Charm CP Violation: Theory

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Can we tell a loop from a tree?

$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\mathrm{QCD}}\,, \quad r_{\mathrm{QCD}} = \mathcal{A}^{\Delta U=0}/\mathcal{A}^{\Delta U=1}$$

Assuming the SM, the data implies $r_{\mathrm{QCD}}^{\mathrm{EXP}} = \mathcal{O}(1)$.

What is $r_{\mathrm{QCD}}^{\mathrm{SM}} \equiv |P/T|$?

- Light Cone Sum Rules (LCSR): $r_{\mathrm{QCD}}^{\mathrm{SM}} \sim 0.1$.
  [Petrov Khodjamirian 1706.07780, Chala Lenz Rusov Scholtz 1903.10490, Lenz Piscopo Rusov 2312.13245]
- Large non-pert. effects like in charm $\Delta I = 1/2$ rule: $r_{\mathrm{QCD}}^{\mathrm{SM}} = \mathcal{O}(1)$.
  [Grossman Schacht 1903.10952, Brod Kagan Zupan 1111.5000, Schacht Soni 2110.07619]
- Predictions based on $\pi\pi/KK$ rescattering data:
  [Franco Mishima Silvestrini 1203.3131, Bediaga Frederico Magalhaes 2203.04056, Pich Solomonidi Vale Silva 2305.11951]

A caveat for the interpretation of the data

“The data implies $|P/T| = \mathcal{O}(1)$”

This statement actually relies on an underlying, commonly made assumption:

- The relative strong phase between P and T is assumed $\mathcal{O}(1)$ (from rescattering).

Reminder: CP violation is an interference effect.

$$A = \left(-V_{cd}^* V_{ud}\right) \times T - \left(\frac{V_{cb}^* V_{ub}}{2}\right) \times P \,.$$

Direct CP asymmetry:

$$a_{CP}^f \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \mathrm{CKM} \times \left|\frac{P}{T}\right| \times \sin\left(\arg\left(\frac{P}{T}\right)\right)$$

How to determine the strong phase from the data

- Strong phases can be obtained from measurements of time-dependent CP violation or quantum-correlated decays. [Xing hep-ph/9606422, Gronau Grossman Rosner hep-ph/0103110, Bevan Inguglia Meadows 1106.5075, Bevan Meadows 1310.0050, Grossman Kagan Zupan 1204.3557, Xing 1903.09566, Grossman Schacht 1903.10952, Schacht 2207.08539]

$$A_{CP}(f,t) \approx a^f_{CP} + \Delta Y^f \frac{t}{\tau_{D^0}}$$

- Analogous strong phase in multi-body decays can be determined from fit to CP violating time-integrated Dalitz plot.

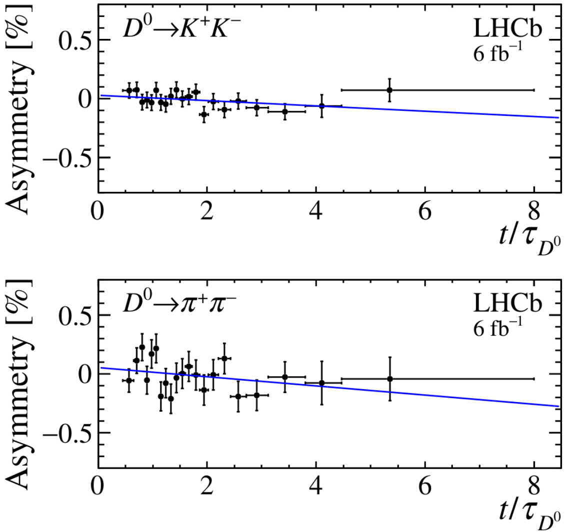
[Dery Grossman Schacht Soffer 2101.02560]

- No measurements of phases yet, despite big experimental advances. ΔY^f have large errors, and sensitivity to phase is subleading.

$$\Delta Y^{K^+K^-} = (-0.3 \pm 1.3 \pm 0.3) \cdot 10^{-4} \,,$$
$$\Delta Y^{\pi^+\pi^-} = (-3.6 \pm 2.4 \pm 0.4) \cdot 10^{-4} \,.$$

[LHCb combination 2105.09889]

$A_{CP}(f,t) \approx a_{CP}^f + \Delta Y^f \times t/\tau_{D^0}$ [LHCb 2105.09889]



Isospin to the rescue: $D \to \pi\pi$

[Gavrilova Grossman Schacht, 2312.10140]

- Assuming SM, isospin allows determination of strong P/T phase from direct CP asymmetries and branching ratios only.

- Also enables extraction of magnitude of P/T without assumptions about phase.

- $D \to \pi\pi$ has same group-theory structure as $B \to \pi\pi$ [Gronau London 1990], however, different approximations are used in the two systems.

Closed-form solution [Gavrilova, Grossman, Schacht 2312.10140]

$$\sin\arg(P/T)^{00} = \frac{-\mathrm{sign}\,(a_{CP}^{00})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{+-}}{a_{CP}^{00}}\sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$\sin\arg(P/T)^{+-} = \frac{-\mathrm{sign}\,(a_{CP}^{+-})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{00}}{a_{CP}^{+-}}\sqrt{2\frac{\mathcal{P}^{+-}}{\mathcal{B}^{+-}}\frac{\mathcal{B}^{00}}{\mathcal{P}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$|P/T|^{00} = \frac{1}{|\mathrm{Im}\,(-\lambda_b/\lambda_d)\,|}\sqrt{(a_{CP}^{00})^2+\frac{(a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}+a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}\cos\delta_d)^2}{2\mathcal{B}^{00}\mathcal{P}^{+-}\sin^2\delta_d}}\,,$$

$$|P/T|^{+-} = \frac{1}{|\mathrm{Im}\,(-\lambda_d/\lambda_d)|}\sqrt{\left(a_{CP}^{+-}\right)^2+\frac{\left(a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}+a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}\cos\delta_d\right)^2}{\mathcal{B}^{+-}\mathcal{P}^{00}\sin^2\delta_d}}\,.$$

Knowledge of $D \to \pi^+\pi^-$ translates into $D \to \pi^0\pi^0$

[Gavrilova, Grossman, Schacht 2312.10140]

$$\frac{\sin\delta^{+-}}{\sin\delta^{00}} = \frac{a_{CP}^{+-}}{a_{CP}^{00}} \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}\,,$$

$$\frac{|P/T|^{00}}{|P/T|^{+-}} = \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}\,.$$

Results

$$|P/T|^{+-} = 5.5^{+14.2}_{-2.7}$$
$$|P/T|^{00} = 5.2^{+13.3}_{-2.4}$$

- Although we have essentially no information about $\sin\delta^{00}$ we can obtain non-trivial information about r^{00}, due to the correlation to r^{+-} from isospin.
- Overall additional relative systematic uncertainty of $\mathcal{O}(10\%)$.
- $|P/T|$ is large. Future data will significantly reduce errors.

Numerical Results: Systematic Uncertainties

- Our results relate P/T to $\mathcal{B}^f$ and a_{CP}^f.
- In experimental extraction of a_{CP}^f additional assumptions are made.
- For extraction of a_{CP}^{+-} LHCb assumes a universal ΔY.
  We use same ΔY for the extraction of a_{CP}^{00}.
- A universal ΔY is motivated by U-spin [Kagan Silvestrini 2001.07207]
  It follows overall systematic theory uncertainty of $\mathcal{O}(\varepsilon^2) \sim 10\%$.
- In order to reach theory uncertainty of $\mathcal{O}(1\%)$ we need extractions of a_{CP}^f without universality assumption: future data on ΔY^{+-} and ΔY^{00}.
- Our numerics with current data include universality assumption, implying an additional overall theory uncertainty of $\mathcal{O}(10\%)$.

Extraction of Penguin over tree in three-body decays

[Dery Grossman Schacht Soffer 2101.02560]

$$\mathcal{A}(D^0 \to \pi^+\rho^-) = -\lambda\, T^{P_1V_2} - V_{cb}^* V_{ub}\, R^{P_1V_2}$$
$$\mathcal{A}(D^0 \to \pi^-\rho^+) = -\lambda\, T^{P_2V_1} - V_{cb}^* V_{ub}\, R^{P_2V_1}$$

- Time-integrated CP asym. of 2-body decays give only combinations

$$|\widetilde{R}^{P_1V_2}|\sin(\delta_{P_1V_2}) \qquad \text{and} \qquad |\widetilde{R}^{P_2V_1}|\sin(\delta_{P_2V_1})\,,$$

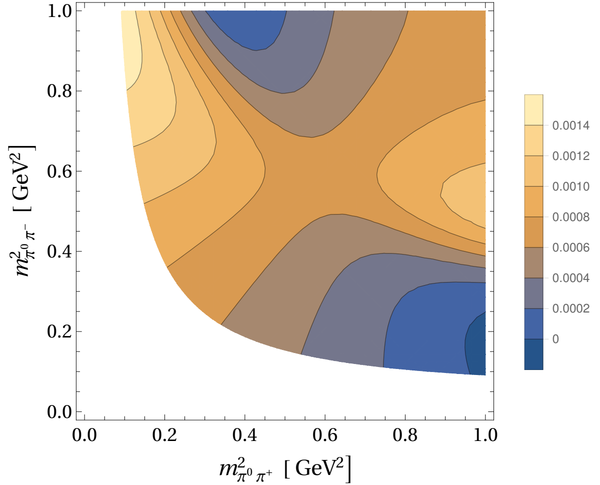
  but **not** magnitudes and phases separately.

- Three body decay changes 2 things:
  - We have additional kinematic dependences.
  - Only in a three-body decay we have interference between $D^0 \to \pi^+(\rho^- \to \pi^-\pi^0)$ and $D^0 \to \pi^-(\rho^+ \to \pi^+\pi^0)$.

➥ Extraction of all parameters from time-integrated CP meas.

Local $a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-\pi^0)$ in overlap region of $\rho^\pm$

[Dery Grossman Schacht Soffer 2101.02560]



Numerical example: $\widetilde{R}^{P_1V_2} = \exp(i\pi/2), \quad \widetilde{R}^{P_2V_1} = \frac{1}{4}\exp(i\pi/3)$

Beyond ΔA_{CP} : A U-spin anomaly?

- Separate measurement of both CP asymmetries allows for first time test of the U-spin expansion in CKM-suppressed amplitudes.

U-spin limit sum rule: Broken at 2.7σ [LHCb, 2209.03179]

$$\Sigma a_{CP}^{dir} \equiv a_{CP}^{dir}(D^0 \to K^+K^-) + a_{CP}^{dir}(D^0 \to \pi^+\pi^-) \overset{\text{U-spin}}{=} 0$$

$$a_{CP}^{\text{dir}}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4}$$

$$a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = (23.2 \pm 6.1) \cdot 10^{-4}$$

- U-spin breaking is expected: Only approximate symmetry.
- Amount goes beyond generic expectations of $\sim 30\%$.

For your next grant

1. Increase precision of $a_{CP}^{\text{dir}}(D_s^+ \to K_S\pi^+)$ and $a_{CP}^{\text{dir}}(D^+ \to K_S K^+)$.
2. Probe U-spin sum rule $a_{CP}^{\text{dir}}(D_s^+ \to K_S\pi^+) = -a_{CP}^{\text{dir}}(D^+ \to K_S K^+)$.
3. Measure also $a_{CP}^{\text{dir}}(D_s^+ \to K^+\pi^0)$ to probe improved sum rule.

[Müller Nierste Schacht 1506.04121]

CP Asymmetry	HFLAV avg. $/10^{-4}$ [https://hflav.web.cern.ch/]
$a_{CP}^{dir}(D_s^+ \to K_S\pi^+)^a$	16 ± 18
$a_{CP}^{dir}(D^+ \to K_S K^+)^a$	-1 ± 7
$a_{CP}^{dir}(D_s^+ \to K^+\pi^0)$	200 ± 300

- Is $a_{CP}^{\text{dir}}(D_s^+ \to K_S\pi^+)$ enhanced, like $D^0 \to \pi^+\pi^-$?
- Is the sum rule broken, like the one between $D^0 \to \pi^+\pi^-, K^+K^-$?

[a] "More precise measurements of these asymmetries can be expected when the data already collected by LHCb in 2018 are included in a future analysis"

[LHCb, 1903.01150]

But is U-spin actually a good symmetry?

Spectroscopy: Eightfold way.

[Gell-Mann, Ne'eman 1961]

- $\mathrm{SU}(3)_F$ limit agrees with baryon octet mass splitting to $\sim 10\%$

[Greiner Müller 1989]

Does it work for rates, too?

- Estimate for breaking on amplitude level: $f_K/f_\pi - 1 \sim 0.2$.
- Two often-cited examples of seemingly $\mathcal{O}(1)$ U-spin breaking:

$$\left.\frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)}\right|_{\mathrm{exp}} \sim 3\,, \qquad \left.\frac{\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+K^-)}\right|_{\mathrm{exp}} \sim 0.03\,.$$

- Strict $\mathrm{SU}(3)_F$ limit (including phase space):

$$\frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} = 1\,, \qquad \frac{\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+K^-)} = 0\,,$$

Yes. (But we keep testing it at every opportunity.)

[detailed review in Schacht 2207.08539]

A closer look

- Amplitude-level $SU(3)_F$ breaking of $\varepsilon \sim 30\%$ suffices in order to explain the data. [Savage 1991]

$$\frac{(1+\varepsilon)^2}{(1-\varepsilon)^2} \sim 3\,.$$

- Amplitude-level $SU(3)_F$ -breaking in $D^0 \to K_S K_S$:

$$\varepsilon' \sim \sqrt{\frac{\mathcal{B}(D^0 \to K^0 \overline{K}^0)}{\mathcal{B}(D^0 \to K^+ K^-)}} = \sqrt{\frac{2\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+ K^-)}} \sim 0.26\,,$$

- Observations agree with global fits.

[Hiller Jung Schacht 1211.3734, Müller Nierste Schacht 1503.06759]

The picture holds at higher order, too.

[Brod Grossman Kagan Zupan 1203.6659]

Ratio of branching ratios:

$$R_{DPP} \equiv \frac{|\mathcal{A}(D^0 \to K^+K^-)/(V_{cs}V_{us})| + |\mathcal{A}(D^0 \to \pi^+\pi^-)/(V_{cd}V_{ud})|}{|\mathcal{A}(D^0 \to K^+\pi^-)/(V_{cd}V_{us})| + |\mathcal{A}(D^0 \to K^-\pi^+)/(V_{cs}V_{ud})|} - 1$$

U-spin prediction

$$R_{DPP}^{\text{th}} = \mathcal{O}(\varepsilon^2)\,.$$

Data

$$R_{DPP}^{\text{exp}} = 0.046 \pm 0.008\,,$$

- If U-spin breaking were $\mathcal{O}(1)$, we would have $R_{DPP}^{\text{exp}} = \mathcal{O}(1)$.
- Instead, perfectly consistent with $\mathcal{O}(\varepsilon^2)$.

More opportunities: Uncharted Territory

CP Asymmetry	HFLAV avg. [https://hflav.web.cern.ch/]	Experiments
$D^0 \to \pi^0\eta'$	—	—
$D^0 \to \eta\eta'$	—	—
$D^+ \to \pi^+\eta'$	$(0.40 \pm 0.20)\%$	LHCb'23, Belle'11, CLEO'10
$D_s^+ \to K^+\eta'$	$(6.0 \pm 18.9)\%$	CLEO'10

- Formalism and branching ratio fit:
  Slight tension in $\mathcal{B}(D_s^+ \to K^+\eta')$ and $\mathcal{B}(D^+ \to K^+\eta')$.
  [Bolognani, Nierste, Schacht, Vos, 2410.08138, accepted by JHEP]
- CP asymmetry predictions: Stay tuned.
  [Bolognani, Nierste, Schacht, Vos, in preparation]

Conclusions

- Charm is a unique gate to flavor structure of up-type quarks.
- This is just the beginning of the exploration of charm CPV.
  - Is the sum rule $a_{CP}^{\mathrm{dir}}(D_s^+ \to K_S\pi^+) = -a_{CP}^{\mathrm{dir}}(D^+ \to K_S K^+)$ broken at $\mathcal{O}(1)$, like the one between $D^0 \to \pi^+\pi^-, K^+K^-$?
  - How large is P/T in $D^0 \to \rho\pi$?
    Many more opportunities in multi-body decays.
  - Basically uncharted territory:
    $$A_{CP}(D^0 \to \pi^0\eta')\,, \quad A_{CP}(D^0 \to \eta\eta')\,, \quad A_{CP}(D_s^+ \to K^+\eta')\,.$$
  - And much more!

BACK-UP

The three $\Delta I = 1/2$ rules for $P \to \pi\pi$

- Relevant ratio of strong isospin matrix elements:

$r_{QCD}^{\Delta I=1/2} \equiv A^{\Delta I=1/2}/A^{\Delta I=3/2}$	Kaon	Charm	Beauty
Data	22	2.5	1.5
"No QCD" limit	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
Enhancement	$\mathcal{O}(10)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

[*D*: Franco Mishima Silvestrini 1203.3131, *B*: Grinstein Pirtskhalava Stone Uttayarat 1402.1164]

- Rescattering most important in K decays, less important but still significant in D decays, and small in B decays.