Clustering in nuclear matter and heavy-ion collisions from kinetic approach

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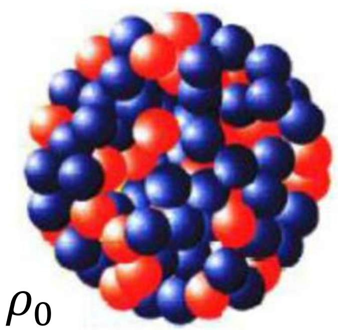
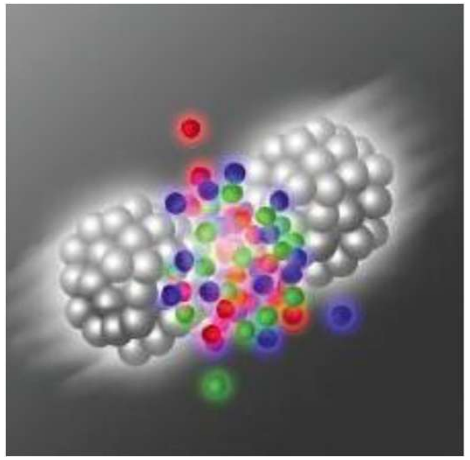
Composition of Nuclear matter

Nuclear matter exists in various environments:

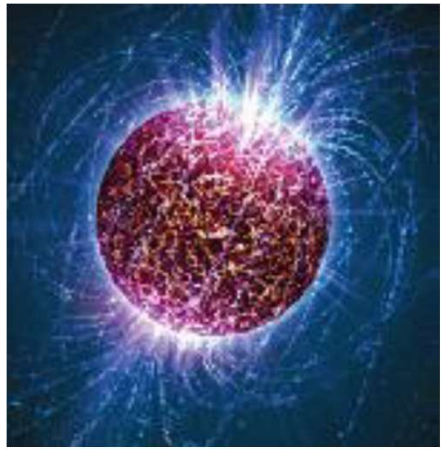


ρ_0
$0.16\ \mathrm{fm}^{-3}$

Finite nuclei

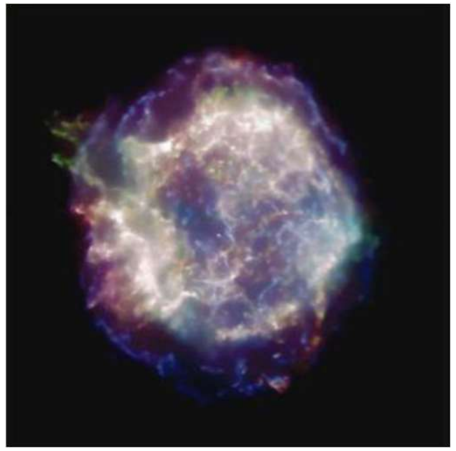


Heavy-ion collisions



Compact stars

Neutron star merges



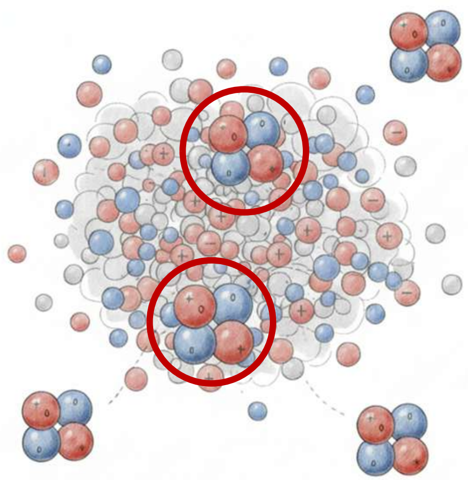
Supernova explosions

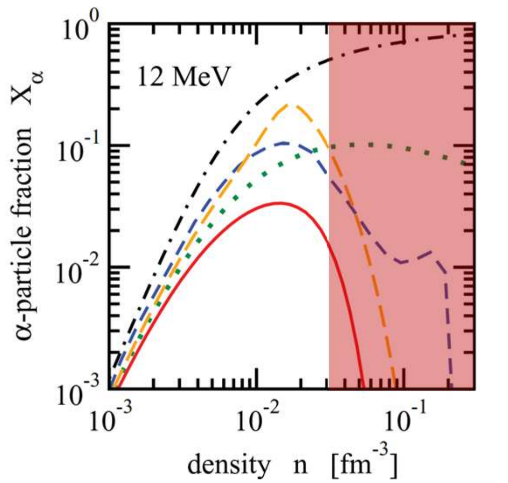
A fundamental question: what is its composition? Or the role of nuclear clustering

Nucleonic liquid?

Boltzmann gas: $\rho_\nu \sim \rho_n^N \rho_p^Z$?

Nuclear clustering may be altered by the surrounding nuclear medium





S. Typel, *et al.*, PRC81, 015803 (2010)

In-medium effects on nuclear clustering at high densities

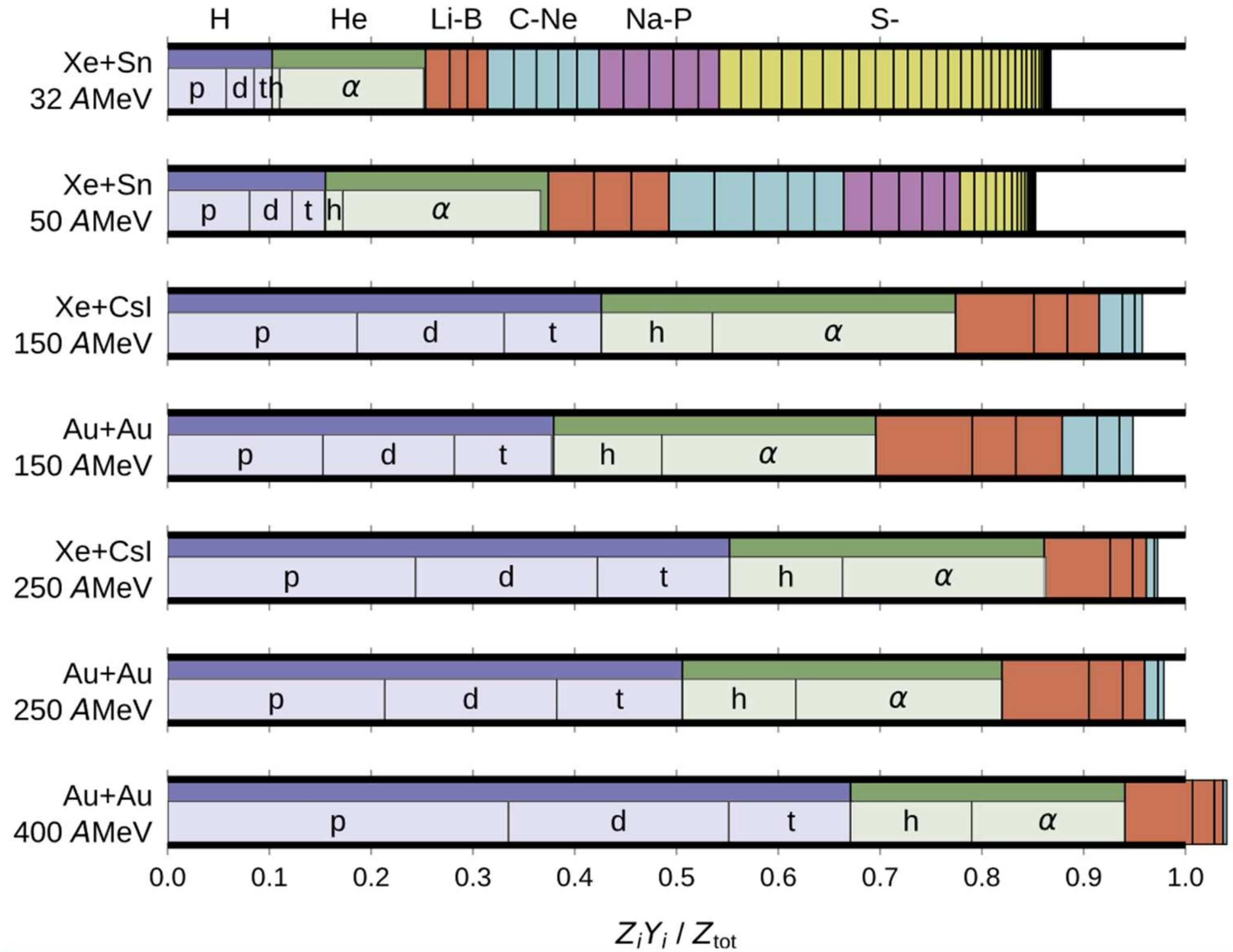
⇕

Heavy-ion collisions

Light clusters in heavy-ion collisions

Light nuclei (mass number ≤4) account for a large portion of the measured final-state charged particles

- Their production mechanism
- Their effects on the reaction dynamics and then on nucleon/pion observables
- They may provide more efficient probes of nuclear equation of state
- They may tell the composition of nuclear matter in certain conditions (in-medium effects)

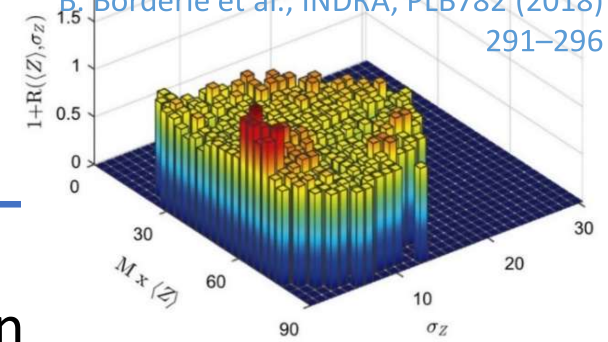


A. Ono, Progress in Particle and Nuclear Physics 105 (2019) 139–179

Realistic modelling of light clusters in heavy-ion collisions is important

Fragments in heavy-ion collisions

Spinodal decomposition -
Nearly equal-sized fragments
B. Borderie et al., INDRA, PRL86, 3252 (2001)
B. Borderie et al., INDRA, PLB782 (2018) 291–296



1. Nucleon correlation

Many-body scatterings like $NNN \leftrightarrow Nd$ In-medium effect

- Quantum/antisymmetrized molecular dynamics
  A. Ono, Journal of Physics: Conference Series 420 (2013) 012103
  H.-G. Cheng and Z.-Q. Feng, Physical Review C109, L021602 (2024)
- Kinetic approach/Boltzmann–Uehling–Uhlenbeck equation
  Light clusters *d*, *t*, ^{3}He, and ^{4}He have been included.
  P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, (1991) 712-748
  R. Wang, Y.-G. Ma, L.-W. Chen, et.al, Physical Review C108, L031601 (2023)

Final-state interactions (coalescence)

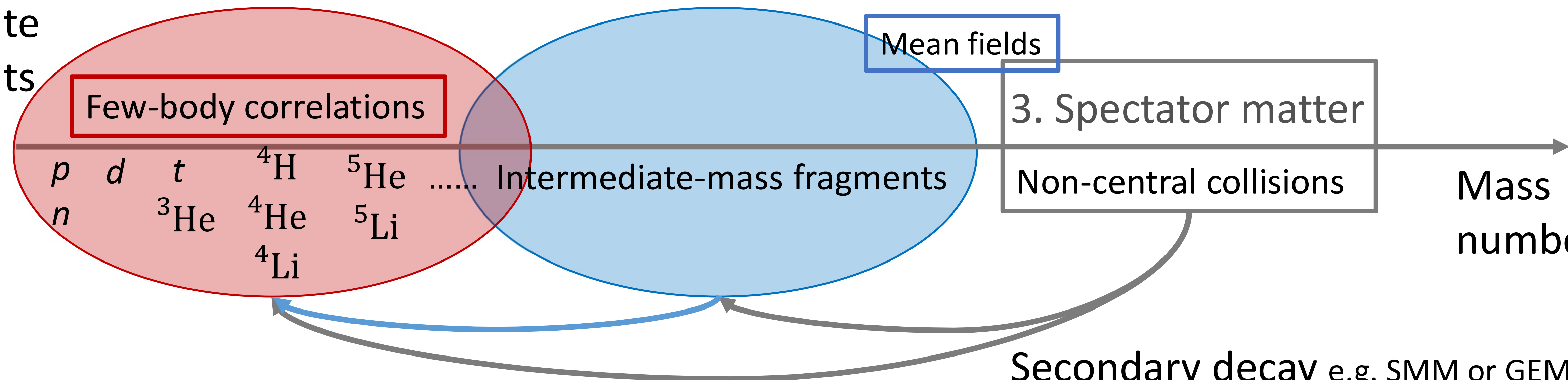
2. Spinodal decomposition

Low incident energies

Phase-space fluctuation

- e.g., Statistical mean-field (SMF) or Boltzmann-Langevin equation
  F.-S. Zhang and E. Suraud, Physical Review C51, 3201-3210 (1995)
  M. Colonna, et. al, Nuclear Physics A642 (1998) 449-460

Final-state fragments



Secondary decay e.g. SMM or GEMINI

Kinetic approach and light clusters

Kinetic equations are derived based on the closed time-path Green's function formulism

For example, in the deuteron case, the two-body Green's function G_2 satisfies an equation

$$G_2 = \mathcal{G}_2 + \frac{1}{4}\mathcal{G}_2 v G_2$$

The light nuclei are realized as poles of the many-body Green's function.
In the vicinity of the pole, we have

$$i\langle x|G_2^<(P,\Omega,R,T)|x'\rangle \sim \langle x|\phi(P,R,T)\rangle\langle\phi(P,R,T)|x'\rangle f_2(P,R,T)2\pi\delta[\Omega - E(P,R,T)]$$

$$i\langle x|G_2^>(P,\Omega,R,T)|x'\rangle \sim \langle x|\phi(P,R,T)\rangle\langle\phi(P,R,T)|x'\rangle[1 + f_2(P,R,T)]2\pi\delta[\Omega - E(P,R,T)]$$

P. Danielewicz and G. F. Bertsch, Nuclear Physics A**533**, 712-748 (1991)

Finally leads to equations of the occupation number f_τ of light nuclei

$$(\partial_t + \vec{\nabla}_p\epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r\epsilon_\tau \cdot \vec{\nabla}_p)f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \cdots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \cdots]f_\tau,$$

Single particle potential

Related to nuclear equation of state

$$\tau = n, p, d, t, h, \alpha, \pi, \Delta, \cdots$$

Kinetic equation and light clusters

Kinetic equation of the occupation number f_τ

$$(\partial_t+\vec{\nabla}_p\epsilon_\tau\cdot\vec{\nabla}_r-\vec{\nabla}_r\epsilon_\tau\cdot\vec{\nabla}_p)f_\tau=\mathcal{K}_\tau^<[f_n,f_p,f_d,\cdots](1\pm f_\tau)-\mathcal{K}_\tau^>[f_n,f_p,f_d,\cdots]f_\tau,\quad \tau=n,p,d,t,h,\alpha$$

For example, the loss term of α particles

$$\begin{aligned}K_\alpha^> f_\alpha =&\frac{\mathcal{S}_{5'}f_\alpha}{2E_\alpha}\int\prod_{i=1'}^{5'}\frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i}\frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N}\overline{|\mathcal{M}_{N\alpha\to NNNNN}|^2}g_N f_N\prod_{i=1'}^{5'}(1\pm f_i)(2\pi)^4\delta^4(\sum_{i=1'}^{5'}p_i-p_N-p_\alpha)\\ &+\frac{\mathcal{S}_{3'}f_\alpha}{2E_\alpha}\int\prod_{i=1'}^{3'}\frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i}\frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N}\overline{|\mathcal{M}_{N\alpha\to NNt}|^2}g_N f_N\prod_{i=1'}^{3'}(1\pm f_i)(2\pi)^4\delta^4(\sum_{i=1'}^{3'}p_i-p_N-p_\alpha)+t\to h\\ &+\frac{\mathcal{S}_{2'}f_\alpha}{2E_\alpha}\int\prod_{i=1'}^{2'}\frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i}\frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N}\overline{|\mathcal{M}_{N\alpha\to dt}|^2}g_N f_N\prod_{i=1'}^{2'}(1\pm f_i)(2\pi)^4\delta^4(\sum_{i=1'}^{2'}p_i-p_N-p_\alpha)+t\to h\\ &+\text{elastic part.}\end{aligned}$$

Light nuclei can be produced and dissociated through many-body scatterings (for intermediate energies, red ones are needed)

- A = 2 $\pi NN\leftrightarrow\pi d$, $NNN\leftrightarrow Nd$
- A = 3 $\pi NNN\leftrightarrow\pi t(h)$, $\pi Nd\leftrightarrow\pi t(h)$, $NNNN\leftrightarrow Nt(h)$, $NNd\leftrightarrow Nt(h)$
- A = 4 $\pi NNNN\leftrightarrow\pi\alpha$, $\pi NNd\leftrightarrow\pi\alpha$, $\pi Nt(h)\leftrightarrow\pi\alpha$, $NNNNN\leftrightarrow N\alpha$, $NNNd\leftrightarrow N\alpha$, $NNt(h)\leftrightarrow N\alpha$, $dt(h)\leftrightarrow N\alpha$

Impulse approximation

- Many body **transition amplitudes** e.g., $\overline{|M_{Npn\leftrightarrow Nd}|^2}$
- The in-medium effects on light clusters – **Mott effect**

See also pBUU, AMD, SMASH, PHQMD, and LQMD

RW, Y.-G. Ma, L.-W. Chen, *et al.*, Physical Review C**108**, L031601 (2023)

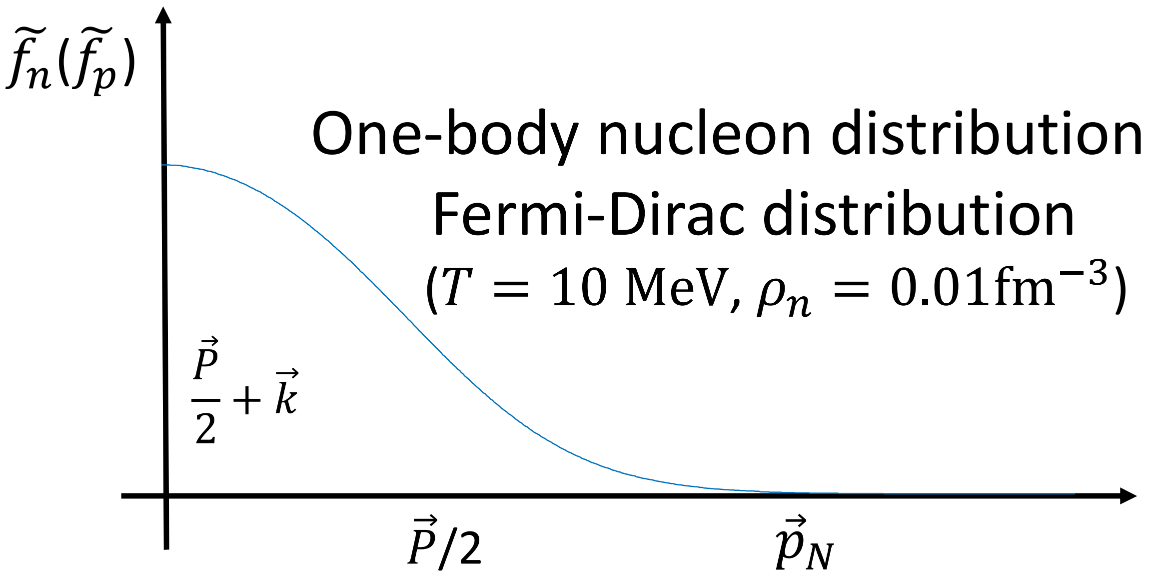
In-medium Schrodinger equation

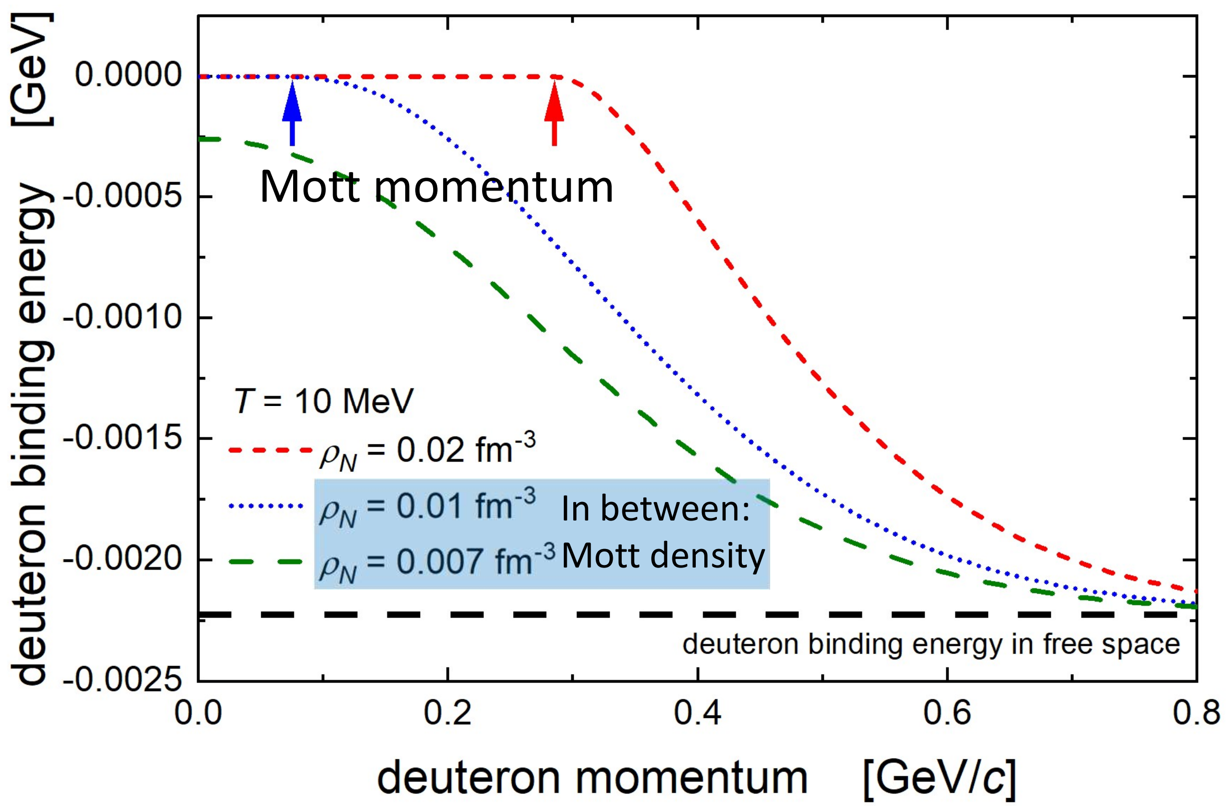
Originates from the Green's function formalism

$$\left[\epsilon_{\tau_1}(\vec{p}_1)+\epsilon_{\tau_2}(\vec{p}_2)+\cdots+\epsilon_{\tau_A}(\vec{p}_A)-\epsilon_\nu(\vec{P})\right]\Psi_{\nu,\vec{P}}(1,2,\cdots,A)$$
$$+\sum_{i<j}\left[1-\tilde{f}_{\tau_i}(\vec{p}_i)-\tilde{f}_{\tau_j}(\vec{p}_j)\right]\int\prod_{l}^{1',2',\cdots,A'}\frac{\mathrm{d}\vec{p}_l}{(2\pi\hbar)^3}V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\Psi_{\nu,\vec{P}'}(1',2',\cdots,A')=0$$

Mott effect: the binding energy of a light cluster is modified by its surrounding nucleons

Right: the deuteron binding energy as a function of its momentum P in nuclear medium, the Mott momentum is recognized as where the binding energy vanishes.





In-medium effects on light nuclei

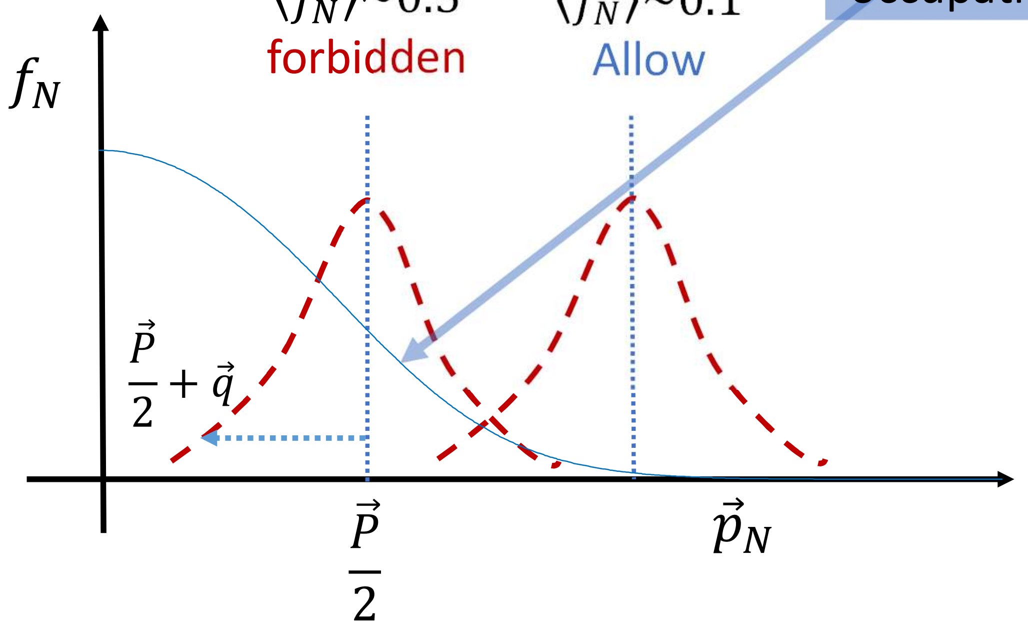
The Mott effect can be effectively introduced into the kinetic approach through a phase-space cutoff parameter.

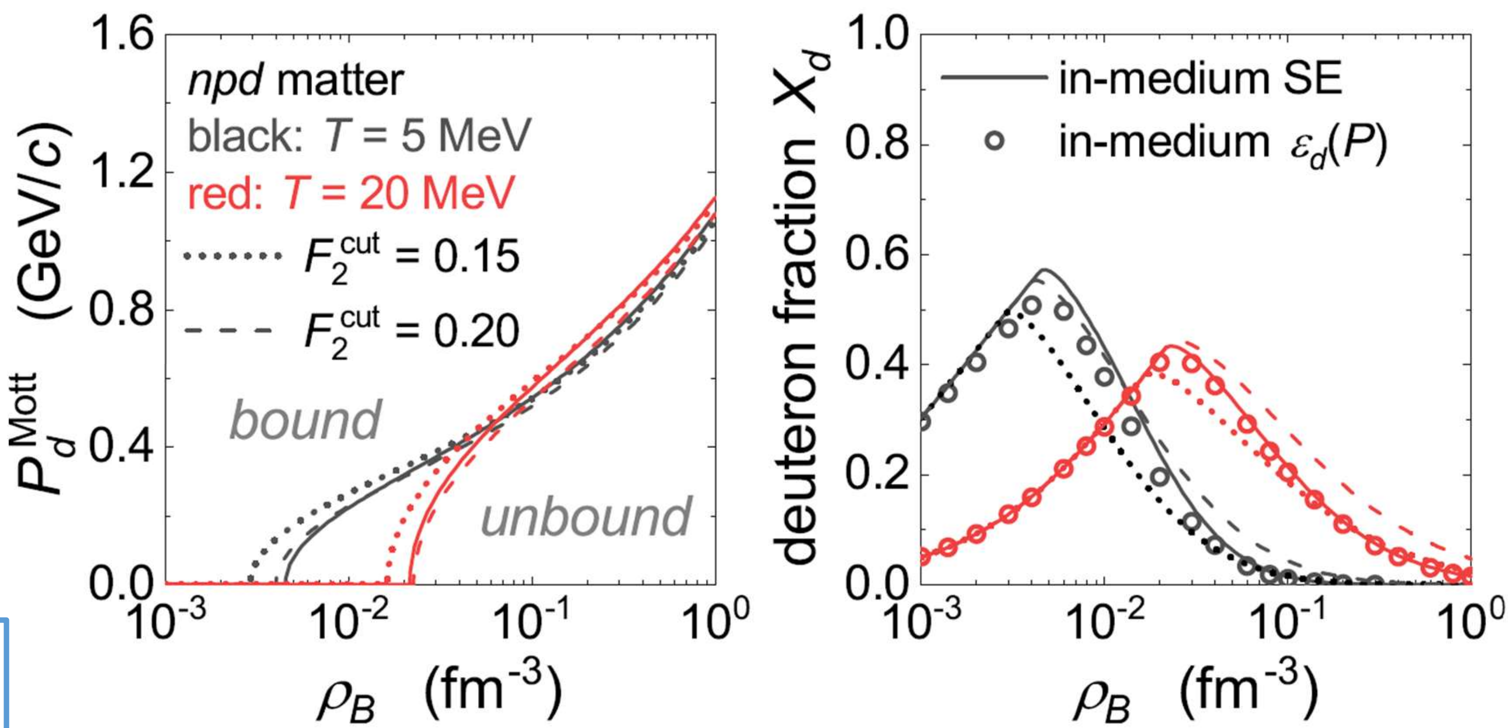
$$\langle f_\tau \rangle_\nu(\vec{P}) \equiv \int \tilde{f}_\tau\left(\frac{\vec{P}}{A}+\vec{q}\right)|\phi_\nu(\vec{q})|^2 \mathrm{d}\vec{q} \leqslant F_A^{\mathrm{cut}}$$

A larger F_A^{cut} corresponds to a weaker Mott effect

One-body nucleon occupation

Momentum distribution of the constituent nucleons, related to the light-nuclei internal wave function





F_2^{cut}, F_3^{cut}, and F_4^{cut} can be treated as surrogates of the strength of light-nuclei Mott effect

RW, Z. Zhang, S. Burrello, *et al.*, arXiv:2506.16437 [nucl-th] (Phys. Rev. C in press)

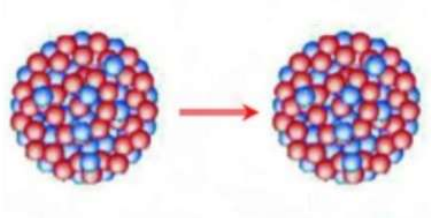
HICs versus nuclear matter

Kinetic equation of the occupation number f_τ

$$(\partial_t+\vec{\nabla}_p\epsilon_\tau\cdot\vec{\nabla}_r-\vec{\nabla}_r\epsilon_\tau\cdot\vec{\nabla}_p)f_\tau=\mathcal{K}_\tau^<[f_n,f_p,f_d,\cdots](1\pm f_\tau)-\mathcal{K}_\tau^>[f_n,f_p,f_d,\cdots]f_\tau,\quad \tau=n,p,d,t,h,\alpha$$

Heavy-ion collisions

By initializing f_τ of each particle species (initially only nucleons) and let them evolve according to the kinetic equation



Parameters characterizing the strength of in-medium effects can be empirically determined by comparison with experimental data, such as light-nuclei yields.

Nuclear matter

By imposing equilibrium conditions on the collision integral→ f_τ^{eq} →Particle fractions (Standard thermal model + in-medium effects on light clusters)

→ Infer light-cluster compositions of nuclear matter at given conditions(densities and temperatures where light clusters freeze out in HICs)

Collision integral → equilibrium

Standard thermal model + in-medium effects on light clusters

Taking the example of $Nnp \leftrightarrow Nd$, at equilibrium

$$f_\tau(\vec{p}_1)f_n(\vec{p}_2)f_p(\vec{p}_3)[1-f_\tau(\vec{p}_4)][1+f_d(\vec{p}_5)] = f_\tau(\vec{p}_4)f_d(\vec{p}_5)[1-f_\tau(\vec{p}_1)][1-f_n(\vec{p}_2)][1-f_p(\vec{p}_3)]$$

In nuclear matter, for given $\beta(T)$, ρ_n^{tot}, and ρ_p^{tot}

- Light clusters f_ν

$$f_\nu^{\text{eq}}(\vec{P}) = \frac{H\left(|\vec{P}| - P_\nu^{\text{Mott}}\right)}{\exp\{\beta[\epsilon_\nu(\vec{P}) - \mu_\nu]\} \pm 1}$$

The in-medium effects enter the light-cluster distribution function by means of the Mott momentum P_ν^{Mott}, i.e., no light clusters exist below P_ν^{Mott}

- Nucleons f_τ

$$f_\tau^{\text{eq}}(\vec{p}) = \frac{1 - f_\tau^{\text{clu}}(\vec{p})}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\} + 1}$$

- Obtain Mott momenta P_ν^{Mott} from the criterion

$$f_\tau^{\text{eq}}(\vec{p}) + f_\tau^{\text{clu}}(\vec{p})$$

$$\langle f_\tau \rangle_\nu(\vec{P}) \equiv \int \tilde{f}_\tau\left(\frac{\vec{P}}{A} + \vec{q}\right)|\phi_\nu(\vec{q})|^2 \mathrm{d}\vec{q} \leqslant F_A^{\text{cut}}$$

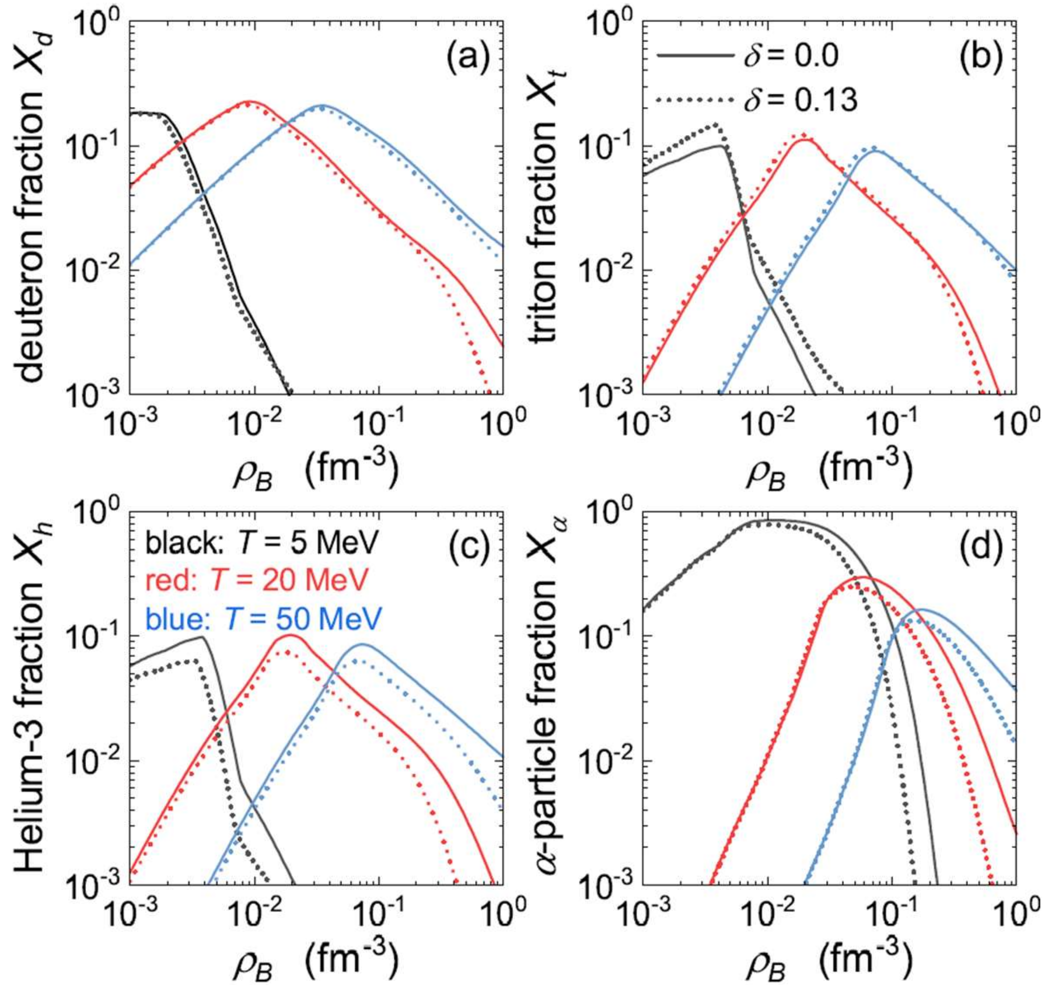
⇨ Obtain μ_n, μ_p, P_d^{Mott}, P_t^{Mott}, P_h^{Mott}, P_α^{Mott}, and thus f_ν and f_τ

Clustered Nuclear matter

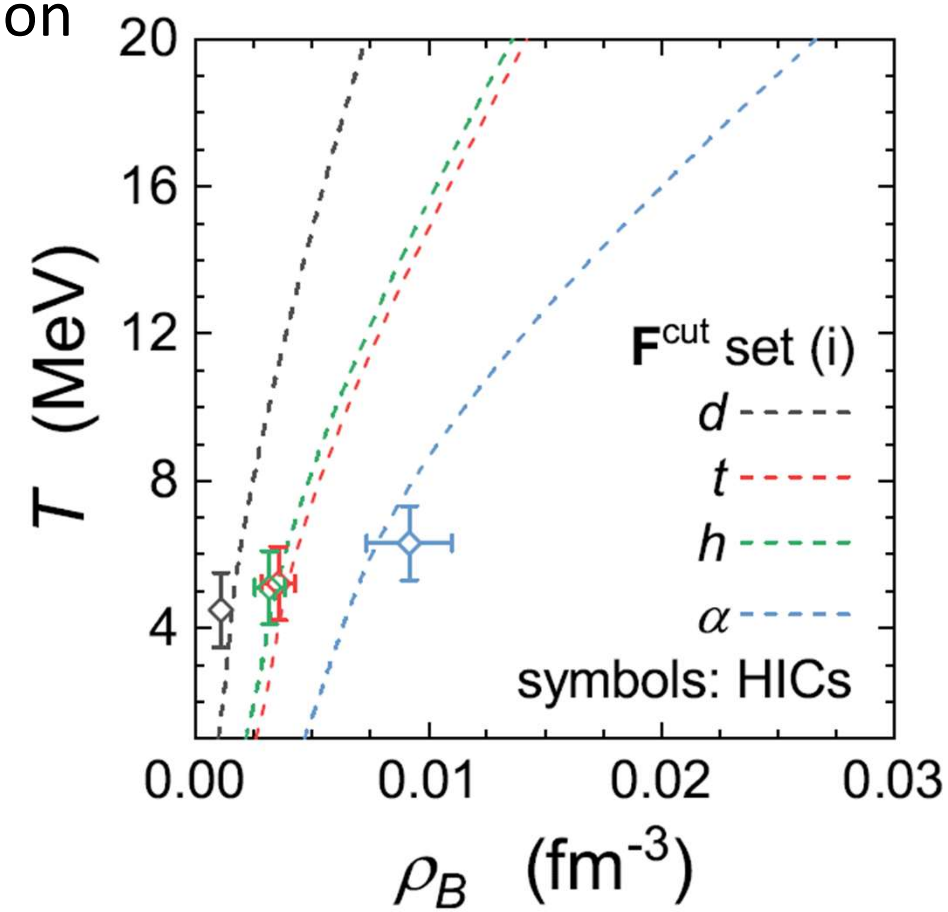
$F_2^{\rm cut} = 0.1, F_3^{\rm cut} = 0.15, F_4^{\rm cut} = 0.25$

Employing standard Skyrme interaction to calculate $\epsilon_\tau(\vec{p})$ and $\epsilon_\upsilon(\vec{P})$

Light-cluster fractions



Mott densities: comparison with experimental deduction



RW, Z. Zhang, S. Burrello, *et al.*, arXiv:2506.16437 [nucl-th] (Phys. Rev. C in press)

TMEP box calculations

Transport Model Evaluation Project:

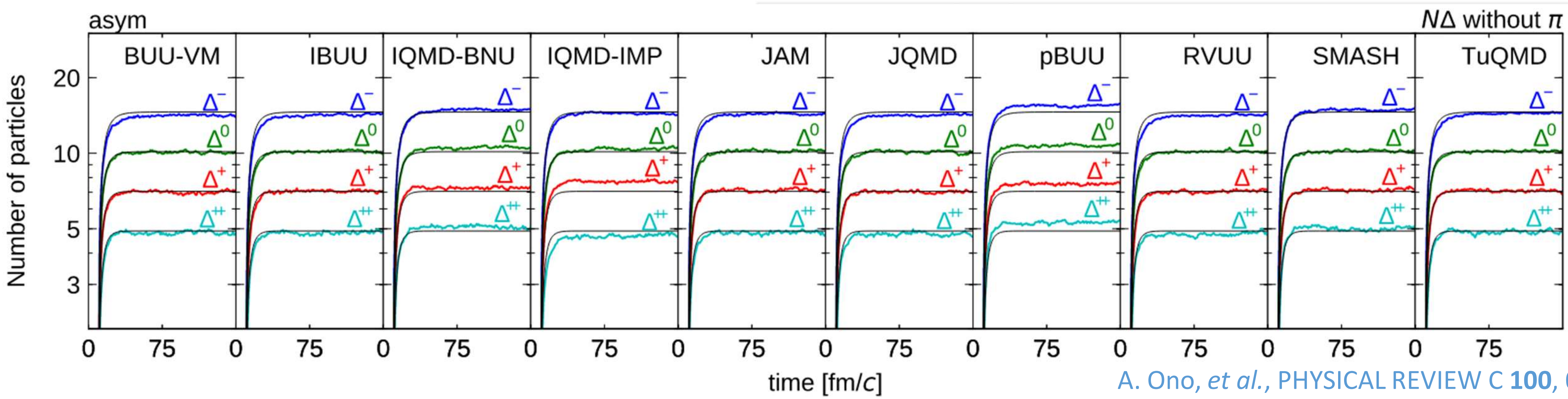
To evaluate, understand, and reduce the uncertainties in transport codes by comparing their results under well-defined conditions (e.g., a system in a box with periodic boundary conditions).

Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box #5
TMEP Collaboration • Akira Ono (Tohoku U.) et al. (Apr 5, 2019)
Published in: *Phys.Rev.C* 100 (2019) 4, 044617 • e-Print: 1904.02888 [nucl-th]
pdf DOI cite claim reference search 83 citations

Comparison of heavy-ion transport simulations: Collision integral in a box #6
TMEP Collaboration • Ying-Xun Zhang (Beijing, Inst. Atomic Energy and Guangxi Normal U.) et al. (Nov 16, 2017)
Published in: *Phys.Rev.C* 97 (2018) 3, 034625 • e-Print: 1711.05950 [nucl-th]
pdf DOI cite claim reference search 134 citations

Transport model comparison studies of intermediate-energy heavy-ion collisions #2
TMEP Collaboration • Hermann Wolter (Munich U.) et al. (Feb 14, 2022)
Published in: *Prog.Part.Nucl.Phys.* 125 (2022) 103962 • e-Print: 2202.06672 [nucl-th]
pdf DOI cite claim reference search 90 citations

Comparison of heavy-ion transport simulations: Mean-field dynamics in a box #3
TMEP Collaboration • Maria Colonna (INFN, LNS) et al. (Jun 23, 2021)
Published in: *Phys.Rev.C* 104 (2021) 2, 024603 • e-Print: 2106.12287 [nucl-th]
pdf DOI cite claim reference search 58 citations

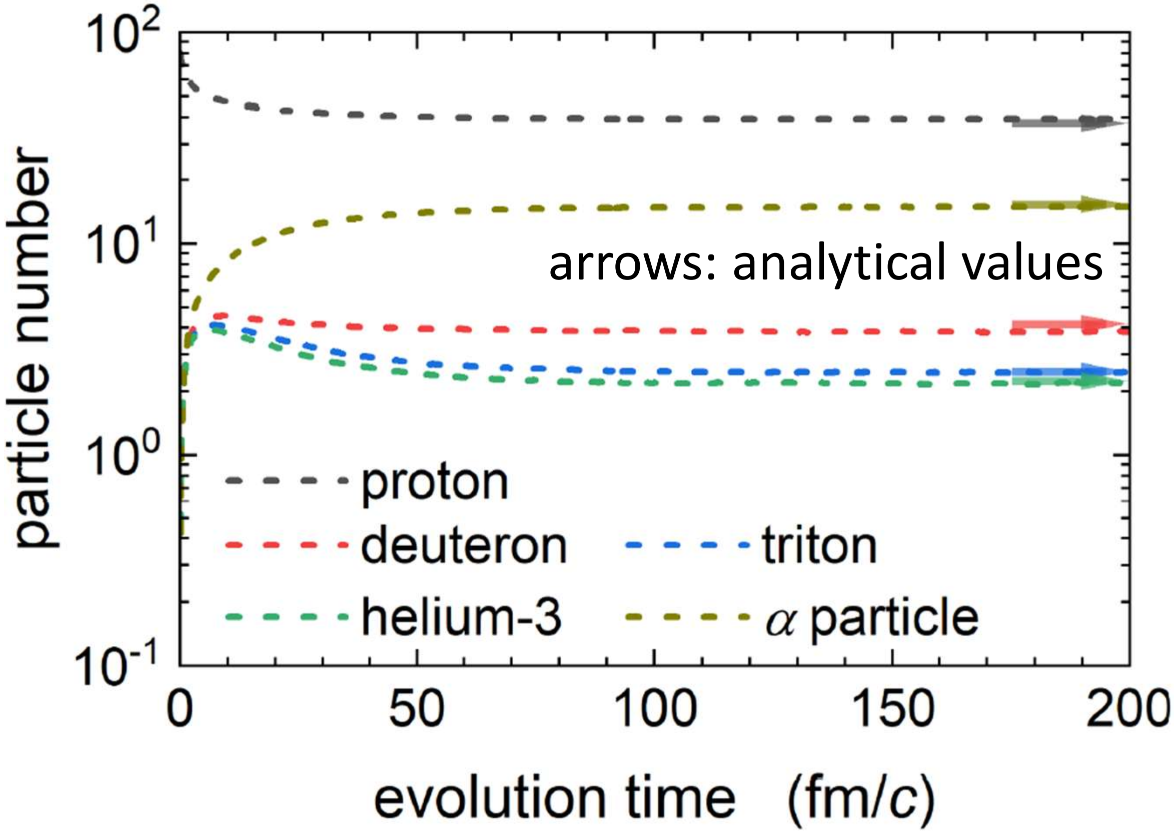


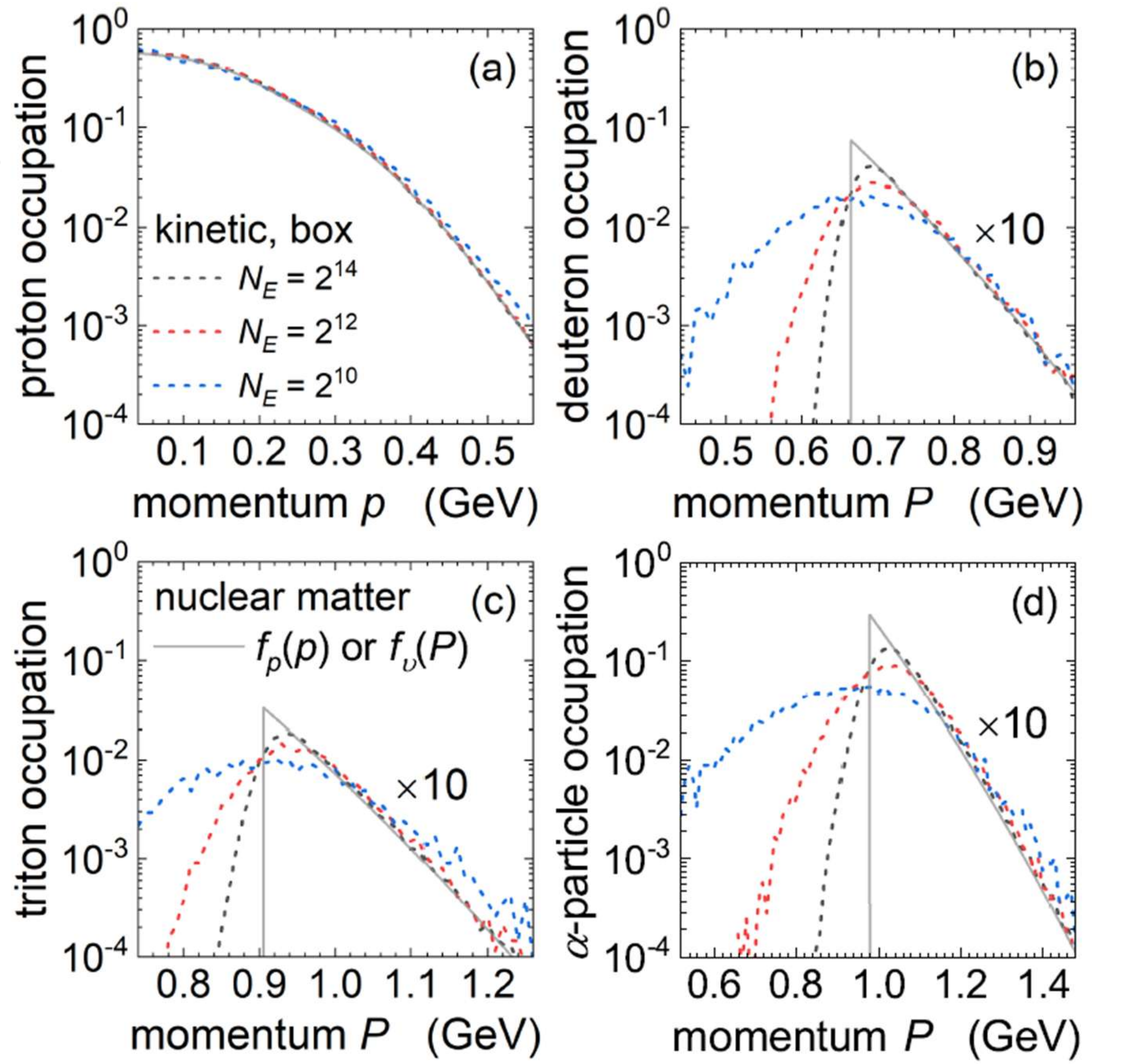
A. Ono, *et al.*, PHYSICAL REVIEW C **100**, 044617 (2019)

Numerical calculation of nuclear matter

Validation: box calculation

As the number of ensembles N_E (or number of test particles) used in the kinetic approach increases, the resulting light-cluster occupations approach their analytical counterparts



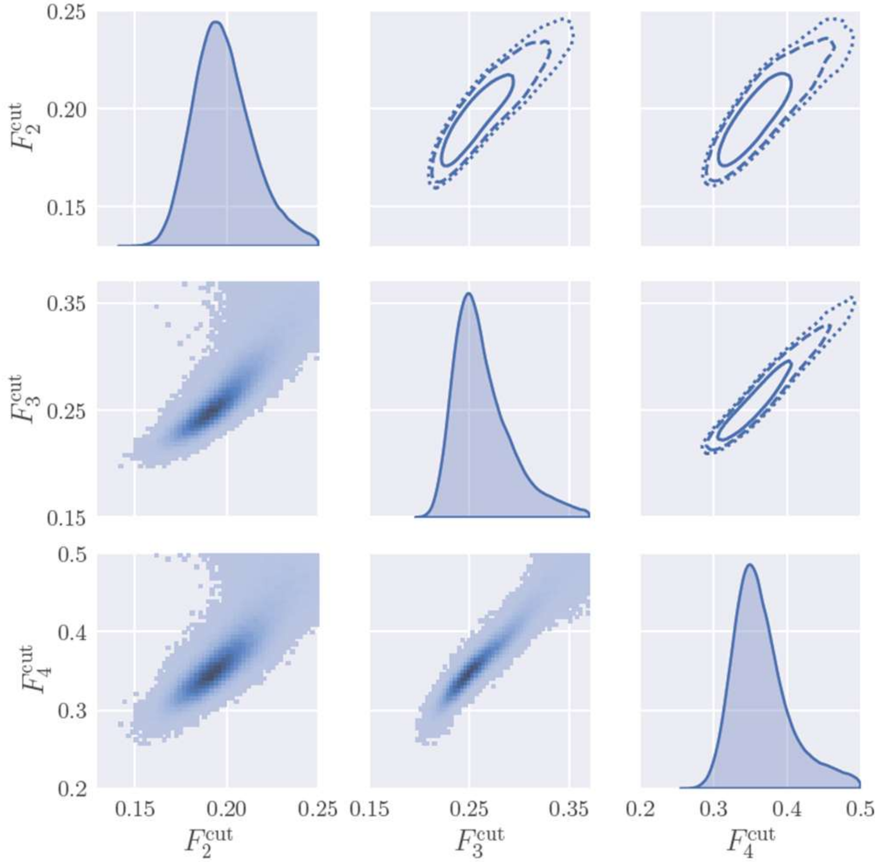


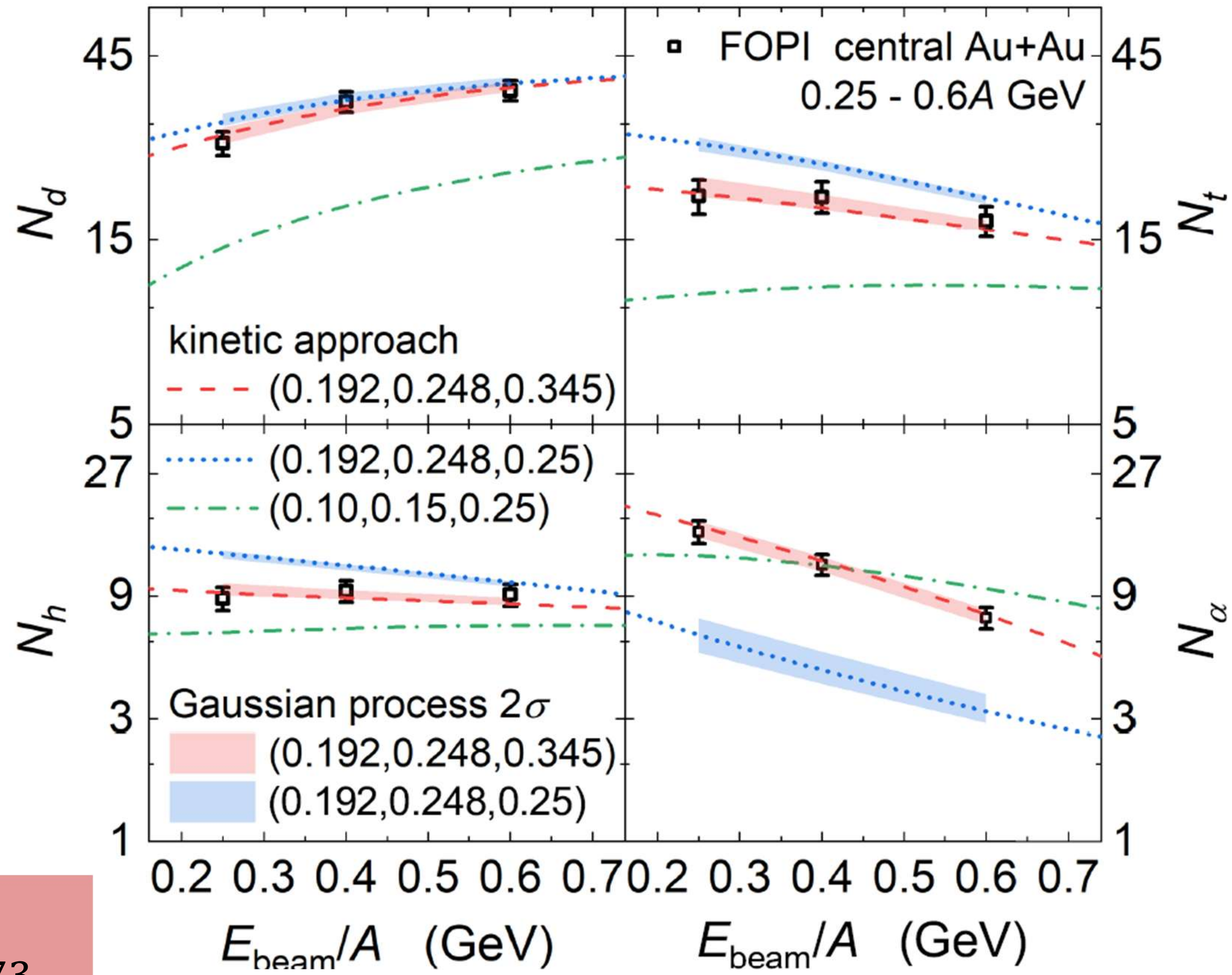
RW, Z. Zhang, Y.-G. Ma, *et al.*, arXiv:2507.16613 [nucl-th]

Light-nuclei yields in heavy-ion collisions

Central Au+Au collisions at energies of 0.25*A* GeV to 0.6*A* GeV:

Bayesian analysis of $F_A^{\rm cut}$:



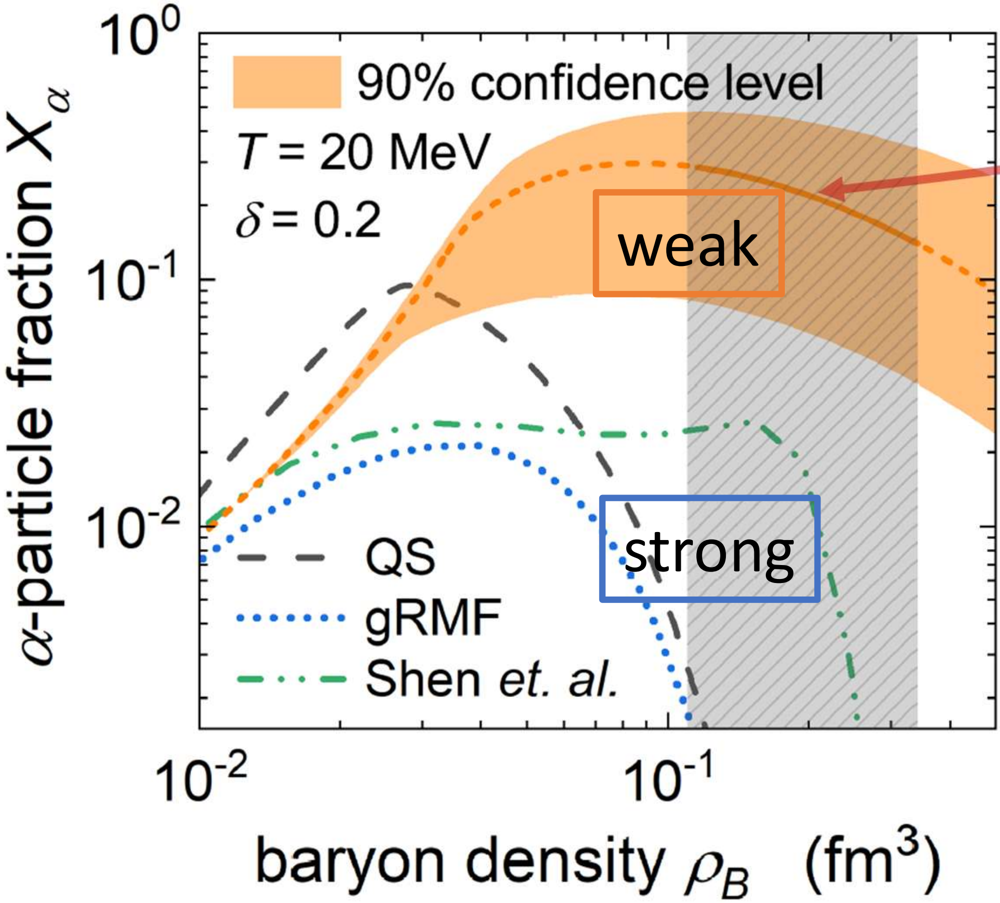


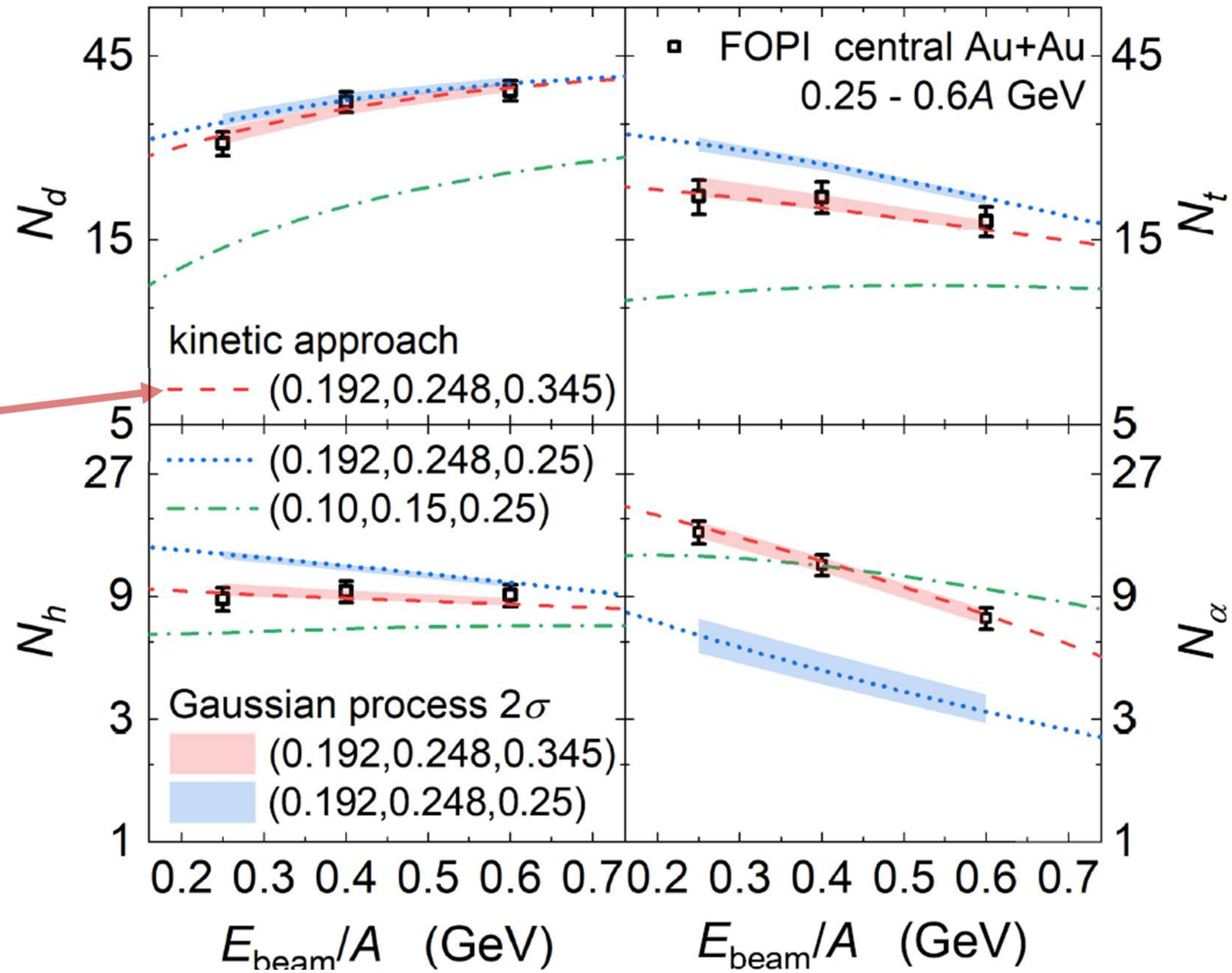
90% confidence level: $F_2^{\rm cut} = 0.192^{+0.023}_{-0.013}$, $F_3^{\rm cut} = 0.248^{+0.046}_{-0.023}$, and $F_4^{\rm cut} = 0.345^{+0.073}_{-0.030}$.

RW, Z. Zhang, Y.-G. Ma, *et al.*, arXiv:2507.16613 [nucl-th]

α-particle fraction

The strength of in-medium effects deduced from light-nuclei yields in heavy-ion collisions can be used to estimate the light-cluster fraction in nuclear matter.





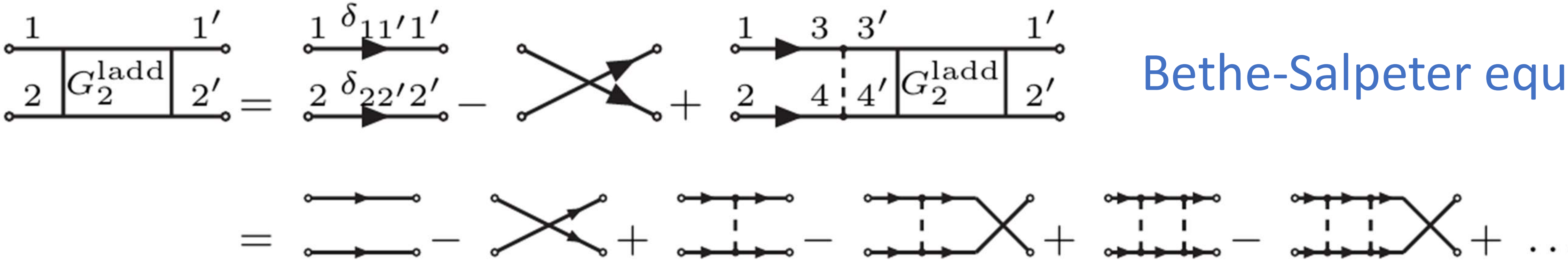
RW, Z. Zhang, Y.-G. Ma, *et al.*, arXiv:2507.16613 [nucl-th]

Summary and outlook

- Based on the kinetic equation, with proper treatment of in-medium effects on light clusters, both the light-nuclei production in heavy-ion collisions and light-cluster compositions of nuclear matter can be studied.
- The strength of in-medium effects on light clusters can be deduced from heavy-ion measurements. Specifically, the light-nuclei yields measured by FOPI suggest that the in-medium effects on alpha clusters in hot and dense nuclear matter are weaker than previously thought.
- The in-medium effects on both light clusters and particle scattering cross sections are being further investigated directly using the thermodynamic Green's function techniques.

Thank you

Thermodynamic Green's function technique



Bethe-Salpeter equation

$$G_A^{\mathrm{ladd}}(1\cdots A, 1'\cdots A') = G_A^0(1\cdots A, 1'\cdots A') + \sum_{('')(*)} G_A^0(1\cdots A, 1''\cdots A'')V(1''\cdots A'', 1^*\cdots A^*)G_A^{\mathrm{ladd}}(1^*\cdots A^*, 1'\cdots A')$$

The free A-(quasi)particle Green function

$$G_A^0(1\cdots A, 1'\cdots A', z_A) = \frac{[1-f(\epsilon_1)]\cdots[1-f(\epsilon_A)] \pm f(\epsilon_1)\cdots f(\epsilon_A)}{z_A - \sum_i \epsilon_i + \mu_A}$$

In-medium modification

The Bethe-Salpeter equation is satisfied by

$$G_A^{\mathrm{ladd}}(1\cdots A, 1'\cdots A', z_A) = \sum_{nP} \Psi_{nP}(1\cdots A)\frac{1}{z_A - \epsilon_{nP} + \mu_A}\Psi_{nP}^*(1'\cdots A')$$

With the internal wave function Ψ_{nP} (two-body case) determined by

$$\left[\sum_i \epsilon_i - \epsilon_{nP}\right]\Psi_{nP}(1\cdots A) + [1 - f_1(\epsilon_1) - f_2(\epsilon_2)]\sum_{1',2'} V(12, 1'2')\Psi_{nP}(1\cdots A) = 0$$