Primordial Graviton Production and Detection Prospects in the Pre-Big Bang Scenario

Speaker:
Eliseo Pavone

Co-authors:
P.Conzinu, G.Fanizza, M.Gasperini, L.Tedesco, G.Veneziano

Bari Theory Xmas Workshop 2024
17/12/2024

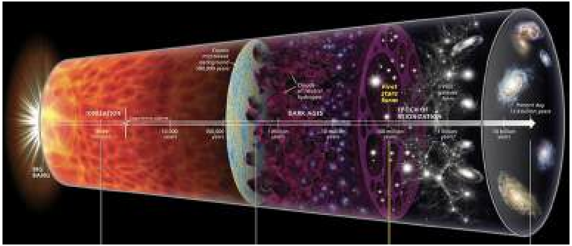
Outline

Introduction

Topics

- Exploring the **pre-Big Bang (pBB) String Cosmology** scenario as an alternative to standard inflationary cosmology.
- Investigating **primordial gravitational waves (pGWs)** as potential observational signatures of the pBB scenario.
- Can the IPTA signal be a signature of pBB evolution? Possibly.

A lightning fast review of the history of our universe



The electromagnetic "wall", the CMB.
Can we have information about earlier epochs?

- **CMB anisotropies.**
- **Primordial Gravitational Wave Background.**
- Other (PBH, Neutrinos, ...).

The metric of the universe (FLRW)

$$ds^2 = a(\tau)^2(d\tau^2 - |d\vec{x}|^2)$$

Cosmological epochs:

- Dark energy $a \sim -\frac{1}{\tau}$.
- Matter domination $a \sim (\tau)^2$.
- Radiation domination $a \sim \tau$.
- **Big Bang: The beginning of the standard radiation phase**.
- Primordial cosmology. Slow-Roll inflation? **Pre-Big-Bang**? Other?

String Effective Action in Cosmology

Bosonic and massless String Tree-Level effective Action:

$$S_0 = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|}\, e^{-\phi} \left[R + \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \mathcal{O}(\alpha') + \mathcal{O}(g_s^2) \right]$$

- ϕ: Dilaton field.
- R: Ricci's scalar.
- $H = dB_2 \to H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]} +$ cyclic: Field strength of the Kalb-Ramond antisymmetric tensor $B_{\mu\nu}$.
- α': Corrections for finite nature of Strings (Curvature corrections).
- g_s^2: String coupling corrections, $g_s^2 = e^{\phi}$.
- In $d = 3$ the Kalb-Ramond field strength has a dual axionic form $H = \star d\sigma e^{\phi} \to H_{\mu\nu\lambda} = \sqrt{-g}\, e^{\phi} \epsilon_{\mu\nu\lambda\rho} \partial^\rho \sigma$.

String Dualities in Cosmological Backgrounds

T-Duality $(R \to \frac{\alpha'}{R})$

$$S = -\frac{1}{2\lambda_s^{d-1}} \int dt\, e^{-\bar{\phi}} \left[\dot{\bar{\phi}}^2 + \frac{1}{8}\mathrm{Tr}\left((\dot{M}_0\eta)^2\right)\right]$$

$$M_0 \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

- $G \equiv g_{ij}(t)$, $B \equiv B_{ij}(t)$ are $d \times d$ matrices.
- Shifted Dilaton $\bar{\phi} = \phi - \frac{1}{2}\ln|G|$.

Global $O(d,d)$ Symmetry

- $\Omega \in O(d,d)$, $\Omega^T\eta\Omega = \eta$, $\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
- Invariant under $M_0 \to \Omega^T M_0 \Omega$, $\bar{\phi} \to \bar{\phi}$
  If $B_{ij} = 0$, then $O(d,d) \to Z_2^d$, namely $g_{ij} \to \frac{1}{g_{ij}}$.

Possibly broken by non-perturbative effects at high string couplings

S-Duality $(g_s \to \frac{1}{g_s})$

$$S = -\frac{1}{2\lambda_s^2} \int d^4x \sqrt{-g}e^{-\tilde{\phi}} \left[R + \frac{1}{4}\mathrm{Tr}\left(\nabla N \nabla N^{-1}\right) + \frac{1}{4}(\nabla_\mu G_{mn})^2\right]$$

$$N \equiv \begin{pmatrix} e^{\tilde{\phi}} & \sigma e^{\tilde{\phi}} \\ \sigma e^{\tilde{\phi}} & e^{-\tilde{\phi}} + \sigma^2 e^{\tilde{\phi}} \end{pmatrix}$$

- $\tilde{\phi} = \phi - \frac{1}{2}\ln|G_{mn}|$, dimensionally reduced dilaton, $G_{mn}(x)$: Internal dimensions metric.

Global $SL(2,R)$ Symmetry

- $\Theta \in SL(2,R)$, $\Theta^{-1} = -J\Theta^T J$, $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
- Invariant under $N \to \Theta N \Theta^T$.
  For a particular Θ, $\tilde{\phi} \to -\tilde{\phi}$, namely $g_4^s \to 1/g_4^s$.

Weak-Strong coupling duality, possibly broken by α' corrections

Pre-Big Bang Scenario

Key Features:

- **Asymptotic Past Triviality (APT):**
  - Universe emerges from the perturbative vacuum of String Theory as $\tau \to -\infty$.
  - Vanishing string coupling ($g_s^2 = e^{\phi} \to 0$) and curvature ($H \to 0$). Both grow in time. Axion VEV $\sigma = 0$.
- **Dual Phases:**
  - *Pre-Big Bang Phase:* Accelerated expansion with increasing curvature and string coupling.
  - *Post-Big Bang Phase:* Standard cosmological evolution with decreasing curvature.

Bianchi I metric:

$$ds^2 = dt^2 - a^2(t)|d\vec{x}|^2 - \sum_i b_i^2(t)dy_i^2, \quad i = 4, \dots, d$$

Phases of Evolution (in conformal time):

1. **Dilaton-Driven Phase** $(\tau < -\tau_s)$ $\qquad a \sim (-\tau)^{\frac{\beta_0}{1-\beta_0}}$
2. **String Phase** $(-\tau_s < \tau < -\tau_1)$ α' correction important, broken S-duality? $\qquad a \sim (-\tau)^{-1}$
3. **Radiation-Dominated Phase** $(-\tau_1 < \tau < \tau_\sigma)$ Axion VEV, $\sigma_i \neq 0$, $\phi = \phi_0$ $\qquad a \sim \tau$
4. **Axion-Dominated Phase** $(\tau_\sigma < \tau < \tau_d)$ Reheating by $U(1)$ coupling with Axion $\qquad a \sim \tau^2$

Generation of Primordial Gravitational Waves and CMB anisotropies

Separate the classical geometric and field background and the quantum fluctuations (QFT in curved space-time)

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \sigma = \bar{\sigma} + \delta\sigma = \delta\sigma$$

Equation of Motion for quantum fluctuations

Tensorial fluctuations, Primordial GW

$$v_h'' + \left(k^2 - \frac{\xi_h''}{\xi_h}\right) v_i = 0$$

- $v_h = h_k \xi_h$: Canonical Mukhanov-Sasaki variable, h_k is the graviton field amplitude at fixed wavenumber.

Axion fluctuations, CMB anisotropies

$$v_\sigma'' + \left(k^2 - \frac{\xi_\sigma''}{\xi_\sigma}\right) = 0$$

- $v_\sigma = \delta\sigma_k \xi_\sigma$: Canonical Mukhanov-Sasaki variable, $\delta\sigma_k$ is the axion field amplitude at fixed wavenumber.

The initial quantum state is chosen to be the Bunch-Davies vacuum state $v_i \sim \frac{1}{\sqrt{2k}} e^{ik\tau}$ as $\tau \to -\infty$.

- $\xi_i(\tau)$: Pump field, related to the background dynamics, the 4 dimensional string coupling dynamics and the amplified perturbation

Amplification Mechanism:

- Quantum vacuum fluctuations are amplified and they freeze when they exit the horizon ($k^2 \ll \xi_i''/\xi_i$).

The Pump Field

The Pump Fields

$$\xi_h(\tau) \sim a g_4^{-1} \times \alpha' \times S\text{−Breaking} \qquad\qquad \xi_\sigma(\tau) \sim a g_4 \times \alpha' \times S\text{−Breaking}$$

- $g_4 \sim \prod_{i=1}^{6} b_i^{-1/2} e^{\phi/2} \sim (-\tau)^{-\beta}$, $4d$ effective string coupling, $\beta > 0$ to have a growing effective string coupling.

Behavior of ξ_i and tensor and axion power spectra at horizon crossing:

Phase	**Dilaton-Driven Phase** $(\tau < \tau_s,\ k < k_s)$	**String Phase** $(\tau_s < \tau < \tau_1,\ k_s < k < k_1)$
$\xi_h(\tau)$	$(-\tau)^{\frac{1}{2}}$	$(-\tau)^{-1+\beta+\gamma} = (-\tau)^{-1+\beta_h}$
$\xi_\sigma(\tau)$	$(-\tau)^{\frac{5\beta_0-1}{2(1-\beta_0}}$	$(-\tau)^{-1-\beta+\delta} = (-\tau)^{-1+\beta_\sigma}$
$P_h(k)$	$\sim k^3$	$\sim k^{3-\lvert 3-2\beta_h\rvert}$
$P_\sigma(k)$	$\sim k^{3-2\left\lvert\frac{3\beta_0-1}{1-\beta_0}\right\rvert} = k^{n_s-1}$	$\sim k^{3-\lvert 3-2\beta_\sigma\rvert}$

- We parametrize phenomenologically α' corrections and other higher order effects with γ and δ.
- Minimal scenario $\gamma = \delta = 0$, unbroken S-duality and no higher order corrections.
- S-duality is preserved if $\beta_\sigma = -\beta_h$, $\epsilon \equiv \gamma + \delta = 0$.

Gravitational Wave Energy Density Spectrum

Energy Density per Logarithmic Frequency Interval:

$$\Omega_{\mathsf{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\mathsf{GW}}}{d\ln f}$$

Full Broken Power-Law Spectrum:

- For $f_\sigma \leq f \leq f_1$: $\Omega_{\mathsf{GW}}(f) = \Omega_{\mathrm{PBB}} \left(\frac{f}{f_1}\right)^{3-|3-2\beta_h|}$
- For $f_d \leq f \leq f_\sigma$: $\Omega_{\mathsf{GW}}(f) = \Omega_{\mathsf{GW}}(f_1) \left(\frac{f_\sigma}{f_1}\right)^{3-|3-2\beta_h|} \left(\frac{f}{f_\sigma}\right)^{1-|3-2\beta_h|}$
- For $f_s \leq f \leq f_d$: $\Omega_{\mathsf{GW}}(f) = \Omega_{\mathsf{GW}}(f_\sigma) \left(\frac{f_d}{f_\sigma}\right)^{1-|3-2\beta_h|} \left(\frac{f}{f_d}\right)^{3-|3-2\beta_h|}$
- For $f \leq f_s$: $\Omega_{\mathsf{GW}}(f) = \Omega_{\mathsf{GW}}(f_d) \left(\frac{f_s}{f_d}\right)^{3-|3-2\beta_h|} \left(\frac{f}{f_s}\right)^{3}$

Amplitude:

$$\Omega_{\mathrm{PBB}} = \Omega_{\mathrm{r0}} \left(\frac{H_1}{M_{\mathrm{Pl}}}\right)^2 \left(\frac{m M_{\mathrm{Pl}}}{\sigma_i^2}\right)^{4/3}$$

ρ_c: Critical energy density today

H_1: Hubble parameter at the bounce $\sim M_s$

m:= Axion mass

Ω_{r0}: Present radiation energy density parameter

β_h: Controls the tensorial pump field in the string phase

Spectrum Parameters and Interpretation

Parameters:

- β_h: Controls the pump field during the string phase.
- z_s, z_σ, z_d: Dimensionless parameters defining the transition scales. $z_i \equiv \tau_i/\tau_1 = f_1/f_i$, they are functions of the parameters of the theory $\{\sigma_i, m, H_1\}$. They encode the duration of each cosmological phase.
- H_1: Hubble parameter at the bounce. It is fixed as a function of the other parameters and β_σ to have the production of CMB anisotropies sourced by the axion fluctuations.

Interpretation of Frequency Bands:

- **High-Frequency Regime** ($f_\sigma \leq f \leq f_1$): Modes exit during string phase and renter in the first radiation phase.
- **Intermediate Regimes**:
  - $f_d \leq f \leq f_\sigma$: Modes re-enter during axion-dominated phase.
  - $f_s \leq f \leq f_d$: Modes re-enter during standard radiation phase.
- **Low-Frequency Regime** ($f \leq f_s$): Modes exit during dilaton-driven phase; spectrum rises as $\Omega_{\mathrm{GW}}(f) \sim f^3$.

Theoretical Constraints on Parameters in the minimal scenario

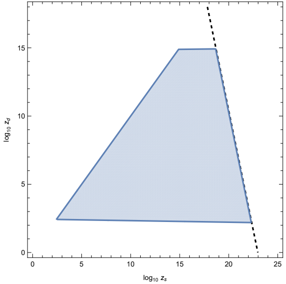


Figure: Allowed region in the $(\log_{10} z_s, \log_{10} z_d)$ for $\beta = 0$ and $\sigma_i = M_{\text{Pl}}$

Theoretical constraints on $\{\beta, z_s, z_d, z_\sigma\}$, minimal scenario $(\gamma, \beta = 0)$:

- β: Increasing dilaton $0 \leq \beta < 3$
- Hierarchy of transition frequency:

$$1 \lesssim z_\sigma < z_d < z_s$$

- Curvaton mechanism $(H_m \geq H_\sigma)$:

$$\log_{10}\left(\frac{H_1}{M_{\text{Pl}}}\right) + \frac{3}{2}\log_{10} z_d - \frac{7}{2}\log_{10} z_\sigma \leq 0$$

- Reheating before BBN $H_d > H_{\text{BBN}}$: $\log_{10}\left(\frac{H_1}{M_{\text{Pl}}}\right) - 3\log_{10} z_d + \log_{10} z_\sigma > -42 - \log_{10} 4$
- CMB normalisation $P_s(f_*) = 2.1 \times 10^{-9}$, involves β_σ.

Spectral Features

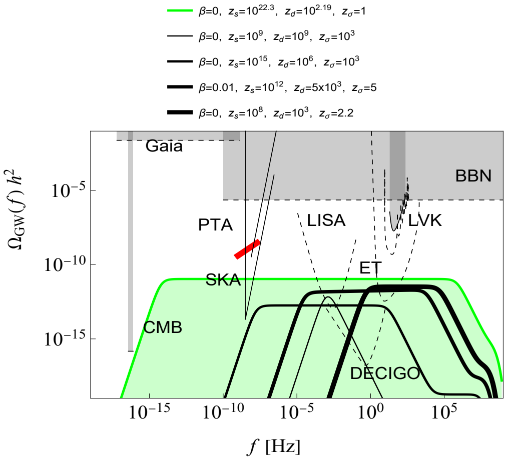


Figure: Maximal allowed spectrum in the minimal pBB scenario. *Pavone et al., JCAP* **09**, *058 (2024)*

Detection Prospects

Maximum Amplitude:

$$\Omega_{\mathsf{GW}}^{\mathsf{max}} = \Omega_{\mathrm{r0}} \left(\frac{H_1}{M_{\mathrm{Pl}}} \right)^2 \approx 10^{-10.6}$$

For $\beta = 0$, $\sigma_i = M_{\mathsf{Pl}}$, $\log_{10}(H_1/M_{\mathsf{Pl}}) \approx -3.29$.

Axion mass range:

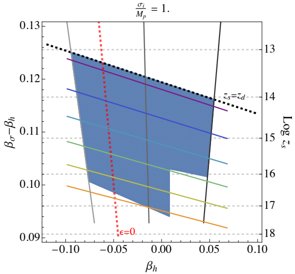
$4 \times 10^{-4} \lesssim \frac{m}{M_{\mathsf{Pl}}} \lesssim 2 \times 10^{-2}$.

Prospects:

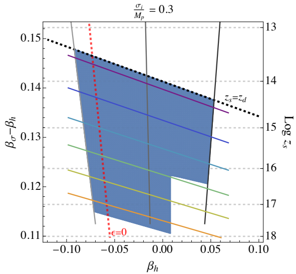
- The predicted amplitude lies within the sensitivity of future detectors like **LISA**, **Einstein Telescope (ET)**, and **DECIGO**.
- Distinct spectral features allow for **multiband detection**, providing a unique signature of the pBB scenario.

What can be said for the more general non-minimal scenario?

The parameter space in the non-minimal scenario



Log $\frac{H_1}{M_p}$	Log $\frac{m}{M_p}$	Log $\frac{z_s}{z_\sigma}$	Log $\frac{z_s}{z_d}$	Log $\frac{z_d}{z_\sigma}$
−1	−12	8.7	0.7	8.0
−2	−10	10.4	3.7	6.7
−3	−8	12.0	6.7	5.3
−2.60	−6	13.7	9.7	4.0
	−4	15.4	12.7	2.7
	−2	17.0	15.7	1.3



Log $\frac{H_1}{M_p}$	Log $\frac{m}{M_p}$	Log $\frac{z_s}{z_\sigma}$	Log $\frac{z_s}{z_d}$	Log $\frac{z_d}{z_\sigma}$
−1	−12	8.0	0.7	7.3
−2	−10	9.7	3.7	6.0
−3	−8	11.4	6.7	4.7
−2.80	−6	13.0	9.7	3.3
	−4	14.7	12.7	2.0
	−2	16.4	15.7	0.7

Figure: Allowed region in the $(\beta_h, \beta_\sigma - \beta_h)$ space for $\frac{\sigma_i}{M_p} = 0.3 - 1$. Red-dashed line unbroken S-duality. *Pavone et al. arXiv:2412.01734 (soon to appear on JCAP)*

The non-minimal scenario

- $\gamma, \delta \neq 0$.
- The recently observed signal detected by the International Pulsar Timing Array (IPTA) can be accounted for by the pBB model.
- We identify the detection frequency $f_s \simeq 1.2 \times 10^{-8}$Hz as the string frequency, with the observed amplitude $\Omega_{\rm GW} \simeq 2.9 \times 10^{-8}$.
- We restricted our analysis to $-3 \leq \log \frac{H_1}{M_p} \leq -1$, in concordance with $H_1 \approx M_s$.
- $\Omega_{\rm GW}(f_{\rm LVK}) < \Omega_{\rm LVK}$, for consistency with LIGO-VIRGO-KAGRA sensitivity bound.

Tensorial and Scalar Spectral Features

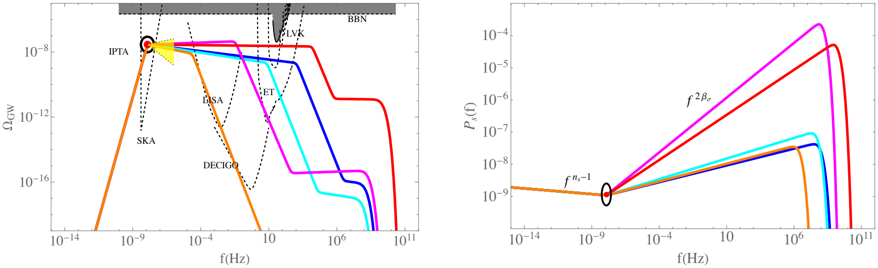


Figure: Left panel: Examples of possible GW spectra consistent with theoretical and phenomenological bounds. Right panel: Possible scalar spectra generated in the non-minimal pBB scenario. *Pavone et al. arXiv:2412.01734 (soon to appear on JCAP)*

	$\log z_s$	$\log z_d$	$\log z_\sigma$	ϵ	β_h	$\log \frac{\sigma_i}{M_P}$	$\log \frac{H_1}{M_P}$	$\log \frac{m}{M_P}$
	15.89	9.44	5.32	0.19	0.02	-1.00	-1.53	-8.18
	16.98	4.88	3.28	0.14	-0.01	-1.00	-1.85	-4.41
	14.14	10.48	4.65	-0.03	-0.08	-0.50	-2.97	-10.30
	15.70	4.74	1.29	0	-0.05	0.00	-2.60	-10.40
	15.46	6.68	2.93	0	-0.06	-0.50	-2.80	-9.10

Conclusion and Future Work

Take Home messages:

- The pBB scenario offers a compelling and phenomenologically rich alternative to the standard inflationary paradigm. We computed and showed that the peculiar pGW background produced may be detected by upcoming gravitational antennas.
- We showed that the signal detected by IPTA may be of primordial origin in this scenario, when higher order effects are accounted (at least phenomenologically) during the string phase. Moreover the signal should be detectable in various frequency branches by LISA, ET and DECIGO.
- If the IPTA signal is interpreted as of cosmological origin in the pBB scenario, the scalar spectra are always growing. This effect may lead to the production of primordial black holes.

Future Directions:

- Estimate the introduced phenomenological parameters β_h and β_σ from higher order stringy effects. (α' corrections, String Hole gas, non equilibrium phenomena in the string phase, ...) Currently working
- Investigate the primordial Black Hole production in the non-minimal scenario.

Thank You!

Questions?

This presentation was based on:

- Constraints on the Pre-Big Bang scenario from a cosmological interpretation of the NANOGrav data, arXiv:2412.01734 submitted to JCAP
- Gravitational-wave background in bouncing models, JCAP **09**, 058 (2024)
- From the string vacuum to FLRW or de Sitter via α' corrections, JCAP **12**, 019 (2023)

E-mail: eliseo.pavone@ba.infn.it