Long-range forces
A theory review

The Low-Energy Frontier of Particle Physics

INFN PADOVA

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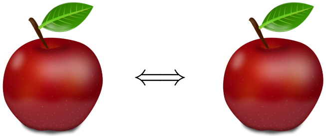
Why do we care about long-range forces?

The low-energy frontier promises hope for progress in physics.
For low-energy, long-range forces can become relevant:

$$m \approx 10^{-5}\ \text{eV} \leftrightarrow \lambda_{\text{compton}} \approx 2\ \text{cm}$$

This allows us to scale up to macroscopic size



10^{23} particles $\Longleftrightarrow$ 10^{23} particles

Coherent enhancement of even a tiny signal!

Main competition: Gravity

How can we discriminate a fifth force from gravity?

The inverse square law:

$$F \propto \frac{1}{r^2}$$

A geometric property characteristic of *massless* mediators.

The (weak) equivalence principle:

Mass and weight are locally in identical ratio for all bodies

- Isaac Newton

Or more simply:

$$m_I = m_g$$

Minimal example: A light scalar

Consider a light scalar ϕ coupling to a fermion f:

$$\mathcal{L} \supset \frac{1}{2} m_\phi^2 \phi^2 + g_s \phi \overline{f} f$$

In the NR limit, this yields a Yukawa-type potential:

$$V_{\text{Yukawa}} \approx \frac{g_s^2}{4\pi r} e^{-m_\phi r}$$

The mass suppresses the range of the force. This is the reason nuclear forces are short-range, suppressed by m_π and $m_{W,Z}$.

Minimal example: A light scalar

V_{Yukawa} competes against gravity:

$$V_G = G\frac{m_f^2}{r}$$

Write $V_{\text{total}} = V_G + V_{\text{Yukawa}}$ in the standard from:

$$V_{\text{total}} = V_G \left(1 + \alpha e^{m_\phi r}\right)$$

where

$$\alpha = \frac{V_{\text{Yukawa}}}{V_G} e^{m_\phi r} = \frac{g_s^2}{4\pi m_f^2 G}$$

Coupling to a whole atom

Violation of the inverse-square law

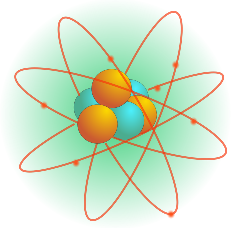
A whole atom consists Z electrons and A nucleons of mass m_N

- Gravity couples to nucleons
- In our example, the new force coupes only to e

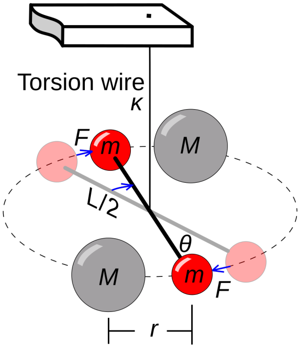
The α parameter between atom 1 and atom 2 is:

If coupling to electrons: $\alpha_{\phi e} = \frac{Z_1 Z_2}{A_1 A_2} \frac{g_s^2}{4\pi m_N^2 G}$,

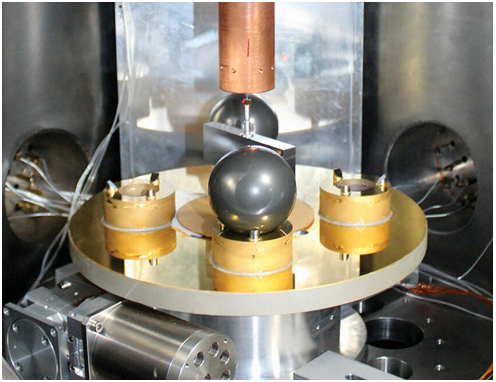
If coupling to nucleons: $\alpha_{\phi N} = \frac{g_s^2}{4\pi m_N^2 G}$



Tests of the inverse square law



Torsion balance sketch

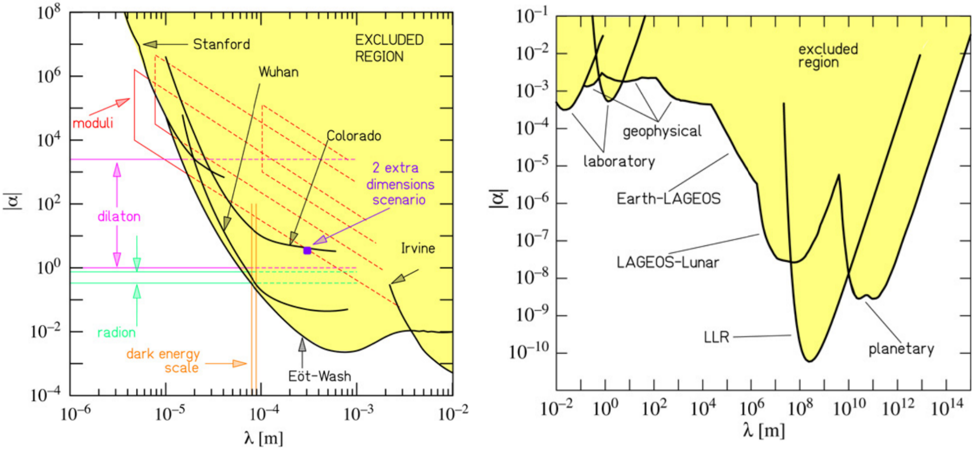


HUST torsion balance

Photo credit: Shanqing Yang

https://doi.org/10.1093/nsr/nwz210

Constraints from inverse-square law



Adelberger et al., Prog. Part. Nucl. Phys. 62 (2009)

Coupling to a whole atom

Violation of equivalence principle

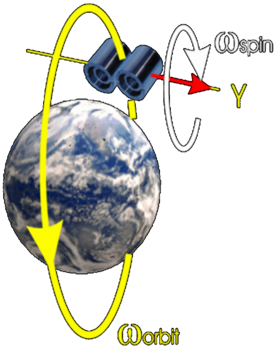
For EP violation, factor out material dependence:

$$V_{\text{total}} = V_G \left(1 + \tilde{\alpha} \left[\frac{\tilde{q}_1}{g_{a\bar{f}f} A_1}\right] \left[\frac{\tilde{q}_2}{g_{a\bar{f}f} A_2}\right] e^{-m_a r}\right)$$

Where $\tilde{q}_1$, $\tilde{q}_2$ are the per-atom couplings to material 1 and 2.

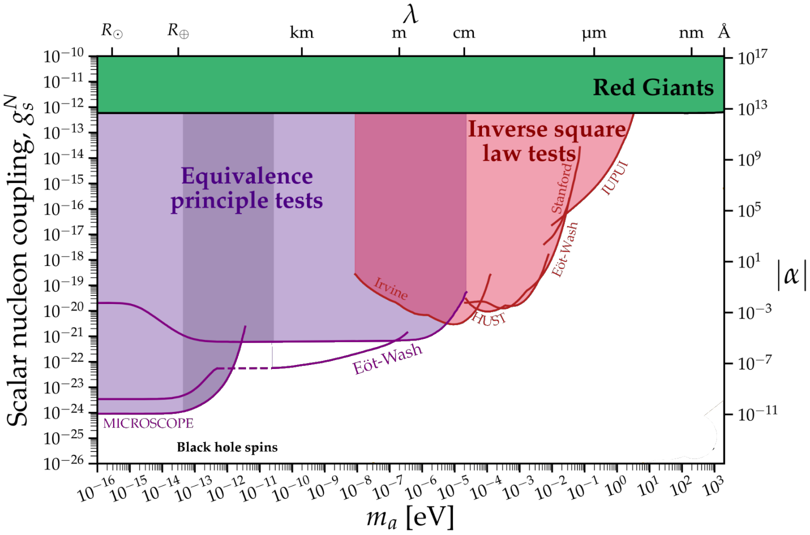
$$\tilde{\alpha}_{\phi e} = \tilde{\alpha}_{\phi N} = \frac{g_s^2}{4\pi m_N^2 G}$$

NB: Because a part of the nucleus mass is binding energy, even universal nucleon couplings are constrained!



MICROSCOPE
Touboul et al 2012
Class. Quantum Grav. 29 184010

Constraints from both inverse-square law and EP



Ohare and Vitagliano, Phys. Rev. D 102, 115026 (2020)

https://cajo-hare.github.io/AxionLimits/

Imperfect QCD Axions

As a pseudo-scalar, scalar interaction for the QCD-axion implies CP violation.
This is heavily constrained,

$$\langle\theta\rangle \lesssim 10^{-10}$$

As pointed out yesterday by Luca, the standard model sets a lower bound

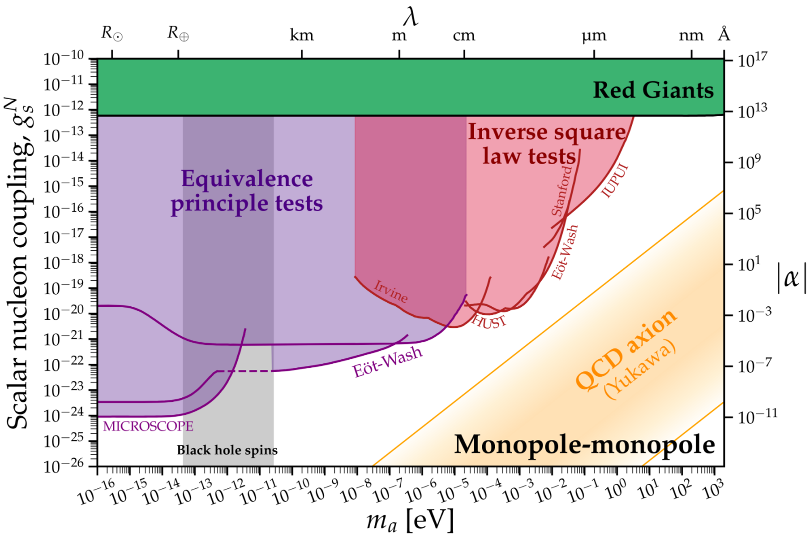
$$10^{-18} \lesssim \langle\theta\rangle$$

This CP violation induces a scalar interaction:

$$g^S_{aN} \simeq \frac{\langle\theta\rangle}{f_a}\frac{m_u m_d}{m_u + m_d}\frac{\langle N|\bar{u}u + \bar{d}d|N\rangle}{2} \simeq 1.3\cdot 10^{-12}\,\langle\theta\rangle\left(\frac{10^{10}\text{GeV}}{f_a}\right)$$

A minimal axion band!

Minimal axion expectation



Ohare and Vitagliano, Phys. Rev. D 102, 115026 (2020)

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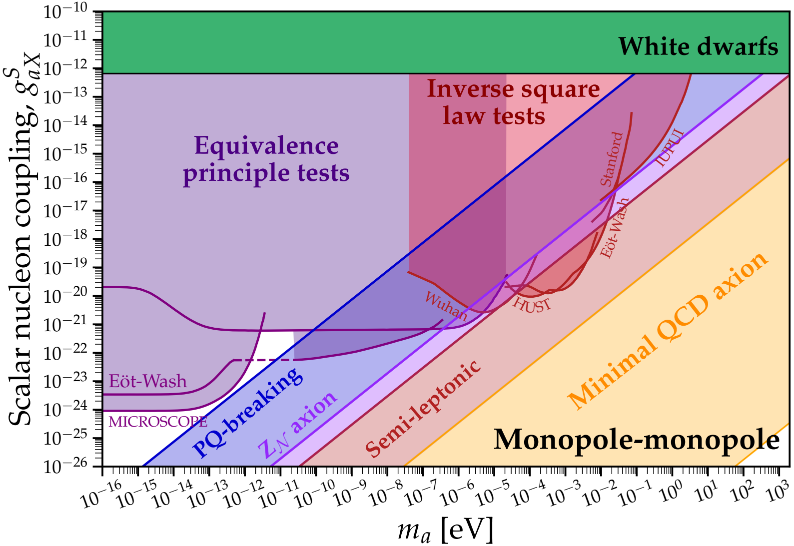
Beyond minimal axion models



Ohare and Vitagliano, Phys. Rev. D 102, 115026 (2020)

https://cajo-hare.github.io/AxionLimits/

+

Di Luzio, Gisbert, Nesti, Sørensen, Phys. Rev. D 110 (2024)

Scalar and pseudoscalar interactions

Consider now both scalar and pseudo-scalar couplings:

$$\mathcal{L} \supset g_s \phi \overline{\psi} \psi + g_p \phi \overline{\psi} i \gamma_5 \psi$$

These couple quite differently:

For scalar:

- $\overline{\psi}\psi$ counts *number* density

For pseudo-scalar:

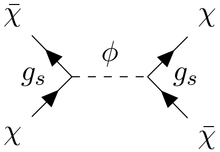
$$\overline{\psi}\gamma_5\psi = i(\xi^\dagger \eta - \eta^\dagger \xi) \quad \text{where} \quad \psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

$$= \xi^\dagger \frac{\vec{\sigma}\cdot\vec{p}}{2m}\xi \quad \text{after applying EOM} \quad \eta = i\frac{\vec{\sigma}\cdot\vec{\nabla}}{2m}\xi$$

- $\overline{\psi}\gamma_5\psi$ couples to the *spin* density

Monopole-Monopole

This is type of coupling we have considered until now:



To derive the NR potential:

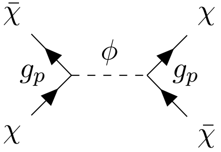
1. Calculate momentum-space amplitude
2. Take non-relativistic limit
3. Furrier transform to position-space

$\rightarrow V_{\text{monopole}-\text{monopole}} = V_{\text{Yukawa}}$

- Strong and broad constraint for true scalars
- Pseudo-scalars require CP-violation
- Suppressed for the QCD axion: $g_s^2 \sim \langle\theta\rangle^2 \lesssim 10^{-20}$

Dipole-dipole

The potential looks slightly more intimidating:

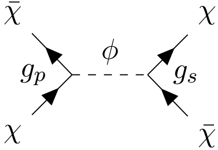


$$\Rightarrow \quad V_{\text{dipole}-\text{dipole}} = -\frac{g_p^2}{16\pi m_\psi^2}\Bigg[\vec{\sigma}\cdot\vec{\sigma}'\left(\frac{m_\phi}{r^2}+\frac{1}{r^3}+\frac{4\pi}{3}\delta^3(\mathbf{x})\right) - (\vec{\sigma}\cdot\hat{\vec{r}})\left(\vec{\sigma}'\cdot\hat{\vec{r}}\right)\left(\frac{m_\phi^2}{r}+\frac{3m_\phi}{r^2}+\frac{3}{r^3}\right)\Bigg]e^{-m_\phi r}$$

Important points:

- Couples *spin-polarized* bodies
- This implies a magnetic interaction $\leftarrow$ potentially huge background!
- Some progress is nonetheless achieved, see e.g. Budker et al. arXiv:2501.07865

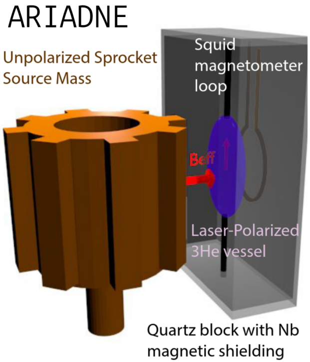
Monopole-dipole



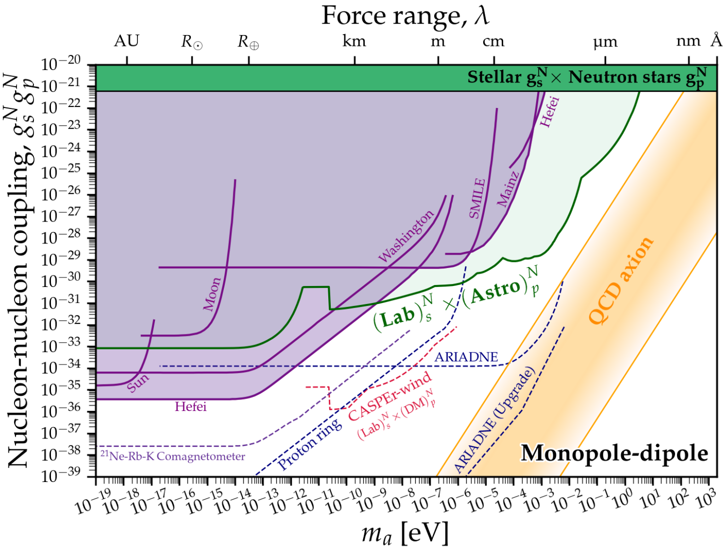
$$\Rightarrow V_{\text{monopole}-\text{dipole}} = \frac{g_s g_p}{8\pi m_\psi} \vec{\sigma} \cdot \vec{r} \left(\frac{m_\phi}{r} \right)$$

- > Avoids strong $\vec{B}$ background
- > Only linear in $\langle \theta \rangle$ for QCD axion
- > New technique: NMR pickup

ARIADNE collab.
arXiv:1710.05413



Monopole-dipole



Ohare and Vitagliano, Phys. Rev. D 102, 115026 (2020)

https://cajo-hare.github.io/AxionLimits/

Which constraints exist?

An overview from Dmitry's review Cong et al., 2408.15691 :

Scalar/Scalar interaction $g_s g_s$
1. e-e
2. e-N
3. e-$\overline{p}$
4. N-N

Pseudoscalar/Pseudoscalar interaction $g_p g_p$
1. e-e, e-e^+, e-μ^+
2. e-n, e-p and e-$\overline{p}$
3. N-N

Pseudoscalar/Scalar interaction $g_p g_s$
1. e-e
2. e-N
3. n-N
4. p-N
5. Spin-gravity

Beyond scalar mediators:

Axial-vector/Vector interaction $g_A g_V$
1. e-e
2. e-N
3. e-n
4. e-p
5. n-N
6. p-N

Axial-vector/Axial-vector interaction $g_A g_A$
1. e-e, e-e^+ & e-μ^+
2. e-N
3. e-n
4. e-p & e-$\overline{p}$
5. N-N

Limits for massless spin-1 bosons
1. $\mathrm{Re}(C_X)\mathrm{Re}(C_Y)/\Lambda^4$
2. $\mathrm{Im}(C_X)\mathrm{Re}(C_Y)/\Lambda^4$
3. $\mathrm{Im}(C_X)\mathrm{Im}(C_Y)/\Lambda^4$

A general basis for spin-dependent operators

$$\mathcal{O}_1 = 1 \ ,$$

$$\mathcal{O}_2 = \vec{\sigma} \cdot \vec{\sigma}\,' \ ,$$

$$\mathcal{O}_3 = \frac{1}{m^2} \left(\vec{\sigma} \cdot \vec{q}\right) \left(\vec{\sigma}\,' \cdot \vec{q}\right) \ ,$$

$$\mathcal{O}_{4,5} = \frac{i}{2m^2} \left(\vec{\sigma} \pm \vec{\sigma}\,'\right) \cdot \left(\vec{P} \times \vec{q}\right) ,$$

$$\mathcal{O}_{6,7} = \frac{i}{2m^2} \left[\left(\vec{\sigma} \cdot \vec{P}\right) \left(\vec{\sigma}\,' \cdot \vec{q}\right) \pm \left(\vec{\sigma} \cdot \vec{q}\right) \left(\vec{\sigma}\,' \cdot \vec{P}\right)\right],$$

$$\mathcal{O}_8 = \frac{1}{m^2} \left(\vec{\sigma} \cdot \vec{P}\right) \left(\vec{\sigma}\,' \cdot \vec{P}\right) \ .$$

$$\mathcal{O}_{9,10} = \frac{i}{2m} \left(\vec{\sigma} \pm \vec{\sigma}\,'\right) \cdot \vec{q} \ ,$$

$$\mathcal{O}_{11} = \frac{i}{m} \left(\vec{\sigma} \times \vec{\sigma}\,'\right) \cdot \vec{q} \ ,$$

$$\mathcal{O}_{12,13} = \frac{1}{2m} \left(\vec{\sigma} \pm \vec{\sigma}\,'\right) \cdot \vec{P} \ ,$$

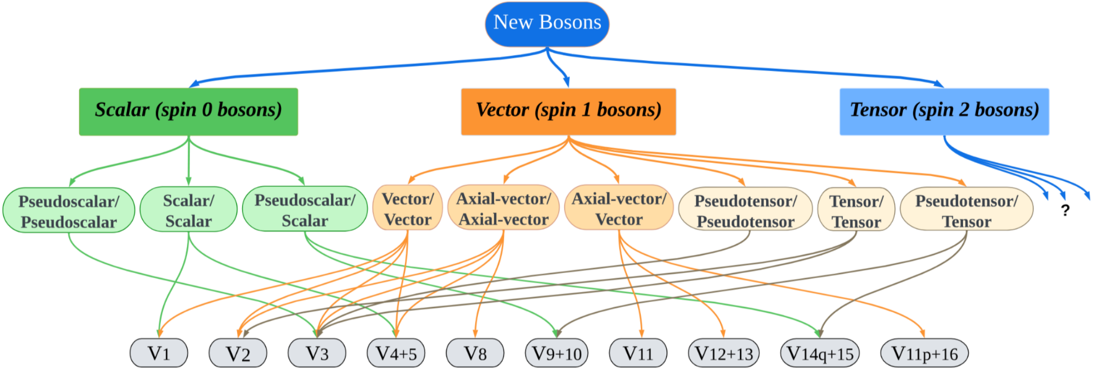
$$\mathcal{O}_{14} = \frac{1}{m} \left(\vec{\sigma} \times \vec{\sigma}\,'\right) \cdot \vec{P} \ ,$$

$$\mathcal{O}_{15} = \frac{1}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] \left(\vec{\sigma}\,' \cdot \vec{q}\right) + \left(\vec{\sigma} \cdot \vec{q}\right) \left[\vec{\sigma}\,' \cdot \left(\vec{P} \times \vec{q}\right)\right] \right\}$$

$$\mathcal{O}_{16} = \frac{i}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right] \left(\vec{\sigma}\,' \cdot \vec{P}\right) + \left(\vec{\sigma} \cdot \vec{P}\right) \left[\vec{\sigma}\,' \cdot \left(\vec{P} \times \vec{q}\right)\right] \right\} \ .$$

Dobrescu and Mocioiu, JHEP11(2006)005

A general basis for spin-dependent operators



Cong et al., 2408.15691

This still only covers spin-dependent interactions!
Other effects are possible.

Power-law modifications

The inverse-square law is a *geometric* effect.

> Instead of messing with fields, we can mess with geometry.

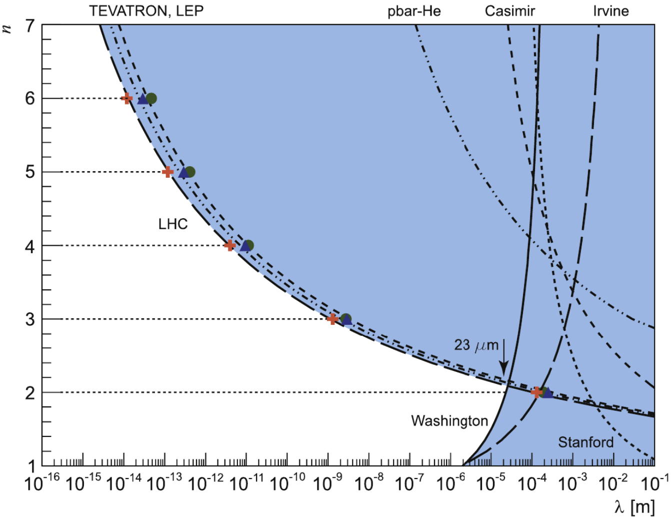
Example: The Arkani-Dimopoulos-Dvali model extends 4D space with $n \gtrsim 2$ extra dimensions to make the weak scale natural:

$$V = \begin{cases} -\frac{G_{4+n} M}{r^{1+n}}, & \text{if } r \ll \lambda, \\ -\frac{G_N M}{r}, & \text{if } r \gg \lambda, \end{cases} \quad \text{such that} \quad G_{4+n} = \lambda^\delta G_N$$

This admits a power law parameterization:

$$V(r) = -\frac{G_N M}{r} \left(1 + \left(\frac{\lambda}{r} \right)^\delta \right)$$

Constraints on power-law modifications



A. S. Lemos 2021 EPL 135 11001

Summary

- Light mediators ($m \ll eV$) interacting with SM fields imply forces on macroscopic length-scales
- A large field:
  - Constraints from EP violation, inverse-square law violation, and source-sensor tests
  - Most common: (psudo)-scalar mediated monopole/dipole forces
  - Frameworks exists to constrain even exotic interactions
- Progress in long-range force experiments have the potential yield important insight into new physics

Thank you!

Discussion!

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Backup slides

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Widening the axion window on long-range forces

- The QCD-band is only a minimal expectation
- Other contributions can exist

Example:

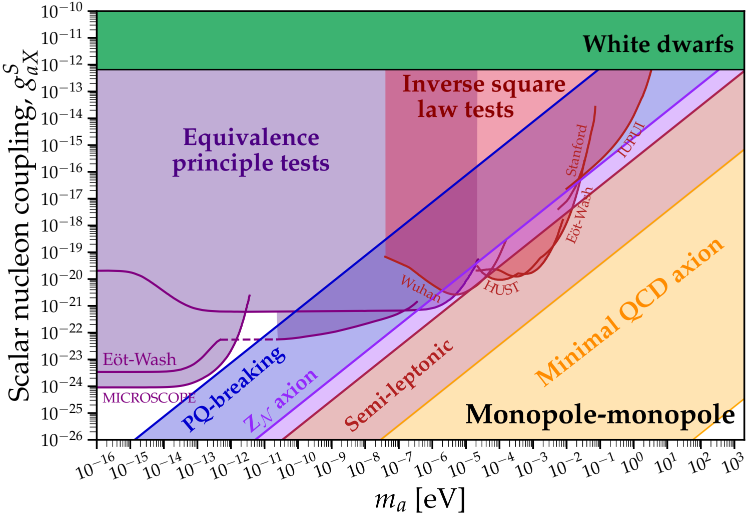
$$\mathcal{L} \supset -e^{i\delta}\left(\frac{\phi}{\Lambda}\right)^n \frac{\sqrt{2}m_e}{v}\bar{L}_L H e_R + \text{h.c.}$$

This implies two contributions

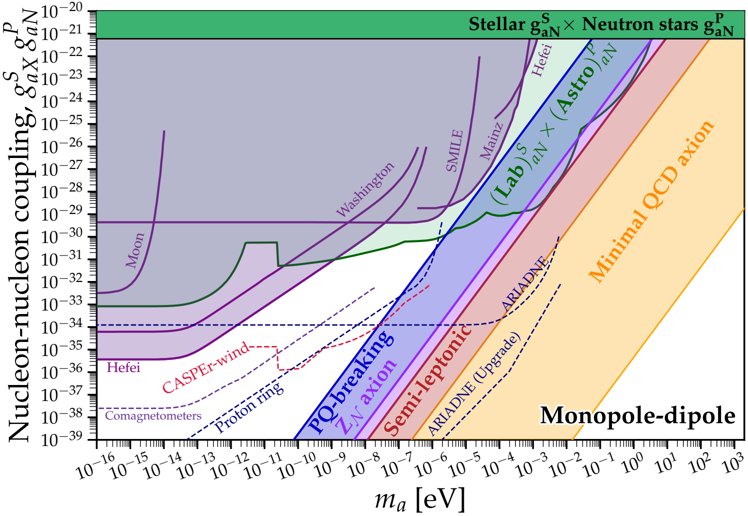
$$g_{a\bar{N}N}^{\text{indirect}} \approx \langle\theta\rangle \frac{13\text{ MeV}}{f_a} \quad \text{and} \quad g_{a\bar{f}f}^{\text{direct}} = n\frac{m_f}{f_a}\left(\frac{f_a}{\sqrt{2}\Lambda_{\text{UV}}}\right)^n \sin\delta_{\not{PQ}}$$

The minimal QCD band is based on the indirect contribution.

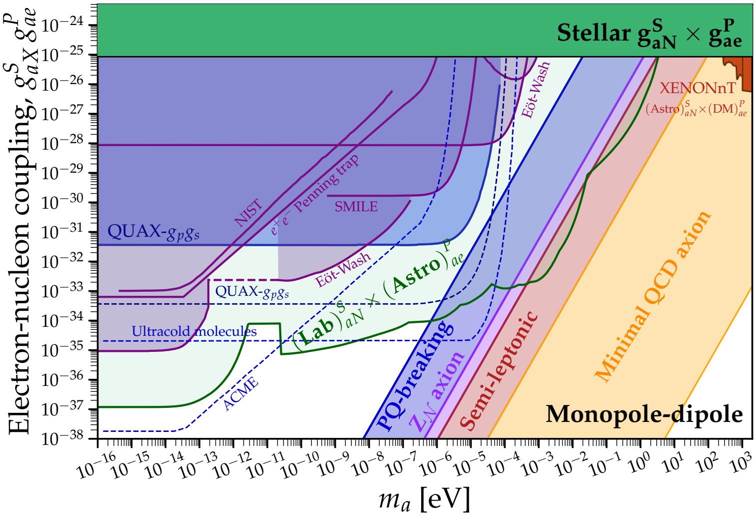
Widening the axion window on long-range forces



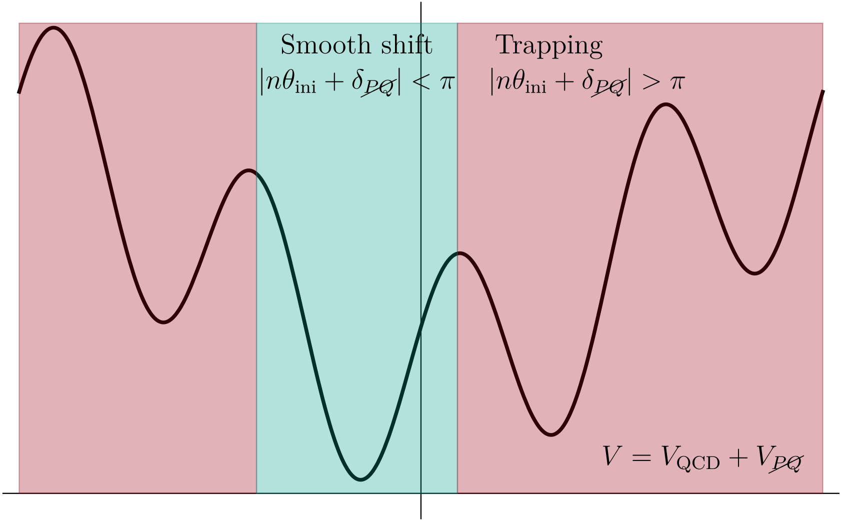
Widening the axion window on long-range forces



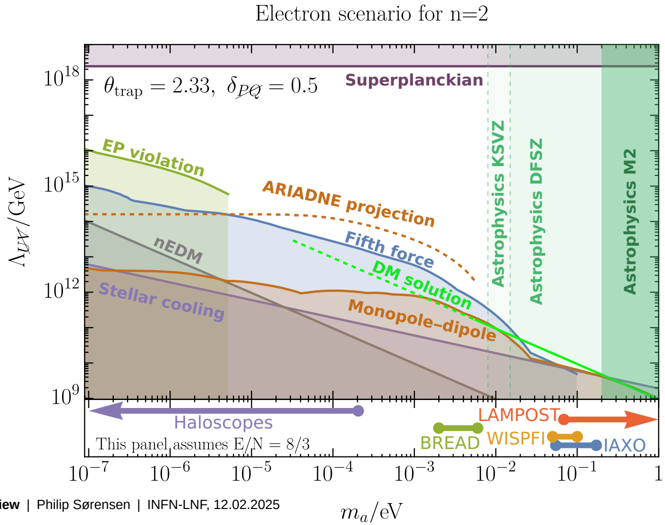
Widening the axion window on long-range forces



Trapped Misalignment



Trapped Misalignment Electron solution: Testable with long range forces



Trapped Misalignment Gluon solution: Testable with nEDM