The Hawking-Unruh Effect

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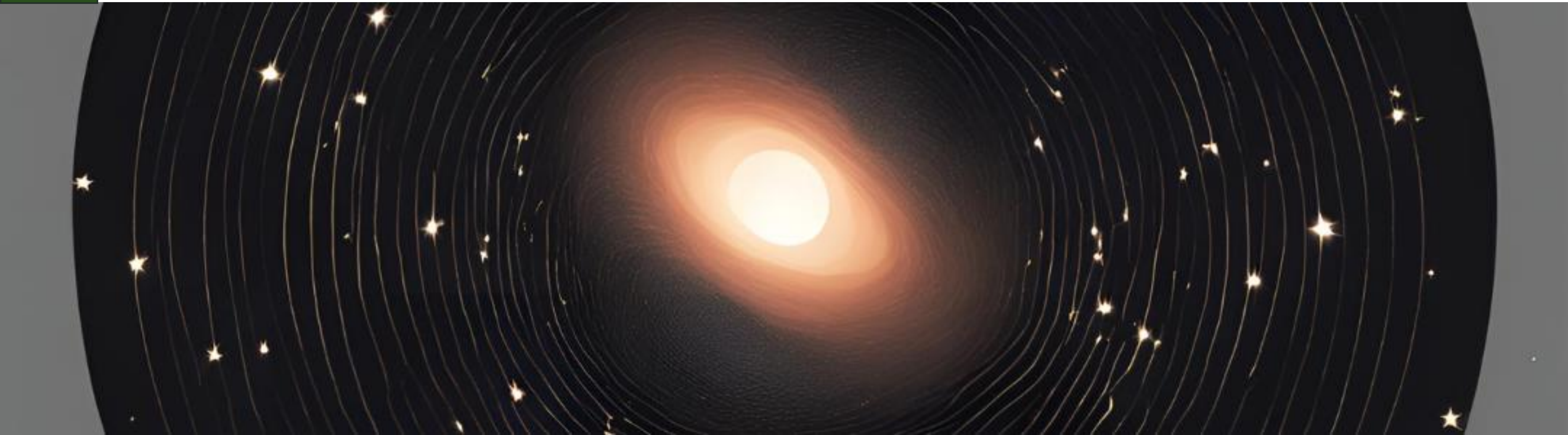
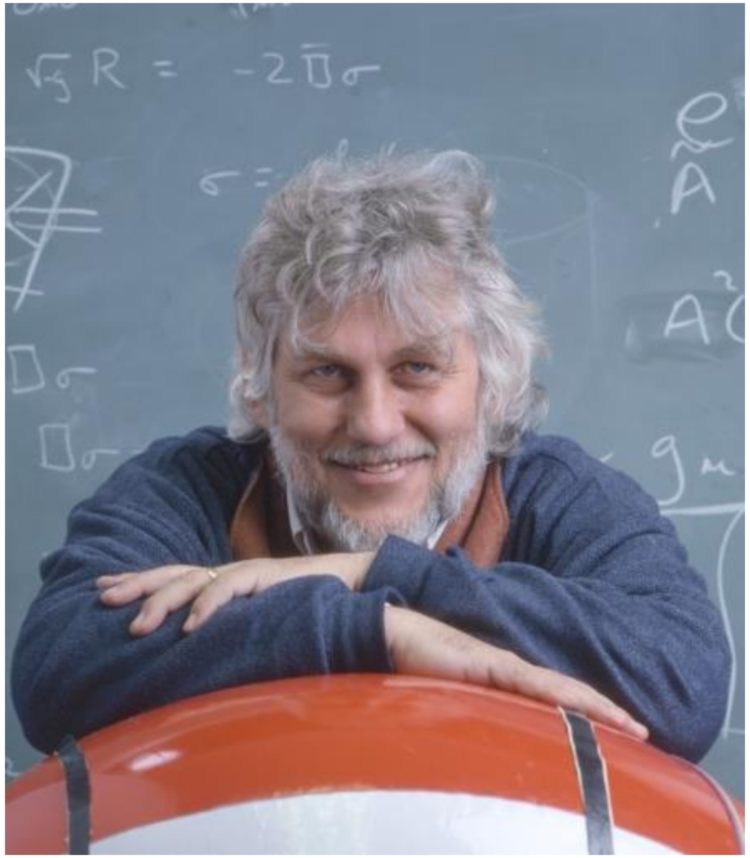
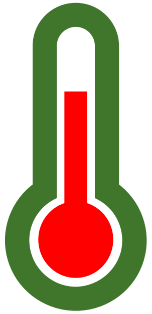


The Unruh Effect



The **Unruh effect** (1976) is a quantum field prediction according to which an accelerating observer in the vacuum detects thermal particles with a temperature directly proportional to its acceleration:

$$T_U = \frac{a}{2\pi}$$



- An accelerating thermometer in empty space will record a non-zero temperature.

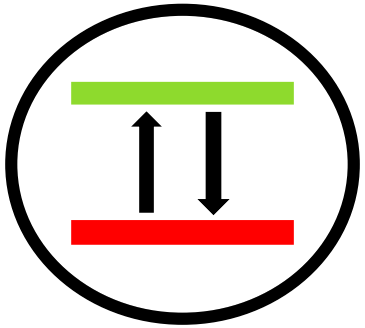
(In Planck units: $c = \hbar = k_B = G = 1$)

Demonstration: Unruh effect for a two-level atom

➢ There is no difference in the detection results between an accelerated observer in the vacuum and an observer at rest immersed in a thermal bath.

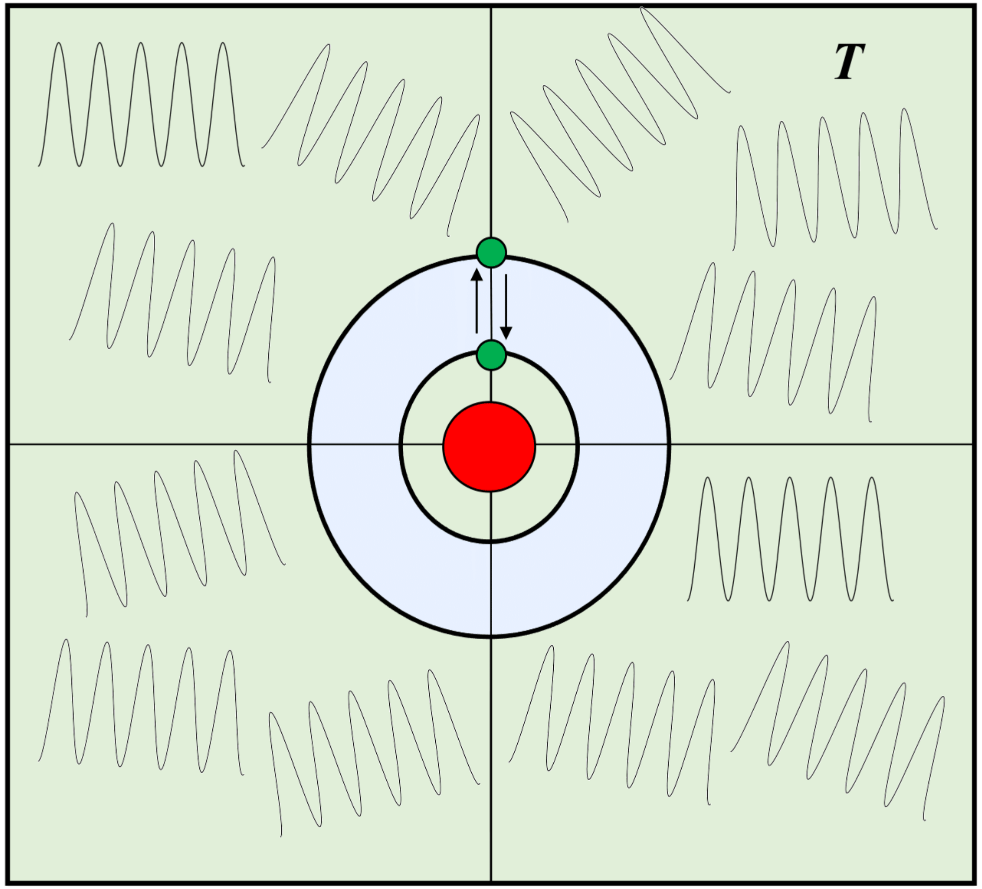
Let us consider a detector, a neutral two-level atom in a 1+1 Minkowski spacetime, whose Hamiltonian has the form

$$\hat{H}_D = \omega_0 \left(\frac{1 + \hat{\sigma}_z}{2} \right)$$



and which is coupled with a massless boson scalar field

$$\hat{\varphi}(t, x) = \hat{\varphi}\big(t(\tau), \boldsymbol{x}(\tau)\big) = \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\pi}} \frac{1}{\sqrt{2k}} \left[\hat{a}\,(\boldsymbol{k}) e^{-i\left(kt(\tau) - \boldsymbol{k}\cdot\boldsymbol{x}(\tau)\right)} + \hat{a}^{+}(\boldsymbol{k}) e^{i\left(kt(\tau) - \boldsymbol{k}\cdot\boldsymbol{x}(\tau)\right)} \right]$$



If the detector is at rest, immersed in a thermal bath at temperature T, the response function is equal to:

$$F_{\beta(1+1)}(\omega) = \frac{1}{\omega}\frac{1}{e^{\omega/T} - 1}$$

Transition probability of the detector per unit of time

Two-level atom at rest in a thermal bath

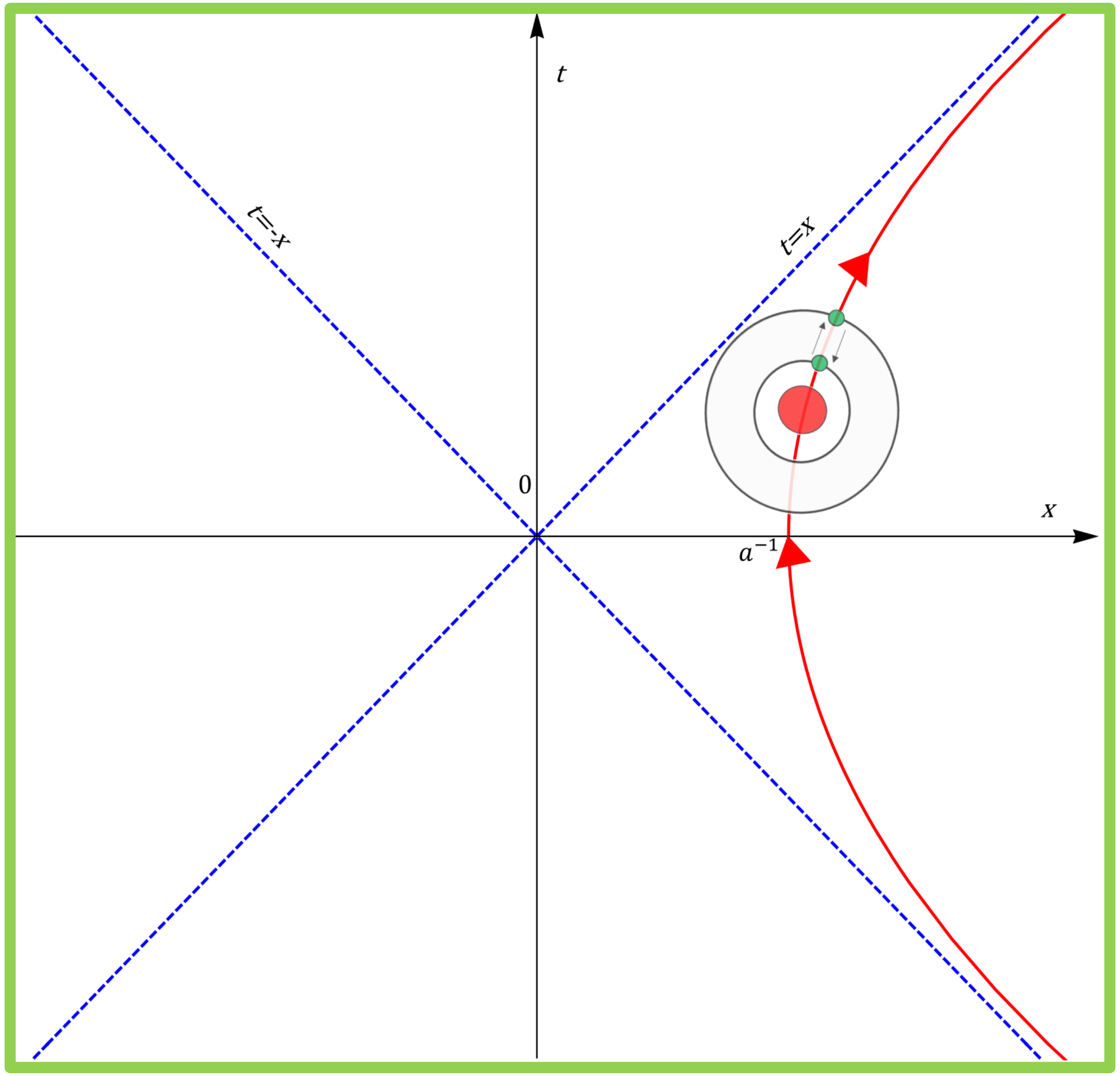
Uniformly accelerating two-level atom

The detector is now uniformly accelerated.

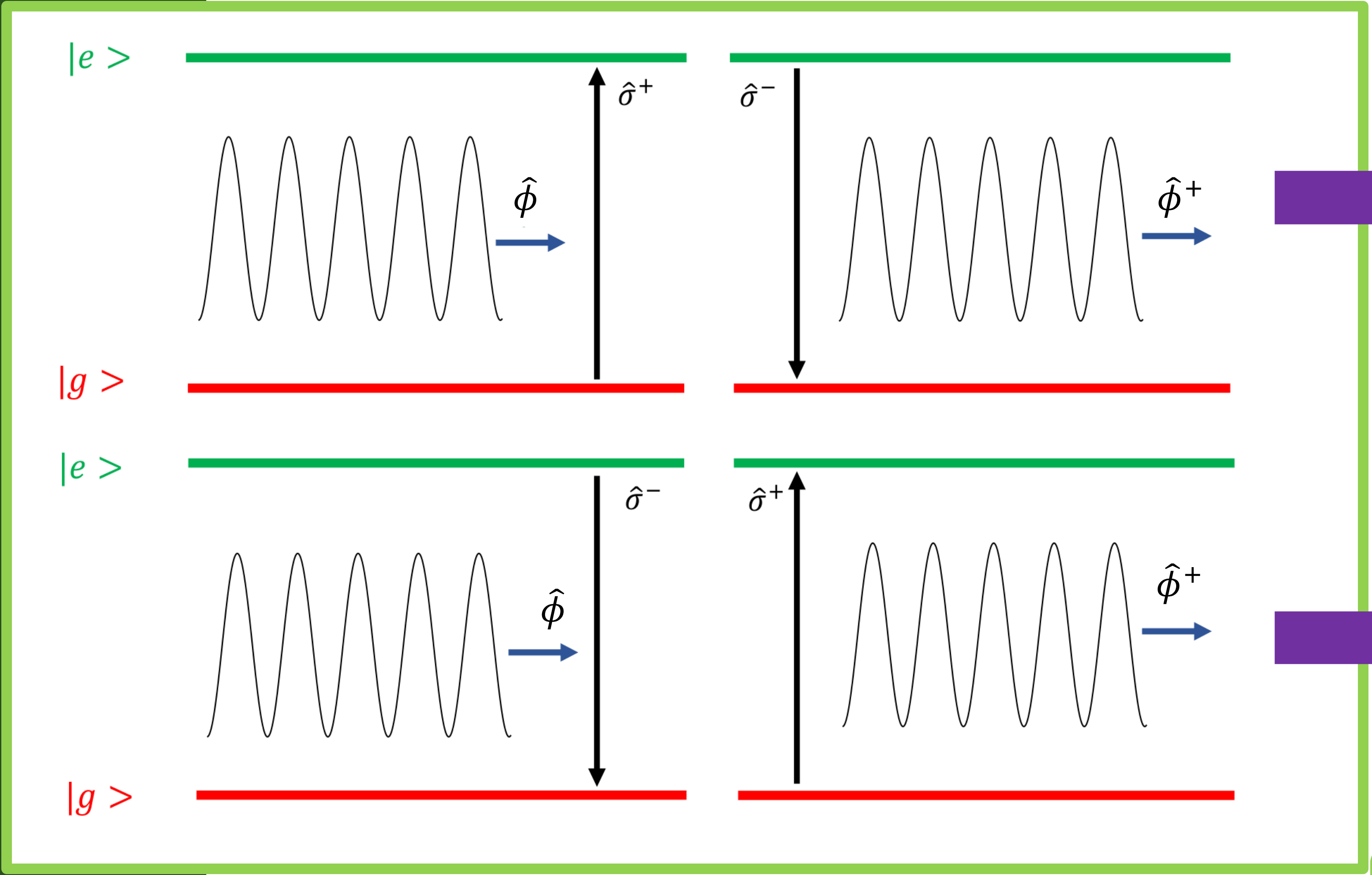
Its trajectory is given by the **Rindler coordinates**:

$$t(\tau) = \frac{1}{a}\sinh(a\tau)$$

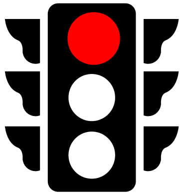
$$x(\tau) = \frac{1}{a}\cosh(a\tau)$$



The detector-field interaction



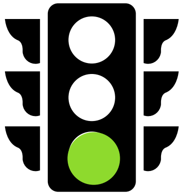
Real Transitions:



$$V = [\hat{\sigma}^+ \otimes \hat{\phi}(t,x) + \hat{\sigma}^- \otimes \hat{\phi}^+(t,x)]$$

$F = 0$

Virtual Transitions:



$$V = [\hat{\sigma}^+ \otimes \hat{\phi}^+(t,x) + \hat{\sigma}^- \otimes \hat{\phi}(t,x)]$$

$F \neq 0$

WE NEED VACUUM FLACTUATIONS!!!

Compare the results

$$F_{(1+1)}(\omega) = \frac{1}{\omega}\frac{1}{e^{2\pi\omega/a} - 1}$$

Response function for the uniformly accelerating detector

=

$$F_{\beta(1+1)}(\omega) = \frac{1}{\omega}\frac{1}{e^{\omega/T} - 1}$$

⟷

$$T = \frac{a}{2\pi}$$

- The Unruh effect is proved: there is no difference between an observer uniformly accelerating in the vacuum and an observer at rest in a thermal bath.

The Hawking Effect

The **Hawking effect** (1974) describes the theoretical thermal radiation emitted by a black hole.

If we take in exam a pair of particle-antiparticle close to the horizon, generated due to the vacuum quantum fluctuations: the particle inside the horizon can not escape from the black hole, while the other one can move on to infinity. Hence, the black hole emits a radiation with a temperature:

$$T_H = \frac{k}{2\pi}$$

k is the surface gravity of the black hole

Demonstration: Hawking effect and Bogolyubov transformations

$$\hat{\phi}(t,x) = \int_0^{+\infty} \frac{d\Omega}{\sqrt{2\pi}} \frac{1}{\sqrt{2\Omega}} \left[\hat{b}\,(\Omega) e^{-i\Omega\tilde{u}} + h.c.\right] + \text{LMM}$$

- Let us consider a 1+1-dimensional black hole which is described by the Schwarzschild metric in terms of tortoise lightcone coordinates $(\tilde{u}, \tilde{v})$.
- An observer at rest far away from the black hole $(r \to \infty)$ can detect particles with frequency Ω.
- The creation and annihilation operators follow the commutaiton rule: $\left[\hat{b}(\Omega), \hat{b}^{+}(\Omega')\right] = \delta(\Omega - \Omega')$

$$\hat{\phi}(t,x) = \int_0^{+\infty} \frac{d\omega}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega}} \left[\hat{a}\,(\omega) e^{-i\omega u} + h.c.\right] + \text{LMM}$$

- Let us consider a 1+1-dimensional black hole which is described in terms of Kruskal – Szekeres coordinates (u, v).

- An observer who is crossing the horizon can detect particles with frequency ω.

- The creation and annihilation operators follow the commutaiton rule: $[\hat{a}(\omega), \hat{a}^{+}(\omega')] = \delta(\omega - \omega')$

➢ To consider the whole spacetime, it is necessary to introduce the Kruskal – Szekeres coordinate system, to have regularity in $r = r_S$ (Schwarzschild radius).

Vacuum states

Through the annihilation operator $\hat{b}(\Omega)$ it is possible to define the vacuum state:

$$\hat{b}(\Omega)|0_B >= 0$$

Boulware vacuum, vacuum state which doesn't contain particles form the point of view of the observer far away from the black hole, physically unacceptable.

Through the annihilation operator $\hat{a}$ (Ω) it is possible to define the vacuum state:

$$\hat{a}(\omega)|0_K >= 0$$

Kruskal vacuum, is regular on the horizon, physically acceptable.

Bogolyubov transformations

It is possible to connect the annihilation and creation operators a and b through the **Bogolyubov transformations**:

$$\hat{b}\,(\Omega) = \int_{0}^{+\infty} d\omega[\alpha_{\omega\Omega}\hat{a}\,(\omega)\; + \beta_{\omega\Omega}\hat{a}^{+}(\omega)]$$

$$\hat{b}^{+}(\Omega) = \int_{0}^{+\infty} d\omega[\alpha^{*}_{\omega\Omega}\hat{a}^{+}(\omega)\; + \beta^{*}_{\omega\Omega}\hat{a}\,(\omega)]$$

We can now compute the expectation value of the number operator $\widehat{N}_{\Omega}$ for the b-particles in the Kruskal vacuum:

$$\widehat{N}_{\Omega} = < 0_K|\hat{b}^{+}\hat{b}|0_K > = \left(e^{2\pi\Omega/k} - 1\right)^{-1} = \left(e^{\Omega/T} - 1\right)^{-1}$$

$$T = \frac{k}{2\pi}$$

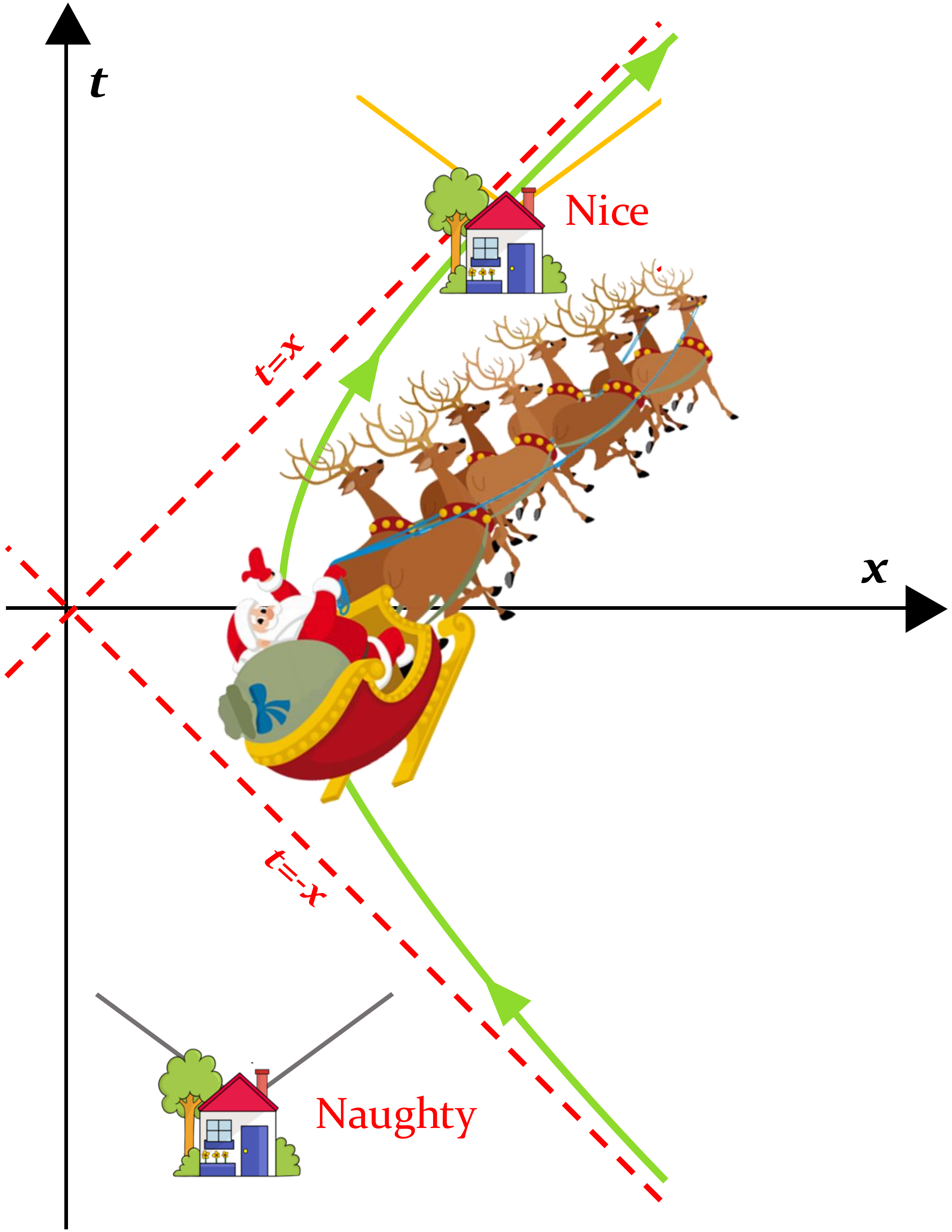
Equivalence principle

Equivalence Principle → A uniformly accelerated observer experiences a uniform gravitational field → The effects of the gravitational interactions are locally indistinguishable from the effects of an accelerated frame

The Hawking-Unruh Effect

Considering the EP → **The two phenomena are the manifestation of the same physcal effect** → **Hawking-Unruh Effect**

- The Unruh temperature and the Hawking temperature present the same expression in which ***a*** is replaced by ***k***.
- A crucial role is played by vacuum fluctuations in both the effects.
- The radiation production takes energy from the kinetic energy of the detector in Unruh, from the gravitational energy of the black hole in Hawking.



This Christmas be nice…stay inside the horizon!

- An accelerating observer in a Rindler spacetime, will have an event horizon analogous to the one we get in the neighborhood of black hole.

Thank you for your attention.