Saturation of the GEM gain

Let's suppose that during the development of the avalanche within the gem multiplication channels a significant amount of electrons and positive ions are produced.

Under the effect of the electric field present in the channel, these slowly migrate toward the lower potential plane of the GEM, tending to partially shield the field itself.

If n₀ is the number of electrons entering a GEM channel and $E_0 = V_{GEM}/d$ the electric field in it:

Multiplication is described by a modified Townsend equation

border of the channel and needed to produce E₀ in it $\beta \propto 1/V_{GEM}$);

 $\frac{dn}{ds} = \alpha E_0 (1 - \beta n) n$

V_{GEM} is the voltage drop between GEM sides d is the GEM thickness

$$
G=\frac{e^{\alpha V}}{1+\beta n_0(e^{\alpha V}-1)}
$$

$$
\int_{n_0}^{n_{tot}} \frac{dn}{(1 - \beta n)n} = \int_0^d \alpha E_0 ds
$$

where $G=n_{tot}/n_0$ is the average gain of the single channel.

It should be noticed that it depends on the amount of primary electrons entering the channel. In particular it decreases with n_0 and:

- if $\beta n_0 \simeq 0$ (i.e. negligible screen effect), $G = e^{\alpha} V$

- if
$$
\beta n_0 \simeq 1
$$
 (i.e. total screen effect), $G = 1$

To fix ideas let us now assume that after a drift over a path z drift and the multiplication process in the first 2 GEMs (GEM#1 and GEM#2) the electron cloud has a distribution in space describable as a Gaussian in 3 dimensions all with RMS equal to σ :

the total volume will then be approximately proportional to σ^3 and the amount of charge collected by each channel will decrease as 1/*σ*³

In the last GEM, the amount of charge collected by each channel n_0 :

- Increases with the primary ionisation in the gas n_e ;
- decreases as $1/\sigma^3$;
- increases as the product of the gains of G_1 and G_2

$$
n_0 \propto n_e G_1 G_2 / \sigma^3
$$

 $\beta n_0 = p_1 G_1 G_2 / \sigma^3$

$$
G_{tot} = \frac{G_1 G_2 e^{\alpha V}}{1 + \beta n_0 (e^{\alpha V} - 1)} = \frac{G_1 G_2 G_3}{1 + p_1 G_1 G_2 / \sigma^3 (G_3 - 1)}
$$

- We can try to fit this last function on the data expecting:
	- po to be the not-saturated gain of the three GEMS;
	- p_1 to almost constant and just slightly dependent on the V GEM

If
$$
G_1 = G_2 = G_3 = e^{\alpha V} = p_0
$$

Let's suppose that only in GEM#3 we have non linear gain because of the larger amount of charges.

$$
G_{tot} = \frac{p_0^3}{1 + p_1 p_0^2 / \sigma_3 (p_0 - 1)}
$$

$$
= \frac{p_0^3 \sigma^3}{\sigma^3 + p_1 p_0^2 (p_0 - 1)}
$$

- From the GIN data we can evaluate the electron gain in 3 different V_{GEM} setup (440, 430 and 420) and the behavior of *σ*
- We can start from the light yield for $55Fe$ spots

- The electron gain is evaluated by taking into account 0.07 γ/e , 150 n_e and $\Omega = 9.2 \times 10^{-4}$

From their ratios we can evaluate two values of alphas: 0.021 and 0.019, close to the one used in the digitization 0.022;

 p_1 is about 80 in all the three fits

PMT saturation

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