Roma 2·12·2011

Giovanni Amelino-Camelia
University of Rome "La Sapienza"

3 questions

I: how much do we know about the geometry of momentum space?

II: Which laws apply to the limit of quantum gravity obtained by

$$G_N \to 0 \quad h \to 0 \quad \textbf{with} \quad \frac{h}{G_N} \ \underline{\textbf{kept fixed}}$$

III: Look around; do you see spacetime?

quantum spacetime, relative locality and entanglement

Roma 2·12·2011
Giovanni Amelino-Camelia
University of Rome "La Sapienza"

3 questions

I: how much do we know about the geometry of momentum space?

II: Which laws apply to the limit of quantum gravity obtained by

$$G_N \to 0 \qquad h \to 0 \qquad \text{with} \quad \frac{h}{G_N} \quad \text{kept fixed}$$

III: Look around; do you see spacetime?

apparently linked

Principle of relative locality
PhysRevD84(2011)084010
with Laurent Freidel, Jerzy Kowalski-Glikman, Lee Smolin

relative locality in kappa-Minkowski noncommutative spacetime
PhysRevLett106(2011)071301 & PhysLettB700(2011)150
with Niccolo' Loret, Marco Matassa, Flavio Mercati, Giacomo Rosati

PART 1:
quantum-gravity snapshots

Planck-scale-modified dispersion relations

attempts to model "quantum spacetime"
(using "spacetime noncommutativity" or certain perspectives on the semiclassical limit of Loop Quantum Gravity) have stumbled upon modifications of the energy-momentum (on-shell) dispersion relation

$$m^2 \approx E^2 - p^2 + \alpha_\# \frac{E^n p^2}{M_{planck}^n}$$

striking!!!
however M_{planck} is ultralarge ($\sim 10^{15} M_{LHC}$) ...difficult to test....
but CAN BE TESTED (see closing remarks)....must be at the forefront of QG research

and what about symmetries? Broken Lorentz Invariance?

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
Gambini+Pullin,PhysRevD59,124021(2000)
Alfaro+Morales-Tecotl+Urrutia,PhysRevLett84,2318(2000)

or rather some sort of "deformed Lorentz invariance" in the sense of "doubly-special relativity"
(both "c" and "M_{planck}" as nontrivial relativistic invariants)

GAC, grqc0012051,IntJModPhysD11,35
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin, hepth0112090,PhysRevLett88,190403
GAC,grqc0207049,Nature418,34

the idea of "deformed symmetries":
the illustrative example "kappa-Minkowski" quantum spacetime
and its kappa-Poincare Hopf-algebra symmetries

kappa-Minkowski $[x_j, x_0] = i\ell x_j$ $\quad [x_j, x_k] = 0$

writing fields in time-to-the-right conventions

$$\Phi(x) = \int d^4x \tilde{\Phi}(k) e^{ik_j x^j} e^{ik_0 x^0}$$

there is a natural implementation of kappa-Poincare' Hopf-algebra transformations

translation generators $P_\mu e^{ik_j x^j} e^{ik_0 x^0} = k_\mu e^{ik_j x^j} e^{ik_0 x^0}$

rotation generators $R_l e^{ik_j x^j} e^{ik_0 x^0} = \epsilon_{lmn} x_m k_n e^{ik_j x^j} e^{ik_0 x^0}$

boost generators

$$\mathcal{N}_l e^{ik_j x^j} e^{ik_0 x^0} = \left[x_0 k_l - x_l \left(\frac{1 - e^{-2\ell k_0}}{2\ell} + \frac{\ell}{2} k_m k^m\right)\right] e^{ik_j x^j} e^{ik_0 x^0}$$

new generators, new "mass Casimir":

$$\mathcal{C} = \left(\frac{2}{\ell}\right)^2 \sinh^2\left(\frac{\ell}{2} P_0\right) - e^{\ell P_0} P_j P^j$$

crucial point for deformed (rather than broken) Lorentz symmetry, in cases where the dispersion relation is modified,
is a deformation of the law of composition of momenta that is consistent with the deformation of the on-shell relation

GAC, grqc0012051,IntJModPhysD11,35
GAC,grqc0207049,Nature418,34

for kappa-Minkowski modified on-shell relation comes accompanied with "funny plane waves"

GAC +Majid,IntJModPhysA15,4301

$$e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} = e^{i(k + e^{\ell k_0} K)x} e^{i(k_0 + K_0)t}$$

notice nonlinear composition of "momenta"

$$k_j + e^{-\ell k_0} K_j$$

symmetries described by a Hopf algebra, essentially codified in the coproduct;for example for translations

$$P_j\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right) = P_j\left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t}\right)$$

$$= \left(k_j + e^{-\lambda k_0} K_j\right)\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right)$$

$$= \left[P_j\left(e^{ikx} e^{ik_0 t}\right)\right]\left(e^{iKx} e^{iK_0 t}\right) + \left[e^{-\lambda P_0}\left(e^{ikx} e^{ik_0 t}\right)\right]P_j\left(e^{iKx} e^{iK_0 t}\right)$$

Nontrivial coproduct!!

momentum-space curvature?

several attempts, though none truly fruitful, to formalize these and other results in terms of the possibility of curvature in momentum space

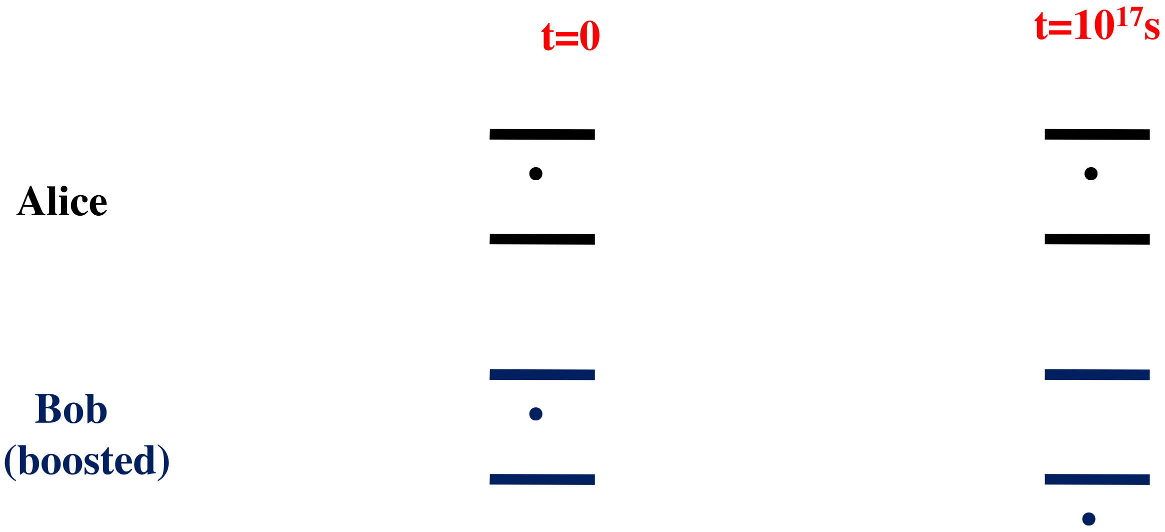
Kadyshevsky +Mateev(1985)
Majid (1992)
KowalskiGlikman (2003)
Girelli+Livine (2005)

and by the way how does one characterize that operatively?
how do we know momentum-space is flat?
is it really sharply flat?

“box problems” for in-vacuo dispersion

if one attempts to proceed heuristically adopting a law for the speed of massless particles with momentum/wavelength dependence <u>as a relativistic law</u> strange things appear to happen to locality

GAC,IntJModPhysD(2002)
Schutzhold +Unruh, JETP Lett (2003)
DeDeo + PrescodWeinstein,arXiv (2008)
Hossenfelder,PhysRevLett (2010)



PART 2:
relative locality

relative simultaneity

19th century Galilean observers/scientists could (should) have asked themselves:

so, do we "see" space? (<u>absoluteness of simultaneity</u>)

relative simultaneity

19th century Galilean observers/scientists could (should) have asked themselves:

so, do we "see" space? (<u>absoluteness of simultaneity</u>)

of course we now know they didn't! we don't!
at best (see later) we "see" spacetime! we "see" our past lightcone...

The properties of Lorentz boosts are such that

"space by itself, and time by itself fade away into mere shadows, and only a kind of union of the two [spacetime] preserves an independent reality" (Minkowski 1908)

And what is responsible for this "union" of space and time?
The nonlinearities of the law of composition of velocities (nonassociativity/noncommutativity)

$$\mathbf{u} \oplus_c \mathbf{v} = \frac{1}{1+\frac{\mathbf{u}\cdot\mathbf{v}}{c^2}}\left(\mathbf{u} + \frac{1}{\gamma_u}\mathbf{v} + \frac{1}{c^2}\frac{\gamma_u}{1+\gamma_u}(\mathbf{u}\cdot\mathbf{v})\mathbf{u}\right)$$

$[N_x, N_y] \approx R_z$

(Wigner-)Thomas rotations

questioning the absoluteness of simultaneity through questioning the linearity of the law of composition of velocities

the principle of relative locality

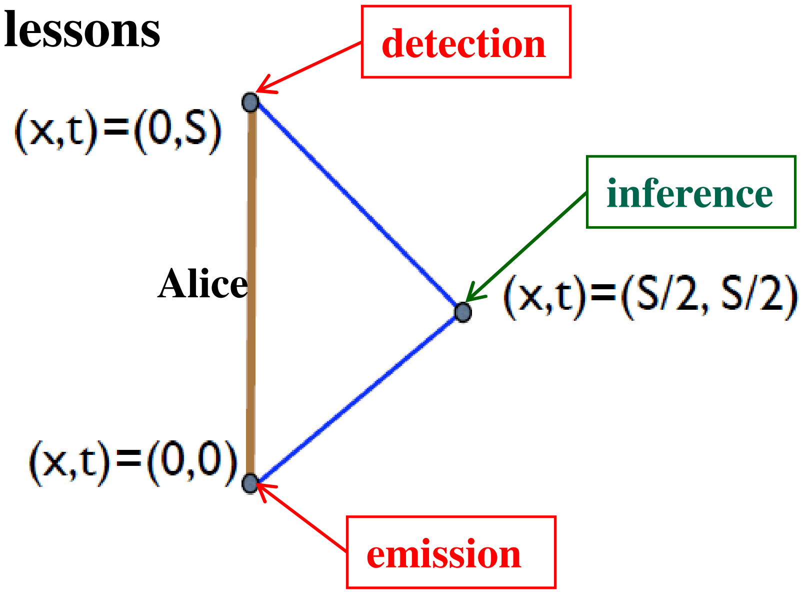
GAC+Freidel+Kowalski-Glikman+Smolin

so, look around: do you "see" spacetime?

NO! you "see" (detect) time sequences of particles
and then <u>abstract</u> a spacetime by inference!

you are more aware of this when you try to set up a <u>macroscopic</u> spacetime/reference frame
(think in particular of the abstraction of a spacetime used to organize logically our inferences for what concerns the observations of distant astros)

This was after all one of Einstein's key lessons

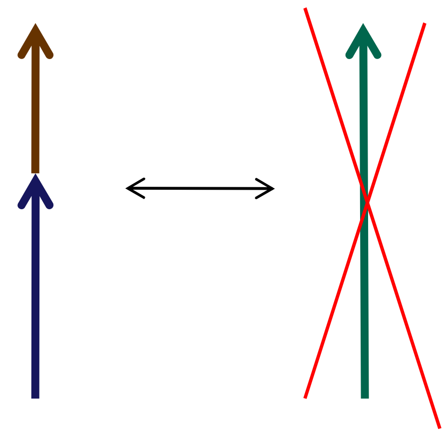


misleading inferences of Galilean Relativity

<u>nonlinearity of special-relativistic laws:</u>

a man runs on a train at speed U
(with respect to the train)
and the train has speed V
with respect to the station

$\Rightarrow$ speed of man with respect to station "must" be U+V

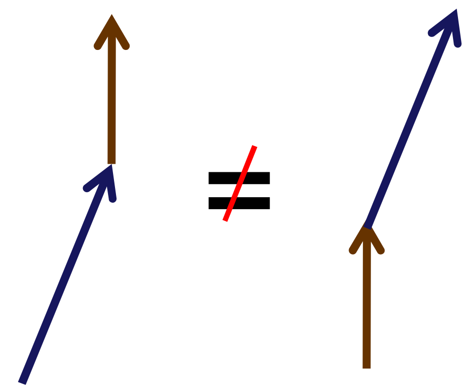


<u>noncommutativity of special-relastivistic laws:</u>

a man runs on a train at velocity U_j
and the train has velocity V_j

"must" be same as

a man runs on a train at velocity V_j
and the train has velocity U_j



the principle of relative locality

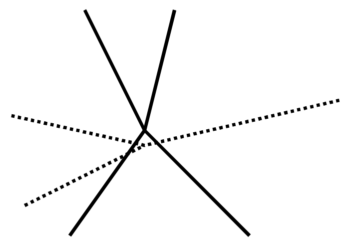
But do macroscopically-distant observers infer/abstract "the same" spacetime?

What does it even mean to infer "the same" spacetime?

the principle of relative locality

But do macroscopically-distant observers infer/abstract “the same” spacetime?

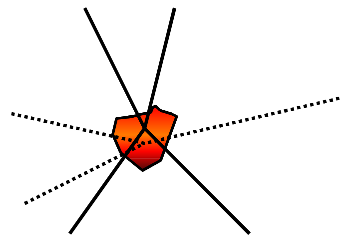
What does it even mean to infer “the same” spacetime?
absolute locality: coincidences of events for one Einsteinian observer are also coincidences of events for all other Einstenian observers



the principle of relative locality

But do macroscopically-distant observers infer/abstract "the same" spacetime?

What does it even mean to infer "the same" spacetime?
<u>absolute locality</u>: coincidences of events for one Einsteinian observer are also coincidences of events for all other Einstenian observers



And what is it that allows absolute locality?
The structureless (linear) law of composition of momenta

$$p_1 \oplus p_2 \oplus p_3 = p_1 + p_2 + p_3$$

link from linear conservation of momentum to locality is most familiar nowadays in the context of field theories

$$\int \mathrm{dk}_j \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3)\delta(k_1 + k_2 + k_3) =$$

$$= \int \mathrm{dk}_j d^4x \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3) e^{i(k_1+k_2+k_3)x} =$$

$$= \int d^4x \Phi_1(x)\Phi_2(x)\Phi_3(x)$$

the principle of relative locality

link from linear conservation of momentum to locality is most familiar nowadays in the context of field theories

$$\int dk_j \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3)\delta(k_1+k_2+k_3) =$$

$$= \int dk_j d^4x \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3) e^{i(k_1+k_2+k_3)x} =$$

$$= \int d^4x \Phi_1(x)\Phi_2(x)\Phi_3(x)$$

Also notice that the conservation law that generates transformations between <u>distant observers</u> are these linear laws of composition of momenta

$$\delta x^a_I = \{\delta x^a_I, b^c \mathcal{P}^{tot}_c\} = b^a$$ **with** $$\mathcal{P}^{tot}_c = \sum_I p^I_c$$

and the fact that these act on coordinates assigned to the event by one observer in a way that is independent of any detail of the specific worldlines and structure of the event is again responsible for the objectivity of the inferred distant coincidences of events

the "planck-scale limit" of quantum gravity

GAC+Freidel+Kowalski-Glikman+Smolin

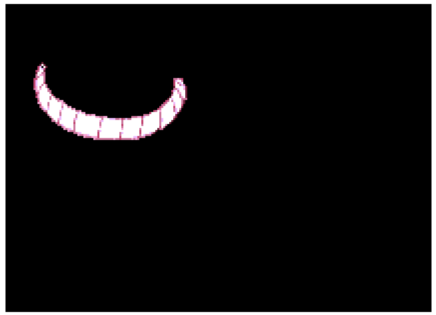
ok, fine, "relative locality"....but how would we find out? quantum gravity is so complex!!! NO. Look at a peculiar limit:

$$G_N \to 0 \qquad h \to 0 \qquad \text{with} \quad \frac{h}{G_N} \approx M^2_{planck} \quad \text{kept fixed}$$

in this limit of quantum gravity roughly speaking
quantum mechanics and gravitation are switched off!!
and the Planck length is switched off!!

But the Planck scale is not switched off and IF the limit is not completely trivial (as implicitly argued by supporters of nonlinearities in momentum space) THEN this limit still contains valuable information about quantum gravity

a sort of Cheshire-cat smile of quantum gravity
described by theories which one should manage to analyze with relatively little effort



also a change of perspective on
how to tackle the quantum-gravity problem

the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

the "Planck-scale regime" is a rather peaceful place

what, if anything, of "interesting" could go on in the Planck-scale regime?

we propose that
in the Planck-scale regime all we have is the geometry of momentum space

special relativity corresponds to flat momentum-space geometry

the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

brief sketch of our proposed description of the geometry of momentum space

let us start from a metric on momentum space

The mass is interpreted as the timelike distance from the origin

$$D^2(p) \equiv D^2(p,0) = m^2.$$

The kinetic energy defines the geodesic spacelike distance between a particle p at rest and a particle p' of identical mass

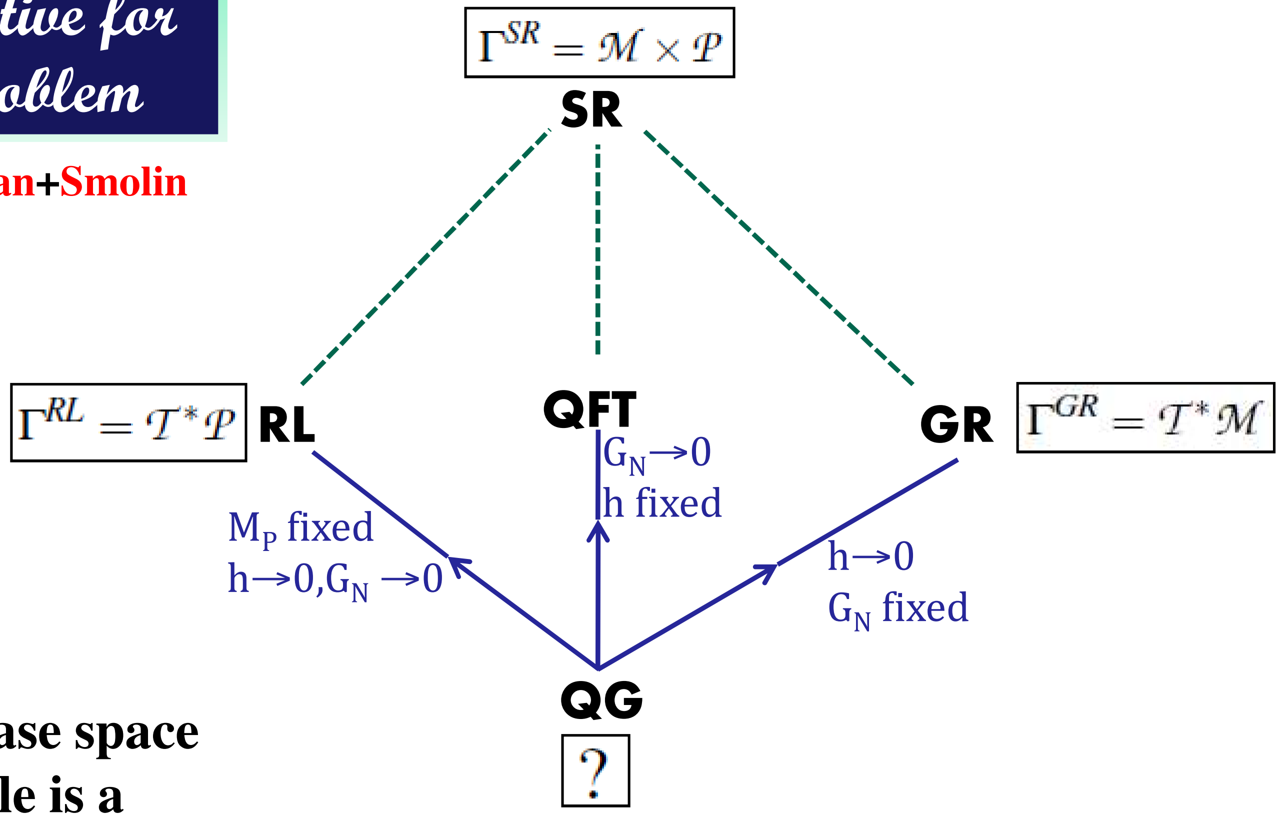
$$D(p) = D(p') = m \longrightarrow D^2(p,p') = -2mK$$

from these measurements we can reconstruct the metric

$$dk^2 = h^{ab}(k)dk_a dk_b$$

more on the new perspective for the quantum-gravity problem

GAC+Freidel+Kowalski-Glikman+Smolin



In special relativity, the phase space associated with each particle is a product of spacetime and momentum space

In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry, and the particle phase space is no longer a product: there is a cotangent space of momenta at each point in the spacetime manifold and the phase space is the cotangent bundle of $\mathcal{M}$.

Within the framework of relative locality, it is the momentum space $\mathcal{P}$ that is curved. Then we have <u>a separate spacetime for each value of momentum, and the whole phase space is then the cotangent bundle over momentum space</u>

the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

we take momentum space curved <u>and operatively primitive</u>
in the "Planck-scale regime"

<u>spacetime locality then is tricky:</u> even if particle of momentum p_I is at x_I and particle of momentum p_{II} is at x_{II} with $x_I = x_{II}$ it still does not necessarily mean the particles are close to each other.....x_I and x_{II} live in different spaces....
before comparing them we need to parallel transport....
ultimately $x_I = x_{II}$ could be a case where the particles spacetime positions <u>do not</u> coincide

the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

we introduce an affine connection, a notion of parallel transport, through the law of composition of momenta

We postulate that there exists a composition of momenta

$$(p,q) \rightarrow p'_a = (p \oplus q)_a$$

notice that it fits the kappaMinkowski

More complicated interaction process are build up by iteration of this composition e.g $(p \oplus q) \oplus k$

We do not assume that it is linear or commutative or associative

Outgoing momenta can be turned in ingoing momenta: there is an operation $p \rightarrow \ominus p$

antipode

satisfying $(\ominus p) \oplus p = 0$

the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

The composition rules defines an affine connection

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p\oplus q)_c|_{q,p=o} = -\Gamma_c^{ab}(0)$$

transform as an affine connexion

Torsion measure non commutativity

$$-\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}((p\oplus q)_c - (p\oplus q)_c)_{q,p=o} = T_c^{ab}(0)$$

the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

Curvature measure non associativity

$$2\frac{\partial}{\partial p_{[a}}\frac{\partial}{\partial q_{b]}}\frac{\partial}{\partial k_c}((p\oplus q)\oplus k - p\oplus(q\oplus k))_d\,|_{q,p,k=o} = R^{abc}{}_d(0)$$

•Non-metricity: if the connection defined by interactions is not the metric connection defined from propagation.

$$N^{abc} = \nabla^a g^{bc}$$

the principle of relative locality: spacetime emerging from dynamics on momentum space

GAC+Freidel+Kowalski-Glikman+Smolin

- Each process has an action principle

$$S^{process} = \sum_{trajectories, I} S_I^{free} + \sum_{interactions, \alpha} S_\alpha^{int}$$

spacetime coordinates by conjugation of momentum-space coordinates of the particles

$$S^J_{free} = \int_{-\infty}^{0} ds \left(x^a_J \dot{k}^J_a + \underline{\mathcal{N}_J C^J(k)} \right)$$

canonical spacetime coordinates

$$\{x^a_I, k^J_b\} = \delta^a_b \delta^J_I$$

mass-shell constraint

$$C^J(k) \equiv D^2(k) - m_J^2$$

Notice that the free particle action makes no reference to a metric for spacetime. Spacetime geometry is inferred from the geometry of momentum space.

the principle of relative locality: spacetime emerging from dynamics on momentum space

GAC+Freidel+Kowalski-Glikman+Smolin

- Each process has an action principle

$$S^{process} = \sum_{trajectories,I} S_I^{free} + \sum_{interactions,\alpha} S_\alpha^{int}$$

$$S^J_{free} = \int_{-\infty}^{0} ds \left(x^a_J \dot{k}^J_a + \mathcal{N}_J C^J(k) \right)$$

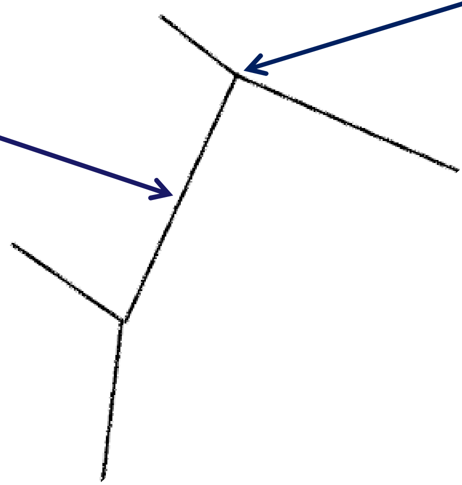
$$S^{int} = \mathcal{K}(k(o))_a z^a$$

$$\dot{k}^J_a = 0$$

$$\dot{x}^a_J = \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k^J_a}$$

$$\mathcal{C}^J(k) = 0$$

free-particle part produces worldlines governed by these equations of motion



the interaction terms only enforce the conservation laws
e.g. $\mathcal{K}_a(p,q,k) = p_a \oplus (q_a \oplus k_a)$

interaction terms only use the connection on momentum space

lagrange multipliers z^a turn out to play a very interesting role...
we call them <u>interaction coordinates</u>

the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

relative locality and the relation between "canonical spacetime coordinates" and "interaction coordinates".......boundary terms....

Is a consequence of the equations of motion at the endpoints

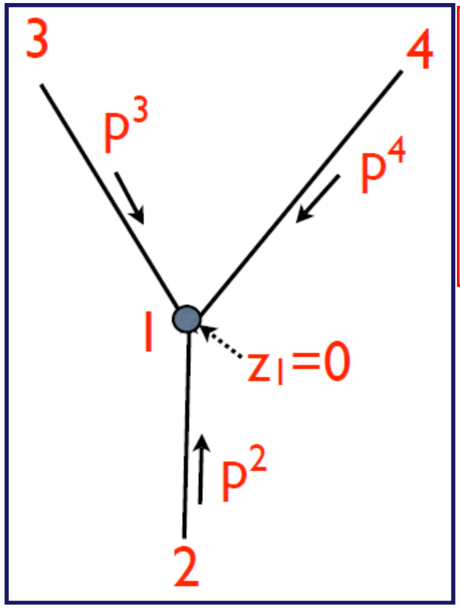
$$\delta S = \left(\frac{\delta \mathcal{K}(k(o))_a}{\delta k^I_a(0)} z^a - x^a(0)\right)\delta k_a(0)$$

The interaction point is related to the endpoint of the worldline by a parallel transport between the spaces where they live.

$$x^a(0) = U(k)^a_b z^b, \qquad U(k)^a_b = \frac{\delta \mathcal{K}_b}{\delta k_a}$$

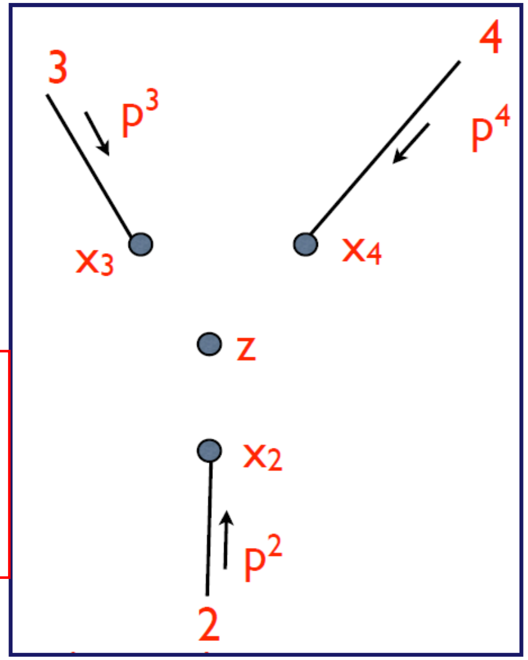
$\longleftrightarrow$

$$x^a_J(0) = z^a - z^b \sum_{L\in\mathcal{J}(J)} C_{J,L}\Gamma^{ac}_b k^L_c + \dots$$



an observer is "local to the event" if her value of z is z=0, in which case the endpoints x(0) of worldlines entering the vertex coincide

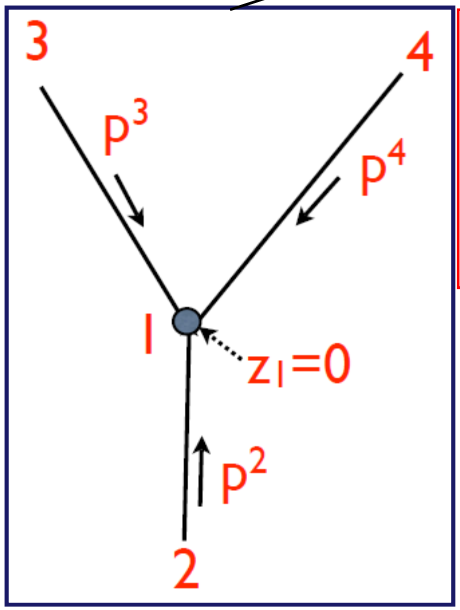
the observers who are "distant from the event" have z≠0 and the endpoints x(0) of worldlines entering the vertex do not coincide



the principle of relative locality

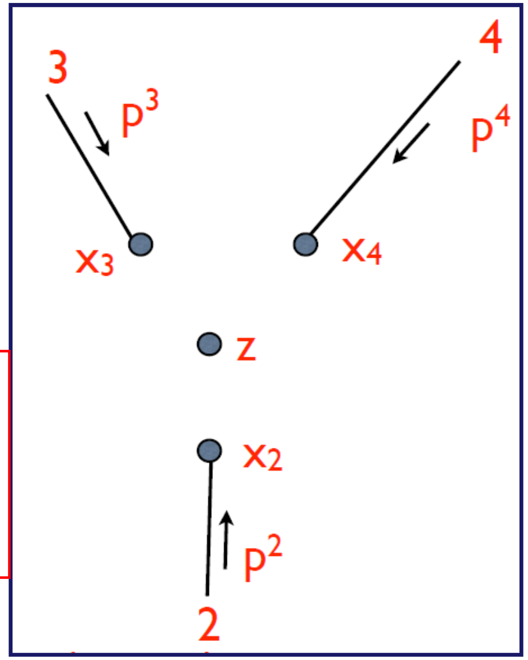
GAC+Freidel+Kowalski-Glikman+Smolin

$$\delta_b x_J^a(0) = b^b\{\mathcal{K}_b, x_J^a\} = -b^a + b^b \sum_{L\in\mathcal{I}(J)} C_{J,L}\Gamma_b^{ac} k_c^L + \dots$$



an observer is "local to the event" if her value of z is z=0, in which case the endpoints x(0) of worldlines entering the vertex coincide

the observers who are "distant from the event" have z≠0 and the endpoints x(0) of worldlines entering the vertex do not coincide



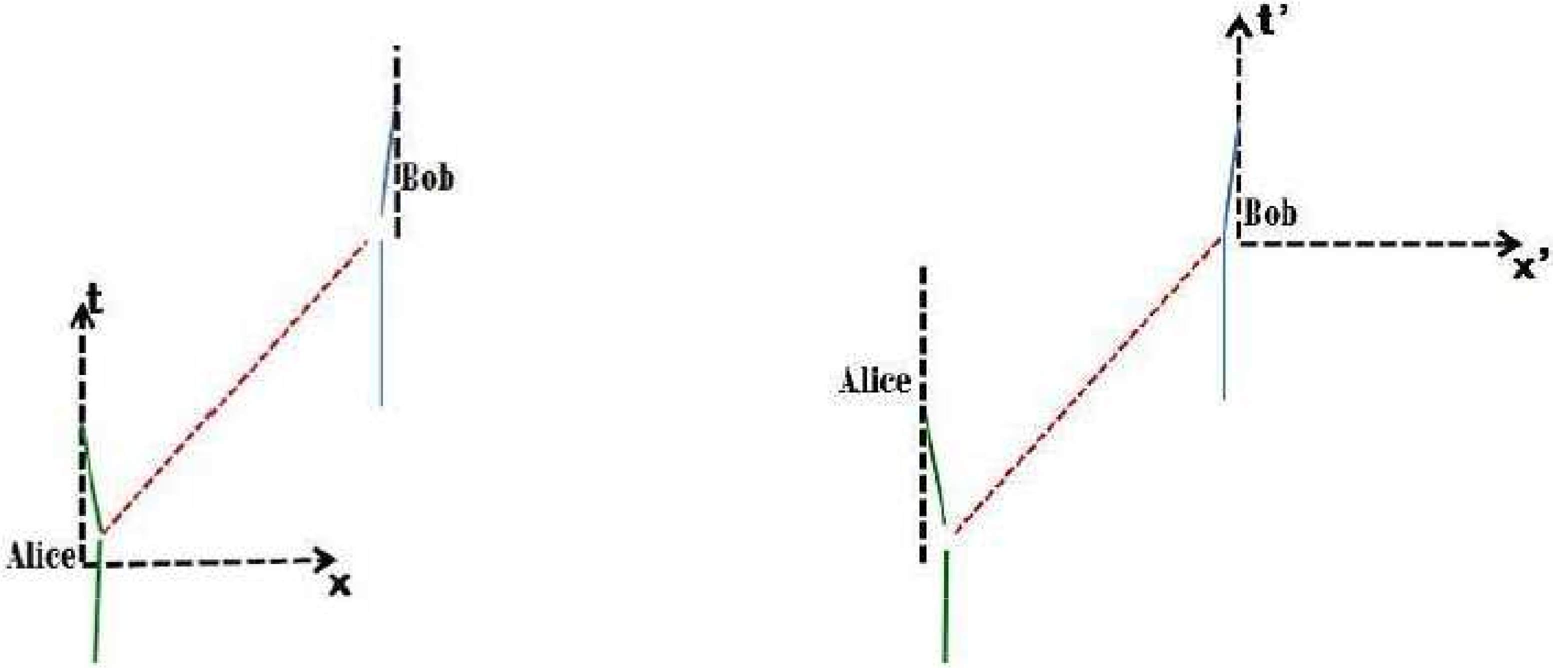


Figure 2. We show the implications of relative locality focusing on the illustrative example of an emission of a photon by a green atom, near Alice, with the absorption of that same photon by a blue atom, near Bob. The causal link between the two processes is still present, and the processes are still local, but the locality of the processes is not manifest in the inferences about distant events of the two observers. According to the coordinates of observer Alice the photon emission by the green atom is indeed a local process but the distant absorption of the photon by the blue atom appears to be a nonlocal process. In reverse, according to the coordinates of observer Bob the photon absorption by the blue atom is indeed a local process but the distant emission of the photon by the green atom appears to be a nonlocal process.

works by GAC+Arzano+Barcaroli+Kowalski-Gikman+Loret+ Matassa+Mercati+Rosati

evolutions of relativity

Galilean →SR

a velocity scale becomes absolute

simultaneity becomes relative

action of boosts depends on "c"

composition of velocity becomes nonlinear, noncommutative,nonassociative

SR →DSR

a momentum scale becomes absolute

locality becomes relative

action of boosts depends on "c" and "*ℓ*"

composition of momenta becomes nonlinear (&noncommutative? &nonassociative?)

"relativistic equilibrium" ←→ trade a relative for an absoluteGalilean →SR

entanglement and multiparticle states with relative locality

$$e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} = e^{i(k + e^{\ell k_0} K)x} e^{i(k_0 + K_0)t}$$

symmetries described by a Hopf algebra,
essentially codified in the coproduct;for example for translations

$$P_j\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right) = P_j\left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t}\right)$$
$$= \left(k_j + e^{-\lambda k_0} K_j\right)\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right)$$
$$= \left[P_j\left(e^{ikx} e^{ik_0 t}\right)\right]\left(e^{iKx} e^{iK_0 t}\right) + \left[e^{-\lambda P_0}\left(e^{ikx} e^{ik_0 t}\right)\right] P_j\left(e^{iKx} e^{iK_0 t}\right)$$

$$k_j + e^{-\ell k_0} K_j$$

$k_j + e^{-\ell k_0} K_j = 0$ **implica che K e' l'antipodo di k ovvero** $K_j = -e^{\ell k_0} k_j \neq -k_j$

Within this setup it is obvious that the description of multiparticle states must require new structures with respect to the usual construction.

Let us consider for example a state with two indistinguishable scalar particles in a 1+1-dimensional κ-Minkowski spacetime. If we measure the energy-momentum of each of the two particles the indistinguishability would require a description of the state of the following form

$$|\Psi^{(2)}_{\{k,\,q\,\}} >= \frac{1}{\sqrt{2}} \left(|\psi_k > \otimes |\psi_q > + |\psi_q > \otimes |\psi_k > \right)$$

However, we are here confronted with a puzzle: this state obtained by "indistinguishability symmetrization", based on the information obtained by measuring the energy-momentum of each of the two particles **is not an eigenstate of total energy-momentum**. In fact the action of K on $|\psi_q > \otimes |\psi_k >$, gives $(q + k\, e^{-\lambda\omega^+(q)})\ |\psi_q > \otimes |\psi_k >$, whereas the action of K on $|\psi_k > \otimes |\psi_q >$ gives $(k + q\, e^{-\lambda\omega^+(k)})\ |\psi_k > \otimes |\psi_q >$.

GAC+Arzano+Marciano', in Frascati volume
Arzano+Marciano',PhysRevD76,125005

Arzano is here now!

$$\epsilon_i = \frac{|\mathbf{k}_i|}{\kappa}$$

there will be **two** 2-particle states

$$|\mathbf{k}_1\mathbf{k}_2\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_1\rangle \otimes |\,\mathbf{k}_2\rangle + |\,(1-\epsilon_1)\mathbf{k}_2\rangle \otimes |\,(1-\epsilon_2)^{-1}\mathbf{k}_1\rangle\right]$$

$$|\mathbf{k}_2\mathbf{k}_1\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_2\rangle \otimes |\,\mathbf{k}_1\rangle + |\,(1-\epsilon_2)\mathbf{k}_1\rangle \otimes |\,(1-\epsilon_1)^{-1}\mathbf{k}_2\rangle\right]$$

same energy and different linear momentum

$$\mathbf{K}_{12} = \mathbf{k}_1 \oplus \mathbf{k}_2 = \mathbf{k}_1 + (1-\epsilon_1)\mathbf{k}_2$$

$$\mathbf{K}_{21} = \mathbf{k}_2 \oplus \mathbf{k}_1 = \mathbf{k}_2 + (1-\epsilon_2)\mathbf{k}_1$$

So do we all share the same spacetime?

as usual it is for experiments to decide:

**entangled states may eventually prove very powerful for constraining torsion of momentum space,
but present understanding too limited for definite predictions**

manifestations of relative locality for observations of distant astros (GRBs....) are more easily analyzed but it seems they are only sensitive to a possible momentum-space nonmetricity

relativity of locality in "kappa-Minkowski phase-space constructions"

GAC+Matassa+Mercati+Rosati, arXiv:1006.2126; PhysRevLett106, 071301

Smolin, arXiv:1007.0718

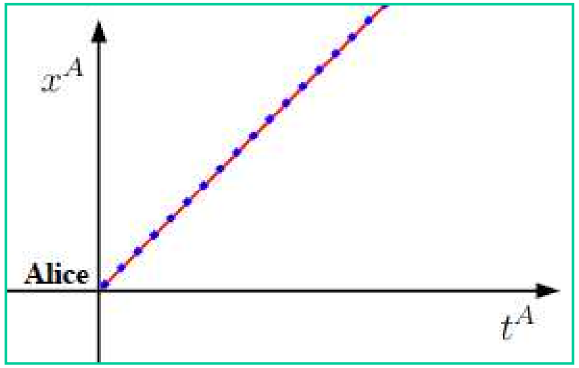
GAC+Loret+Rosati, arXiv:1102.4637 (PhysLettB, in press)

so situation was

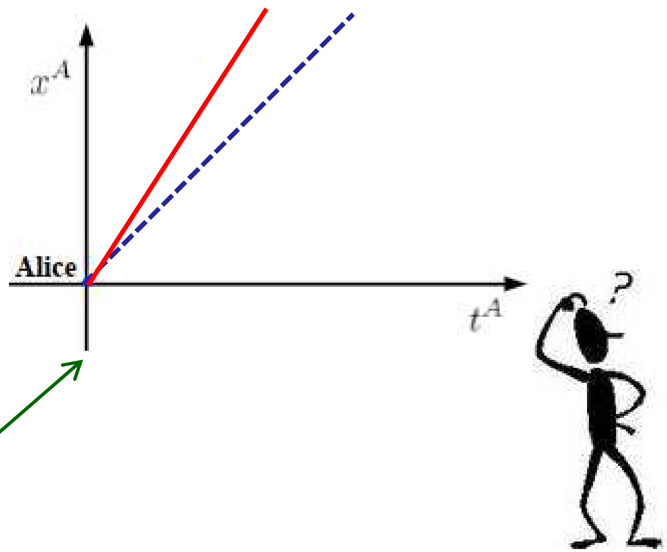
$$\{x,t\} = -\ell x$$

$$\{\Omega, P\} = 0\,, \qquad \{\mathcal{N}, \Omega\} = P\,, \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\,,$$
$$\{P, t\} = \ell P, \qquad \{P, x\} = -1$$



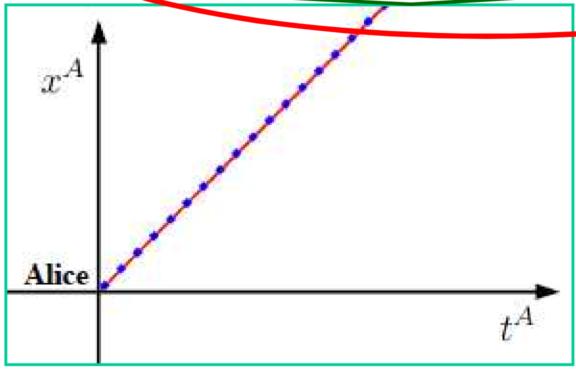
whereas from the "noncommutative Klein-Gordon equation" one would have expected this

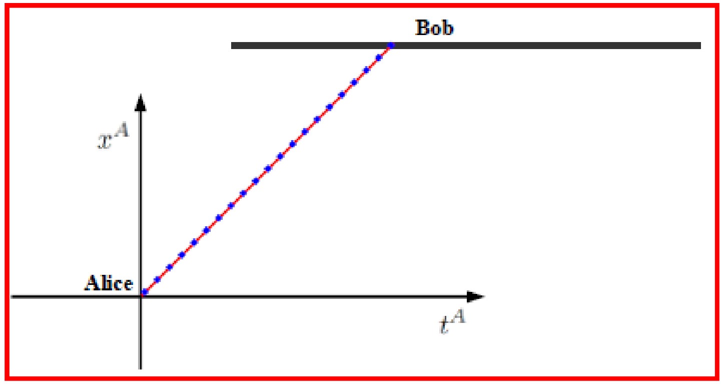


$$\{x,t\} = -\ell x$$

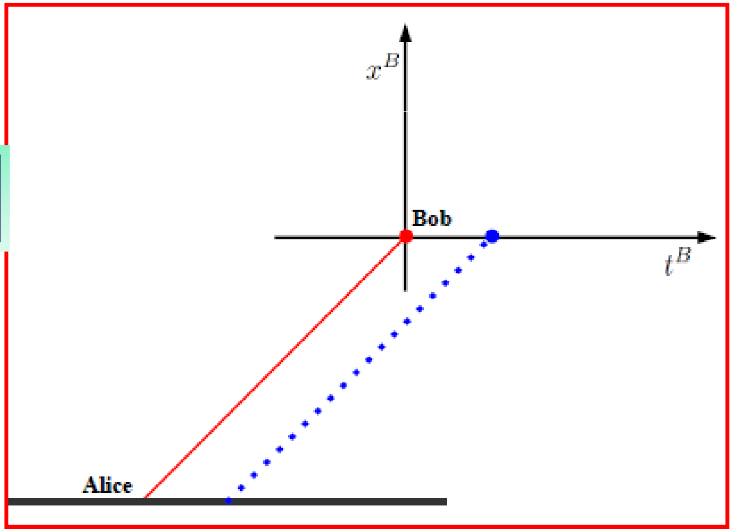
$$\{\Omega,P\}=0\ ,\qquad \{\mathcal{N},\Omega\}=P\ ,\qquad \{\mathcal{N},P\}=\Omega+\ell\Omega^2+\frac{\ell}{2}P^2$$

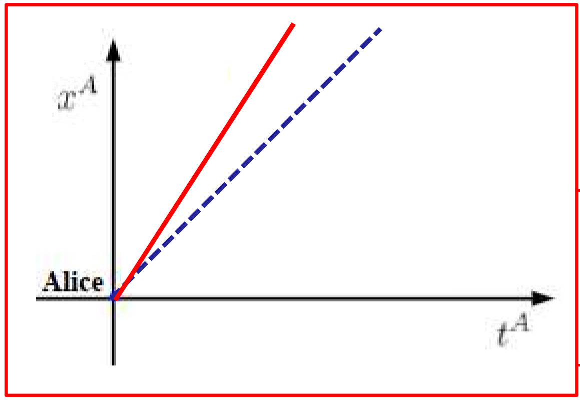
$$\{\Omega,t\}=1,\qquad \{\Omega,x\}=0\ ,$$
$$\{P,t\}=\ell P,\qquad \{P,x\}=-1$$





what about Bob?





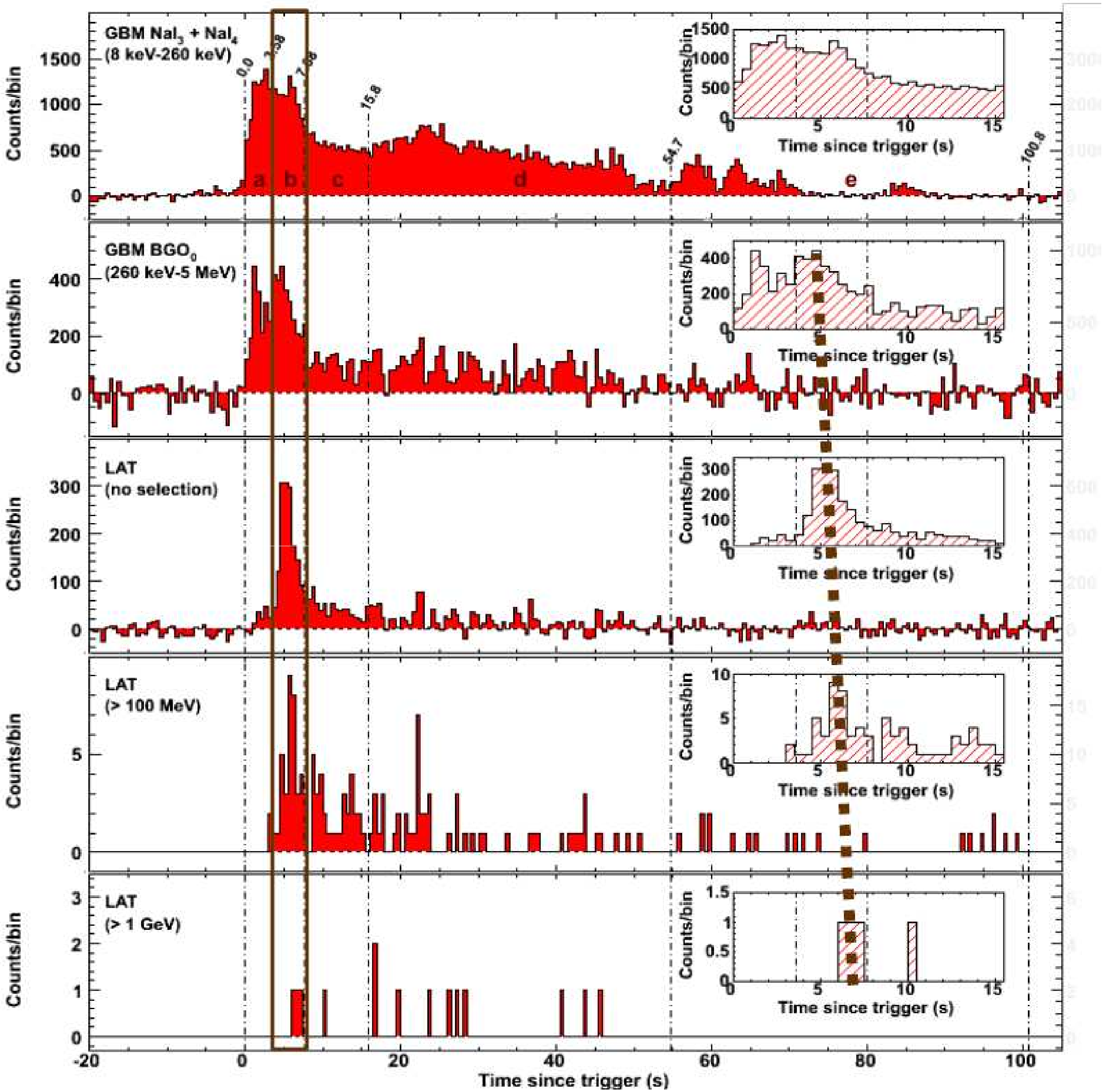
whereas from the "noncommutative Klein-Gordon equation"
one would have expected this

paraphrasing Minkowski we could argue that
“spacetime by itself fades away into a mere shadow,
and only a kind of union of spacetime and momentum space
preserves an independent objectivity”

this is plenty for today
more details and additional observations in arXiv:1101.0931

important point is that this is the natural framework for stating the questions about geometry of momentum space and absoluteness of locality!!
they MUST be viewed as experimental issues
and we cannot test them without a framework for formalization

first ideas on how to test separately the cases of
torsionless metric connection
metric connection with torsion
non-metricity are also in arXiv:1101.0931



GBM NaI$_3$ + NaI$_4$
(8 keV-260 keV)
GBM BGO$_0$
(260 keV-5 MeV)
LAT
(no selection)
LAT
(> 100 MeV)
LAT
(> 1 GeV)
Counts/bin
Time since trigger (s)
0.0
3.58
7.68
15.8
54.7
100.8
a
b
c
d
e

symmetries describe a Hopf algebra
essentially codified in the coproduct;for example for translations

$$P_j\left(e^{ikx}e^{ik_0t}e^{iKx}e^{iK_0t}\right)=P_j\left(e^{i(k+e^{\lambda k_0}K)x}e^{i(k_0+K_0)t}\right)$$

$$=\left(k_j+e^{-\lambda k_0}K_j\right)\left(e^{ikx}e^{ik_0t}e^{iKx}e^{iK_0t}\right)$$

$$=\left[P_j\left(e^{ikx}e^{ik_0t}\right)\right]\left(e^{iKx}e^{iK_0t}\right)+\left[e^{-\lambda P_0}\left(e^{ikx}e^{ik_0t}\right)\right]P_j\left(e^{iKx}e^{iK_0t}\right)$$

Nontrivial coproduct!!

Baker
Campbell
Hausdorff

notice nonlinear composition of momenta

$$k_j+e^{-\lambda k_0}K_j$$

rather unusual form of boost generator due to requirement of closing Hopf algebra and it leads to a deformed mass Casimir

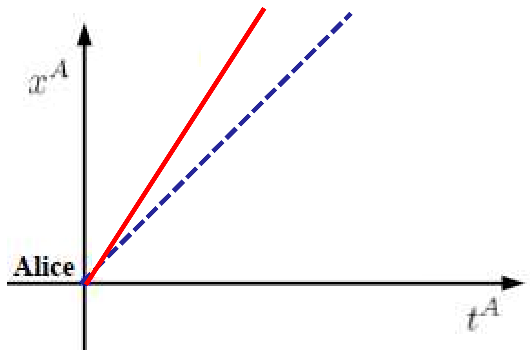
Note that
$\ell\equiv\lambda$

$$\mathcal{C}=\left(\frac{2}{\ell}\right)^2\sinh^2\left(\frac{\ell}{2}P_0\right)-e^{\ell P_0}P_jP^j$$

notice connection with modified dispersion relation

wave equation governed by this Casimir operator and the properties of the "kappa-Minkowski noncommutative differential calculus" describes massless waves that propagate at speed

$$v=e^{-\ell|\vec{p}|}\simeq 1-\ell|\vec{p}|$$



GAC +Majid,IntJModPhysA15(2000)4301

snapshot 1, page 3

kappa-Minkowski also studied in terms of some "kappa-Minkowski phase-space constructions"

basically take the commutators on previous slides and turn them into Poisson brackets:

Note that $\ell \equiv \lambda$

$$\{x,t\} = -\ell x$$

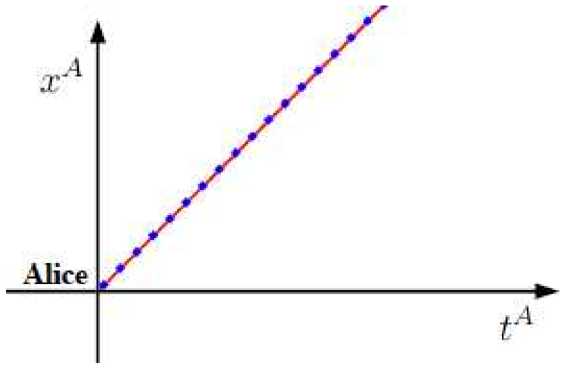
$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\ ,$$
$$\{P, t\} = \ell P, \qquad \{P, x\} = -1$$

$$\{\Omega, P\} = 0\ , \qquad \{\mathcal{N}, \Omega\} = P\ , \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

then derive worldlines of massless particles within a rather standard Hamiltonian analysis

$$x = x_0 + \left(\frac{p}{\sqrt{p^2+m^2}} - \ell p\left(1 - \frac{p^2}{p^2+m^2}\right)\right)(t-t_0)$$

and for massless particles



snapshot 1, page 3

kappa-Minkowski also studied in terms of some "kappa-Minkowski phase-space constructions"

basically take the commutators on previous slides and turn them into Poisson brackets:

Note that $\ell \equiv \lambda$

$$\{x,t\} = -\ell x$$

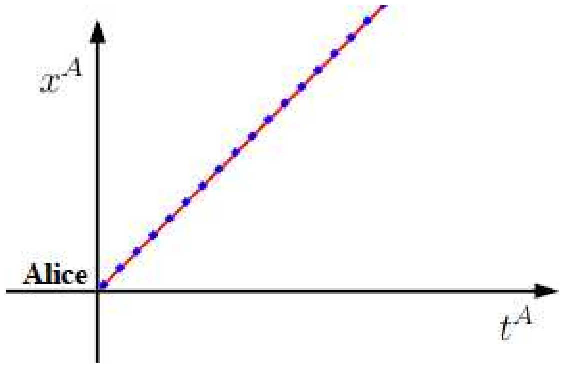
$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\ ,$$
$$\{P, t\} = \ell P, \qquad \{P, x\} = -1$$

$$\{\Omega, P\} = 0\ , \qquad \{\mathcal{N}, \Omega\} = P\ , \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

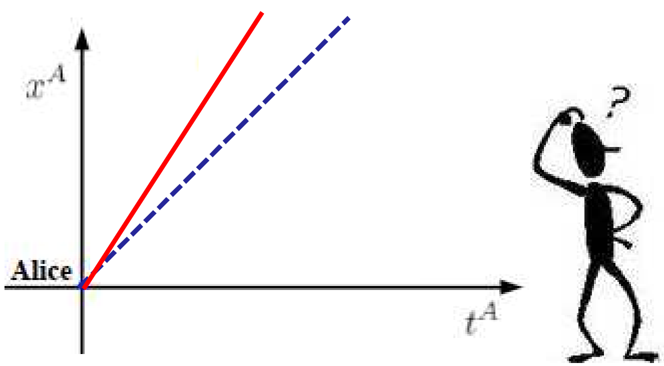
then derive worldlines of massless particles within a rather standard Hamiltonian analysis

$$x = x_0 + \left(\frac{p}{\sqrt{p^2+m^2}} - \ell p\left(1 - \frac{p^2}{p^2+m^2}\right)\right)(t-t_0)$$

and for massless particles



while on previous slide



the principle of relative locality and momentum-space geometry

GAC+Freidel+Kowalski-Glikman+Smolin

note that under a diffeomorphism $p \rightarrow p' = \phi(p)$

The operator τ_p^0

$$(\tau_p^0)^{\alpha}_{\mu}(p) \equiv \partial_q^{\alpha}(p \oplus q)_{\mu}|_{q=0}$$

parallel transport on the tangent bundle

transform as a map from $T_0 P$ to $T_p(P)$

It can be interpreted as a paralell transport operator