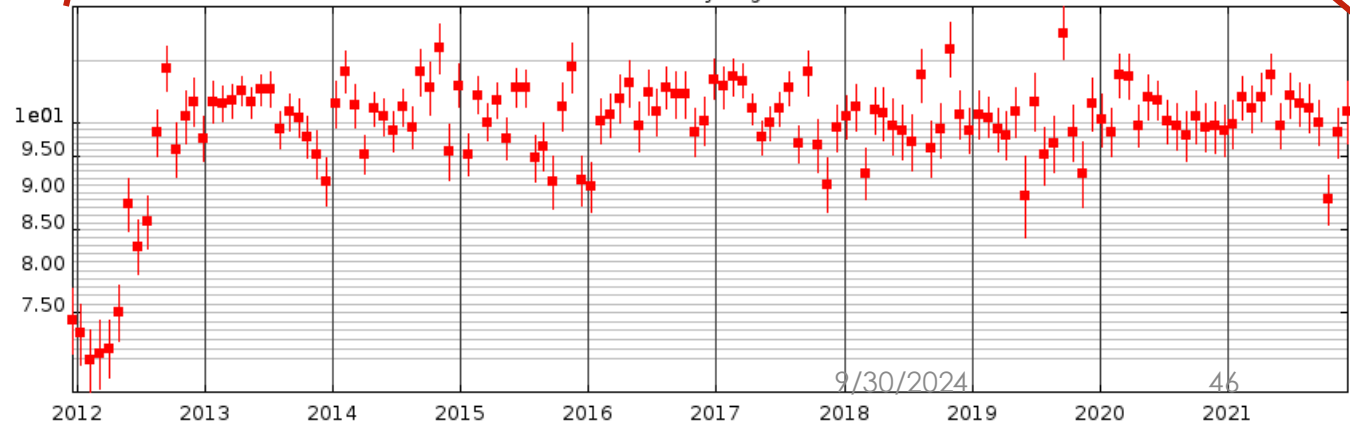


Termination Shock

Heliopause

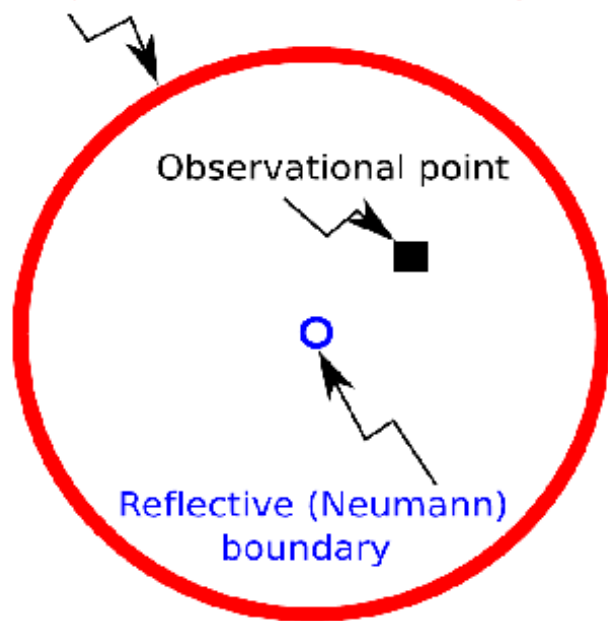
Voyager-1 CRS (27 day average Flux ((m²_ster_sec)⁻¹))

■ 270.050 - 346.034 MeV/n Hydrogen flux derived from IIPH



Se rivediamo il Metodo MC alla luce del nostro Caso Fisico (i.e. Trasporto dei raggi cosmici)

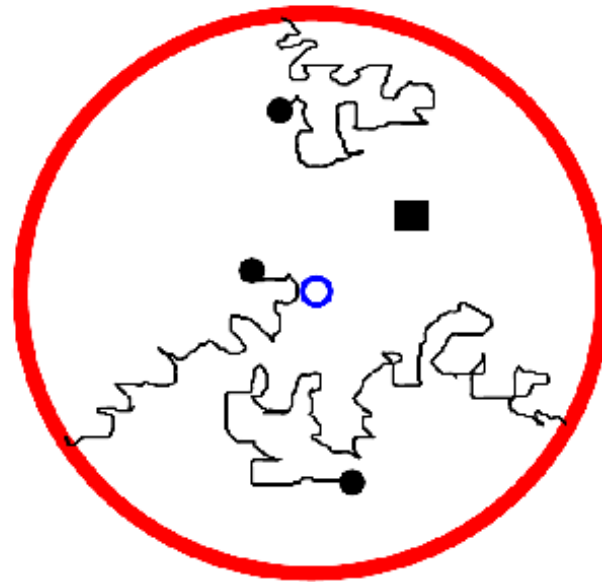
Heliopause (Dirichlet boundary)



Observational point

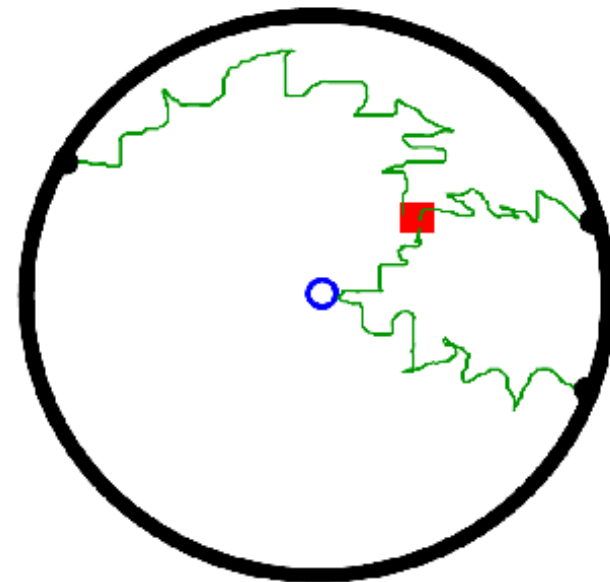
Reflective (Neumann) boundary

Time-forward integration



Particles released uniformly from the heliopause

Time-backward integration

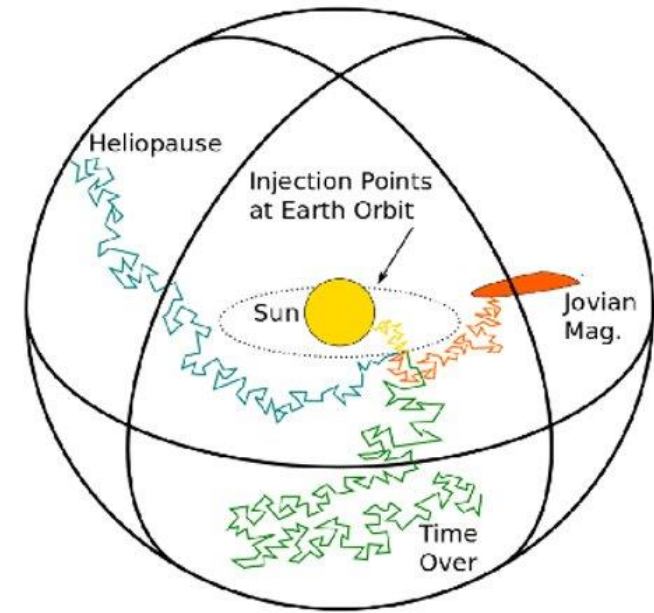


Particles released from the observational point

P. Denzloff et al. / Computer Physics Communications 192 (2015) 156–165

SDE n-dimensionale

$$dx_i = a_i(x_i, s)ds + \sum_{j=1}^n b_{ij}(x_i, s)dW_j(s)$$



Backward Kolmogorov equation - Fokker Plank

$$\frac{\partial \rho}{\partial s} = \sum_{i=1}^n a_i(x_i, s) \frac{\partial \rho(x_i, s)}{\partial x_i} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ij}(x_i, s) \frac{\partial^2 \rho(x_i, s)}{\partial x_i \partial x_j}$$

Forward Kolmogorov equation - Fokker Plank Equation

$$\frac{\partial \rho(x_i, t)}{\partial t} = - \sum_{i=1}^n \frac{\partial}{\partial x_i} [\tilde{a}_i(x_i, t) \rho(x_i, t)] + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2}{\partial x_i \partial x_j} [\tilde{C}_{ij}(x_i, t) \rho(x_i, t)].$$

$$\frac{\partial U}{\partial t} = -\nabla \cdot (U \vec{V}) + \nabla \cdot [K^S \cdot \nabla U] + \frac{(\nabla \cdot \vec{V}_{sw})}{3} \frac{\partial}{\partial T} (\alpha_{rel} T U)$$

$$\frac{\partial F}{\partial t} = \frac{1}{2} \sum_{i,j} [BB^\top]^{ij} \frac{\partial^2 F}{\partial x_i \partial x_j} + \sum_i A_B^i \frac{\partial F}{\partial x_i} + LF.$$

$$dx_i = A_B^i dt + \sum_j B^{ij} d\omega_j.$$

$$\Delta r = \left[\frac{1}{r^2} \frac{\partial r^2 K_{rr}}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (K_{\theta r} \sin \theta) \right] \Delta t$$

$$- [V_{sw} + v_{drift,r}] \Delta t + (2K_{rr})^{1/2} \omega_r \sqrt{\Delta t},$$

$$\Delta \theta = \left[\frac{1}{r^2} \frac{\partial r K_{r\theta}}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{K_{\theta\theta} \sin \theta}{r} \right) - \frac{V_{drift,\theta}}{r} \right] \Delta t$$

$$+ \frac{2K_{r\theta}}{r \sqrt{2K_{rr}}} \omega_r \sqrt{\Delta t}$$

$$+ \left[\frac{2K_{\theta\theta}}{r^2} - \frac{2K_{r\theta}^2}{r^2 K_{rr}} \right]^{1/2} \omega_\theta \sqrt{\Delta t},$$

$$\Delta T = \frac{\alpha_{rel} T V_{sw}}{3r} \Delta t,$$

$$L = \frac{2V_{sw}}{r} \left(\frac{1}{3} \frac{\partial \alpha_{rel} T}{\partial T} - 1 \right),$$

$$A_B^r = \frac{1}{r^2} \frac{\partial r^2 K_{rr}}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta K_{\theta r}) + \frac{1}{r \sin \theta} \frac{\partial K_{\phi r}}{\partial \phi} - V_{sw} - V_{dr}$$

$$A_B^\theta = \frac{1}{r^2} \frac{\partial r K_{r\theta}}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{K_{\theta\theta} \sin \theta}{r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} (K_{\phi\theta}) - \frac{V_{d\theta}}{r}$$

$$A_B^\phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\frac{r K_{r\phi}}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{K_{\theta\phi}}{r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} (K_{\phi\phi}) - \frac{V_{d\phi}}{r \sin \theta}$$

$$A_B^T = \frac{2}{3} \frac{\alpha T V_{sw}}{r}$$

$$BB^\top = \begin{bmatrix} 2K_{rr} & \frac{2K_{r\theta}}{r} & \frac{2K_{r\phi}}{r \sin \theta} \\ \frac{2K_{\theta r}}{r} & \frac{2K_{\theta\theta}}{r^2} & \frac{2K_{\theta\phi}}{r^2 \sin \theta} \\ \frac{2K_{\phi r}}{r \sin \theta} & \frac{2K_{\phi\theta}}{r^2 \sin \theta} & \frac{2K_{\phi\phi}}{r^2 \sin^2 \theta} \end{bmatrix}$$

$$L = \frac{2V_{sw}}{r} \left(\frac{1}{3} \frac{\partial \alpha T}{\partial T} - 1 \right).$$

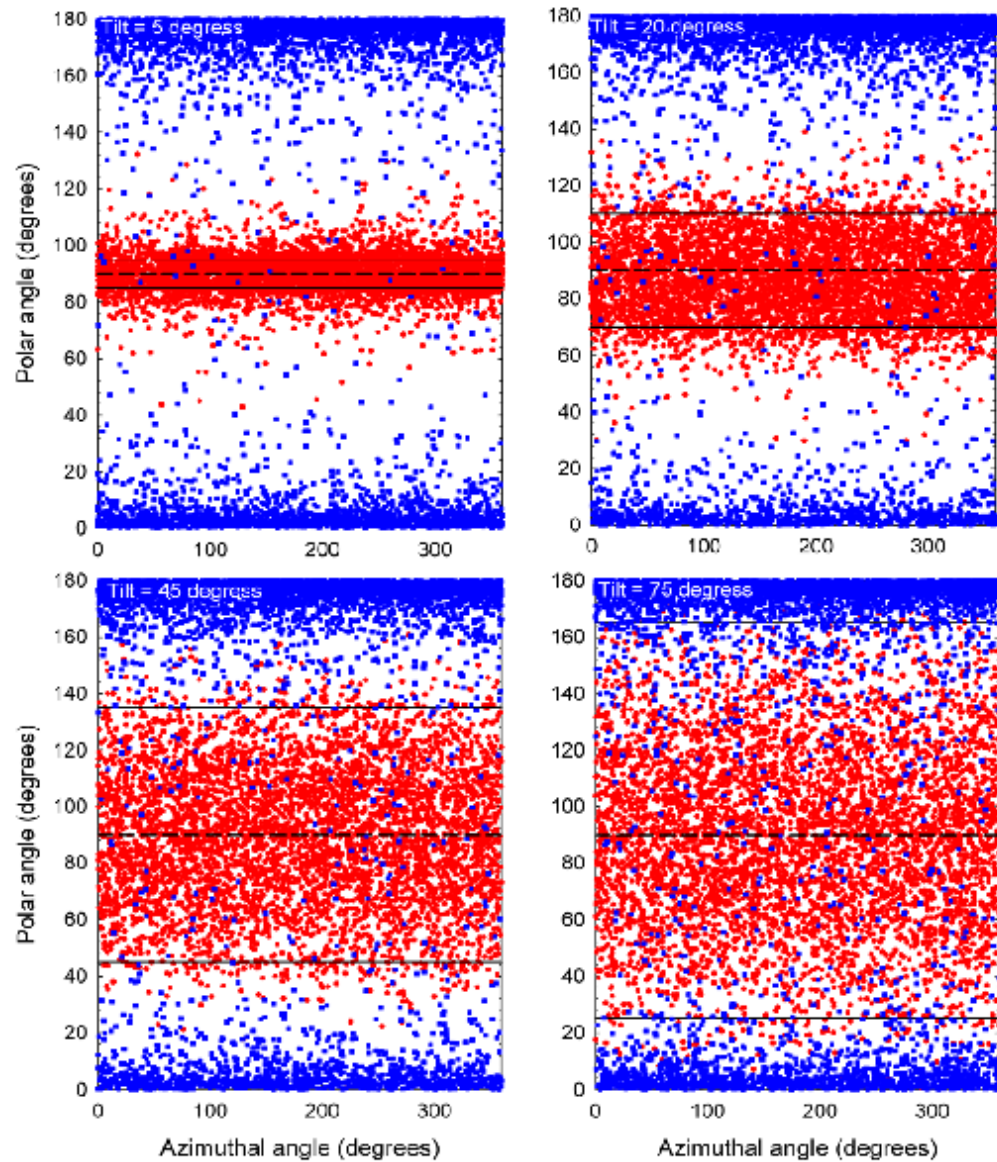
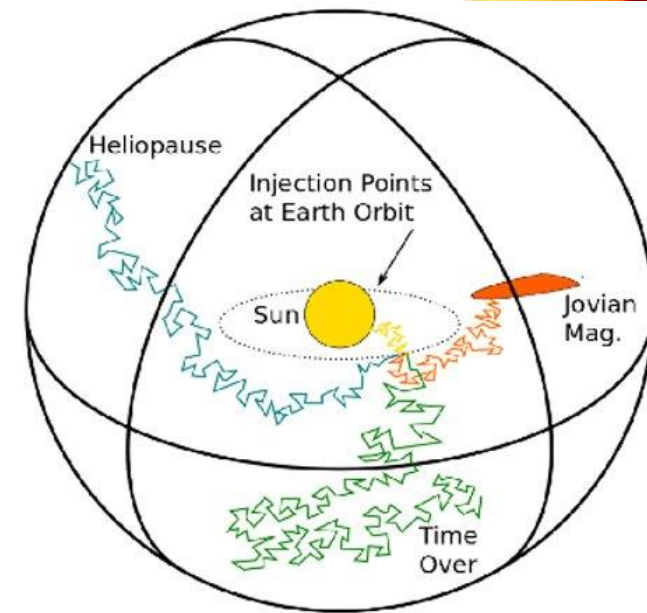


Fig. 7 Illustrating drift effects in the heliosphere by plotting the exit position of protons in the so-called $A > 0$ (blue) and $A < 0$ (red) drift cycles for various tilt angles. The solid and dashed lines show the latitudinal extend of the wavy HCS, with the different tilt angles indicated on each panel. The figure is taken from Strauss et al. (2012). With permission of Springer



Usando il Codice Monte Carlo per cercare gli "exit point"

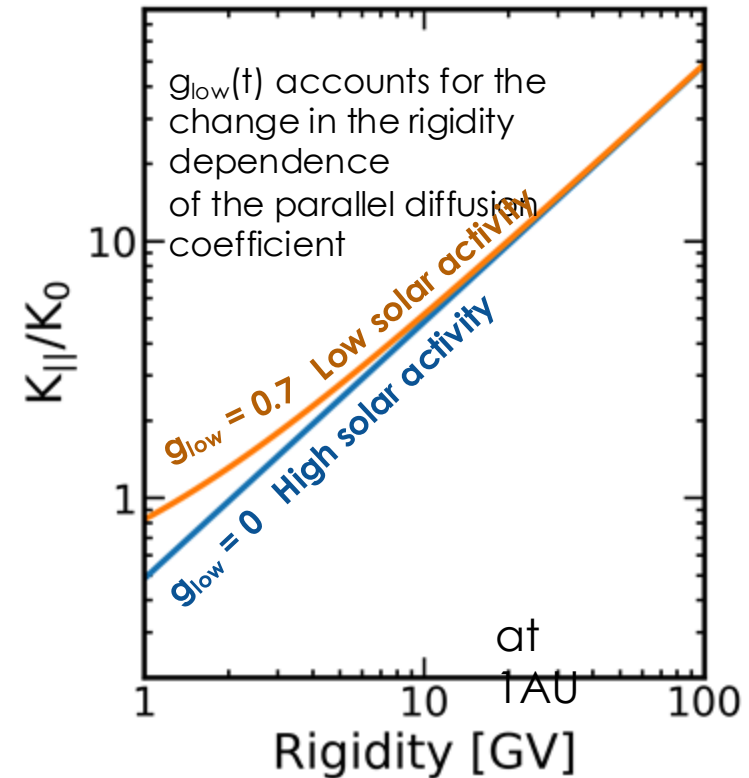
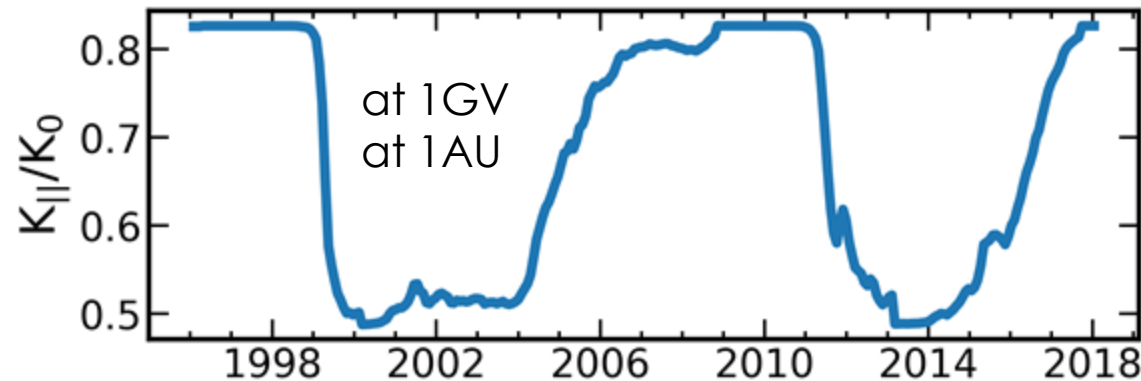
DIFFUSION TERM

Parametrization of diffusion coefficients:

P is rigidity in GV

$$\frac{K_{\parallel}}{K_0} = \frac{\beta}{3} \left[\frac{P}{1 \text{ GV}} + g_{\text{low}}(t) \right] \left(1 + \frac{r}{1 \text{ AU}} \right)$$

$$K_{\perp,i} / K_{\parallel} = \rho_i$$



$K_0(t)$ is the diffusion parameter obtained using cosmic ray fluxes measured with neutron monitor at different latitudes.

see

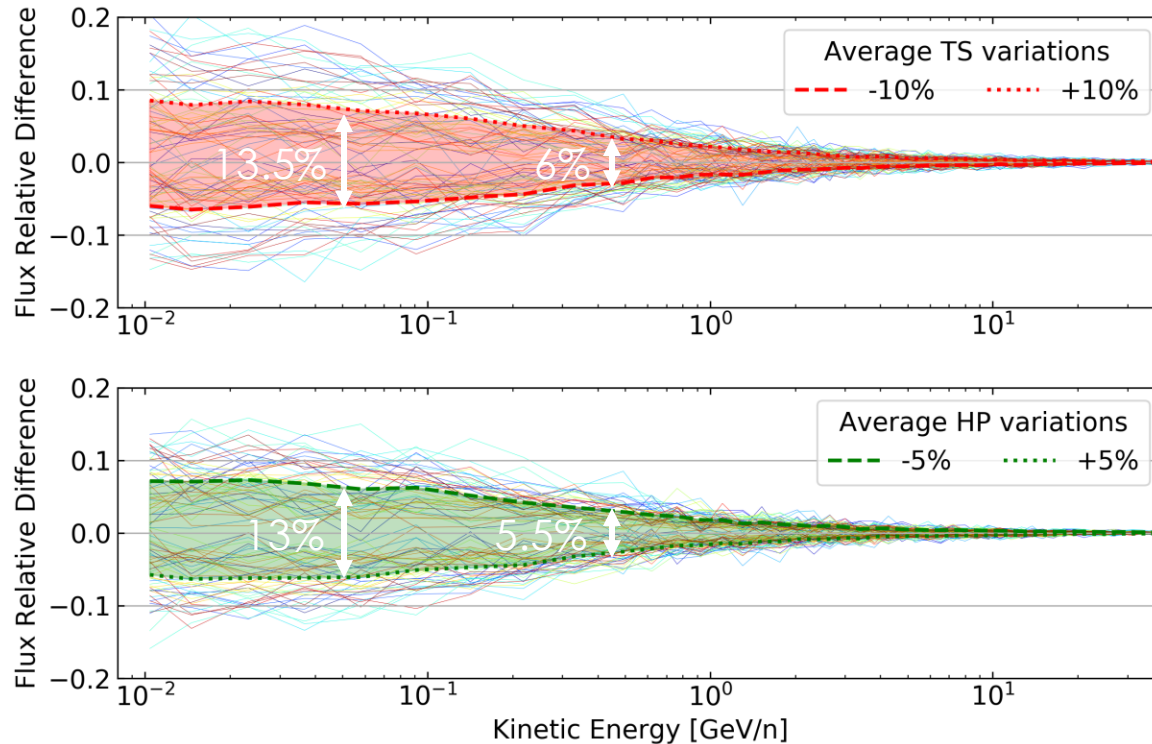
Bobik et al. ApJ 745:132 (2012)

Bobik et al. AdsAst, ID 793072 (2013)

Boschini et al. Adv. S. Res. (2017,2019,2022,2024)

EFFECTS OF TS & HP TIME-VARIATION ON GCR MODULATION

The accurate prediction of the solar modulation of GCRs requires a time-dependent heliosphere structure



Some degeneracy with GCR transport properties

→ “A priori” knowledge of TS & HP time variation is needed.

GCR solar modulation usually calculated on Carrington rotation time intervals:

→ no large time resolution needed

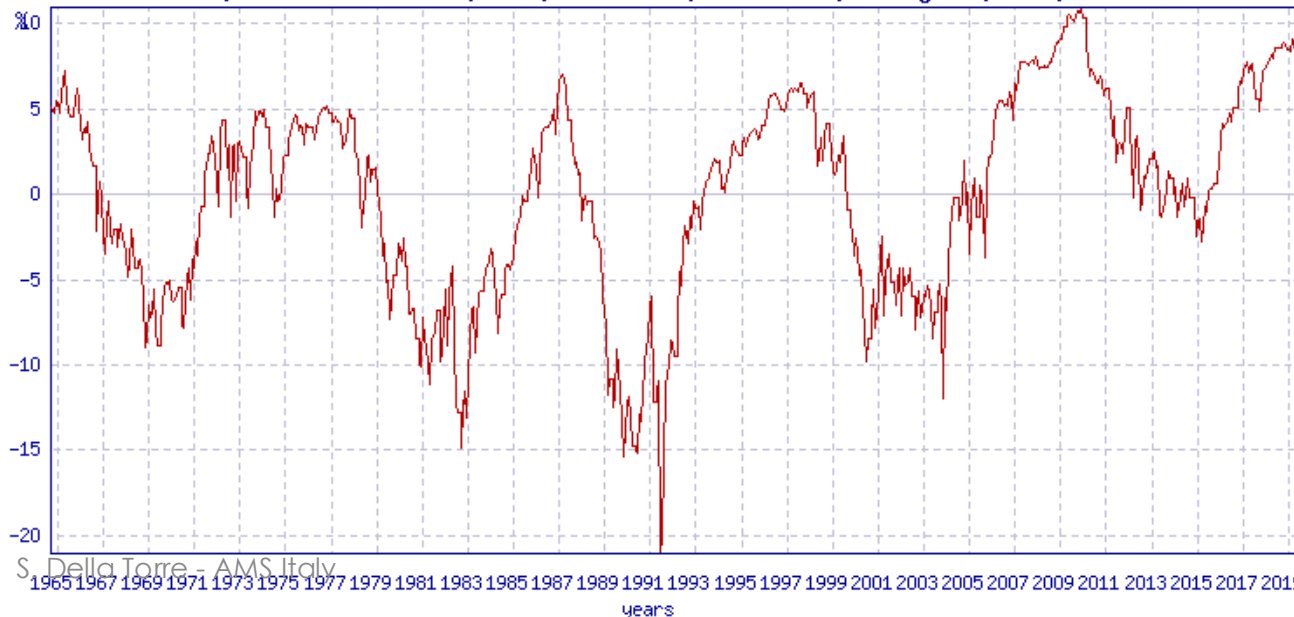
- impact on the long-term variation of cosmic rays
- appreciable effects at energies ≤ 1 GeV/n

AFTER >100Y OBSERVATIONS

Nowadays we improved our knowledge on cosmic rays (CR) thank to a global network of observatory both on ground and on space

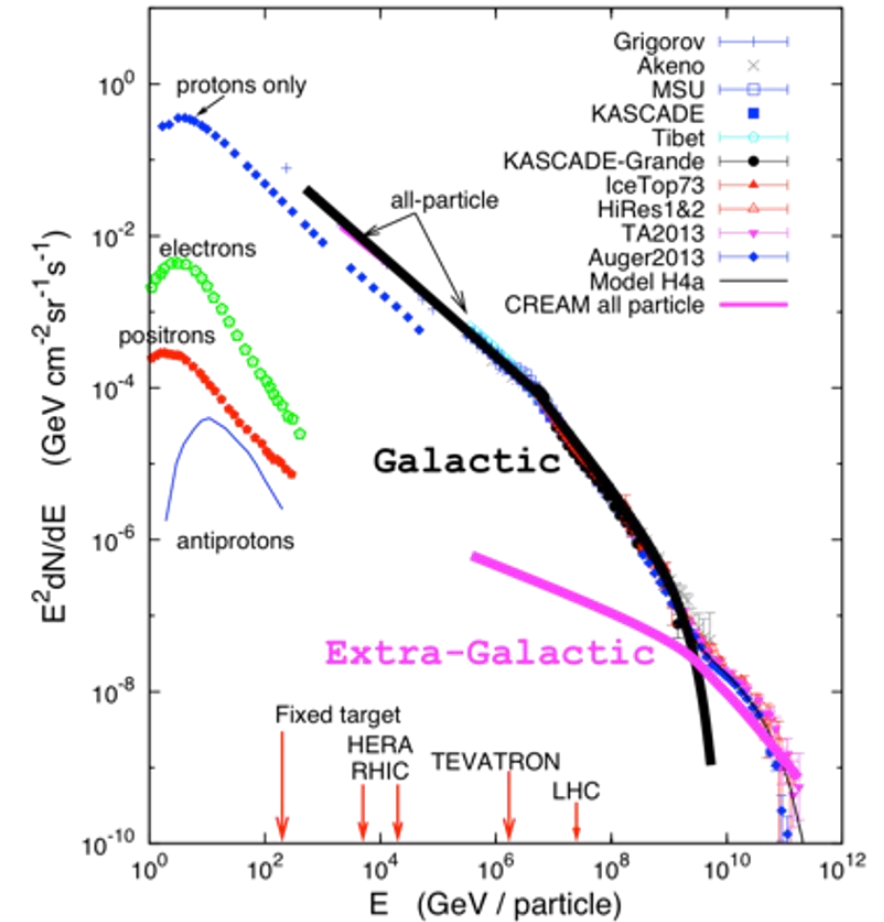
Oulu Neutron Monitor

1964/09/24 00:00 - 2019/10/24 00:00 UT. Resolution: 1 month(s). Average CR: 6175.03



S. Della Torre - AMS Italy

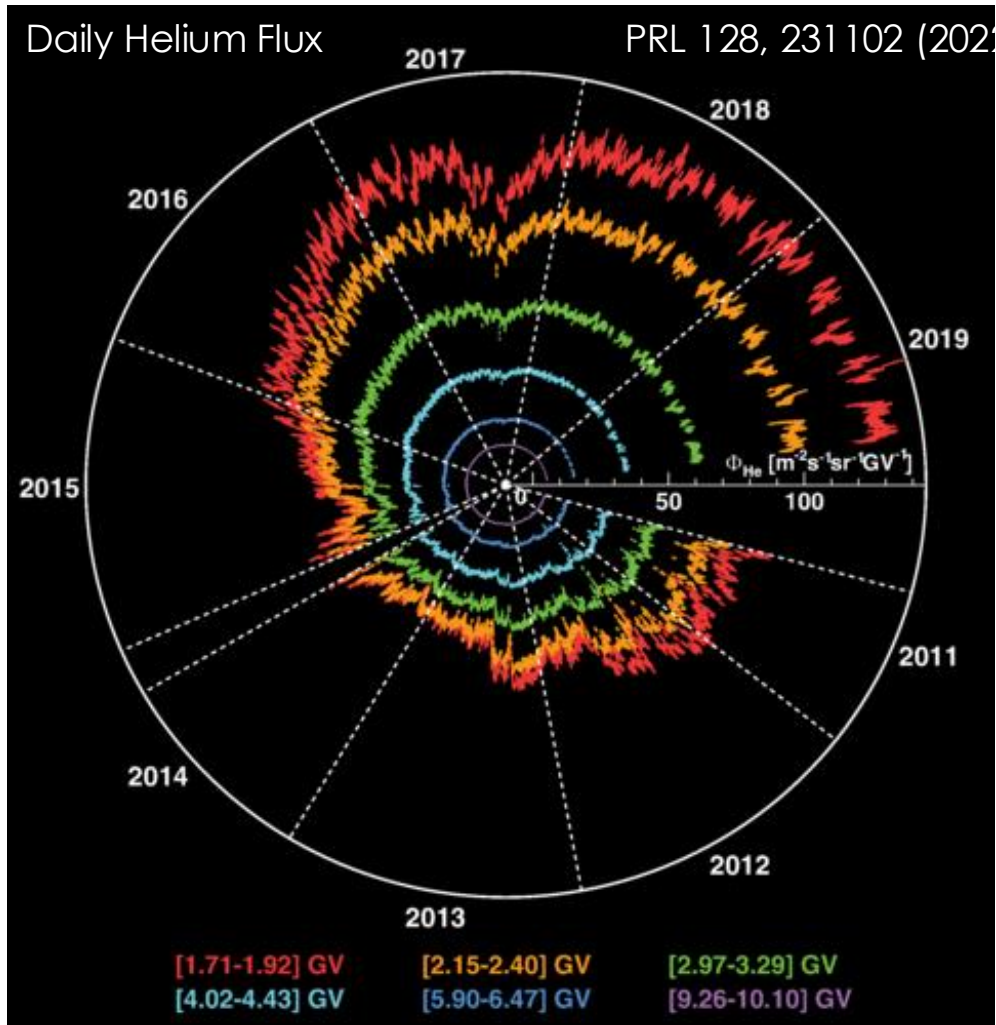
Energies and rates of the cosmic-ray particles



The intensity of CR varies with time with a cycle of 11y (22y considering solar polarity)

CR SOLAR MODULATION MEASUREMENTS

PAMELA, on space from 2006 to 2016, and AMS-02, on International Space Station (ISS) since 2011, represented a *game changer* in this field.



They provide the highest quality Cosmic ion measurements in the GV region.

They provide high quality CR spectra