Recent developments in $c \to s l \nu$

Carolina Da Silva Bolognani[1], Camille Normand[2]

[1]Maastricht University, [2]University of Bristol

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Beyond the Flavour Anomalies

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CKM matrix

- Experimentally determine CKM elements

$$V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Study of $c \to s\ell\nu$ transitions $\Rightarrow V_{cs}$
- Test of unitarity

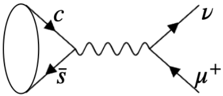
$$\text{Second row: } |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$\text{Second column: } |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$

(Semi)leptonic decays

All hadronic information factorises from leptonic: $\mathcal{B} \propto |V_{cs}|^2 |\text{Had. mat. el.}|^2$

Leptonic

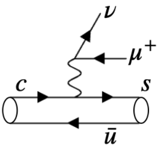


$D_s^+ \to \mu^+ \nu$

$$\langle 0|\bar{s}\gamma^\mu \gamma_5 c|D_s^+(p)\rangle = i f_{D_s} p^\mu$$

Decay constant

Semileptonic



$D^0 \to K^- \mu^+ \nu$

$$\langle K(k)|\bar{s}\gamma^\mu c|D(p)\rangle = f_0^{D\to K}(q^2) q^\mu \frac{M_D^2 - M_K^2}{q^2} + f_+^{D\to K}(q^2)\left[(p+k)^\mu - q^\mu \frac{M_D^2 - M_K^2}{q^2}\right]$$

Form factors

The current $|V_{cs}|$ by PDG

- Leptonic: HFLAV 2021 average of branching ratios of $D_s \to \{\mu, \tau\}\nu^*$ combined with PDG 2024 averages for mass, lifetime, decay constant
  - Average: $|V_{cs}|_{PDG, D_s \to \ell^+\nu} = 0.984 \pm 0.012$

- Semileptonic: HFLAV 2021 average of $f_+^{D\to K}(0)|V_{cs}|$ of $D^{+,0} \to K\{e, \mu\}\nu^*$ combined with FLAG 2021 average of ETM and (old) HPQCD form factor calculations*
  - No shape distribution information
  - Average: $|V_{cs}|_{PDG, D\to K\ell^+\nu} = 0.972 \pm 0.007$

- No electroweak corrections in Wilson coefficients
- Combined into $|V_{cs}|_{PDG} = 0.975 \pm 0.006$

*: since updated

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Framework

Bolognani, Reboud, van Dyk, Vos *JHEP 09 (2024) 099*

- SM description of $c \to s\ell\nu$ including Sirlin factor:

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_F}{\sqrt{2}}\; V_{cs}^*\, \mathcal{C}^{\ell}_{V,L}(\mu_c)\mathcal{O}^{\ell}_{V,L}$$

$$\mathcal{O}^{\ell}_{V,L} = [\bar{s}\gamma^{\mu} P_L c][\bar{\nu}\gamma_{\mu} P_L \ell]$$

$$\mathcal{C}^{\ell}_{V,L}(\mu_c) = 1 + \frac{\alpha_e}{\pi} \ln\left(\frac{M_Z}{\mu_c}\right) \simeq 1.01$$

- Bayesian model comparison between different fit models
- All models share hadronic nuisance parameters and experimental likelihood

Models: SM, CKM, WET

Eur.Phys.J.C 82 (2022) 6, 569

Experimental data

Bolognani, Reboud, van Dyk, Vos *JHEP 09 (2024) 099*

Branching ratio:

$D_s \to \tau^+ \nu_\tau$

$D_s \to \mu^+ \nu_\mu$

$D_s^* \to e^+ \nu_e$

$D^0 \to K^- \mu^+ \nu_\mu$

$D^0 \to K^- e^+ \nu_e$

$D^+ \to K_S^0 e^+ \nu_e$

$D^+ \to K_S^0 \mu^+ \nu_\mu$

$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$

$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$

Shape distribution:

$D^0 \to K^- \mu^+ \nu_\mu$

$D^0 \to K^- e^+ \nu_e$

$D^+ \to K_S^0 e^+ \nu_e$

Updated measurements here

New!
not included in PDG value

Hadronic matrix elements for $c \to s\ell\nu$ transitions

Bolognani, Reboud, van Dyk, Vos *JHEP 09 (2024) 099*

- Mostly calculated using LQCD
- Most competitive calculations: $f_{D_s}, f_{D_s^*}, \mathrm{FF}(D \to K), \mathrm{FF}(\Lambda_c^+ \to \Lambda^0)$
- Dispersive bounds: ensure unitarity, correlate most hadronic parameters through perturbatively calculated quantities χ related to hadronic representation
- BGL-like parametrisation of FF

$$\chi_A^{(J=0)}\Big|_{1\mathrm{pt}} = \frac{M_{D_s}^2 f_{D_s}^2}{(M_{D_s}^2 - Q^2)^2}$$

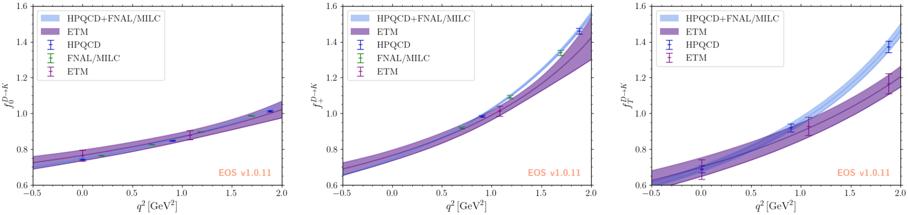
$$f(q^2) = \frac{1}{\phi_f(z)B(z)} \sum_{k=0}^{K} a_k^{(f)} p_k^{(f)}(z)\Bigg|_{z=z(q^2)}, \sum_f \sum_{k=0}^{K} |a_k^{(f)}|^2 < 1$$

Further discussion on FF approach: Gubernari, (Reboud), van Dyk, Virto 2021 & 2022; Blake et al. 2022; Flynn et al. 2023

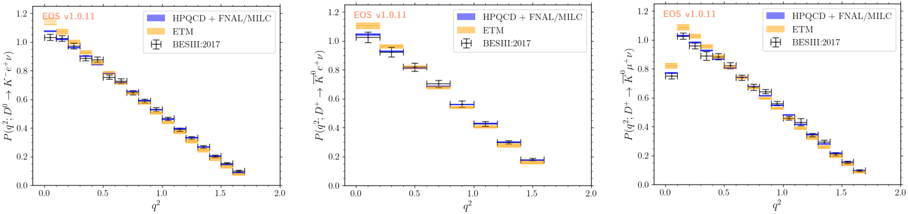
Incompatibility between LQCD calculations

Bolognani, Reboud, van Dyk, Vos *JHEP 09 (2024) 099*

- HPQCD+FNAL/MILC: p-value = 4% , +ETM: p-value<0.1%



- ETM does not reproduce experimental results well

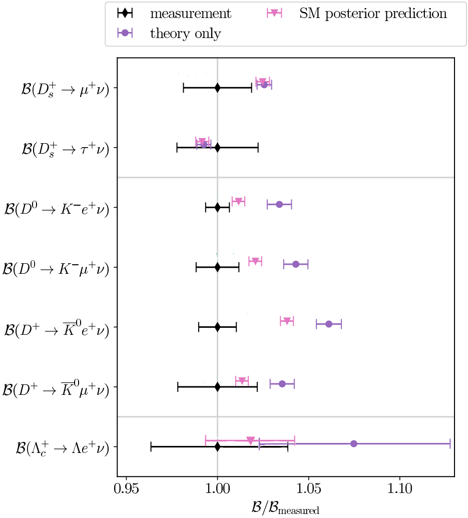


- Two scenarios: *nominal* (HPQCD+FNAL/MILC) and *scale factor* (all)

Compatibility of current $|V_{cs}|$ with data

Bolognani, Reboud, van Dyk, Vos *JHEP 09 (2024) 099*

- Results for *nominal* scenario
- Fixed value of $|V_{cs}| = 0.975$ from PDG
- Theory only: calculating the predictions for the branching ratios using the central values of the theoretical determinations
- SM model: fit to the experimental data allowing the theory determinations to vary within their uncertainties



Extraction of $|V_{cs}|$

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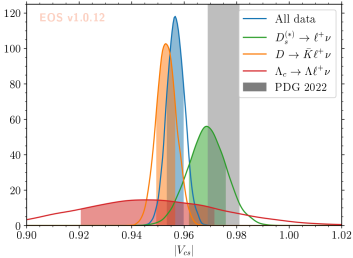

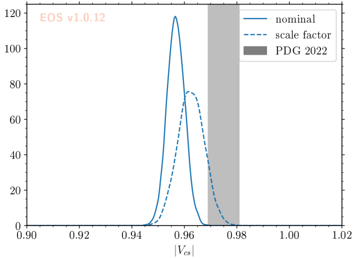

nominal: $|V_{cs}| = 0.957 \pm 0.003$

2.7σ away from PDG

scaled: $|V_{cs}| = 0.963 \pm 0.005$

1.5σ away from PDG

Unitarity

$$2^{\text{nd}} \text{ row}: \sum_{D=d,s,b} |V_{cD}|^2 \qquad 2^{\text{nd}} \text{ column}: \sum_{U=u,c,t} |V_{Us}|^2$$

	PDG	nominal	scale factor
$\lvert V_{cs}\rvert$	0.975 ± 0.006	0.957 ± 0.003	0.963 ± 0.005
2^{nd} row	1.00 ± 0.014 $(0.08\,\sigma)$	0.966 ± 0.008 $(4.3\,\sigma)$	0.978 ± 0.012 $(1.9\,\sigma)$
2^{nd} column	1.00 ± 0.012 $(0.22\,\sigma)$	0.968 ± 0.006 $(5.2\,\sigma)$	0.979 ± 0.010 $(2.0\,\sigma)$

Available space for BSM physics

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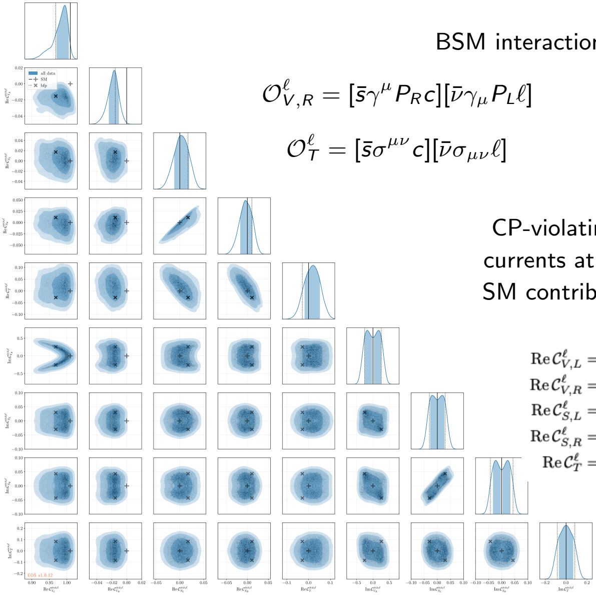

BSM interactions:

$$\mathcal{O}^{\ell}_{V,R} = [\bar{s}\gamma^{\mu} P_R c][\bar{\nu}\gamma_{\mu} P_L \ell] \qquad \mathcal{O}^{\ell}_{S,L} = [\bar{s} P_L c][\bar{\nu} P_L \ell]$$

$$\mathcal{O}^{\ell}_{T} = [\bar{s}\sigma^{\mu\nu} c][\bar{\nu}\sigma_{\mu\nu} \ell] \qquad \mathcal{O}^{\ell}_{S,R} = [\bar{s} P_R c][\bar{\nu} P_L \ell]$$

CP-violating effects in right-handed currents at the level of $\sim 23\%$ of the SM contribution are not yet excluded

$$\begin{aligned}
\mathrm{Re}\,\mathcal{C}^{\ell}_{V,L} &= [\ \ 0.957, \ \ 1.002]\,, & & \\
\mathrm{Re}\,\mathcal{C}^{\ell}_{V,R} &= [-0.026, -0.012]\,, & \mathrm{Im}\,\mathcal{C}^{\ell}_{V,R} &= [-0.225, 0.225]\,, \\
\mathrm{Re}\,\mathcal{C}^{\ell}_{S,L} &= [-0.019, \ \ 0.014]\,, & \mathrm{Im}\,\mathcal{C}^{\ell}_{S,L} &= [-0.030, 0.030]\,, \\
\mathrm{Re}\,\mathcal{C}^{\ell}_{S,R} &= [-0.026, \ \ 0.006]\,, & \mathrm{Im}\,\mathcal{C}^{\ell}_{S,R} &= [-0.028, 0.028]\,, \\
\mathrm{Re}\,\mathcal{C}^{\ell}_{T} &= [-0.021, \ \ 0.046]\,, & \mathrm{Im}\,\mathcal{C}^{\ell}_{T} &= [-0.068, 0.068]\,.
\end{aligned}$$

Conclusions of analysis

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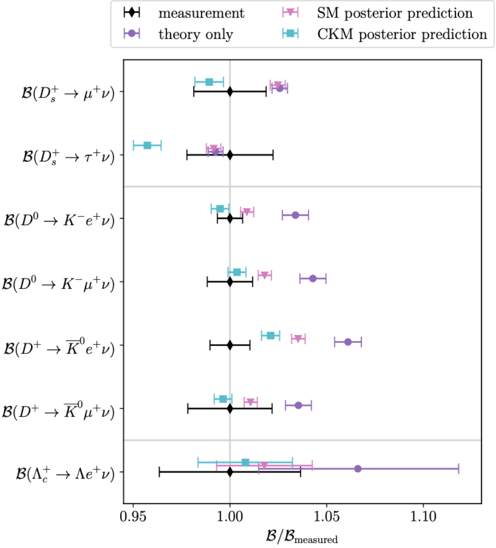
- Dispersive bounds connecting hadronic parameters
- Tensions between $D \to K$ f.f. determinations
- Extraction of $|V_{cs}| = 0.957 \pm 0.003$ deviating by 2.7σ from PDG due to Sirlin factor + f.f. factor input
- Preference for CKM model w.r.t. WET model is barely worth mentioning $\Rightarrow$ cannot distinguish

Scenario	Fit model M	χ^2	d.o.f.	p value [%]	$\ln P(D, M)$
nominal	SM	60.9	51	16.1	240.3 ± 0.3
	CKM	51.7	50	40.9	251.7 ± 0.3
	WET	48.4	42	23.1	250.3 ± 0.3
scale factor	SM	67.4	51	6.2	232.8 ± 0.3
	CKM	48.2	50	54.5	249.1 ± 0.3
	WET	46.6	42	29.0	248.7 ± 0.3



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Data sets

- BESIII: hermetic detector dedicated to τ and charm physics located at BEPCII (Beijing).
- Centre-of-mass energy ranging from 2 to 4.7 GeV.

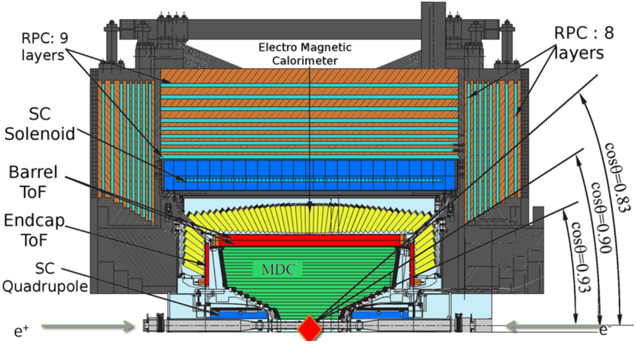


Figure: Credits: Alberto Bortone

Data sets and "double-tag" method

- Based on $7.33\,\mathrm{fb}^{-1}$ of e^+e^- collisions recorded at $E_{\mathrm{cm}} \in [4.128, 4.226]$ GeV.
- At this E_{cm}, $D_S^{\pm}$ mesons are produced through $e^+e^- \to D_s^+ D_s^{*-}$.
- Double tag method used to cancel the luminosity and the $e^+e^- \to D_s^+ D_s^{*-}$ cross-section dependence:
  - Reconstruct D_s^- through several *tag* modes, e.g. $K\pi\pi, KK\pi\pi^0$ [1]: N_{ST}, single tag.
  - Among these candidates, reconstruct $D_s^{*-} \to D_s^- \gamma/\pi^0$ and the decay of interest $D_s^+ \to \mu^+\nu$: N_{DT}, double tag.
  - Branching fraction computed as

$$N_{\mathrm{DT}} = N_{\mathrm{ST}} \times \epsilon_{\gamma/\pi^0\mu\nu} \times \mathcal{B}(D_s^- \to \mu\nu) \quad (1)$$

where

$$\epsilon_{\gamma/\pi^0\mu\nu} \equiv \sum_{\text{tag modes}} (N^i_{\mathrm{ST}}/N_{\mathrm{ST}}) \times (\epsilon^i_{\mathrm{DT}}/\epsilon^i_{\mathrm{ST}}) \quad (2)$$

[1] Complete list of 16 modes in the back-ups

Analysis steps (That I will skip)

- Selections of the single tag modes
  $\epsilon_{\mathrm{ST}} \in [9, 52]\%$
- Determination of the number of single tag events with a fit to the (mode-dependent) invariant mass of the final state particles
  $N^i_{\mathrm{ST}} \in [5\mathrm{k}, 42\mathrm{k}]$
- Selections of the double tag candidates
  $\epsilon_{\gamma/\pi^0\mu\nu} \in [50, 66]\%$
- Estimation of systematic uncertainties

Extraction of N_{DT}

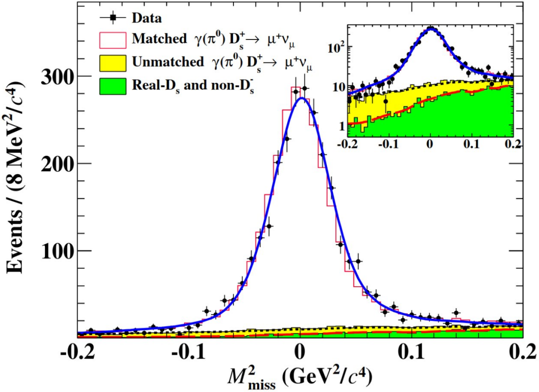
- Fit to the missing mass defined as

$$\begin{aligned} M^2_{\mathrm{miss}} &= (E_{cm} - E_{tag} - E_{\gamma/\pi^0} - E_{\mu})^2/c^4 \\ &-(-\vec{p}_{tag} - \vec{p}_{\gamma/\pi^0} - \vec{p}_{\mu})^2/c^2 \end{aligned} \quad (3)$$

- Two types of signal events **are modelled separately.**
  - Matched: the γ/π^0 is properly reconstructed.
    Peaking in the missing mass.
  - Unmatched: it is not.
    Not peaking, but still contains signal candidates.
- Two dominant background components
  - non-D_s^- background (wrongly single-tagged D_s^-) extracted from inclusive MC samples.
  - real-D_s^- background dominated by $D_s^- \to \tau\nu, \tau \to \pi\nu$

Branching fraction measurement

- Signal is modeled from MC with a (free) Gaussian to account for data-MC discrepancies.



- $N_{\rm DT} = 2514.5 \pm 51.6$ events gives

$$\mathcal{B}(D_s^\pm \to \mu^\pm \nu) = (0.5294 \pm 0.0108)\%$$

Interpretation

- The partial decay width is given by

$$\Gamma_{D_s^\pm \to \mu\nu} = \frac{G_F^2}{8\pi} |V_{cs}|^2 f_{D_s^\pm}^2 m_\mu^2 m_{D_s^\pm} \left(1 - \frac{m_\mu^2}{m_{D_s^\pm}^2}\right)^2$$

- With G_F , m_μ, $m_{D_s^\pm}$ and the $D_s^\pm$ lifetime from the PDG,

$$f_{D_s^\pm} \times |V_{cs}| = 241.8 \pm 2.5(\mathrm{stat}) \pm 2.2(\mathrm{syst})\,\mathsf{MeV}$$

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Data sets and “double-tag” method

- Based on $7.93\,\mathrm{fb}^{-1}$ of e^+e^- collisions recorded at $E_{\mathrm{cm}} = 3.773\,\mathrm{GeV}$.

- The dominant production mechanism of charged or neutral D mesons is

$$e^+e^- \to \psi(3770) \to D\bar{D}$$

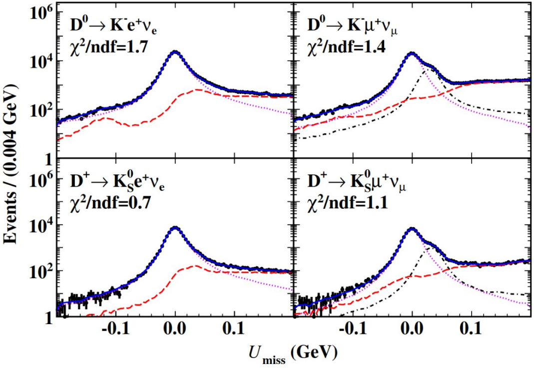
- The double tag method is applied here again.

Signal yields extraction

- Simultaneous fit in all four channels to the distribution of U_{miss} defined as

$$U_{\mathrm{miss}} = E_{\mathrm{miss}} - \vec{p}_{\mathrm{miss}}$$

- The main background components come from mis-ID'd hadronic or semileptonic decays of D mesons.



Branching fraction measurement

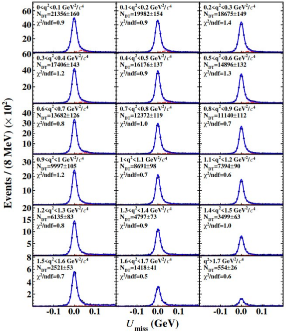
- Branching fraction results are (in %)

Channel	This work	Bolognani et al.
$D^0 \to K^- e\nu$	$3.521 \pm 0.009 \pm 0.016$	3.525 ± 0.023
$D^0 \to K^- \mu\nu_\mu$	$3.419 \pm 0.011 \pm 0.016$	3.41 ± 0.04
$D^+ \to \bar{K}^0 e\nu_e$	$8.864 \pm 0.039 \pm 0.082$	8.72 ± 0.09
$D^+ \to \bar{K}^0 \mu\nu_\mu$	$8.665 \pm 0.046 \pm 0.084$	$8.72 \pm 0.07 \pm 0.18$

- This shows that the tension presented before with the "theory only" prediction persists with this new measurement.

Bolognani, Reboud, van Dyk, Vos JHEP 09 (2024) 099

- The U_{miss} fit is reproduced in bins of the di-lepton invariant mass to extract partial decay rates $\Delta\Gamma^{\mathrm{meas.}}$ to extract N^i_{DT}, and compute the efficiency and bin-migration corrected signal yields N^i_{prod}



q^2 (GeV$^2/c^4$)	N^i_{DT}	N^i_{prod}	$\Delta\Gamma$ (ns^{-1})
(0.00, 0.10)	21356 ± 160	29580 ± 236	9.100 ± 0.073
(0.10, 0.20)	19982 ± 154	28248 ± 247	8.690 ± 0.076
(0.20, 0.30)	18675 ± 149	26707 ± 249	8.216 ± 0.076
	⋮		
(1.40, 1.50)	3499 ± 63	5627 ± 118	1.731 ± 0.036
(1.50, 1.60)	2521 ± 53	4356 ± 105	1.340 ± 0.032
(1.60, 1.70)	1418 ± 41	2621 ± 86	0.806 ± 0.026
(1.70, 1.88)	554 ± 26	1378 ± 72	0.424 ± 0.022

Form factors parameterisation

- The form factors are extracted through a least-χ^2 fit, taking into account the statistical and systematic covariances between the various q^2 bins,

$$\chi^2 = \sum_{i,j}^{N_{\mathrm{bins}}} \Delta\Gamma_j C_{ij} \Delta\Gamma_i$$

  with $\Delta\Gamma_i = \Delta\Gamma^{\mathrm{meas.}} - \Delta\Gamma^{\mathrm{pred.}}$ the difference between measured and predicted partial decay rates, and $\Delta\Gamma^{\mathrm{pred.}}$ depends on the form factors.

- Form-factor parametrised with a two (one) parameter z-expansion for f_+ (f_0)

$$\begin{aligned} f_+^K(q^2) &= \frac{1}{P(q^2)\Phi(q^2)} \frac{f_+^K(0)P(0)\Phi(0)}{1 + r_1(t_0)z(0,t_0)} \times (1 + r_1(t_0)[z(q^2,t_0)]) \\ f_0^K(q^2) &= \frac{1}{P(q^2)\Phi(q^2)} f_0^K(0)P(0)\Phi(0) \end{aligned}$$

Form factors parameterisation

- With $f_+^K(0) = f_0^K(0)$, there are **two free parameters:**

$$r_1(t_0) \text{ and } V_{cs} \times f_+^K(0)$$

- Results from the combined fits are

Fit procedure	Simultaneous to all $D \to K\ell\nu$
$f_+^K(0) \times \lvert Vcs \rvert$	$0.7171 \pm 0.0011 \pm 0.0013$
$r_1(t_0)$	$-2.28 \pm 0.04 \pm 0.02$
Correlation	0.44
χ^2/ndf	60.9/70

- **Good fit quality** from the two parameters z-expansion.

Comparison with LQCD

- Comparison with LQCD results from the FermiLab/MILC collaborations[2]

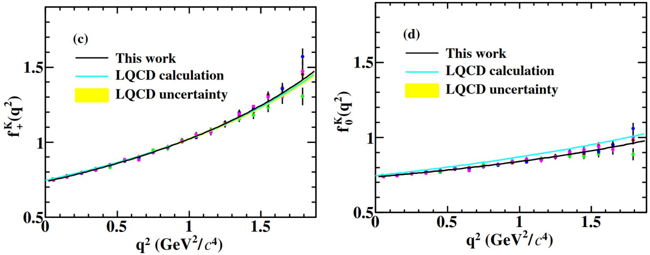


Figure: (left) f_+ and (right) f_0

- Fairly good agreement !
- All information to reproduce the least-χ^2 fits (and to do your own !) are available in the paper.

[2] Fermilab Lattice, MILC collaboration, *Phys. Rev. D 107 (2023) 094516, [hep-lat/2212.12648].*

Experimental conclusion - Discussion

- Single-measurement determination of $V_{cs} \times f_{D_s}$ is **reaching the 2% level precision.**
- Increased statistics allow for **fully experimental determination of the form factors in various $c \to s l \nu$ channels.**
  - How to provide results in a way that would be the most useful to theorists/long-term reinterpretation ?
  - Possibly an unbinned measurement ? In what form ?
- Concerning excited strange meson states, BESIII can also provide some interesting studies measuring the **contributions from S- and P-waves** in e.g. $D \to K^* \ell \nu$.
  - How could these results interplay with theoretical predictions/provide inputs for theoretical predictions (i.e. data-driven determination) ?
  - Again, how to provide results in a way that would be the most useful to theorists/long-term reinterpretation ?
  - What's your favourite channel ?
  - Shapes for parameterising S- and P-waves ?

Theory inputs

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Decay constants f_{D_s} , $f_{D_s^*}$ and $f^T_{D_s^*}$ $\longrightarrow$ $\left.\dfrac{f_{D_s^*,T}}{f_{D_s}}\right|_{\mathrm{QCDSR}}$

ETM
FNAL/MILC

CLQCD

FLAG Review 2021
ETM, Phys.Rev.D 91 (2015)
FNAL/MILC, Phys.Rev.D 98 (2018)
CLQCD, Phys.Rev.D 109 (2024)
Pullin, Zwicky, JHEP 09 (2021) 023

$D \to K$ form factors $\longrightarrow$ HPQCD, FNAL/MILC, ETM

HPQCD, Phys.Rev.D 107 (2023)
FNAL/MILC, Phys.Rev.D 107 (2023)
ETM, Phys.Rev.D 96 (2017)
ETM, Phys.Rev.D 98 (2018)

$\Lambda_c \to \Lambda$ form factors

$\nearrow$ (Axial)vector and (pseudo)scalar: LQCD — Meinel, Phys.Rev.Lett. 118 (2017)

$\searrow$ Tensor: HQET + SCET relations to (axial)vector FF's — Our work

$\Lambda_c \to \Lambda$ FFs theory

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$$\begin{aligned}
\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,\gamma^\mu\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle &= \bar{u}_\Lambda(k,s_\Lambda)\Big[f_{V,t}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}-m_\Lambda)\frac{q^\mu}{q^2}\\
&+ f_{V,0}^{\Lambda_c\to\Lambda}(q^2)\frac{m_{\Lambda_c}+m_\Lambda}{s_+}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)\\
&+ f_{V,\perp}^{\Lambda_c\to\Lambda}(q^2)\left(\gamma^\mu-\frac{2m_\Lambda}{s_+}p^\mu-\frac{2m_{\Lambda_c}}{s_+}k^\mu\right)\Big]u_{\Lambda_c}(p,s_{\Lambda_c})\,,\\
\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,\gamma^\mu\gamma_5\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle &= -\bar{u}_\Lambda(k,s_\Lambda)\,\gamma_5\Big[f_{A,t}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}+m_\Lambda)\frac{q^\mu}{q^2}\\
&+ f_{A,0}^{\Lambda_c\to\Lambda}(q^2)\frac{m_{\Lambda_c}-m_\Lambda}{s_-}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)\\
&+ f_{A,\perp}^{\Lambda_c\to\Lambda}(q^2)\left(\gamma^\mu+\frac{2m_\Lambda}{s_-}p^\mu-\frac{2m_{\Lambda_c}}{s_-}k^\mu\right)\Big]u_{\Lambda_c}(p_{\Lambda_c},s_{\Lambda_c}),\\
\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\,b\,|\Lambda_c(p,s_{\Lambda_c})\rangle &= -\bar{u}_\Lambda(k,s_\Lambda)\Big[f_{T,0}^{\Lambda_c\to\Lambda}(q^2)\frac{q^2}{s_+}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)\\
&+ f_{T,\perp}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}+m_\Lambda)\left(\gamma^\mu-\frac{2m_\Lambda}{s_+}\,p^\mu-\frac{2m_{\Lambda_c}}{s_+}\,k^\mu\right)\Big]u_{\Lambda_c}(p,s_{\Lambda_c})\,,\\
\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\gamma_5\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle &= -\bar{u}_\Lambda(k,s_\Lambda)\,\gamma_5\Big[f_{T5,0}^{\Lambda_c\to\Lambda}(q^2)\,\frac{q^2}{s_-}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)\\
&+ f_{T5,\perp}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}-m_\Lambda)\left(\gamma^\mu+\frac{2m_\Lambda}{s_-}\,p^\mu-\frac{2m_{\Lambda_c}}{s_-}\,k^\mu\right)\Big]u_{\Lambda_c}(p,s_{\Lambda_c})\,,
\end{aligned}$$

Heavy Quark limit ⇒ expansion in α_s/π , $\Lambda_{\mathrm{QCD}}/m_c$

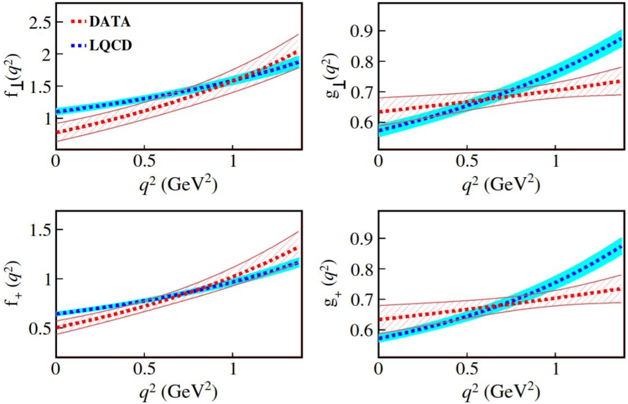
Large Energy limit ⇒ expansion in α_s/π , $\Lambda_{\mathrm{had}}/m_c$, $\Lambda_{\mathrm{had}}/E_\Lambda$

$$\begin{aligned}
\frac{\xi}{m_{\Lambda_c}} &= f_{V,t}(0) = f_{V,\perp}(0) = f_{V,0}(0) = f_{A,t}(0) = f_{A,\perp}(0) = f_{A,0}(0)\\
&= f_{T,\perp}(0) = f_{T,0}(0) = f_{T5,\perp}(0) = f_{T5,0}(0)\,,\\
\frac{\xi_1-\xi_2}{m_{\Lambda_c}} &= f_{V,\perp}(q^2_{\max}) = f_{V,0}(q^2_{\max}) = f_{A,t}(q^2_{\max}) = f_{T,\perp}(q^2_{\max}) = f_{T,0}(q^2_{\max})\,,\\
\frac{\xi_1+\xi_2}{m_{\Lambda_c}} &= f_{A,\perp}(q^2_{\max}) = f_{A,0}(q^2_{\max}) = f_{V,t}(q^2_{\max}) = f_{T5,\perp}(q^2_{\max}) = f_{T5,0}(q^2_{\max})\,.
\end{aligned}$$

Constraint through relations

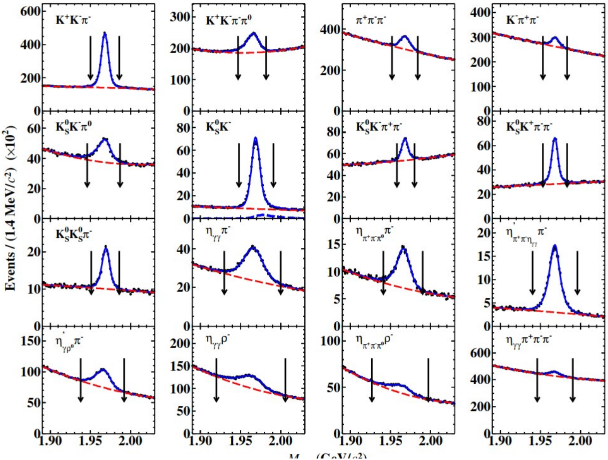
$$\begin{aligned}
f_{T,\perp}/f_{V,\perp} &= 1\pm 0.35\,, \quad f_{T,0}/f_{V,0} = 1\pm 0.35\,,\\
f_{T5,\perp}/f_{A,\perp} &= 1\pm 0.35\,, \quad f_{T5,0}/f_{A,0} = 1\pm 0.35\,,
\end{aligned}$$

- $\Lambda_c \to \Lambda$ FFs
  BESIII: arXiv:2306.02624
  LQCD: S. Meinel, Phys. Rev. Lett. 118, 082001 (2017).



$D_s \to \mu\nu$ STs

- 16 channels



$D_s \to \mu\nu$ systematics

- Uncertainties

Source	Uncertainty (%)
ST yield	0.44
μ tracking	0.24
μ PID	0.19
Transition γ/π^0 reconstruction	1.00
Least $\lvert\Delta E\rvert$ selection	0.70
$E_{\mathrm{max}}^{\mathrm{extrafl}}$ and $N_{\mathrm{ncharged}}^{\mathrm{extra}}$ requirements	0.29
M_2 miss fit	0.72
Quoted BFs	0.34
Contribution from $D_s^+ \to \gamma\mu\nu$	0.30
Total	1.61

$D \to K\ell\nu$ STs

- 12 channels