

DIPARTIMENTO  
INTERATENEO  
DI FISICA

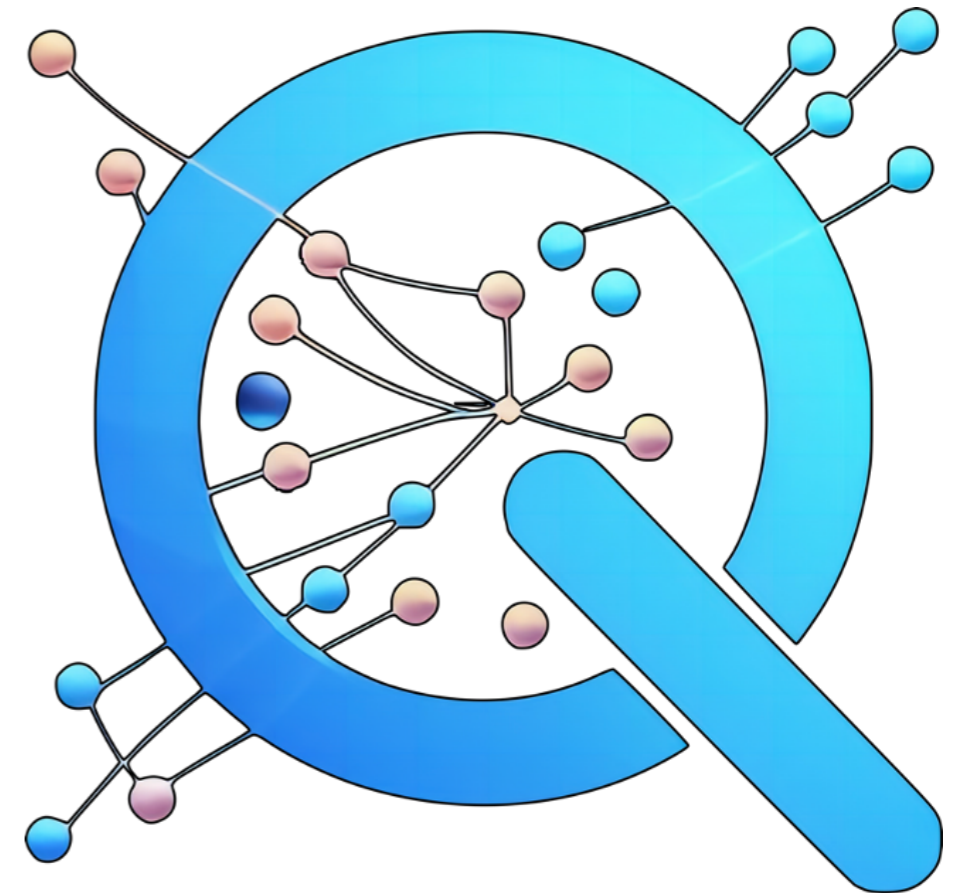


**TUM**

# Dynamical cluster-based optimization of tensor network algorithms for simulating quantum circuits with finite fidelity

Andrea De Girolamo

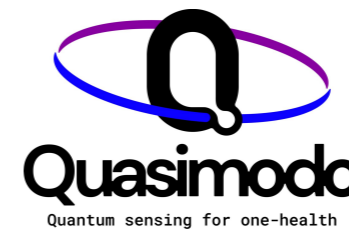
Bari Theory Group XMas Workshop  
Bari, December 17<sup>th</sup> 2024



In collaboration with:

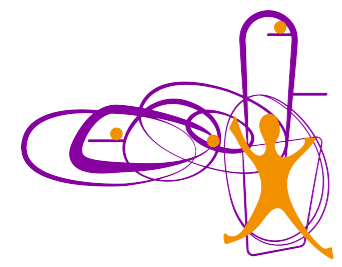


Acknowledgments:

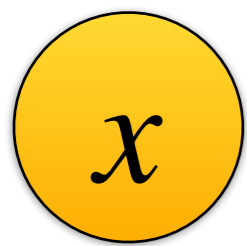


Leibniz Supercomputing Centre  
of the Bavarian Academy of Sciences and Humanities

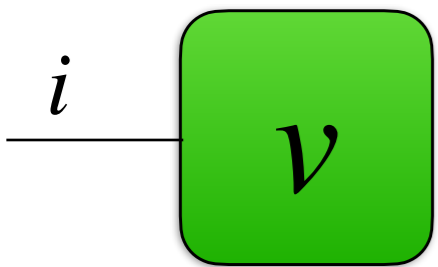




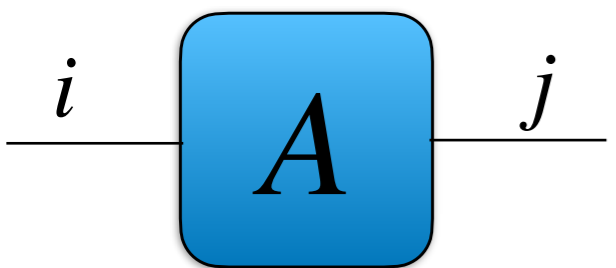
# Tensor diagrams



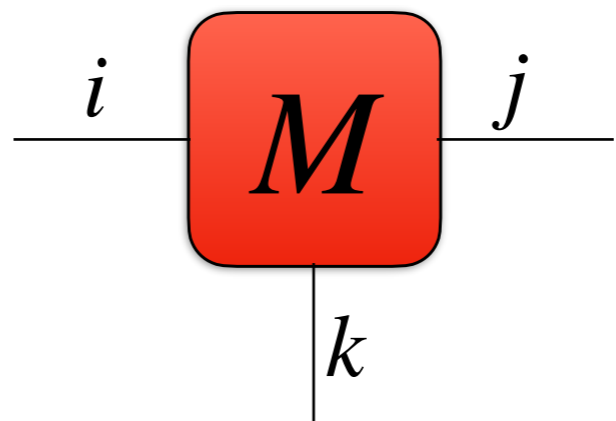
number



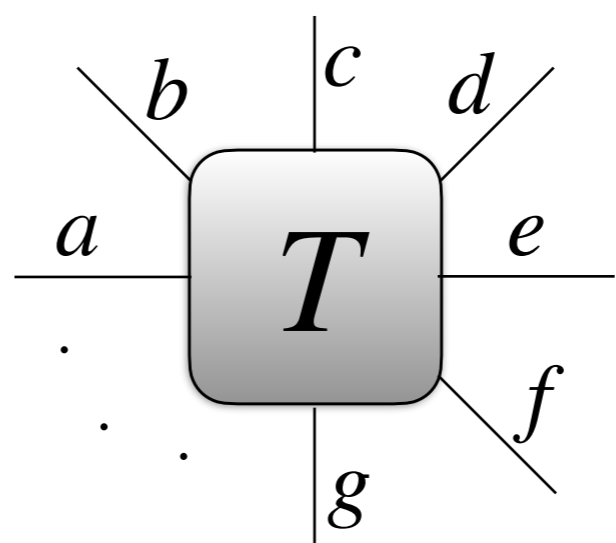
vector



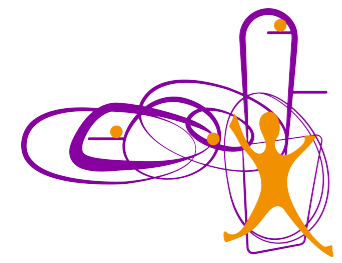
matrix



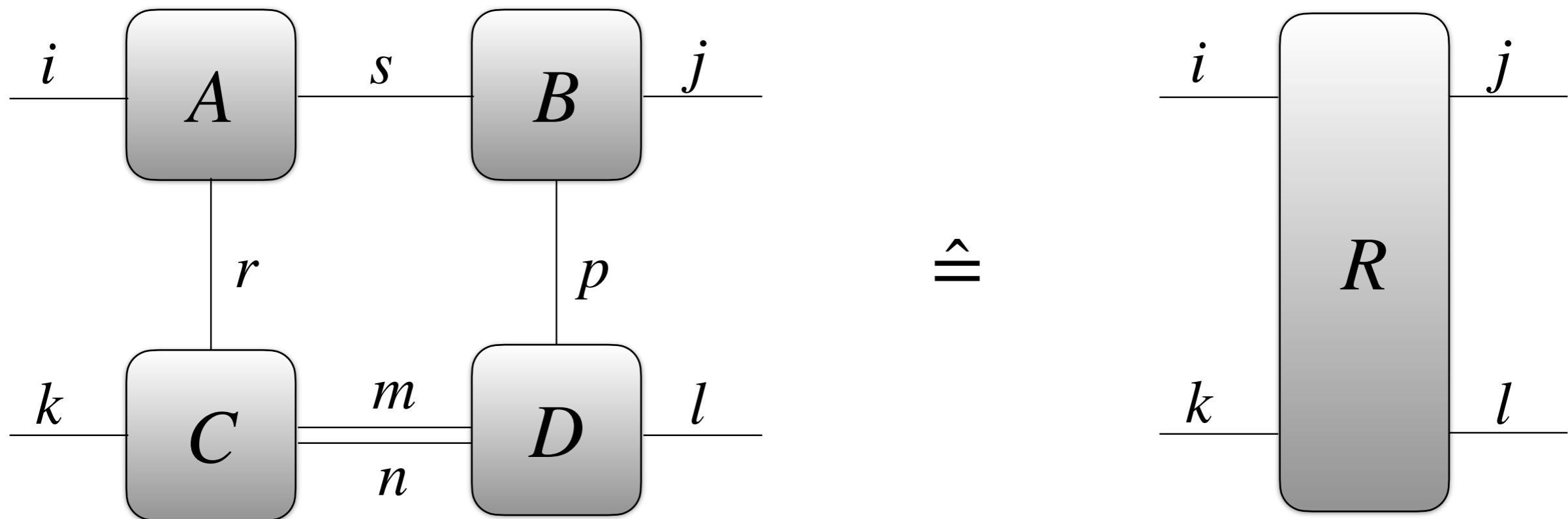
rank-3 tensor

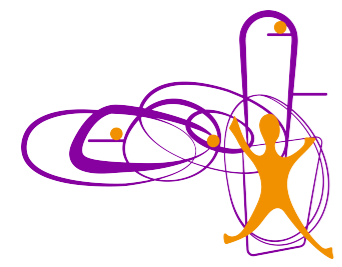


rank- $n$  tensor

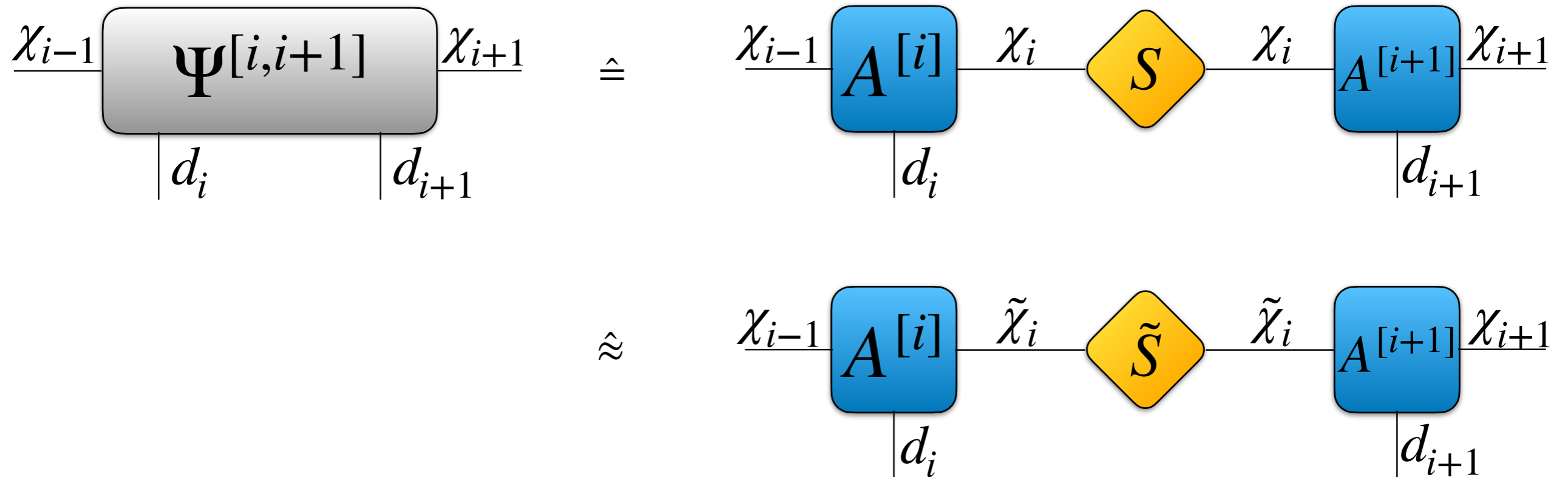


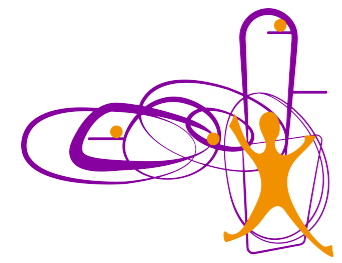
# Tensor network



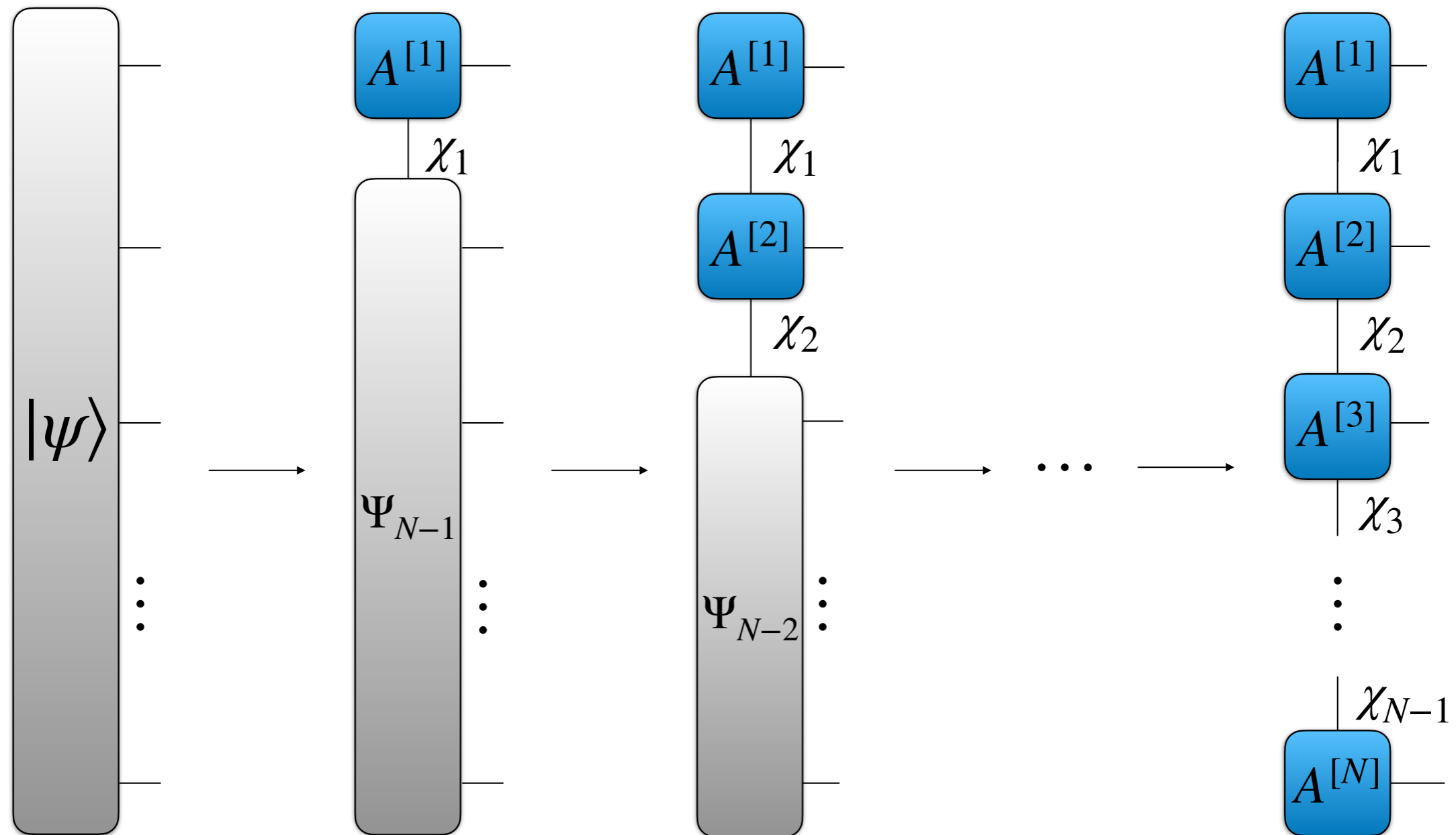


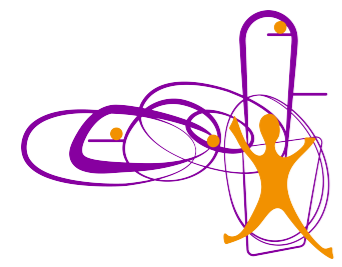
# Singular Value Decomposition (SVD) [3,4]



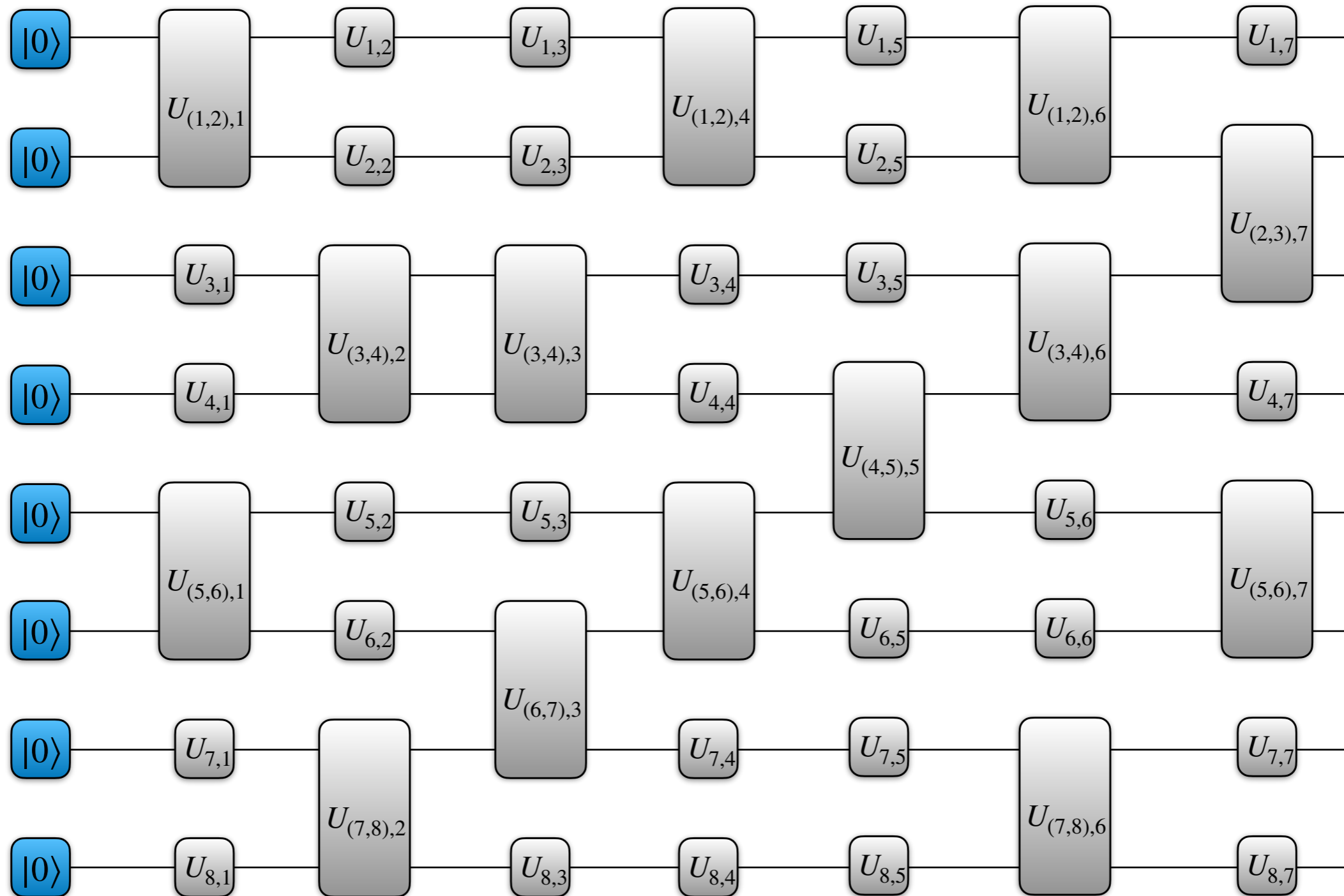


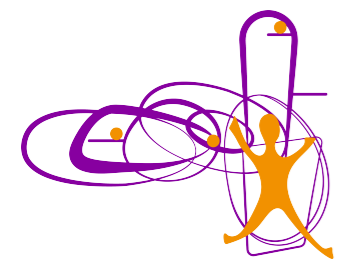
# Matrix-Product State (MPS) [3,4]



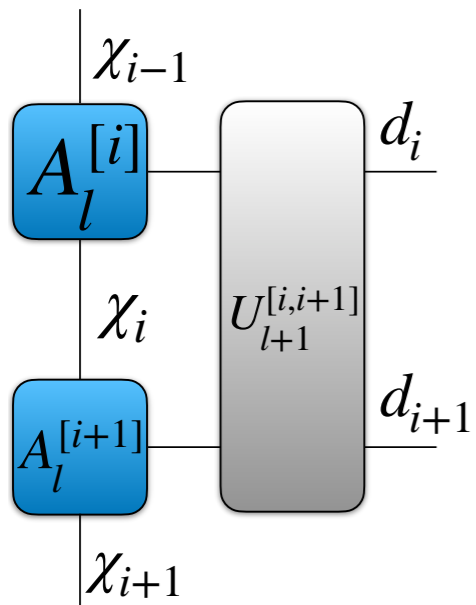


# Quantum circuit modelled as a tensor network [1]

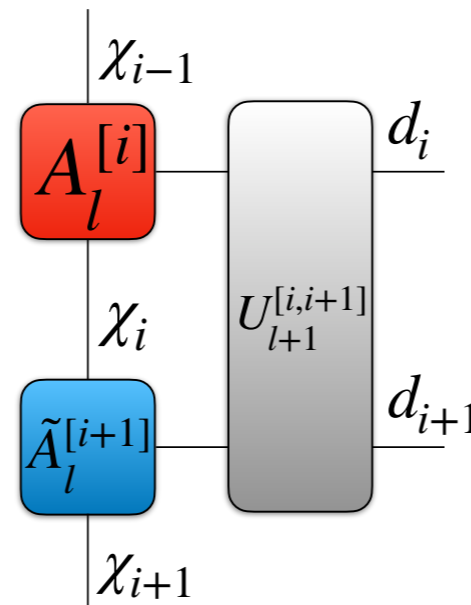




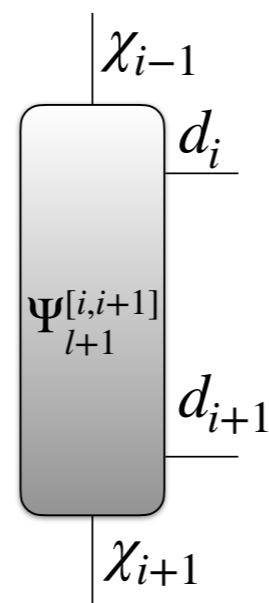
# Time-Evolving Block Decimation (TEBD) algorithm [2]



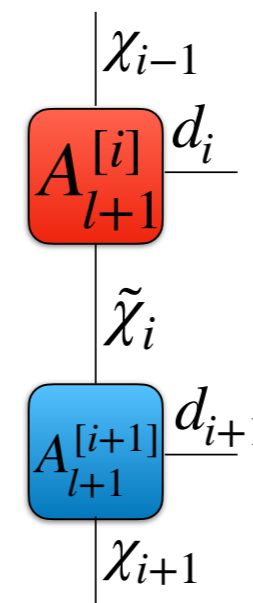
1.  
 $\hat{=}$

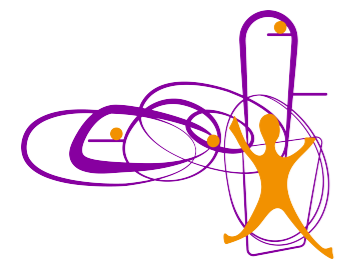


2.  
 $\hat{=}$

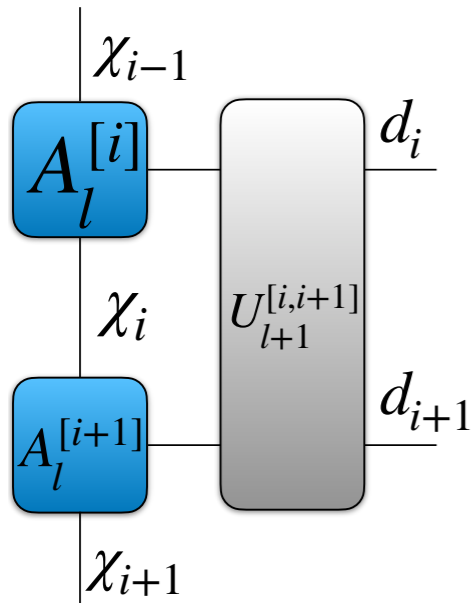


3.  
 $\approx$

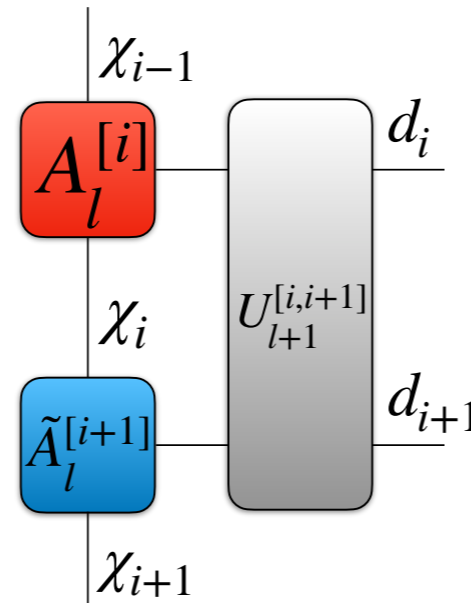




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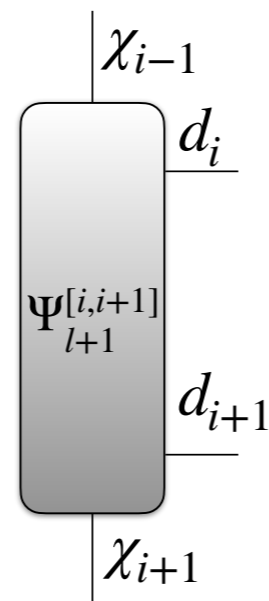


1.  
 $\hat{=}$

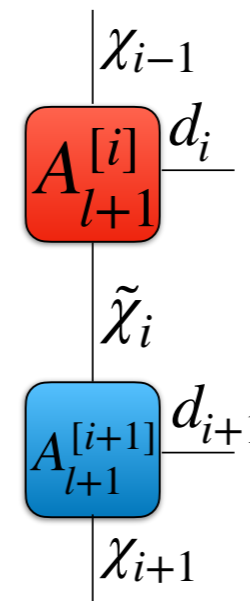


Repeat for all gates in the quantum circuit

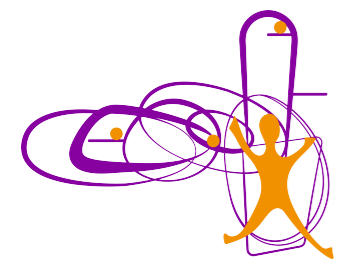
2.  
 $\hat{=}$



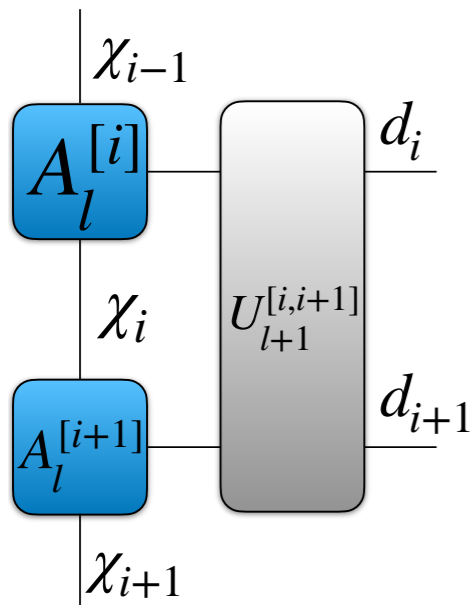
3.  
 $\approx$



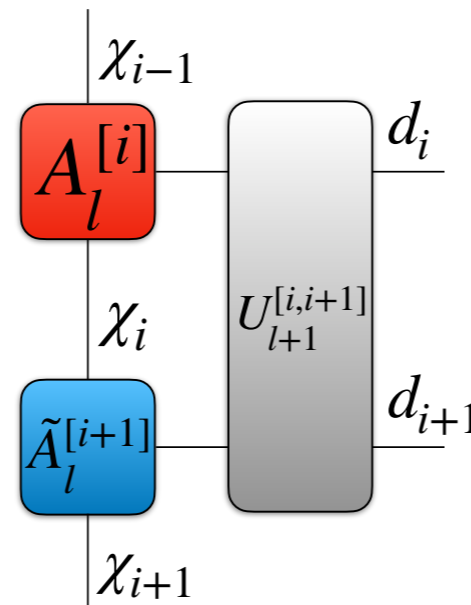




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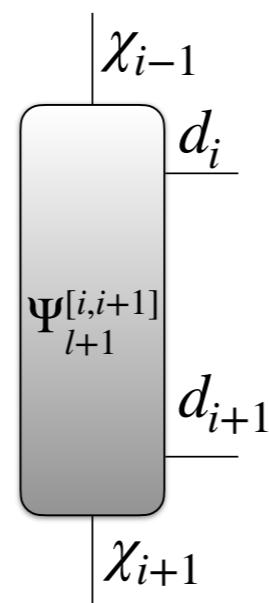


1.  
 $\hat{=}$

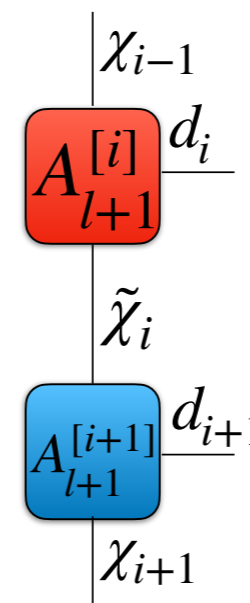


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 $\hat{=}$

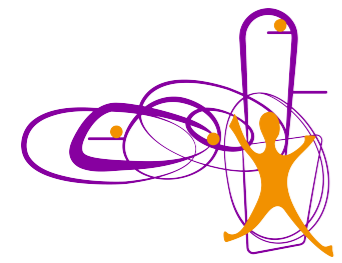


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 $\approx$

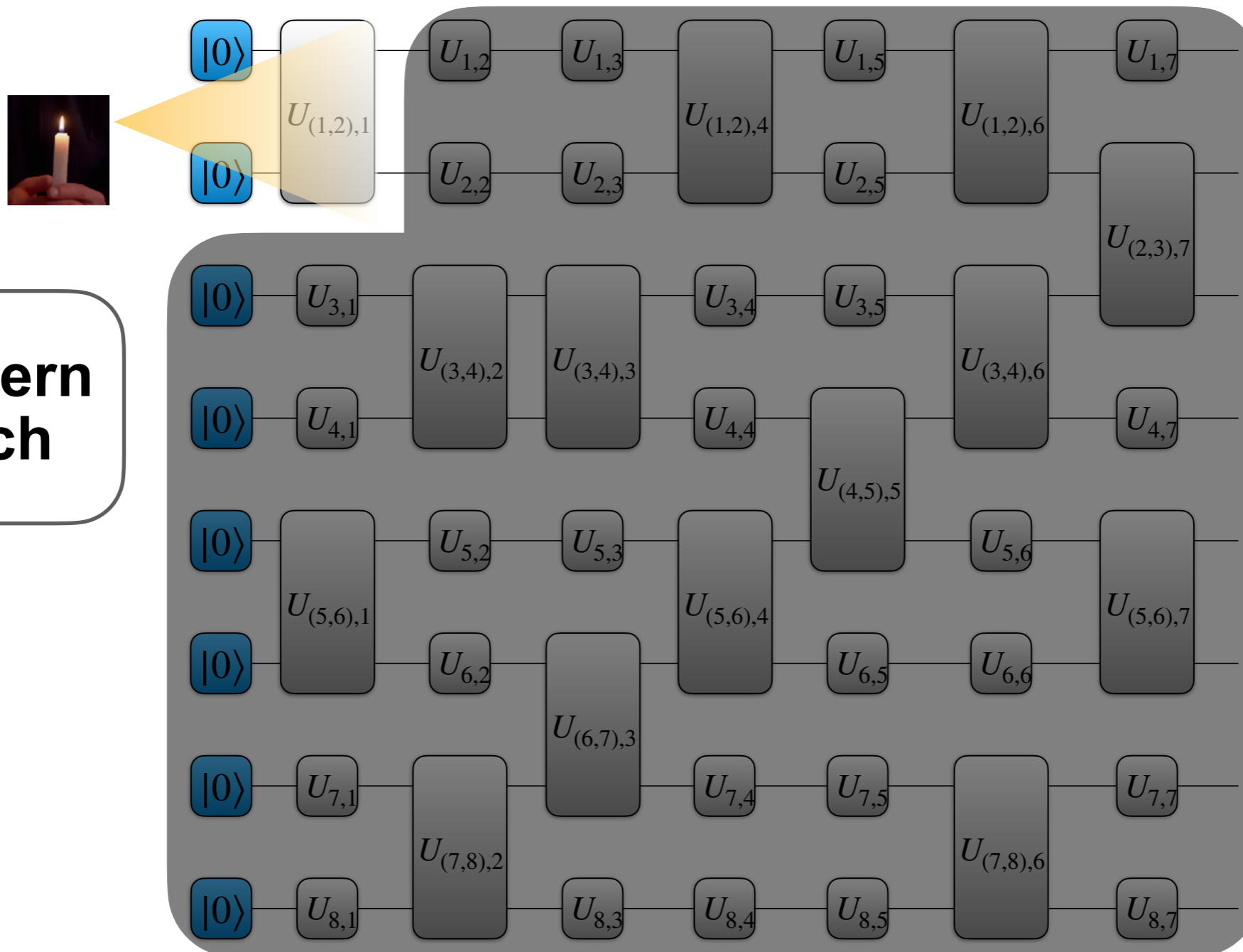


Let's talk about philosophy!

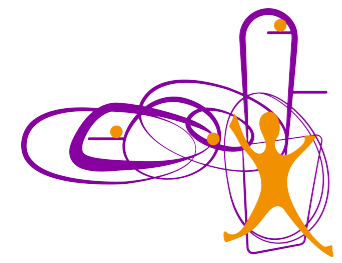




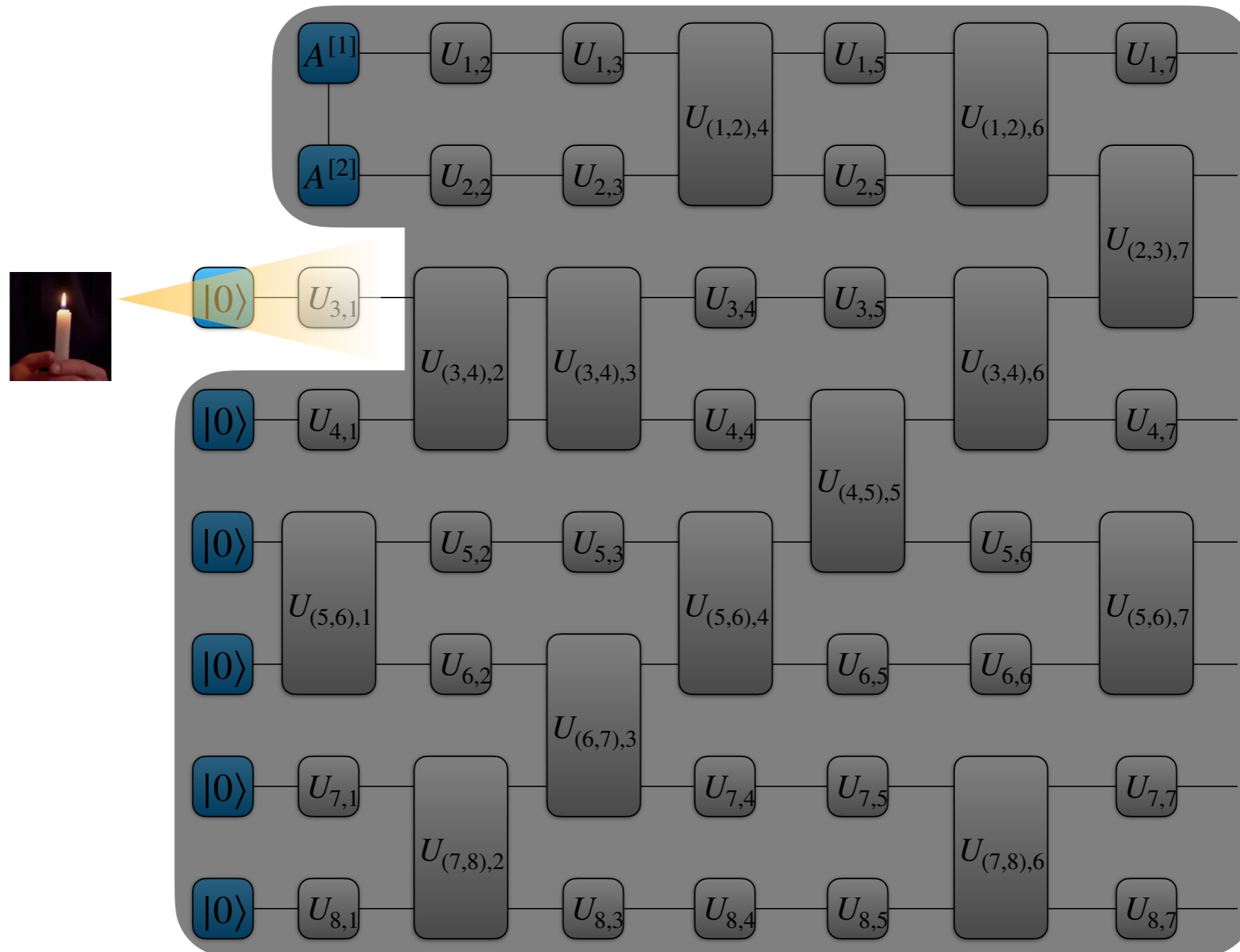
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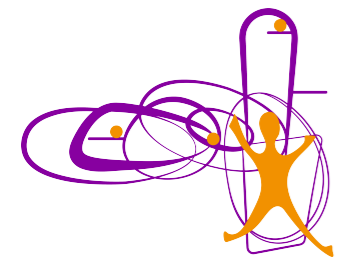


The western  
approach

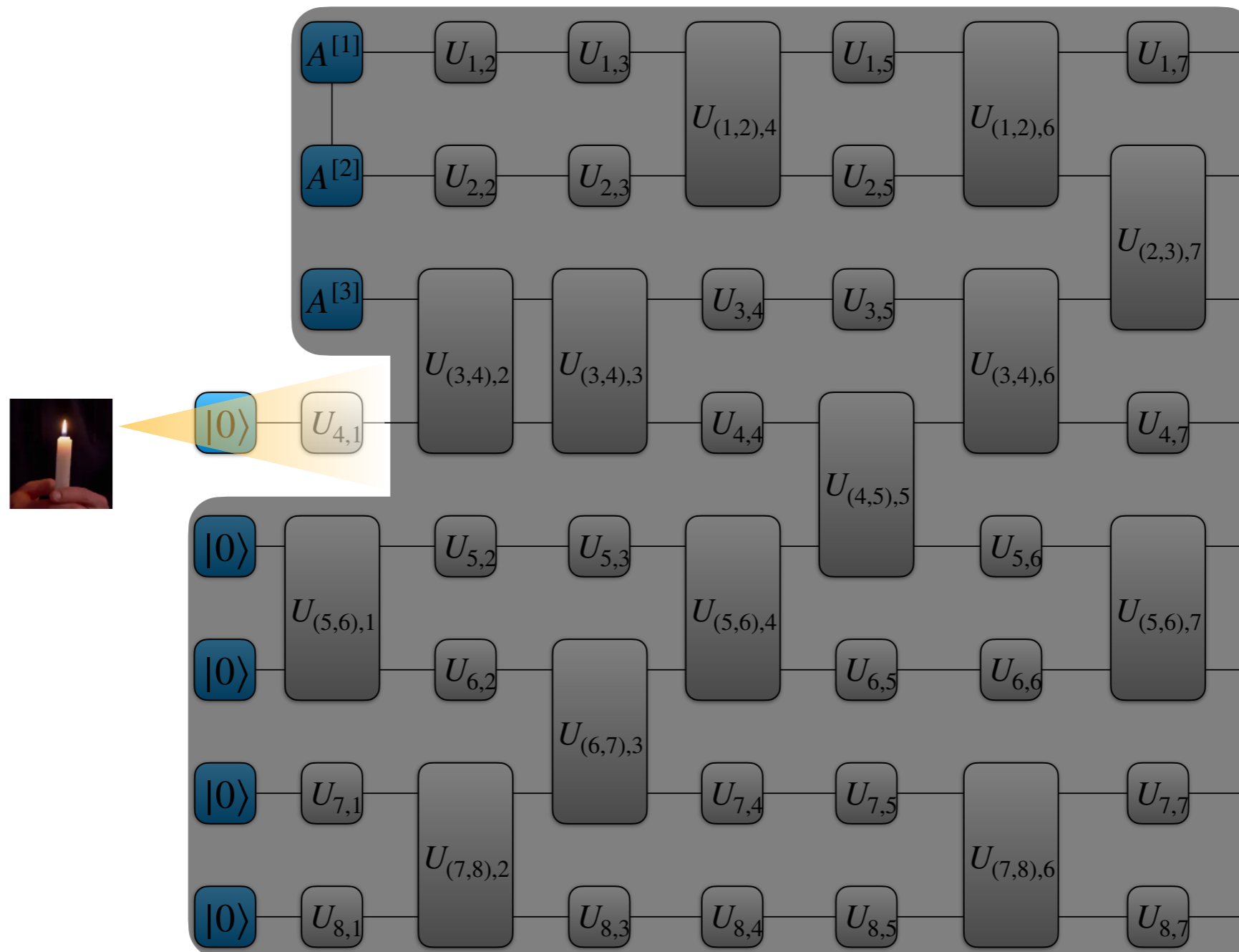


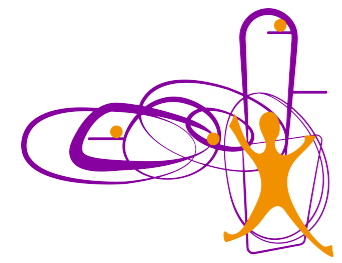
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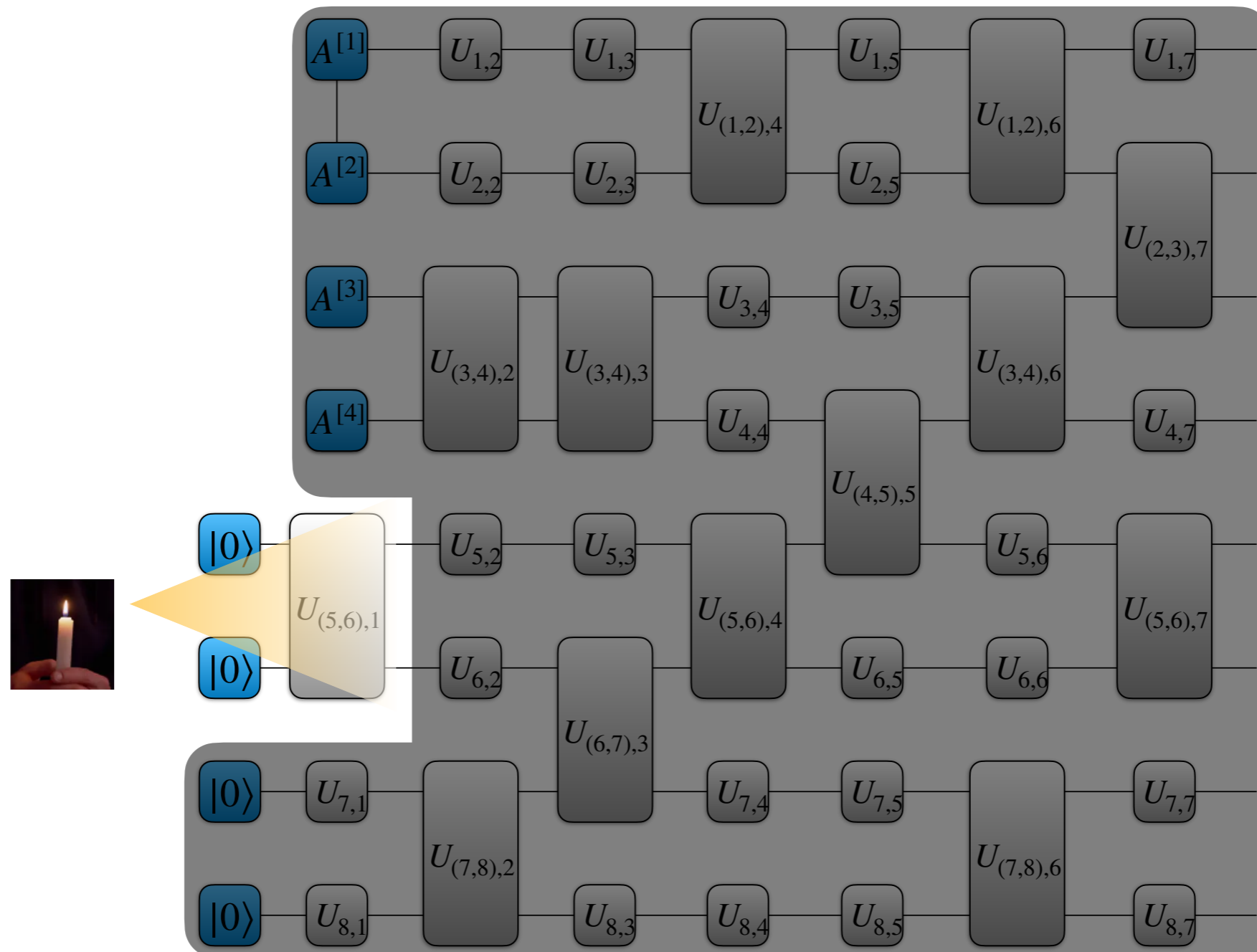


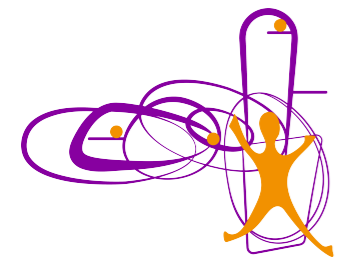
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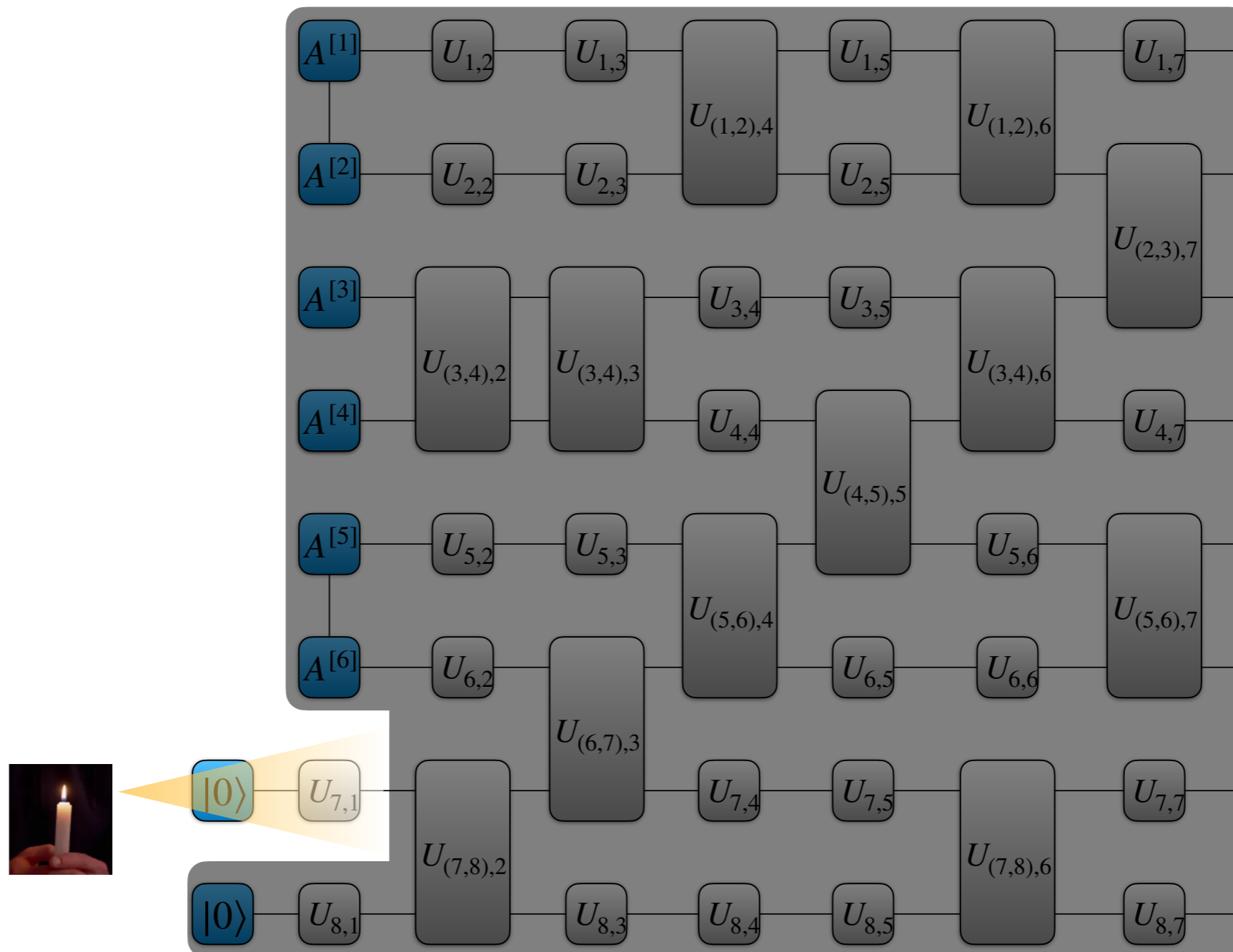


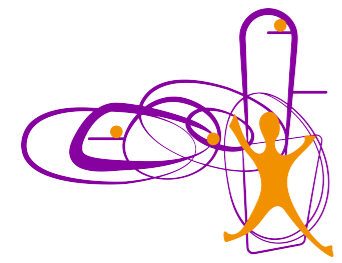
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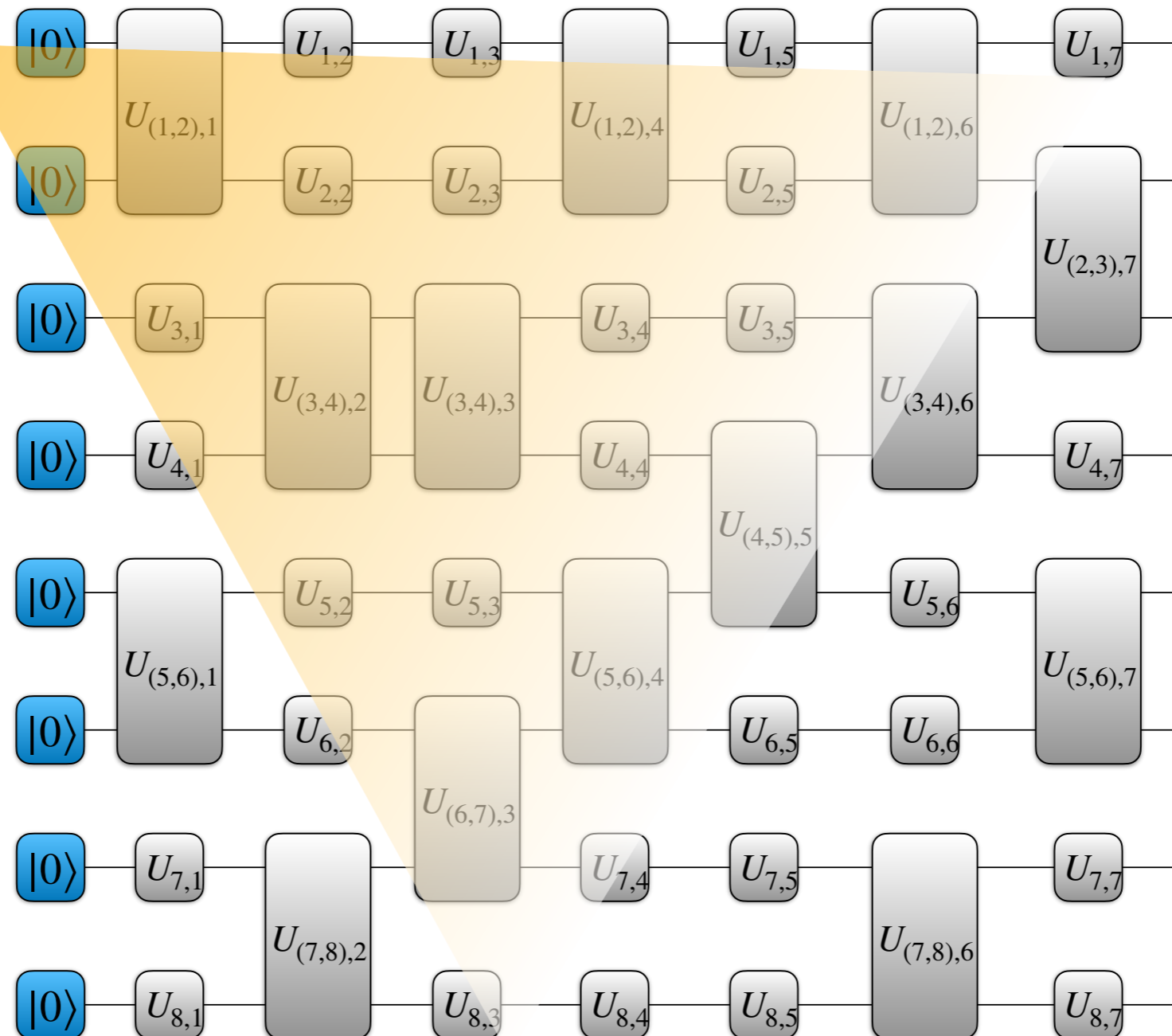


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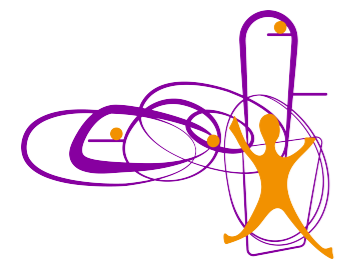




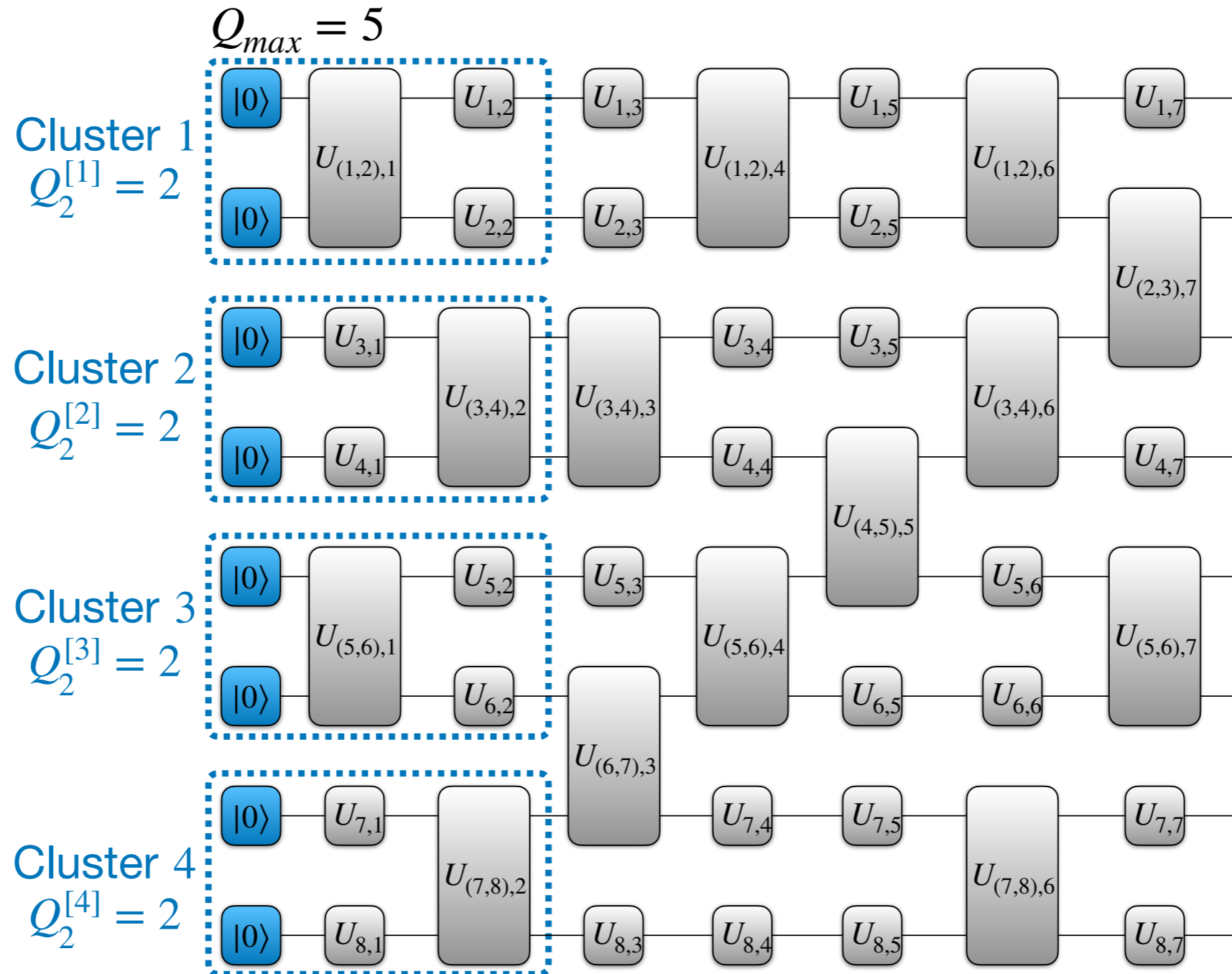
# Cluster-TEBD algorithm [1]



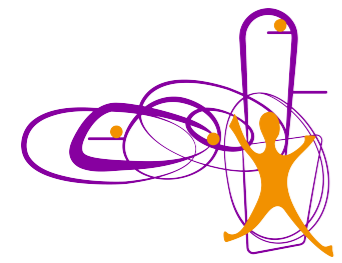
**The zen  
approach:  
switch on the  
light!**



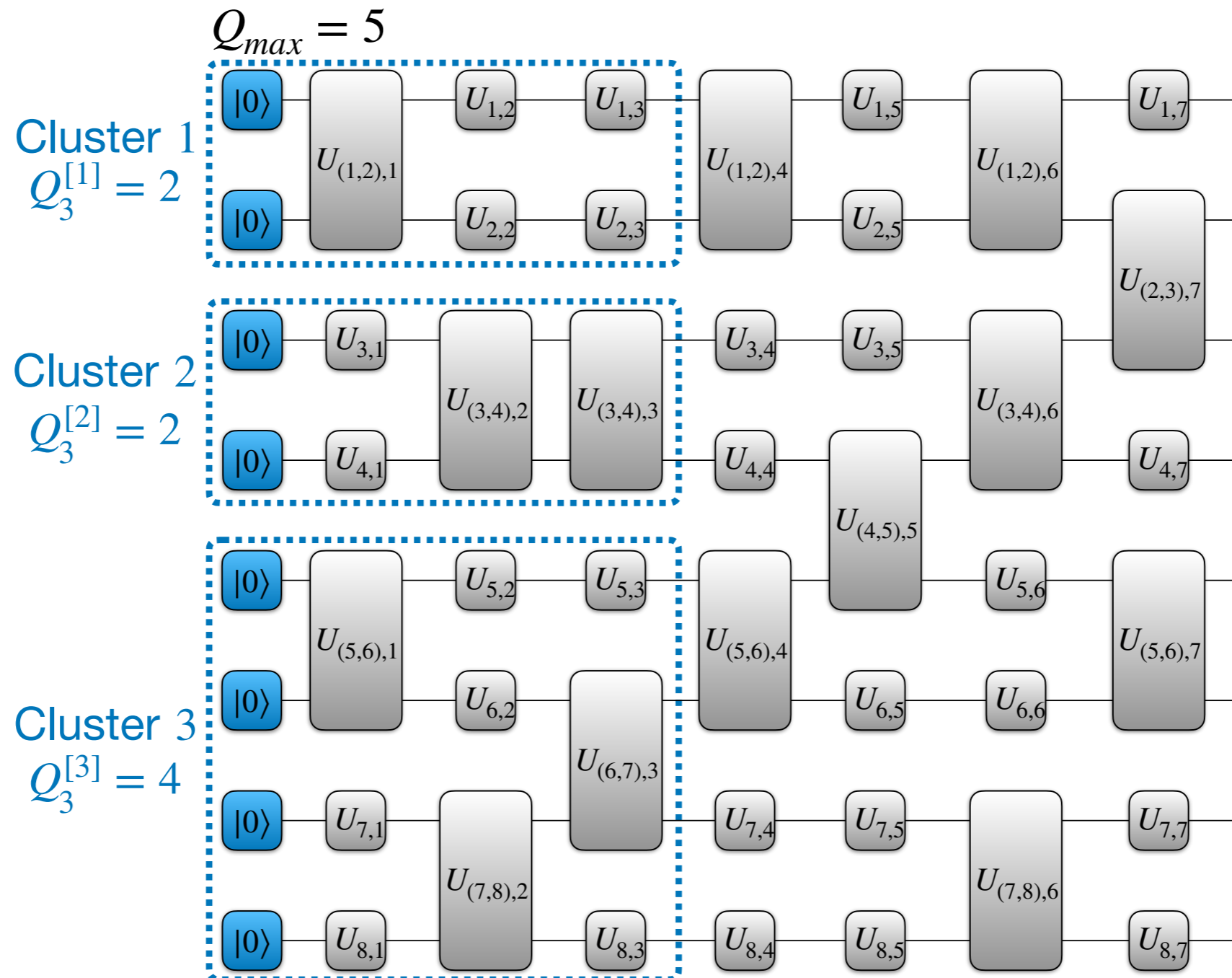
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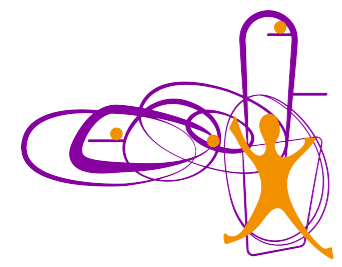




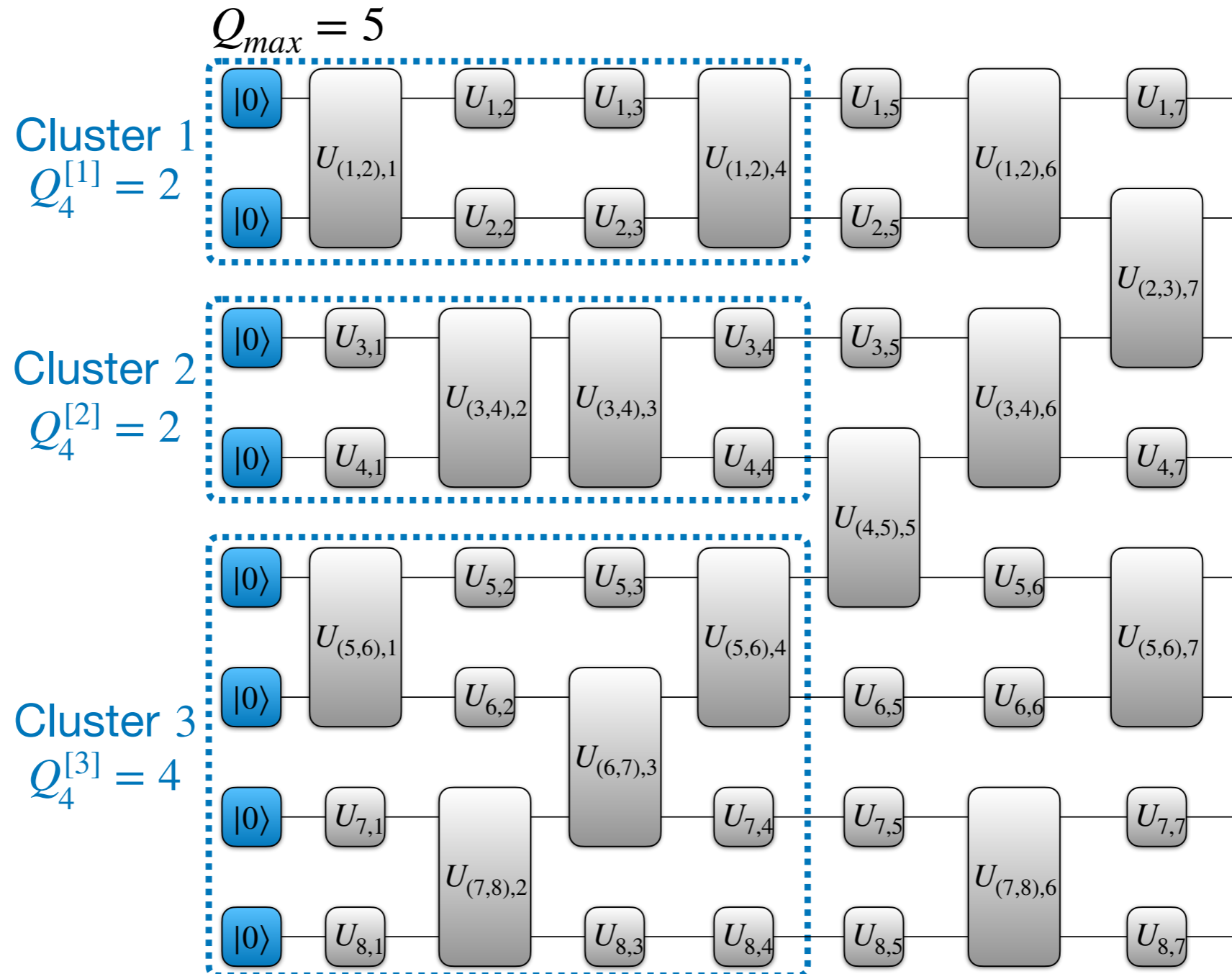


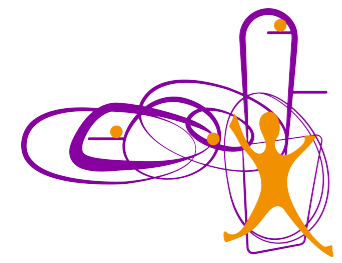
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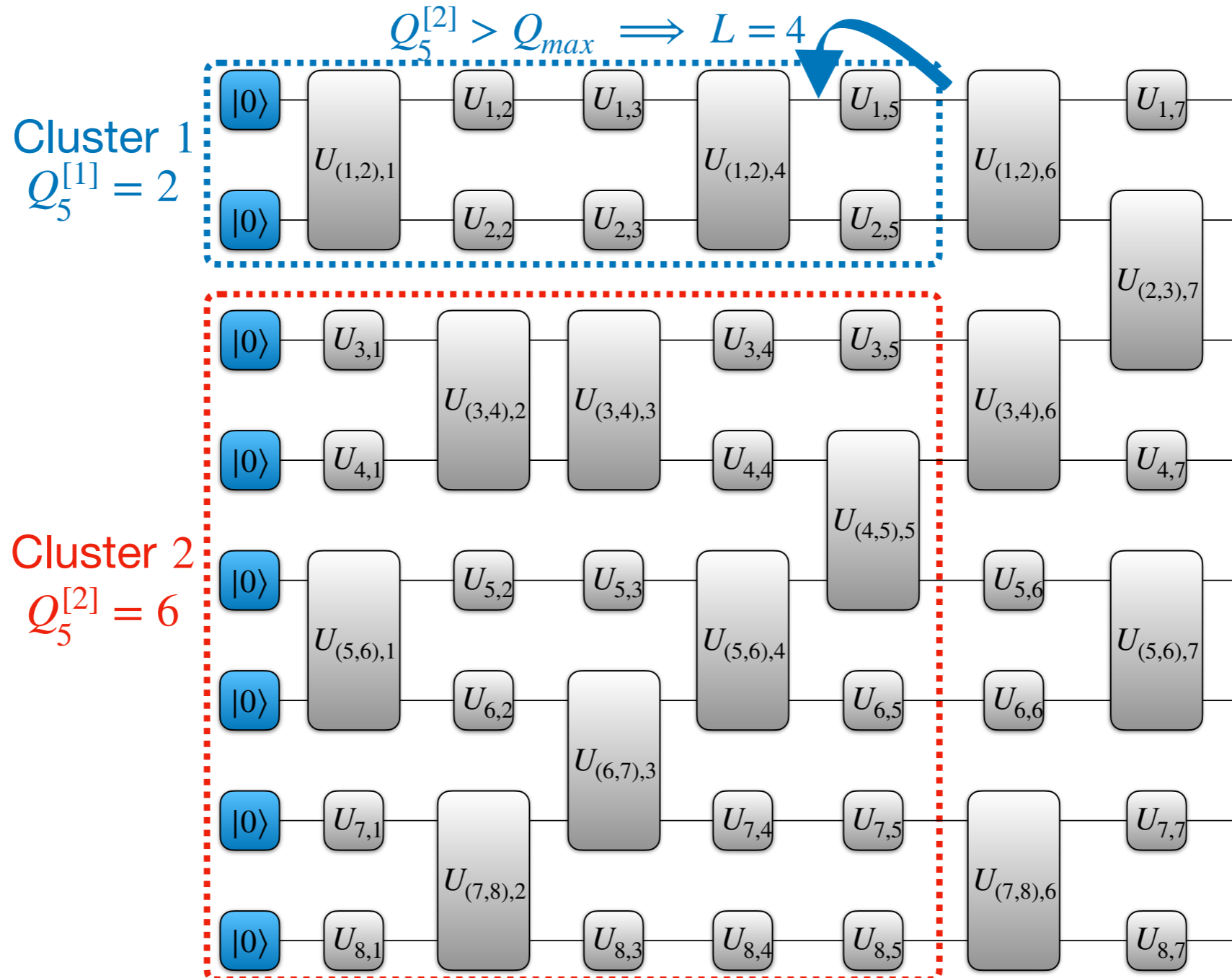


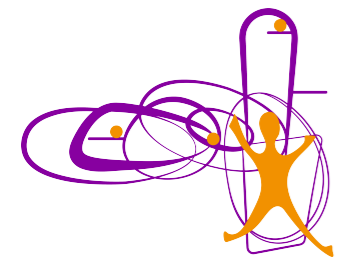
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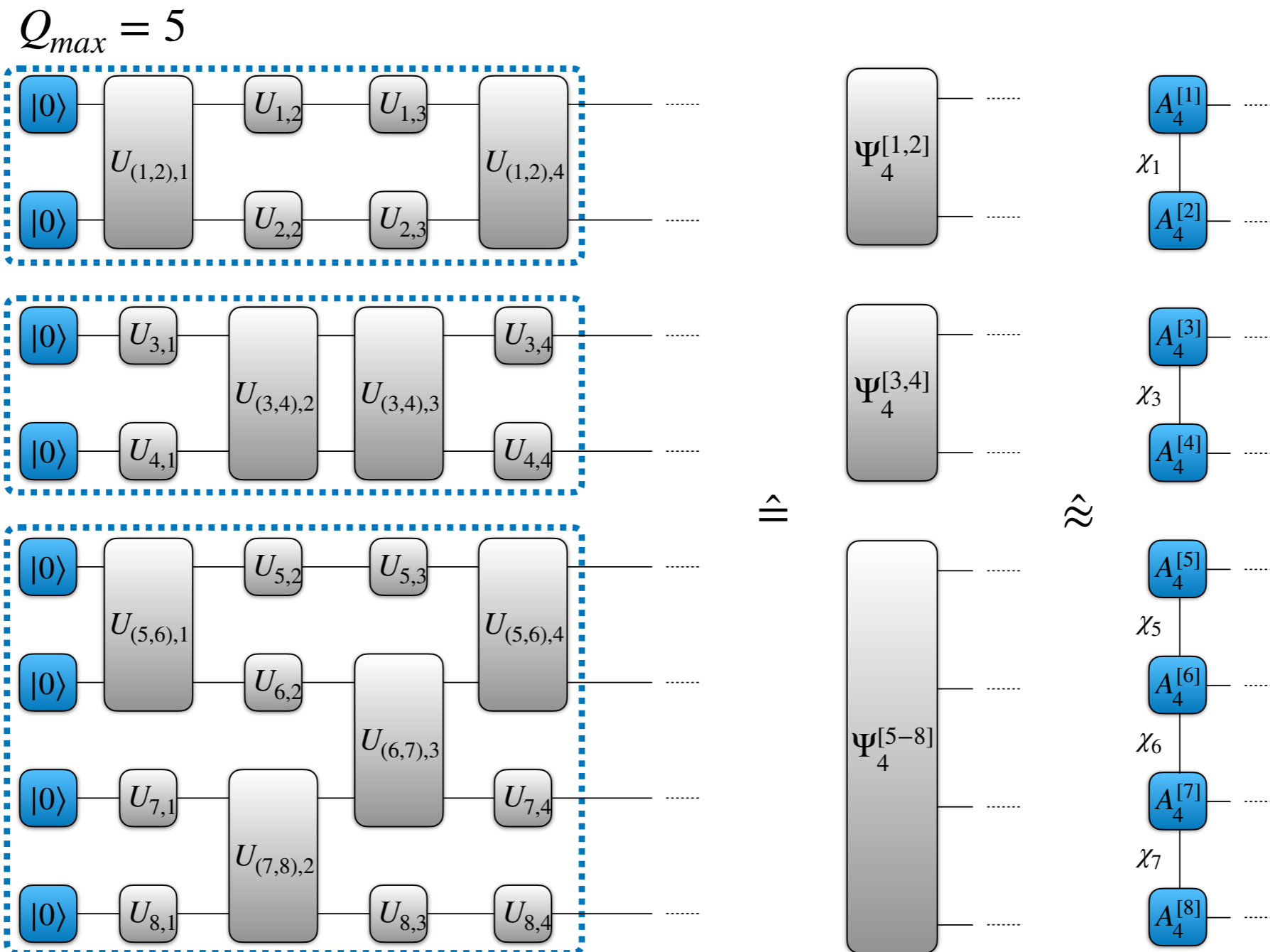


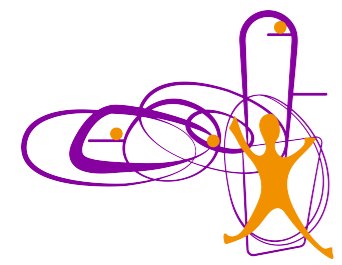
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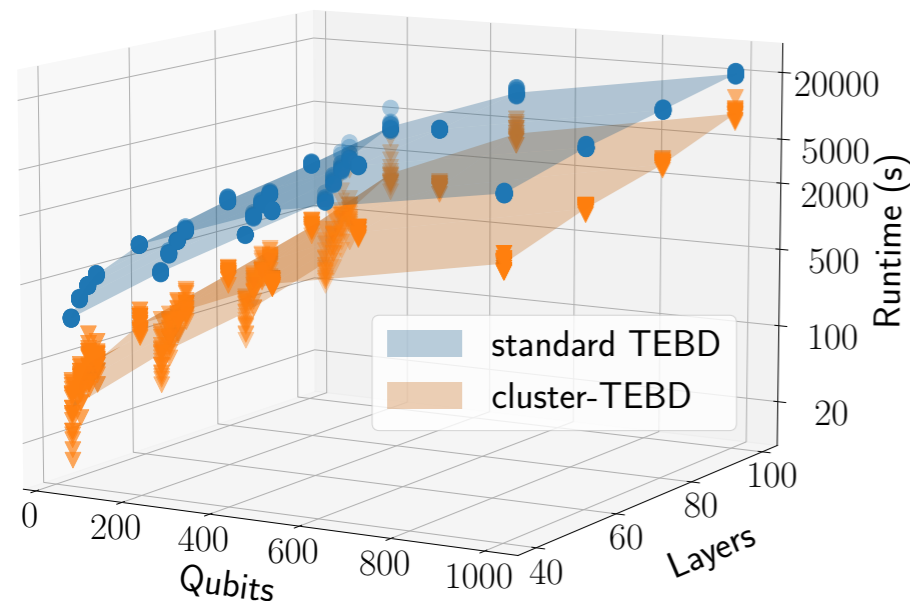




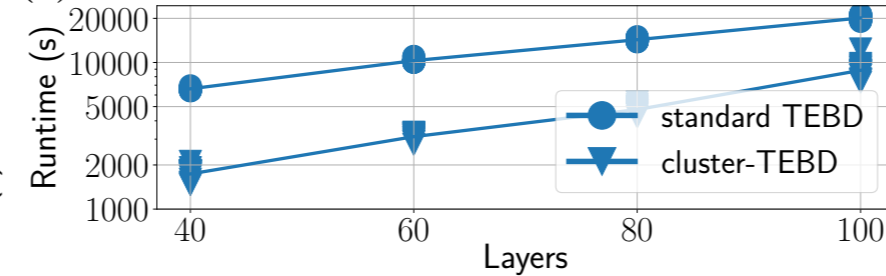
# Results - cluster-TEBD vs. standard TEBD [1]

Clifford<sup>[4]</sup> random-structure quantum circuits:

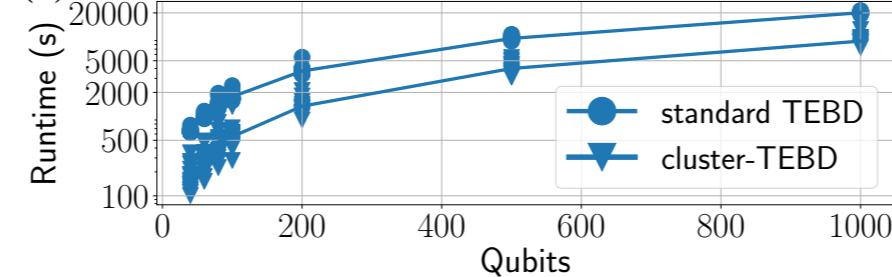
(a)



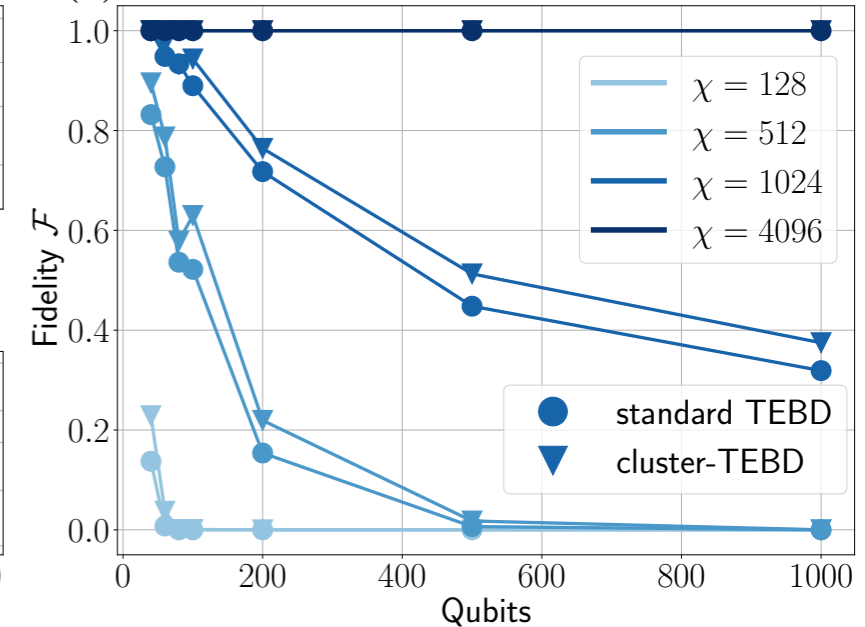
(b)



(c)

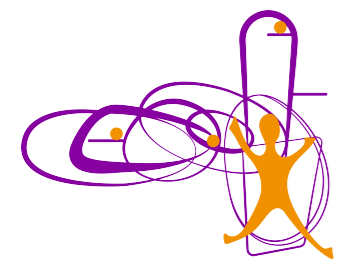


(d)



[1] A. De Girolamo, P. Facchi, P. Rabl, S. Pascazio, C. Lupo, G. Magnifico, *TBA*

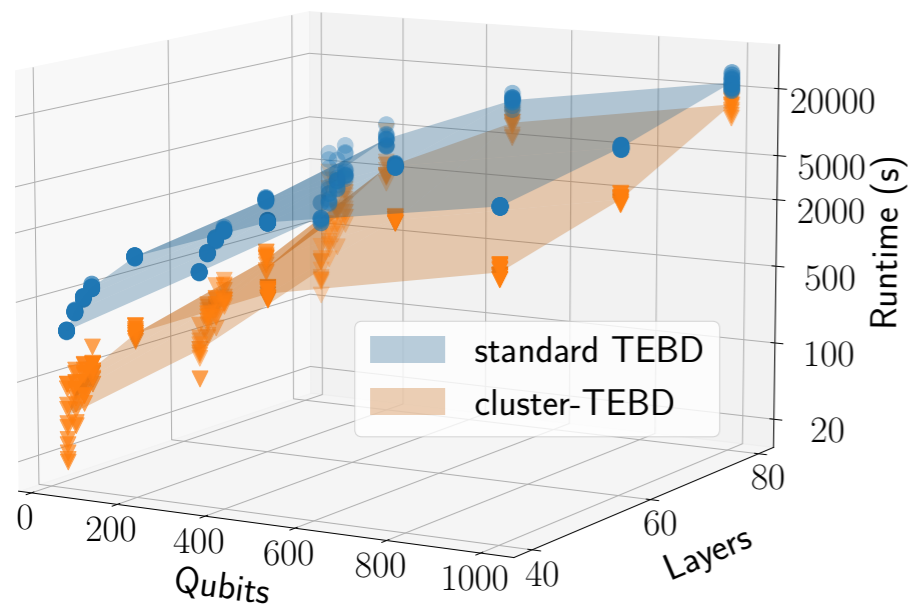
[4] D. Gottesman, *arXiv:quant-ph/9807006 [quant-ph]*, (1998).



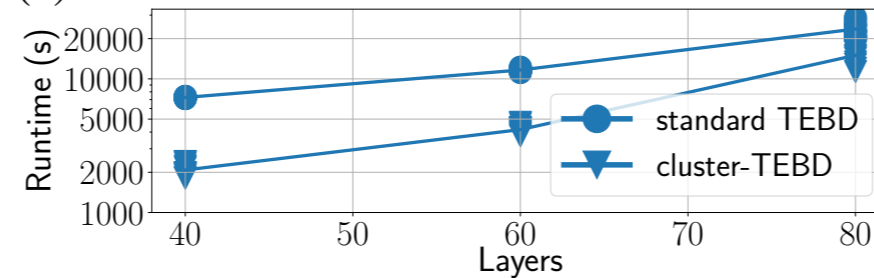
# Results - cluster-TEBD vs. standard TEBD [1]

Non-Clifford<sup>[4]</sup> random-structure quantum circuits:

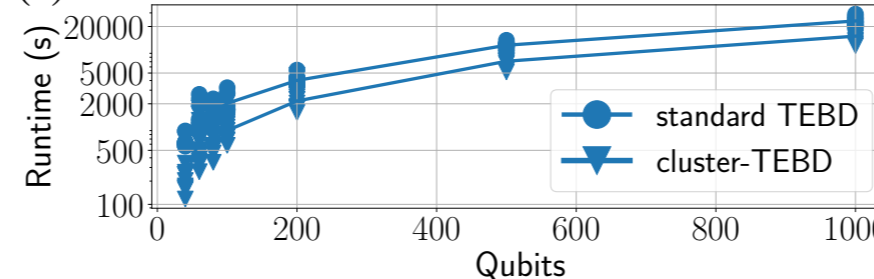
(a)



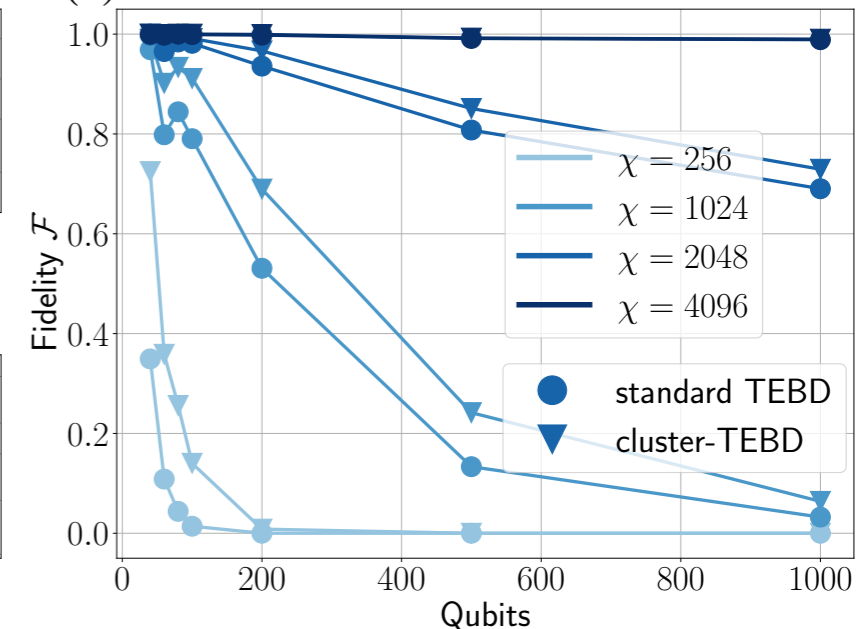
(b)



(c)

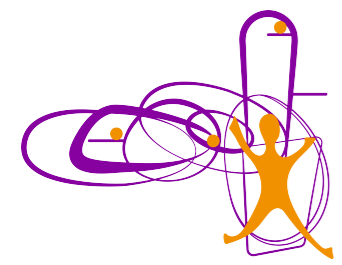


(d)



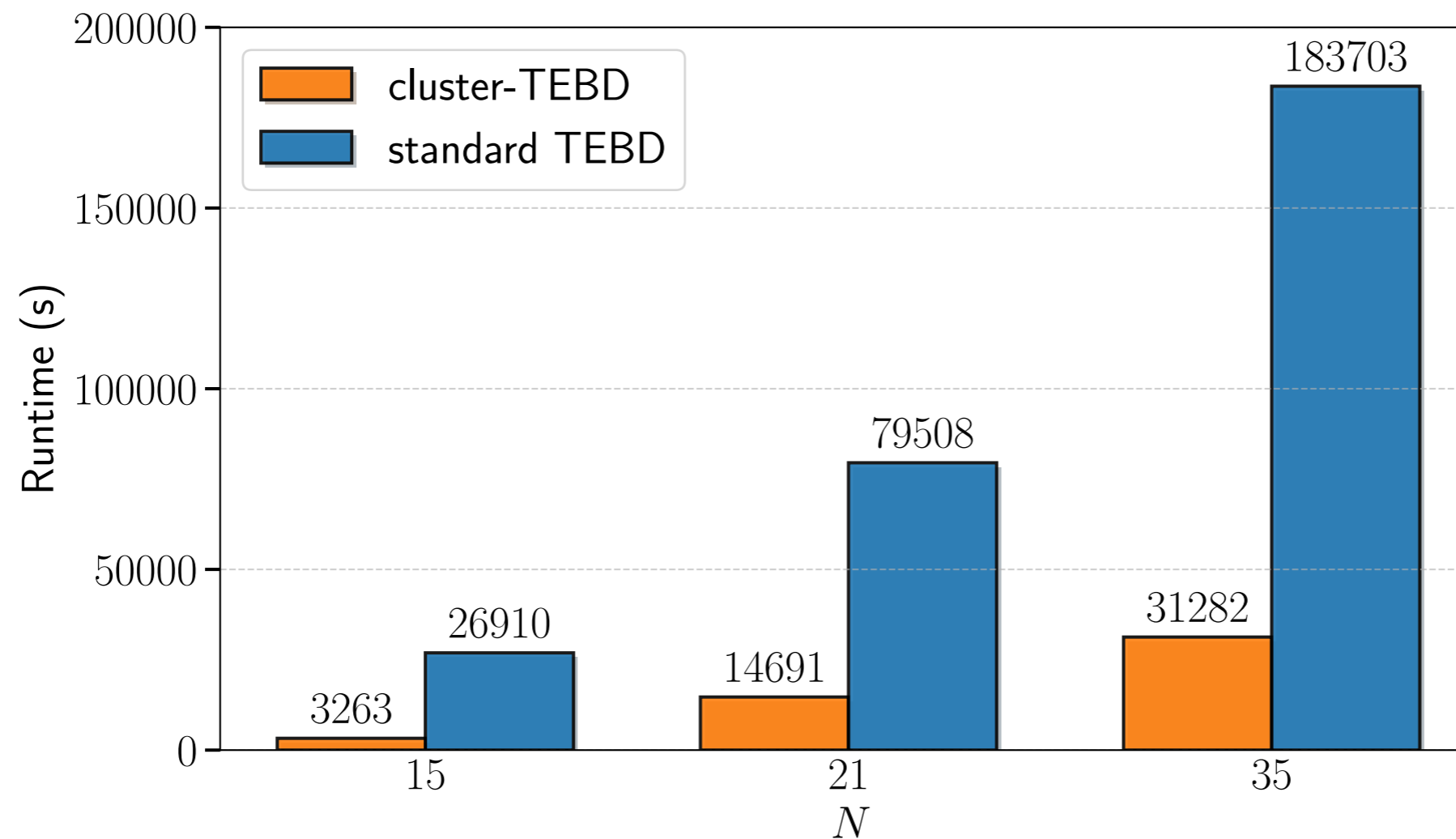
[1] A. De Girolamo, P. Facchi, P. Rabl, S. Pascazio, C. Lupo, G. Magnifico, *TBA*

[4] D. Gottesman, *arXiv:quant-ph/9807006 [quant-ph]*, (1998).



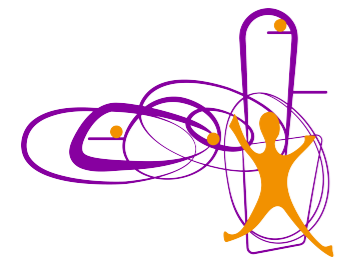
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Shor's algorithm for factoring<sup>[5]</sup>:



[5] P. W. Shor, *SIAM Review* 41, 303 (1999)

[1] A. De Girolamo, P. Facchi, P. Rabl, S. Pascazio, C. Lupo, G. Magnifico, *TBA*



## Conclusions

- Cluster-TEBD significantly outperforms standard TEBD in both runtime and fidelity

## Next steps

- Apply framework for new ansätze (tree tensor networks) for 2D topologies
- Exploiting clustering techniques in quantum compilation
- Digital twin of realistic quantum computers

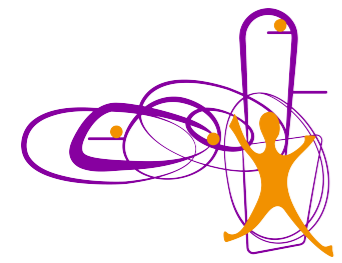


### QuanTeN.jl

A Julia package for optimized simulations of quantum circuits with tensor networks

**COMING SOON**





DIPARTIMENTO  
INTERATENEIO  
DI FISICA



# THANK YOU!



Giuseppe Magnifico



Cosmo Lupo



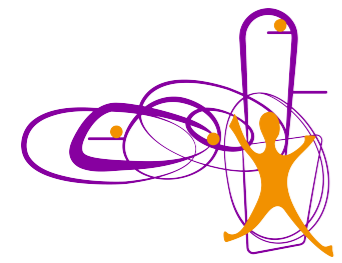
Peter Rabl



Paolo Facchi



Saverio Pascazio

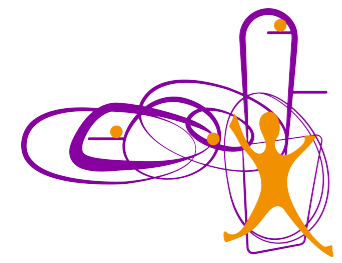


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# **Dynamical cluster-based optimization of tensor network algorithms for simulating quantum circuits with finite fidelity**

## **SUPPLEMENTARY MATERIAL**

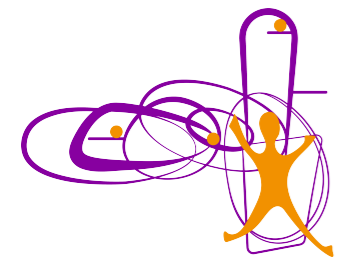


# Generating random-structure quantum circuits

Toss a coin and apply either a single or a two qubit gate with 50% probability



GATES	Clifford	Non-Clifford
Single-qubit	$H, X, Y, Z$	$T, P(3\pi/4), \sqrt[4]{X}, \sqrt{(X+Y)/2}$
Two-qubit	$CNOT, C-Y, C-Z, SWAP$	$C-H, C-S, C-T, \sqrt{SWAP}$

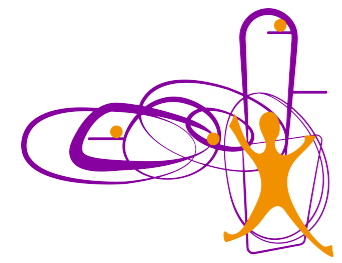


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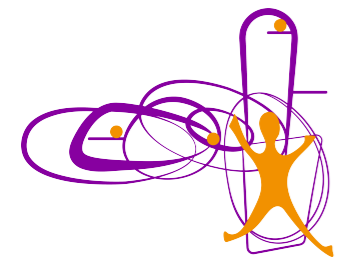


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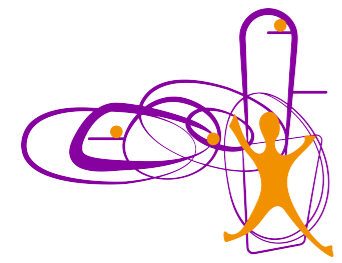


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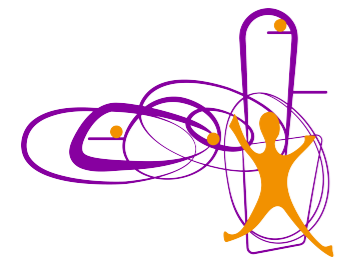


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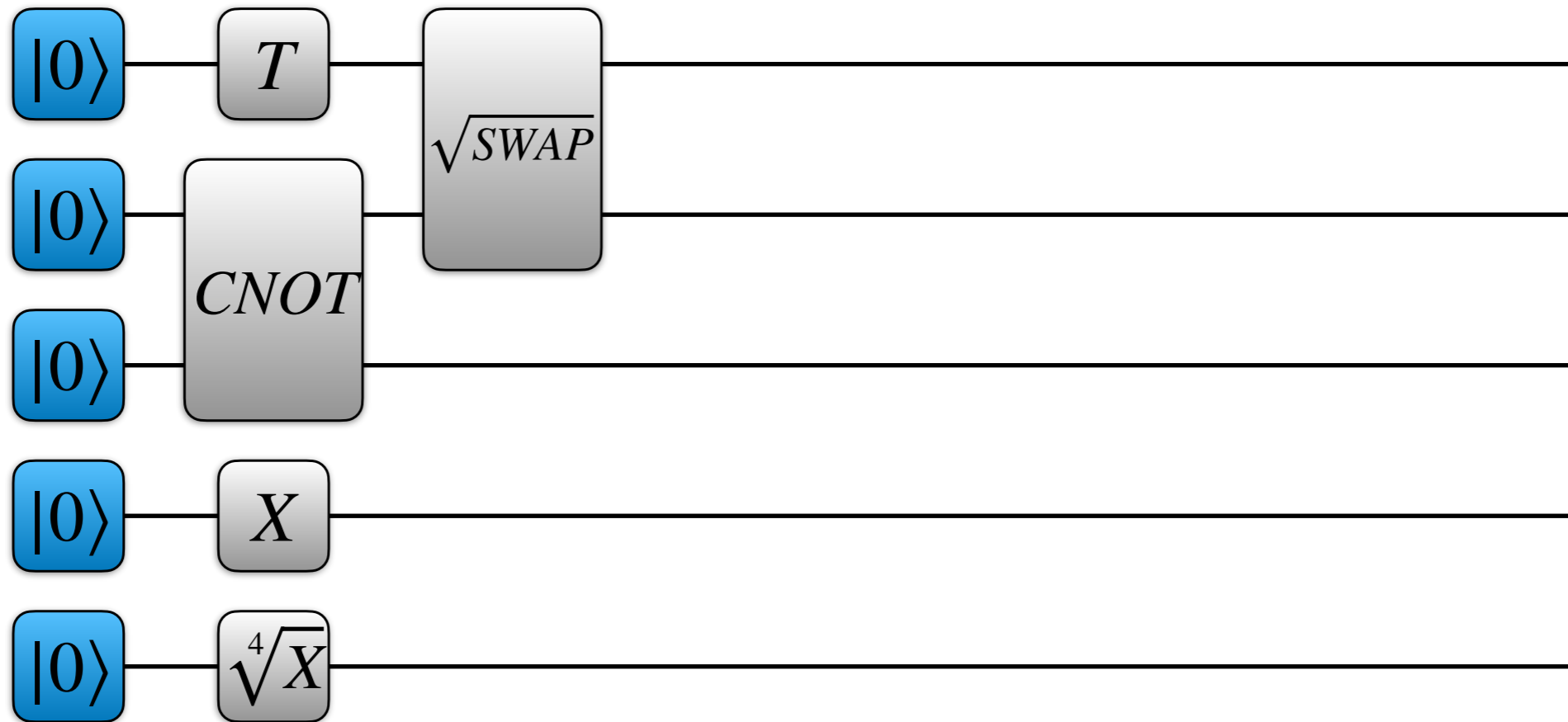


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Two-qubit	$CNOT, C-Y, C-Z, SWAP$	$C-H, C-S, C-T, \sqrt{SWAP}$



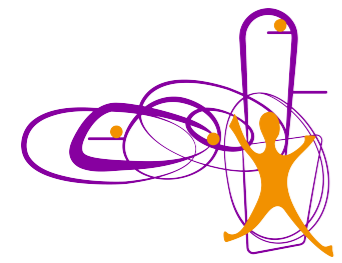
# Generating random-structure quantum circuits

Toss a coin and apply either a single or a two qubit gate with 50% probability



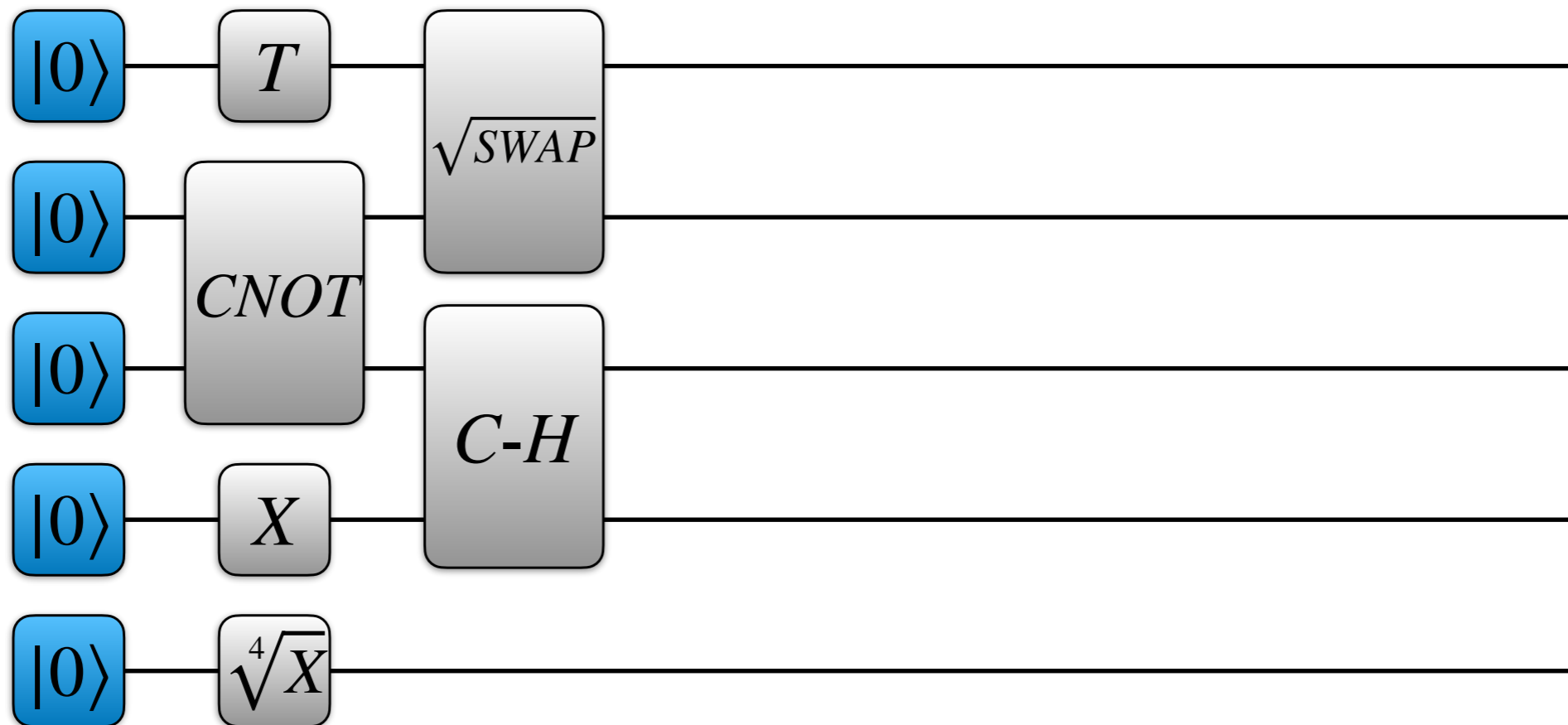
GATES	Clifford	Non-Clifford
Single-qubit	$H, X, Y, Z$	$T, P(3\pi/4), \sqrt[4]{X}, \sqrt{(X+Y)/2}$
Two-qubit	$CNOT, C-Y, C-Z, SWAP$	$C-H, C-S, C-T, \sqrt{SWAP}$



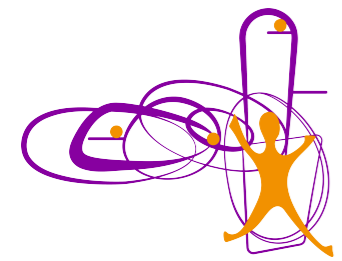


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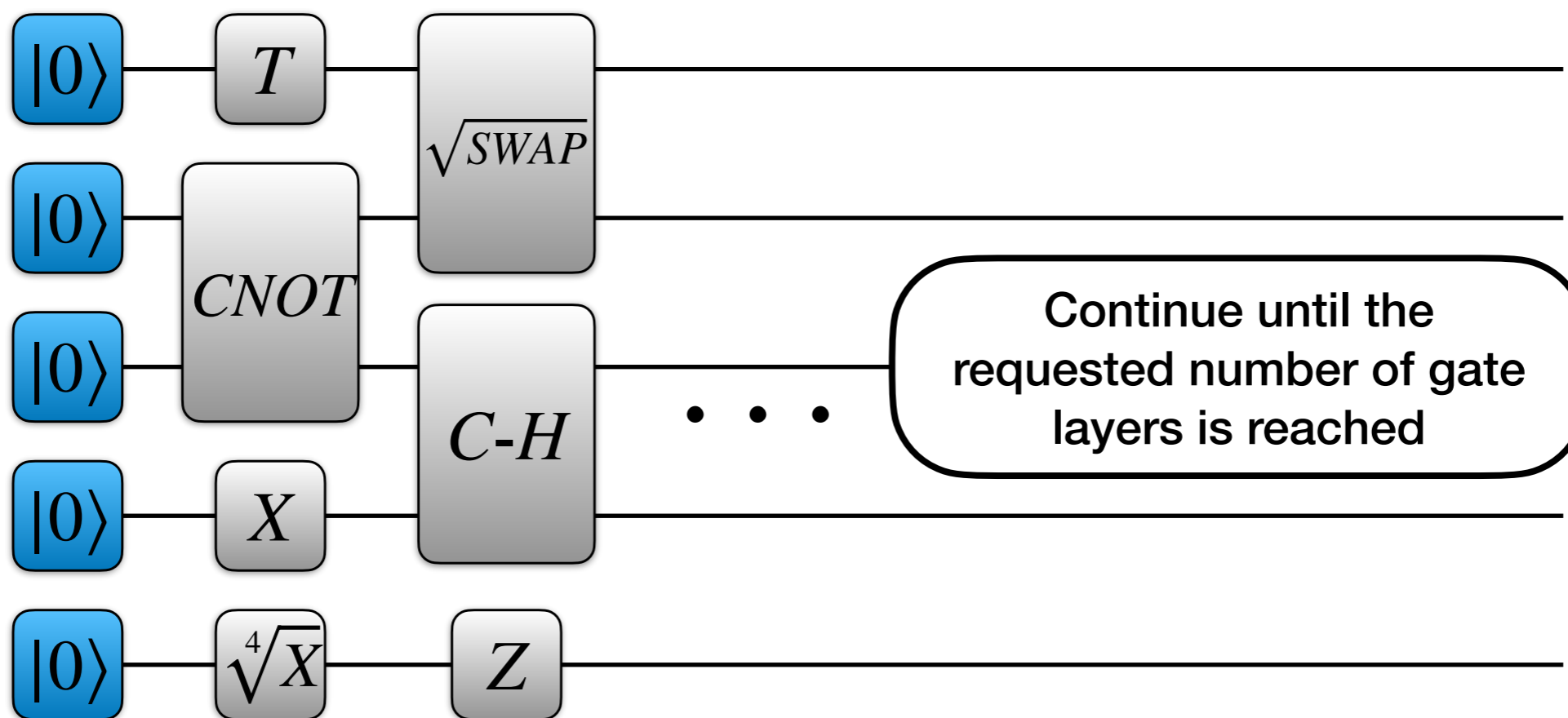


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