News on $D - \overline{D}$

6th edition of "Beyond the flavour anomalies"

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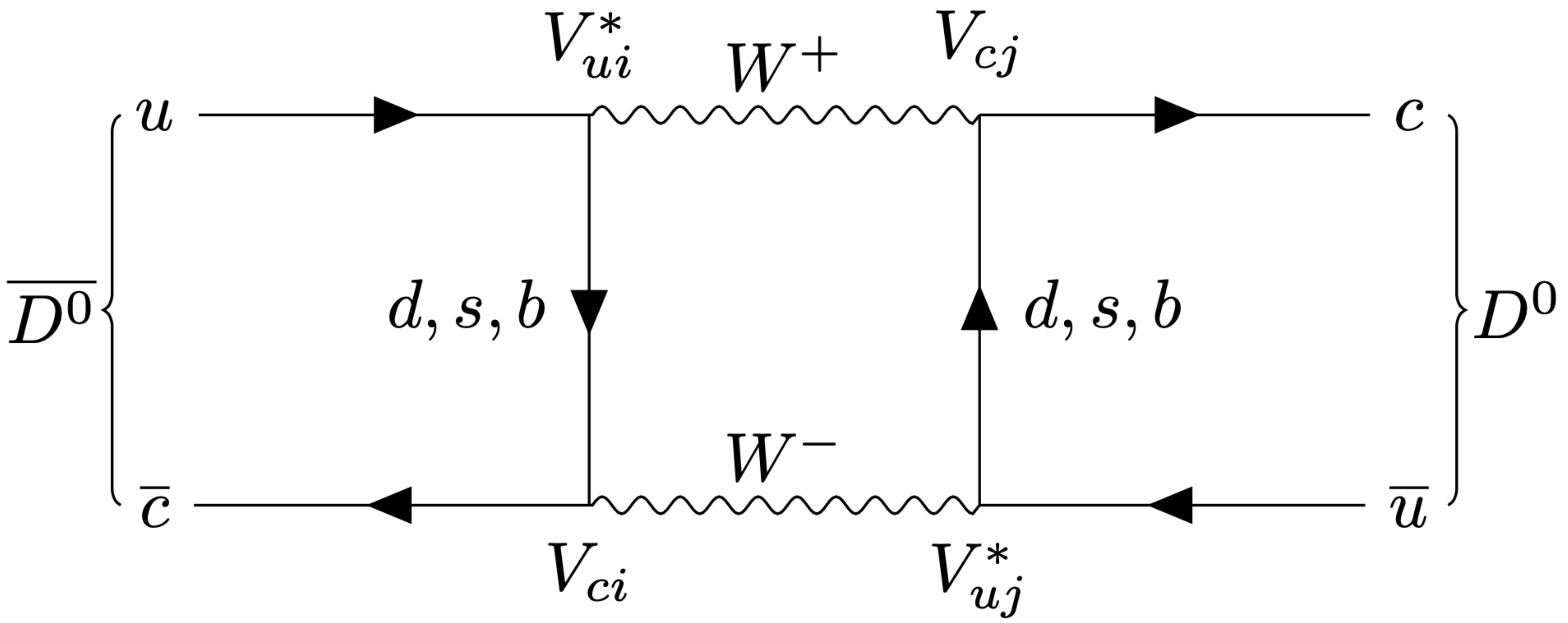
Why charm mixing?

- NO FCNC at the tree-level in the SM

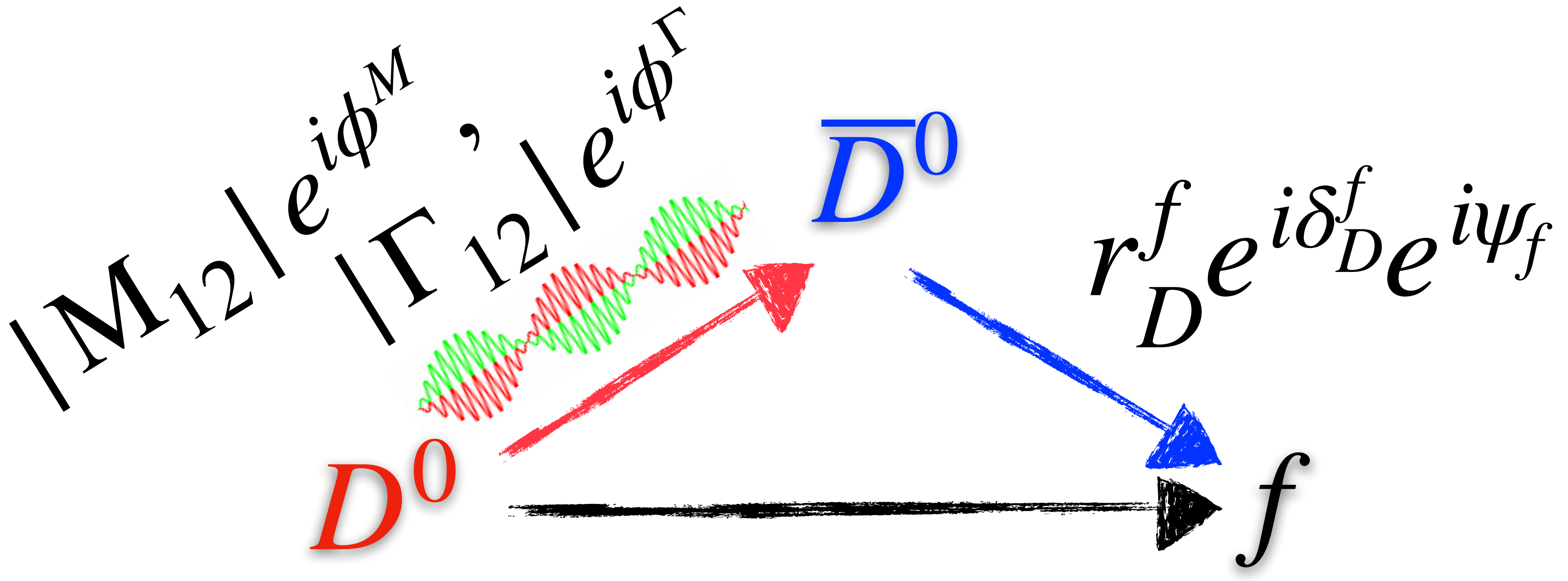
- From GIM + CKM: CPV is very suppressed $O(\theta_c^4)$



$$\mathrm{H} = \mathrm{M} - i/2\Gamma$$

Description of indirect CPV

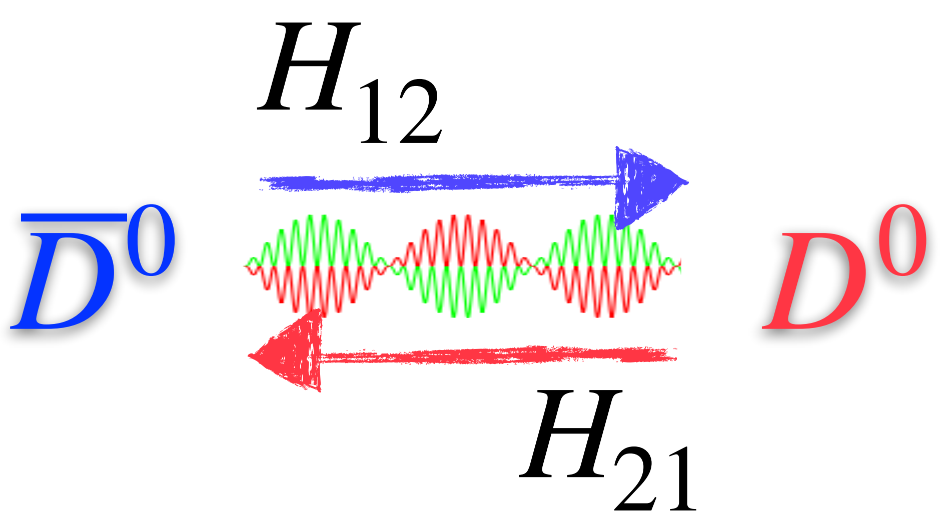
- CPV in the interference between mixing and decay to a state f



$$\phi_f^M = \phi^M + \psi_f$$

$$\phi_f^\Gamma = \phi^\Gamma + \psi_f$$

- CPV in pure mixing



$$\phi_{12} = \phi_f^M - \phi_f^\Gamma$$

Experimental measurements of CPV

More than 15 years of experiments...

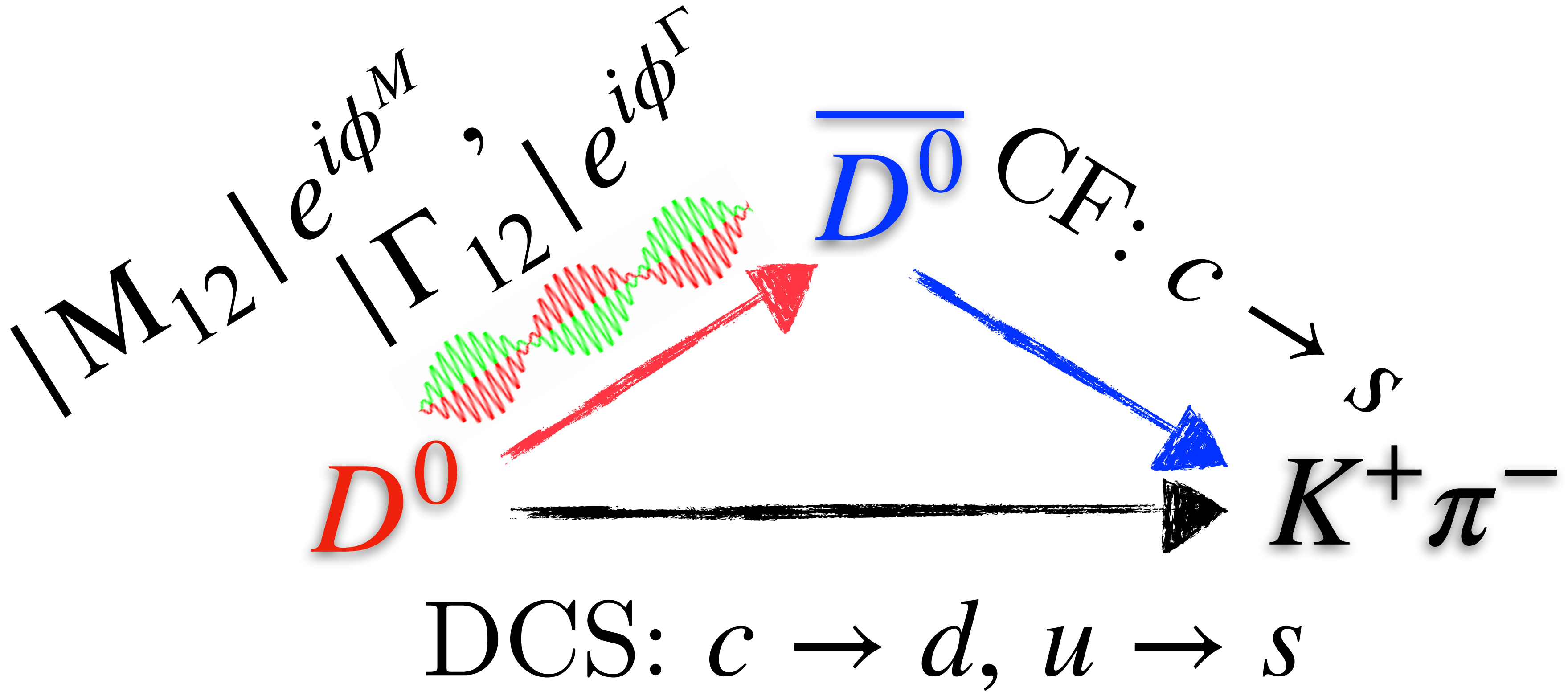
- CF/DCS decays to $K^{\mp}\pi^{\pm}(\pi^0)$

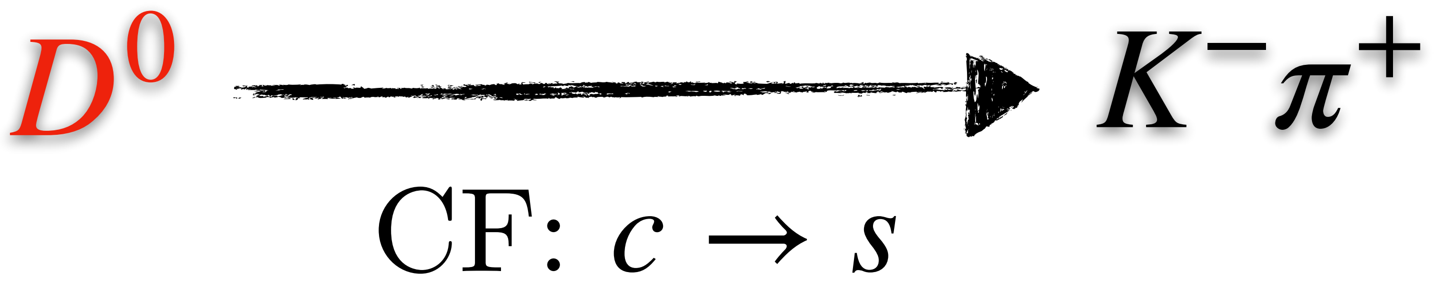
- Phase-space analyses of the three-body mode $K_S^0\pi^+\pi^-$

- SCS decays to $\pi^+\pi^-$, K^+K^- (also direct CPV: $|A_D^f| \neq |A_{\bar{D}}^f|$)

CF/DCS decays to $K^{\mp}\pi^{\pm}$

- Exploiting the interference between the CF and the DCS decays





- They fit the ratio of these two modes in time

(e.g. Belle PRL(2006), Babar PRL(2007), LHCb(2024, 2025))

$$\frac{d\Gamma(D^0 \to K^+\pi^-)}{d\Gamma(D^0 \to K^-\pi^+)}(t) = (r_D^f)^2 + r_D^f L^+(\phi^{\rm M}_{K\pi}, \phi^{\Gamma}_{K\pi})t/\tau + Q^+(\phi^{\rm M}_{K\pi}, \phi^{\Gamma}_{K\pi})(t/\tau)^2$$

Three-body decays

- They measure the Dalitz distribution of the events

$$d\Gamma_i(D^0 \to K_S^0\pi^+\pi^-)(t) \qquad d\Gamma_i(\overline{D}^0 \to K_S^0\pi^+\pi^-)(t)$$

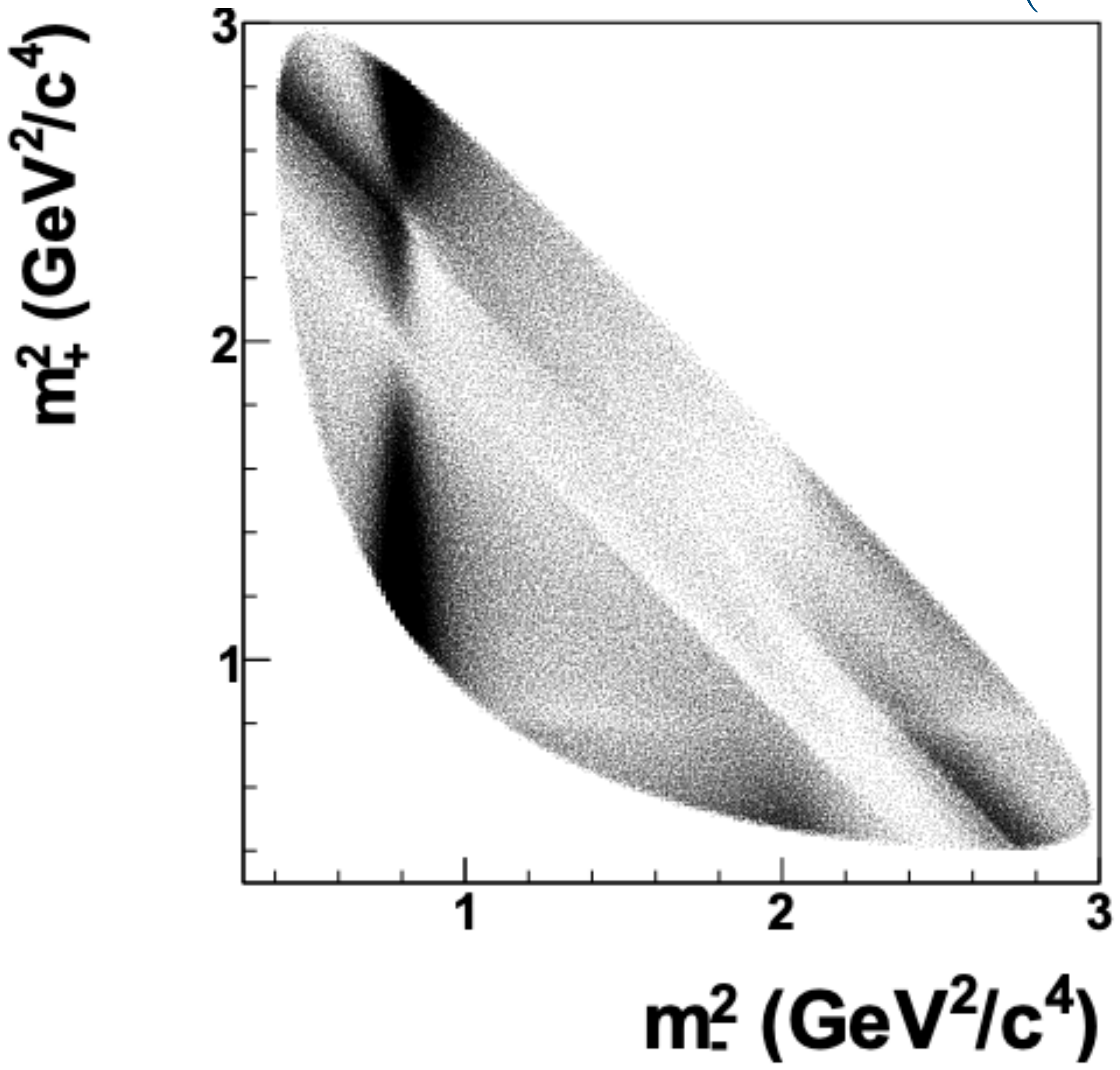
- Fitting CP-conserving obs.

$$x_{CP}^f = 2|M_{12}|/\Gamma\cos(\phi^M_{K_S^0\pi\pi}) \qquad y_{CP}^f = |\Gamma_{12}|/\Gamma\cos(\phi^\Gamma_{K_S^0\pi\pi})$$

- Fitting CPV obs.

$$\Delta x^f = -|\Gamma_{12}|/\Gamma\sin(\phi^\Gamma_{K_S^0\pi\pi}) \qquad \Delta y^f = 2|M_{12}|/\Gamma\sin(\phi^M_{K_S^0\pi\pi})$$

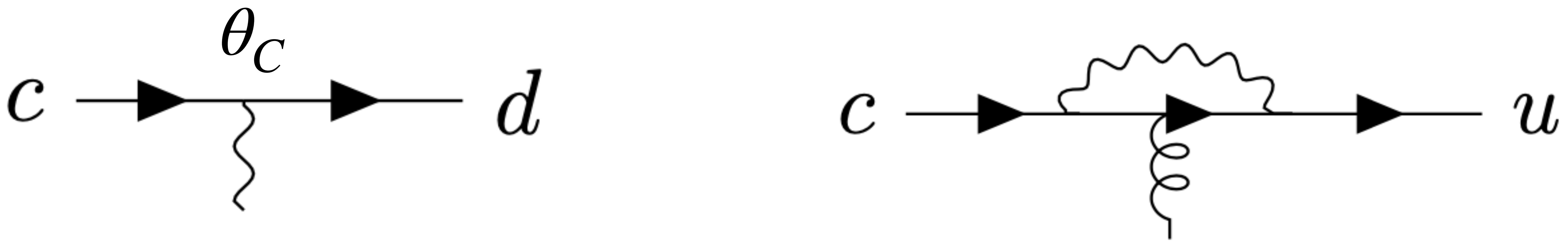
From Belle PRD(2014)



(e.g. Babar PRL(2010), LHCb PRD(2023))

SCS decays to CP eigenstates

- SCS decays to $\pi^+\pi^-(K^+K^-)$ in the SM has tree-level + penguin



- Measurements of the following time-dependent CP asymmetry

(e.g. CDF PRD(2014), LHCb PRD(2021))

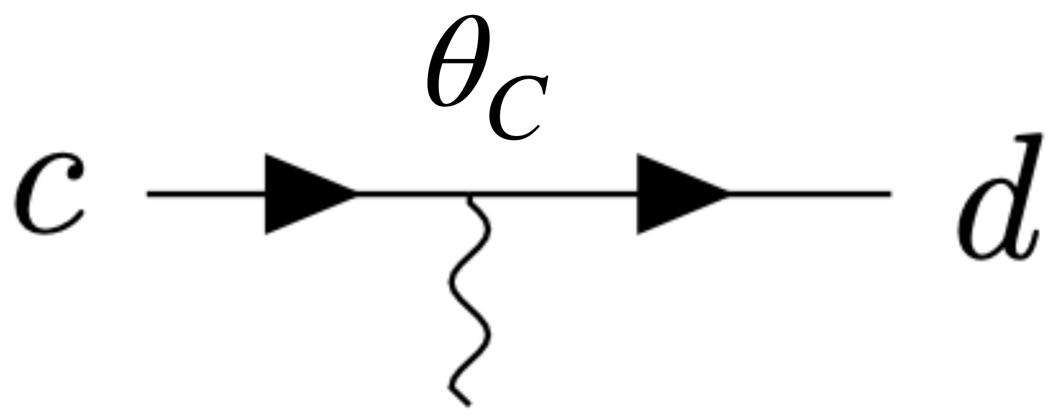
$$A_f(t) = \frac{d\Gamma(D^0 \to f) - d\Gamma(\overline{D}^0 \to f)}{d\Gamma(D^0 \to f) + d\Gamma(\overline{D}^0 \to f)}(t) = a_f + \Delta Y_f t/\tau$$

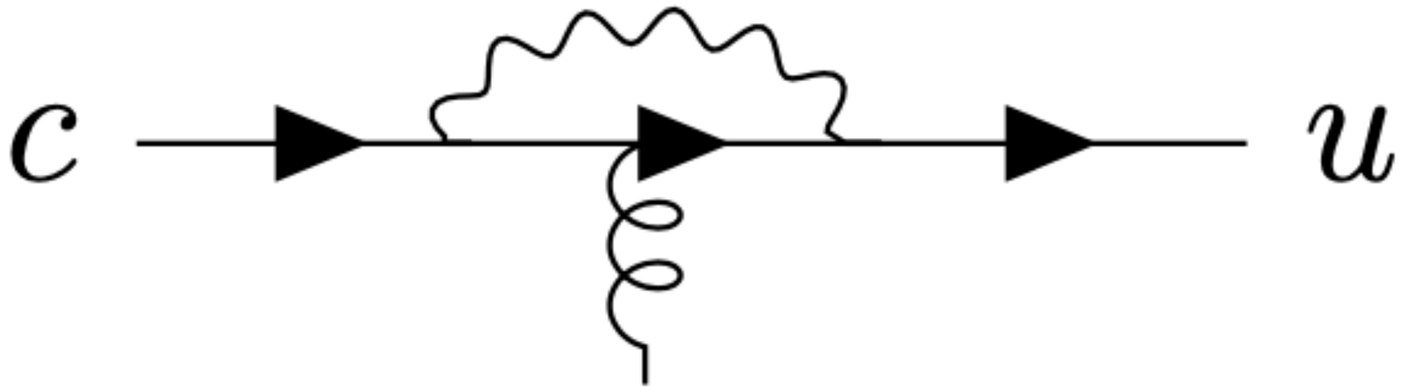
- Fitting the following CPV observables

$$a_f \qquad \Delta Y_f = (-2|\mathrm{M}_{12}|/\Gamma \sin\phi_f^M + a_f|\Gamma_{12}|/\Gamma)$$

SCS decays to CP eigenstates

- SCS decays to $\pi^+\pi^-(K^+K^-)$ in the SM has tree-level + penguin





- Measurements of the following time-integrated CP asymmetry

(e.g. CDF PRD(2012), Babar PRL(2008), Belle Phys. Lett. B (2008), LHCb PRL(2023))

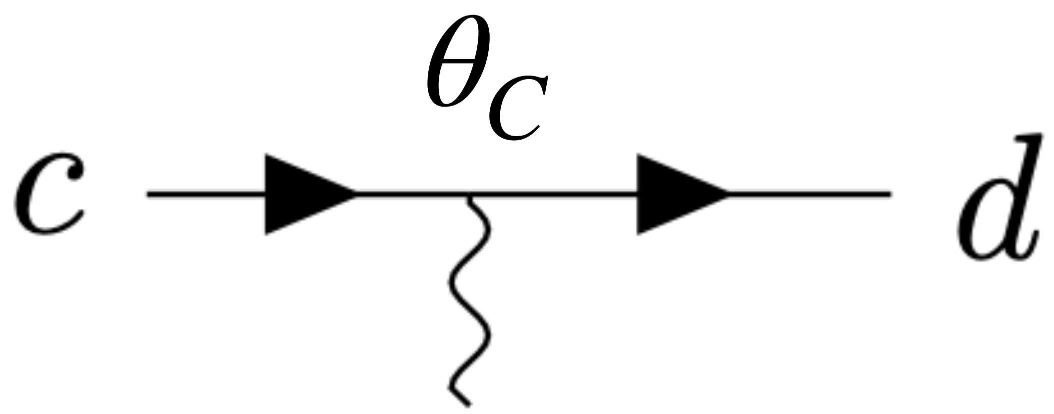
$$A_f = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to f)} = a_f + \Delta Y_f \langle t/\tau \rangle$$

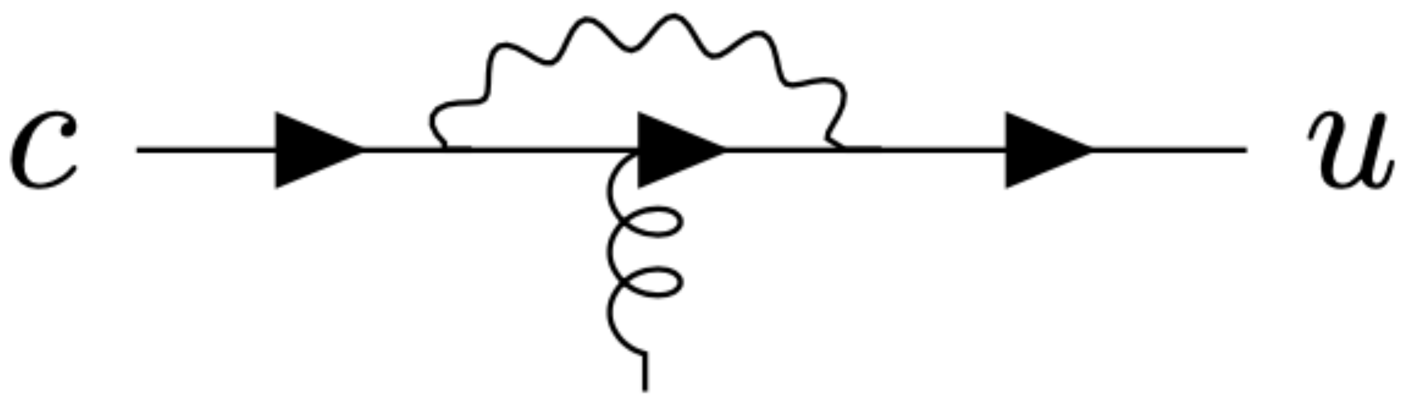
- Fitting the following CPV observables

$$A_f \qquad \Delta A^{\mathrm{CP}} = A_{KK} - A_{\pi\pi}$$

SCS decays to CP eigenstates

- SCS decays to $\pi^+\pi^-(K^+K^-)$ in the SM has tree-level + penguin





- Measurements of the following time-dependent CP asymmetry

(e.g. E791 PRL(1999), CLEO PRD(2002), Babar PRD(2013), BESIII Phys. Lett. B (2015), Belle PRD(2020), LHCb PRD(2022))

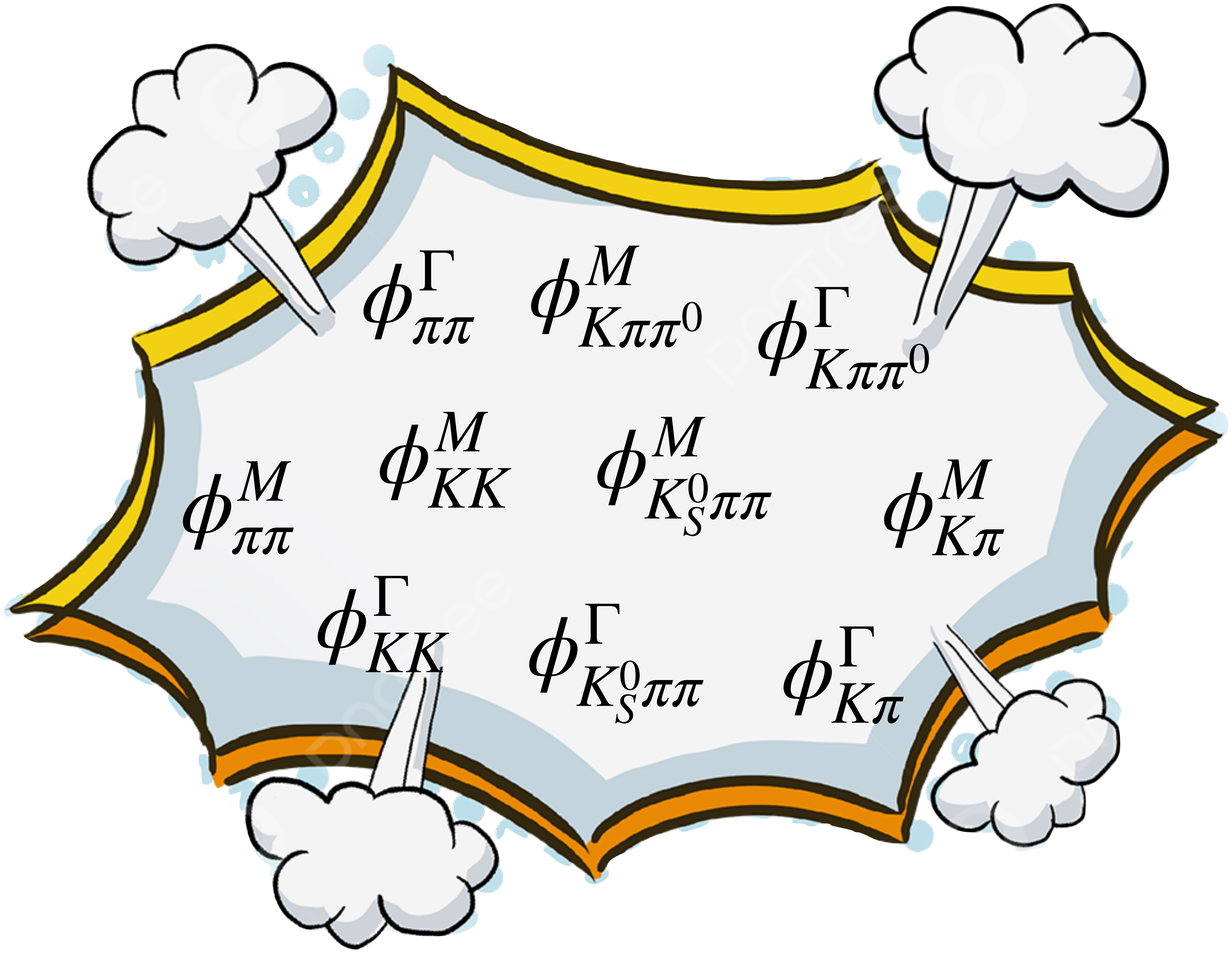
$$R^f(t) = \frac{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to f)}{\Gamma(D^0 \to K^-\pi^+) + \Gamma(\overline{D}^0 \to K^+\pi^-)} \propto 1 - \tilde{y}^f_{\rm CP} t/\tau$$

- Fitting the following CP conserving observable

$$\tilde{y}^f_{\rm CP} = |\Gamma_{12}|/\Gamma \cos\phi^\Gamma_f + r_D^{K\pi}(\cdots)$$

Theoretical framework

- Two CPV phases for each of the final states: how can we fit them?



Approximate universality $\longrightarrow$ ϕ^{M}_{2}, ϕ^{Γ}_{2}

Kagan, Silvestrini PRD(2020)

Approximate universality

- In the SM, the contributions to H_{12} reads

$$\Gamma_{12}^{\mathrm{SM}} = \sum_{i,j=d,s} \lambda_{uc}^{i}\lambda_{uc}^{j}\Gamma_{ij} \qquad \mathrm{M}_{12}^{\mathrm{SM}} = \sum_{i,j=d,s,b} \lambda_{uc}^{i}\lambda_{uc}^{j}\mathrm{M}_{ij}$$

- Employing CKM unitarity + U-spin decomposition, we get

$$\Gamma_{12}^{\mathrm{SM}} = \frac{(\lambda_{uc}^{s} - \lambda_{uc}^{d})^2}{4}\Gamma_2 \times \left[1 + O(10^{-3})\left(\frac{0.3}{\varepsilon}\right) + O(10^{-7})\left(\frac{0.3}{\varepsilon}\right)^2\right]$$

CPV in Approximate universality

- Two universal CPV phases can be defined w.r.t. the dominant terms

$$\phi_2^{M} = \arg\left[\frac{\boxed{M_{12}}}{M_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right], \qquad \phi_2^{\Gamma} = \arg\left[\frac{\Gamma_{12}}{\Gamma_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right]$$

- Dispersive mixing $\mathbf{M}_{12}$ receives contributions from both SM + possible short distance New Physics

Is it a good approximation?

- They provide very good approximations for:

  - CF/DCS decays to $K^-\pi^+$: $\phi^{M,\Gamma}_{K\pi} - \phi^{M,\Gamma}_{2} = O(\lambda^b_{uc}/\lambda^s_{uc})^2 \longrightarrow O(10^{-6})$
  - Three-body mode $K^0_S\pi\pi$: $\phi^{M,\Gamma}_{K^0_S\pi\pi} - \phi^{M,\Gamma}_{2} - 2\epsilon_I - 2|\lambda^b_{uc}/\lambda^s_{uc}|\sin(\gamma) = -2\mathrm{Im}[r_0] \longrightarrow O(10^{-4})$
  - SCS decays to $\pi^+\pi^-(K^+K^-)$: In the U-spin symmetric limit $(\phi^{M,\Gamma}_{KK} - \phi^{M,\Gamma}_{2}) = -(\phi^{M,\Gamma}_{\pi\pi} - \phi^{M,\Gamma}_{2}) \approx \phi^{M,\Gamma}_{2} O(\varepsilon)$.
    Then, we get $(\phi^{M,\Gamma}_{KK} + \phi^{M,\Gamma}_{\pi\pi})/2 = \phi^{M,\Gamma}_{2}(1 + O(\varepsilon^2))$

- In this framework, the CPV phases are estimated to be

$$\phi^M_2 \sim \phi^\Gamma_2 \sim \phi^{\mathrm{U-spin}} = 0.13^\circ \qquad\qquad |\phi^\Gamma_2| < 0.3^\circ$$

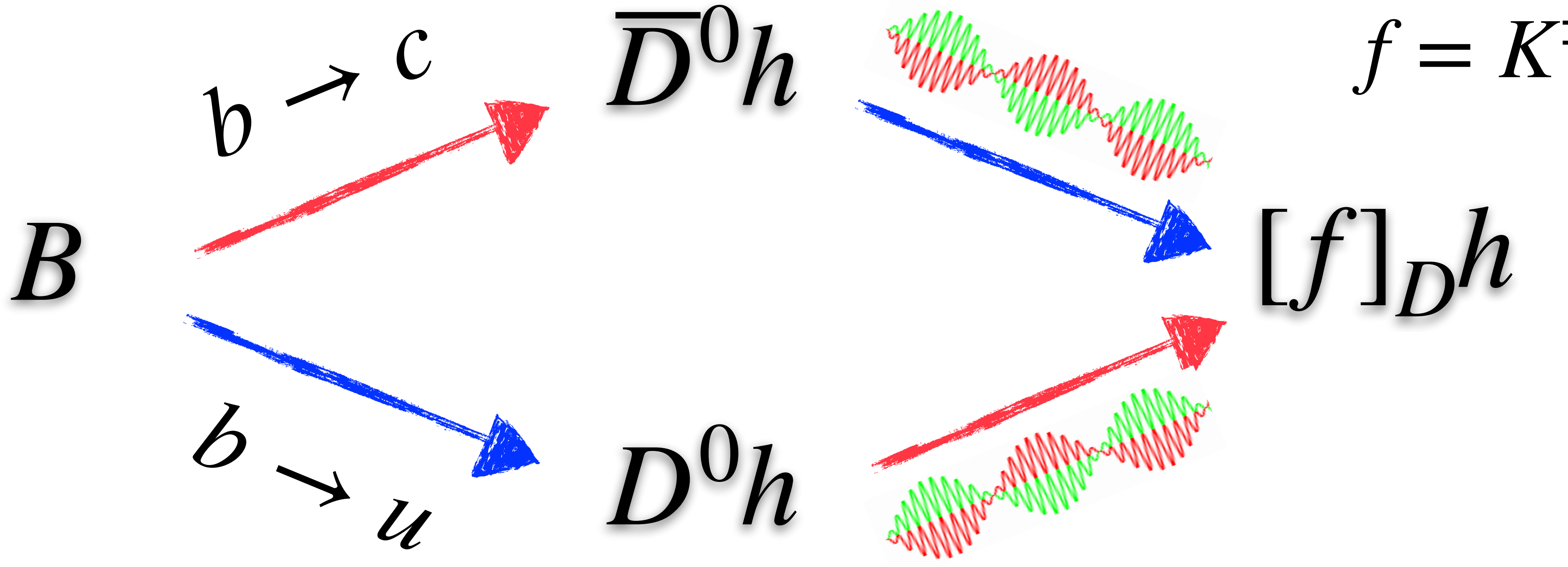
Beauty observables?

- Determining $\phi_2^{\mathrm{M},\Gamma}$ requires to know $r_D^{K\pi(\pi^0)}$, $\delta_D^{K\pi(\pi^0)}$, ...

- Additional information are provided by LHCb JHEP(2021)

$h = K,\ \pi, K\pi\pi, K^*, \ldots$

$f = K^\pm\pi^\mp, K_S^0\pi^+\pi^-, \pi^+\pi^-, K^+K^-, \ldots$



- We can determine the CKM angle $\gamma = \arg[-V_{ud}V_{ub}^* V_{cb}V_{cd}^*]$

Combination

Charm obs.

Share $|\mathbf{M}_{12}|$, $|\Gamma_{12}|$, $r_D^{K\pi}, \delta_D^{K\pi}, \ldots$

Beauty obs.

Share γ

Time-dependent $B^0 \to D^- \pi^+$, $B_s^0 \to D_s^- K^+$ obs.

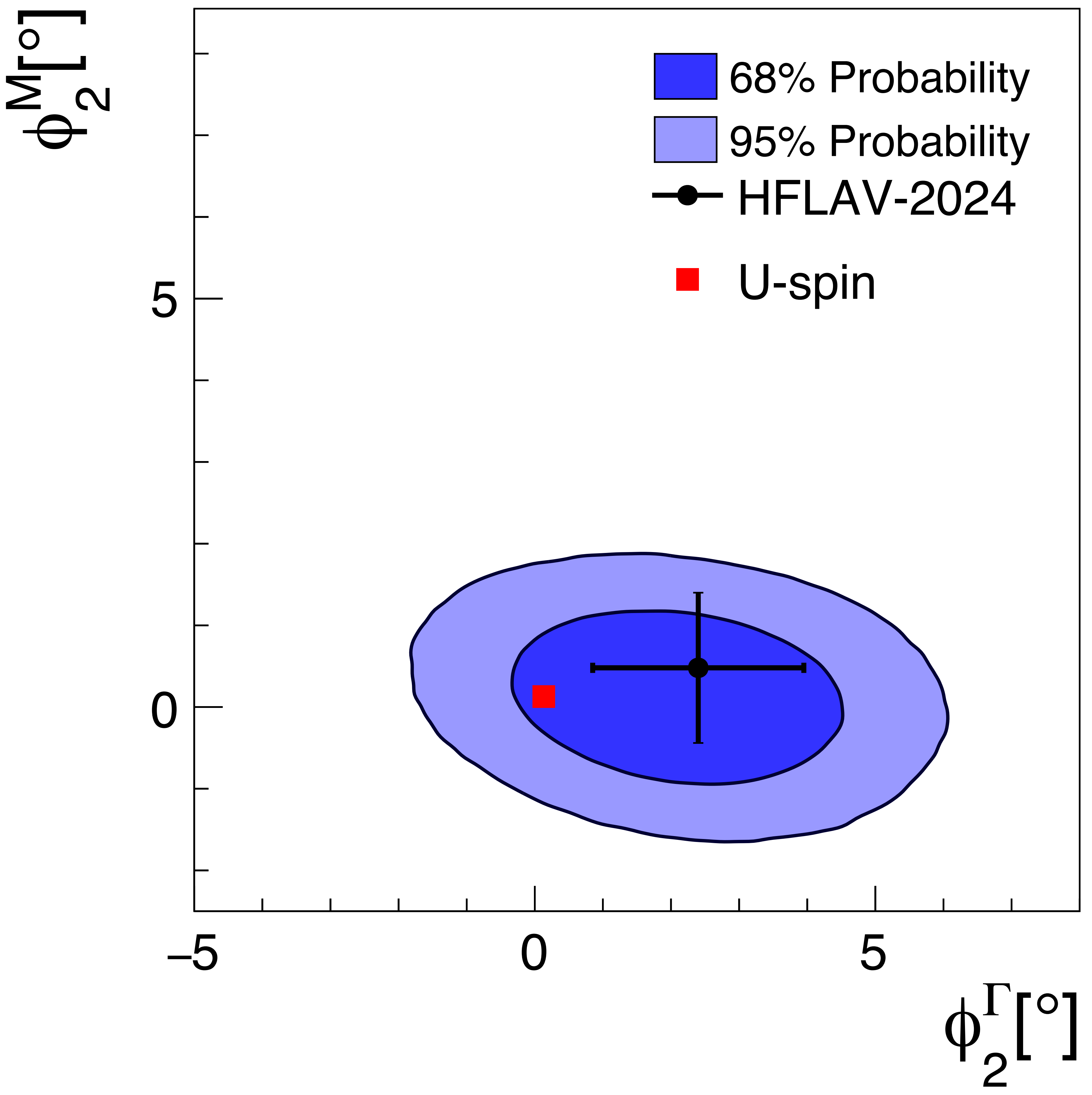
Bayesian Combination

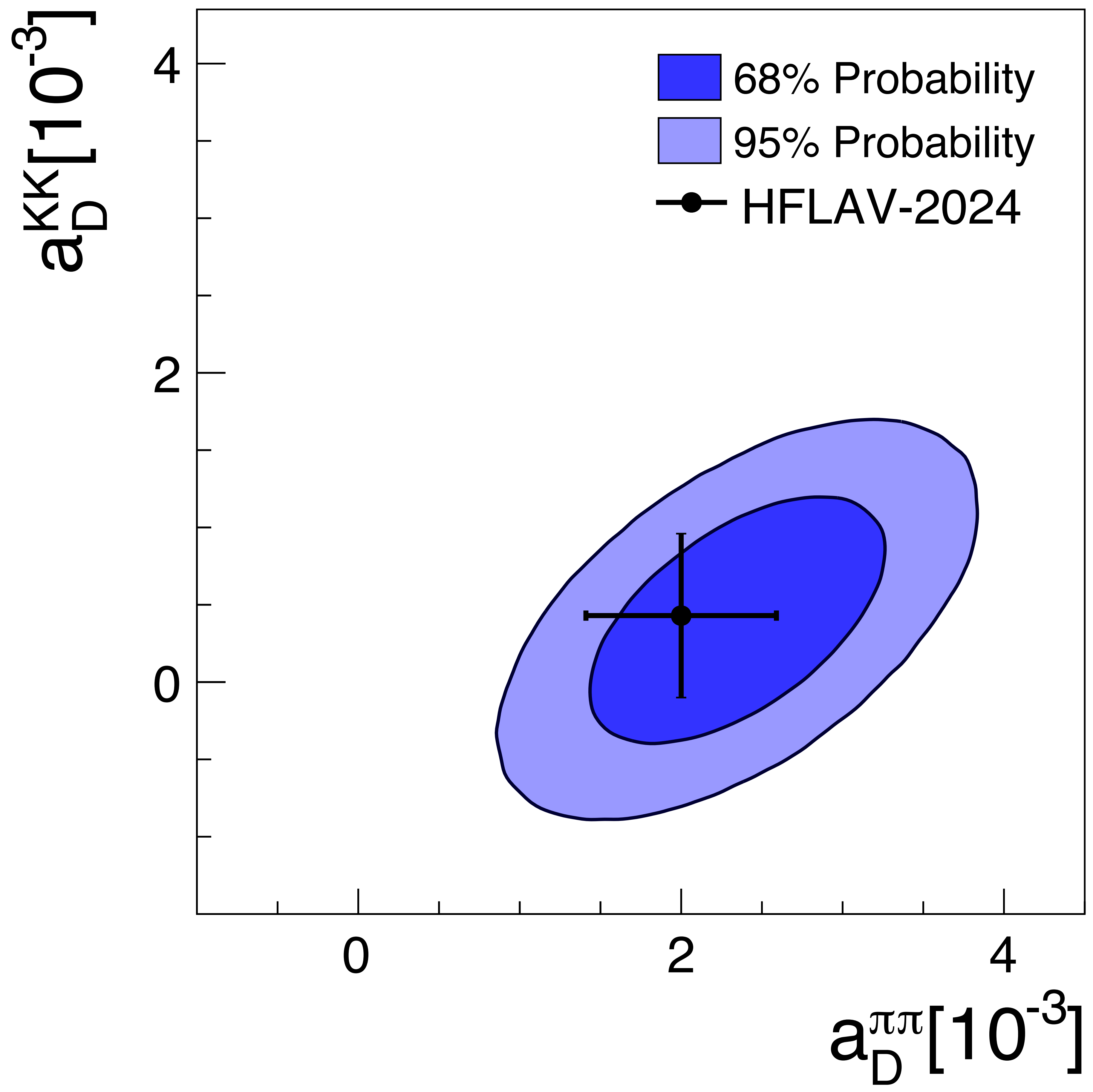
$$P(\vec{\theta}\,|\,\mathbf{O}) \propto P(\mathbf{O}\,|\,\vec{\theta})\; P_0(\vec{\theta})$$

Our work
(Submitted to PRD)

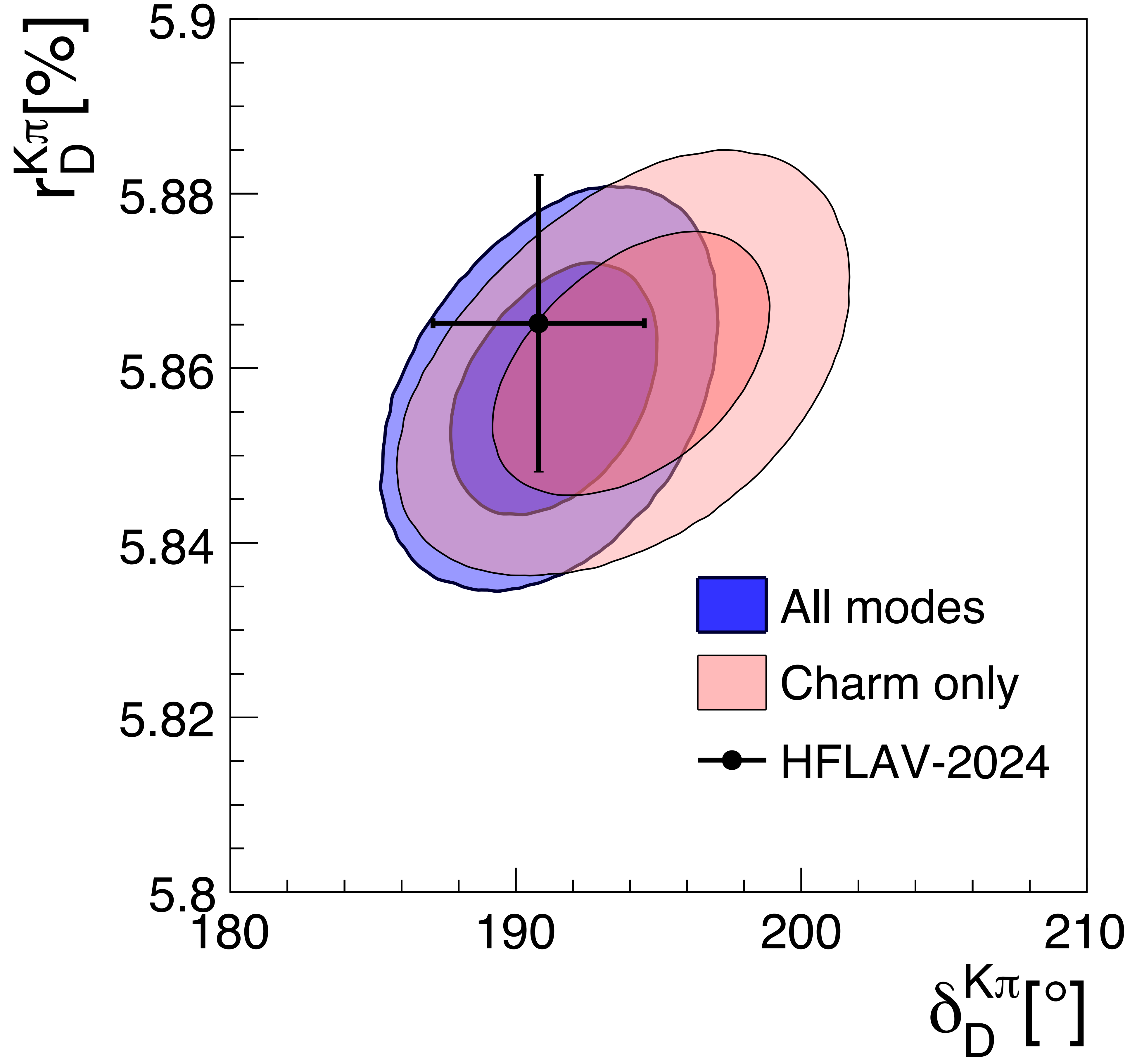
Posterior pdf

CPV parameters





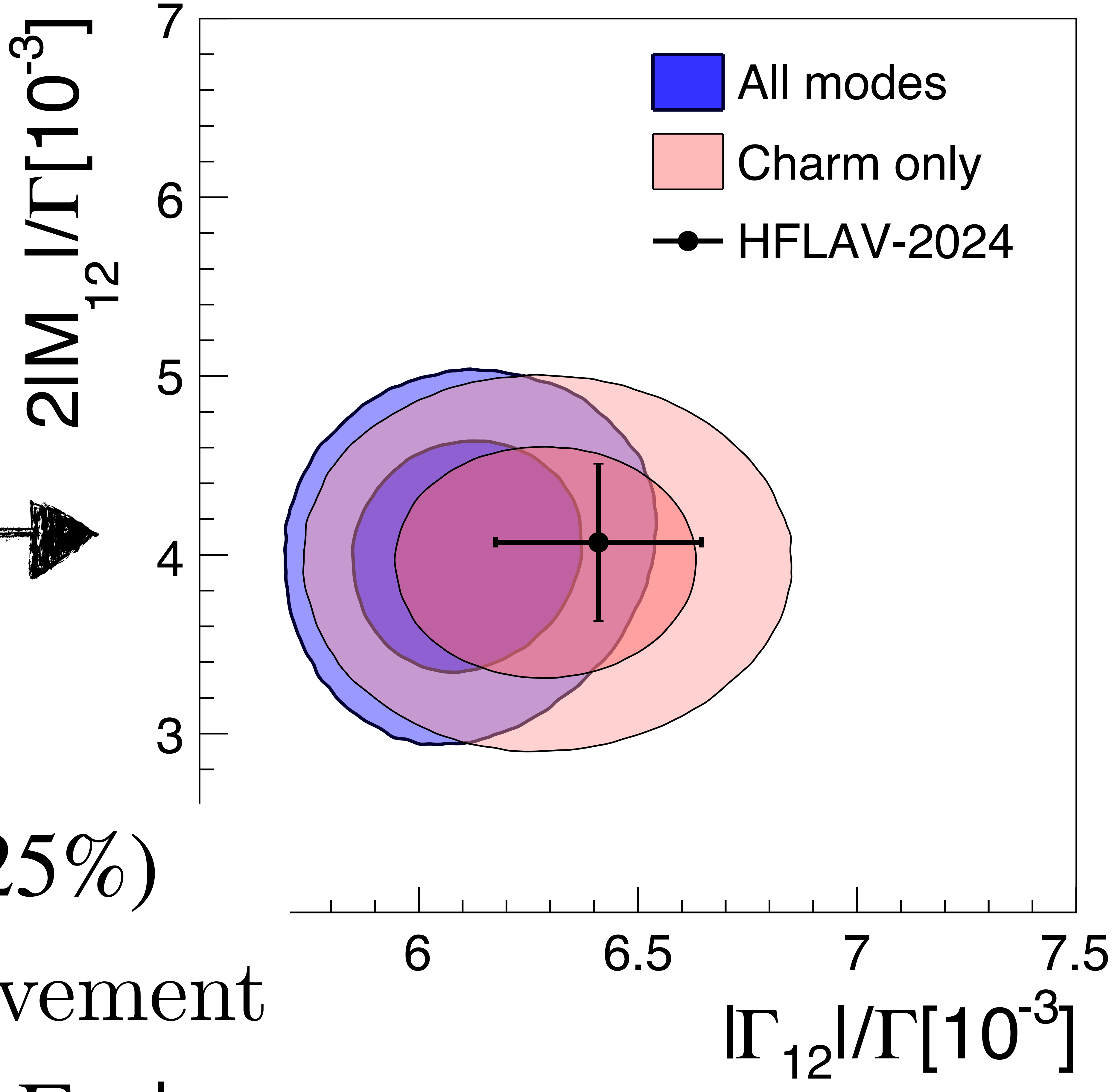
Impact of the combination



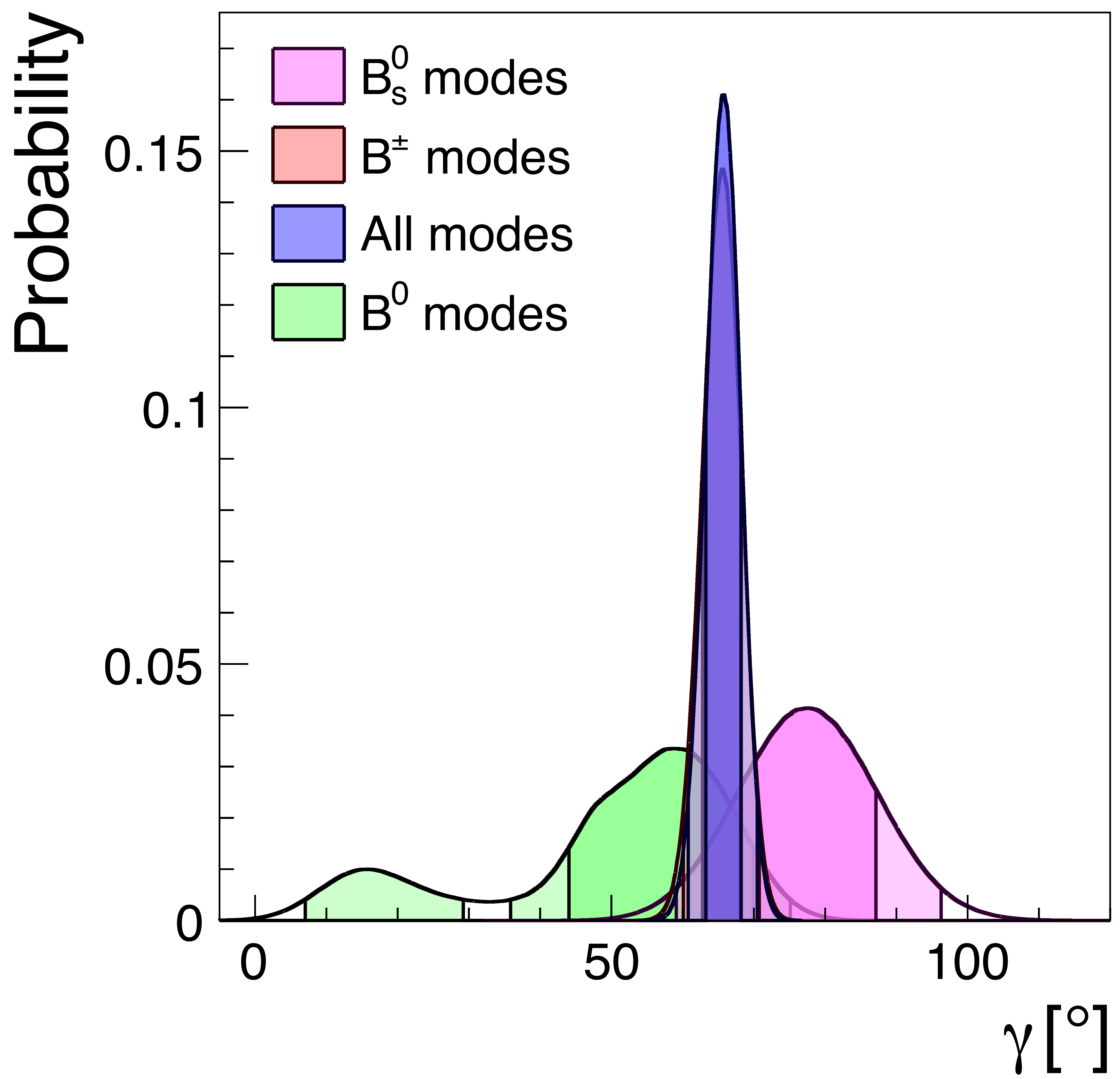
$\mathcal{O}(25\%)$

Improvement

on $|\Gamma_{12}|$



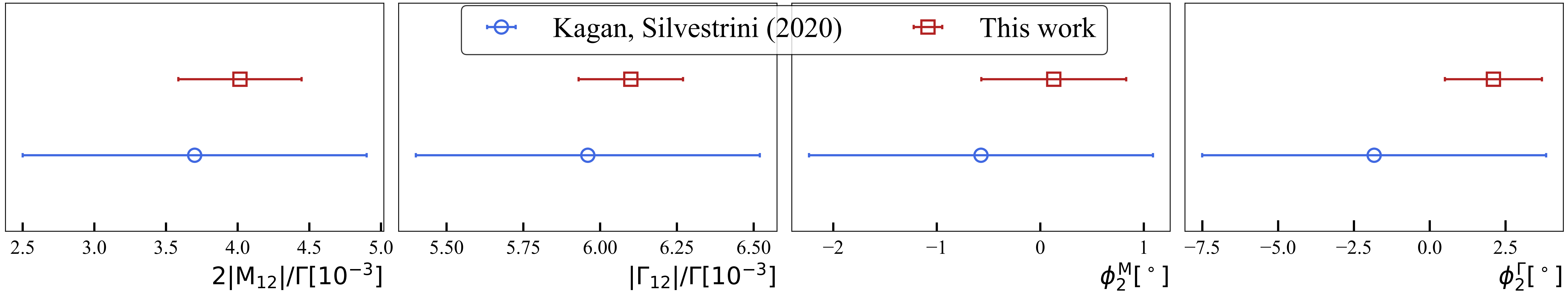
CKM angle γ



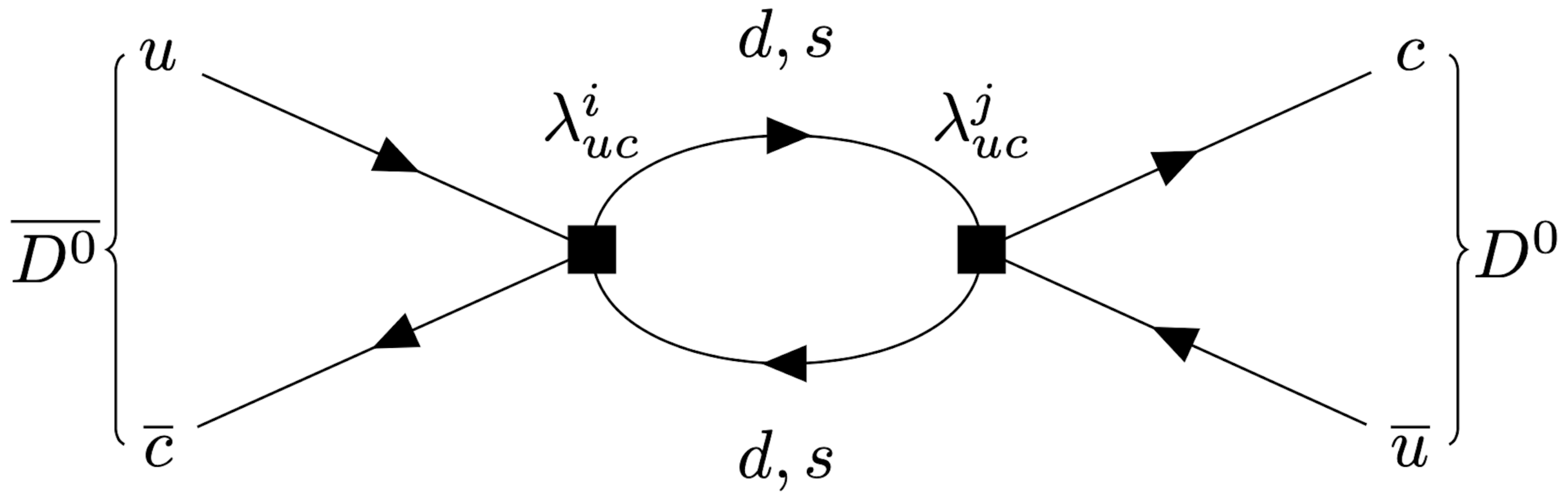
$$\gamma = 65.8(2.5)^\circ$$

Next steps?

- Experimental uncertainties have been reduced by more than a factor of 2(3) on dispersive (absorptive) mixing parameters



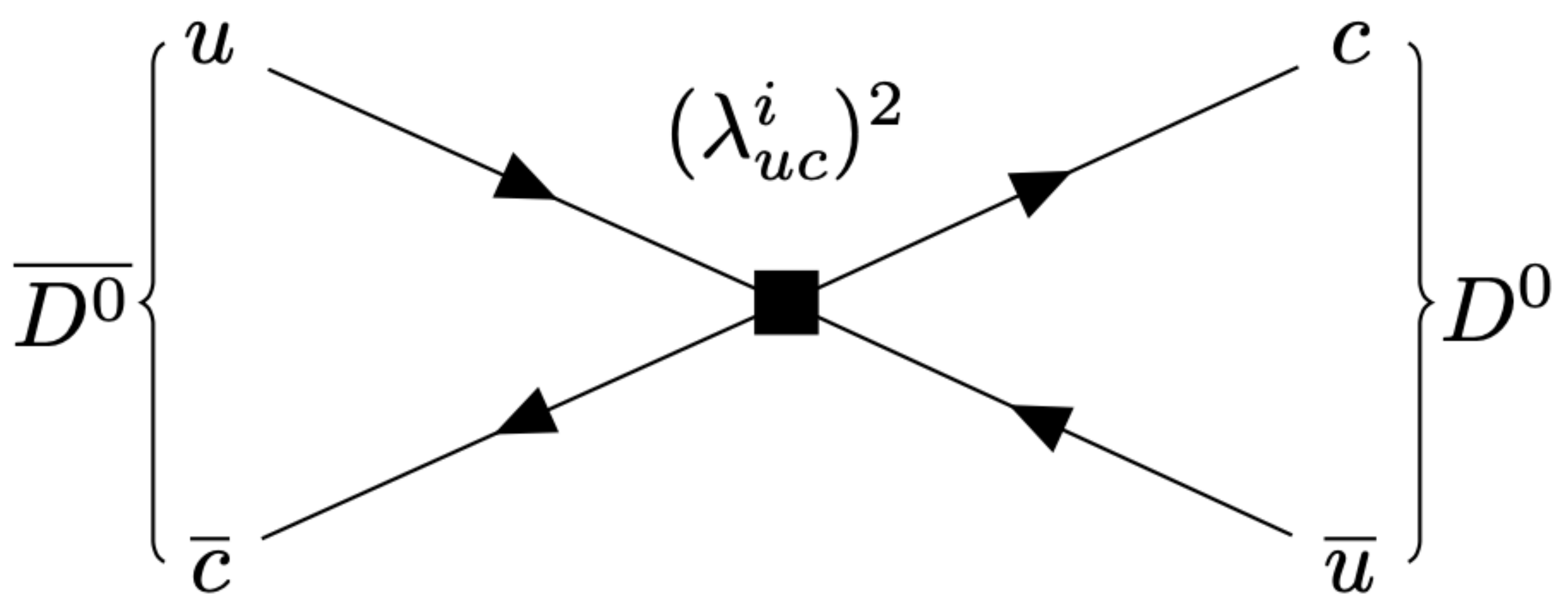
- Can we get SM estimates for CPV in charm mixing?



Backup

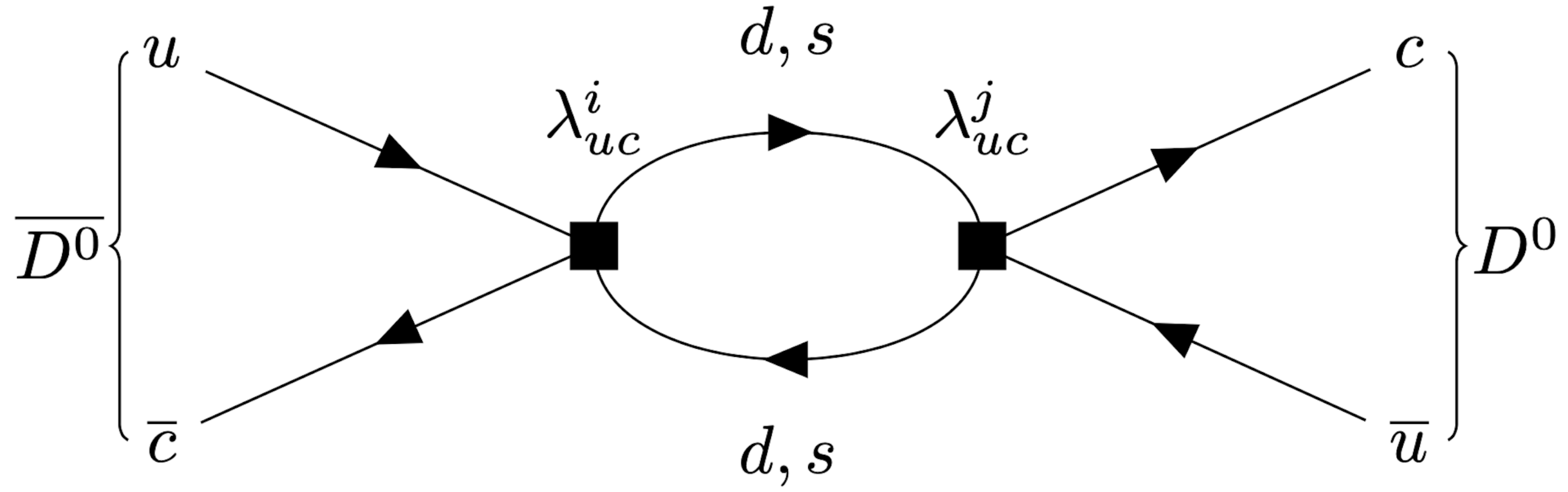
Short vs long distance

- The short distance contribution is given by



SM: $\propto (\lambda^b_{uc} m_b)^2 \approx (\theta^5_C m_b)^2$

- The long distance contribution is given by



SM: $\propto (\lambda^s_{uc} m_s)^2 \approx (\theta_c m_s)^2 \approx 10^2 \times \mathrm{SD}$

Amplitude decomposition

- The absorptive part of the mixing hamiltonian reads

$$\Gamma_{12}^{\mathrm{SM}} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\Gamma_2 + \frac{(\lambda_{uc}^s - \lambda_{uc}^d)\lambda_{uc}^b}{2}\Gamma_1 + \frac{(\lambda_{uc}^b)^2}{4}\Gamma_0$$

- The amplitudes and CKM matrix elements satisfy

$$\Gamma_0 = (\bar{s}s + \bar{d}d)^2 = O(1) \qquad \Gamma_1 = (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = O(\varepsilon) \qquad \Gamma_2 = (\bar{s}s - \bar{d}d)^2 = \mathcal{O}(\varepsilon^2)$$

$$\lambda_{uc}^s - \lambda_{uc}^d \approx 0.44 - i1.2 \times 10^{-4} \qquad \lambda_{uc}^b \approx (5.7 + i12) \times 10^{-5}$$

- We get the expansion

$$\Gamma_{12}^{\mathrm{SM}} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\Gamma_2 \times \left[1 + (0.86 + i1.8) \times 10^{-3}\left(\frac{0.3}{\epsilon}\right) + (-6.4 + i7.8) \times 10^{-7}\left(\frac{0.3}{\epsilon}\right)^2\right]$$

Estimates

- Estimates of $\phi_2^{\mathrm{M},\Gamma}$ can be obtained by using the SM definitions

$$\left.\phi_2^{\Gamma}\right|_{SM} = \arg\left[1 + \frac{2\lambda_{uc}^{b}}{\lambda_{uc}^{s} - \lambda_{uc}^{d}}\frac{\Gamma_1}{\Gamma_2}\right] = \arg\left[1 - \frac{V_{ub}^{*}V_{ud}}{V_{cb}^{*}V_{cd}} \times \left(\frac{2}{1 - \frac{V_{us}^{*}V_{cs}}{V_{ud}^{*}V_{cd}}}\right)\varepsilon^{-1}\right]$$

$$\simeq \left|\frac{\lambda_{uc}^{b}}{\lambda_{uc}^{d}}\right| \sin(\gamma)\varepsilon^{-1} \approx (2.2 \times 10^{-3}) \times \left[\frac{0.3}{\varepsilon}\right]$$

Upper bound

$$|\phi_2^\Gamma| = \left|\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right| \sin(\gamma)\frac{\Gamma_1}{\Gamma_2}$$

- Now, we have that $|\Gamma_2| = y_{12}\Gamma/(\lambda_{uc}^d)^2$ and

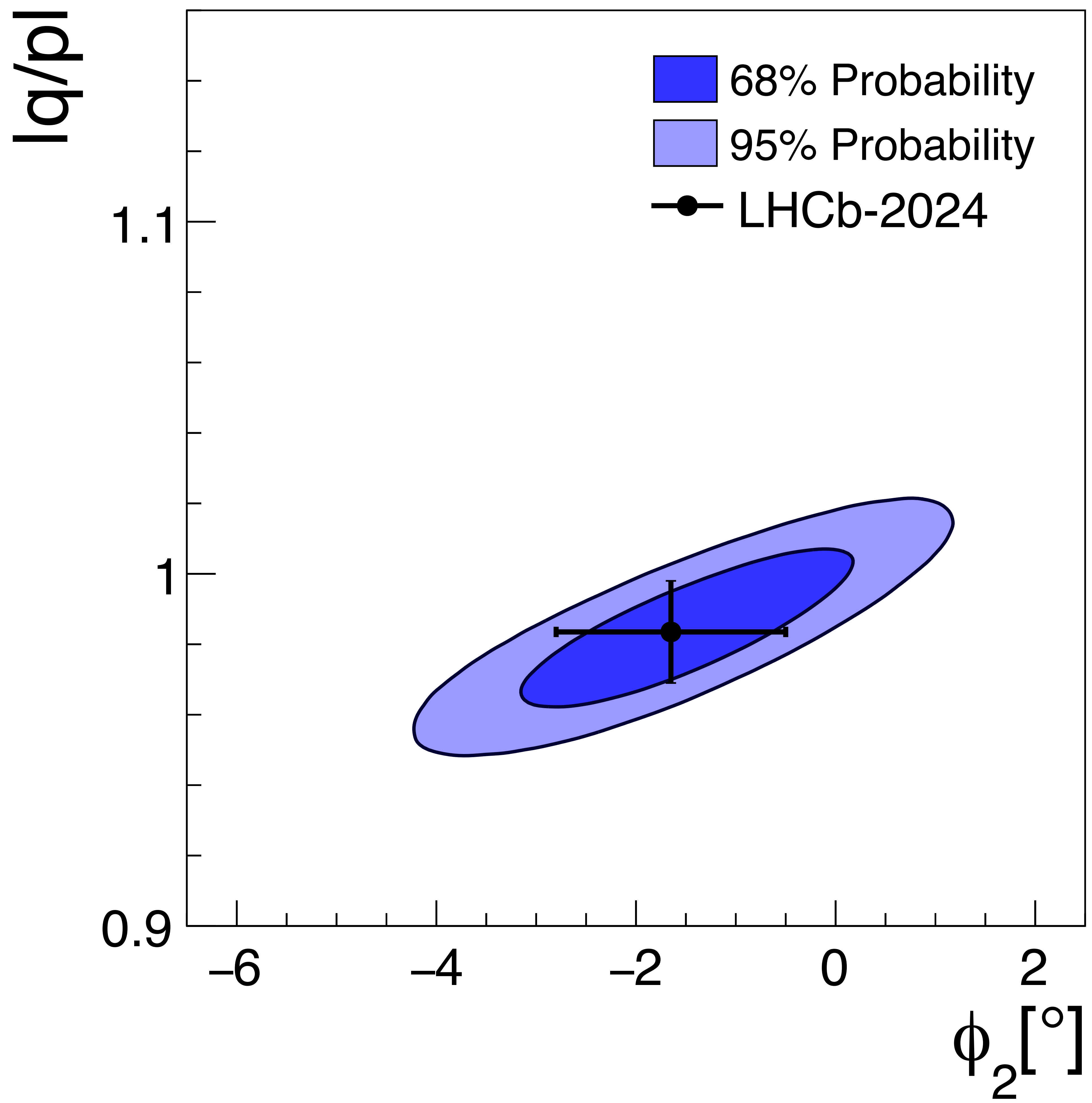
$$|\phi_2^\Gamma| = \left|\frac{\lambda_{uc}^b \lambda_{uc}^d}{y_{12}}\right| \sin(\gamma)\frac{|\Gamma_{sd}|}{\Gamma}\frac{|\Gamma_{ss}-\Gamma_{dd}|}{|\Gamma_{sd}|} \longrightarrow O(\varepsilon)$$

$$< 1 + O(\varepsilon) \quad \xleftarrow{SU(3)_f}$$

Also see M. Bobrowski, A. Lenz, J. Riedl, J. Rohrwild (2010)

$$|\phi_2^\Gamma| < 5 \times 10^{-3}\varepsilon(1+O(\varepsilon)) < 0.3^\circ$$

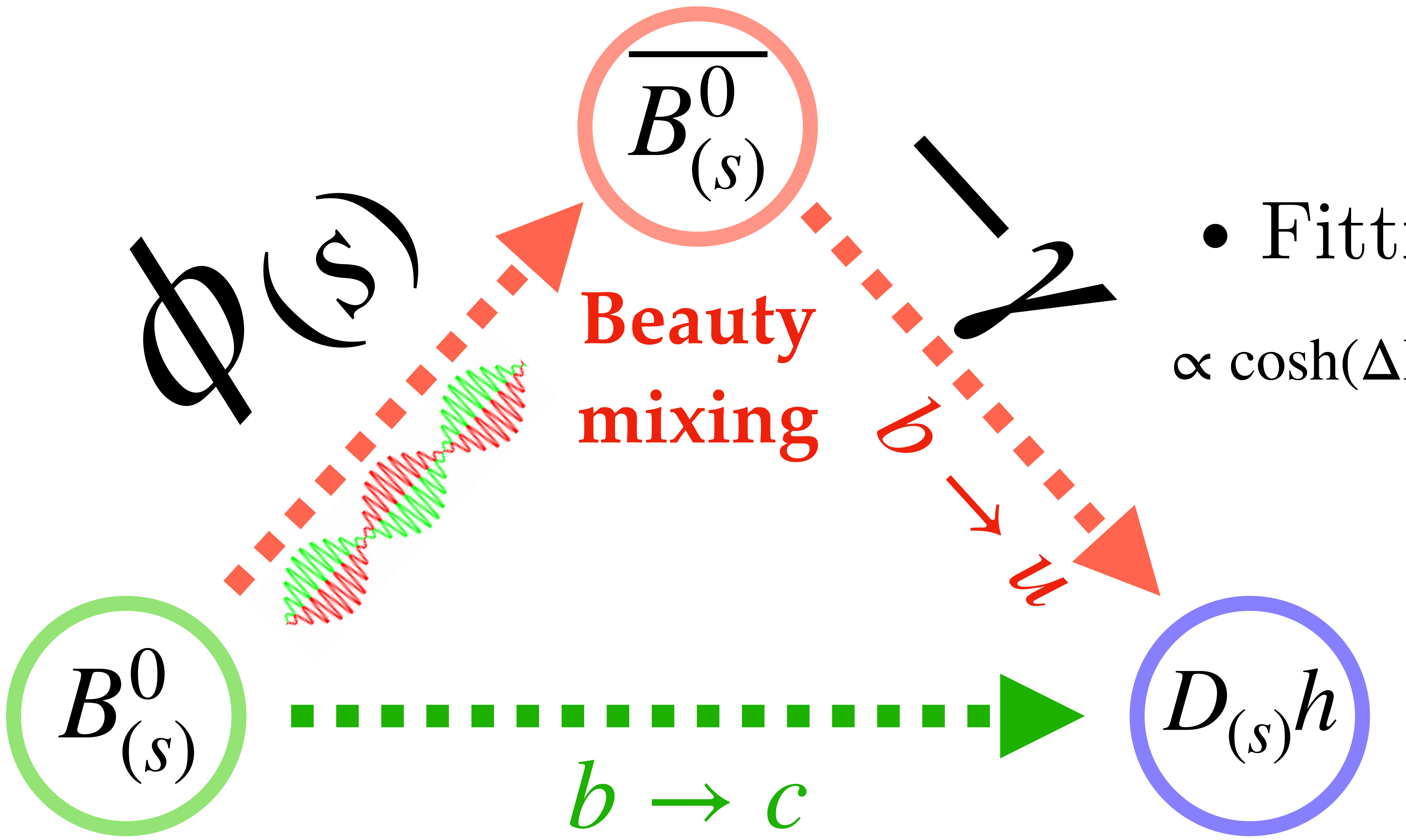
Alternative parametrisation



Neutral B meson obs.

- Exploiting the **CPV phase of the interference** between $B^0_{(s)}$ mixing and decay to charmed mesons $D^{\mp}_{(s)}h^{\pm}$

Mixing phases $\phi = -2\beta$ $\phi_s = 2\beta_s$



- Fitting the time-dependent decay rates

$$\propto \cosh(\Delta\Gamma_{(s)}t/2) - G_f \sinh(\Delta\Gamma_{(s)}t/2) + C_f \cos(\Delta m_{(s)}t) - S_f \sin(\Delta m_{(s)}t)$$

Observables!!

C_f $G_f \propto \cos(\delta^f_{B^0_{(s)}} + (\phi_{(s)} - \gamma))$

$S_f \propto \sin(\delta^f_{B^0_{(s)}} + (\phi_{(s)} - \gamma))$