Probing the structure of the $\chi_{c1}(3872)$ meson: heavy quark symmetries at work

Giuseppe Roselli

Università degli Studi di Bari
Dipartimento interateneo di Fisica "Michelangelo Merlin"

Based on: P.Colangelo, F. De Fazio, and G.Roselli

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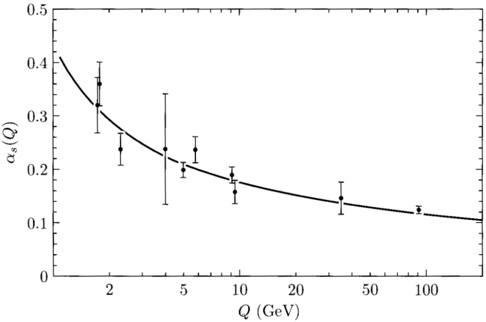
Summary

- Difficulties in the application of perturbative methods.
- 1973: Non-Abelian gauge theories $\Longrightarrow$ **asymptotic freedom**.
- Deep inelastic scattering $\Longrightarrow$ evidences of almost free elementary constituents in the nucleon.
- 1973: **QCD** $\Longrightarrow$ quarks.

Light quarks	**Heavy quarks**
up (u)	charm (c)
down (d)	bottom (b)
strange (s)	top (t)

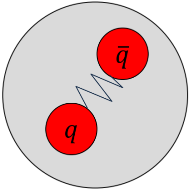
- Free quarks not observed $\Longrightarrow$ all hadronic states are color singlets. $\Longrightarrow$ confinement (non perturbative regime).
- Coupling constant is running:



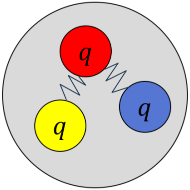
Quark Model

- Since 1964 hadronic states classified within the quark model:

  - $q\bar{q}$ (mesons)

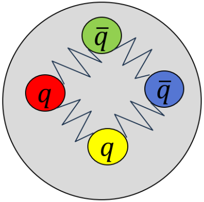


  - qqq (baryons)

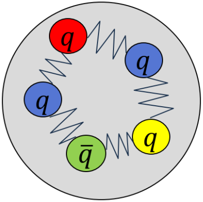


More complex structures, called **exotic** states, include:

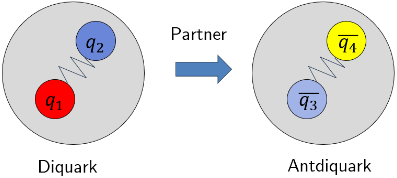
- $qq\bar{q}\bar{q}$ (tetraquarks)



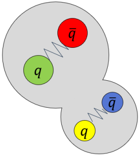
- $qqq\bar{q}q$ (pentaquarks)



- **Tetraquarks and pentaquarks are compact multiquark states:**
  - Quarks form *qq* pairs (*diquarks*), which combine with antidiquarks.



- **Molecular Picture:**
  - Loosely bound systems of two color-neutral objects.



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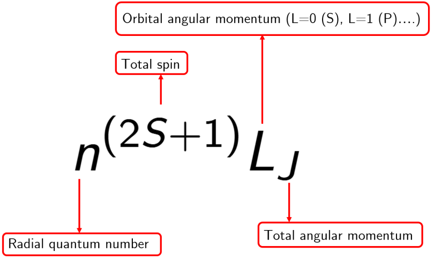
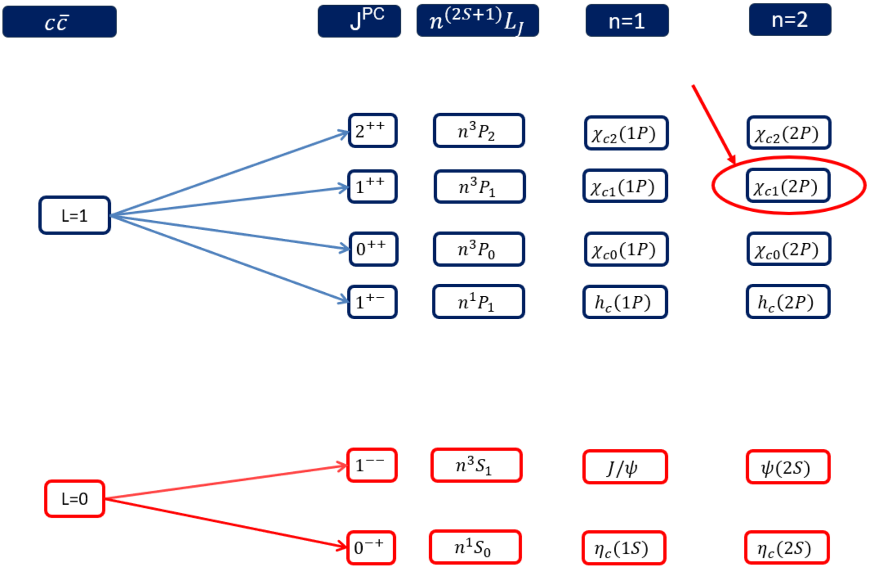
Spectroscopic notation

- Classification using the Spectroscopic notation:

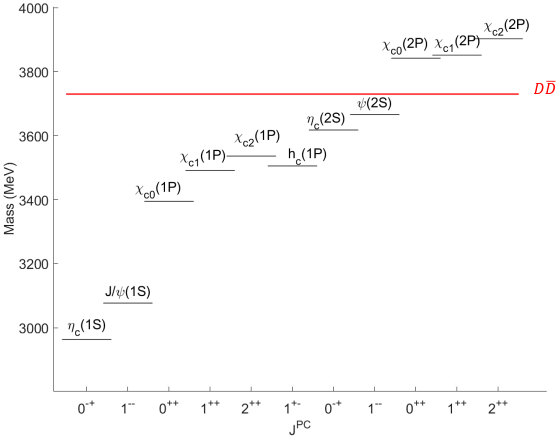


- For a meson $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \implies S = $ 0 or 1
- $P = (-1)^{L+1}$
- $C = (-1)^{L+S}$

Conventional states classification

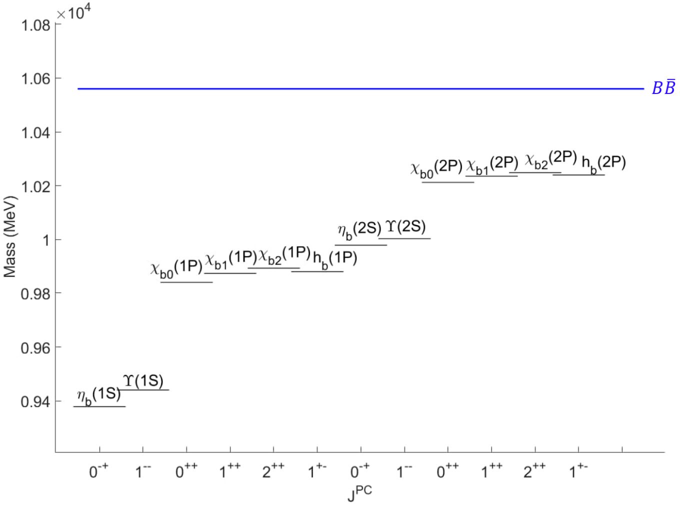


- Below the open-charm threshold, all states have been identified.
- Above this threshold, some conventional states remain unobserved, e.g., $2^1P_1 \rightarrow h_c(2P)$.



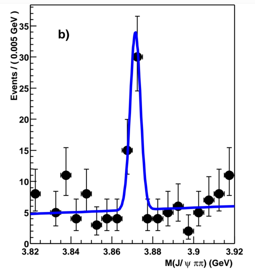
Bottomonium States

- Open-bottom threshold $M \approx 2m_B$ ($= 10560\,\text{MeV}$)



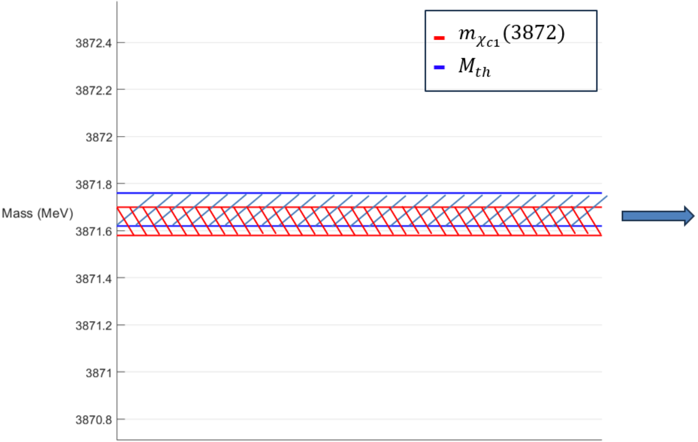
$\chi_{c1}(3872)$

- Since 2003: New observations challenge the quark model.
- My thesis: the case of $\chi_{c1}(3872)$.
- Observed by the Belle collaboration in the decay $B \to K\pi^+\pi^- J/\psi$ $\to$narrow peak in the $\pi^+\pi^- J/\psi$ invariant mass distribution.



- Confirmed by BaBar, BESIII, LHCb, CDF, D0, ATLAS, CMS, ALICE
- Fixed $J^{PC} = 1^{++}$.
- No isospin partners found.

- Ordinary charmonium interpretation: most likely $\chi_{c1}(2P)$
- $m_{\chi_{c1}(3872)} = 3871.64 \pm 0.06$ MeV is very close to the $D^0 \bar{D}^{0*}$ threshold: $M_{th} = m(D^0) + m(\bar{D}^{0*}) = 3871.69 \pm 0.07$ MeV.



MOLECULE?

- ...

Isospin Violation in $\chi_{c1}(3872)$ Decays

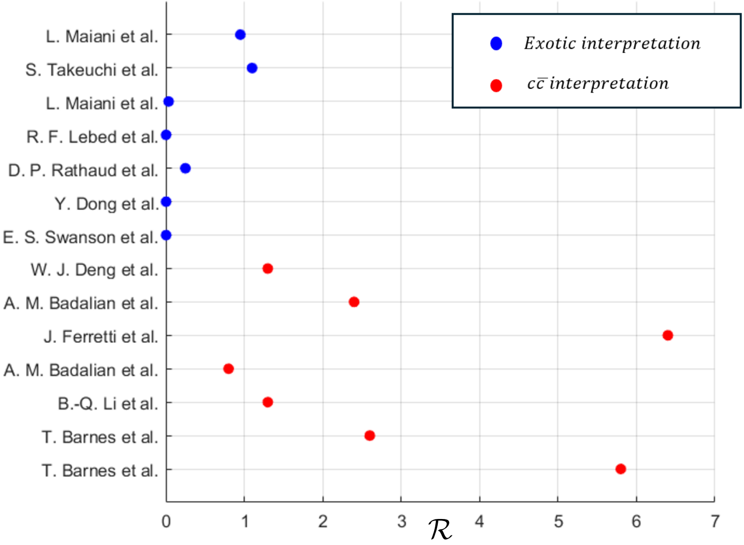
⋆ Isospin violation in decays: Non-conventional interpretation?

$$\frac{\mathcal{B}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 & \text{(Belle)}, \\ 0.8 \pm 0.3 \ (1.7 \pm 1.3) & \text{(BaBar)}, \\ 1.6^{+0.4}_{-0.3} \pm 0.2 & \text{(BESIII)}. \end{cases}$$

- χ_{c1} as ordinary charmonium ($I = 0$):
  - Decay into ω conserves isospin ($I(\omega) = 0$).
  - Decay into ρ violates isospin ($I(\rho) = 1$).
  - Large phase space suppression of ω mode vs ρ mode.
  - LHCb collaboration analysis: ratio of isospin-violating to isospin-conserving $\chi_{c1}(3872)$ couplings: 0.29 ± 0.04 (six times larger than in isospin-violating decays of $\psi(2S)$).
- Molecular interpretation proposed: not well-defined isospin.
  - ⋆ Small production cross section disfavours the molecular model (being similar to the one expected for ordinary charmonium)

Isospin violation in $\chi_{c1}(3872)$ decays remains unresolved.

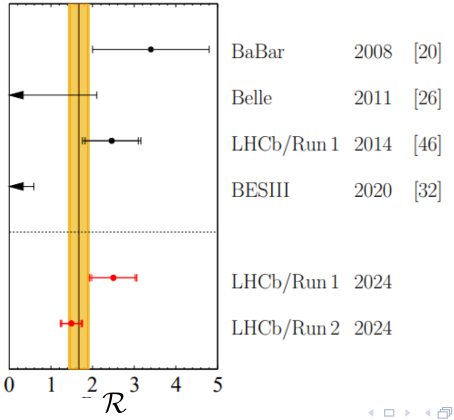
$\mathcal{R} \equiv \frac{\Gamma(\chi_{c1}(3872)\to\psi(2S)\gamma)}{\Gamma(\chi_{c1}(3872)\to J/\psi\gamma)}$ sensitive to the structure of $\chi_{c1}(3872)$



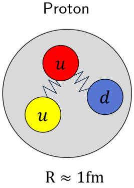
- Experimental results for the ratio $\mathcal{R}$:

LHCb Average 2024:

$$\mathcal{R} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04$$



- HQET: an effective theory of QCD valid in the large mass limit of heavy quarks (c, b).
- The hadron radius $R_{\text{had}} \approx 1\,\text{fm}$ corresponds to $\Lambda_{\text{QCD}} \approx 200\,\text{MeV}$,



- For a heavy quark:

$$m_Q \gg \Lambda_{\text{QCD}} \implies \lambda_Q \ll R_{\text{had}}$$

- HQ acts like a static source of color field $\implies$ New symmetries arise.
- Remaining degrees of freedom have no sensitivity to HQ flavour or spin.

- QCD Lagrangian for a heavy quark:

$$\mathcal{L}_Q = \bar{Q}(i\not{D} - m_Q)Q,$$

- HQ nearly on-shell ($v^2 = 1$):

$$P_Q^\mu = m_Q v^\mu + k^\mu,$$

where $k \approx \mathcal{O}(\Lambda_{\mathrm{QCD}})$.

- $Q(x)$ is redefined as:

$$Q(x) = e^{-im_Q v\cdot x}[h_v(x) + H_v(x)],$$

$h_v(x) \rightarrow e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x)$, $H_v(x) \rightarrow e^{im_Q v\cdot x}\frac{1-\not{v}}{2}Q(x)$

- $H_v(x)$ integrated out (EOM).

- The effective Lagrangian becomes:

$$\mathcal{L}_{\text{eff}} = \underbrace{\bar{h}_v iv\cdot D h_v}_{\mathcal{L}_{\text{HQET}}} + \underbrace{\frac{1}{2m_Q}\boxed{\bar{h}_v (iD_\perp)^2 h_v} + \frac{1}{2m_Q}\boxed{g\bar{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v}}_{\frac{\mathcal{L}_1}{2m_Q}} + \mathcal{O}\left(\frac{1}{m_Q^2}\right),$$

KINETIC OPERATOR

O_π Breaks flavour simmetry

CHROMOMAGNETIC OPERATOR

O_G Breaks flavour simmetry and HQ spin simmetry

$\mathcal{L}_{\text{HQET}}$ →

No Dirac matrices $\Longrightarrow$ HQ Spin invariance.

No m_Q $\Longrightarrow$ Flavour invariance.

Heavy-Light Multiplets

- Due to these new symmetries, hadrons, which differ only for the orientation of SQ, can be collected in multiplets
- $\vec{J} = \vec{S} + \vec{L}$, for mesons: $S =$ 0 / 1
- e.g. with $L = 0$:
  - $J = (0, 1) \implies$ Doublet: $J^P = (0^-, 1^-)$,
  - Denoted as:

$$H_a = \frac{1 + \not{v}}{2} \left[P^*_{a\mu} \gamma^\mu - P_a \gamma_5 \right] .$$

$P^*_{a\mu}\gamma^\mu$ → 1^-, e.g D*

$P_a\gamma_5$ → 0^-, D

- Identify states obtained rotating the spin of both HQs.
- The number of states in a multiplet depends on L.

- L=1 multiplet (P-wave):

$$J^\mu = \frac{1+\not{v}}{2}\Big\{ \underbrace{H_2^{\mu\alpha}\gamma_\alpha}_{\chi_{Q2}} + \underbrace{\frac{1}{\sqrt{2}}\epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta H_{1\gamma}}_{\chi_{Q1}} + \underbrace{\frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu) H_0}_{\chi_{Q0}} + \underbrace{K_1^\mu \gamma_5}_{h_Q} \Big\} \frac{1-\not{v}}{2} \; ;$$

- L=0 multiplet (S-wave):

$$J = \frac{1+\not{v}}{2} \left[\underbrace{H_1^\mu \gamma_\mu}_{\psi(\Upsilon)} - \underbrace{H_0 \gamma_5}_{\eta_Q} \right] \frac{1-\not{v}}{2} \; .$$

Effective Lagrangians can be constructed in terms of the just presented multiplets:

- Radiative $nP \to mS$ transitions are described by:

$$\mathcal{L}_{n_P \leftrightarrow m_S} = \delta_Q^{n_P m_S} \mathrm{Tr}\left[\bar{J}(m_S) J_\mu(n_P)\right] v_\nu F^{\mu\nu} + \text{h.c.} \; .$$

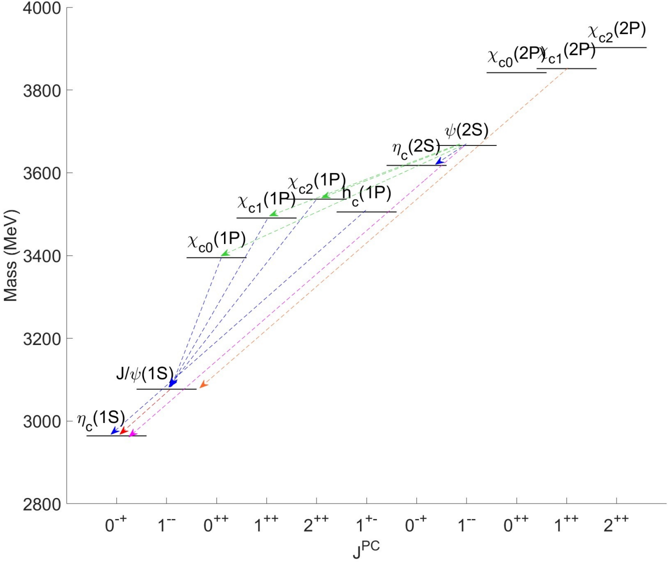
Hadron velocity

Electromagnetic field tensor:
$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

- Invariant under the symmetries:
    - C, P, T
    - $SU(2)_{HQ}$

δ_Q^{nPmS} describes **all** the transitions among the members of the nP multiplet to the mS one.

Considered radiative decays



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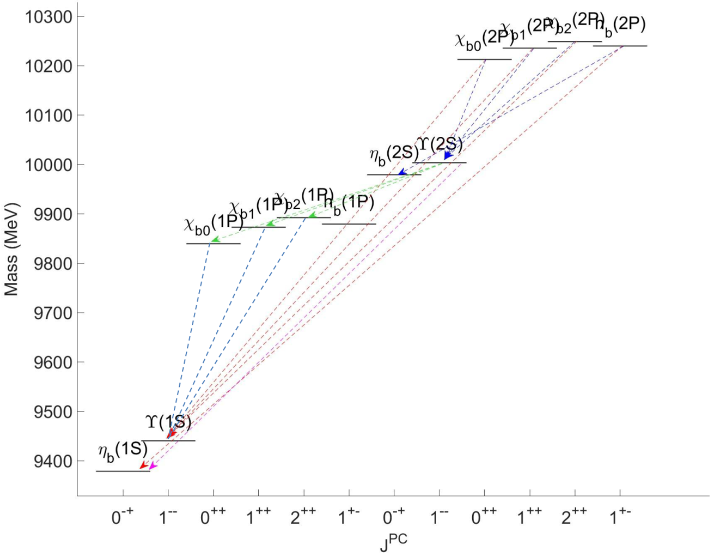

Test HQ limit

I analyze the radiative decays:

- $(\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P), h_c(1P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \to (\eta_c(2S), \psi(2S))$,
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \to (\eta_c(2S), \psi(2S))$.

Coupling δ_Q^{nPmS} determined comparing HQ predictions to experimental data:

Results

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \to n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

Process	Branching ratio (Γ_i/Γ)
$J/\psi \to \gamma\eta_c(1S)$	$(1.41 \pm 0.14)\%$
$\chi_{c0}(1P) \to \gamma J/\psi$	$(1.41 \pm 0.09)\%$
$\chi_{c1}(1P) \to \gamma J/\psi$	$(34.3 \pm 1.3)\%$
$\chi_{c2}(1P) \to \gamma J/\psi$	$(19.5 \pm 0.8)\%$
$h_c(1P) \to \gamma\eta_c(1S)$	$(60 \pm 4)\%$
$\eta_c(2S) \to \gamma J/\psi$	$< 1.4\%$
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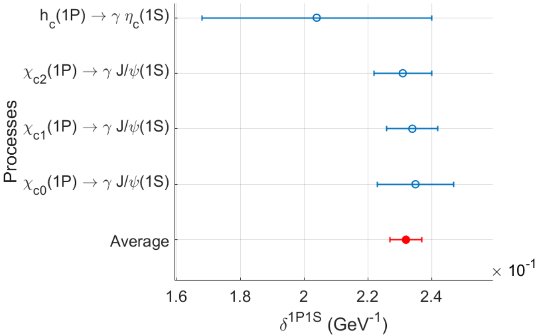
$$\delta_c^{1P1S} = (2.00 \pm 0.40) \times 10^{-1}\,\mathrm{GeV}^{-1}$$



Average: $\delta_c^{1P1S} = (0.232 \pm 0.005)\ \text{GeV}^{-1}$.

- Results fulfill the expectations based on HQ symmetries.

My thesis

Goal: calculate ratio $\mathcal{R} \equiv \frac{\Gamma(\chi_{c1}(3872)\to\psi(2S)\gamma)}{\Gamma(\chi_{c1}(3872)\to J/\psi\gamma)}$

Identifying $\chi_{c1}(3872) \equiv \chi_{c1}(2P)$

I analyze the radiative decays:

- $(\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P), h_c(1P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \to (\eta_c(2S), \psi(2S))$,
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \to (\eta_c(1S), J/\psi)$,
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \to (\eta_c(2S), \psi(2S))$.

Coupling δ_Q^{nPmS} determined comparing HQ predictions to experimental data:

Bottom Sector Predictions

- Predictions limited by incomplete knowledge of total widths for bottomonia states.
- Introducing:

$$\tilde{\delta}_b^{nPmS}(P_b) = \frac{\delta_b^{nPmS}}{[\Gamma_{\rm tot}(P_b)]^{1/2}}, \qquad P_b = \chi_{Q0}, \chi_{Q1}, \chi_{Q2}, h_Q$$

and

$$R_b^{\delta}(P_b) \equiv \frac{\delta_b^{2P2S}(P_b)}{\delta_b^{2P1S}(P_b)} = \frac{\tilde{\delta}_b^{2P2S}(P_b)}{\tilde{\delta}_b^{2P1S}(P_b)}$$

Bottom production section

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \to n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

Process	Branching ratio (Γ_i/Γ)
$\chi_{b0}(2P) \to \gamma\Upsilon(1S)$	$(3.8 \pm 1.7) \times 10^{-3}$
$\chi_{b0}(2P) \to \gamma\Upsilon(2S)$	$(1.38 \pm 0.30)\%$
$\chi_{b1}(2P) \to \gamma\Upsilon(1S)$	$(9.9 \pm 1.0)\%$
$\chi_{b1}(2P) \to \gamma\Upsilon(2S)$	$(18.1 \pm 1.9)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \to \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \to \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b0}(2P)) = 13 \pm 4$$

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

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$\chi_{b1}(2P) \to \gamma\Upsilon(2S)$	$(18.1 \pm 1.9)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \to \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \to \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b1}(2P)) = 8 \pm 1$$

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

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$\chi_{b2}(2P) \to \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \to \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \to \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b2}(2P)) = 7 \pm 1$$

Bottom production section

$$\Gamma(n^3P_J \to m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \to n^3P_J\gamma) = (2J+1)\frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \to m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \to n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

Process	Branching ratio (Γ_i/Γ)
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$\chi_{b2}(2P) \to \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \to \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \to \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \to \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(h_b(2P)) = 8 \pm 2$$

- Average result:

$$R_b^\delta = 9.0 \pm 0.7$$

Differences between $Q\bar{Q}$ and $Q\bar{q}$ symmetries:

- ✓ spin symmetry
- ✗ flavour symmetry $\Longrightarrow$ $\frac{1}{m_Q}$ suppression $\Longrightarrow$ cancels in ratios $\Longrightarrow$ $R_c^\delta \approx R_b^\delta$.

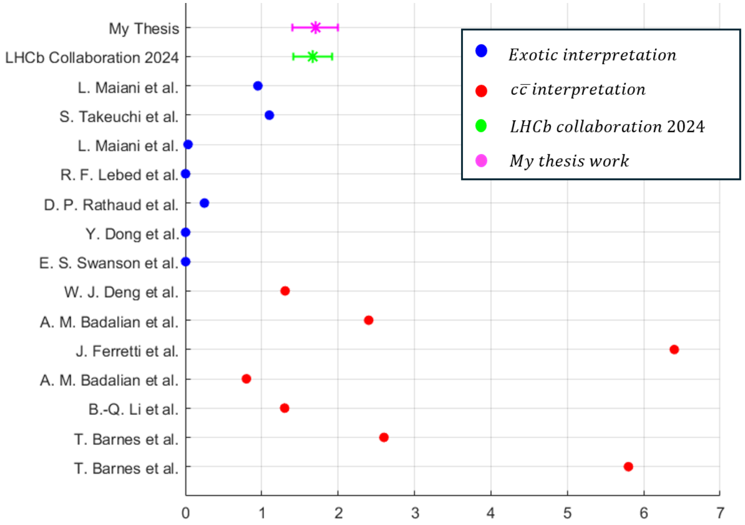
- My Prediction for the ratio:

$$\mathcal{R}(\chi_{c1}(3872)) = \frac{\mathcal{B}(\chi_{c1}(3872) \to \psi(2S)\gamma)}{\mathcal{B}(\chi_{c1}(3872) \to J/\psi\gamma)} \propto (R_c^\delta)^2$$

$$\boxed{\mathcal{R}(\chi_{c1}(3872)) = 1.7 \pm 0.3}$$

P.Colangelo, F. De Fazio, and G. Roselli
BARI-TH/770-24, in preparation (2024)

Results



My Thesis
LHCb Collaboration 2024
L. Maiani et al.
S. Takeuchi et al.
L. Maiani et al.
R. F. Lebed et al.
D. P. Rathaud et al.
Y. Dong et al.
E. S. Swanson et al.
W. J. Deng et al.
A. M. Badalian et al.
J. Ferretti et al.
A. M. Badalian et al.
B.-Q. Li et al.
T. Barnes et al.
T. Barnes et al.
0
1
2
3
4
5
6
7
Exotic interpretation
$c\bar{c}$ interpretation
LHCb collaboration 2024
My thesis work

Other predictions

The predicted δ allows to find several other observables e.g.:

- Branching ratios:
  - $\mathcal{B}(\chi_{c2}(2P) \to J/\psi\gamma) = (3.1 \pm 1.5)\, 10^{-4}$
  - $\mathcal{B}(\chi_{c1}(3872) \to \psi(2S)\gamma) = (9.0 \pm 3.4) \times 10^{-3}$.
- The analogous $\mathcal{R}$ for $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$:
  - $\mathcal{R}(\chi_{c0}(2P)) = 1.5 \pm 0.2$
  - $\mathcal{R}(\chi_{c2}(2P)) = 2.9 \pm 0.4$

Conclusions

- Large HQ mass limit: HQ spin and HQ flavour symmetries.
- $\chi_{c1}(3872)$:
  - many observables, many puzzles, many interpretations.
- Under the assumption $\boxed{\chi_{c1}(3872) \equiv \chi_{c1}(2P)} \implies$ the prediction follows:

$$\mathcal{R} = \frac{\mathcal{B}(\chi_{c1}(3872) \to \psi(2S)\gamma)}{\mathcal{B}(\chi_{c1}(3872) \to J/\psi\gamma)} = 1.7 \pm 0.3$$

- Experimental LHCb 2024 average:

$$\mathcal{R} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04.$$

This agreement supports the assumed identification.

Implications and Future Questions

- Open questions:
  - Size of isospin-breaking in $\chi_{c1}(3872)$ decays.
  - Understanding production cross sections.
- Observation of missing conventional states preliminary and necessary step.
- Predictions for unobserved radiative decays to be compared to experimental data still to come $\rightarrow$further test of the method and the results.

Thank You!

Other predictions

Further predictions are possible by constructing an nSmS interaction Lagrangian:

- from the decay $J/\psi \to \eta_c(1S)\gamma$:
  $\delta_c^{1S1S} = 0.0485 \pm 0.0024 \text{ GeV}^{-1}$
- from the decay $\psi(2S) \to \eta_c(1S)\gamma$:
  $\delta_c^{2S1S} = (3.42 \pm 0.26) \times 10^{-3} \text{ GeV}^{-1}$
- from the decay $\psi(2S) \to \eta_c(2S)\gamma$:
  $\delta_c^{2S2S} = (0.66 \pm 0.24) \times 10^{-1} \text{ GeV}^{-1}$
- from the decay $\Upsilon(2S) \to \eta_b(1S)\gamma$:
  $\delta_b^{2S1S} = (4.50 \pm 0.05) \times 10^{-4} \text{ GeV}^{-1}$.

The large number of predictions confirms the important role of the effective theory approach.

Mass splitting

- Degeneracy among spin multiplet members is broken by the chromomagnetic operator.
- Effective Lagrangian for spin symmetry breaking:

$$\mathcal{L}^{SpSB} = \frac{\lambda_J}{2m_Q}\frac{1}{\mathcal{N}}\mathrm{Tr}[\bar{J}_{\mu_1\ldots\mu_n}\sigma_{\alpha\beta}J_{\nu_1\ldots\nu_n}\Phi^{\mu_1\ldots\mu_n\alpha\beta\nu_1\ldots\nu_n}]$$

  with normalization $\mathcal{N} = \mathrm{Tr}[\bar{J}_{\mu_1\ldots\mu_n}J^{\mu_1\cdots\mu_n}]$.
- Function Φ ensures invariance under P, C, T and satisfies antisymmetry of $\sigma_{\alpha\beta}$.
- For S-wave states, hyperfine splitting arises:

$$\mathcal{L}_S^{SpSB} = \frac{\lambda_S}{2m_Q}\frac{1}{\mathcal{N}}\mathrm{Tr}[\bar{J}\sigma_{\alpha\beta}J\sigma^{\alpha\beta}]$$

  with spin, parity, and charge quantum numbers $J^{PC} = (0^{-+}, 1^{--})$.
- Mass formula for S-wave states:

$$m_H = m_Q + \bar{\Lambda}_S + \frac{-\lambda_1 + d_H\lambda_S}{2m_Q}$$

Mass splitting

- For P-wave states:

$$\mathcal{L}_P^{SpSB} = \frac{\lambda_P}{2m_Q}\frac{1}{\mathcal{N}}\mathrm{Tr}[\bar{J}_\mu \sigma_{\alpha\beta} J_\nu \Phi^{\alpha\beta\mu\nu}]$$

- P-wave multiplet states $J^{PC} = (0^{++}, 1^{++}, 2^{++}, 1^{+-})$:

$$m_H = m_Q + \bar{\Lambda}_P + \frac{-\lambda_1}{2m_Q} + \frac{d_H^{(1)}\lambda_P^{(1)} + d_H^{(2)}\lambda_P^{(2)} + d_H^{(3)}\lambda_P^{(3)}}{2m_Q}$$

- Parameters for mass splitting:

$$\lambda_P^{(1)} = \frac{1}{24}(-m_0^2 + 3m_1^2 + m_2^2 - 3m_h^2), \quad \lambda_P^{(2)} = \frac{1}{24}(-2m_0^2 - 3m_1^2 + 5m_2^2),$$

$$\lambda_P^{(3)} = \frac{1}{24}(-2m_0^2 + 3m_1^2 - m_2^2)$$

where m_0, m_1, m_2, m_h are masses of states in the multiplet.

- Recent discoveries of exotic hadrons include:
  - $P_c(4380)^+$, $P_c(4440)^+$, $P_c(4457)^+$, and $P_c(4312)^+$, observed in $\Lambda_b^0 \to J/\psi p K^-$ decays.
  - $X(6900)$ in $J/\psi J/\psi$ final states.
  - $X_0(2900)$ and $X_1(2900)$ in $B^+ \to D^+ D^- K^+$.
  - $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ in $B^+ \to J/\psi \phi K^+$.
  - $T_{cc}(3875)^+$ in $D^0 D^0 \pi^+$.

Transformations under C, P and T

Multiplets transform under the discrete symmetries C, P, and T:

$$J^{\mu_1 \dots \mu_k} \xrightarrow{P} \gamma^0 J_{\mu_1 \dots \mu_k} \gamma^0$$

$$J^{\mu_1 \dots \mu_k} \xrightarrow{C} \hat{C}(J^{\mu_1 \dots \mu_k})^T \hat{C}^\dagger (-1)^L$$

$$J^{\mu_1 \dots \mu_k} \xrightarrow{T} -\hat{T} J_{\mu_1 \dots \mu_k} \hat{T}^{-1}$$

$\hat{C} = i\gamma^2\gamma^0$ and $T = i\gamma^1\gamma^3$.
The electromagnetic field strength tensor transforms as:

$$F^{\mu\nu} \xrightarrow{P} F_{\mu\nu}, \quad F^{\mu\nu} \xrightarrow{C} -(F^{\mu\nu}), \quad F^{\mu\nu} \xrightarrow{T} -F_{\mu\nu}.$$