Leptons masses and mixing from S3 modular symmetry - an mcmc analysis

Francesco Marcone

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Fundamental problems

- Mixing problem:

  PMNS mixing angles for neutrino seems much larger than the CKM mixing angles for quarks. Still unclear origin of such mixing.

- Hierarchy problem:

  How to address the presence of a strong hierarchy between charged leptons and neutrino masses.

- Flavor problem:

  How are flavor symmetries involved in lepton masses and mixing generation.

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The aim of innumerable works is to derive both the mass hierarchy between charged leptons and the mixing angles in the neutrino sector, possibly deriving from a common symmetry.

Many efforts have been directed towards modular groups and modular-invariant SUSY models, addressing possible resolutions to the following issues:

- CP violation problem
- θ_{23} octant
- Toward NO or IO for neutrino masses
- Lightest neutrino mass value
- Are neutrino Dirac or Majorana

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A promising approach to the flavor problem based on modular invariance was put forward in 2017[1] and largely explored from 2018 .

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General modular symmetry

Homogeneous modular group:

$$\Gamma = SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & e \end{pmatrix} \mid a,b,c,d \in \mathbb{Z} \wedge ad - cb = 1 \right\}$$

Inhomogeneous modular group:

$$\overline{\Gamma} = SL(2,\mathbb{Z})/\left\{\pm\mathbb{I}\right\} = PSL(2,\mathbb{Z})$$

Given $\gamma \in \Gamma$ acting on the modulus:

$$\gamma : \tau \to \gamma(\tau) = \frac{a\tau+b}{c\tau+d}$$

it is clear that γ and $-\gamma$ act on the modulus in the same way, therefore the faithful action group on it is $\overline{\Gamma}$.

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Modular forms

Given the group

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{Z}) \,\middle|\, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\},$$

where N is called the *level*, acting on $\tau \in \mathbb{C}$ with $\mathrm{Im}(\tau) > 0$, we call a function $f(\tau)$ a **modular form** of

weight k with respect to $\Gamma(N)$ if it satisfies:

$$f(\gamma(\tau)) = (c\tau + d)^k f(\tau),$$

for all $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma(N)$, and $f(\tau)$ is holomorphic on the upper half-plane and at infinity.

Finite modular group:

$\Gamma_N \equiv SL(2, \mathbb{Z}/ \pm \Gamma(N)$ and $\Gamma'_N \equiv SL(2, \mathbb{Z})/\Gamma(N)$ For N $= 2$ both groups are isomorphic to S3.

S3 modular group properties

$\Gamma(2) \equiv S3$ is the group of all possible permutations of three objects.

- S3 has three irreducible representations, one two-dimensional and two one-dimensional, namely **2, 1, 1'**;
- It admits only **even-weighted** modular forms;
- There are only two k=2 modular forms, which form an S3 doublet:

$$Y_{\mathbf{2}}^{(1)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

  From the tensor decomposition one finds that all higher-weighted modular forms transform under **1** and **1'** can be obtained from the doublet.

Modular forms will be expressed in the **fundamental domain**:

$$D = \{\tau \in \mathbb{C} : Im(\tau) > 0, |Re(\tau)| \leq \frac{1}{2}, |\tau| \geq 1\}$$

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Minimal super symmetric standard model

$$S = \int d^4x \int d^2\theta d^2\overline{\theta} K(\varphi, \overline{\varphi}, \tau, \overline{\tau}) + [\int d^4x \int d^2\theta W(\varphi, \tau) + h.c.]$$

[2]The modular group acts as follow on the various therms:

- Modulus:

$$\tau \to \gamma(\tau) = \frac{a\tau + b}{c\tau + d}$$

- Matter supermultiplets:

$$\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

- Yukawa couplings:

$$Y_{I_1 \dots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \dots I_n}(\tau)$$

Where $\rho^{(I)}$ is a unitary irreducible representation of Γ_N.
The Superpotential invariance condition is met if the following are satisfied:

$$\begin{cases} \rho \bigotimes \rho_{I_1} \bigotimes \rho_{I_2} \bigotimes \dots \bigotimes \rho_{I_n} \supset \mathbf{1} \\ k_Y = k_{I_1} + k_{I_1} + \dots + k_{I_n} \end{cases}$$

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Model building

Particle content

- Right-handed charged leptons: E_i^c
- Left-handed SU(2) doublets: $\begin{pmatrix} \nu_i \\ E_i \end{pmatrix}$
- Higgs doublets: H_d, H_u

Model assignations

- Doublet: $D_l \equiv \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \sim \mathbf{2}$
- Pseudo-singlet: $l_3 \sim \mathbf{1'}$
- Singlet: $H_{u,d} \sim \mathbf{1}$

Superfields assignments [3]						
	E_1^c	E_2^c	E_3^c	D_l	l_3	$H_{d,u}$
$SU(2)_L x U(1)_Y$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, \mp 1/2)$
$\Gamma_2 \equiv S3$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1'}$	$\mathbf{2}$	$\mathbf{1'}$	$\mathbf{1}$
k_I	k_{E_1}	k_{E_2}	k_{E_3}	k_D	k_l	$k_{u,d} = 0$

It has to be noticed that the Higgs doublet are **sterile** with respect to the modular symmetry.

[3] Meloni and Parriciatu, "A simplest modular S_3 model for leptons"

Superpotential therms

The Lagrangian of the model contains the most general superpotential with all possible singlets under S3 built with the modular forms organized in various multiplets of different weights.

$$W_l = \alpha E_1^c H_d (D_l Y_{\mathbf{2}}^{(2)})_{\mathbf{1}} + \beta E_2^c H_d (D_l U_{\mathbf{2}})_{\mathbf{1'}} + \gamma E_3^c H_d l_3$$

$$W_\nu^{k_l=2} \supset \frac{g}{\Lambda} H_u H_u (D_l D_l)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)} + \frac{g'}{\Lambda} H_u H_u D_l l_3 (Y_{\mathbf{2}}^{(2)}) + \frac{g''}{\Lambda} H_u H_u (D_l D_l)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)} + \frac{g_p}{\Lambda} H_u H_u l_3 l_3 (Y_{\mathbf{1}}^{(2)})$$

Where Λ is the scale of new physics.

$$M_l = m_d \cdot \begin{pmatrix} \alpha (Y_{\mathbf{2}}^{(2)})_1 & \alpha (Y_{\mathbf{2}}^{(2)})_2 & 0 \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix}$$

$$m_\nu^{k_l=2} = \frac{2 g v_u^2}{\Lambda} \cdot \left[\begin{pmatrix} -(Y_2^2 - Y_1^2) & 2Y_1Y_2 & \frac{g_1}{2g} 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) & -\frac{g_1}{2g}(Y_2^2 - Y_1^2) \\ \frac{g_1}{2g} 2Y_1Y_2 & -\frac{g_1}{2g}(Y_2^2 - Y_1^2) & 0 \end{pmatrix} + \begin{pmatrix} \frac{g_2}{g}(Y_1^2 + Y_2^2) & 0 & 0 \\ 0 & \frac{g_2}{g}(Y_1^2 + Y_2^2) & 0 \\ 0 & 0 & \frac{g_p}{g}(Y_1^2 + Y_2^2) \end{pmatrix} \right]$$

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Parameters and Observables

The model provided is effectively controlled by a set of six dimensionless parameters and by two mass scales:

$$(\tau, \beta/\alpha, \gamma/\alpha, g_1/g, g_2/g, g_p/g)(\alpha v_d, g v_u^2/\Lambda)$$

Which in the following are referred to as:

$$(\tau, \beta, \gamma, g_1/g, g_2/g, g_p/g)(\alpha v_d, g v_u^2/\Lambda)$$

It has than been tested on the following set of observables:

$$(sin^2\theta_{12}, sin^2\theta_{13}, sin^2\theta_{23}, m_e/m_\mu, m_e/m_\tau, \Delta m^2_{sol}, \Delta m^2_{atm})$$

Best-fit value and 1σ range	
Parameter	Value (NH)
$sin^2\theta_{12}$	0.303 ± 0.013
$sin^2\theta_{13}$	0.02219 ± 0.00063
$sin^2\theta_{23}$	0.473 ± 0.024
m_e/m_μ	0.0048 ± 0.0002
m_e/m_τ	0.0565 ± 0.0045
$\Delta^2 m_{atm}/10^{-3}eV^2$	2.48498 ± 0.031
$\Delta^2 m_{sol}/10^{-5}eV^2$	7.362112 ± 0.166798

The following definitions for masses differences squared have been used:

$\Delta^2 m_{sol} \equiv m_2^2 - m_1^2$

$\Delta^2 m_{atm} \equiv m_3^2 - \frac{(m_1^2+m_2^2)}{2}$

A particular choice for $sin^2\theta_{23}$ has been made, due to the lack of degeneracy resolutions.
The model is still sensible to the degeneracy.

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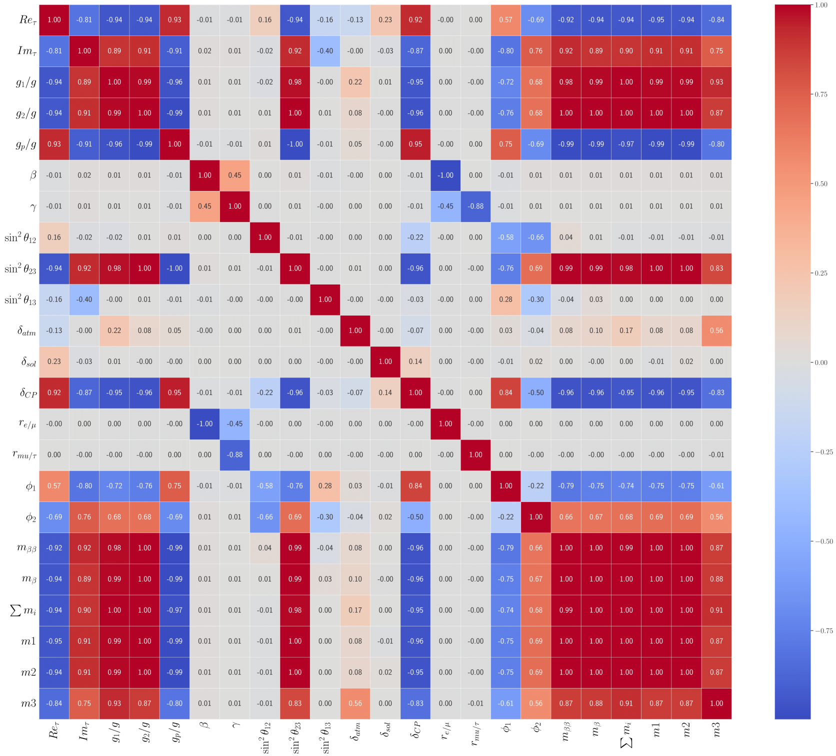
Correlation matrix



Correlation matrix among parameters and observables.

- $Re(\tau), Im(\tau), g_1/g, g_2/g, g_p/g$ are mostly correlated to m_1, m_2, m_3 and to the Majorana masses and angles along with δ_{CP} and $sin^2\theta_{23}$;
- β results in a strong anti correlation with the ratio $r_{e/\mu}$;
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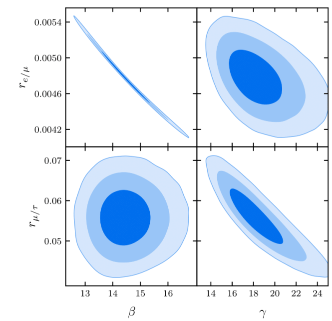

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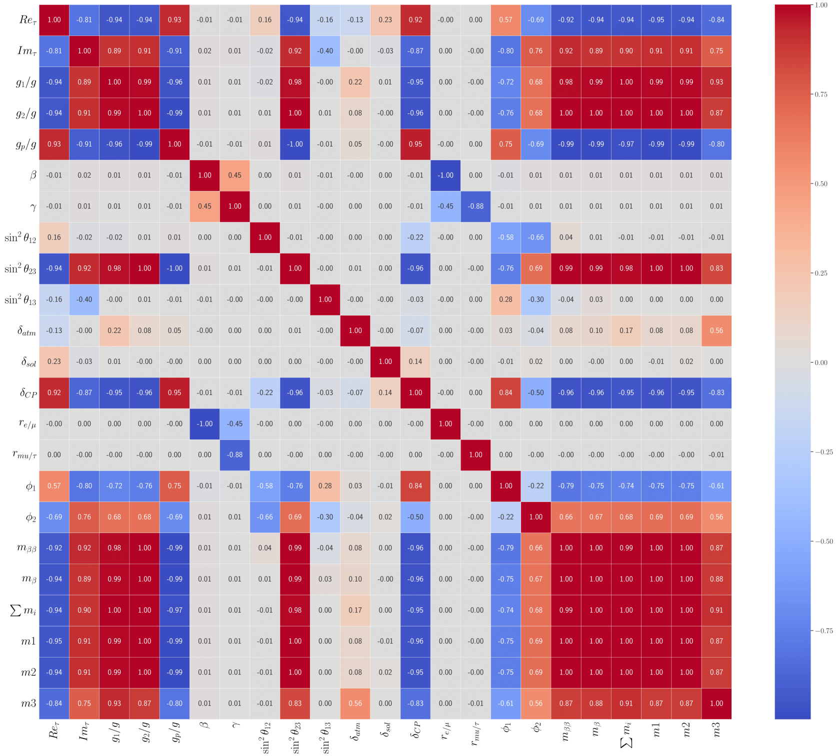

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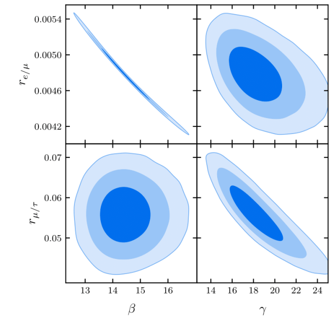

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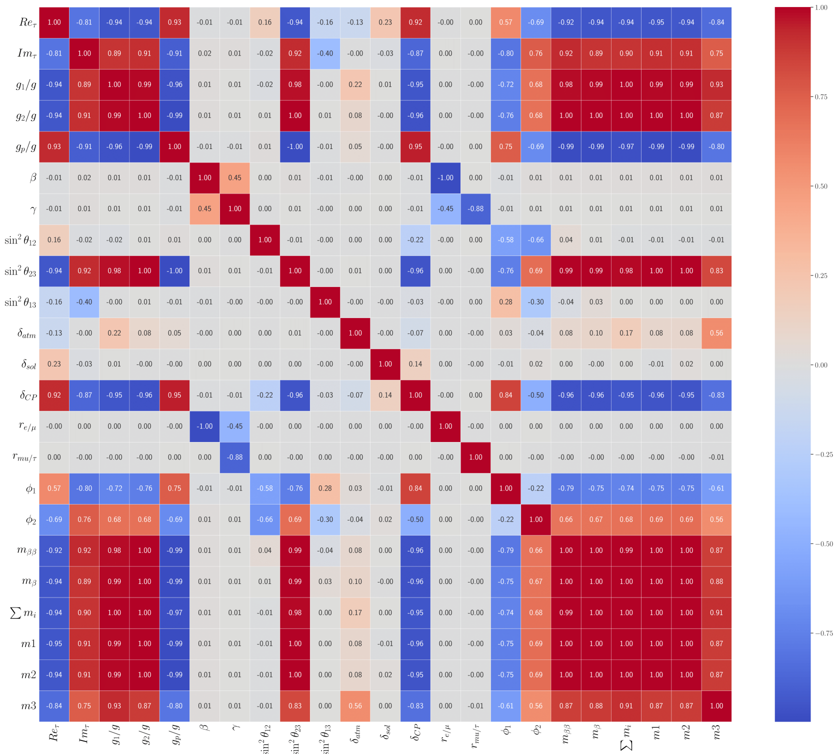

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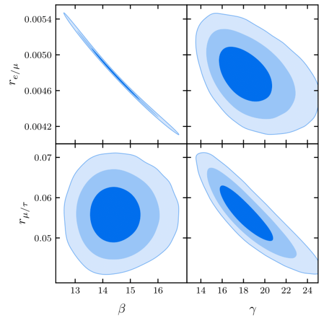
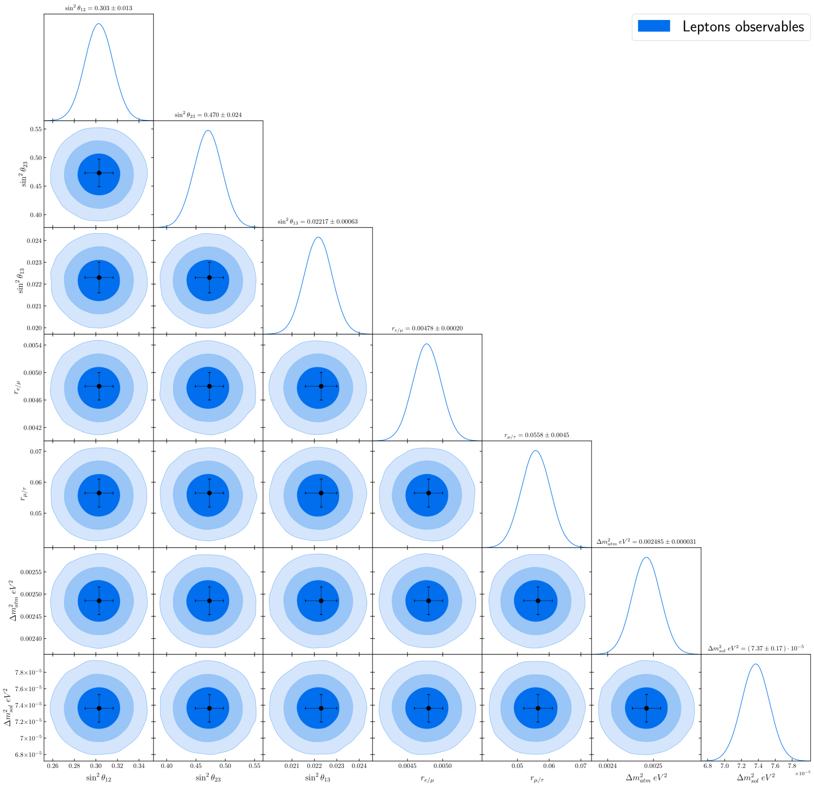


Selected Observables - Fit quality



Parameter	NO Best-fit (1σ)	χ^2
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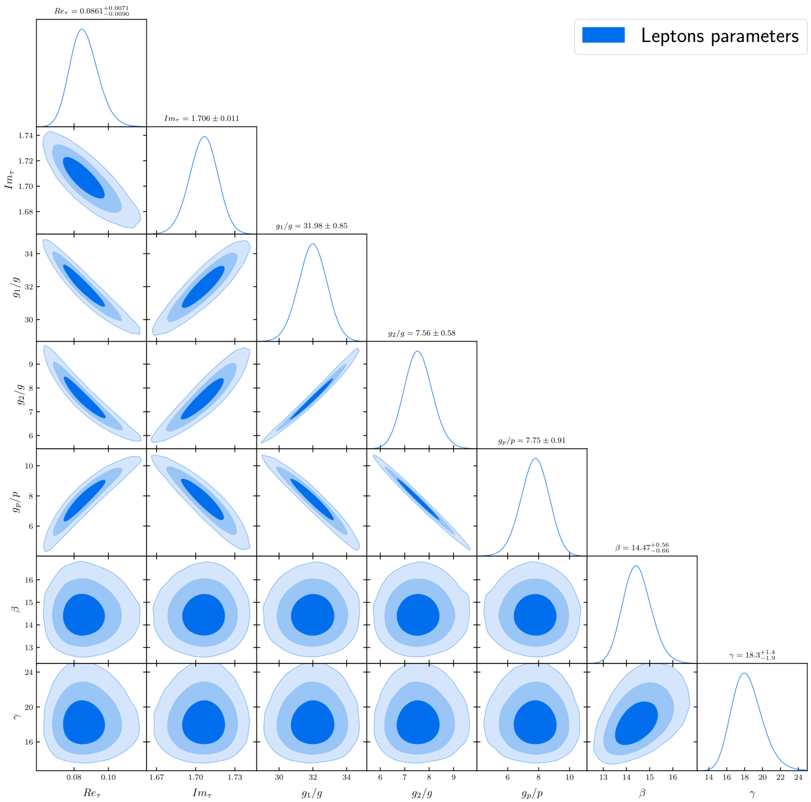
$\sin^2\theta_{23}$	$0.470^{+0.018}_{-0.021}$	0.0035
$\sin^2\theta_{13}$	$0.02217^{+0.00065}_{-0.00066}$	0.0005
$r_{e/\mu}$	0.00478 ± 0.00020	0.0137
$r_{\mu/\tau}$	0.0558 ± 0.0045	0.0251
Δm^2_{sol} [eV^2]	$(7.37 \pm 0.17) \times 10^{-5}$	0.0014
Δm^2_{atm} [eV^2]	0.002485 ± 0.000031	0.0089
		0.2368

All the observables are well reproduced, with a slight tension on $sin^2\theta_{12}$

Model parameters

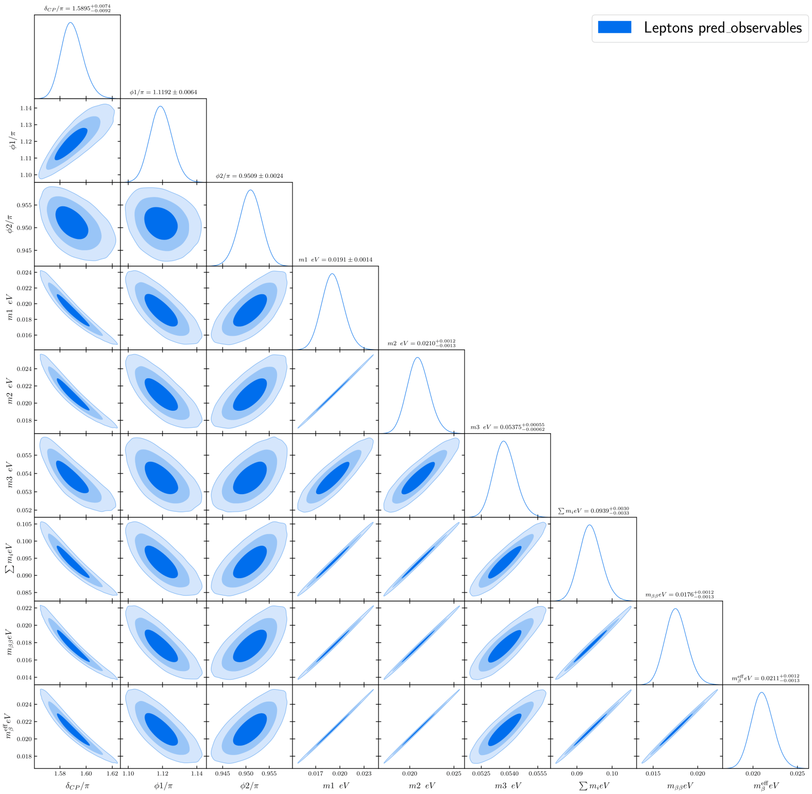


Parameter	NO Best-fit (1σ)
Re_τ	$0.0861^{+0.0071}_{-0.0090}$
Im_τ	1.706 ± 0.011
g_1/g	31.98 ± 0.85
g_2/g	7.56 ± 0.58
g_p/g	7.75 ± 0.91
β	$14.47^{+0.56}_{-0.66}$
γ	$18.3^{+1.4}_{-1.9}$

Table: Best-fit and error at 1σ.

All parameters lie in a narrow interval centered in the peak value.
Weak correlation between charged lepton sector's parameters and neutrino ones.

Model prevision

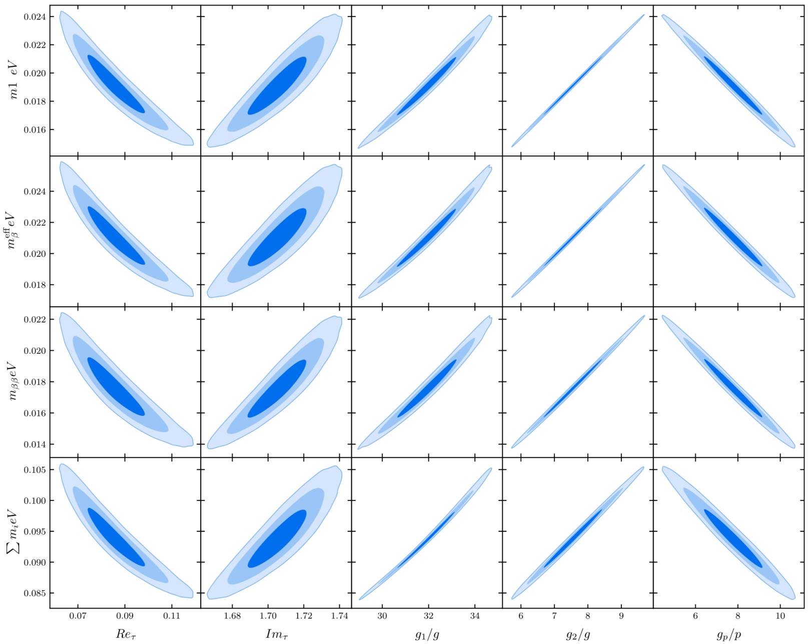


Parameter	NO Best-fit (1σ)
δ_{CP}/π	$1.5895^{+0.0074}_{-0.0092}$
ϕ_1/π	1.1192 ± 0.0064
ϕ_2/π	0.9509 ± 0.0024
$m_1 eV$	0.0191 ± 0.0014
$\sum_i m_i eV$	$0.0939^{+0.0030}_{-0.0033}$
$m_{\beta\beta} eV$	$0.0176^{+0.0012}_{-0.0013}$
$m_\beta^{eff}\, eV$	$0.0211^{+0.0012}_{-0.0013}$

Table: Best-fit and error at 1σ.

Strong correlations between observables are to be intended as deriving from $(Re(\tau), Im(\tau), g_1/g, g_2/g, g_p/g)$

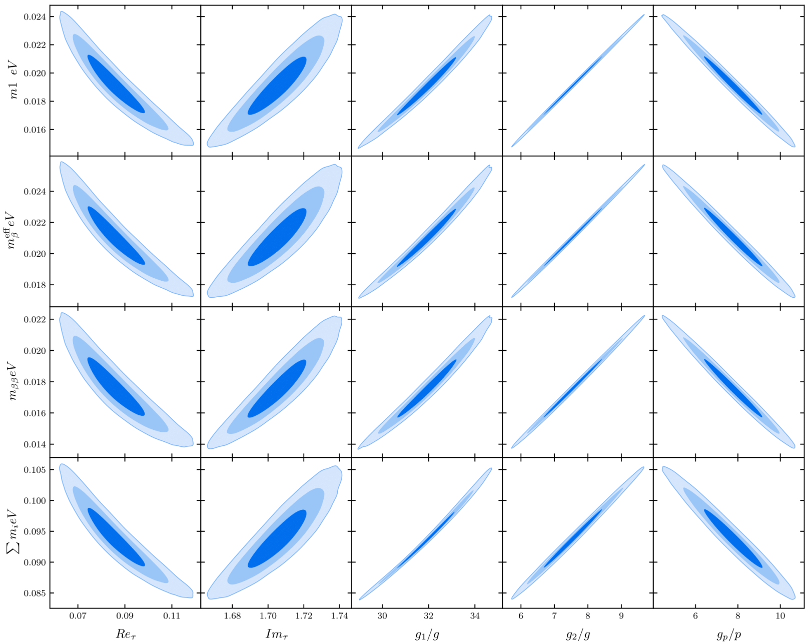
Correlations



As shown from the correlation matrix:

- Strong anti-correlation between $(Re(\tau), \frac{g_p}{g})$ and mass parameters;
- Strong correlation between $(Im(\tau), \frac{g_1}{g}, \frac{g_2}{g})$ and mass parameters.

Correlations



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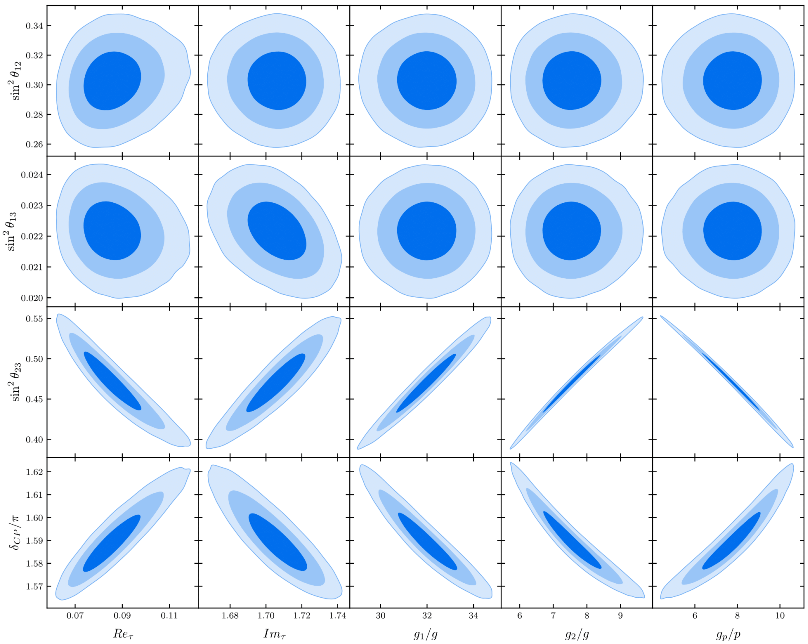
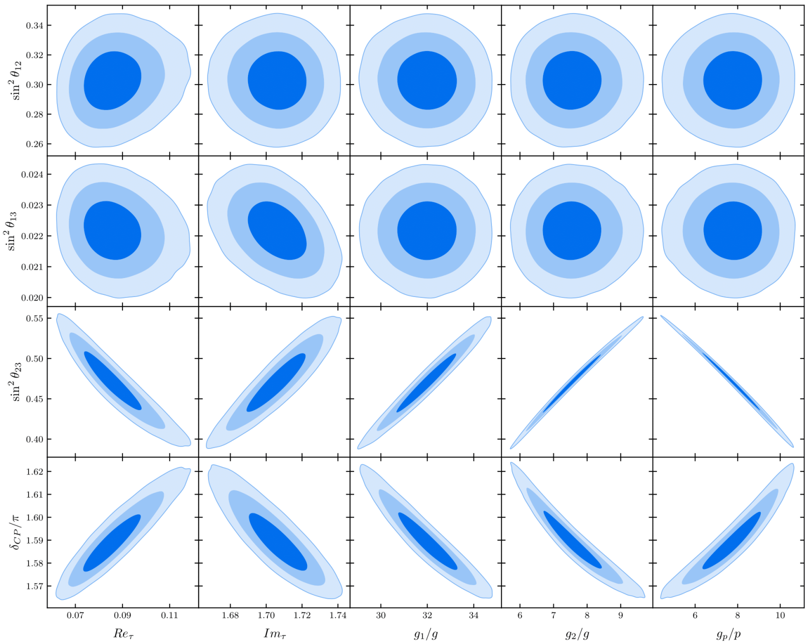
Correlations



As shown from the correlation matrix:

- Weak correlation between model $(\tau, \frac{g_1}{g}, \frac{g_2}{g}, \frac{g_p}{g})$ and $(sin^2\theta_{12}, sin^2\theta_{13})$;
- Strong correlations (and anti-correlations) beween the model parameters and $(sin^2\theta 23, \delta_{CP})$.

Correlations



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- Weak correlation between model $(\tau, \frac{g_1}{g}, \frac{g_2}{g}, \frac{g_p}{g})$ and $(sin^2\theta_{12}, sin^2\theta_{13})$;
- Strong correlations (and anti-correlations) beween the model parameters and $(sin^2\theta 23, \delta_{CP})$.

Conclusions

The modular symmetry framework represents a promising approach to origin of lepton masses and mixing. The

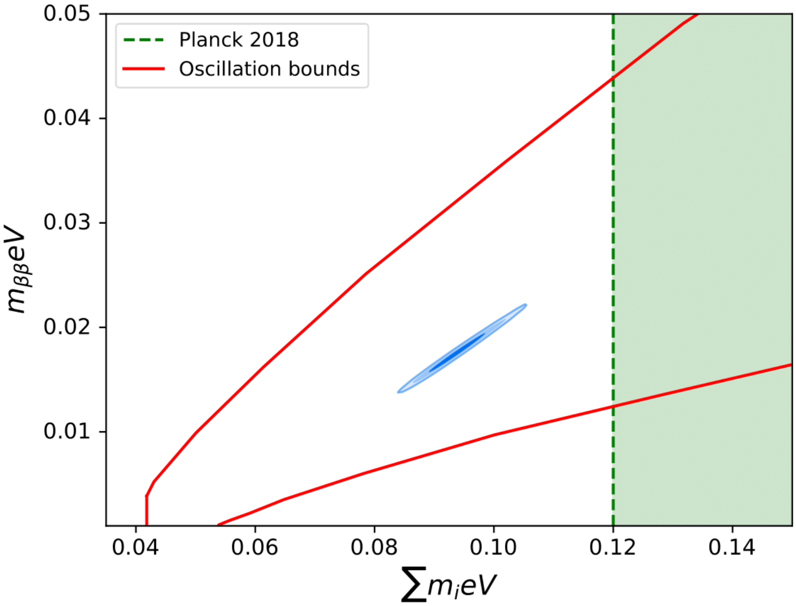
S3 modular invariant model consist of **6 free dimensionless parameters** (including a complex modulus) and two mass scales, and correctly describes 8 observables. Predictive power:

- Correlations among parameters and observables;
- Experimental predictions for: $\delta_{CP}, m_1, m_\beta, m_{\beta\beta}, \sum_i m_i$;
- Further implementations including quarks may lead to cross-correlations between lepton and quark sector.

Thank you for the attention

Non oscillation boundaries

Monte Carlo simulation, confronted with current bounds from oscillation experiments[4].



- Good fit within the oscillation bounds;
- $\sum_i m_i$ value consistent with limits from Planck[a]:

$$\sum_i m_i < 0.12 \text{ eV}$$

- $m_{\beta\beta}$ consistent with limits from Gerda[b] and KamLAND-Zen[c];

$$m_{\beta\beta} < 0.079 - 0.180 \text{ eV}$$

[a] Aghanim et al., "Planck 2018 results. VI. Cosmological parameters"

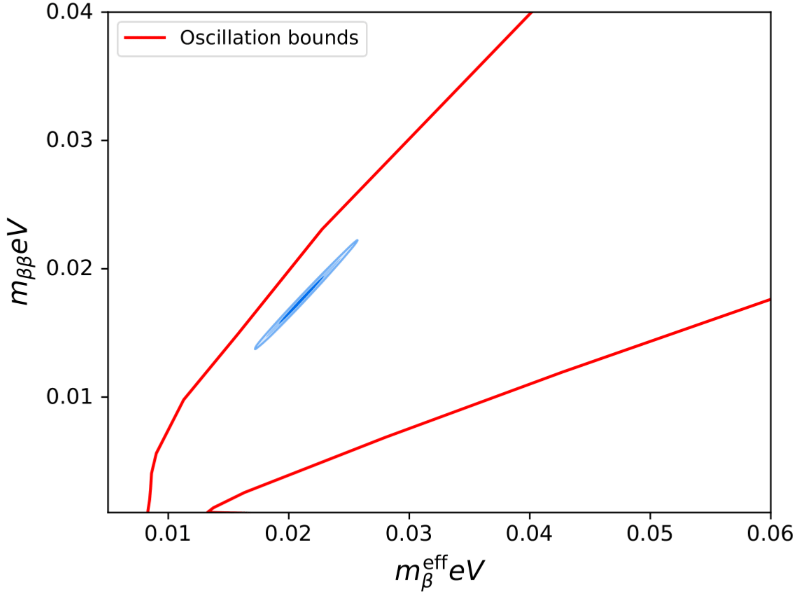
[b] Agostini et al., "Final Results of GERDA on the Search for Neutrinoless Double-β Decay"

[c] Abe et al., "Search for the Majorana Nature of Neutrinos in the Inverted Mass Ordering Region with KamLAND-Zen"

[4] Capozzi et al., "Addendum to "Global constraints on absolute neutrino masses and their ordering""

Non oscillation boundaries

Monte Carlo simulation, confronted with current bounds from oscillation experiments.



- Good fit within the oscillation bounds;
- m_β value consistent with limits from Katrin[a]:

$$m_\beta < 0.8 \text{ eV}$$

- $m_{\beta\beta}$ consistent with limits from Gerda[b];

$$m_{\beta\beta} < 0.079 - 0.180 \text{ eV}$$

[a] Aker et al., "Direct neutrino-mass measurement with sub-electronvolt sensitivity"

[b] Agostini et al., "Final Results of GERDA on the Search for Neutrinoless Double-β Decay"