



UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO



Axions from Neutron stars mergers: production and detection signatures

FRANCESCA LECCE



BARI THEORY XMAS WORKSHOP 2024

DECEMBER 17, 2024

Overview

① AXION AND AXION
LIKE PARTICLES

② BINARY NEUTRON
STAR MERGERS

③ PRIMAKOFF EFFECT

④ ALP-PHOTON
CONVERSION

⑤ EXPERIMENTS
SENSITIVITY

⑥ WHAT'S NEXT?

Axion and Axion-like particles

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Non trivial structure of the QCD vacuum

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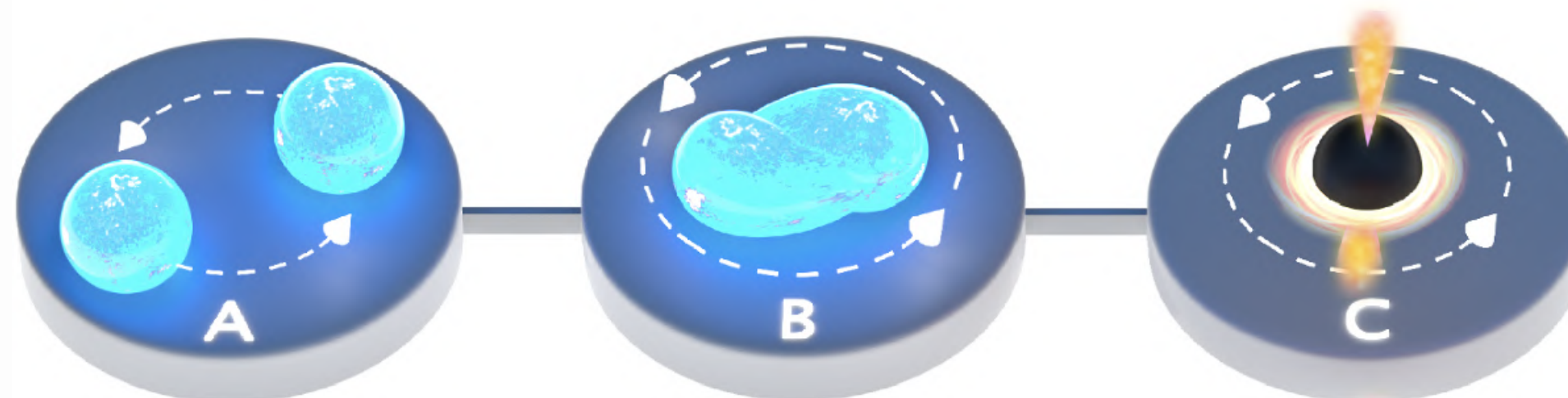
Binary Neutron Star Mergers

Binary Neutron Star systems can form dynamically from pre-existing binary systems or dynamically in dense environments [*M. Renzo et al., 624, A66 (2019), S. Ye et al., Astr. Jou. 888, L10 (2019)*].

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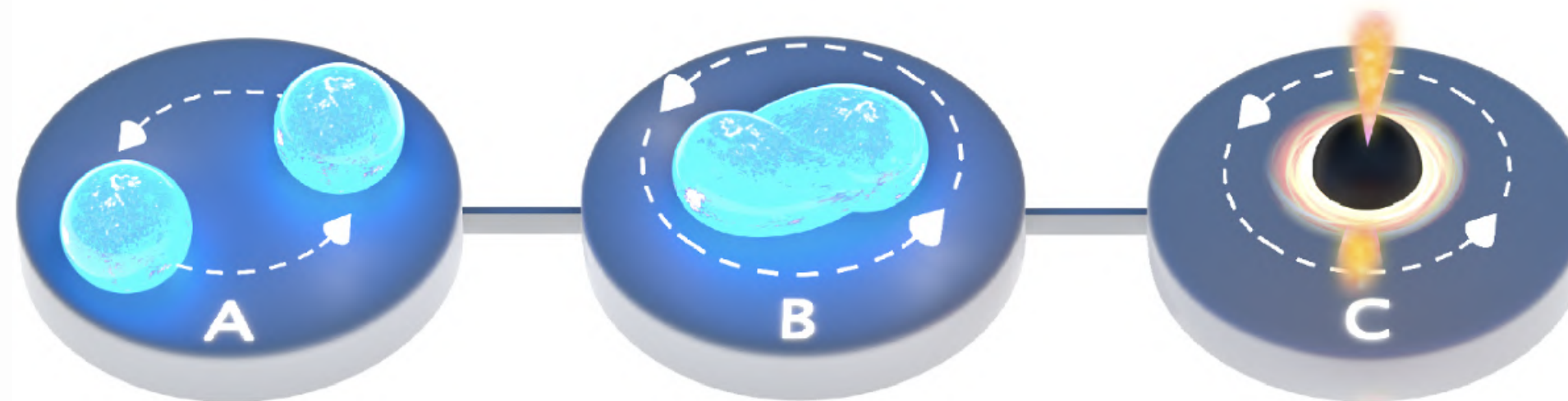
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Inspiral phase

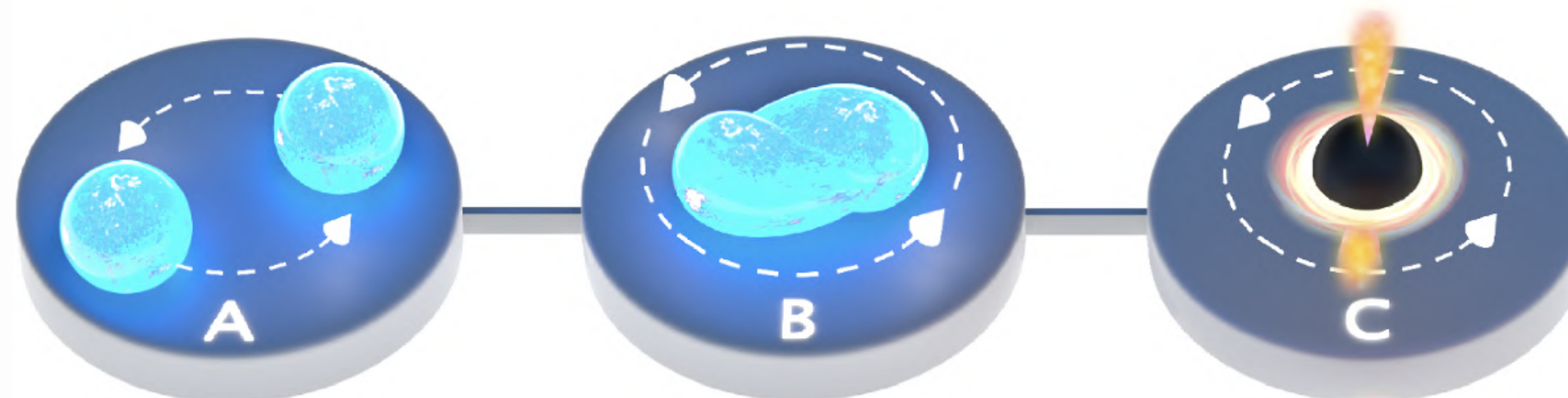
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Latter phase $O(1) \text{ min} \lesssim \Delta t \lesssim O(1) \text{ hrs}$

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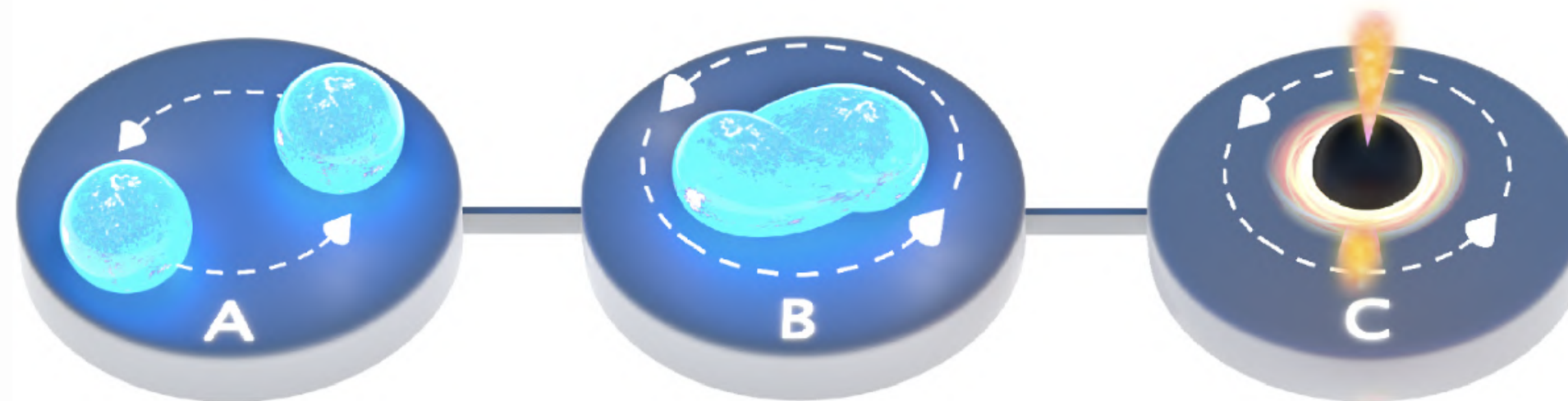
Merger phase

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Merger phase

$\Delta t \sim O(20) \text{ ms}$

Ringdown phase

$\Delta t \sim O(1) \text{ s}$

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The landmark in Binary Neutron Merger is the observation of *GW170817* [B.P. Abbott et al., *Phys. Rev. Lett.* 119, 16 (2017)]

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Simulations of NSM are based on hydrodynamics equations including General Relativity effects.

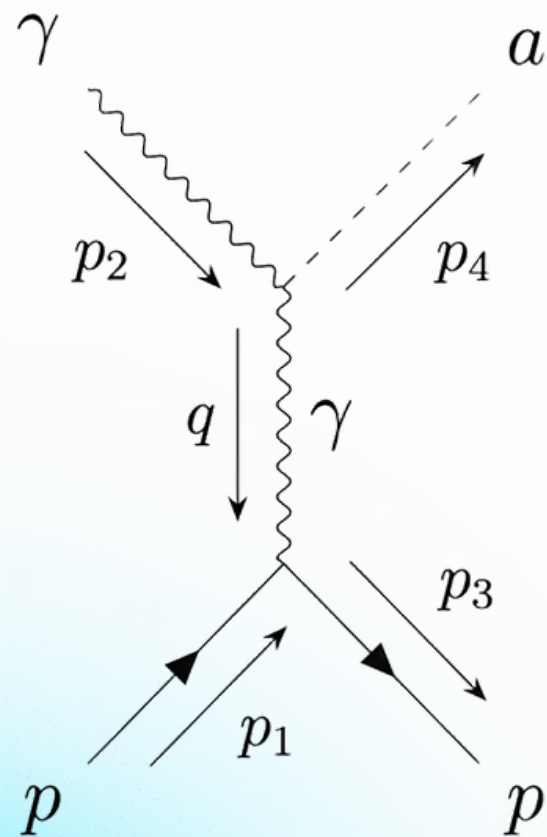
The simulations used in this work were performed by the ***Relativistic Group at the GSI Helmotz Centre in Darmstadt.***

Aim of the Work

- ① Study the production of ALPs from Binary Neutron Star Mergers, via Primakoff effect.
- ② Determine the sensitivity of experiments and proposed experiment to the gamma-ray signal produced by ALP-photon conversion

Primakoff effect

Conversion of a thermal photon in a stellar plasma into an ALP in the fluctuation of electromagnetic field, generated by charged particles in the plasma. [*H. Primakoff, Phys. Rev. 81, 899.(1951)*]

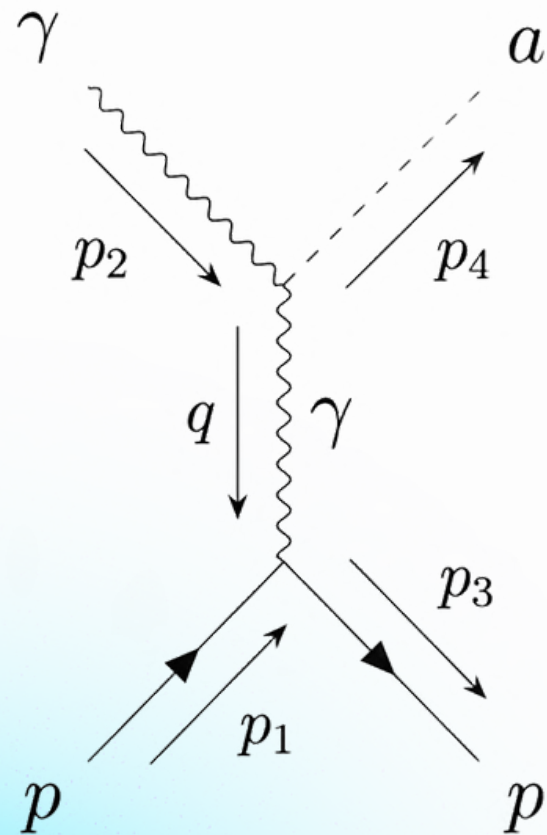


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We considered of infinite proton mass, massless ALP and $m_\gamma = \sqrt{\frac{3}{2}}\omega_{\text{pl}} \approx 16.3 \text{ MeV } Y_e^{1/3} \rho_{14}^{1/3}$

[*E. Braaten and D. Segel. Phys. Rev. D 48.4, 1478 (1993), A. Kopf and G. Raffelt, Phys. Rev. D 57.6, 3235 (1998)*]

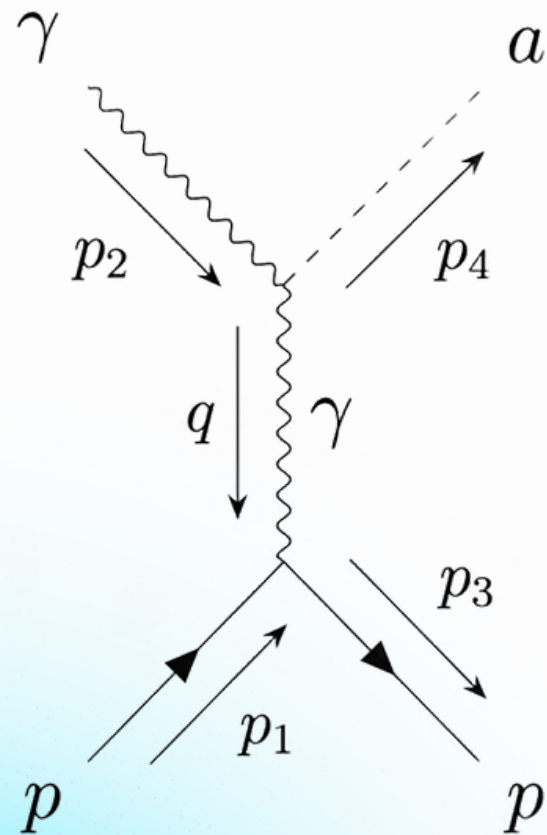


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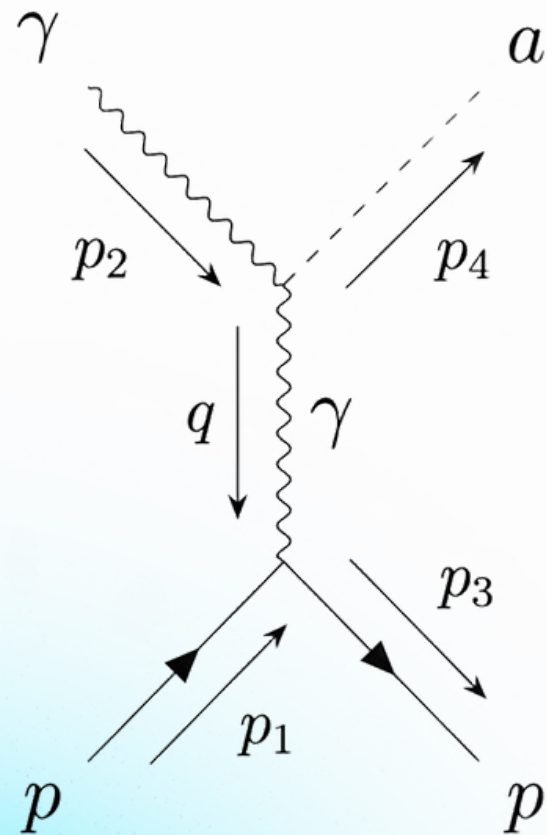
$$\sum_{\text{spin, pol}} |\mathcal{M}|^2 = \frac{32\pi\alpha}{\mathbf{q}^2(\mathbf{q}^2 + \kappa_s^2)} g_{a\gamma\gamma}^2 m_p^2 [\mathbf{p}_2^2 \mathbf{p}_4^2 - (\mathbf{p}_2 \cdot \mathbf{p}_4)^2]$$

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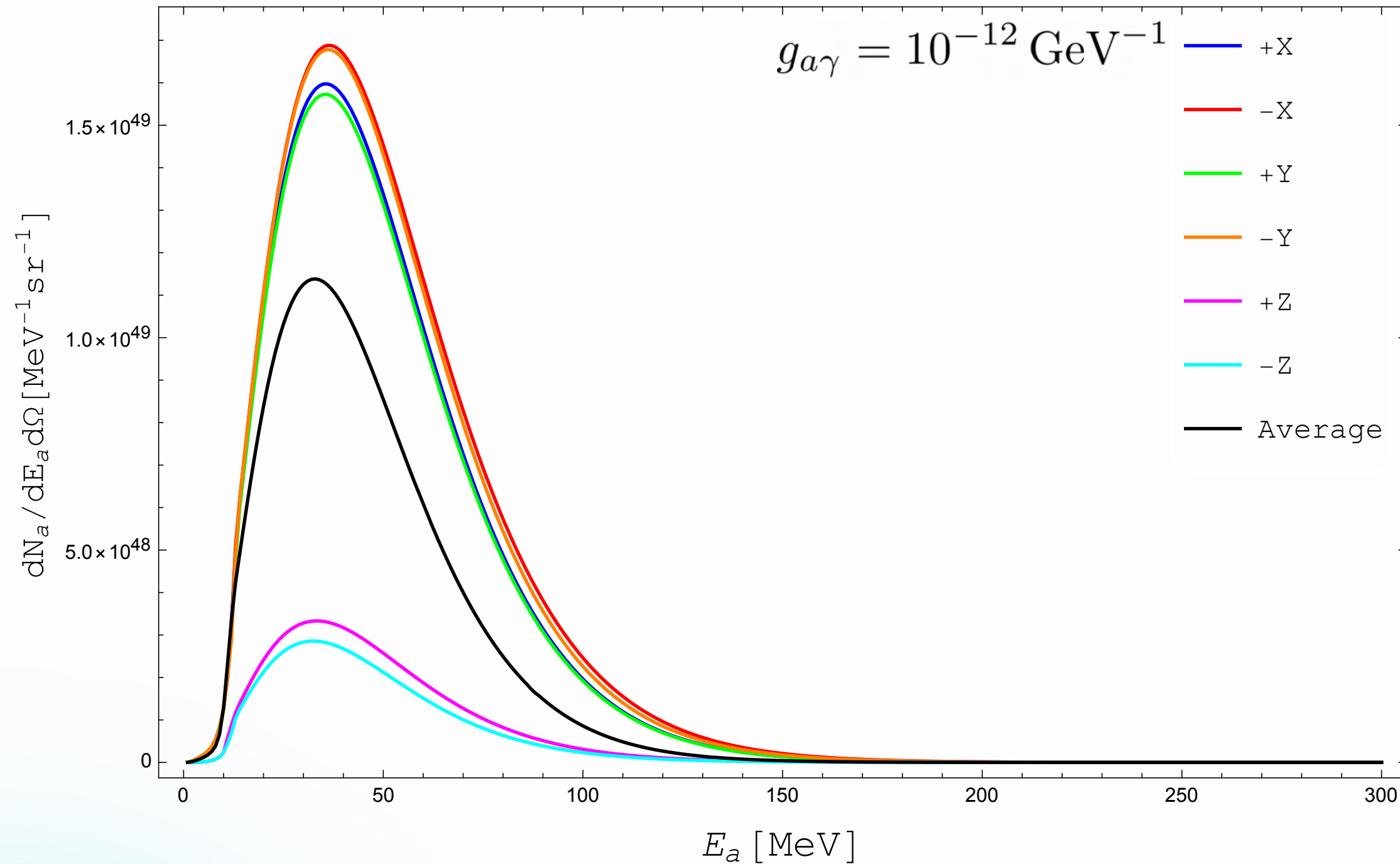
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Screening effect in the Debye expression

$$\kappa_s^2 = \frac{4\pi\alpha n_p}{T}$$

[P. Debye and E. Hückel, Phys. Zeit. 24.9, 185 (1923)]

ALP spectrum



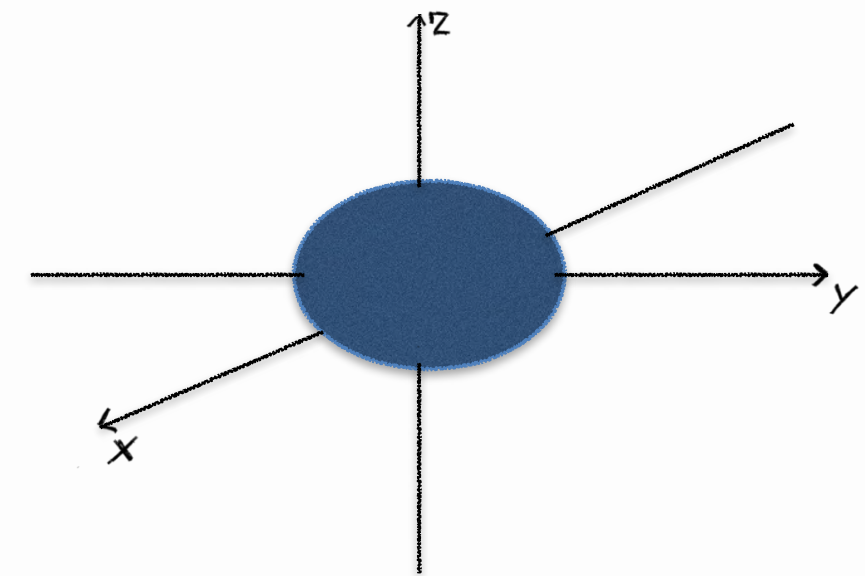
The lapse factor encodes the strong gravitational field effects and affects

$$E = E^*(r)e^{\Phi(r)}$$

$$dt = dt^*(r)e^{-\Phi(r)}$$

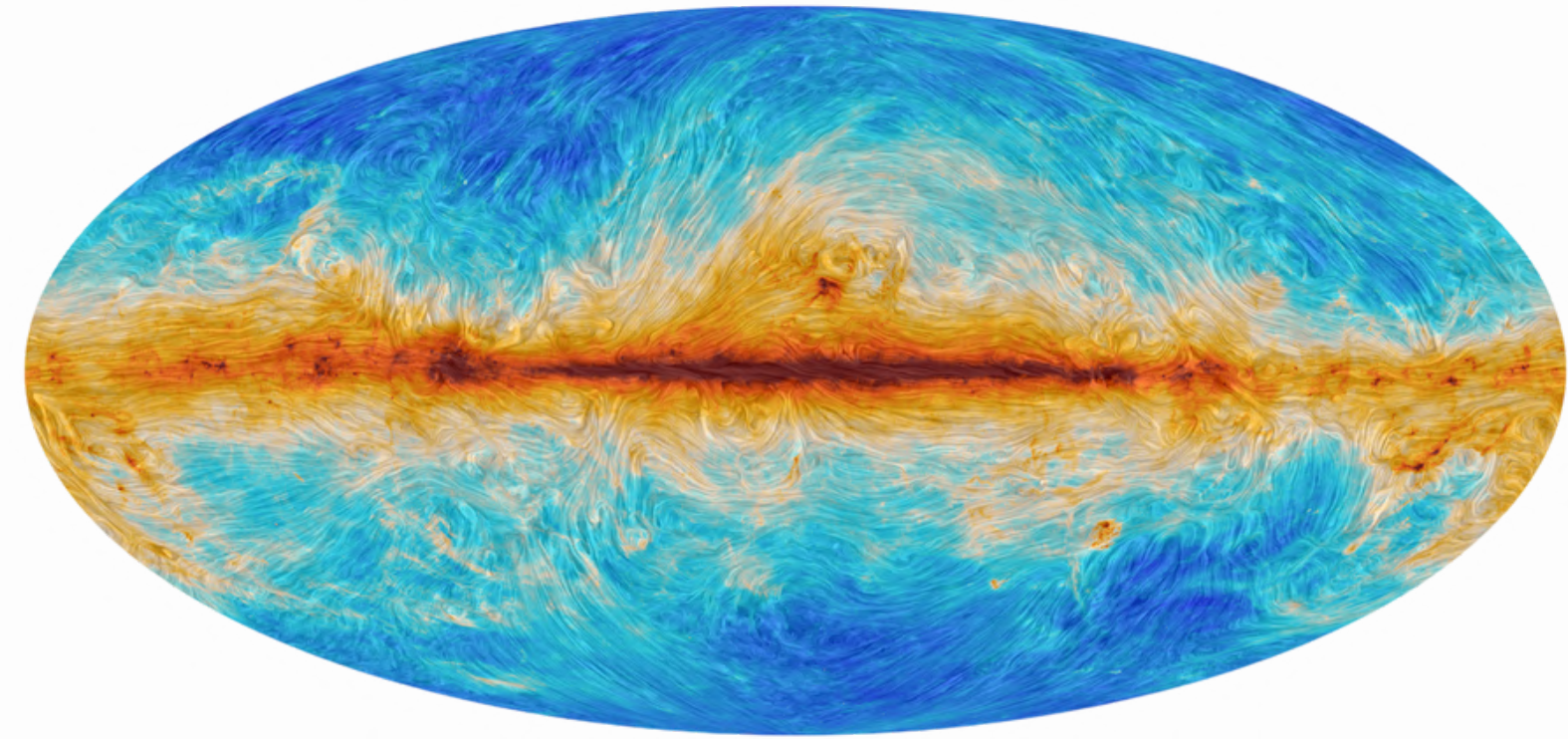
$$\frac{dN_a}{dE_a^*} = \frac{1}{4\pi} \int dr r^2 dt^* \frac{d\Gamma_{a\gamma}}{dE_a^*} \exp[-\Phi(r)]$$

$$\frac{dN_a}{dE_a^* d\Omega} = \int dr \cos \gamma r^2 dt^* \frac{d\Gamma_{a\gamma}}{dE_a^*} \exp[-\Phi(r)]$$



ALP-photon conversion

ALPs can convert into photons while propagating in external magnetic fields thanks to the ALP-photon coupling [*G. Raffelt and L. Stodolsky, Phys. Rev. D 37 , 1237 (1988)*].

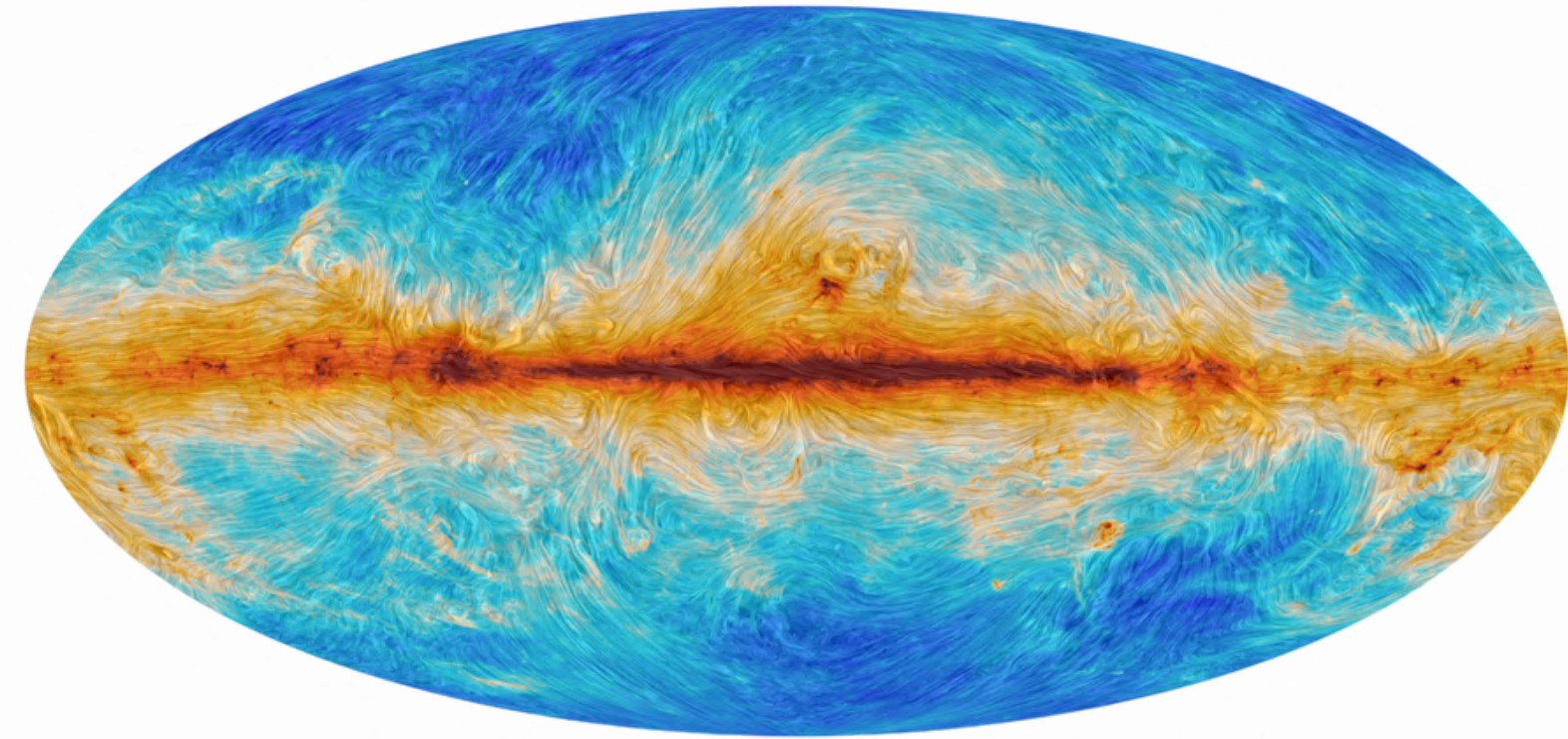


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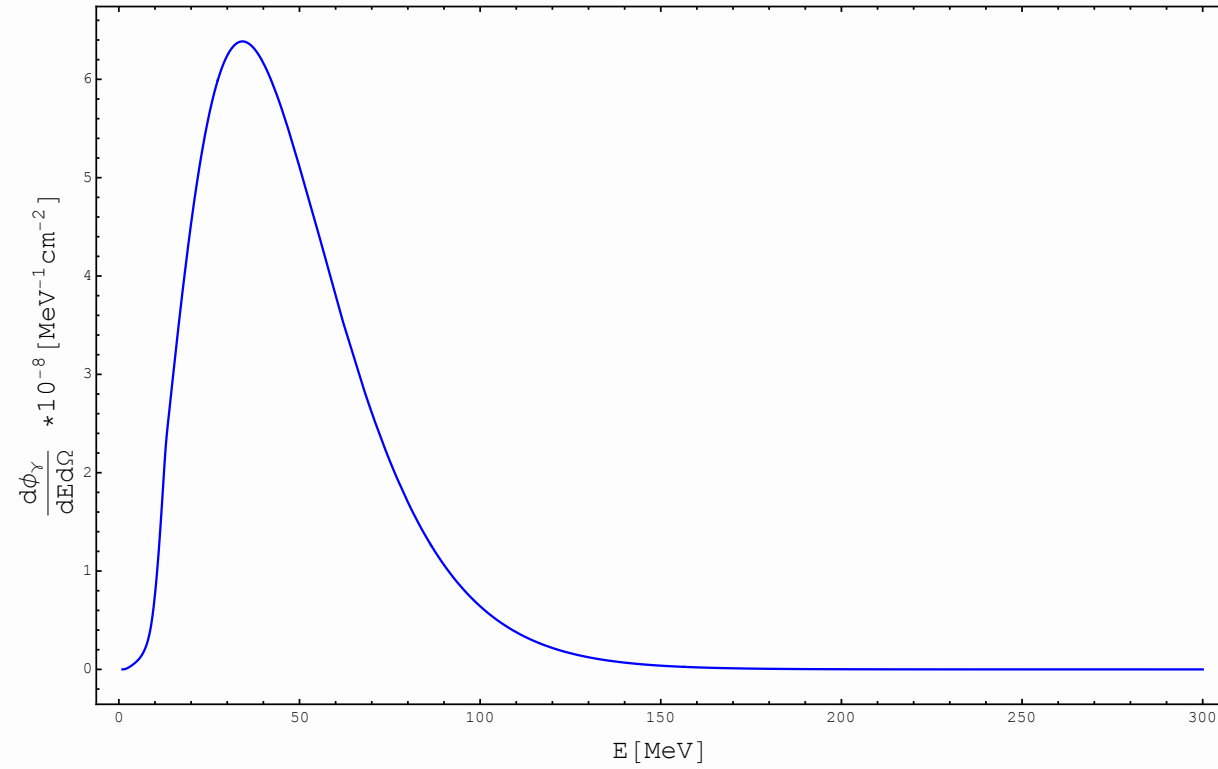
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$$i\partial_z \begin{pmatrix} A_{\perp}(z) \\ A_{\parallel}(z) \\ a(z) \end{pmatrix} = \begin{pmatrix} \Delta_{\parallel} \sin^2 \theta + \Delta_{\perp} \cos^2 \theta & (\Delta_{\parallel} - \Delta_{\perp}) \cos \theta \sin \theta & \Delta_{a\gamma} \sin \theta \\ (\Delta_{\parallel} - \Delta_{\perp}) \cos \theta \sin \theta & \Delta_{\parallel} \cos^2 \theta + \Delta_{\perp} \sin^2 \theta & \Delta_{a\gamma} \cos \theta \\ \Delta_{a\gamma} \sin \theta & \Delta_{a\gamma} \cos \theta & \Delta_a \end{pmatrix} \begin{pmatrix} A_{\perp}(z) \\ A_{\parallel}(z) \\ a(z) \end{pmatrix}$$

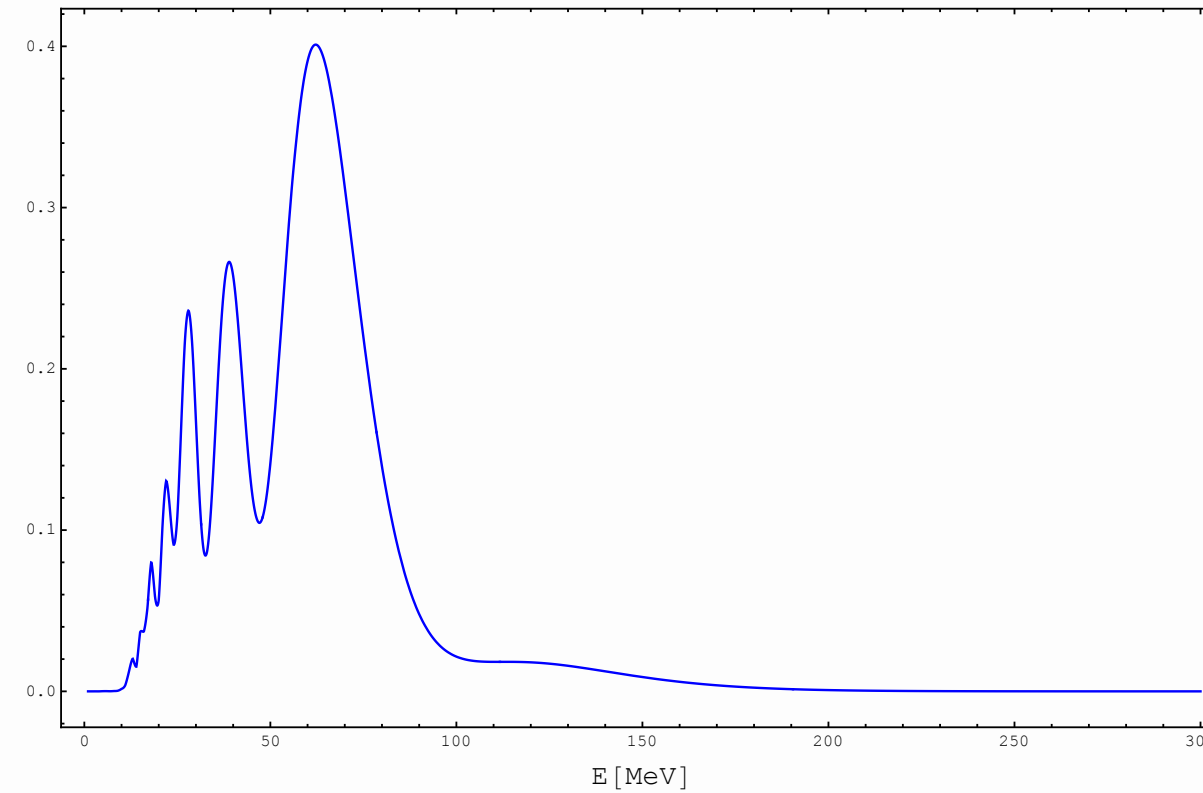
$$\text{With } \Delta_{\parallel,\perp} = \Delta_{\text{pl}} + \Delta_{\text{QED}}^{\parallel,\perp} + \Delta_{\text{CMB}}, \quad \Delta_a = -\frac{m_a^2}{2E_a} \quad \text{and} \quad \Delta_{a\gamma} = \frac{1}{2} g_{a\gamma} B_T$$



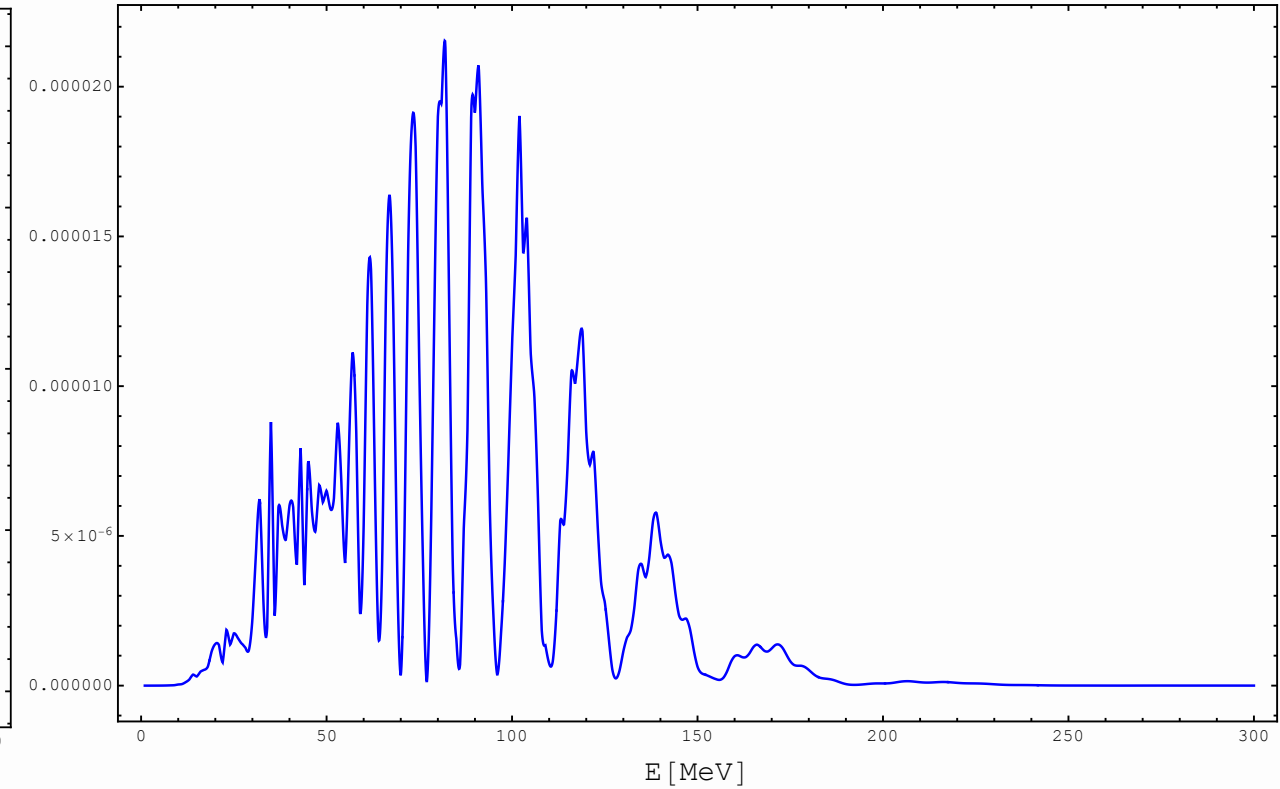
Observable photon flux



$m_a = 0.1 \text{ neV}$



$m_a = 1 \text{ neV}$

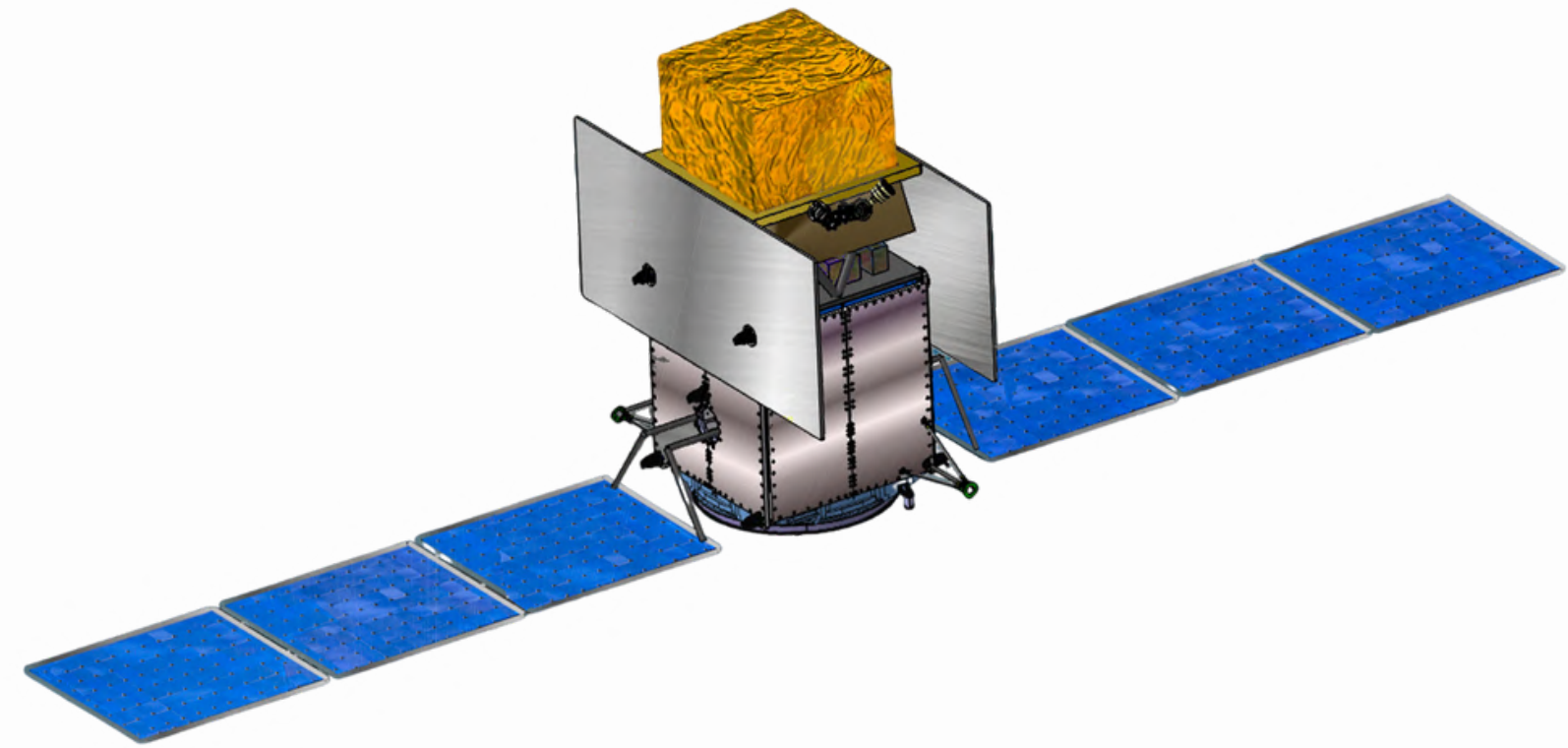


$m_a = 10 \text{ neV}$

Observable photon flux: $\frac{d\phi_\gamma}{dE} = \frac{1}{4\pi L^2} \frac{dN}{dE} P_{a\gamma}(E, m, d, l, b, g_{a\gamma})$ with $L = 40 \text{ Mpc}$,
 $d = 20 \text{ kpc}$ and $g_{a\gamma} = 10^{-12} \text{ GeV}^{-1}$

Experiments sensitivity

We quantified the sensitivity of Fermi-LAT and of the proposed e-ASTROGRAM, AMEGO-X and GRAMS experiment to the photon-ALP coupling, by studying the observed gamma-ray flux [[A. De Angelis et al., Exp. Astr. 44.1 , 25 \(2017\)](#)],[[T. Aramaki et al., Astr. Phy. 114 , 107-114 \(2020\)](#)],[[R. Caputo et al., Jou. Astr. Tel. \(2022\)](#)]

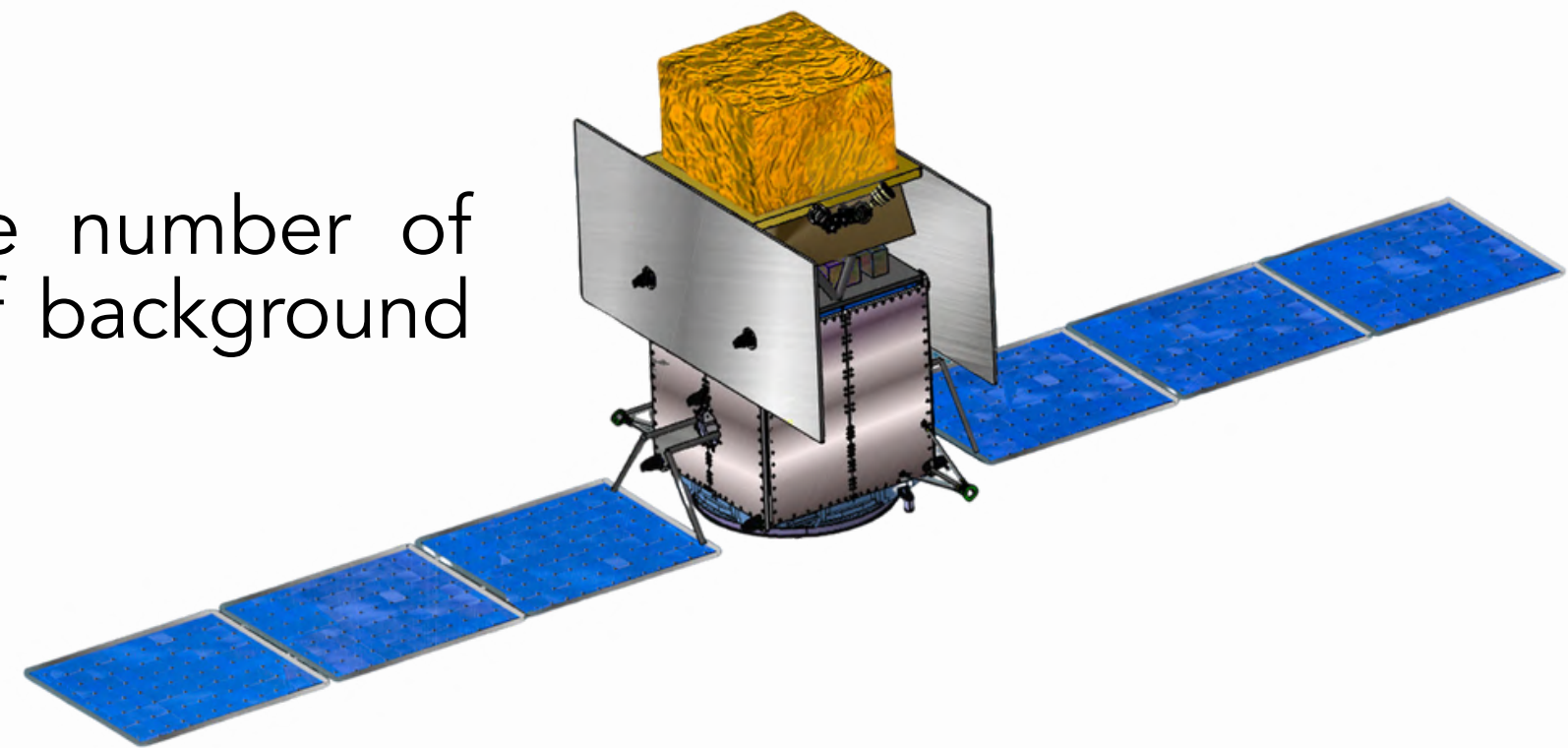


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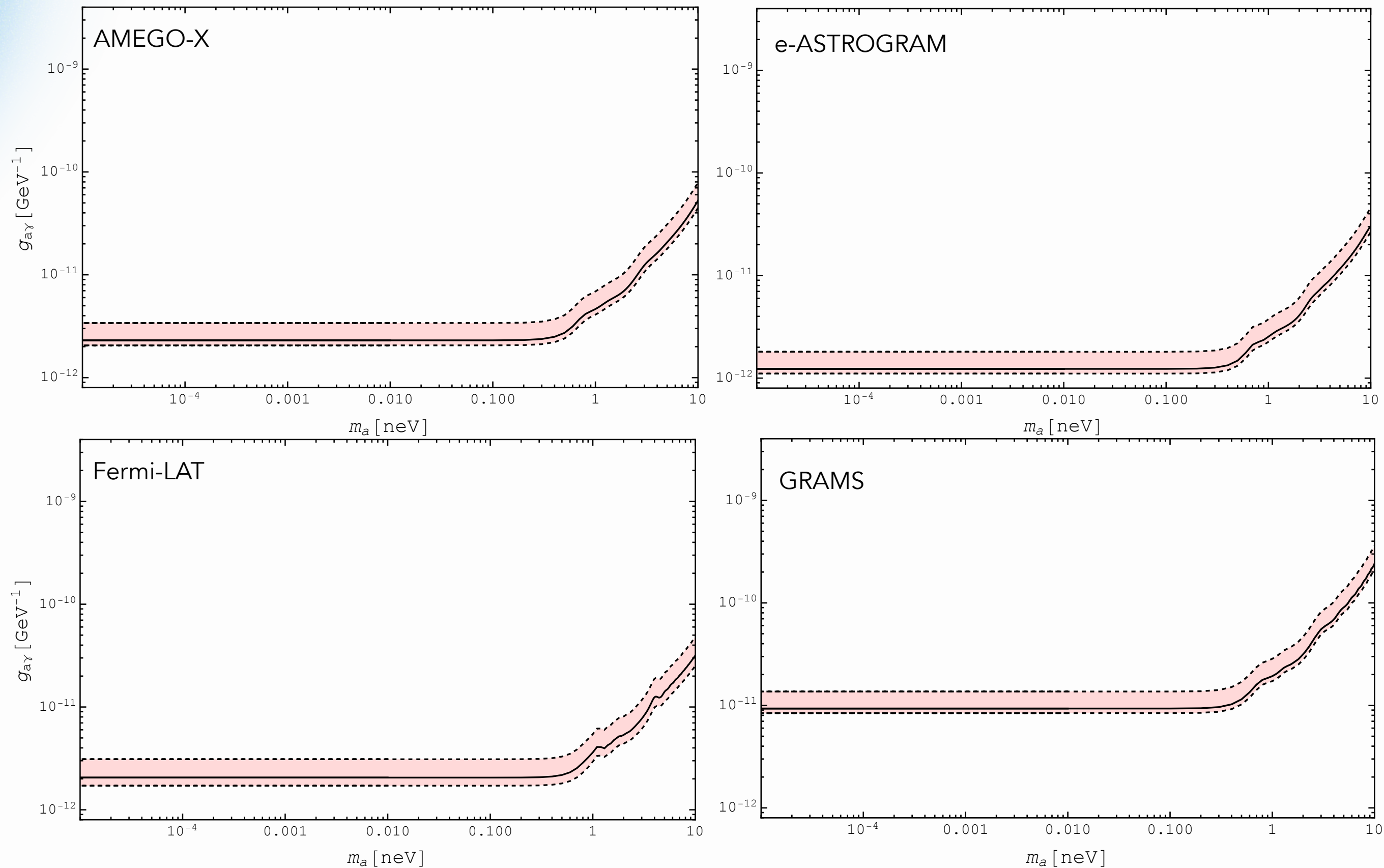
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The sensitivity was obtained by requiring that the number of expected signal events is larger than the number of background events.

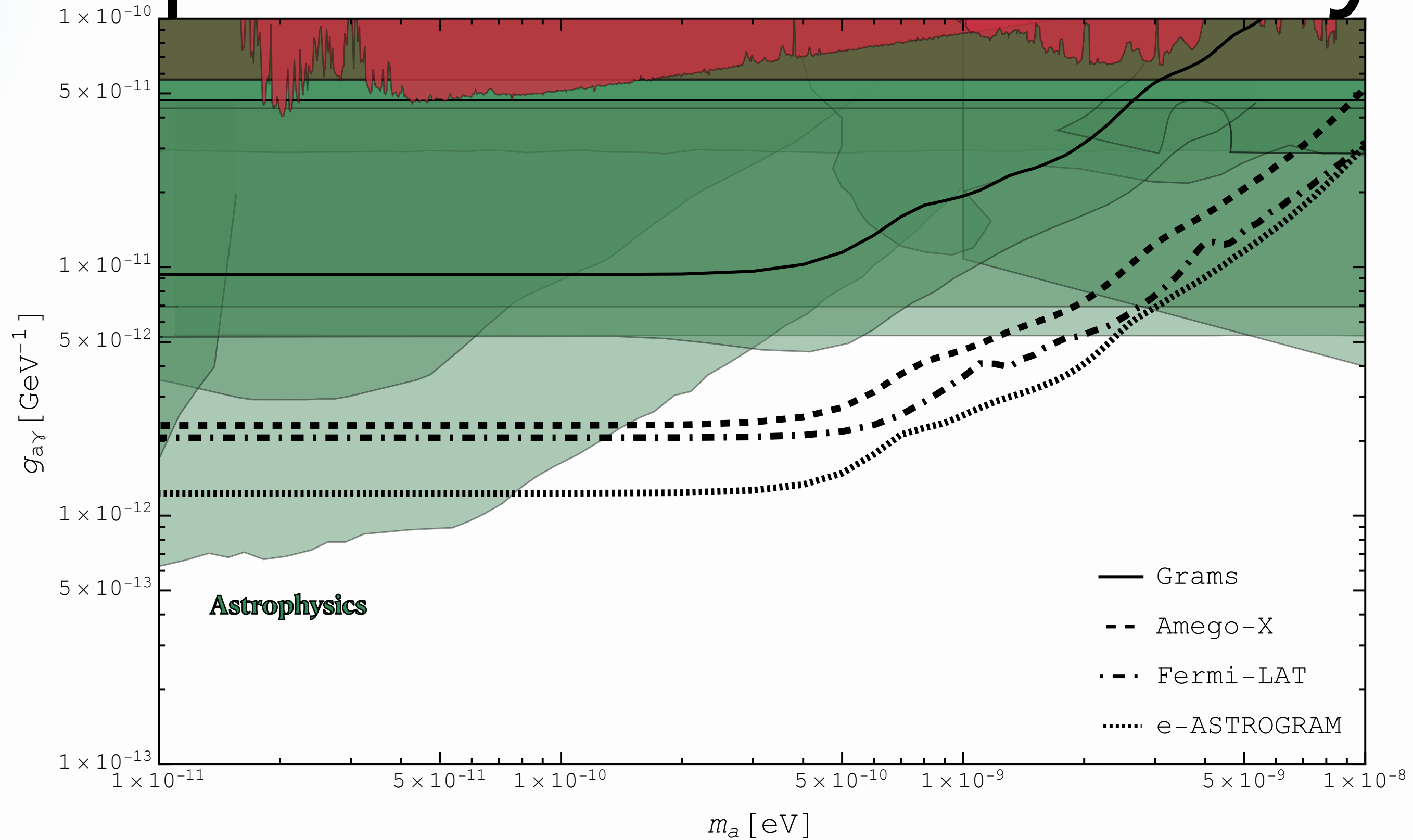
$$g_{a\gamma} \gtrsim 10^{-12} \left(\frac{N_{background}}{N_{event}} \right)^{\frac{1}{4}}$$



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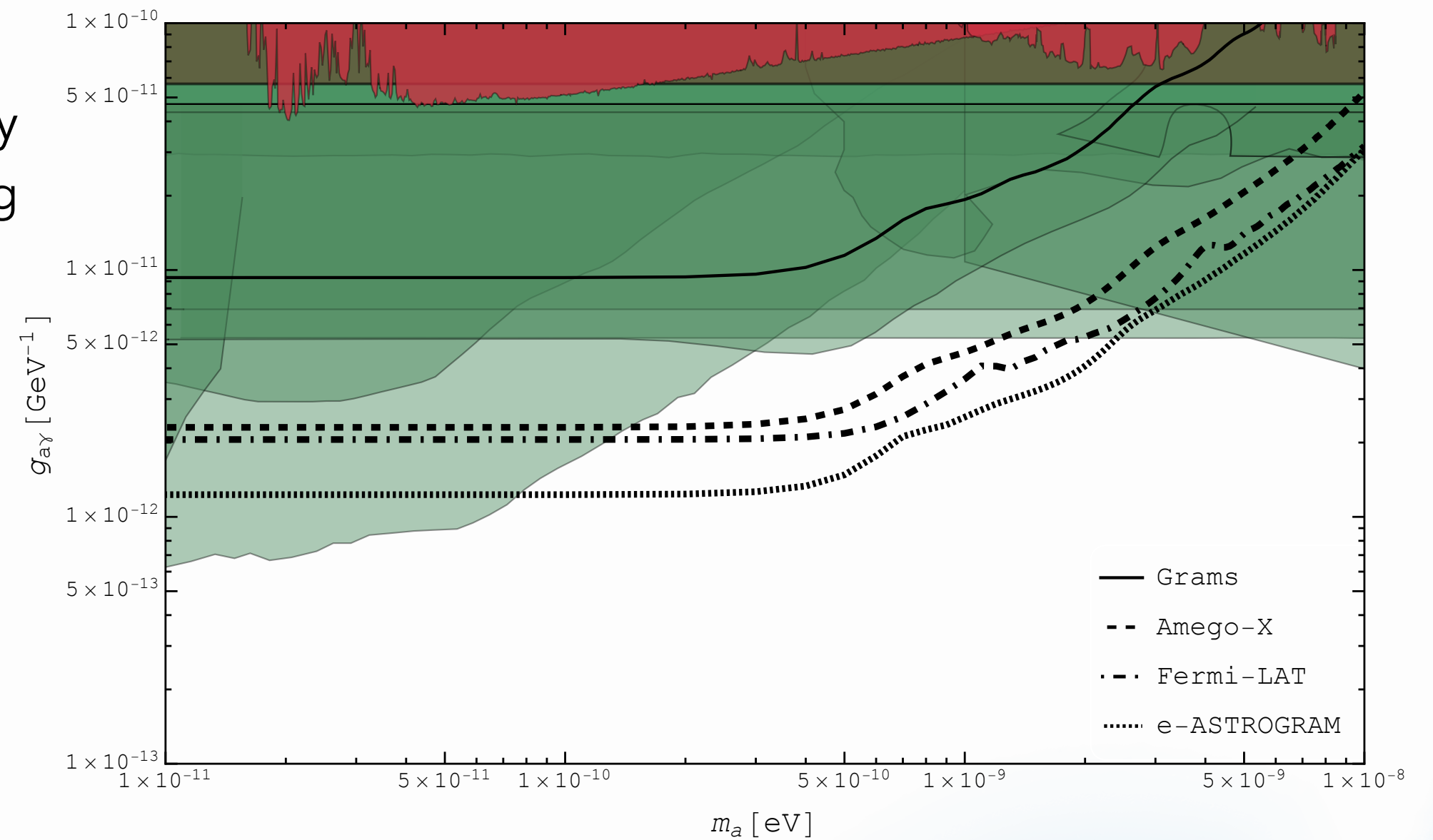


Conclusions

We have evaluated the flux of ALPs from Binary Neutron Star Merger and the corresponding gamma-ray flux reaching the Earth.

The sensitivities obtained show a potential improvement also with respect to the current gamma-ray constraints from SN 1987A [*S. Hoof and L. Schulz., Jour. of Cosm. and Astro. Phys. 2023.03, 054 (2023)*]

$$g_{a\gamma} \lesssim 4.2 \times 10^{-12} \text{GeV}^{-1}$$



What's next?

The detection rate for LIGO of a BNSM signal is

$$\mathcal{R}_{LIGO} = 0.09^{+0.12}_{-0.06} \times \left(\frac{D_r}{100 \text{ Mpc}} \right)^3 \text{ yr}^{-1}$$

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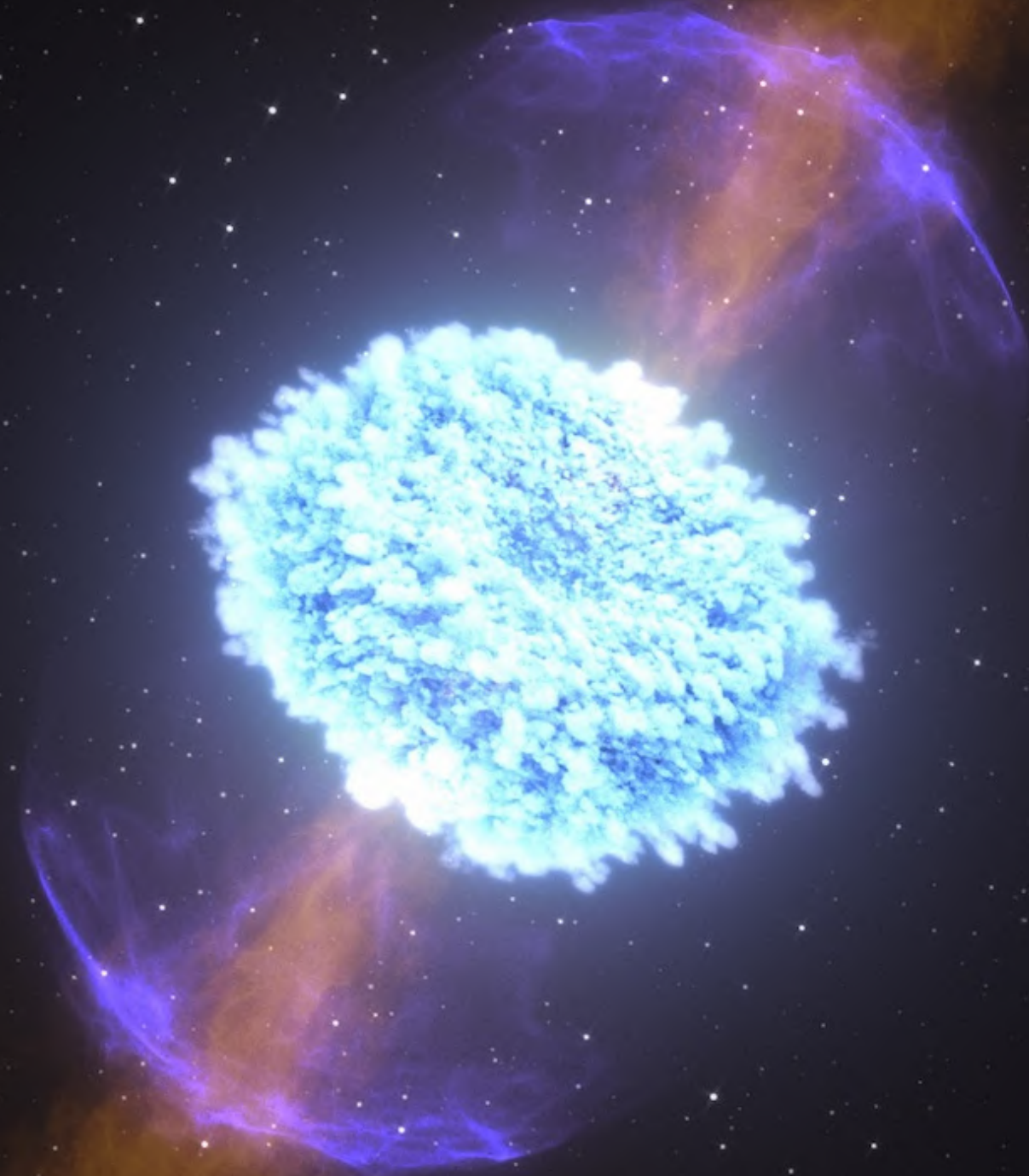
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Hence, with the current experiments and proposed ones we could see a joint event of GW and γ -ray experiment every 54 – 186 years!



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PRESENTATION BY FRANCESCA LECCE



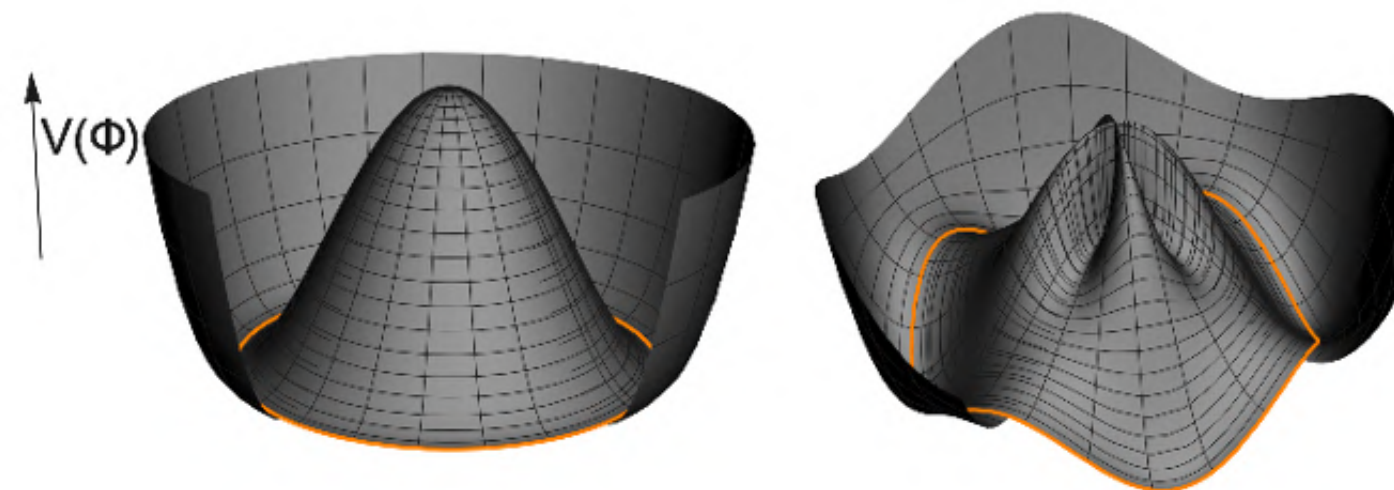
DECEMBER 17, 2024

The QCD axion

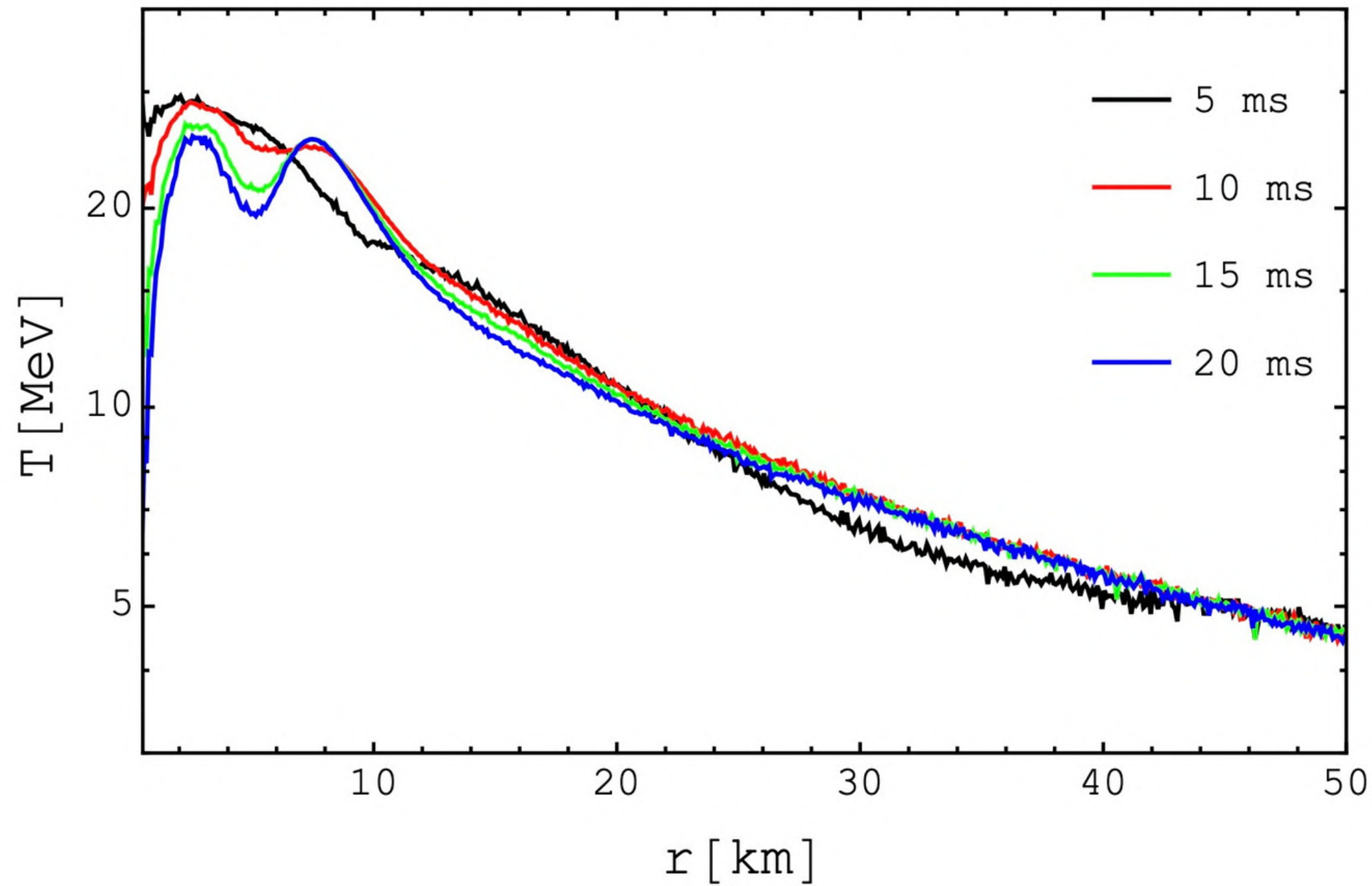
The QCD axion is a pseudoscalar particle postulated in relation to the *Peccei-Quinn (PQ) mechanism* to solve the *Strong-CP problem* in QCD.

↪ The introduction of a global symmetry $U(1)_{PQ}$ spontaneously broken at f_a and the Goldstone boson is the axion
[Peccei & Quinn, Phys. Rev. Lett. 38 (1977)]

The coupling constant depends on the energy scale

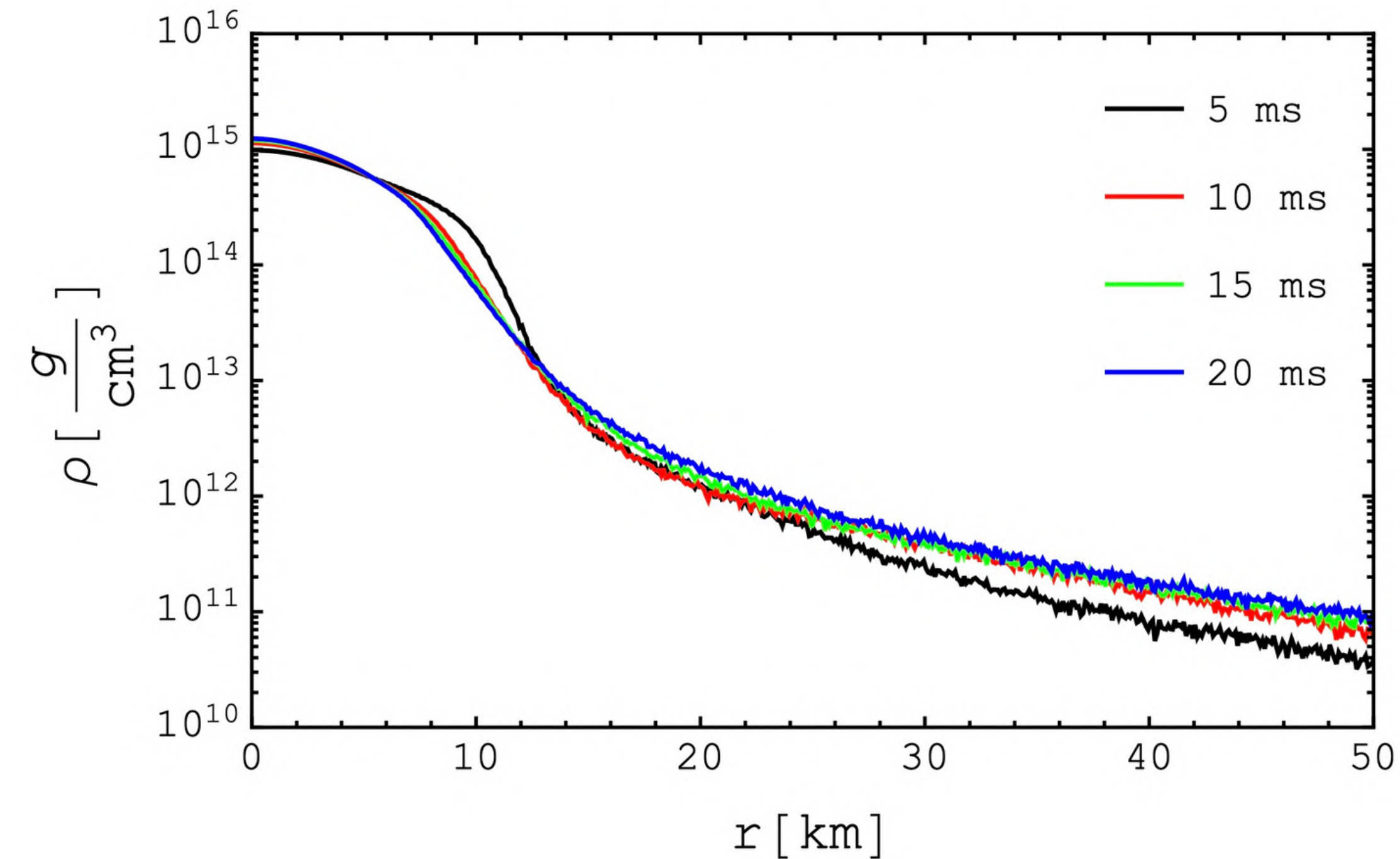


Temperature



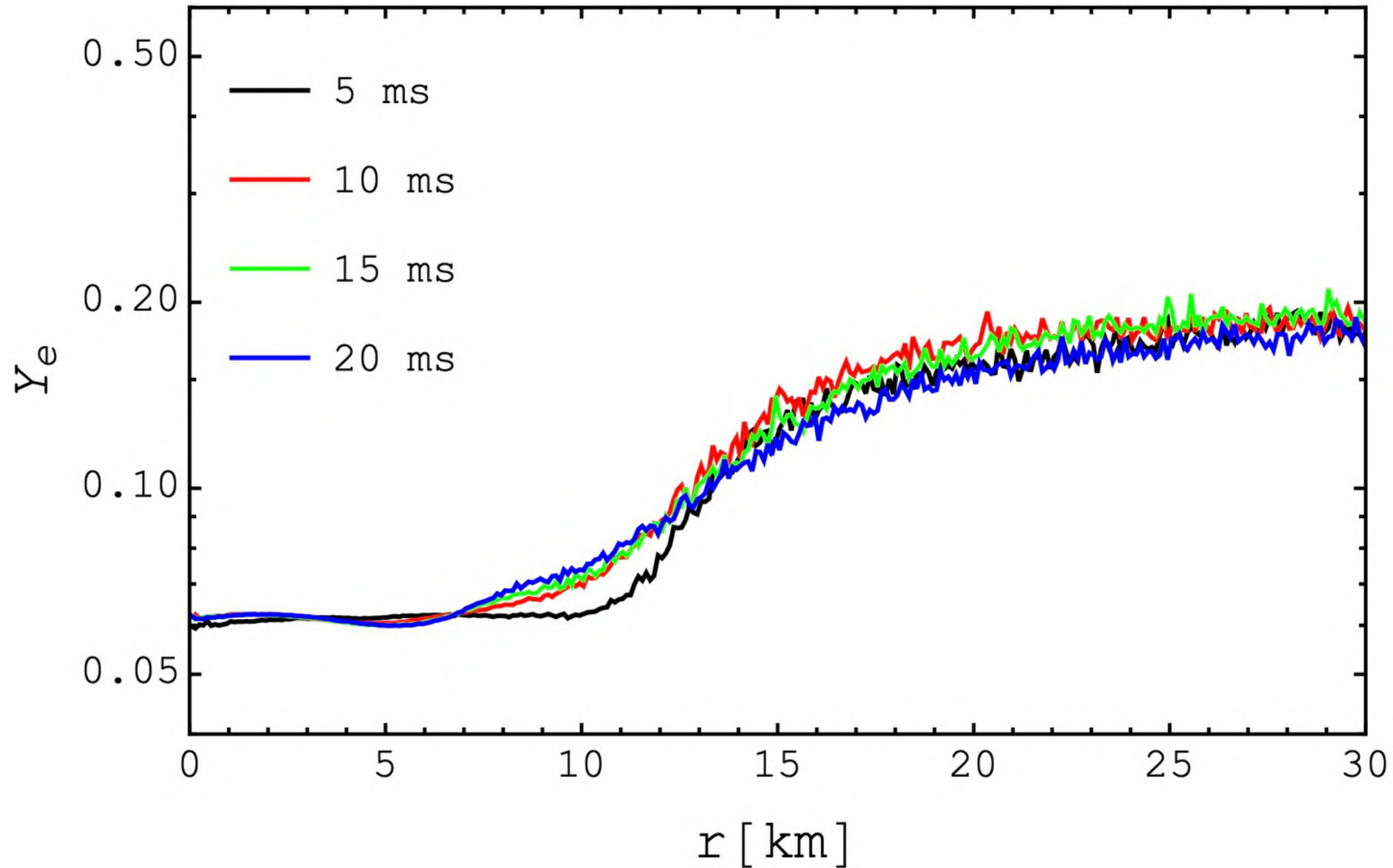
Radial evolution of the Temperature at $t=5, 10, 15, 20$ ms after merging.

Matter density



Radial evolution of the matter density at $t=5, 10, 15, 20$ ms after merging.

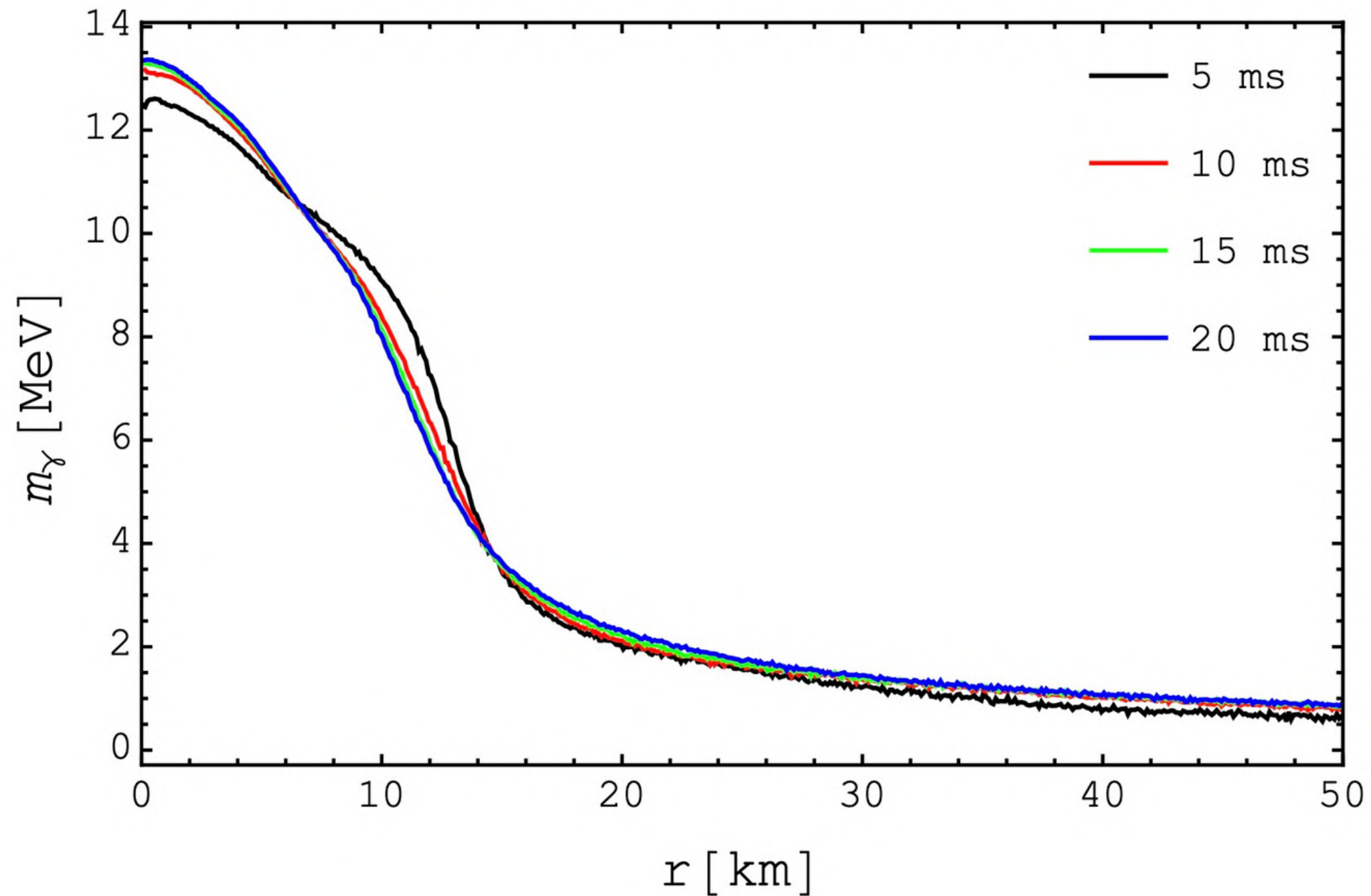
Electron fraction



Radial evolution of the electron fraction at $t=5, 10, 15, 20$ ms after merging.

$$Y_e = \frac{n_e}{n_b}$$

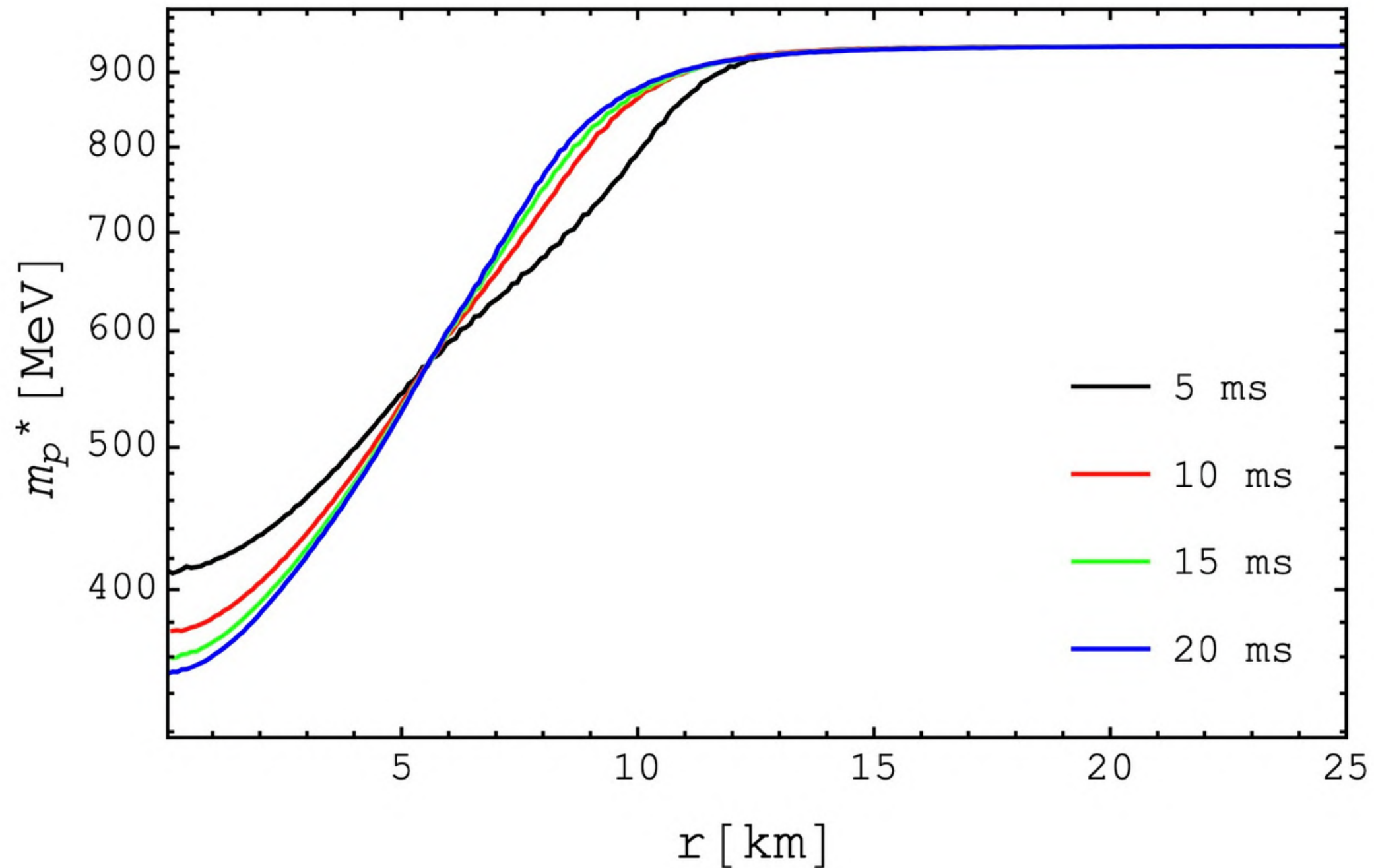
Photon mass



Radial evolution of the mass of the photon at $t=5, 10, 15, 20$ ms after merging.

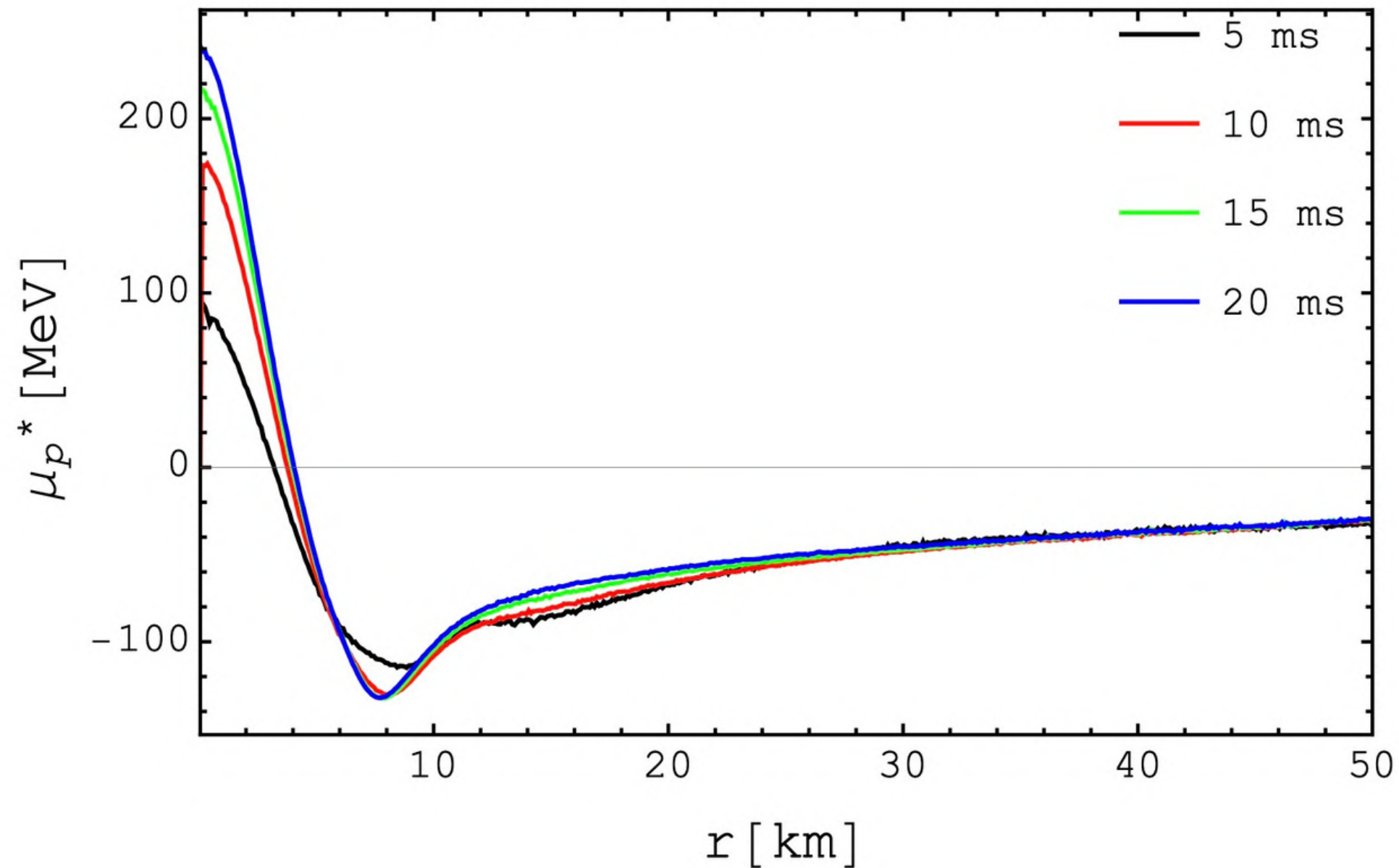
$$m_\gamma = \sqrt{\frac{3}{2}} \omega_{\text{pl}} \approx 16.3 \text{ MeV } Y_e^{1/3} \rho_{14}^{1/3}$$

Proton effective mass



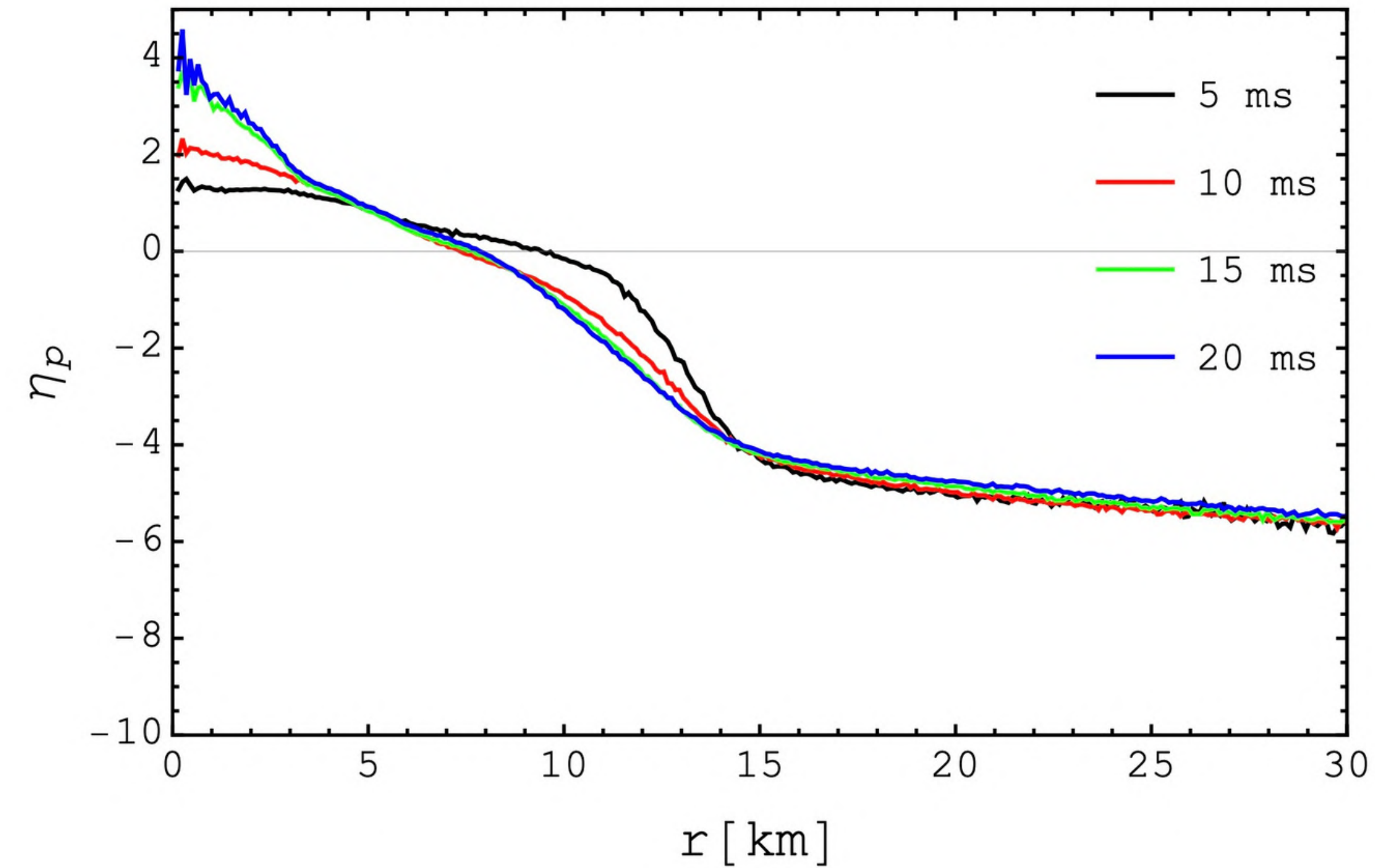
Radial evolution of the effective mass of the proton at $t=5, 10, 15, 20$ ms after merging.

Proton effective chemical potential



Radial evolution of the effective chemical potential of the proton at $t=5, 10, 15, 20$ ms after merging.

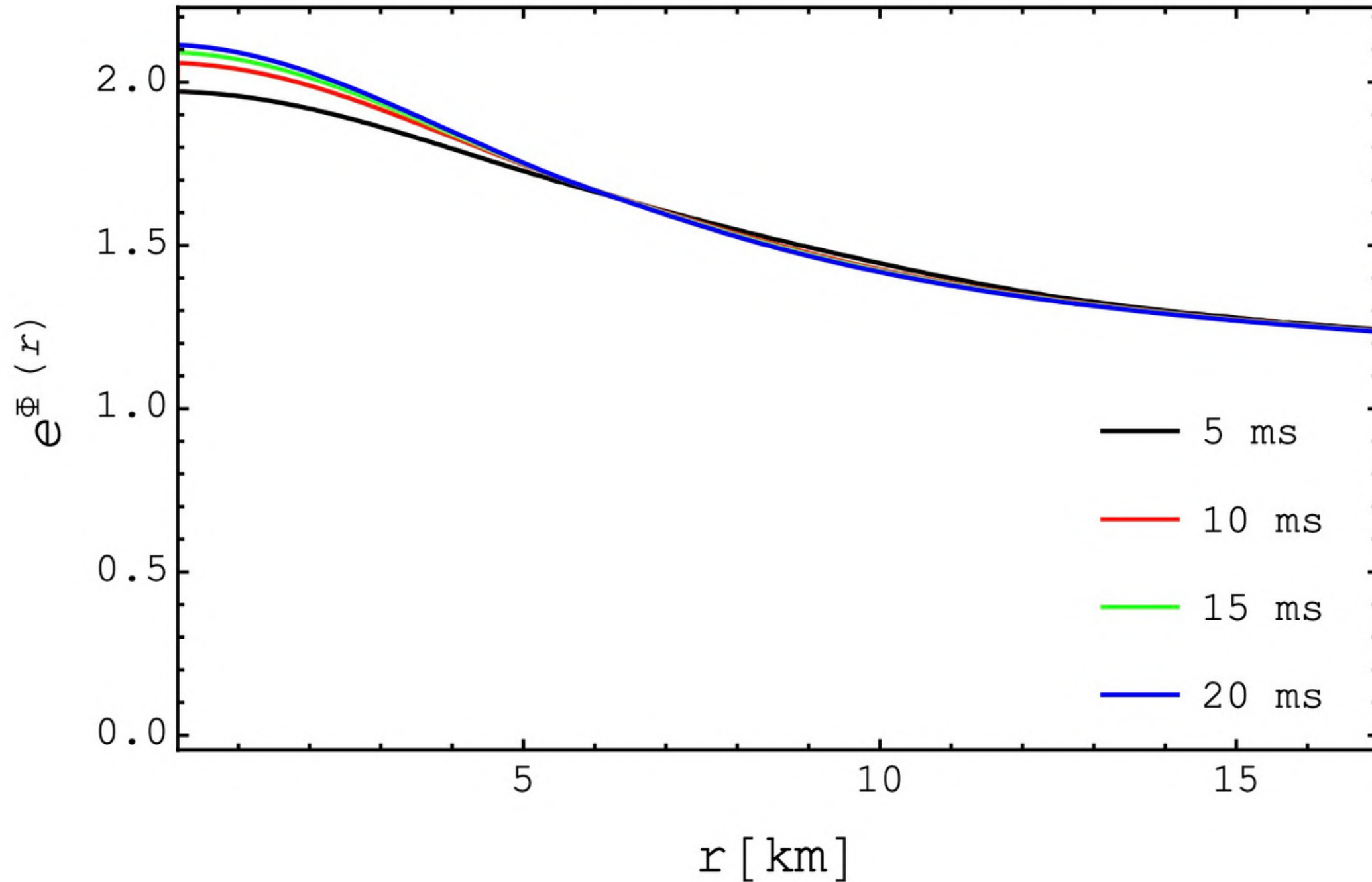
Degeneracy parameter



$$\eta_p = \frac{\mu_p^* - m_p^*}{T}$$

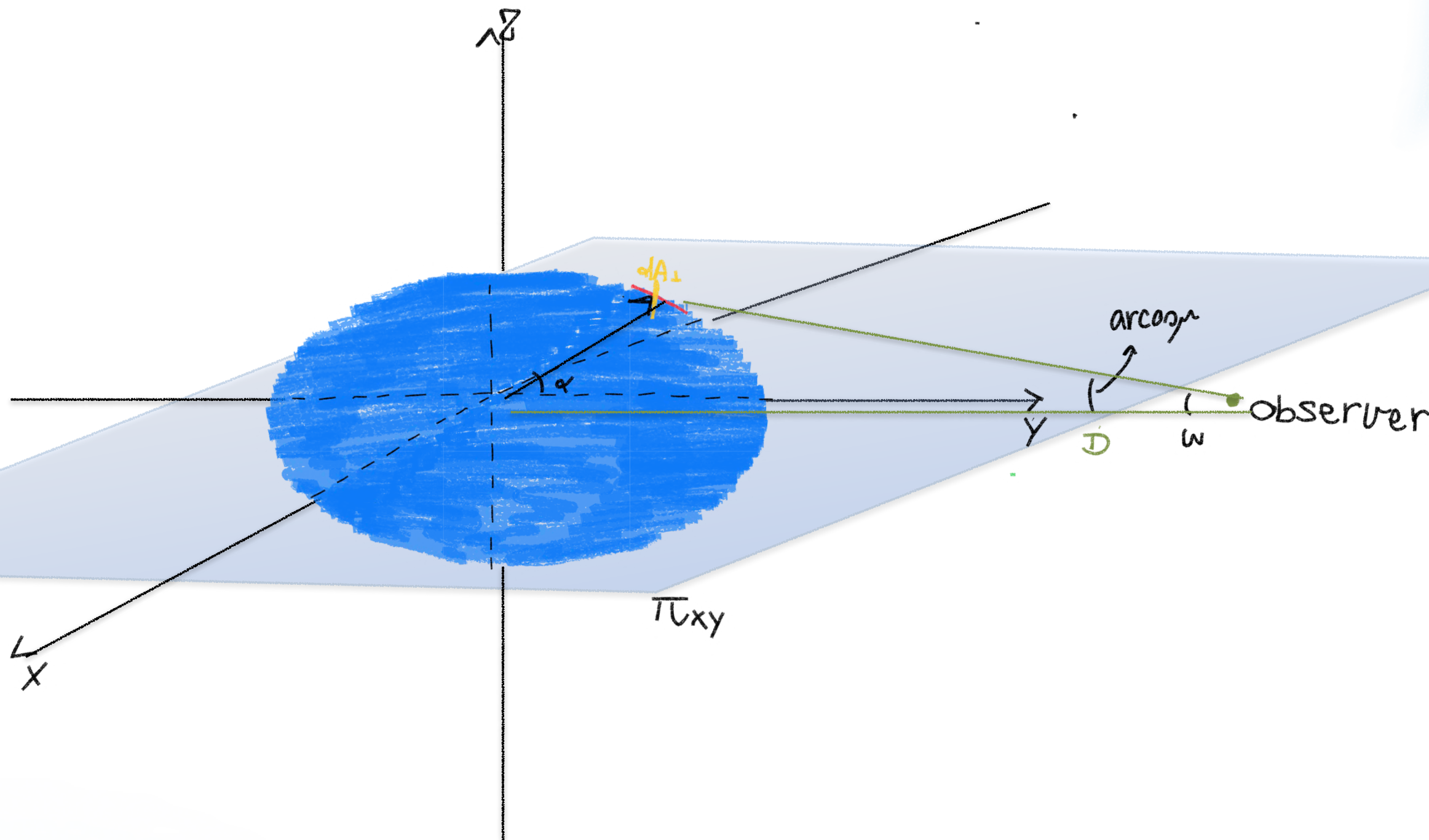
Radial evolution of the degeneracy parameter of the proton at $t=5, 10, 15, 20$ ms after merging.

Lapse factor



Radial evolution of the lapse factor at $t=5, 10, 15, 20$ ms after merging.

ALP spectrum



We are in the case $D \gg R$ and the observer is in an arbitrary direction $\Omega(\Theta, \Phi)$. The angle $\omega(\theta, \phi)$ describes the angular emission characteristic relative to the direction \vec{R} on the surface. In order to obtain the energy flux at the location of the observer we have to integrate over $d\Omega'$, the surface of the source as seen by the observer, which is given by

$$d\Omega' = \cos \gamma \frac{dA}{D^2}$$

ALP production

We know the rules concerning the external legs and the lower vertex of the diagram but we don't know the rule for the upper vertex. We know the Lagrangian interaction term

$$\mathcal{L}_I = -\frac{1}{4} g_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a$$

In order to get the Feynman rule for the vertex we can use the generating functional formalism, where the rule is associated to the truncated connected three point Green function in the momentum space.

$$\begin{aligned} G^{(3)}(y_1, y_2, y_3) &= \frac{1}{(i)^3} \frac{i}{2} \int -\frac{1}{2} g_{a\gamma\gamma} \left(\partial_\mu \frac{1}{i} D^{\mu\nu}(x - y_1) \right) \\ &\quad \times \left(\partial_\rho \frac{1}{i} D^{\rho\sigma}(x - y_2) \right) \frac{1}{i} D(x - y_3) d^4x \end{aligned}$$

Therefore, the matrix element of the process

$$-i\mathcal{M} = \bar{u}(p_3) (i\sqrt{4\pi\alpha}\gamma^\mu) u(p_1) \frac{-ig_{\mu\nu}}{q^2} (-ig_{a\gamma\gamma} \epsilon^{\alpha\nu\rho\sigma} q_\rho p_{2\sigma} \epsilon_\alpha(p_2))$$

ALP production

Summing over the spins and the two transverse polarization of photons

$$\begin{aligned} \sum_{\text{spin, pol}} |\mathcal{M}|^2 &= \frac{32\pi\alpha}{q^4} g_{a\gamma\gamma}^2 [m_p^2 m_\gamma^2 q^2 - m_p^2 (p_2 \cdot q)^2 - m_\gamma^2 (p_1 \cdot q)(p_3 \cdot q) + \\ &\quad -q^2 (p_1 \cdot p_2)(p_2 \cdot p_3) + (p_1 \cdot q)(p_2 \cdot q)(p_2 \cdot p_3) \\ &\quad + (p_2 \cdot q)(p_3 \cdot q)(p_1 \cdot p_2)] \end{aligned}$$

In this computation we are interested in the limit of infinite proton mass. The momentum transferred $q = p_1 - p_3$ becomes $q \approx (0, \mathbf{q})$

as the proton mass dominates the proton energy and allows little energy transfer.

So the matrix element becomes

$$\sum_{\text{spin, pol}} |\mathcal{M}|^2 = \frac{32\pi\alpha}{\mathbf{q}^4} g_{a\gamma\gamma}^2 m_p^2 [\mathbf{p}_2^2 \mathbf{p}_4^2 - (\mathbf{p}_2 \cdot \mathbf{p}_4)^2]$$

ALP production

We must take into account the screening of the Coulomb interaction in order to do so we have to take in the screening. Moreover to get the ALP spectrum we need to integrate over the phase space

$$\Gamma_{a\gamma} = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p_3}{(2\pi)^3} \frac{d^3 p_4}{(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{32\pi\alpha}{\mathbf{q}^2(\mathbf{q}^2 + k_s^2)} g_{a\gamma\gamma}^2 m_p^2 \\ \times \frac{[\mathbf{p}_2^2 \mathbf{p}_4^2 - (\mathbf{p}_2 \cdot \mathbf{p}_4)^2]}{16E_1 E_2 E_3 E_4} f_1 g_2 [1 - f_3]$$

Now we can work with spherical coordinates. Hence, we have the freedom to chose the coordinates such that the photon momentum lies along the z axis and the axion momentum lies in the same plane as the photon momentum.

$$\Gamma_{a\gamma} = \frac{\alpha 16 m_p^*}{128 \pi^7} g_{a\gamma\gamma}^2 \int \int \int dp_1 dp_2 dp_4 du \frac{p_1 p_2^4 p_4^4 (1 - u^2)}{E_2 E_4 (p_2^2 + p_4^2 - 2p_2 p_4 u)^{3/2}} \\ \frac{f_1 g_2 [1 - f_3]}{(p_2^2 + p_4^2 - 2p_2 p_4 u + k_s^2)}$$

ALP production

As a result we obtain, introducing also the lapse factor

$$\frac{d\Gamma_{a\gamma}}{dE_a^*} = \frac{\alpha m_p^{*2}}{8\pi^4} g_{a\gamma\gamma}^2 (E_a^*)^3 \times \int_0^\infty dp_2 \int_{-1}^1 du \frac{p_2^4 (1-u^2)}{E_2^* (p_2^2 + E_a^{*2} - 2p_2 E_a^* u)^{3/2} (p_2^2 + E_a^{*2} - 2p_2 E_a^* u + \kappa_s^2)}$$
$$\times \frac{E_2^* - T \log \xi}{(e^{\beta E_2^*} - 1)(1 - e^{\beta(E_a^* - E_2^*)})}$$

with

$$\xi = \frac{\exp[\beta(E_a + \mu_p^* - m_p^*)] + \exp\left[\beta\left(E_2 + \frac{p_{1min}^2}{2m_p^*}\right)\right]}{\exp[\beta(\mu_p^* - m_p^*)] + \exp\left[\beta\frac{p_{1min}^2}{2m_p^*}\right]}$$

ALP-photon conversion

From the wave equation for time-varying part of the vector potential and for the ALP field we obtain the following Klein-Gordon equation of motion

$$\left[E_a^2 + \partial_z^2 + \begin{pmatrix} 2E_a^2(n_{\perp} - 1) & 2E_a^2 n_R & 0 \\ 2E_a^2 n_R & 2E_a^2(n_{\parallel} - 1) & g_{a\gamma} B_T E_a \\ 0 & g_{a\gamma} B_T E_a & -m_a^2 \end{pmatrix} \right] \begin{pmatrix} A_{\perp}(z) \\ A_{\parallel}(z) \\ a(z) \end{pmatrix} = 0$$

We considered a photon beam traveling through a single magnetic domain, where the field is assumed to be homogeneous. Additionally, the optical activity is disregarded

$$n_R = 0$$

Moreover, since we are focusing on the regime where $E_a \gg m_a$ the short-wavelength approximation is valid and the Klein-Gordon can be linearized

$$\left(i \frac{d}{dz} + E_a + \mathcal{M} \right) \begin{pmatrix} A_{\parallel}(z) \\ a(z) \end{pmatrix}$$

ALP-photon conversion

it is more convenient to work with the polarization density matrix and in a single magnetic domain the mixing matrix can be brought into a diagonal form. By introducing a rotating matrix we obtain

$$D = \begin{pmatrix} \Delta_{\text{pl}} & 0 & 0 \\ 0 & \Delta_{\text{pl}} \cos^2 \theta + \Delta_{a\gamma} \sin 2\theta + \Delta_a \sin^2 \theta & -\frac{1}{2} \Delta_{\text{pl}} \sin 2\theta + \Delta_{a\gamma} (\cos^2 \theta - \sin^2 \theta) + \frac{1}{2} \Delta_a \sin 2\theta \\ 0 & (\Delta_a - \Delta_{\text{pl}}) \frac{1}{2} \sin 2\theta + \Delta_{a\gamma} \cos 2\theta & -\Delta_{\text{pl}} \sin^2 \theta - \Delta_{a\gamma} \sin 2\theta + \Delta_a \cos^2 \theta \end{pmatrix}$$

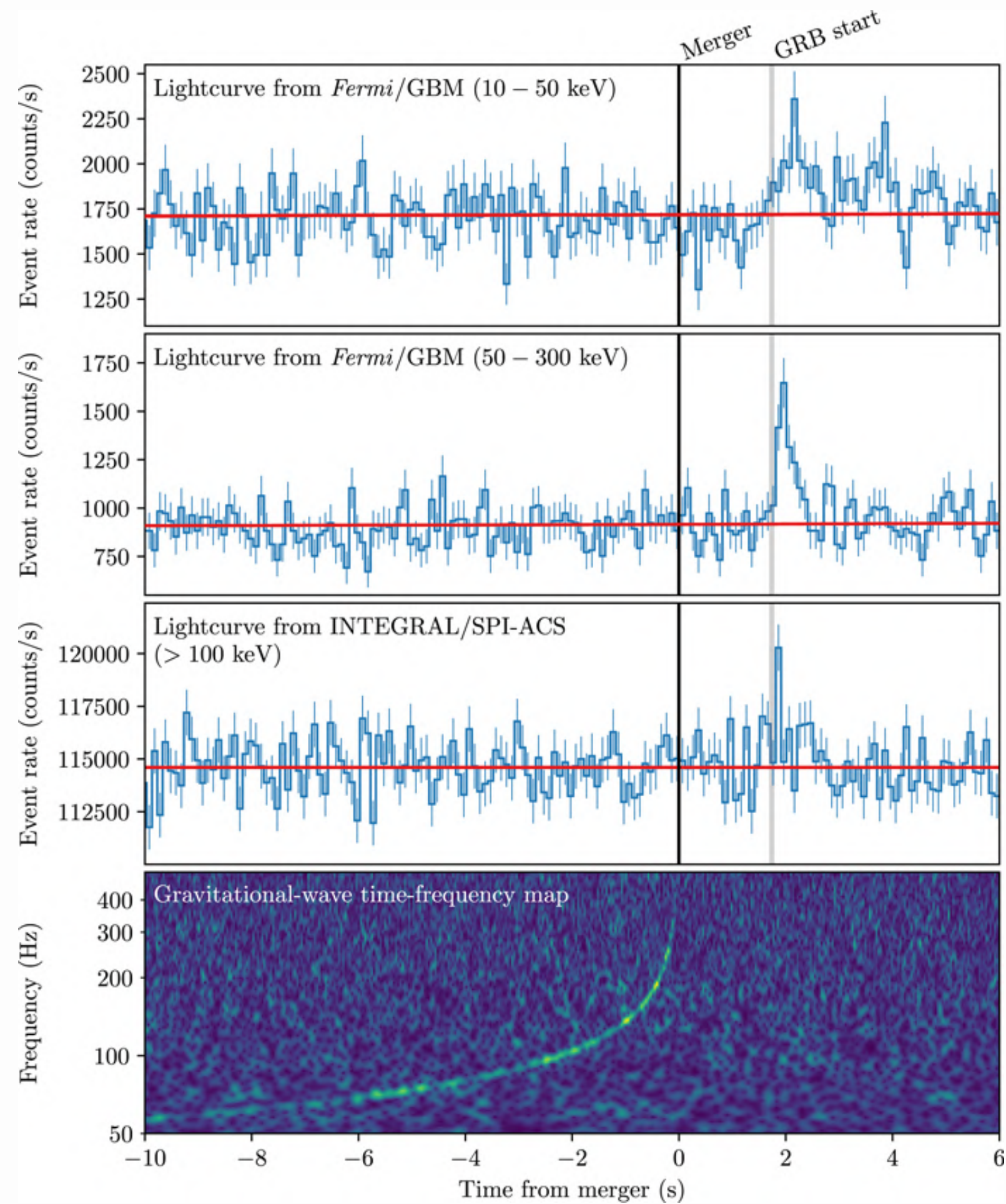
from which we obtain the mixing matrix

$$\theta = \frac{1}{2} \arctan \left(\frac{2\Delta_{a\gamma}}{\Delta_{\text{pl}} - \Delta_a} \right)$$

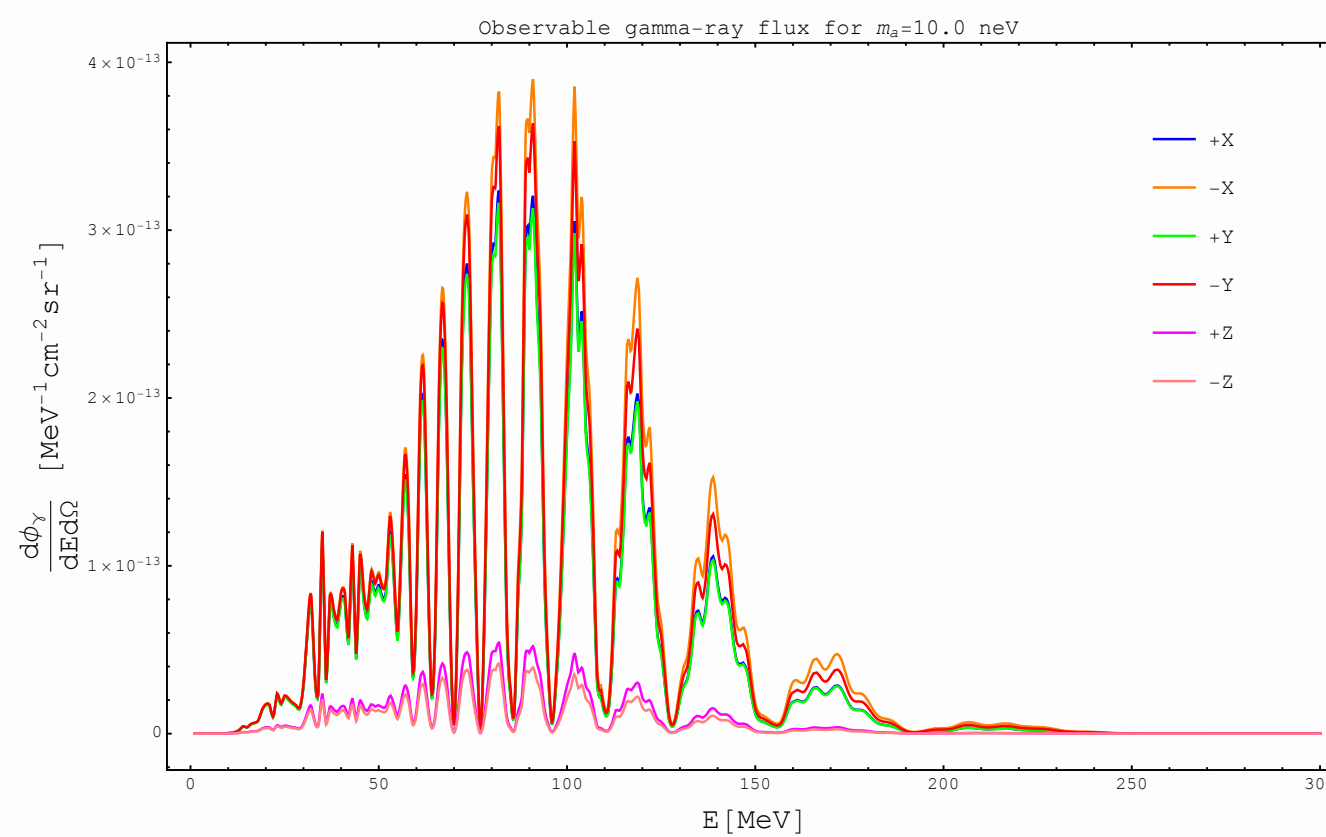
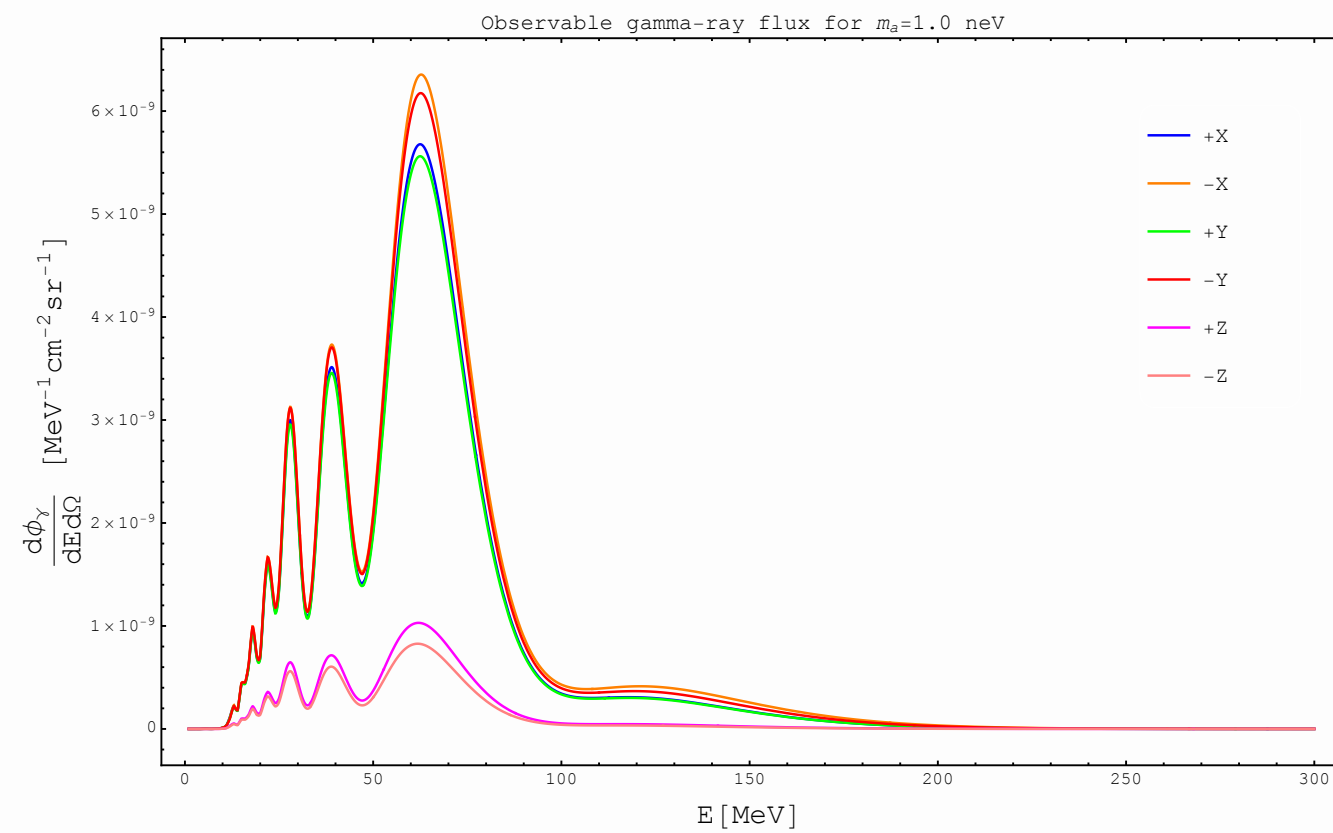
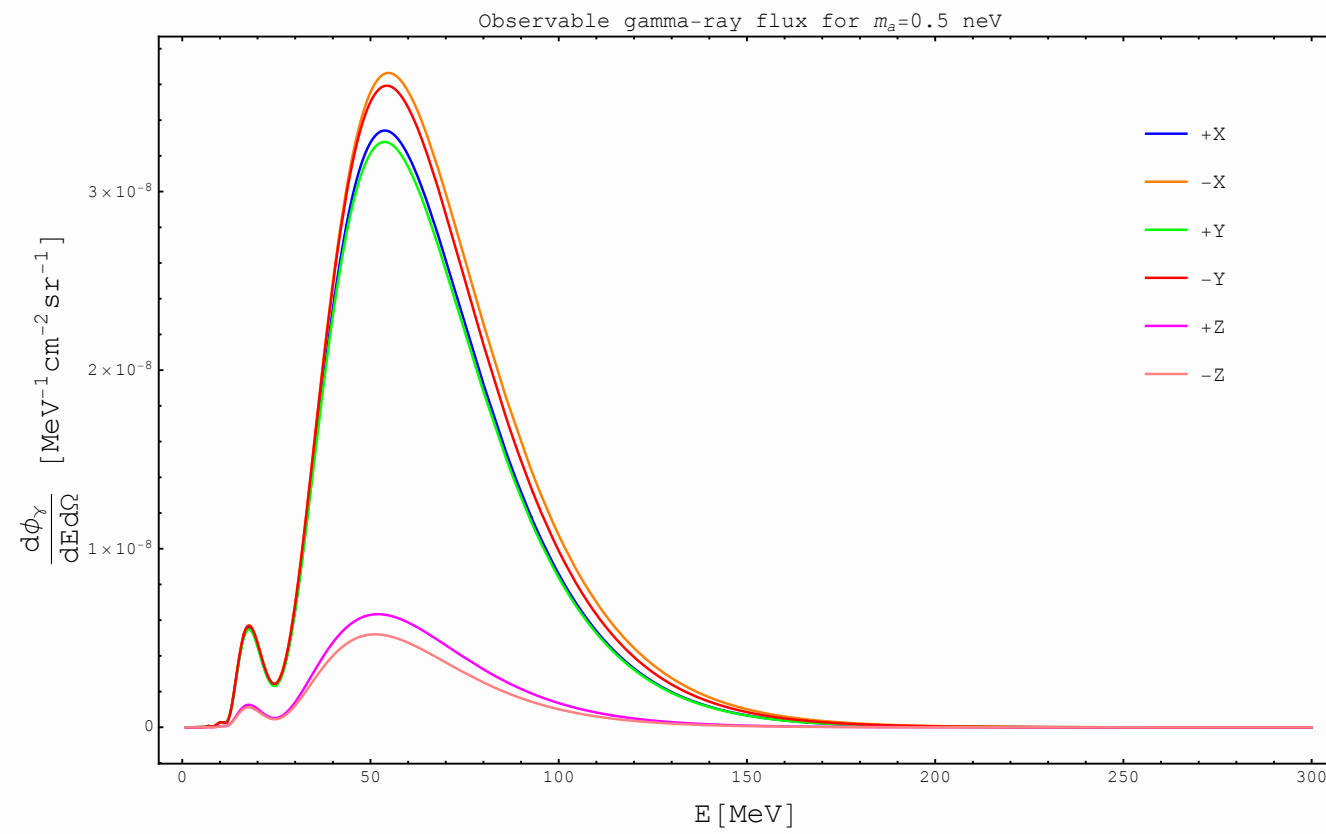
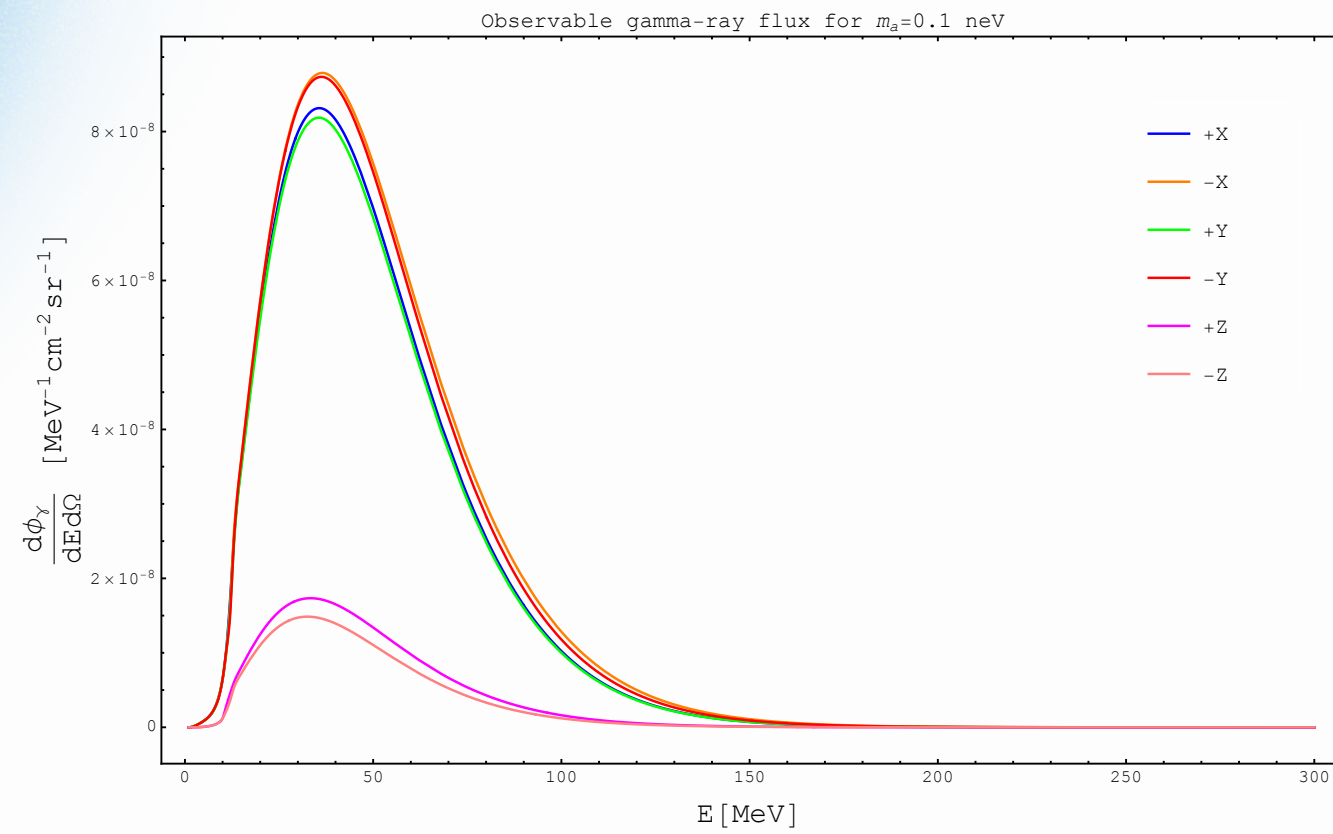
the resulting density matrix will be

$$\rho = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (e^{iD_2d} - e^{iD_3d})(e^{-iD_2d} - e^{-iD_3d}) \sin^2 \theta \cos^2 \theta & (e^{iD_2d} - e^{iD_3d})(e^{-iD_2d} \sin^2 \theta - e^{-iD_3d} \cos^2 \theta) \\ 0 & (e^{iD_2d} - e^{iD_3d})(e^{-iD_2d} - e^{-iD_3d}) \sin^2 \theta \cos^2 \theta & (e^{iD_2d} \sin^2 \theta + e^{iD_3d} \cos^2 \theta)(e^{-iD_2d} \sin^2 \theta - e^{-iD_3d} \cos^2 \theta) \end{pmatrix}$$

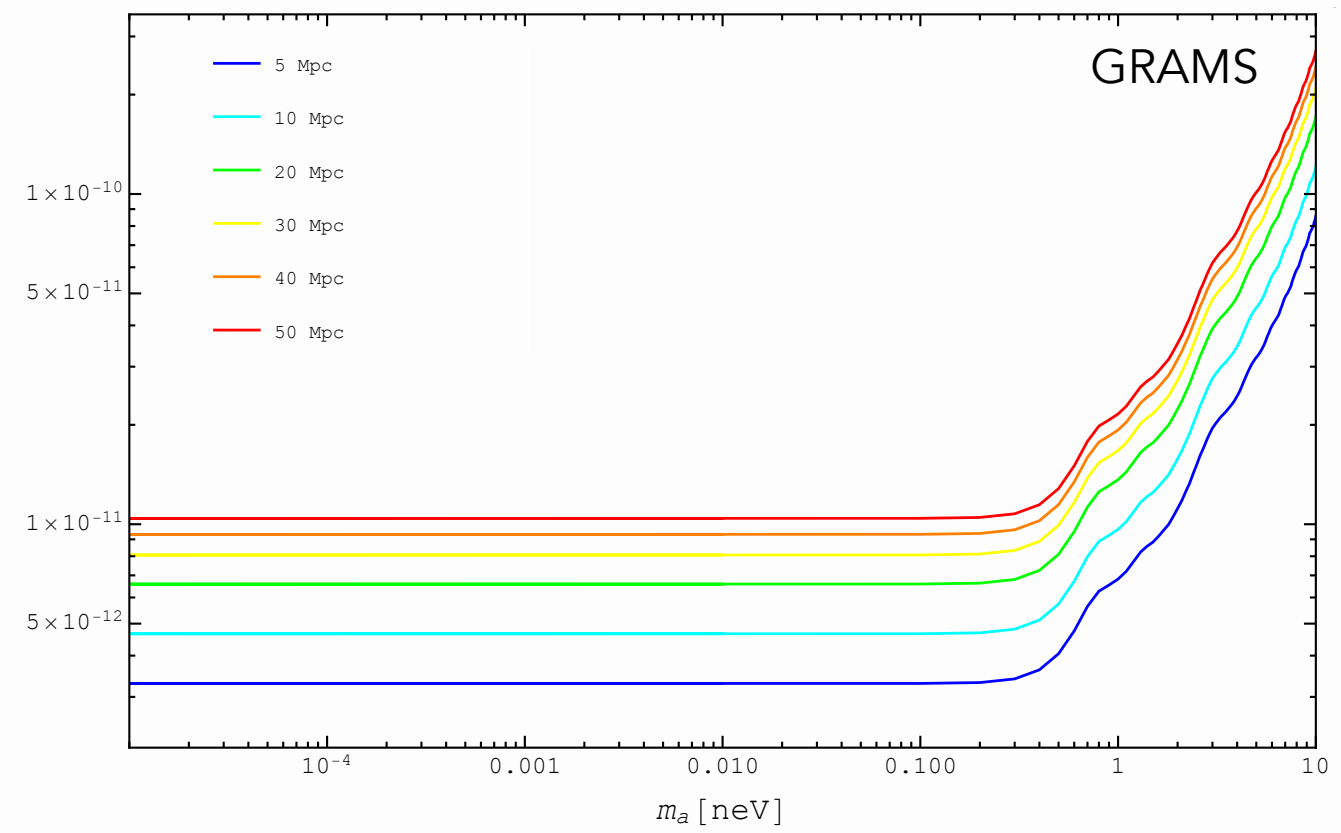
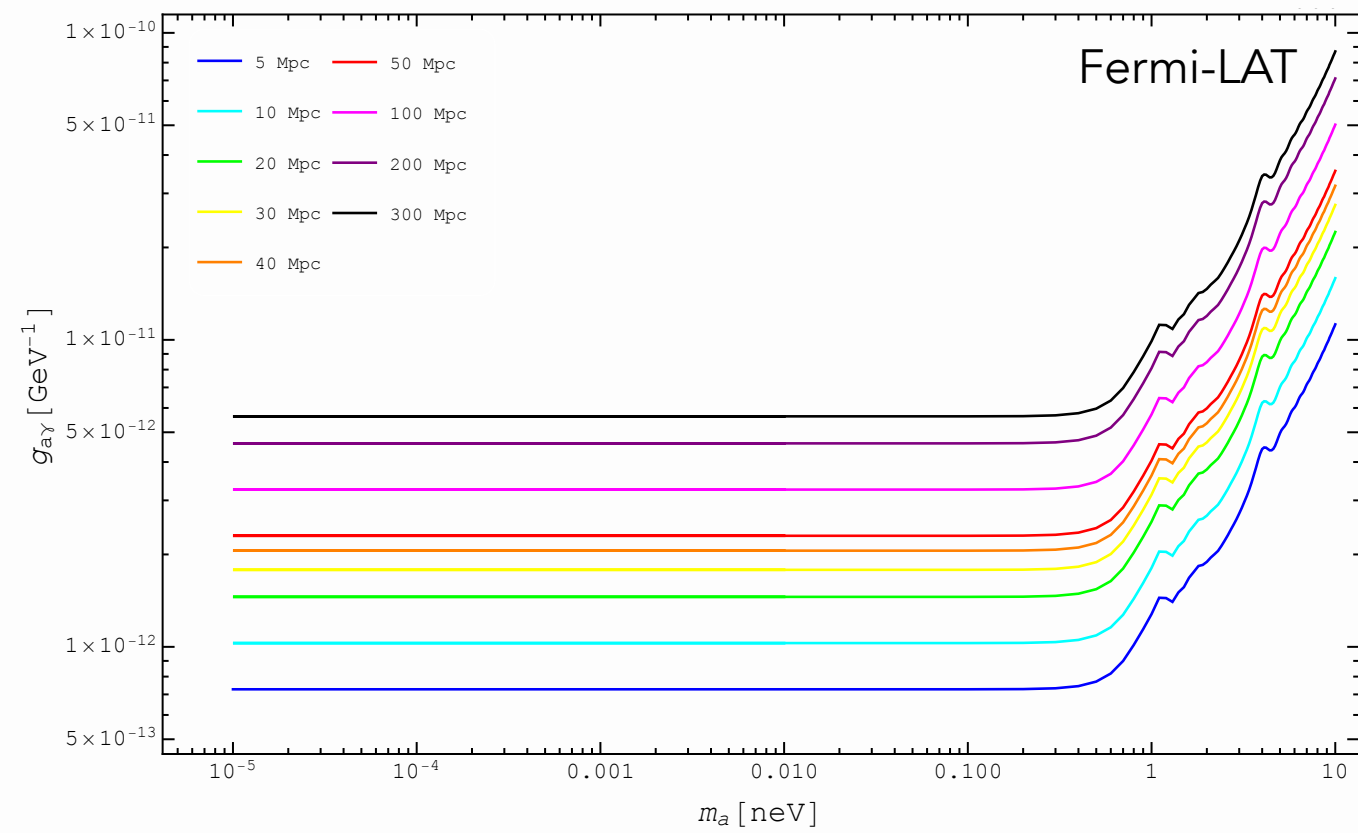
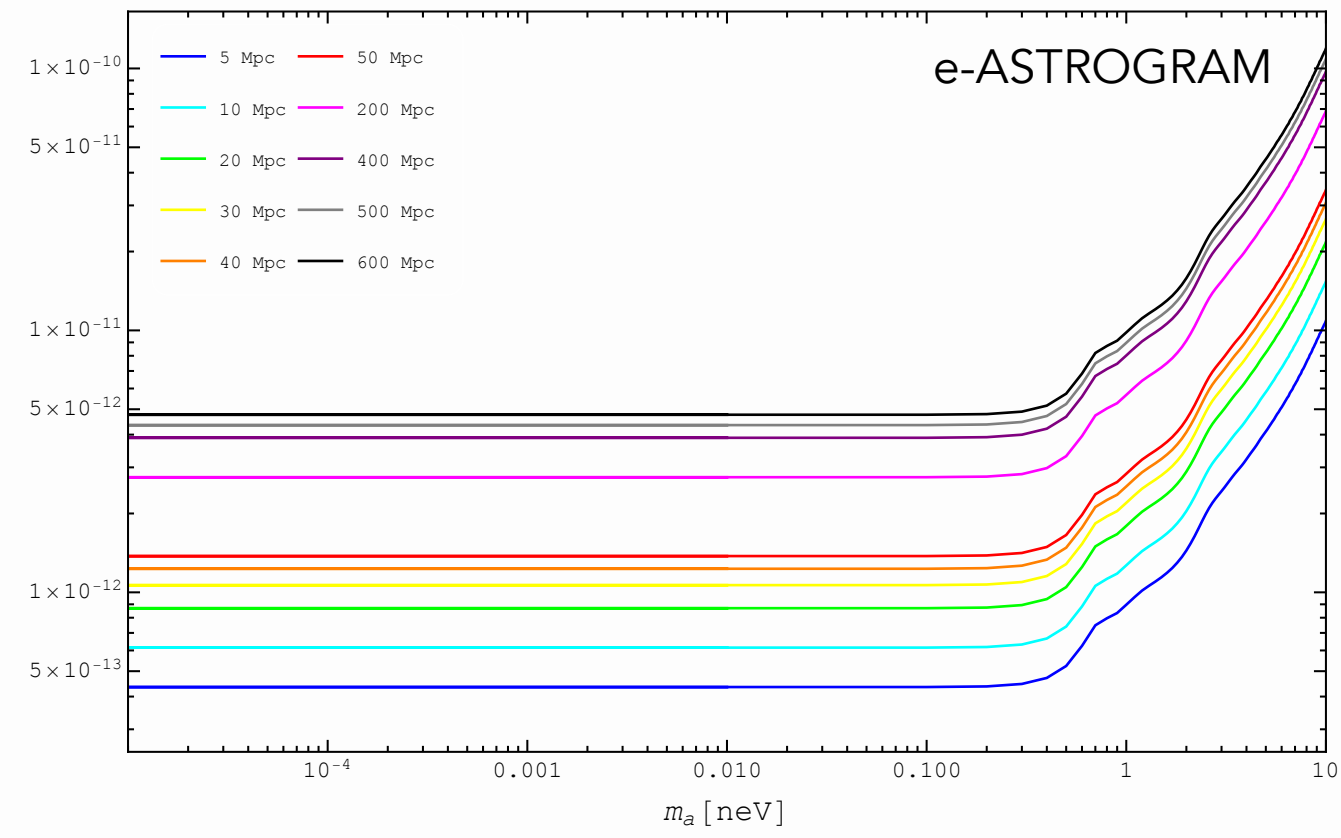
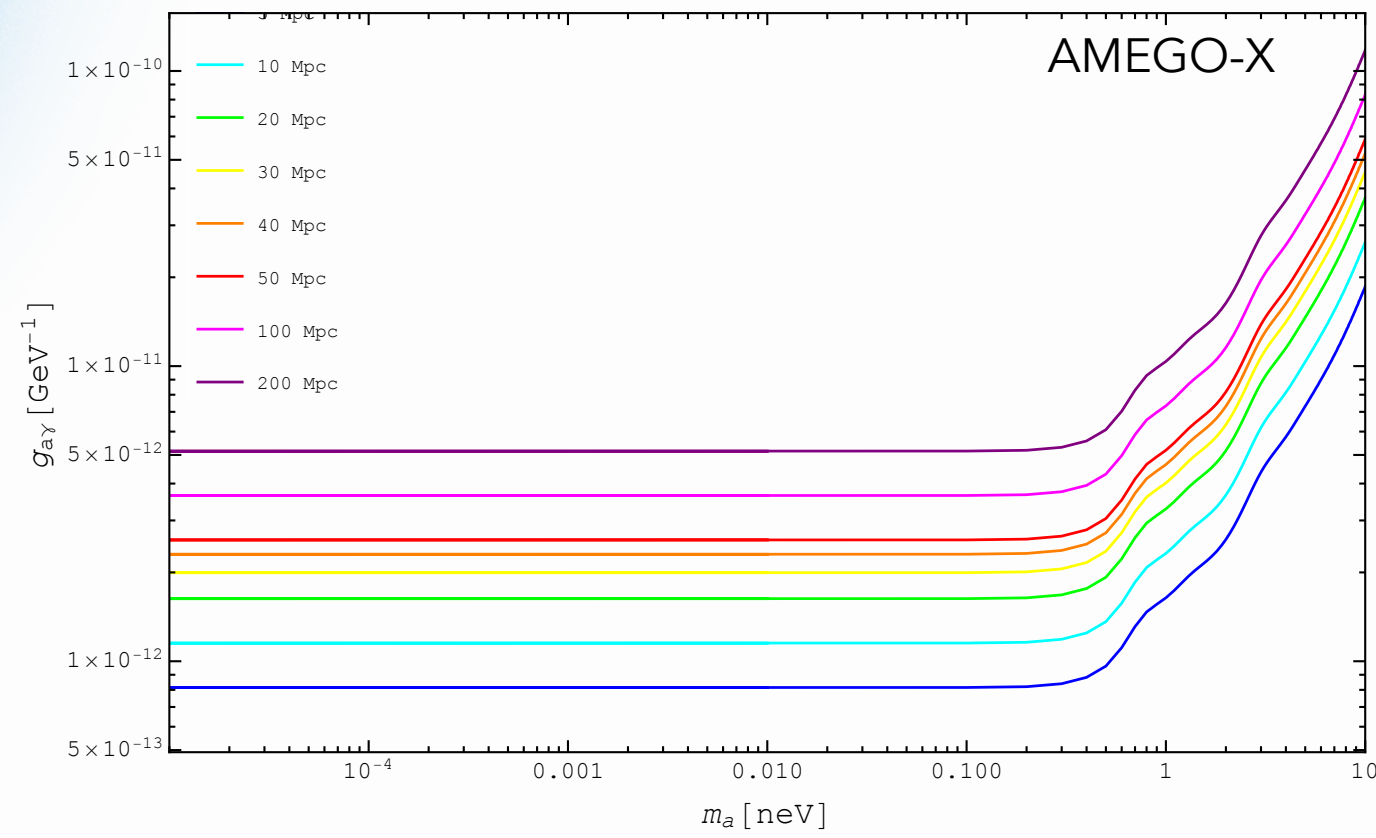
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Photon flux



Experiments sensitivity



Light-ALP parameter space

