Numerical exploratory calculation of charming penguin contributions to $B(B_s) \to K(\eta_s)\ell^+\ell^-$

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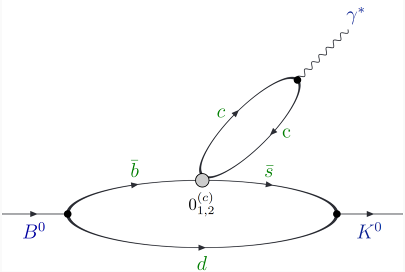
Beyond the Flavour Anomalies - April 9th 2025

A (very) challenging calculation

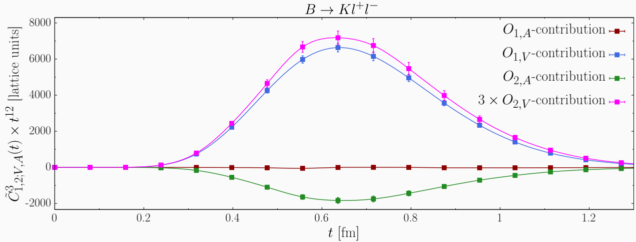


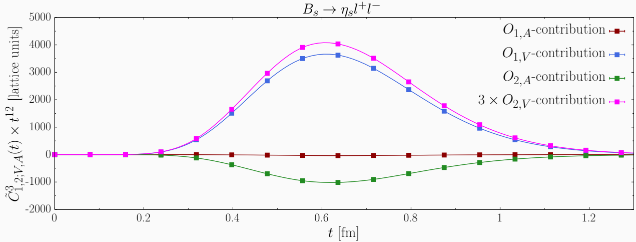
- We focus on the charming penguin diagram in the figure.
- We have investigated the possibility of computing this diagram on a single Extended Twisted Mass (ETM) gauge ensemble ($a \simeq 0.08$ fm).

- We considered a single heavy quark mass $m_h = 2m_c < m_b$, and single photon momentum $\boldsymbol{q} \simeq 250$ MeV in the decaying meson rest frame.
- A full calculation requires handling both the IR part (through the SFR/HLT method) and the UV part (renormalization of the relevant matrix elements).
- For now we only performed a proof-of-principles calculation to show that the IR part (the previously-considered limiting factor) can be controlled.
- However, this is the first time charming penguin diagrams are investigated on the lattice.

The lattice correlator for $t > 0$ (where SFR/HLT is required)

PRELIMINARY results for $\tilde{C}^{\nu+}_{1,2}(t,\boldsymbol{q}) = e^{-E_{K(\eta_s)}t} C^{\nu+}_{1,2}(t,\boldsymbol{q})$



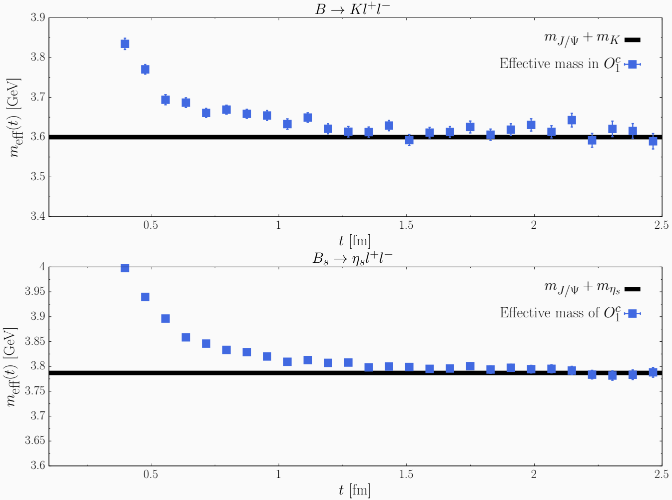


- In the factorization approximation: $C^{\nu+}_{1,V} = 3C^{\nu+}_{2,V}$, $C^{\nu+}_{1,2,A} = 0$.
- We find a very small $C^{\nu+}_{1,A}$, but $C^{\nu+}_{2,A}$ (purely non-fact.) is sizable.
- No renormalization performed: these are the bare correlators.

Sanity check: the lowest intermediate-state energy

- Correlators $\tilde{C}^{\nu+}_{1,2}(t, \boldsymbol{q})$ are of the form: $\tilde{C}^{\nu+}_{1,2}(t, \boldsymbol{q}) = \sum_n A^{\nu+(n)}_{1,2} e^{-E_n t}$.
- Lowest expected energy E_0 is:

$$E_0 = m_{J/\Psi} + m_K \ \ (B \to K\ell^+\ell^-), \qquad E_0 = m_{J/\Psi} + m_{\eta_s} \ \ (B_s \to \eta_s \ell^+\ell^-)$$



From the Euclidean correlators to the amplitude

The charming-penguin contribution to the hadronic part ($H^{\nu}_{1,2}(\boldsymbol{q})$) of the $B \to K\ell^+\ell^-$ (or $B_s \to \eta_s \ell^+\ell^-$) amplitude can be written as (q_0 is γ^* energy)

$$H^{\nu}_{1,2}(\boldsymbol{q}) = \underbrace{\int_{E_0^-}^{\infty} \frac{\rho^{\nu -}_{1,2}(E,\boldsymbol{q})}{E - m_B + q_0 - i\epsilon}}_{H^{\nu -}_{1,2}(\boldsymbol{q})} + \underbrace{\int_{E_0}^{\infty} \frac{dE}{2\pi}\, \frac{\rho^{\nu +}_{1,2}(E,\boldsymbol{q})}{E - m_B - i\epsilon}}_{H^{\nu +}_{1,2}(\boldsymbol{q})}$$

The spectral densities $\rho^{\pm}_{1,2}$ are related to the Euclidean correlators through

$$C^{\nu -}_{1,2}(t,\boldsymbol{q}) = \int_{E_0^-}^{\infty} \frac{dE}{2\pi} e^{-E|t|} \rho^{\nu -}_{1,2}(E,\boldsymbol{q}) \;, \qquad \tilde{C}^{\nu +}_{1,2}(t,\boldsymbol{q}) = \int_{E_0}^{\infty} \frac{dE}{2\pi} e^{-Et} \rho^{\nu +}_{1,2}(E,\boldsymbol{q})$$

- As shown by Chris, only the second term is problematic: the integrand develops singularities in the integration-range for $\varepsilon \to 0^+$. In the SFR method* one promotes ε to a non-zero energy scale (which regularizes the singularities). This defines the smeared amplitude

$$H^{\nu +}_{1,2}(\boldsymbol{q}, \varepsilon) \equiv \int_{E_0}^{\infty} \frac{dE}{2\pi}\, \frac{\rho^{\nu +}_{1,2}(E,\boldsymbol{q})}{E - m_B - i\epsilon}$$

* R. Frezzotti, G. Gagliardi, V. Lubicz, F. Sanfilippo, S. Simula, N. Tantalo, arXiv:2306.07228

SFR/HLT applied to the charming penguin diagram

It is convenient to organize the calculation in such a way that the (difficult) part treated with SFR/HLT does not have contact divergencies:

$$H^{\nu}_{1,2}(\boldsymbol{q}) = \underbrace{\int_{-\infty}^{\infty} dt\, C^{\nu}_{1,2}(t,\boldsymbol{q})\, f(t)}_{\text{easy-part, contact-log-divergence}} + \underbrace{\lim_{m\to m_{B_{(s)}}} \lim_{\varepsilon\to 0^+} H^{\nu+;3-\text{subs}}_{1,2}(\boldsymbol{q},m;\varepsilon)}_{\text{handled via SFR/HLT}}$$

$$H^{\nu+;3-\text{subs}}_{1,2}(\boldsymbol{q},m;\varepsilon) = \int_{E_0}^{\infty} \frac{dE}{2\pi} \rho^{\nu+}(E,\boldsymbol{q}) K^{3-\text{subs}}(E,m;\varepsilon)\ ,$$

where the three-times-subtracted kernel is given by

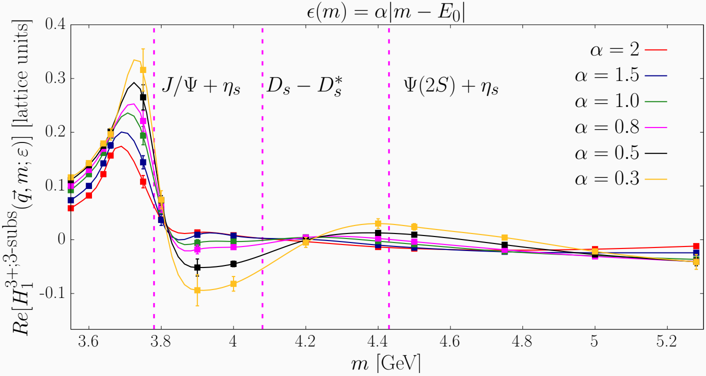
$$K^{3-\text{subs}}(E,m;\varepsilon) = \frac{1}{E-m-i\varepsilon} + \frac{3}{E+m-i\varepsilon} - \frac{3}{E-i\varepsilon} - \frac{1}{E+2m-i\varepsilon}$$

We have done the exercise of computing $H^{\nu+;3-\text{subs}}_{1,2}(\boldsymbol{q},m;\varepsilon)$ as a function of m, for different ε.

PRELIMINARY numerical results

We show, as an example, the results for $B_s \to \eta_s \ell^+ \ell^-$.

- The results are shown as a function of m for the O_1^c-contribution.
- ε is taken to be $m-$dependent: $\varepsilon(m) = \alpha|m - E_0|$, for different choices of the parameter α (at fixed α the difficulty of the spectral reconstruction is basically $m-$independent).



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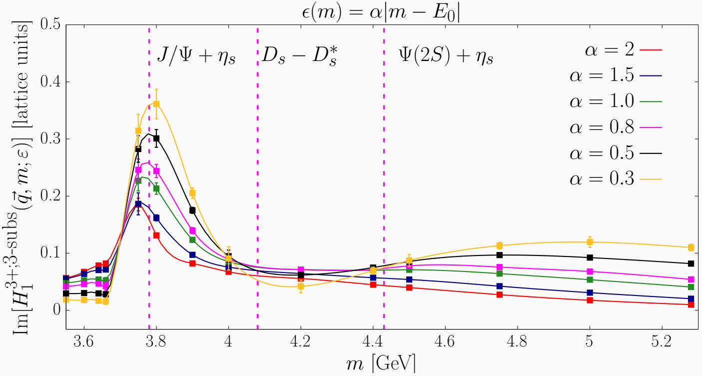


A simple model for $\rho_1^{\nu+}(E,\boldsymbol{q})$

Let's try to compare our results with a rough model for the spectral density $\rho_1^{\nu+}$, assuming the factorization approximation $(V^\mu(x) = \bar{c}(x)\gamma^\mu c(x))$

$$\rho_1^{\nu+;\mathrm{FA}}(E,\boldsymbol{q},m_H) = \langle \eta_s(-\boldsymbol{q})|\bar{s}\gamma^\mu b|B_s(\mathbf{0})\rangle \times Q_c\langle 0|V_\mu(0)\,\delta(E-\mathbb{H})\,V^\nu(0,\boldsymbol{q})|0\rangle$$

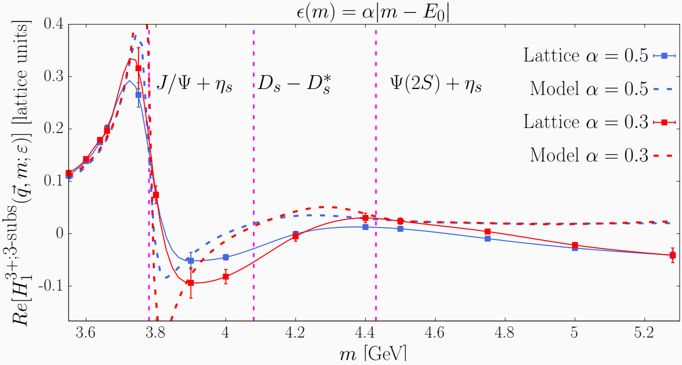
The first term parametrized by local FF f_0 and f_+

$$\langle \eta_s(-\boldsymbol{q})|\bar{s}\gamma^\mu b(0)|B_s(\mathbf{0})\rangle = f_+(q^2)\left(p_H^\mu + p_\eta^\mu - \frac{m_H^2 - m_{\eta_s}^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_H^2 - m_{\eta_s}^2}{q^2}q^\mu$$

- $f_+(q^2)$, $f_0(q^2)$ directly evaluated on the lattice.
- The $c\bar{c}$ part modeled as a sum of charmonium resonances (V) as:
$$\langle 0|V_\mu(0)\,\delta(E-\mathbb{H})\,V^\nu(0,\vec{q})|0\rangle = \sum_V \frac{f_V^2 m_V^2}{2E_V}\left(\delta_\mu^\nu - \frac{1}{m_V^2}k_{V\mu}k_V^\nu\right)\delta(E-E_V)$$
- $E_V = \sqrt{m_V^2 + |\boldsymbol{q}|^2}$. Masses and decay constants taken from PDG for the first few resonances. We also added to $\rho_1^{\nu+;\mathrm{FA}}$ a perturbative high-energy part $\propto E^2$.

Comparison with the simple model

The comparison between model and lattice data is qualitative. Overall scale (due to the missing renormalization in our results) adjusted to make model and data agree at low values of m (turns out to be $\simeq 1$). Discretization effects are present in the data (especially relevant at large m).



No orders-of-magnitude difference between model and lattice data for all m and ε explored. However, this is only the O_1^c contribution!

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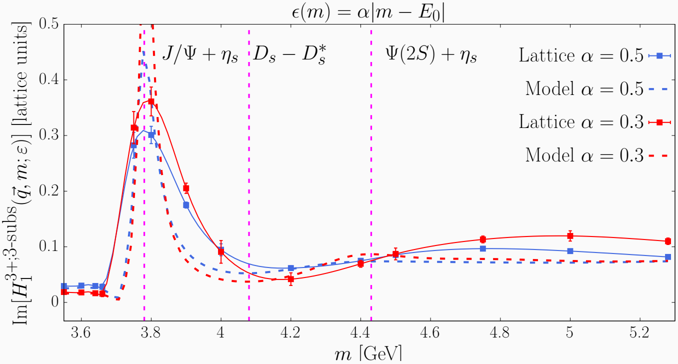

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Conclusions

Todo list (Short-term):

- Perform SFR/HLT analysis also for $B \to K\ell^+\ell^-$.
- In the region of m where the spectral density is smooth (away from narrow $c\bar{c}$ resonances) attempt $\varepsilon \to 0$ extrapolation.

Todo list (Long-term):

- Producing phenomenologically relevant results for $B \to K\ell^+\ell^-$ requires performing a series of very demanding calculations, e.g.: computations must be performed for several heavy-quark masses to extrapolate to physical b quark, several momenta $\boldsymbol{q}$ must be considered, all contractions of the four-fermion operators must be included (not only the charming-penguin).
- Non-perturbative renormalization must be performed, and calculations must be done on at least three gauge ensemble to perform continuum-limit extrapolation.

Thank you for the attention