Robustness of quantum symmetries against perturbations

V.Viesti[1] P.Facchi[1] M.Ligabò[2]

[1]Department of Physics
University Aldo Moro of Bari

[2]Department of Mathematics
University Aldo Moro of Bari

INFN Bari Theory Group Christmas Workshop, December 17th 2024

Istituto Nazionale di Fisica Nucleare

Table of Contents

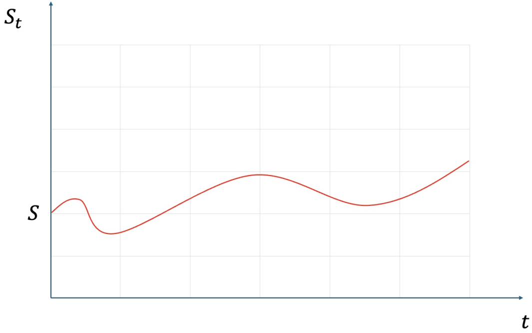
Table of Contents

1. Symmetries

2. Fragile and robust symmetries

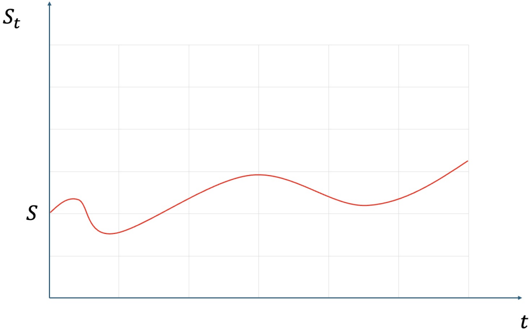
The evolution is unitary

- S bounded operator



The evolution is unitary

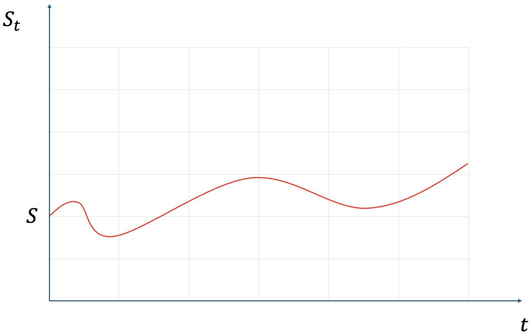
- S bounded operator
- $H = H^\dagger$ Hamiltonian



The evolution is unitary

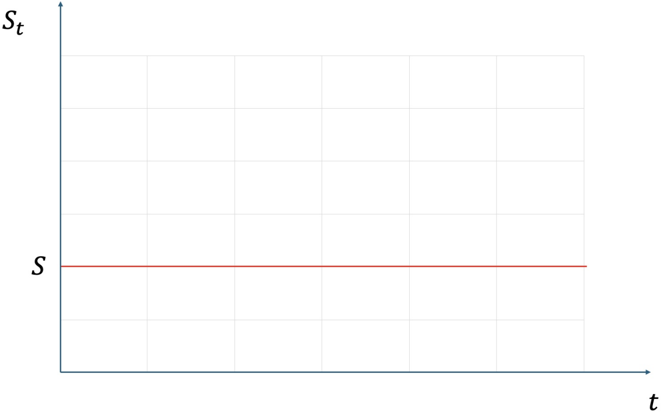
- S bounded operator
- $H = H^\dagger$ Hamiltonian
- Heisenberg evolution

$$S_t = \mathrm{e}^{\mathrm{i}tH} S \mathrm{e}^{-\mathrm{i}tH},$$



Conserved quantities are time-independent

$$S_t = S, \quad \forall t \in \mathbb{R}. \tag{1}$$



Symmetries and conserved quantities

- Noether's Theorem

$$S \text{ conserved quantity} \iff e^{-i\theta S} \text{ continuous symmetry of } H$$

$$e^{i\theta S} H e^{-i\theta S} = H \quad \forall \theta \in \mathbb{R}$$

Symmetries and conserved quantities

- Noether's Theorem

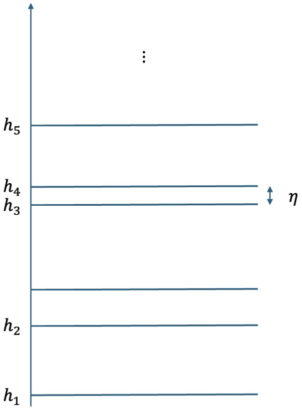
$$S \text{ conserved quantity} \iff \mathrm{e}^{-\mathrm{i}\theta S} \text{ continuous symmetry of } H$$

$$\mathrm{e}^{\mathrm{i}\theta S} H \mathrm{e}^{-\mathrm{i}\theta S} = H \quad \forall \theta \in \mathbb{R}$$

- Conserved quantities $\iff$ Symmetries

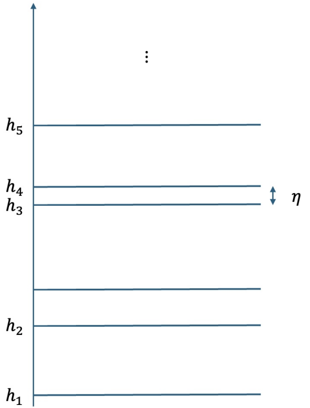
The Hamiltonian has discrete spectrum

- H with purely-point spectrum



The Hamiltonian has discrete spectrum

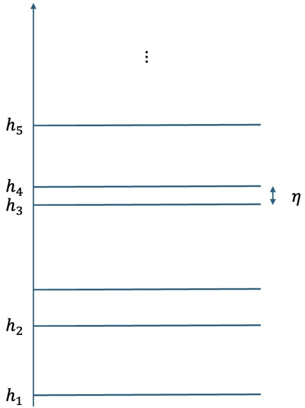
- H with purely-point spectrum
- Non-vanishing gap



The Hamiltonian has discrete spectrum

- H with purely-point spectrum
- Non-vanishing gap
- Spectral decomposition

$$H = \sum_{k \geq 1} h_k P_k$$



In matrix form we have...

$$H = \begin{pmatrix} \begin{matrix} h_1 & & \\ & \ddots & \\ & & h_1 \end{matrix} & \mathbf{0} & \cdots \\ \mathbf{0} & \begin{matrix} h_2 & & \\ & \ddots & \\ & & h_2 \end{matrix} & \\ \vdots & & \ddots \end{pmatrix}$$

Symmetries commute with H

- S symmetry of $H \iff [S, H] = 0$

Symmetries commute with H

- S symmetry of $H \iff [S, H] = 0$
- S block-diagonal in H-representation

Symmetries commute with H

- S symmetry of $H \iff [S,H]=0$
- S block-diagonal in H-representation
- In matrix form we have

$$S = \begin{pmatrix} S_1 & \mathbf{0} & \cdots \\ \mathbf{0} & S_2 & \\ \vdots & & \ddots \end{pmatrix}$$

Table of Contents

Perturbation of the Hamiltonian

- S symmetry of H

Perturbation of the Hamiltonian

- S symmetry of H
- V hermitian bounded operator

Perturbation of the Hamiltonian

- S symmetry of H
- V hermitian bounded operator
- $H \to H + \varepsilon V$, $\varepsilon \ll 1$, perturbed Hamiltonian

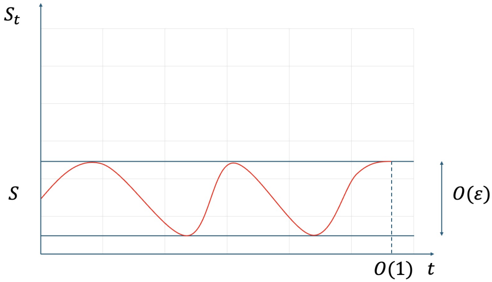
Perturbation of the Hamiltonian

- S symmetry of H
- V hermitian bounded operator
- $H \to H + \varepsilon V$, $\varepsilon \ll 1$, perturbed Hamiltonian
- $[S, V] \neq 0$ in general

Symmetries are almost conserved for small times

- S no longer conserved

$$\mathrm{e}^{\mathrm{i}t(H+\varepsilon V)} S \mathrm{e}^{-\mathrm{i}t(H+\varepsilon V)} \neq S$$



Symmetries are almost conserved for small times

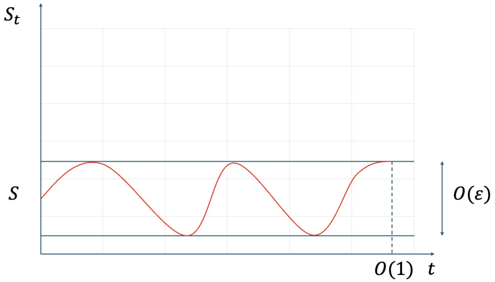
- S no longer conserved

  $$\mathrm{e}^{\mathrm{i}t(H+\varepsilon V)} S \mathrm{e}^{-\mathrm{i}t(H+\varepsilon V)} \neq S$$

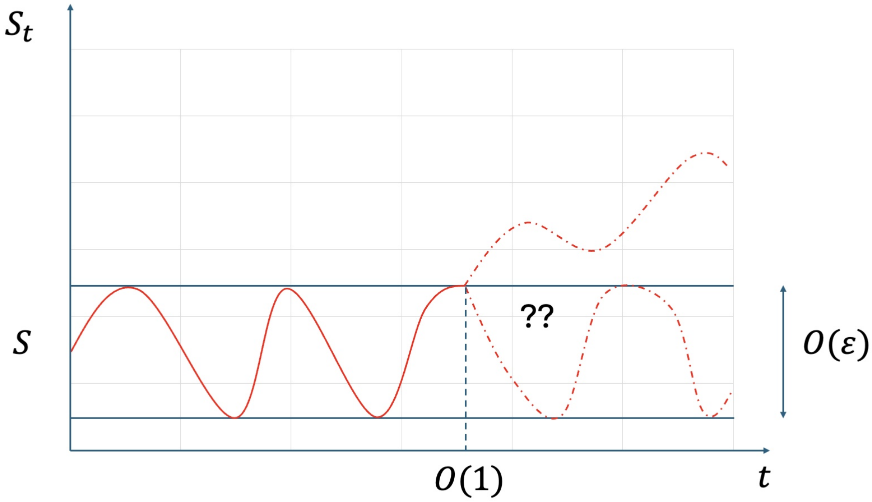
- However

  $$\mathrm{e}^{\mathrm{i}t(H+\varepsilon V)} S \mathrm{e}^{-\mathrm{i}t(H+\varepsilon V)} = S + O(\varepsilon)$$

  for $t = O(1)$

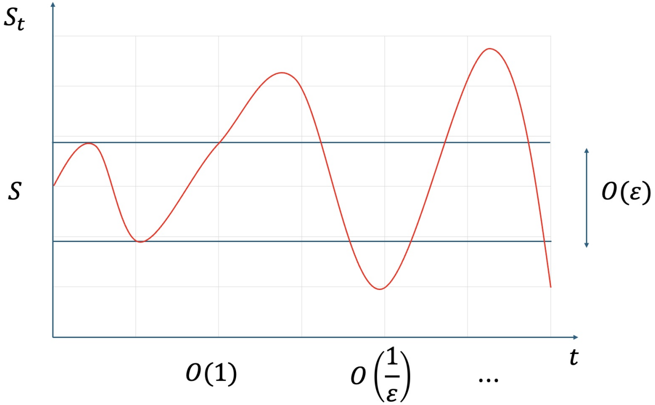


What happens for long times?



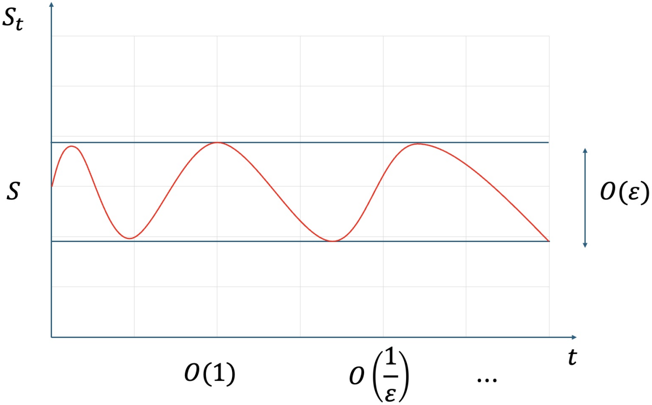
Fragile symmetries are lost for long times

$$e^{it(H+\varepsilon V)} S e^{-it(H+\varepsilon V)} = S + O(1) \quad (2)$$



Robust symmetries are almost conserved for all times

$$\mathrm{e}^{\mathrm{i}t(H+\varepsilon V)} S \mathrm{e}^{-\mathrm{i}t(H+\varepsilon V)} = S + O(\varepsilon) \tag{3}$$



Kato's Theorem

- $H + \varepsilon V$ self-adjoint:

$$H + \varepsilon V = \sum_{n \geq 1} h_n(\varepsilon) P_n(\varepsilon) \tag{4}$$

Kato's Theorem

- $H + \varepsilon V$ self-adjoint:

$$H + \varepsilon V = \sum_{n \geq 1} h_n(\varepsilon) P_n(\varepsilon) \tag{4}$$

- $\{P_n(\varepsilon)\}_{n \geq 1}$ analytic

Kato's Theorem

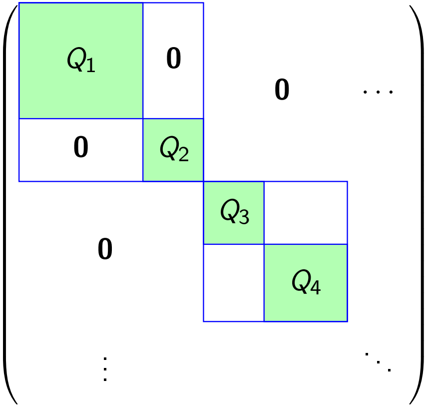
- $H + \varepsilon V$ self-adjoint:

$$H + \varepsilon V = \sum_{n \geq 1} h_n(\varepsilon) P_n(\varepsilon) \tag{4}$$

- $\{P_n(\varepsilon)\}_{n \geq 1}$ analytic
- $Q_n := P_n(0)$ family of subprojections of the projections of H

Perturbations induces subprojections

$V \to$



Robust symmetries commute with the subprojections

$$
S = \begin{pmatrix}
\begin{matrix} S_1 & \mathbf{0} \\ \mathbf{0} & S_2 \end{matrix} & \mathbf{0} & \cdots \\
\mathbf{0} & \begin{matrix} S_3 & \\ & S_4 \end{matrix} & \\
\vdots & & \ddots
\end{pmatrix}
$$

Conclusions

- Symmetries commute with the projections of H

Conclusions

- Symmetries commute with the projections of H
- Robust symmetries commute with the family of subprojections induced by V

Thanks for your attention!

[1] P. Facchi, M. Ligabò, V. Viesti *Robustness of quantum symmetries against perturbations*, arXiv:2411.18529 [quant-ph]