QUANTUM DYNAMICS
AND CONFORMAL GEOMETRY:

THE "AFFINE QUANTUM MECHANICS"

From Dirac's equation to the EPR Quantum Nonlocality

Francesco De Martini
Accademia dei Lincei, Rome, Italy

Enrico Santamato
University of Naples, Italy

Quantum Mechanics meets Gravity, La Sapienza, Roma
December 2, 2011

AFFINE QUANTUM MECHANICS

According to Felix Klein "Erlangen Program" the "affine" geometry deals with intrinsic geometric properties that remain unchanged under "affine transformations" (affinities). These consist of collinearity transformations, e.g. sending parallels into parallels and preseving ratios of distances along parallel lines.

The "conformal geometry" by Hermann Weyl is considered a kind of affine geometry preserving angles.

Cfr: H. Coxeter, *Introduction to Geometry* (Wiley, N.Y. 1969).

We aim at showing that the wave equation for quantum spin (in particular Dirac's spin ½ equation) may have room in the classical mechanics of the "relativistic Top"

A.J.Hanson, T. Regge, Ann.Phys (NY) 87, 498 (1974); L.Shulman, Nucl.Phys B18, 595(1970).
E. Sudarshan, N. Mukunda, *Classical Dynamics, a modern perspective* (Wiley, New York, 1974)

Niels Bohr,
Wolfgang Pauli
and the
spinning Top

(Lund, 1951)

PROGRAM:

.1) DERIVATION OF DIRAC's EQUATION

.2) WEYL's CURVATURE AND THE ORIGIN OF QUANTUM NONLOCALITY

3) ELECTRON "ZITTERBEWEGUNG"

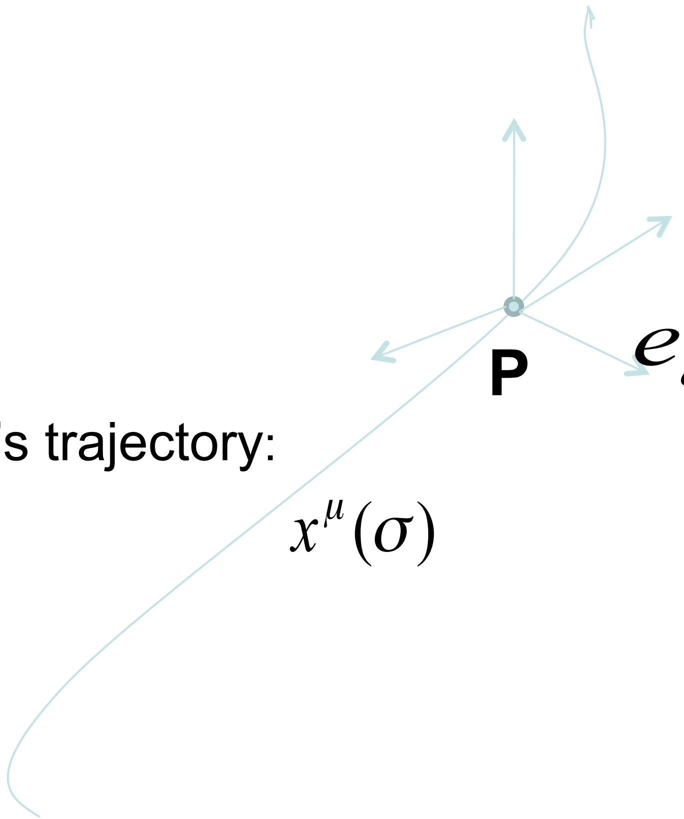
4) KALUZA-KLEIN –JORDAN THEORIES

5) ISOTOPIC SPIN

E.Santamato, F. De Martini , ArXiv 1107.3168v1 [quant-ph]

Tetrad (a) (fourlegs, vierbein)

Top's trajectory:



$e_a^{\mu}(\sigma)$ Tetrad vector = vector's components on Tetrad 4-legs

$$\sigma : parameter \quad (e.g.\, proper \;\; time)$$

$$g_{\mu\nu} e_a^{\mu} e_b^{\nu} = g_{ab} \quad (tetrad's \;\; normalization \;\; equation)$$

$$g_{\mu\nu} = g_{ab} = diag(-1, 1, 1, 1); \quad \{\mu, \nu; a, b : 0, 1, 2, 3\}$$

$$(d\, e_a^{\mu} / d\sigma) = \omega_{\nu}^{\mu}\, e_a^{\nu}; \;\; 6\, Euler \; angles : \theta^{\alpha} (\alpha = 1, \ldots .6)$$

$$\omega^{\mu\nu} = \omega_{\rho}^{\mu} g^{\rho\nu} \;\; : top's \; angular \; velocity (skewsymm.)$$

LAGRANGIANS in V_{10}

$$L_0 = mc\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} - a^2 g_{\mu\nu} g^{ab}\frac{de^\mu_a}{d\sigma}\frac{de^\nu_b}{d\sigma}} =$$

$$= mc\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} - a^2\omega_{\mu\nu}\omega^{\mu\nu}},$$

a: constant $\sim \lambda = (h/mc)$ Compton wl

$$= mc\frac{ds}{d\sigma} = mc\sqrt{-g_{ij}\frac{dq^i}{d\sigma}\frac{dq^j}{d\sigma}},$$

$q^i = \{x^\mu, \theta^\alpha\}$ $(i = 0, \ldots, 9)$ span the dynamical configuration space:

$$V_{10} = \mathcal{M}_4 \times SO(3,1).$$

$$g_{ij} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{\alpha\beta} \end{pmatrix}$$

$$L_{em} = \frac{e}{c}A_\mu\frac{dx^\mu}{d\sigma} + \frac{\kappa e}{4c}\, a^2 F_{\mu\nu}\omega^{\mu\nu} = -(e/c)A_i dq^i/d\sigma,$$

$A_\mu = (-\phi, \boldsymbol{A}), A_i = (A_\mu, A_\alpha)$ is a 10-D vector. A_α of A_i are linear combinations of the fields $\boldsymbol{H}(x)$ and $\boldsymbol{E}(x)$,

$$L = L_0 + L_{em}$$ is σ-parameter independent:

[*H. Rund, The Hamilton-Jacobi equation* (Krieger, NY; 1973)]

Weyl's affine connection and curvature:

$$\Gamma^i_{jk} = -\left\{ {i \atop jk} \right\} + \delta^i_j \phi_k + \delta^i_k \phi_j + g_{jk}\phi^i,$$

where $\left\{ {i \atop jk} \right\}$ are the Cristoffel symbols out of the metric g_{ij}, and $\phi^i = g^{il}\phi_l$ is the Weyl potential ϕ_i as $\phi_i = \chi^{-1}\partial\chi/\partial q^i$.

By Weyl's af f ineonnection Γ^i_{jk} *the overall Weyl's scalar curvature* R_W *can be calculated in* $D = n = 10$ dim*ensions* :

$$\begin{aligned} R_W &= R + 2(n-1)\nabla_k\phi^k - (n-1)\phi_k\phi^k = \\ &= R + 2(n-1)\frac{\nabla_k\nabla^k\chi}{\chi} - n(n-1)\frac{\nabla_k\chi\nabla^k\chi}{\chi^2} \end{aligned}$$

Hermann Weyl gauge - invariant Geometry

In **Riemann geometry:** change of component of a contravariant vector under *"parallel transport"* along dx^{μ} : $\delta\eta^{\mu} = -\Gamma^{\mu}_{\rho\sigma}\eta^{\sigma}dx^{\sigma}$; $\Gamma^{\mu}_{\rho\sigma} = \Gamma^{\mu}_{\sigma\rho}$: "*affine connection*"

Scalar product of covariant - contravariant vector : $\delta l^2 = \delta(\eta_{\mu}\eta^{\mu}) = 0$ leading to:

$\delta\eta_{\mu} = \Gamma^{\rho}_{\mu\sigma}\eta_{\rho}dx^{\sigma}$. Then, *covariant differentiation* : $\eta^{\mu}_{\ |\sigma} = \partial_{\sigma}\eta^{\mu} + \Gamma^{\mu}_{\rho\sigma}\eta^{\rho}$,

$\eta_{\mu|\sigma} = \partial_{\sigma}\eta_{\mu} - \Gamma^{\rho}_{\mu\sigma}\eta_{\mu}$; *Differentiation of metric tensor* : $g_{\mu\nu|\rho} = 0$.

In **Weyl's geometry :** $\delta l^2 = \delta(\eta^{\mu}\eta_{\mu}) = \varphi_{\sigma}\delta x^{\sigma} l$; $g_{\mu\nu,\rho} = 2g_{\mu\nu}\varphi_{\rho}$;

$\Rightarrow$ *New "gauge" vector field* φ_{ρ} : "*Weyl's potential*"!

Weyl conformal transformation : $l \to e^{\lambda} l$ "*local gauge field*": $\lambda(q^i)$

$$\varphi_{\mu} = \partial\lambda / \partial q^{\mu} \equiv grad\,\lambda = \chi(q^i)^{-1} \times \partial\chi(q^i)/\partial q^i$$

WEYL's CONFORMAL MAPPING (CM) (*)

= spacetime dependent change of the unit of length L:

$$ds \rightarrow e^{\lambda(q)} ds$$

Under (*CM*) *any physical quantity with* dim*ensions* L^W *is assigned a transformation law:* $X \rightarrow e^{W\lambda} X$ (W = "*weight*" *or* "dim*ensional number*" *of the object* (*like the electric charge e in electrodynamics*)).

Thus, (*CM*) *is a* "*unit transformation*" *amounting to a space-time redefinition of the* "*unit of length*".

Examples: $g_{ik} : W = 2;\quad g^{ik} : W = -2;\quad \sqrt{-g} : W = 2;\quad \Gamma^{l}_{ik} : W = 0;\quad R : W = -2;$ *etc.*

Conformal Mapping preserves angles between 4 – *vectors.*

(*) H. Weyl, *Ann. der Physik,* 59, 101 (1919); *Time, Space, Matter* (Dover, NY, 1975)

CONFORMAL GROUP :

.1) 6 – parameters Lie group, isomorphic to the proper, orthochronous, homogeneous Lorentz group.

.2) Preserves the angle between two curves in space time and its direction.

.3) In flat spacetime of Special Relativity the relevant group structure is the inhomogeneous Lorentz group (Poincare' group).

In General Relativity , if spacetime is only "conformally flat" (i.e. Weyl's conformal tensor: $C^{\mu}{}_{\nu\rho\mu} = 0$) we obtain a larger group (15 parameters) of which the Poincaré group is a subgroup.

Weyl's conformal tensor:

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \frac{1}{2}(R_{\mu\rho}g_{\nu\sigma} - R_{\mu\sigma}g_{\nu\rho} + R_{\nu\sigma}g_{\mu\rho} - R_{\nu\rho}g_{\mu\sigma})$$

$$- \frac{1}{6}R(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}); \qquad R_{\mu\nu\rho\sigma} = Riemann\ \ curvature\ \ tensor$$

INTRODUCTION
TO THE
THEORY OF RELATIVITY

by
PETER GABRIEL BERGMANN
PROFESSOR OF PHYSICS
SYRACUSE UNIVERSITY

WITH A FOREWORD
BY
ALBERT EINSTEIN

DOVER PUBLICATIONS, INC.
NEW YORK

P. Bergmann, *Theory of Relativity, (*Dover 1976, Pg. 250).

Physical interpretation of Weyl's geometry. In Weyl's geometry, the geometric structure of space is characterized by the symmetric tensor density $g_{\mu\nu}$ and the "pseudovector" φ_μ. It appears reasonable to assume that the $g_{\mu\nu}$ represent the gravitational field, and that the φ_μ are the components of the world vector potential. In Weyl's original formalism, the φ_μ transform as a vector with respect to coördinate transformations, but are changed by a gradient when a gauge transformation is carried out. This is the historical reason for calling the addition of a gradient to the electromagnetic world vector potential a gauge transformation.

We shall now attempt to set up field equations for the $g_{\mu\nu}$ and φ_μ.

Weyl's variational principle. Weyl considered it desirable to derive the field equations as the Euler-Lagrange equations of a variational principle. We shall show that the Euler-Lagrange equations of a varia-

$$A_\varpi \to A_\mu - \partial_\mu \alpha(x)$$

In place of L assume a $g_{\mu\nu} \to \rho(q^i) g_{\mu\nu}$ conformally - invariant Lagrangian:

$$\bar{L} = \gamma\hbar\sqrt{-R_W g_{ij} \frac{dq^i}{d\sigma}\frac{dq^j}{d\sigma}} + L_{em},$$

where the particle's mass is replaced by the Weyl's scalar curvature: R_W

$$mc \to \gamma\hbar\sqrt{R_W}.$$

R_W acts as a scalar potential on the top and, because it depends on χ and its derivatives, the field χ acts on the top as a sort of pre-potential. The paths followed by the top in the configuration space $V_{10} = \mathcal{M}_4 \times SO(3,1)$ are assumed to be the extremal curves of the action integral $\int \bar{L}\, d\sigma$.

HAMILTON – JACOBI EQUATION

Search for a family of equidistant hypersurfaces S = constant as bundles of extremals $\int \bar{L}\, d\sigma$. via Hamilton-Jacobi equation:

$$g^{ij}\left(\frac{\partial S}{\partial q^i} - \frac{e}{c}A_i\right)\left(\frac{\partial S}{\partial q^j} - \frac{e}{c}A_j\right) =$$

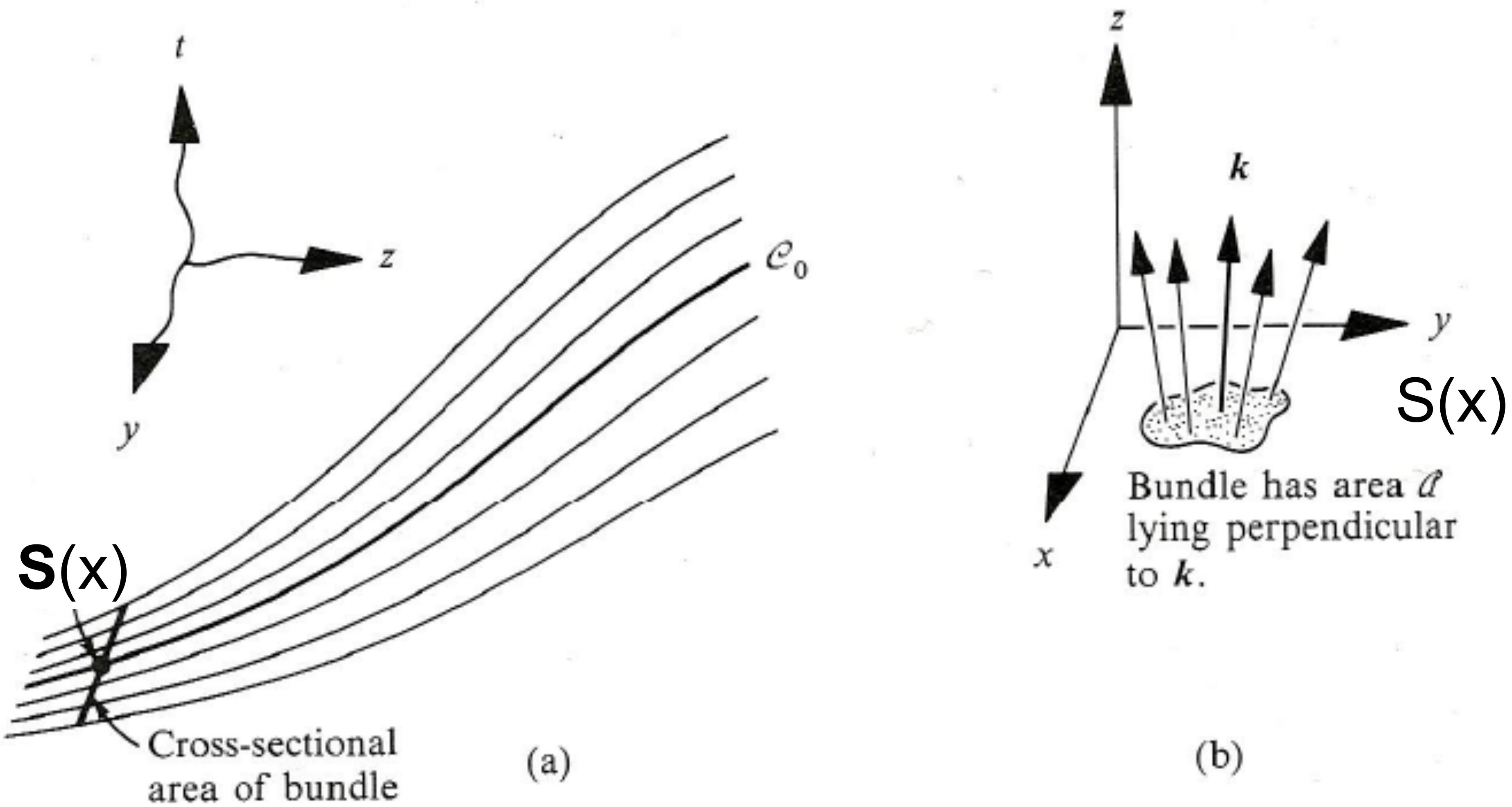
$$= g^{ij}\left(D_i S - \frac{e}{c}A_i\right)\left(D_j S - \frac{e}{c}A_j\right) = -\hbar^2\gamma^2 R_W$$

by integrating the differential equations

$$\frac{dq^i}{ds} = \frac{g^{ij}(\frac{\partial S}{\partial q^j} - \frac{e}{c}A_j)}{[g^{mn}(\frac{\partial S}{\partial q^m} - \frac{e}{c}A_m)(\frac{\partial S}{\partial q^n} - \frac{e}{c}A_m)]^{1/2}}.$$

Nonlinear partial differential equations for the unknown S(q) and $\chi(q)$ once the metric tensor $g_{ik}(q)$ is given.

S(q) is the Hamilton's "Principal function".



S(x) : Hamilton's Principal Function $\Longrightarrow \quad p_i = \dfrac{\partial S(x)}{\partial q^i}$

By the "ansatz" solution, with Weyl "weight": W = (2 – n)/2

$$\Rightarrow \quad \psi(q) = \chi(q)^{-\frac{n-2}{2}} e^{i\frac{S(q)}{\hbar}}$$

and, by fixing, for D=10 : $\gamma^2 = \frac{n-2}{4(n-1)} = \frac{2}{9}$.

the "classical" **Hamilton - Jacobi** equation is **linearized** leading to:

$$\Rightarrow \quad g^{ij}\left(-i\hbar\nabla_i - \frac{e}{c}A_i\right)\left(-i\hbar\nabla_j - \frac{e}{c}A_j\right)\psi + \hbar^2\gamma^2 R\psi = 0.$$

In the absence of the e.m. field , $A_j = 0$, the above equation reduces to:

$\hat{L}\psi = (-\Delta + \gamma^2 R)\psi$ where Δ is the Laplace–Beltrami operator and

$\hat{L}$ is the "conformal" Laplacian , i.e. a Laplace- de Rham operator. $R = 6/a^2$.

STANDARD DEFINITION OF THE:

"HAMILTON's PRINCIPAL FUNCTION" S(x) :

$$p_i = \frac{\partial S(x)}{\partial x^i} \Rightarrow [-i\hbar\nabla_i] \times [i S(x) / \hbar]$$

$$\Uparrow \qquad\qquad \Uparrow$$

$$Momentum\ \ operator(\hat{p}) \times\ complex(!)"\ phase"$$

Moreover, we assume that the action function S obeys the auxiliary divergence condition

$$D_k \left(D^k S - \frac{e}{c} A_k \right) = 0. \tag{9}$$

expressing the Weyl's invariant current – density:

$$j^i = \chi^{-(n-2)} \sqrt{g}\, g^{ij} (\partial S/\partial q^j - (e/c) A_j).$$

which can be written in the alternative, significant form:

$$j^i = |\psi|^2 \sqrt{g}\, g^{ij} (\partial S/\partial q^j - (e/c) A_j).$$

This is done by introducing the <u>same</u> "ansatz":

$$\Longrightarrow \quad \psi(q) = \chi(q)^{-\frac{n-2}{2}} e^{i \frac{S(q)}{\hbar}}$$

The above results show that the scalar density $|\psi|^2 \equiv \rho$ is transported along the particle's trajectory in the configuration space, allowing a possible statistical interpretation of the wavefunction according to Born's quantum mechanical rule.

The quantum equation appears to be mathematically equivalent to the classical Hamilton-Jacobi associated with the conformally – invariant Lagrangian $\hat{L}$ and the Born's rule arises from the conformally invariant zero - divergence current along any Hamiltonian bundle of trajectories in the configuration space.

It is possible to show that the **Hamilton – Jacobi** equation can account for the **quantum Spin-1/2.**

$\Rightarrow$ *Homomorphism*: $SO(3,1) \to SL(2)$

$$\downarrow$$

[*eigenvalues of the Casimir operators*: $u(u+1),\ v(v+1)$]

$$(0, 1/2) \oplus (1/2, 0)$$

$$\uparrow \qquad \uparrow$$

$$(u,v) \qquad (u,v)$$

$\odot$ *Dotted / undotted spinors* $\Rightarrow$ $\eta^{\alpha} \qquad \eta^{\dot{\alpha}}$

$\Rightarrow$ *Space inversion* I_s (*Parity*):

$$I_s \xi^{\alpha} \xrightarrow{\Lambda} \eta^{\dot{\alpha}} \ ; \ I_s \eta^{\dot{\alpha}} \xrightarrow{\Lambda} \xi^{\alpha}$$

$$I_s \begin{pmatrix} \xi^{\alpha} \\ \eta^{\dot{\beta}} \end{pmatrix} \xrightarrow{\Lambda} \lambda \begin{pmatrix} \eta^{\dot{\alpha}} \\ \xi^{\beta} \end{pmatrix} \quad \{\lambda = 1,\ i\}$$

$\Rightarrow$ *Because of equation's* I_s *invariance*: 4 – *Dimension Dirac's spinor.*

4 - D solution, invariant under Parity :

$$\psi_{uv}(q) = D^{(u,v)}(\Lambda^{-1})^{\sigma}_{\sigma'}\psi^{\sigma'}_{\sigma}(x) \ + \ D^{(v,u)}(\Lambda^{-1})^{\dot{\sigma}}_{\dot{\sigma}'}\psi^{\dot{\sigma}'}_{\dot{\sigma}}(x)$$

$D^{(u,v)}(\Lambda)^{\sigma}_{\sigma'}$: (2u+1) ×(2v+1) matrices accounting for transf.s : $\Lambda(\theta) = \{e^{\mu}_{a}(\theta)\}$

$\psi^{\sigma'}_{\sigma}(x)$, $\psi^{\dot{\sigma}'}_{\dot{\sigma}}(x)$: 2-component spinors accounting for space-time coord.s: x^{μ}

The two matrices are related by $[D^{(u,v)}(\Lambda)]^{\dagger} = [D^{(v,u)}(\Lambda)]^{-1}$.

$$\psi_{uv}(q) = D^{(u,v)}(\Lambda^{-1})^{\sigma}_{\sigma'}\psi^{\sigma'}_{\sigma}(x) \;+\; D^{(v,u)}(\Lambda^{-1})^{\dot\sigma}_{\dot\sigma'}\psi^{\dot\sigma'}_{\dot\sigma}(x)$$
$$(u \leq v)$$

where $D^{(u,v)}(\Lambda)^{\sigma}_{\sigma'}$ are the $(2u+1)\times(2v+1)$ matrices representing the Lorentz transformation $\Lambda(\theta) = \{e^{\mu}_{a}(\theta)\}$ in the irreducible representation labeled by the two numbers u, v given by $2u, 2v = 0, 1, 2, \ldots$, and the $\psi^{\sigma'}_{\sigma}(x)$ and $\psi^{\dot\sigma'}_{\dot\sigma}(x)$ are expansion coefficients depending on the space-time coordinates x^{μ} alone. The matrices $D^{(u,v)}(\Lambda)$ and $D^{(v,u)}(\Lambda)$ depend on the Euler angles θ^{α} only, and provide conjugate representations of the Lorentz transformations

The two matrices are related by $[D^{(u,v)}(\Lambda)]^{\dagger} = [D^{(v,u)}(\Lambda)]^{-1}$.

4 components Dirac's equation

$$\left[g^{\mu\nu}\left(-i\hbar\partial_\mu - \frac{e}{c}A_\mu\right)\left(-i\hbar\partial_\nu - \frac{e}{c}A_\nu\right) - \frac{e\hbar}{c}(\boldsymbol{\Sigma}\cdot\boldsymbol{H} - i\alpha\cdot\boldsymbol{E})\right]\Psi_D +$$
$$+\left[\frac{e^2a^2}{c^2}(H^2 - E^2) + \frac{3\hbar^2}{2a^2}(1+4\gamma^2)\right]\Psi_D = 0,$$

Where:

$$\Psi_D = \begin{pmatrix}\psi_\sigma^{\sigma'}\\ \psi_{\dot\sigma}^{\dot\sigma'}\end{pmatrix} \quad \boldsymbol{\Sigma} = \begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{pmatrix}, \quad \alpha = \begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & -\boldsymbol{\sigma}\end{pmatrix} \quad \boldsymbol{\sigma} = \{\sigma_x, \sigma_y\sigma_z\}$$

By the replacement : $R_W \to R_W - (ea/c\hbar\gamma)^2(\frac{1}{2}F_{\mu\nu}F^{\mu\nu})$ the e.m. term: $(ea/c)^2(H^2 - E^2)$ cancels, and by setting: $a = (\hbar/mc)\sqrt{3(1+4\gamma^2)/2}$ the equation reproduces exactly the quantum – mechanical results given by:

L.D. Landau, E.M. Lifschitz, *Relativistic Quantum Theory* (Pergamon, NY, 1960)
L.S. Schulman, Nucl. Phys. B18, 595 (1970).

THE SQUARE OF THE DIRAC'S EQUATION FOR THE SPIN ½

CAN BE CAST IN THE EQUIVALENT FORM:

$$\gamma^\mu \gamma^\nu [\hat{p}_\mu - (e/c)A_\mu + m][\hat{p}_\nu - (e/c)A_\nu - m]\psi = 0$$

WHERE THE GAMMA MATRICES OBEY TO THE CLIFFORD's ALGEBRA:

THE 4 – MOMENTUM OPERATOR IS: $\hat{p}_\mu = i\hbar(\partial_t, -\nabla)$

IN SUMMARY: our results suggest that:

.1) The methods of the classical **Differential Geometry** may be considered as an inspiring context in which the relevant paradigms of modern physics can be investigated satisfactorily by a direct , logical, (likely) “complete” theoretical approach.

.2) Quantum Mechanics may be thought of as a “**gauge theory”** based on “fields” and “potentials” arising in the context of differential geometry.
Such as in the geometrical theories by: *Kaluza, Klein, Heisenberg, Weyl, Jordan, Brans-Dicke, Nordström, Yilmaz, etc. etc.*

.3) Viewed from the above quantum - geometrical perspective,

“**GRAVITATION”**

is a “monster” sitting just around the corner…….

QUANTUM MECHANICS : A WEYL's GAUGE THEORY ?

LOOK AT THE DE BROGLIE – BOHM THEORY

Max Jammer,

The Philosophy of Quantum Mechanics

(Wiley, 1974; Pag. 51)

DE BROGLIE - BOHM

Thus the quantum mechanical equation

$$\frac{1}{2m}\left(-i\hbar\nabla-\frac{e}{c}A\right)^2\psi+(e\varphi+V)\psi=i\hbar\frac{\partial\psi}{\partial t}, \tag{66}$$

which describes the motion of a charged particle in a field, determined by the vector potential A and the scalar potential φ, could be replaced by the hydrodynamic equations

$$\operatorname{div}(\rho v)+\frac{\partial\rho}{\partial t}=0, \tag{67}$$

the equation of continuity, and

$$\rho_m\frac{dv}{dt}=-\rho\operatorname{grad}(V+Q)+\frac{\rho e}{c}(v\times H)+\rho_e E, \tag{68}$$

where

$$\rho_m=m\rho,\quad \rho_e=e\rho,\quad \text{and}\quad Q=-\frac{\hbar^2}{2m}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}}. \tag{69}$$

[49] In their subsequent paper "Relativistic hydrodynamics of rotating fluid masses," *Physical Review* **109**, 1881–1889 (1958), Bohm and Vigier generalized their approach to a hydrodynamical interpretation of the Dirac and Kemmer wave equation, hoping to provide thereby a physical basis for a causal interpretation also of relativistic wave equations.

[50] H. W. Franke, "Ein Strömungsmodell der Wellenmechanik," *Acta Physica Academiae Scientiarum Hungaricae* **4**, 163–172 (1954).

[51] L. Jánossy, "Zum hydrodynamischen Modell der Quantenmechanik," *Zeitschrift für Physik* **169**, 79–89 (1962). L. Jánossy and M. Ziegler, "The hydrodynamical model of wave mechanics," *Acta Physica Academiae Scientiarum Hungaricae* **16**, 37–48 (1963); *ibid.*, 345–354 (1964); **20**, 233–251 (1966); **25**, 99–109 (1968); **26**, 223–237 (1969); **27**, 35–46 (1969); **30**, 131–137 (1971); *ibid.*, 139–143 (1971).

(Max Jammer, *The Philosophy of Quantum Mechanics, Wiley 1974; Pag. 52)*

(Q, often referred to as the "quantum mechanical potential," has to be interpreted in the present context as an "elastic potential" whose gradient yields the interior force which, together with an exterior force, produces the acceleration of the fluid.) The last equation manifests ostensibly the action of the Lorentz force on the elements of the fluid.

Jánossy and his collaborators also showed how the hydrodynamic interpretation can be extended to account for particles described by the Pauli equation. By expressing the Pauli equation in terms of hydrodynamic variables as a system of equations which describe motions in an elastic medium they succeeded in proving that there exists a one-to-one correspondence between the normalized solutions of the wave equation and the solutions of the hydrodynamic equations which satisfy the appropriate initial conditions. Even the spin-orbit coupling can be accounted for on this interpretation. The difficulties which arise in extending this interpreta-

$\Rightarrow$ *De Broglie – Bohm "Quantum Potential":*

$$Q = -\frac{\hbar^2}{2m} \times \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \qquad \textit{For wavefunction } \psi = \sqrt{\rho} \times \exp(iS/\hbar)$$

$$Q = -\frac{\hbar^2}{2m} \times \left(\frac{1}{2} \frac{\nabla_k \nabla^k \rho}{\rho} - \frac{1}{4} \frac{\nabla_k \rho \, \nabla^k \rho}{\rho^2} \right) \qquad [\textit{In space} \dim : n = 3]$$

$\Rightarrow$ *Overall "Curvature in" Weyl's geometry:*

$$R_W = R + \frac{1}{4}(n-1) \times \left(\frac{1}{2} \frac{\nabla_k \nabla^k \chi}{\chi} - \frac{n}{4} \frac{\nabla_k \chi \, \nabla^k \chi}{\chi^2} \right).$$

$[\textit{In spacetime} \dim : n = 10 \quad \textit{FULLY RELATIVISTIC THEORY}]$

TOTAL SPACE TIME CURVATURE

DUE TO AFFINE CONNECTION
(i. e. TO CHRISTOFFEL SYMBOLS)
IMPLIED BY 10-D METRIC TENSOR G

$$Q \Rightarrow (R_W - R) = Curvature\ due\ to$$

$$Weyl's\ gauge\ field$$

Physical effects of Weyl coupling amo

particles via space-time curvature:

1) *Quantum Interference (Young's IF)*

2) *Quantum Nonlocality, etc. etc.*

Einstein-Podolsky-Rosen “paradox” (EPR 1935)

I.8 CAN QUANTUM-MECHANICAL DESCRIPTION OF PHYSICAL REALITY BE CONSIDERED COMPLETE?

ALBERT EINSTEIN, BORIS PODOLSKY, AND NATHAN ROSEN

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

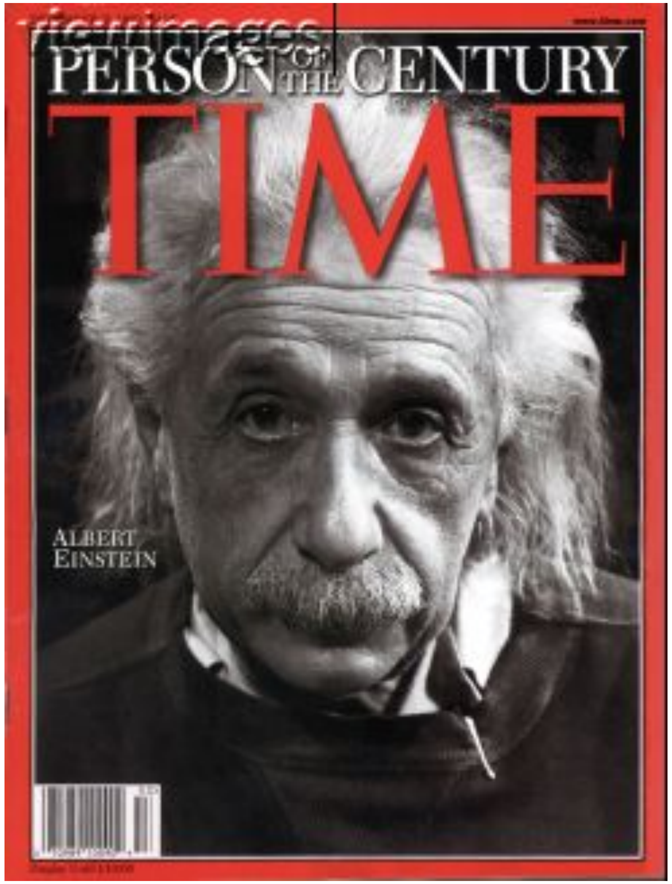
In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

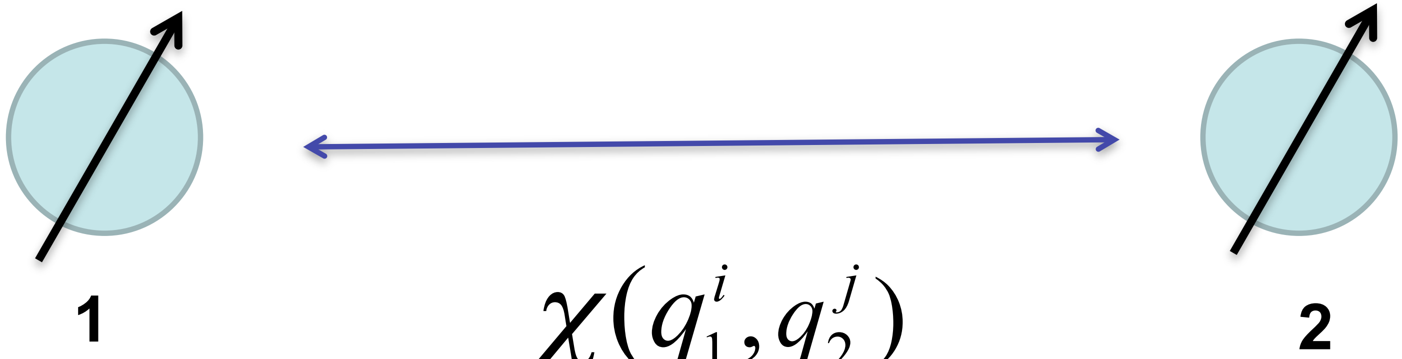
Originally published in *Physical Review*, 47, 777-80 (1935).

$$|\psi\rangle = \sum_K \underbrace{\varphi(1)\,\vartheta(2)}_{\text{eigenstates of } \hat{x}} = \sum_e \underbrace{\chi(1)\,\eta(2)}_{\text{eigenstates of } \hat{p}}$$



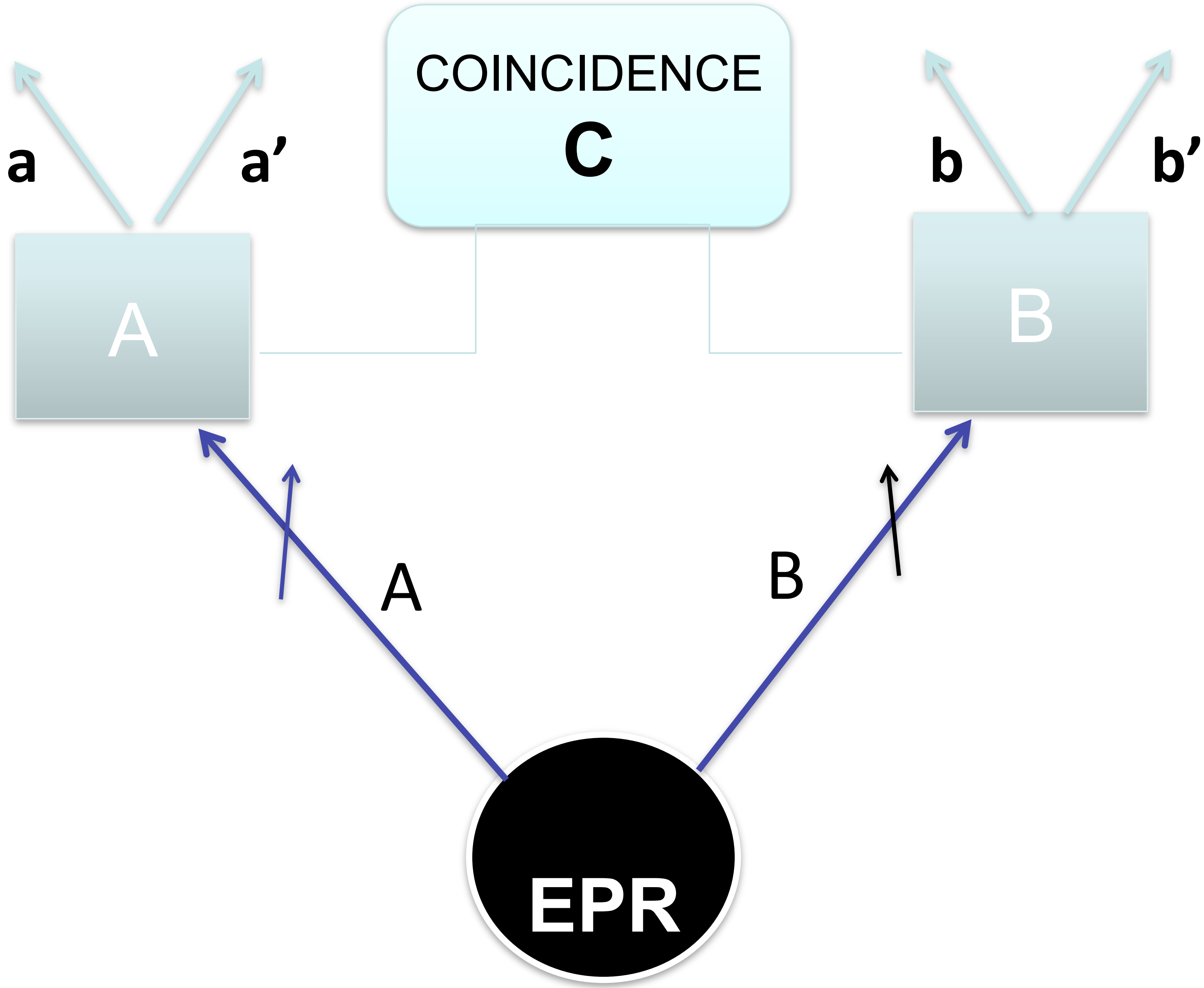
"SPOOKY ACTION – AT – A – DISTANCE"

A. Einstein

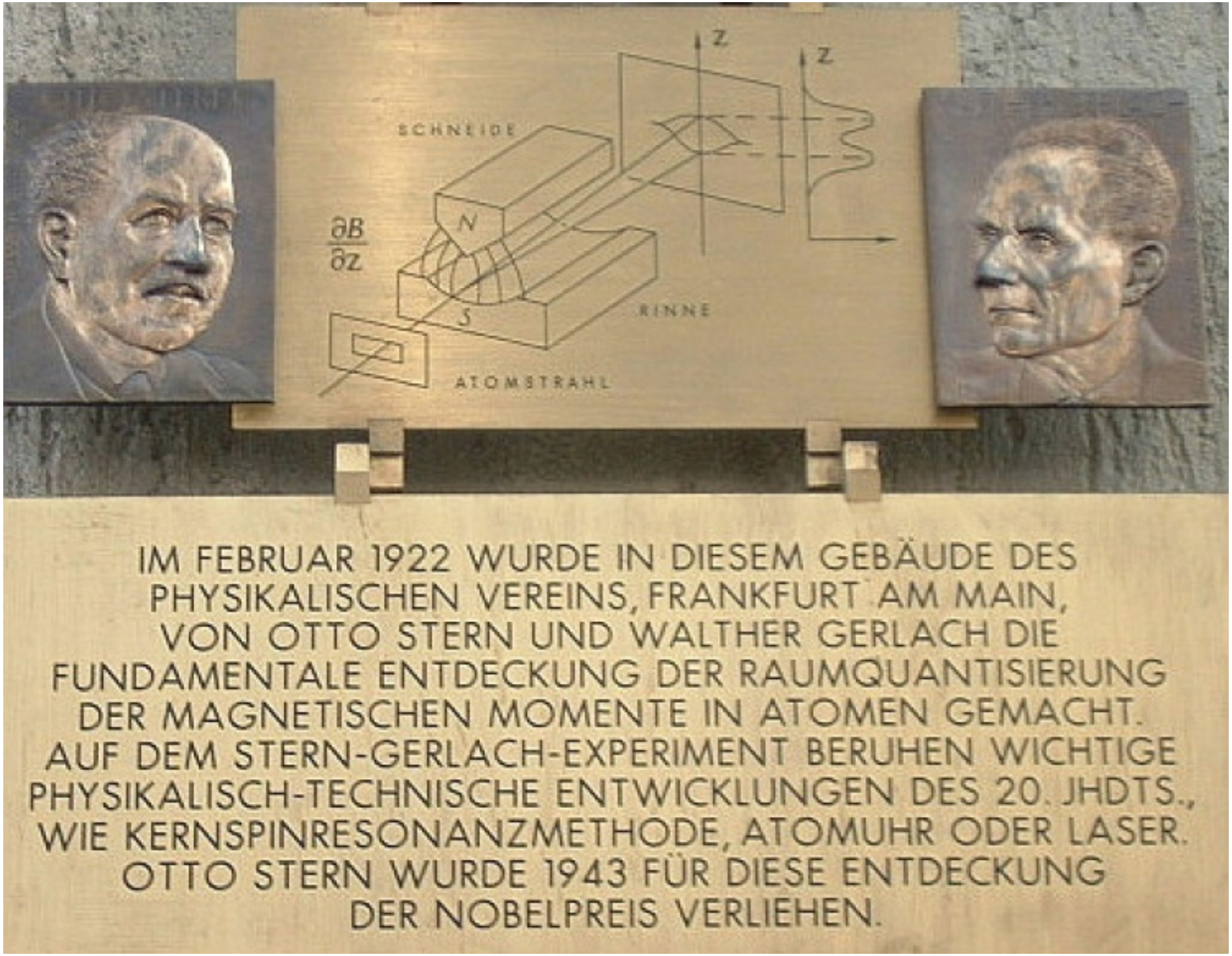


$$\chi(q_1^i, q_2^j)$$

The two-particle configuration space $\mathcal{V}_{20}$ is the product of two copies of the 10-D space $\mathcal{V}_{10} = \mathcal{M}_4 \times SO(3,1)$ of each particle. We take as coordinates in $SO(3,1)$ the six Euler angles θ_1^α and θ_2^α ($\alpha = 1, \ldots, 6$) for particle 1 and 2, respectively. We set also $q_1^i = \{x_1^\mu, \theta_1^\alpha\}$ and $q_2^i = \{x_2^\mu, \theta_2^\alpha\}$ ($i = 1, \ldots, 10$) the coordinates of each particle and $Q^A = \{q_1^i, q_2^i\}$ ($A = 1, \ldots, 20$) the coordinates of the whole configuration space $\mathcal{V}_{20}$.

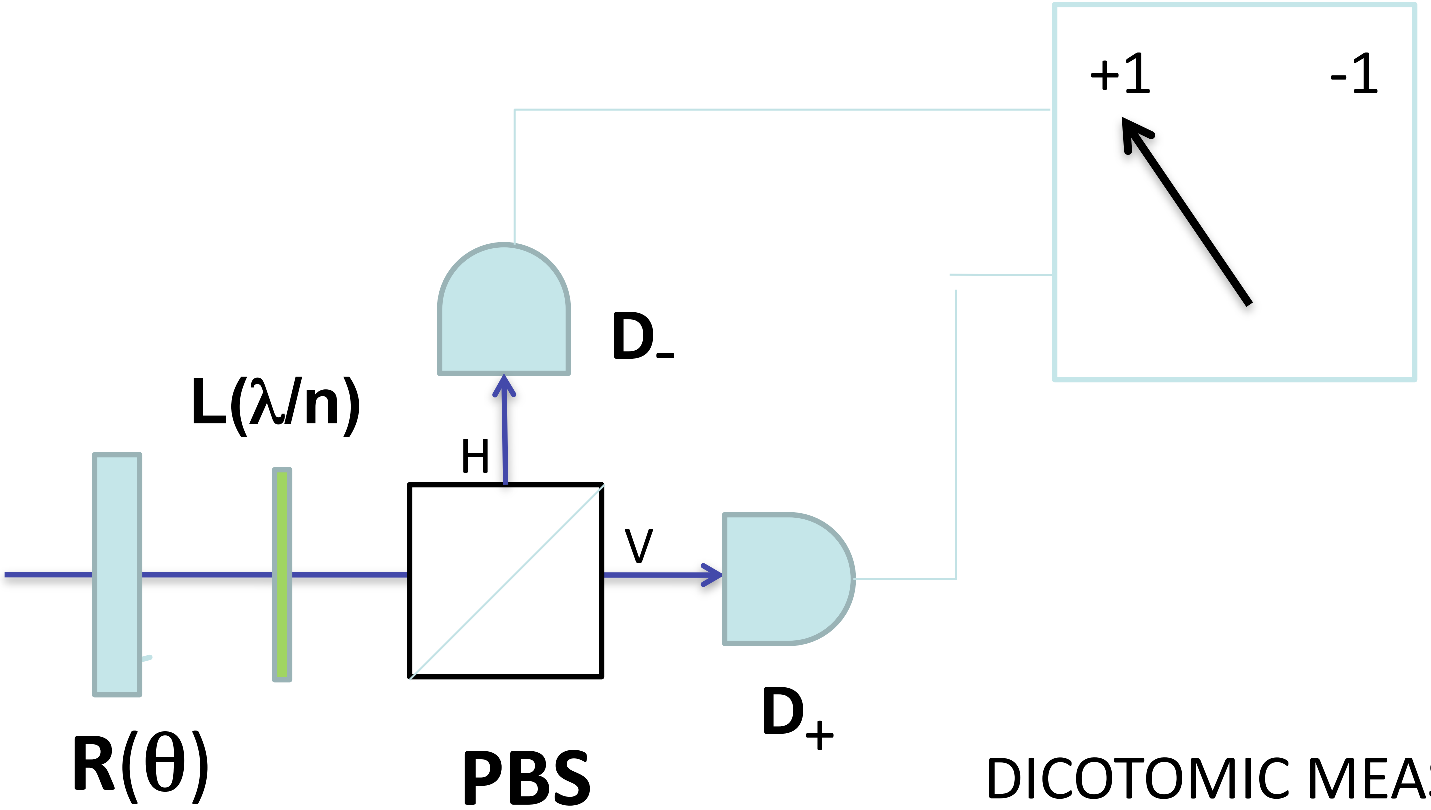


F11-3


SCHNEIDE
N
S
RINNE
ATOMSTRAHL
∂B/∂z
z
z
IM FEBRUAR 1922 WURDE IN DIESEM GEBÄUDE DES
PHYSIKALISCHEN VEREINS, FRANKFURT AM MAIN,
VON OTTO STERN UND WALTHER GERLACH DIE
FUNDAMENTALE ENTDECKUNG DER RAUMQUANTISIERUNG
DER MAGNETISCHEN MOMENTE IN ATOMEN GEMACHT.
AUF DEM STERN-GERLACH-EXPERIMENT BERUHEN WICHTIGE
PHYSIKALISCH-TECHNISCHE ENTWICKLUNGEN DES 20. JHDTS.,
WIE KERNSPINRESONANZMETHODE, ATOMUHR ODER LASER.
OTTO STERN WURDE 1943 FÜR DIESE ENTDECKUNG
DER NOBELPREIS VERLIEHEN.

F5-3

OPTICAL STERN - GERLACH



DICOTOMIC MEASUREMENT
ON A SINGLE PHOTON:
Click (+) : a = +1
Click (-) : a = -1

SINGLE SPIN:

Generalized coordinates: $q^i = \{x, y, z, \alpha, \beta, \gamma\}\ (i = 1, \ \ldots, 6)$

(Euler angles)

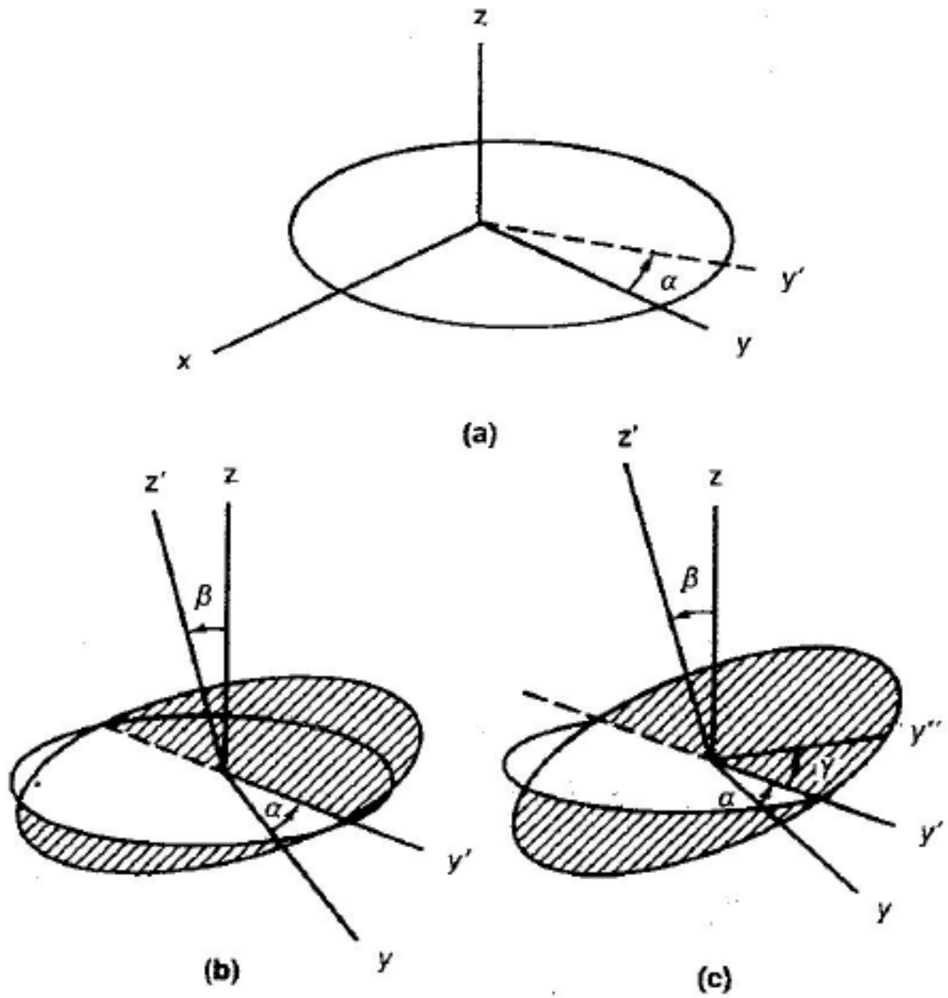
Lagrangian: $L_{\text{cl}} = \frac{1}{2} m v^2 + \frac{1}{2} I_C \omega^2 = \frac{1}{2} g_{ij} \dot{q}^i \dot{q}^j.$

where

Metric Tensor:

$$\{g_{ij}\} = \begin{pmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_C & 0 & I_C \operatorname{Cos}[\beta] \\ 0 & 0 & 0 & 0 & I_C & 0 \\ 0 & 0 & 0 & I_C \operatorname{Cos}[\beta] & 0 & I_C \end{pmatrix}.$$

Riemann scalar curvature of the Top: $R = \frac{3}{2 I_C}$



Euler angles for a rotating body in space

TWO IDENTICAL SPINS

Metric Tensor:

$$\begin{pmatrix}
m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & IC & 0 & IC\,Cos[\beta 1] & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & IC & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & IC\,Cos[\beta 1] & 0 & IC & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & IC & 0 & IC\,Cos[\beta 2] \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & IC & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & IC\,Cos[\beta 2] & 0 & IC
\end{pmatrix}$$

EPR 2-SPIN STATE

The EPR state can be symbolically written as

$$|\psi 12\rangle_{\text{EPR}} = \frac{1}{\sqrt{2}}\left(|\uparrow_{z1}, \downarrow_{z2}\rangle_{\text{Fermi}} - |\downarrow_{z1}, \uparrow_{z2}\rangle_{\text{Fermi}}\right)$$

(**Singlet :** invariant under spatial rotations).

where the antisymmetric Fermi states of the two particles have been defined previously.

The EPR state has the following properties

1) the spin and space coordinates degrees of freedom are factorized (not entangled)
2) the spin degrees of freedom of the two particles are entangled
3) the space degrees of freedom of the two particles are entangled
4) the EPR wavefunction is symmetric (even) in the coordinates exchange, i.e. under $\{x_1, y_1, z_1, \alpha_1, \beta_1, \gamma_1\} \leftrightarrow \{x_2, y_2, z_2, \alpha_2, \beta_2, \gamma_2\}$
5) the EPR wavefunction is antisymmetric (odd) in the coordinates *and spin* exchange

---

$$|\uparrow_{z1}, \downarrow_{z2}\rangle_{\text{Fermi}} = \frac{1}{\sqrt{2}}\left(|\psi_1, \uparrow_{z1}\rangle\, |\psi_2, \downarrow_{z2}\rangle - |\psi_2, \uparrow_{z1}\rangle\, |\psi_2, \downarrow_{z2}\rangle\right) =$$

$$= \frac{1}{\sqrt{2}} \begin{vmatrix} \psi[x_1, y_1, z_1, \uparrow_1] & \psi[x_2, y_2, z_2, \downarrow_2] \\ \varphi[x_1, y_1, z_1, \uparrow_1] & \varphi[x_2, y_2, z_2, \downarrow_2 \end{vmatrix}$$

WEYL's POTENTIAL CONNECTING TWO DISTANT SPINS (in entangled state):

Spatial (x,y,z) terms: (non entangled)

$$\phi EPR = \text{Log}\left[\text{Abs}[\psi 1[x2, y2, z2]]^2\right] + \text{Log}\left[\text{Abs}[\psi 2[x2, y2, z2]]^2\right] + \text{Log}[1 - \text{Cos}[\beta 1]\,\text{Cos}[\beta 2] - \text{Cos}[\alpha 1 - \alpha 2]\,\text{Sin}[\beta 1]\,\text{Sin}[\beta 2]]$$

Euler – angles term (ENTANGLED !)

But the Euler's angles degrees of freedom in the last term of ϕ_{EPR} cannot be split in the sum of two terms dependong on $\{\alpha_1, \beta_1, \gamma_1\}$ and $\{\alpha_2, \beta_2, \gamma_2\}$ separately. Then, we see that the angular degrees of freedom of the two particles remain entangled irrespective of their mutual distance in space. This provides a geometric angular interaction between the two spins and this interaction is *nonlocal*. This nonlocal interaction cannot be removed by space distance and it is no visible in the wave equation. Only the HJE approach unveils the presence of this nonloal interaction and, hence, provides an explaination to the EPR paradox: effectively, when one of the particle is made to collapse by a measurement apparatus, the other particle perceives this through the uneliminable nonlocal interaction due to the Weyl curvature and collapses itself.

WEYL's CURVATURE ASSOCIATED TO THE EPR STATE:

Rw =

$$(2\ (-13 + 24\ \mathrm{Cos}[\beta 1]\ \mathrm{Cos}[\beta 2] + 24\ \mathrm{Cos}[\alpha 1]\ \mathrm{Cos}[\alpha 2]\ \mathrm{Sin}[\beta 1]\ \mathrm{Sin}[\beta 2] + 24\ \mathrm{Sin}[\alpha 1]\ \mathrm{Sin}[\alpha 2]\ \mathrm{Sin}[\beta 1]\ \mathrm{Sin}[\beta 2]))\ /\ (5\ \mathrm{IC}\ (-1 + \mathrm{Cos}[\beta 1]\ \mathrm{Cos}[\beta 2] + \mathrm{Cos}[\alpha 1]\ \mathrm{Cos}[\alpha 2]\ \mathrm{Sin}[\beta 1]\ \mathrm{Sin}[\beta 2] + \mathrm{Sin}[\alpha 1]\ \mathrm{Sin}[\alpha 2]\ \mathrm{Sin}[\beta 1]\ \mathrm{Sin}[\beta 2]))$$

We see that R_W depends on the Euler angles of both tops in a complicated way. The top are then interacting and this interaction cannot be removed by setting the particles apart each other.

State of two spinning particles acted upon by Stern – Gerlach (SG) apparatus # 1

ψ12EPRfinal[x1, y1, z1, α1, β1, γ1, x2, y2, z2, α2, β2, γ2]

$$-\frac{1}{2}\, e^{-\frac{1}{2} i\, (\alpha 1+\alpha 2-\gamma 1-\gamma 2)} \left(e^{i\, \alpha 2}\, \mathrm{Cos}\left[\frac{\beta 2}{2}\right] \mathrm{Sin}\left[\frac{\beta 1}{2}\right] - e^{i\, \alpha 1}\, \mathrm{Cos}\left[\frac{\beta 1}{2}\right] \mathrm{Sin}\left[\frac{\beta 2}{2}\right]\right) \psi 1[x1, y1, z1]\, \psi 2[x2, y2, z2]$$

Changed by SG into:

Collect[ψ12EPRszmeas[x1, y1, z1, α1, β1, γ1, x2, y2, z2, α2, β2, γ2], ψ2[___]]

$$\left(-\frac{1}{2}\, e^{-\frac{i\,\alpha 1}{2}+\frac{i\,\alpha 2}{2}+\frac{i\,\gamma 1}{2}+\frac{i\,\gamma 2}{2}}\, \mathrm{Cos}\left[\frac{\beta 2}{2}\right] \mathrm{Sin}\left[\frac{\beta 1}{2}\right] \psi 1\mathrm{down}[x1, y1, z1] + \frac{1}{2}\, e^{\frac{i\,\alpha 1}{2}-\frac{i\,\alpha 2}{2}+\frac{i\,\gamma 1}{2}+\frac{i\,\gamma 2}{2}}\, \mathrm{Cos}\left[\frac{\beta 1}{2}\right] \mathrm{Sin}\left[\frac{\beta 2}{2}\right] \psi 1\mathrm{up}[x1, y1, z1]\right) \psi 2[x2, y2, z2]$$

Then, the apparatus changes the S and ρ functions of our system and changes, in particular, the Weyl curvature R_W of the configuration space too. This change affects both particles, because they *do interact* through the Weyl curvature, which acts as an potential on the two particles. In fact, the Weyl curvature does not split in the sum of two functions depending on the Euler angles of the two particles separately.

Under detection of, say ψ1down[x1, y1, z1] , the Weyl curvature acts nonlocally on apparatus **# 2 and determines the EPR correlation !**

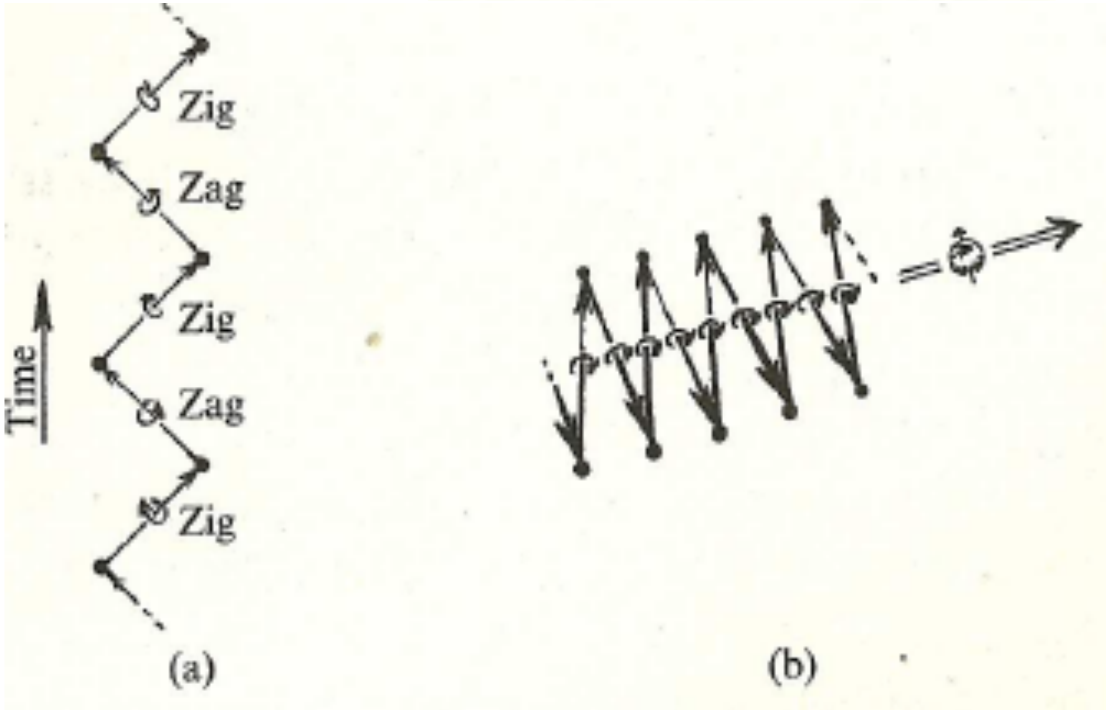
"Zitterbewegung" (Trembling motion *)

$$\psi(x) = \psi_L(x) + \psi_R(x) \equiv \tfrac{1}{2}(1-\gamma^5)\psi(x) + \tfrac{1}{2}(1+\gamma^5)\psi(x).$$

$$\mathcal{L} = \bar{\psi}(i \not\partial - m)\psi = \bar{\psi}_L i \not\partial \psi_L + \bar{\psi}_R i \not\partial \psi_R - m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$$m = \frac{\hbar}{c}\sqrt{R_W} \approx \frac{\hbar\sqrt{6}}{a \times c}$$

Oscillation frequency: $\frac{2p_0c}{\hbar} > \frac{2mc^2}{\hbar} = 2 \times 10^{21}\ \text{sec}^{-1}$

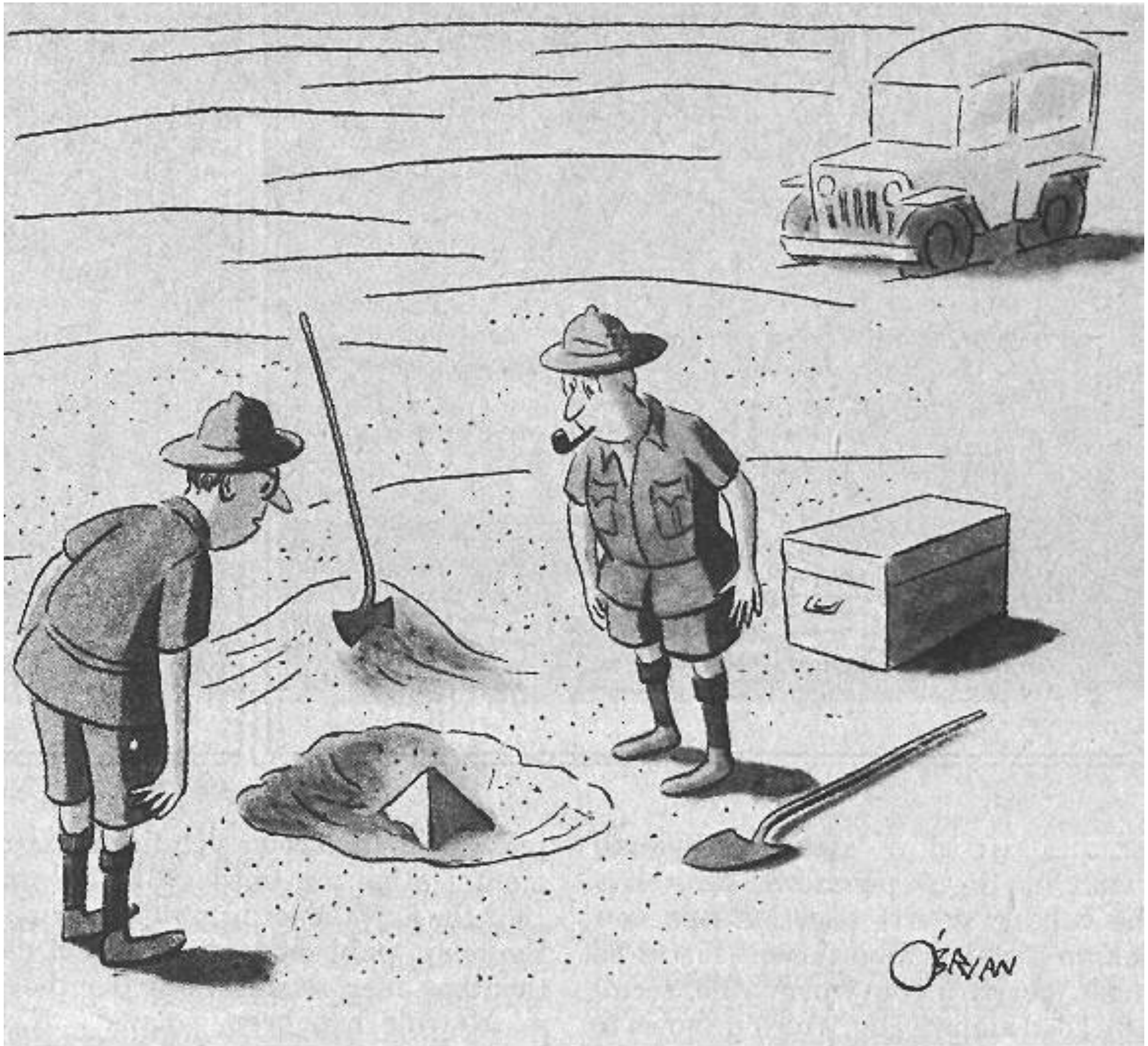


$$a = \frac{\hbar\sqrt{6}}{mc} \approx \frac{h}{mc} = \lambda\ Compton\ w.l.$$

Klein paradox on $\bar{e}$ localization

(*O. Klein, Z. Phys* 53, 157 (1929).

*) E. Schrödinger, *Sitzber. Preuss. Akad. Wiss Physik-Math* 24, 418 (1930)



"This could be the discovery of the century. Depending, of course, on how far down it goes."

HAMILTON– JACOBI EQUATION

$$Lagrangian: L = \gamma \hbar \sqrt{- R_W \, g_{ij} \, \dot{q}^i \, \dot{q}^j}\,; \qquad L^2 \equiv \overline{L}$$

$$\delta \int L \, d\sigma = 0 \quad \rightarrow \quad \delta \int L^2 \, d\sigma = -\frac{1}{2} R_W \, g^{ij} \, \frac{\partial S}{\partial q^i} \frac{\partial S}{\partial q^j} + \gamma^2 \hbar^2 R_W = 0$$

Find the Hamiltonian Function:

$$\overline{H}(p,q) = \dot{q}^i \, \frac{\partial \overline{L}}{\partial \dot{q}^i} - \overline{L} = \dot{q}^i \, (- R_W \, g_{ij} \, \dot{q}^j + A_i) - \overline{L} = -\frac{1}{2} R_W \, \dot{q}^i \dot{q}^j \, g_{ij}$$

$$p_i = \frac{\partial \overline{L}}{\partial \dot{q}^i} = - R_W \, g_{ij} \dot{q}^j + A_i \quad \rightarrow \quad \dot{q}^j = - R_W^{-1} \, g^{ij} (p_j - A_j)$$

$Hamiltonian:$

$$\bar{H} = -\frac{1}{2} R_W g_{ij} \partial R_W^{-2} g^{ik} (p_k - A_K)(p_l - A_l) g^{jl} = -\frac{1}{2} R_W^{-1} g^{kl} (p_k - A_K)(p_l - A_l)$$

$$p_i = \nabla_i S \rightarrow \bar{H}(\nabla_i S, q_i) = -R_W$$

$S(q) = Solution\ of\ the\ Hamilton-Jacobi\ Equation.$

$$\dot{q}^i = \frac{\partial q^i}{\partial \sigma} = f^i(q^i);\ Choose\ as\ parameter:\ \sigma \rightarrow ds = \sqrt{(-g_{ij} \frac{\partial q^j}{\partial s} \frac{\partial q^i}{\partial s})}$$

$$\frac{\partial q^i}{\partial s} = [g^{ij} (\frac{\partial S}{\partial q^j} - \frac{e}{c} A_j)] / [g^{mn} (\frac{\partial S}{\partial q^m} - \frac{e}{c} A_m)(\frac{\partial S}{\partial q^n} - \frac{e}{c} A_n)]^{\frac{1}{2}} \rightarrow Eq.(8)$$

The tensor $\omega_{\mu\nu}$ has the explicit form

$$\omega_{\mu\nu} = \begin{pmatrix} 0 & a_1 & a_2 & a_3 \\ -a_1 & 0 & -\omega_3 & \omega_2 \\ -a_2 & \omega_3 & 0 & -\omega_1 \\ -a_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

and $\omega^{\mu\nu}$ has the form

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & -a_1 & -a_2 & -a_3 \\ a_1 & 0 & -\omega_3 & \omega_2 \\ a_2 & \omega_3 & 0 & -\omega_1 \\ a_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

Then we have $\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} = \omega^2 - a^2$.

The tensor $F_{\mu\nu}$ has the explicit form

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B1 & 0 \end{pmatrix}$$

and $F^{\mu\nu}$ has the form

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B1 & 0 \end{pmatrix}$$

Notice the change of sign in the space components of the tensors $F_{\mu\nu}$ and $\omega_{\mu\nu}$. This is the usual convention. Then we have $\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = B^2 - E^2$ and $\frac{1}{2}F_{\mu\nu}\omega^{\mu\nu} = -(\boldsymbol{B}\cdot\boldsymbol{\omega} + \boldsymbol{E}\cdot\boldsymbol{a})$. Usually $\boldsymbol{\omega}$ is interpreted as the intrinsic particle magnetic moment and $\boldsymbol{a}$ as the intrinsic particle electric dipole.

ELECTROMAGNETIC LAGRANGIAN

$$L_{em} = \frac{e}{c} A_\mu \frac{dx^\mu}{d\sigma} + \frac{\kappa e}{4c}\, a^2 F_{\mu\nu} \omega^{\mu\nu}$$

where e is the particle charge and $F_{\mu\nu}$ is given by $F_{\mu\nu} = \partial A_\nu / \partial x^\mu - \partial A_\mu / \partial x^\nu$ with four-potential A_μ given by $A_\mu = (-\phi, \boldsymbol{A})$, ϕ, $\boldsymbol{A}$ being the scalar and vector electromagnetic potentials, respectively. Finally, κ is a numeric constant that will be identified as the particle gyromagnetic ratio. The fourleg components e^μ_a (and the $SO(3,1)$ group) are parametrized by six "Euler angles" θ^α ($\alpha = 1, \ldots, 6$), so that the configuration space spanned by the space-time coordinates and the Euler angles is ten dimensional.

In place of L assume a $g_{\mu\nu} \rightarrow \rho(q^i)\, g_{\mu\nu}$ conformally - invariant Lagrangian:

$$\bar{L} = \gamma\hbar\sqrt{-R_W\, g_{ij}\frac{dq^i}{d\sigma}\frac{dq^j}{d\sigma}} + L_{em},$$

where the particle's mass is replaced by the Weyl's scalar curvature: R_W

$$mc \rightarrow \gamma\hbar\sqrt{R_W}.$$

R_W acts as a scalar potential on the top and, because it depends on χ and its derivatives, the field χ acts on the top as a sort of pre-potential. The paths followed by the top in the configuration space $V_{10} = \mathcal{M}_4 \times SO(3,1)$ are assumed to be the extremal curves of the action integral $\int \bar{L}\, d\sigma$.

DO EXTEND THEORY TO 2 - PARTICLE ASSEMBLY !

2 – PARTICLE LAGRANGIAN

$$L_W(1,2) = \xi\hbar\sqrt{-R_W(Q)G_{AB}(Q)\frac{dQ^A}{d\sigma}\frac{dQ^B}{d\sigma}}$$

2 – PARTICLE METRIC TENSOR

$$G_{AB}(Q) = \begin{pmatrix} g_{\mu,\nu}(x_1) & 0 & 0 & 0 \\ 0 & \gamma_{ab}(\theta_1) & 0 & 0 \\ 0 & 0 & g_{\mu,\nu}(x_2) & 0 \\ 0 & 0 & 0 & \gamma_{ab}(\theta_2) \end{pmatrix}$$

2 – PARTICLE LAGRANGIAN

$$L_W(1,2) = \xi\hbar\sqrt{R_W}\left[-g_{\mu\nu}(x_1)\frac{dx_1^\mu}{d\sigma}\frac{dx_1^\nu}{d\sigma} - a^2\omega_{1\mu\nu}\omega_1^{\mu\nu} - \right.$$
$$\left. - g_{\mu\nu}(x_2)\frac{dx_2^\mu}{d\sigma}\frac{dx_2^\nu}{d\sigma} - a^2\omega_{2\mu\nu}\omega_2^{\mu\nu}\right]^{\frac{1}{2}}$$

For integrable Weyl's connection, the scalar curvature $R_W(Q)$ is given by

$$R_W = R + 2(n-1)\frac{\nabla_K\nabla^K\chi}{\chi} - n(n-1)\frac{\nabla_K\chi\nabla^K\chi}{\chi^2}$$

The motion of the two rotating particles is obtained from the Hamilton-Jacobi equation associated to the Lagrangian (5), namely

$$G^{AB}\frac{\partial S}{\partial Q_A}\frac{\partial S}{\partial Q_B} + \xi^2\hbar^2 R_W = 0$$

and solving the following ODEs for $Q^A(s)$

$$\frac{dQ^A}{ds} = \frac{G^{AB}\frac{\partial S}{\partial Q_A}}{\left[-G^{MN}\frac{\partial S}{\partial Q_M}\frac{\partial S}{\partial Q_N}\right]^{1/2}} = \left(\frac{G_{AB}}{\xi\hbar\sqrt{R_W}}\right)\frac{\partial S}{\partial Q_A}$$

with (time-like) parameter s given by $ds^2 = -G_{AB}dQ^A dQ^B$. We notice that the Weyl's curvature depends on the space-time coordinates of both particles, thus introducing a nonlocal interaction of geometric origin. As we shall see, this geometric interaction is at the basis of nonlocal quantum correlation phenomena.

2 - PARTICLE BELTRAMI - DE – RHAM - KLEIN – GORDON EQUATIONS

Klein-Gordon-like wave equation for ψ

$$(\hat{p}^2 + \hbar^2 \xi^2 R_W)\psi = 0$$

where $\hat{p}^2 = \hat{p}_K \hat{p}^K = -\Delta$ with $\hat{p}_K = -i\hbar\nabla_K$, and Δ and R are the Laplace-Beltrami and Riemann scalar curvature of $\mathcal{V}_{10}$, respectively. As shown elsewhere, the value of ξ is fixed by the requirement of Weyl's conformal invariance of the wave equation (14) according to

$$\xi = \frac{n-2}{4(n-1)}$$

with $n = 20$. We notice that any reference to the underlying Weyl's structure of the space is disappeared in Eq. (14), which has a pure Riemannian form. Owing to the block form of the metric tensor (2), Eq. (14) can be also written as

$$(\hat{H}_1 + \hat{H}_2)\psi = 0$$

where $\hat{H}_k = \hat{p}_k^2 + \hbar^2 \xi^2 R_k$ $(k = 1, 2)$ are the Hamiltonians of the two particles.

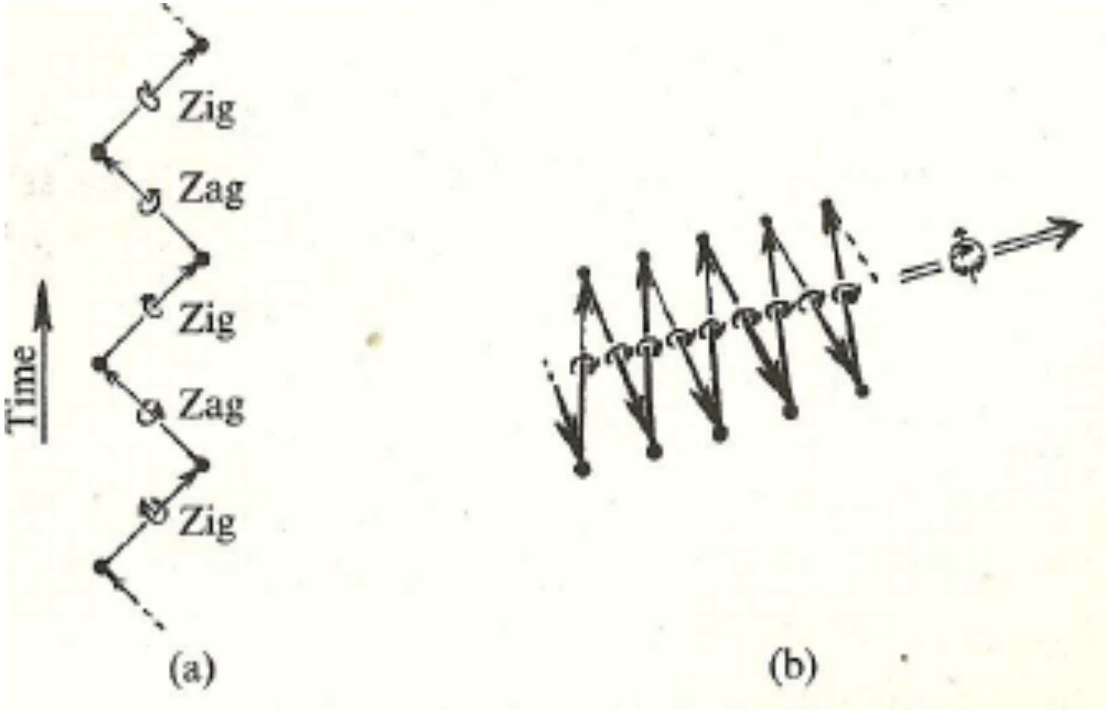
“Zitterbewegung” (Trembling motion *)

$$\psi(x) = \psi_L(x) + \psi_R(x) \equiv \tfrac{1}{2}(1-\gamma^5)\psi(x) + \tfrac{1}{2}(1+\gamma^5)\psi(x).$$

$$\mathcal{L} = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi = \bar{\psi}_L i\,\partial\!\!\!/\,\psi_L + \bar{\psi}_R i\,\partial\!\!\!/\,\psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$m = \frac{\hbar}{c}\sqrt{R_W} \approx \frac{\hbar\sqrt{6}}{a\times c}$$

Oscillation frequency: $\frac{2p_0c}{\hbar} > \frac{2mc^2}{\hbar} = 2 \times 10^{21}\ \mathrm{sec}^{-1}$



$$a = \frac{\hbar\sqrt{6}}{mc} \approx \frac{h}{mc} = \lambda\ \textit{Compton w.l.}$$

Klein paradox on $\bar{e}$ localization

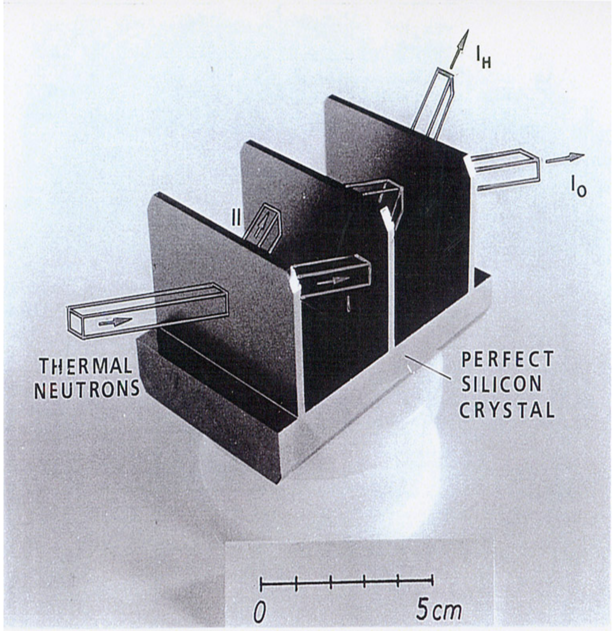
(*O.Klein, Z.Phys* 53, 157 (1929).

*) E. Schrödinger, *Sitzber. Preuss. Akad. Wiss Physik-Math* 24, 418 (1930)

THE NEW YORKER COLLECTION Ch..Addams 1940

$$I_0 \propto \left|\psi_0^{\,I} + \psi_0^{\,II}\right|^2 \propto A + B\cos\chi$$

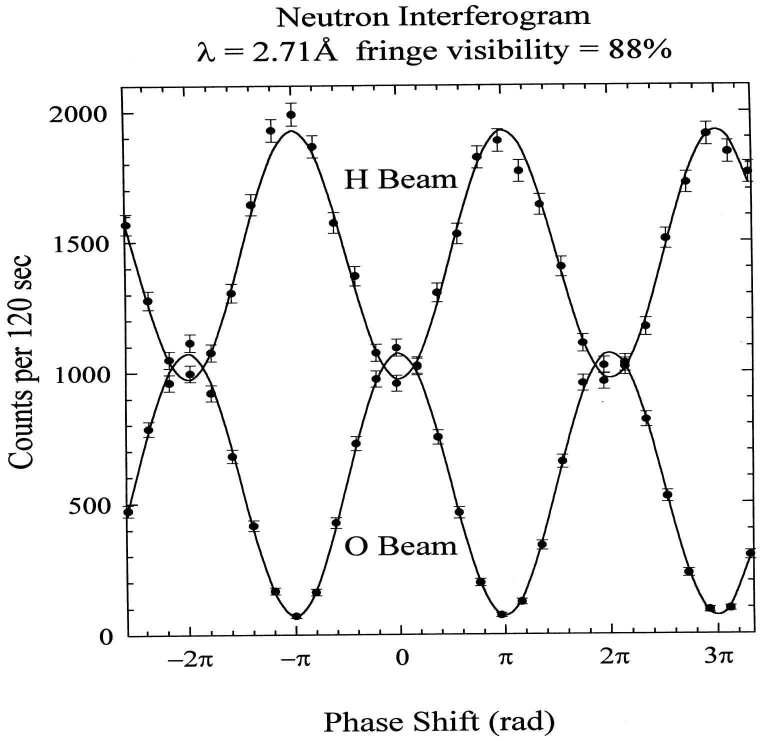
$$\chi = \oint \vec{k}\, d\vec{s} = (1-n)kD_{eff} \equiv -Nb_c\lambda D_{eff} = \Delta\cdot k = \Delta k \cdot D_{eff}$$



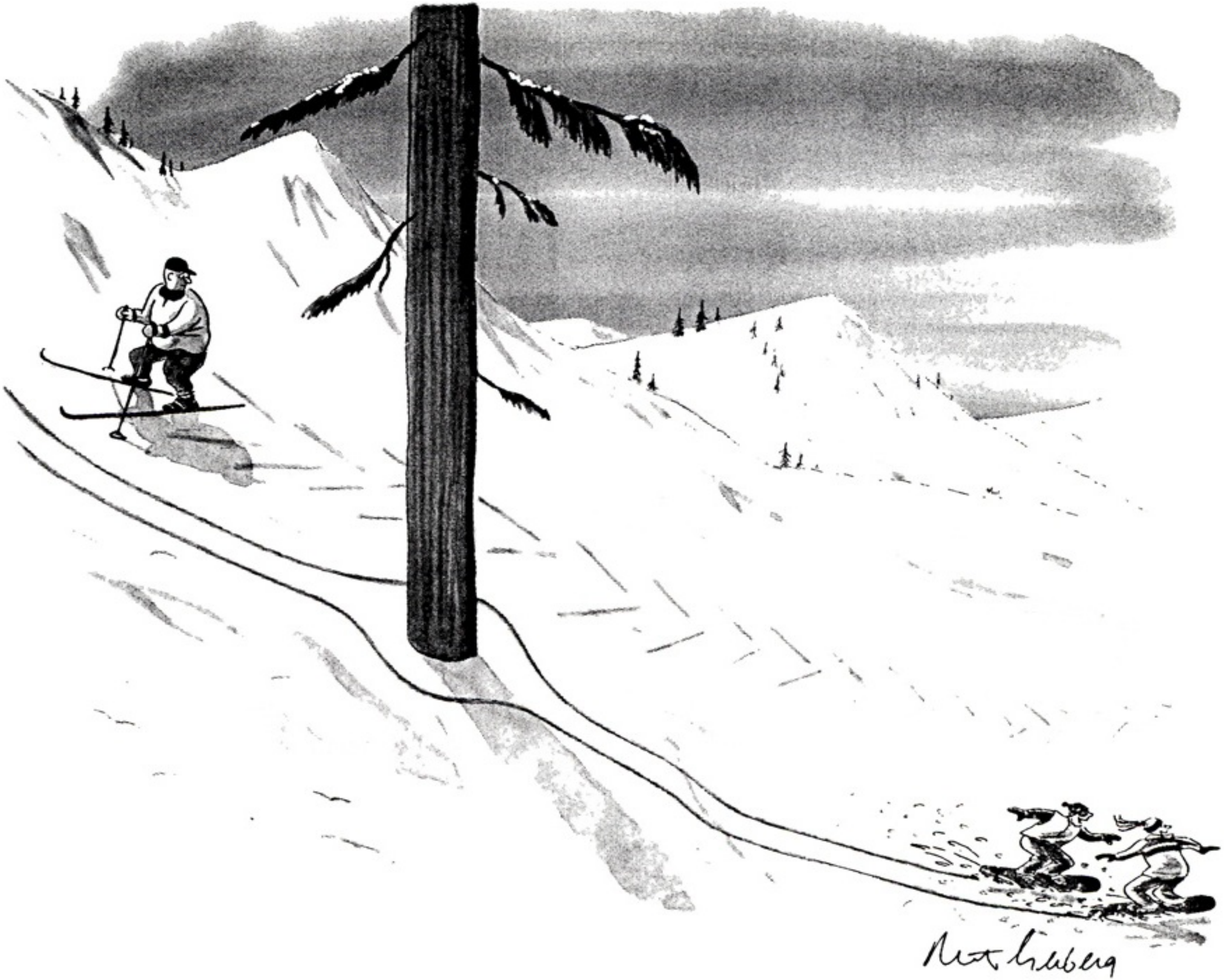
Self interference

(phase space density ~10^{-14})

Efficiency of detectors, polarizers, flippers >99%



H. Rauch, W. Treimer, U. Bonse, Phys.Lett. A47 (1974) 369



THE NEW YORKER, FEBRUARY 19 & 26, 2001

The tensor $\omega_{\mu\nu}$ has the explicit form

$$\omega_{\mu\nu} = \begin{pmatrix} 0 & a_1 & a_2 & a_3 \\ -a_1 & 0 & -\omega_3 & \omega_2 \\ -a_2 & \omega_3 & 0 & -\omega_1 \\ -a_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

and $\omega^{\mu\nu}$ has the form

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & -a_1 & -a_2 & -a_3 \\ a_1 & 0 & -\omega_3 & \omega_2 \\ a_2 & \omega_3 & 0 & -\omega_1 \\ a_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

Then we have $\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} = \omega^2 - a^2$.

The tensor $F_{\mu\nu}$ has the explicit form

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B1 & 0 \end{pmatrix}$$

and $F^{\mu\nu}$ has the form

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B1 & 0 \end{pmatrix}$$

Notice the change of sign in the space components of the tensors $F_{\mu\nu}$ and $\omega_{\mu\nu}$. This is the usual convention. Then we have $\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = B^2 - E^2$ and $\frac{1}{2}F_{\mu\nu}\omega^{\mu\nu} = -(\boldsymbol{B} \cdot \boldsymbol{\omega} + \boldsymbol{E} \cdot \boldsymbol{a})$. Usually $\boldsymbol{\omega}$ is interpreted as the intrinsic particle magnetic moment and $\boldsymbol{a}$ as the intrinsic particle electric dipole.

Hamiltonian:

$$\bar{H} = -\frac{1}{2} R_W g_{ij} \partial R_W^{-2} g^{ik} (p_k - A_K)(p_l - A_l) g^{jl} = -\frac{1}{2} R_W^{-1} g^{kl} (p_k - A_K)(p_l - A_l)$$

$$p_i = \nabla_i S \rightarrow \bar{H}(\nabla_i S, q_i) = -R_W$$

$S(q) =$ *Solution of the Hamilton – Jacobi Equation.*

$$\dot{q}^i = \frac{\partial q^i}{\partial \sigma} = f^i(q^i);\ \text{Choose as parameter: } \sigma \rightarrow ds = \sqrt{\left(-g_{ij} \frac{\partial q^j}{\partial s} \frac{\partial q^i}{\partial s}\right)}$$

$$\frac{\partial q^i}{\partial s} = [g^{ij}(\frac{\partial S}{\partial q^j} - \frac{e}{c} A_j)] / [g^{mn}(\frac{\partial S}{\partial q^m} - \frac{e}{c} A_m)(\frac{\partial S}{\partial q^n} - \frac{e}{c} A_n)]^{\frac{1}{2}} \rightarrow Eq.(8)$$

THE PHILOSOPHY OF QUANTUM MECHANICS

The Interpretations of Quantum Mechanics in Historical Perspective

MAX JAMMER

A Wiley-Interscience Publication

New York · London · Sydney · Toronto

Weyl’s affine connection and curvature:

$$\Gamma^i_{jk} = -\left\{ {i \atop jk} \right\} + \delta^i_j \phi_k + \delta^i_k \phi_j + g_{jk}\phi^i,$$

where $\left\{ {i \atop jk} \right\}$ are the Cristoffel symbols out of the metric g_{ij}, and $\phi^i = g^{il}\phi_l$ is the Weyl potential ϕ_i as $\phi_i = \chi^{-1}\partial\chi/\partial q^i$.

By Weyl's af f ineonnection Γ^i_{jk} *the overall Weyl's scalar curvature* R_W *can be calculated in* $D = n = 10$ dim*ensions* :

$$\begin{aligned} R_W &= R + 2(n-1)\nabla_k\phi^k - (n-1)\phi_k\phi^k = \\ &= R + 2(n-1)\frac{\nabla_k\nabla^k\chi}{\chi} - n(n-1)\frac{\nabla_k\chi\nabla^k\chi}{\chi^2} \end{aligned}$$