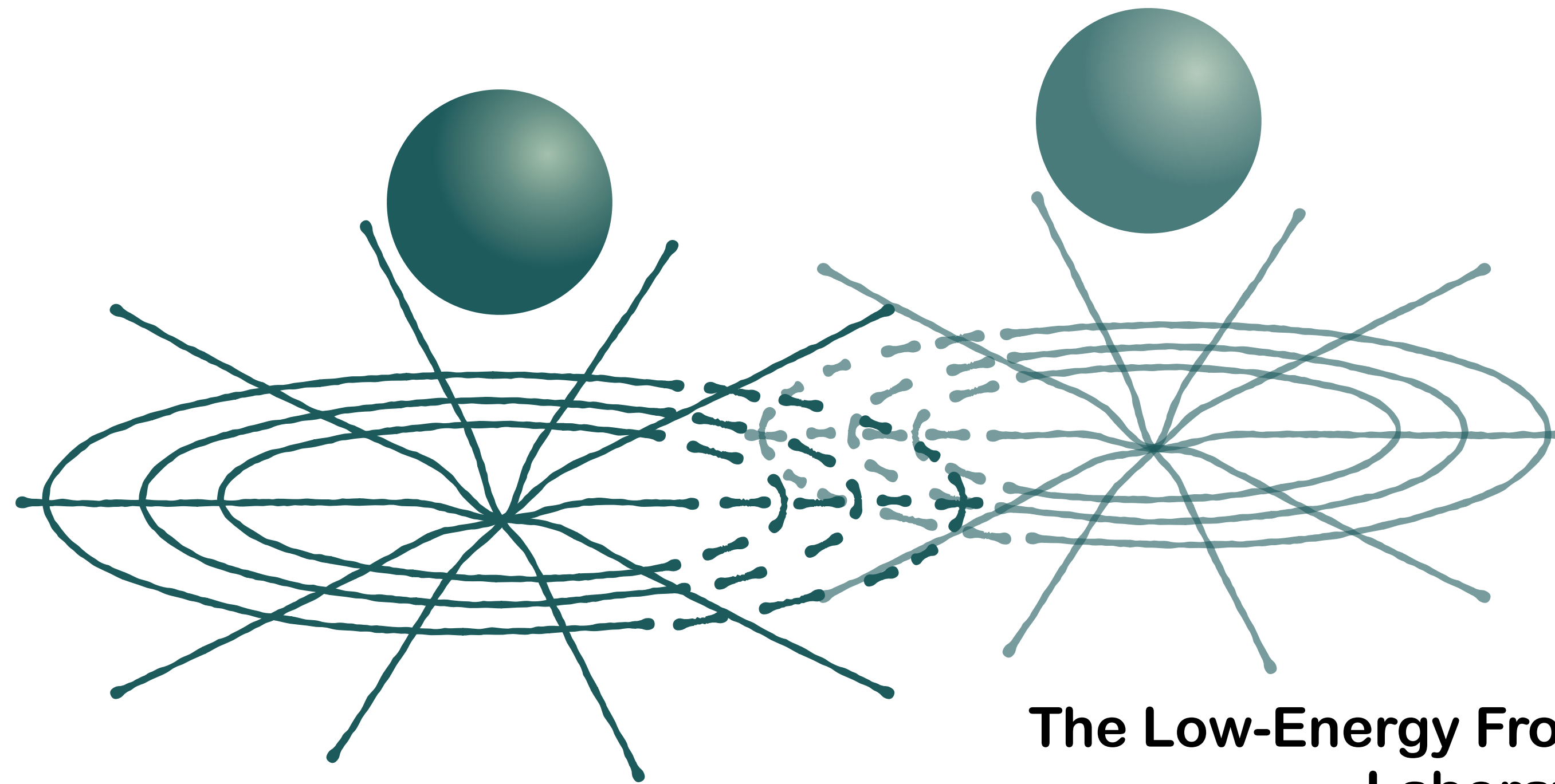


**QUANTUM EFFECTS IN GRAVITY FOM A DELOCALISED QUANTUM SOURCE**

**Flaminia Giacomini**

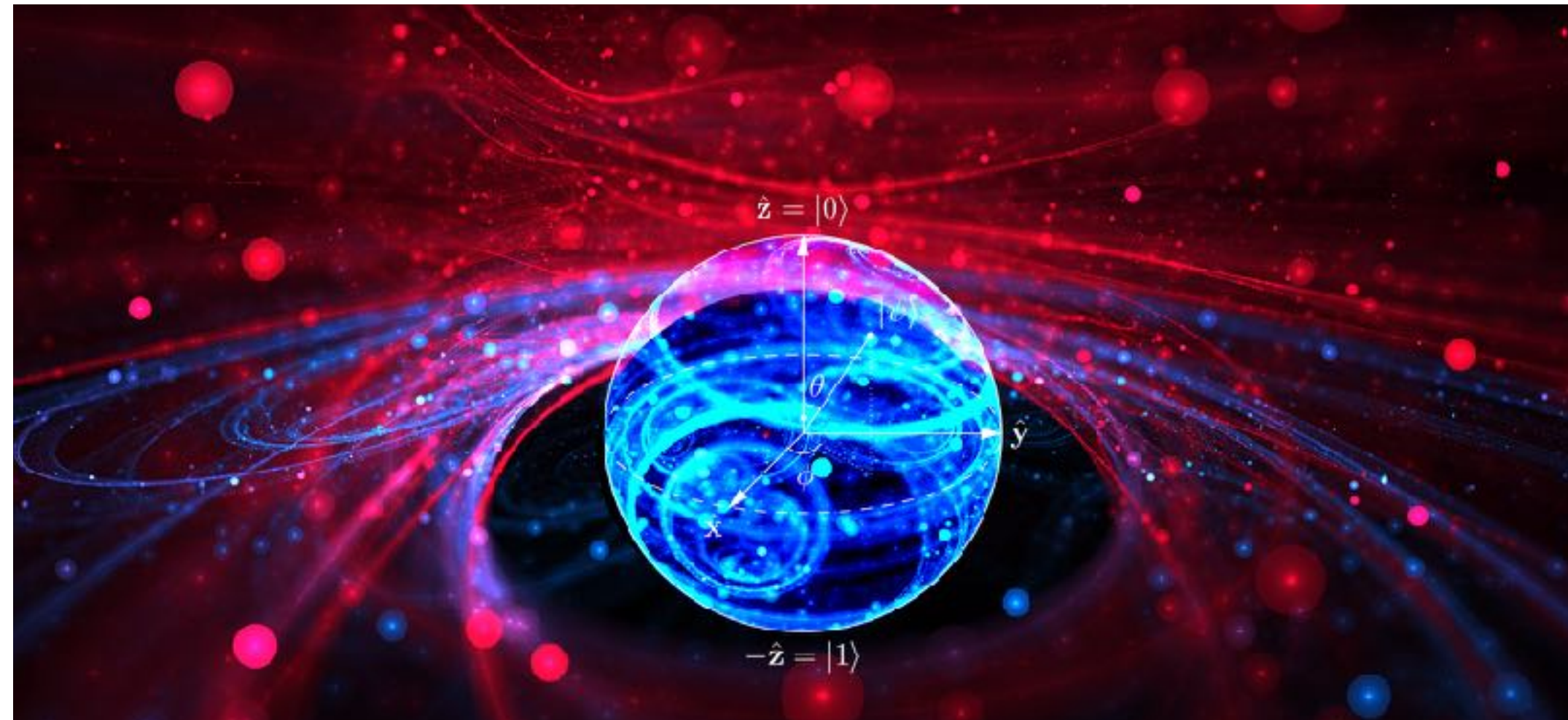
**SNSF Ambizione Fellow, ETH Zürich**



**The Low-Energy Frontier of Particle Physics  
Laboratori Nazionali di Frascati  
10-12 February 2025**

# TWO SUCCESSFUL THEORIES, DIFFERENT PRINCIPLES

## QUANTUM THEORY



## GENERAL RELATIVITY

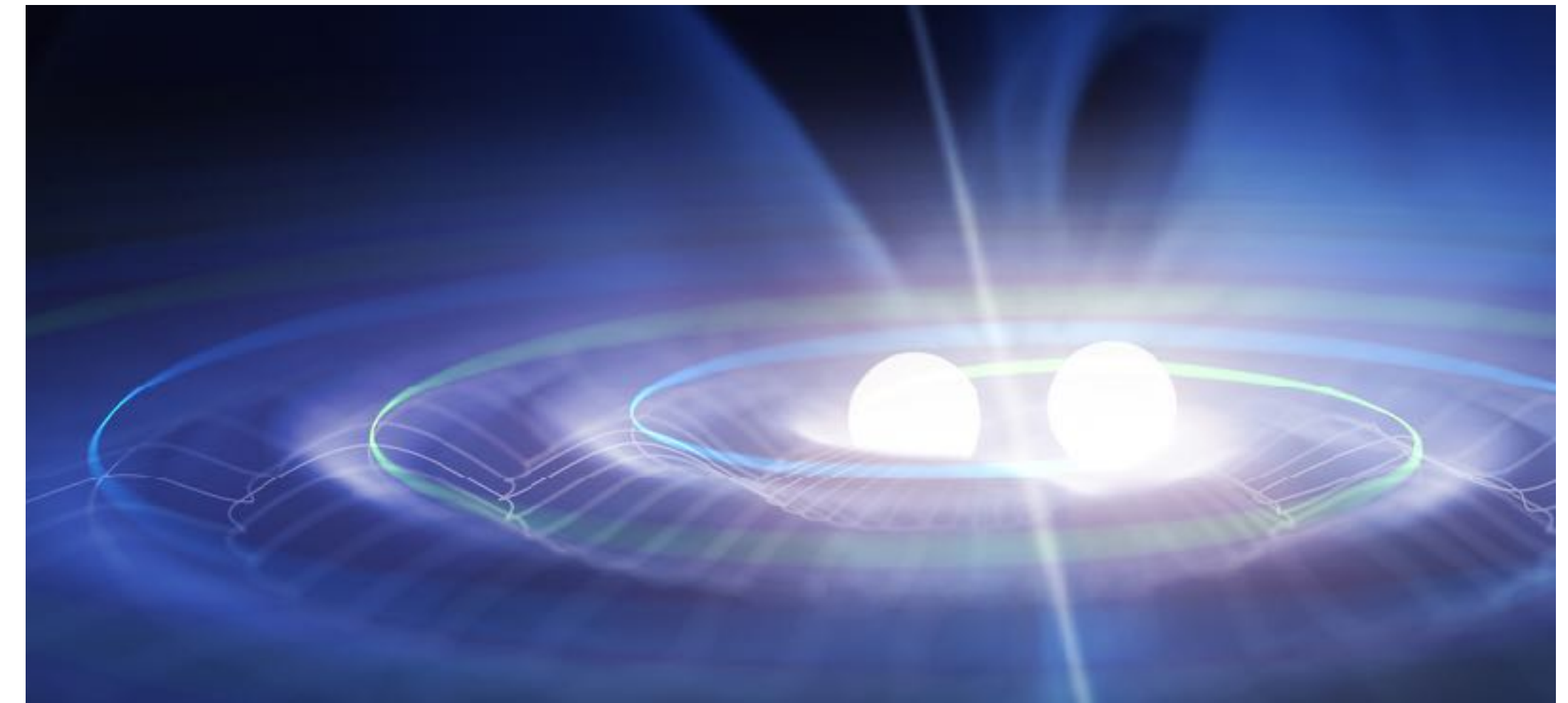
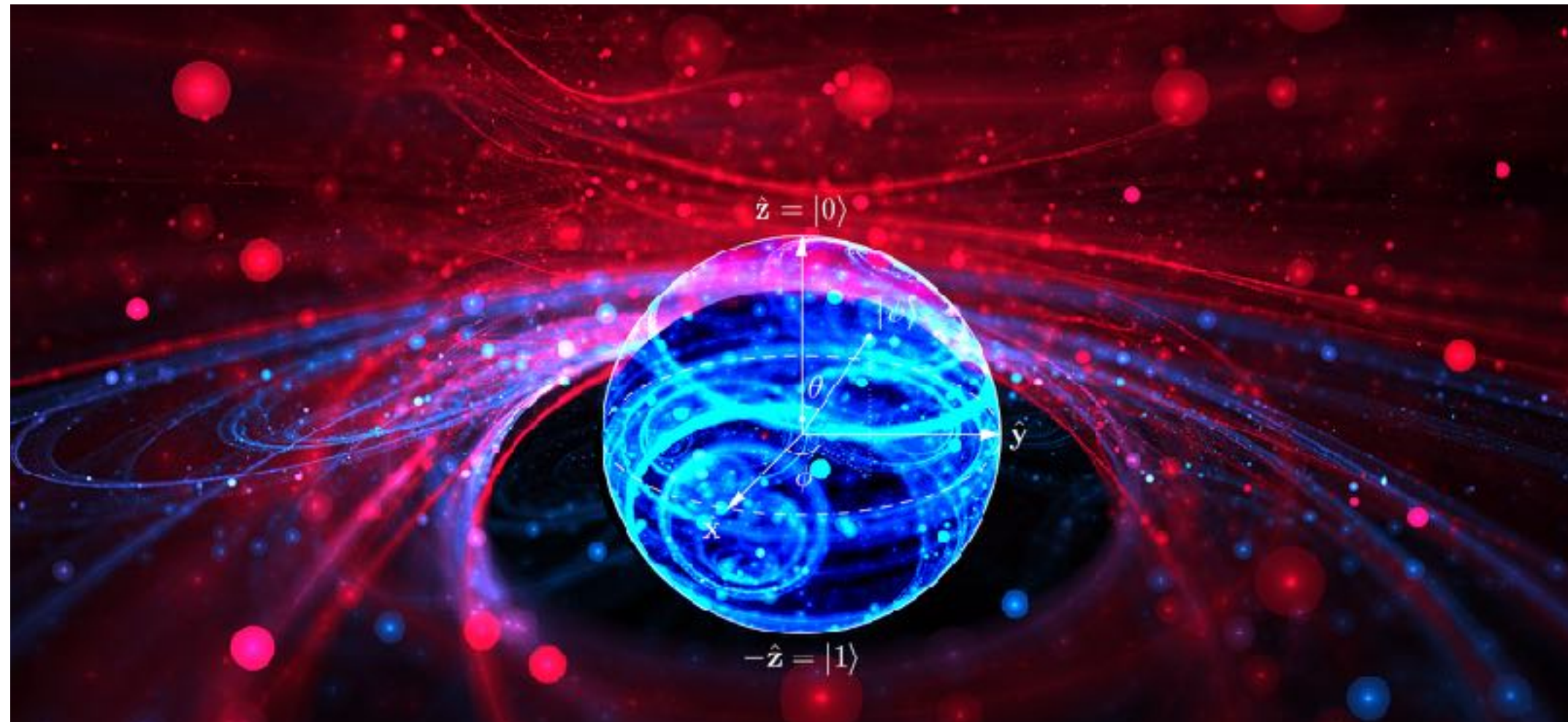


Image credits: Perimeter Institute

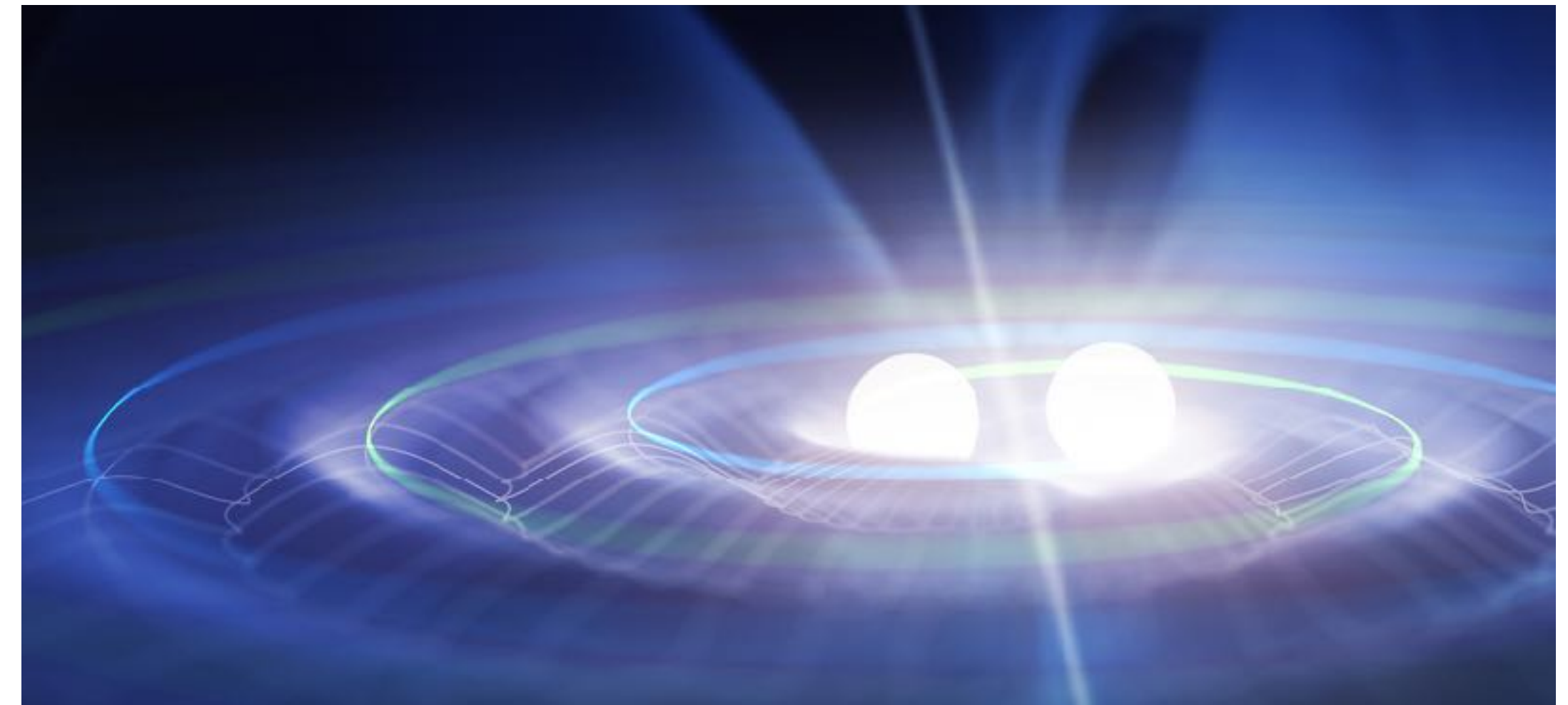
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## QUANTUM THEORY



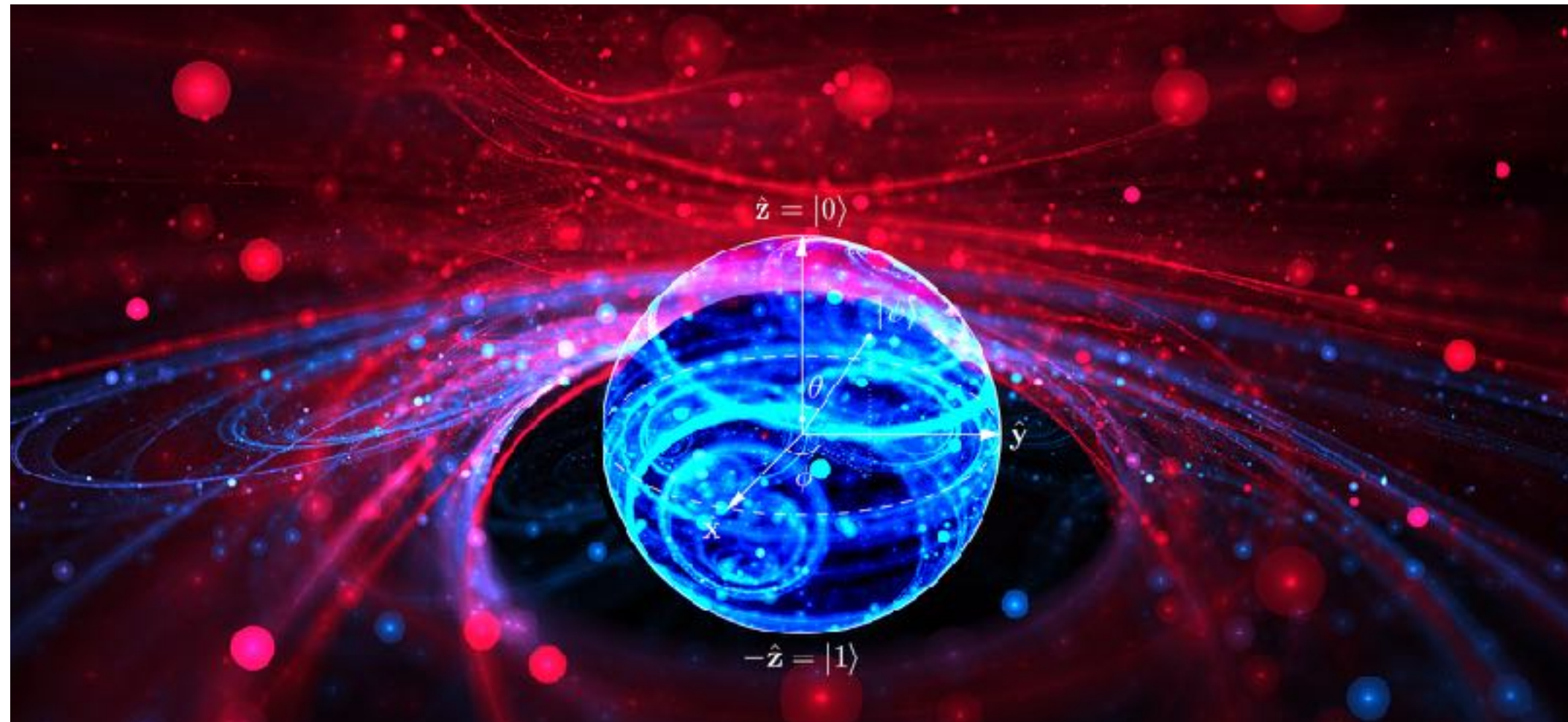
Entanglement, superposition...

## GENERAL RELATIVITY



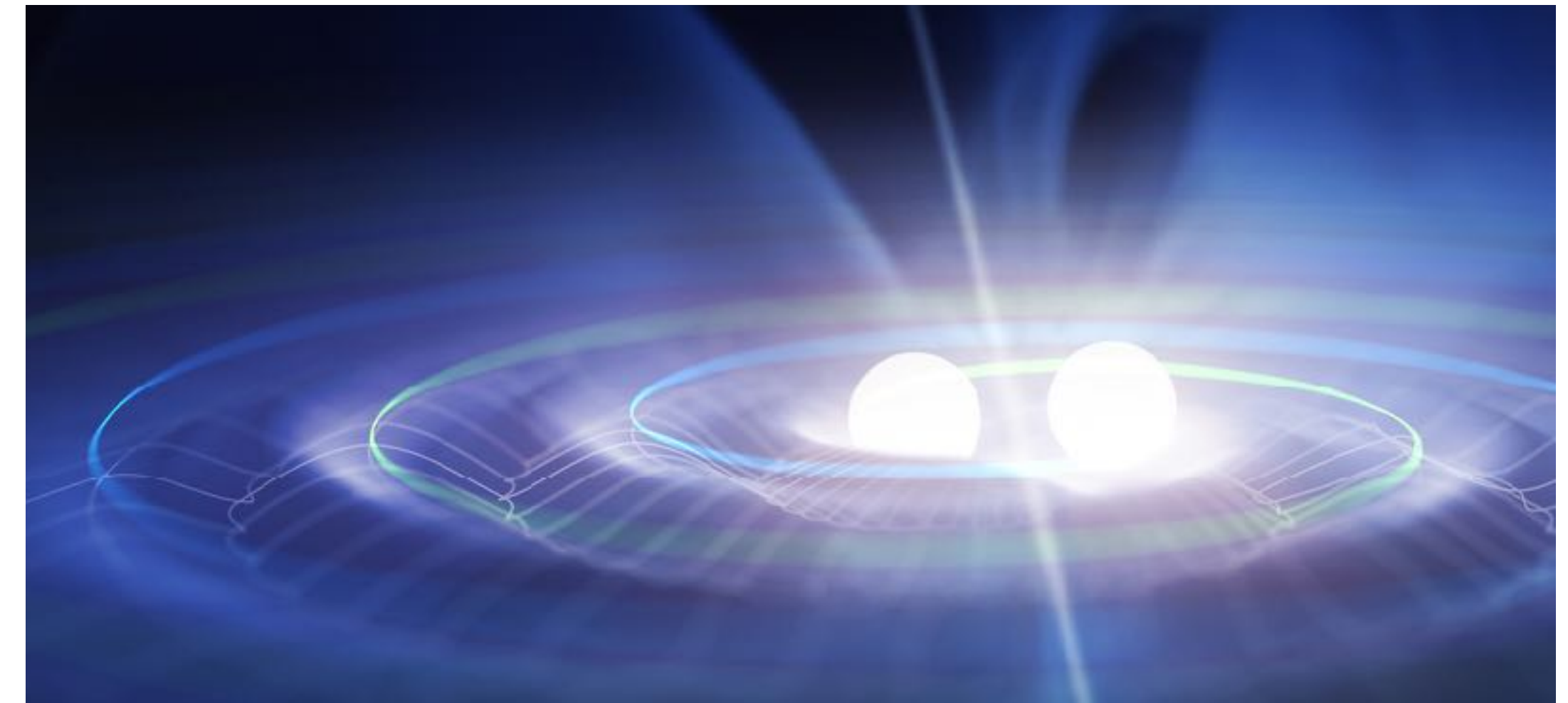
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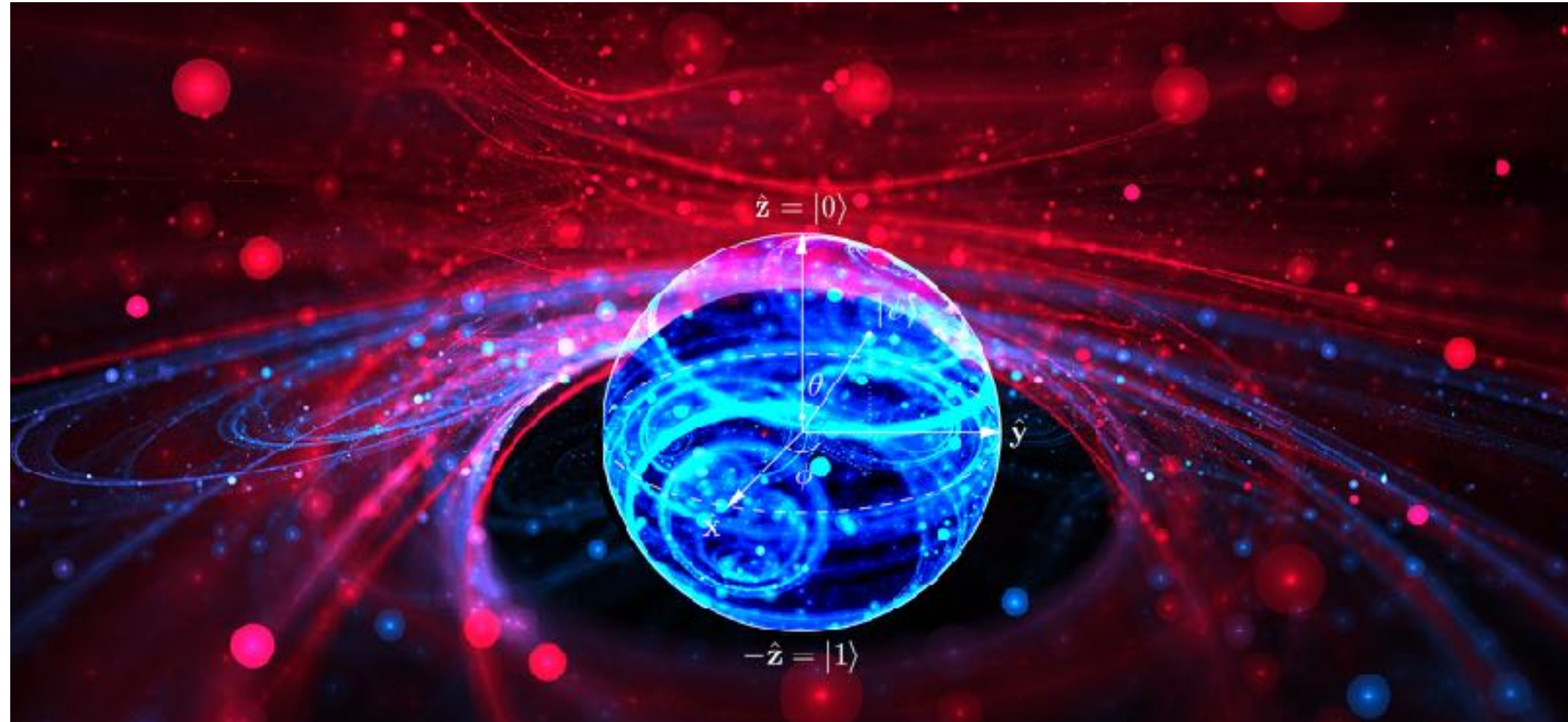


Relation between gravity and matter

*Matter tells spacetime how to curve;  
spacetime tells matter how to move.*

# TWO SUCCESSFUL THEORIES, DIFFERENT PRINCIPLES

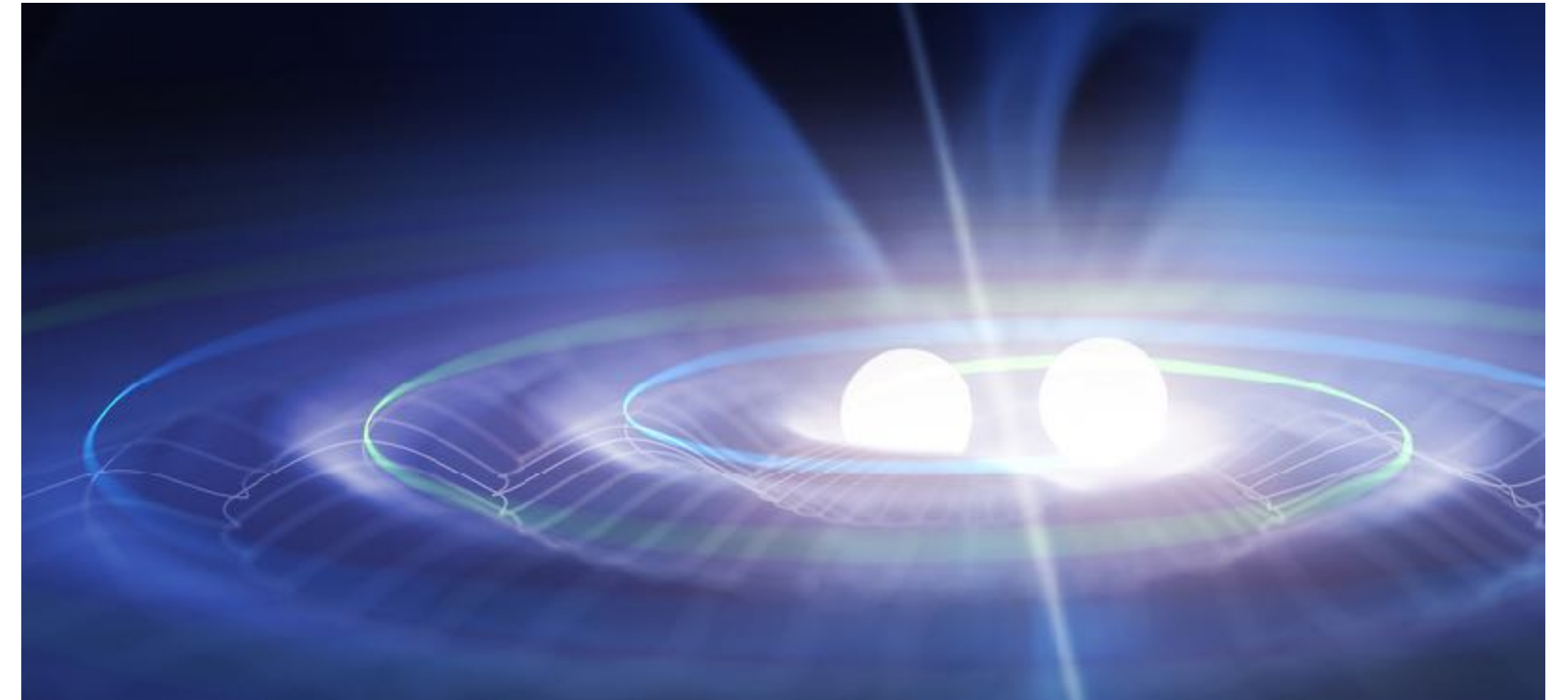
## QUANTUM THEORY



Entanglement, superposition...

Spacetime is the stage

## GENERAL RELATIVITY



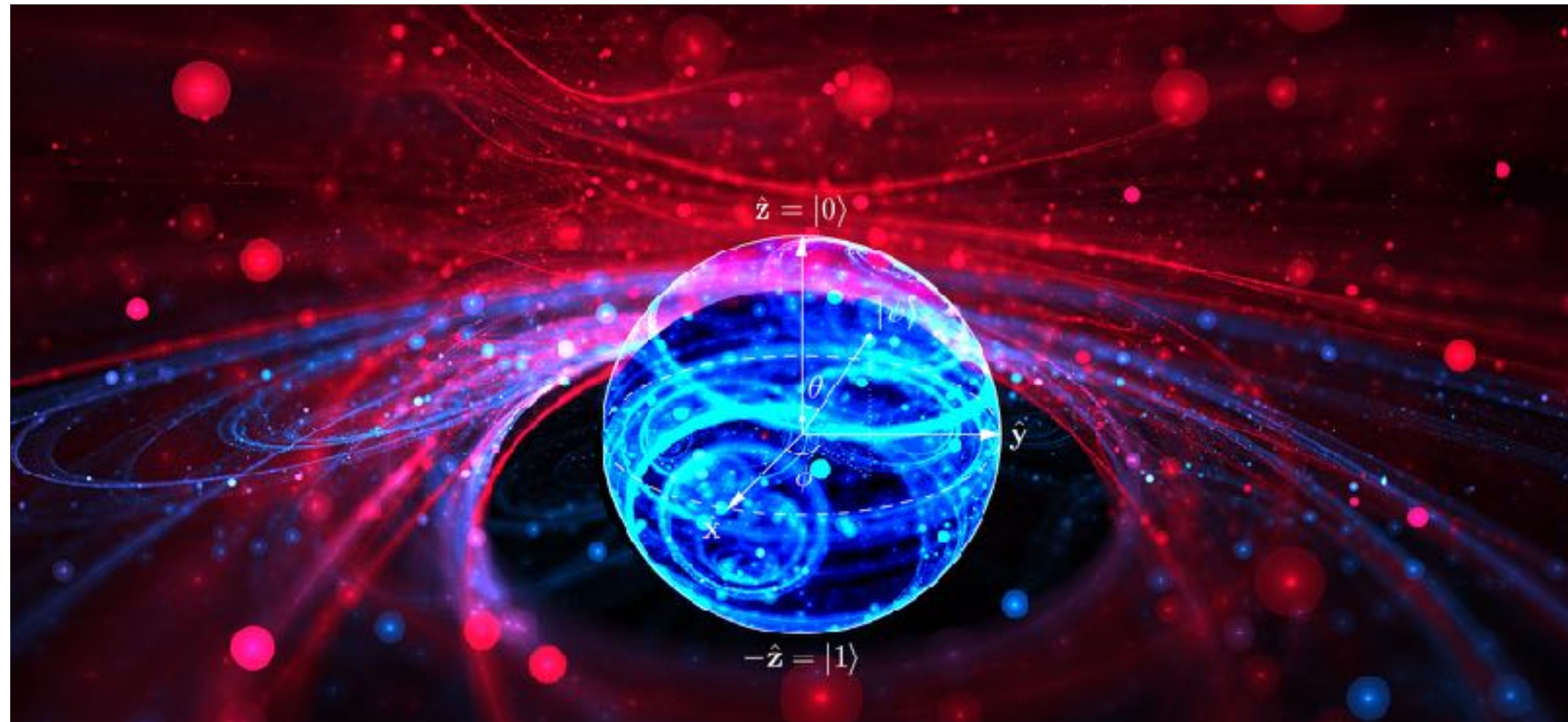
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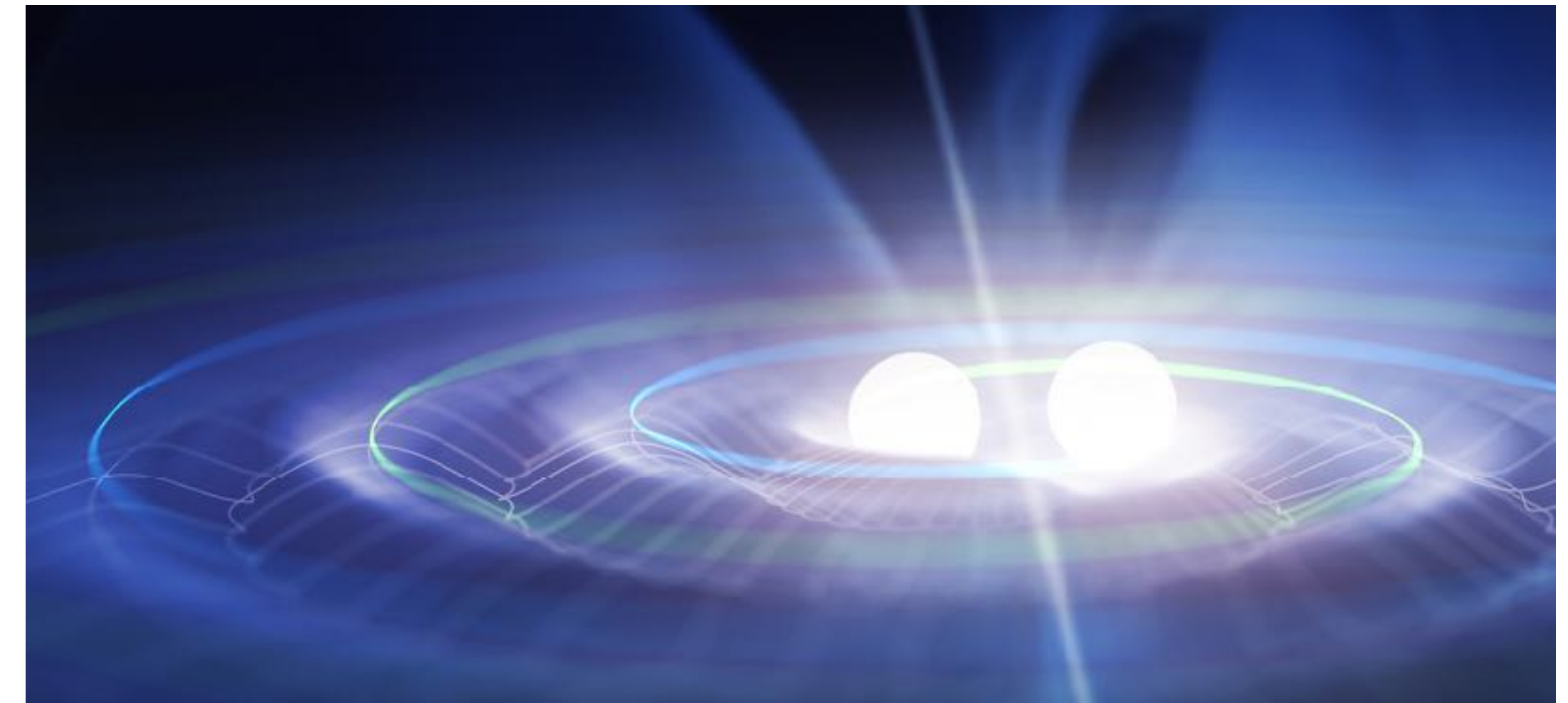
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Entanglement, superposition...

Spacetime is the stage

## GENERAL RELATIVITY



Relation between gravity and matter

*Matter tells spacetime how to curve;  
spacetime tells matter how to move.*

Spacetime is the actor

All experiments are compatible with these theories.

# WHERE SHALL WE LOOK FOR QUANTUM EFFECTS IN GRAVITY?

**HIGH ENERGIES:  
STRONG GRAVITATIONAL  
AND QUANTUM EFFECTS**



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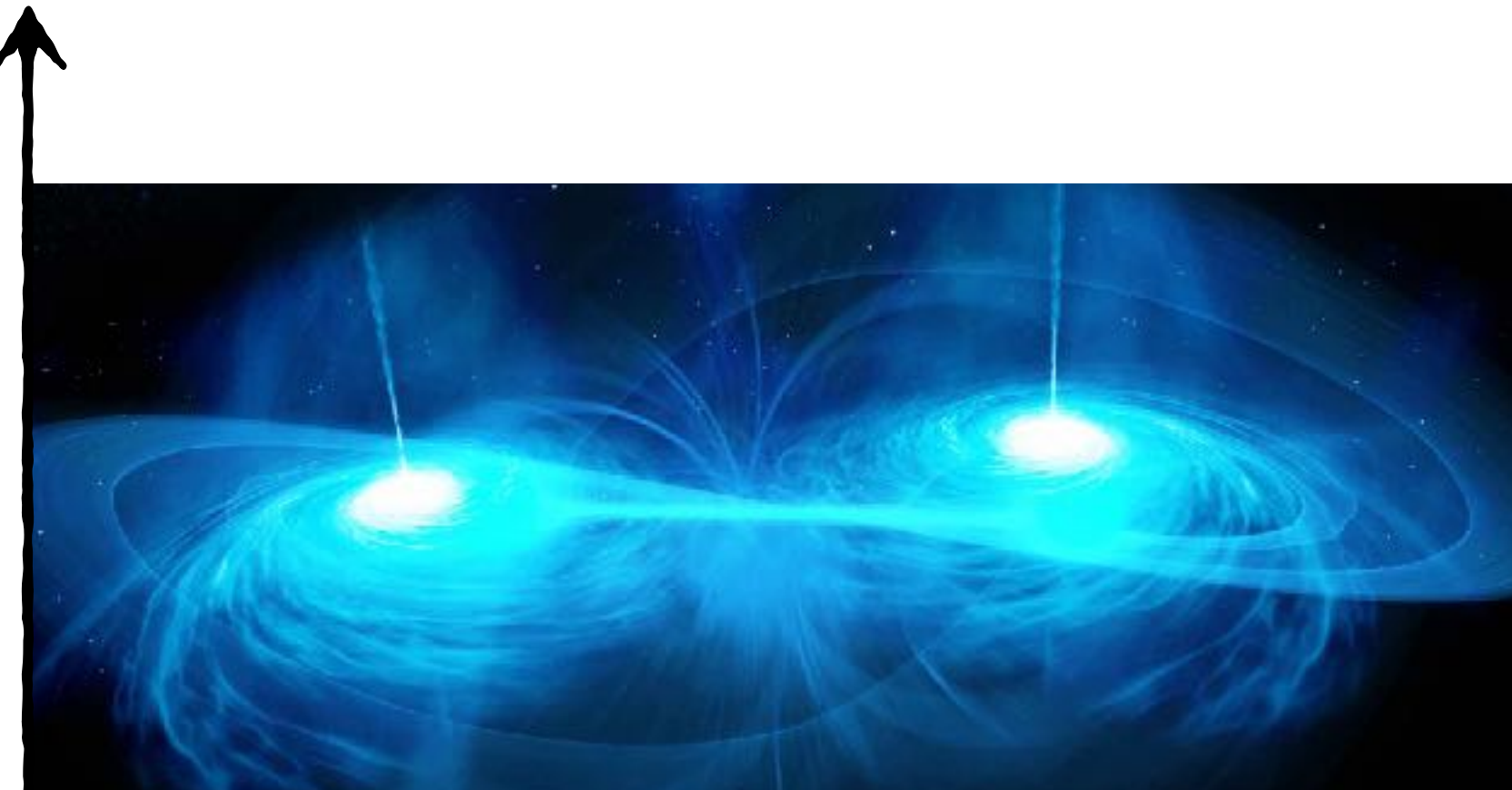
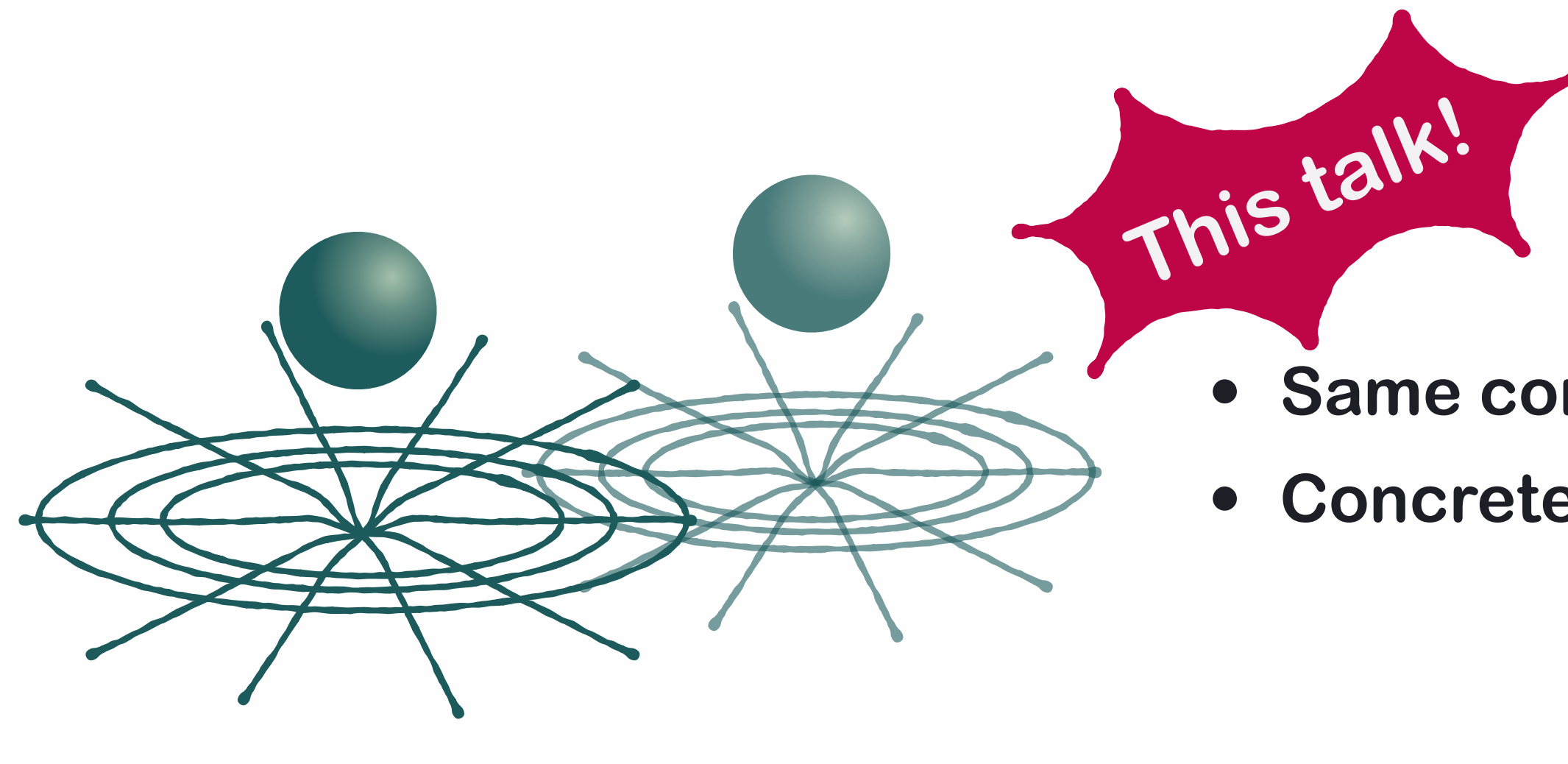


Image credits: Perimeter Institute

QUANTUM GRAVITY

$$\ell_P \approx 10^{-35} m$$

LOW ENERGIES:  
WEAK-FIELD GRAVITY  
QUANTUM PARTICLES



- Same conceptual questions as in QG
- Concrete physical situations

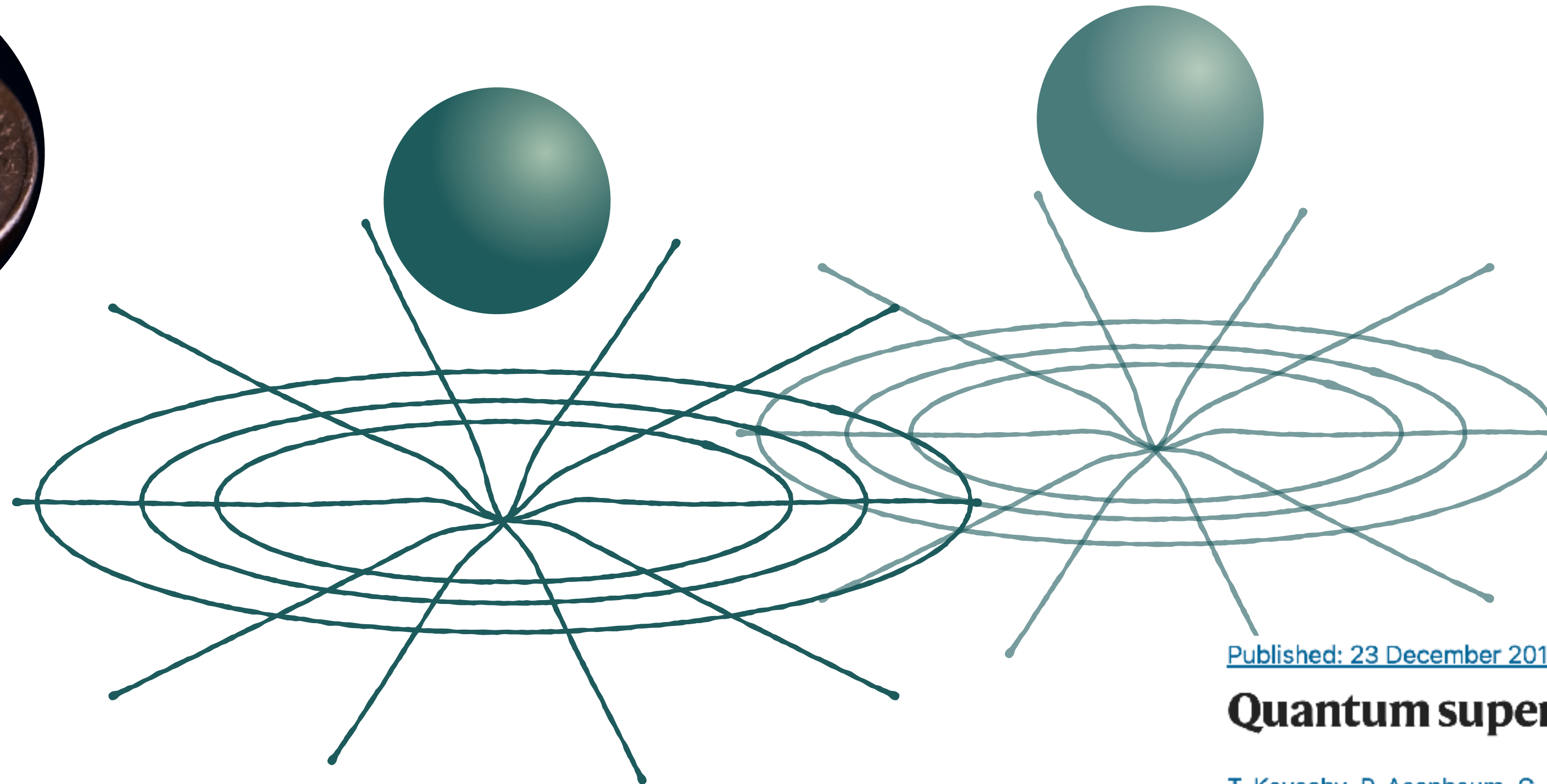
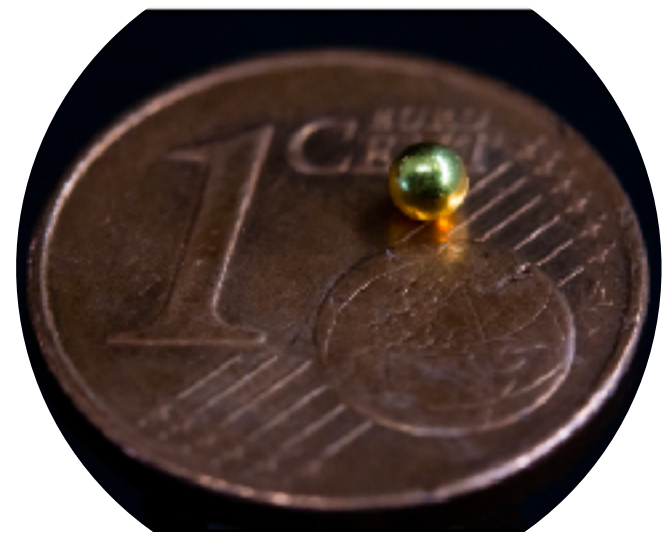


# NONCLASSICAL SPACETIME FROM A QUANTUM SOURCE

Article | [Published: 10 March 2021](#) **GRAVITY SOURCE: 90 mg**  
**Measurement of gravitational coupling between millimetre-sized masses**

[Tobias Westphal](#) , [Hans Hepach](#), [Jeremias Pfaff](#) & [Markus Aspelmeyer](#) 

[Nature](#) **591**, 225–228 (2021) | [Cite this article](#)



**Schrödinger cat states of a 16-microgram mechanical oscillator**  
**SUPERPOSED MASS:  $10^{-5}g$**

[MARIUS BILD](#) , [MATTEO FADEL](#) , [YU YANG](#) , [UWE VON LÜPKE](#) , [PHILLIP MARTIN](#) , [ALESSANDRO BRUNO](#) , AND [YIWEN CHU](#)  [Authors Info & Affiliations](#)

[SCIENCE](#) • 20 Apr 2023 • Vol 380, Issue 6642 • pp. 274-278 • DOI: 10.1126/science.adf7553

[Published: 23 December 201](#) **QUANTUM SUPERPOSITION: 0.5 m**  
**Quantum superposition at the half-metre scale**

[T. Kovachy](#), [P. Asenbaum](#), [C. Overstreet](#), [C. A. Donnelly](#), [S. M. Dickerson](#), [A. Sugarbaker](#), [J. M. Hogan](#) & [M. A. Kasevich](#) 

[Nature](#) **528**, 530–533 (2015) | [Cite this article](#)

# DO WE REQUIRE A QUANTUM DESCRIPTION OF GRAVITY?

*“... it seems to me that we are in trouble if we believe in quantum mechanics but do not quantize gravitational theory”*

R. Feynman, Chapel Hill Conference (1957)

## The Role of Gravitation in Physics

Report from the 1957 Chapel Hill Conference

Cécile M. DeWitt and Dean Rickles (eds.)



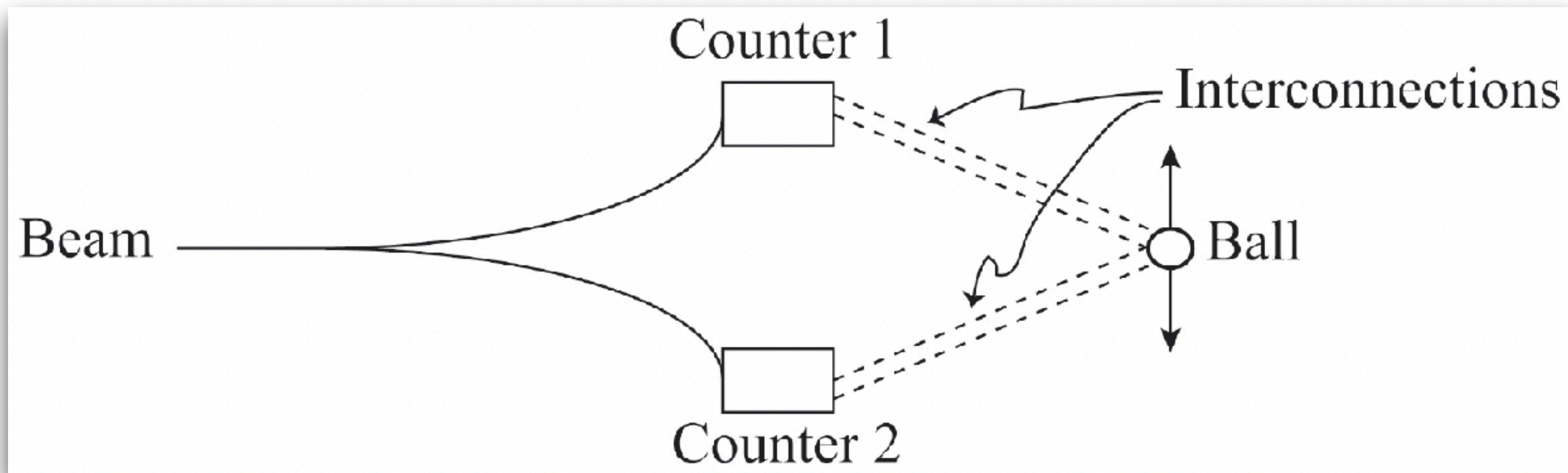
Max Planck Institute for the History of Science  
Sources in the Development of Knowledge 5

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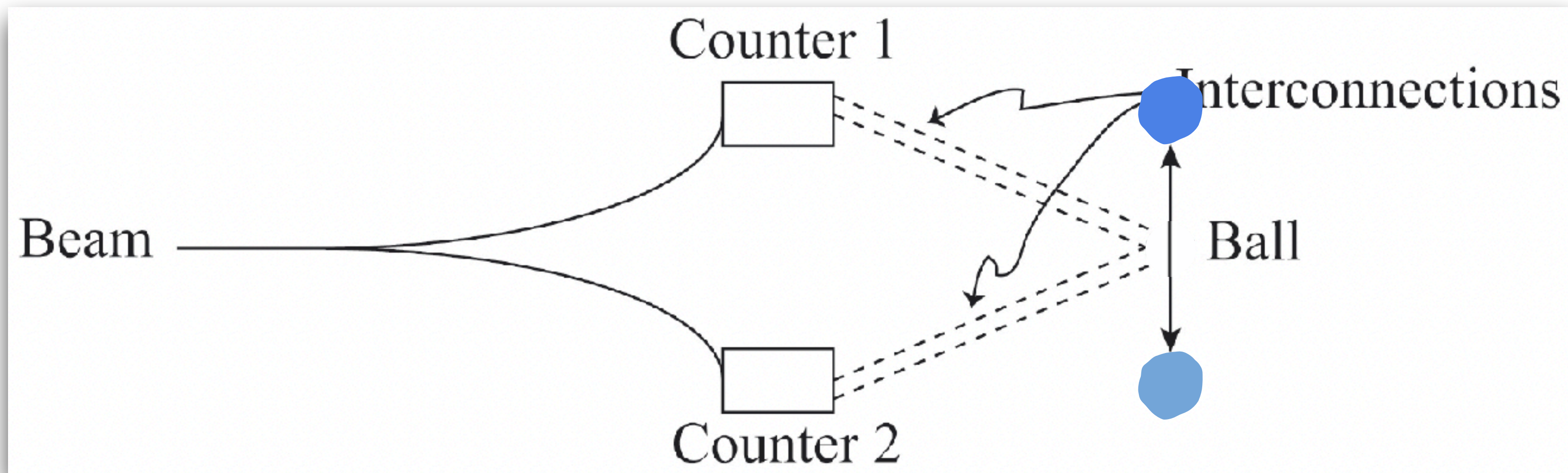
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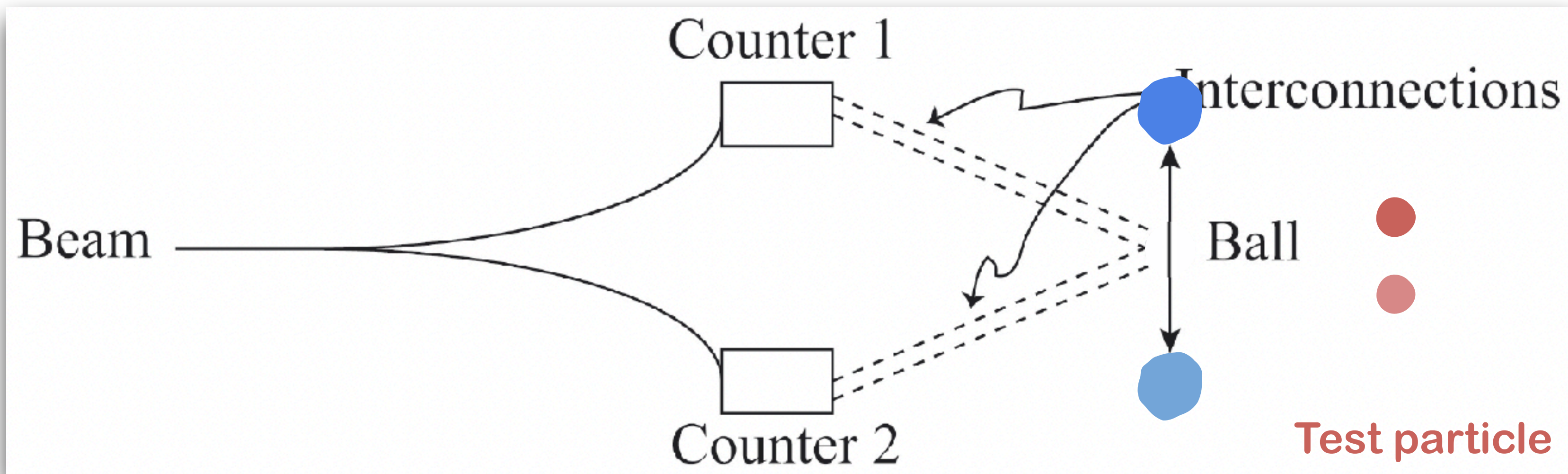
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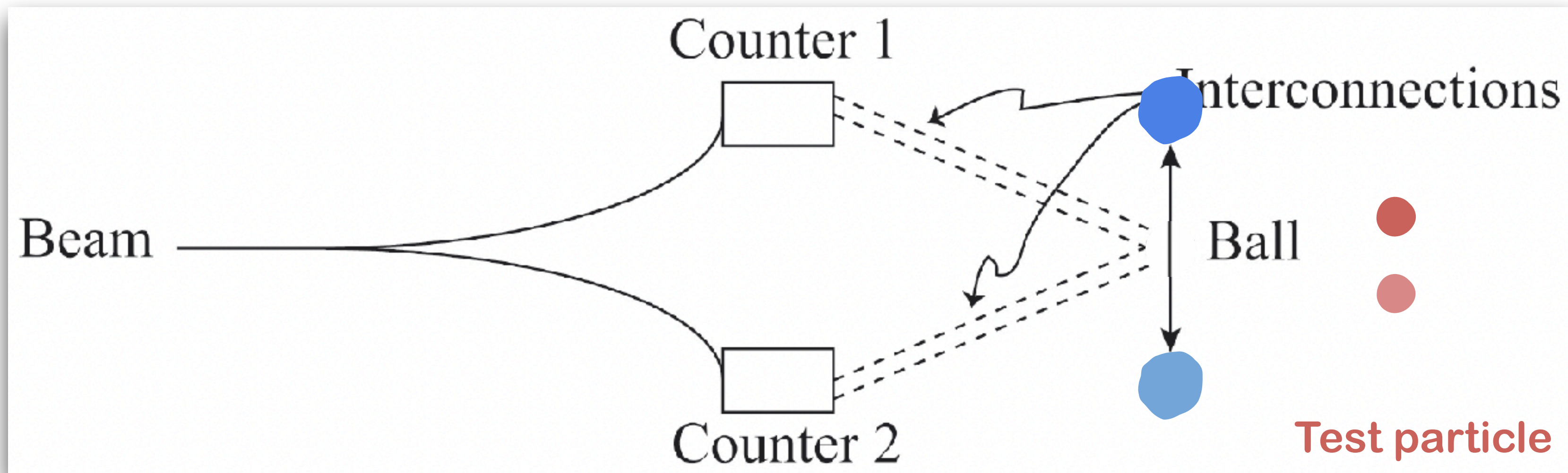
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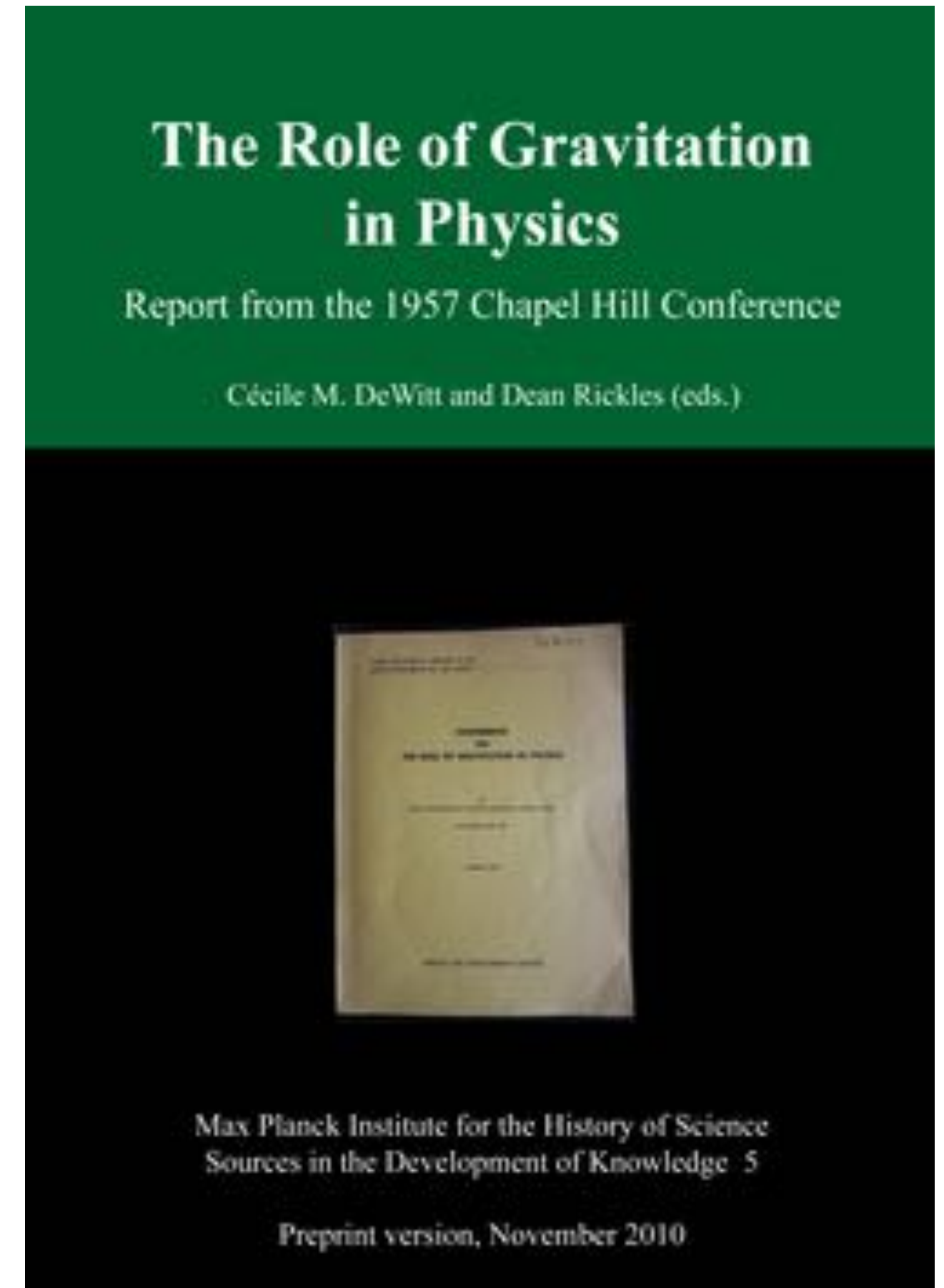
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*“If you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment.”*



# SIDE REMARK: WHAT PROVES THAT ELECTROMAGNETISM IS QUANTUM?

1905: photoelectric effect (Einstein)

1923: photoelectric effect does not require quantum (Millikan)

1923: Compton effect

1960s: semiclassical theory of radiation (Jaynes)

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

$$\tau = 0 \quad g^{(2)}(0) = \frac{\langle I^2(t) \rangle}{\langle I(t) \rangle^2} \geq 1$$

For a single photon source

$$g^{(2)}(0) = 0 \not\geq 1$$

Signature of nonclassicality

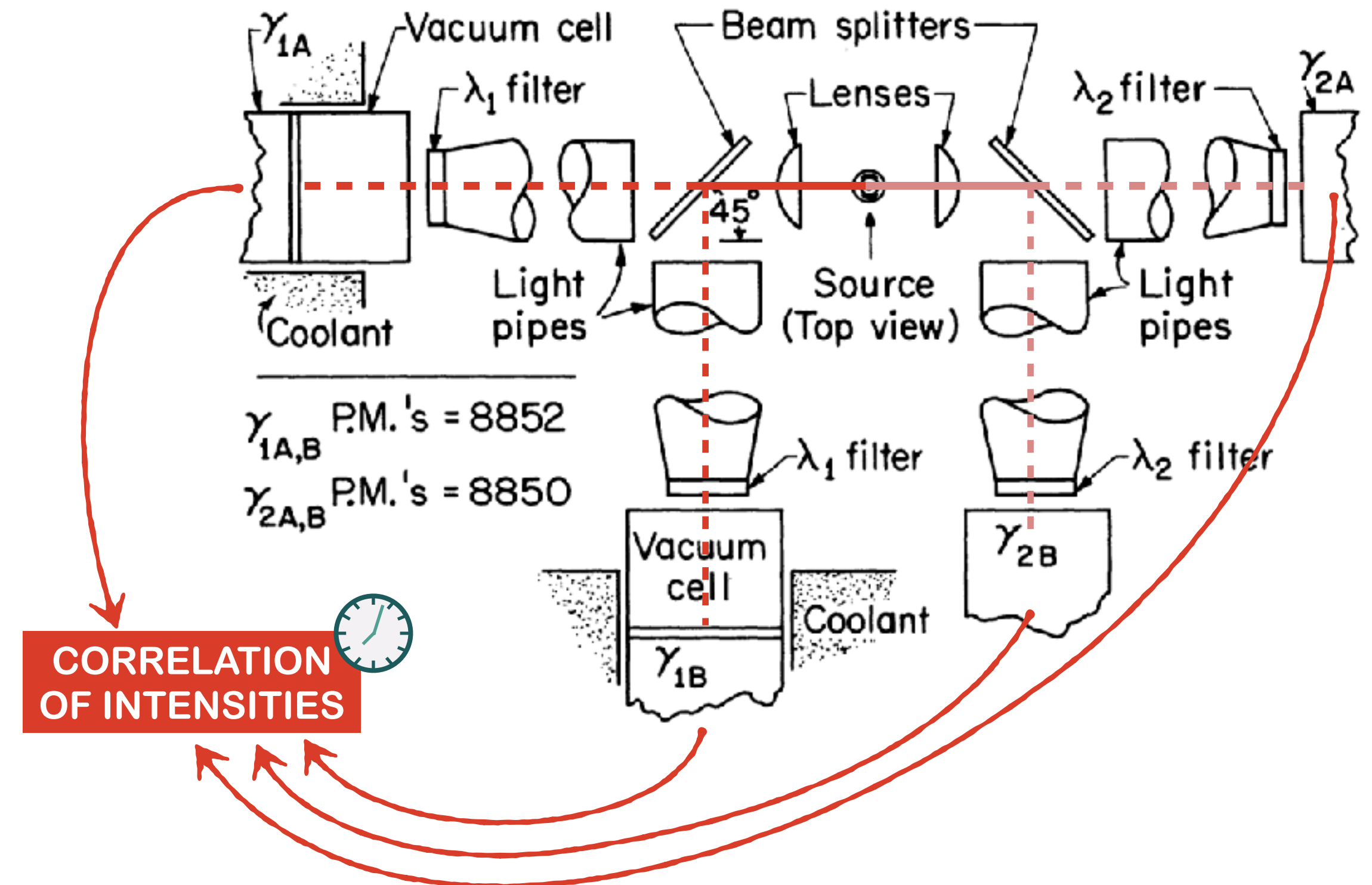
PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

## Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effect\*

John F. Clauser

Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 30 October 1973)



# GRAVITATIONALLY INDUCED ENTANGLEMENT

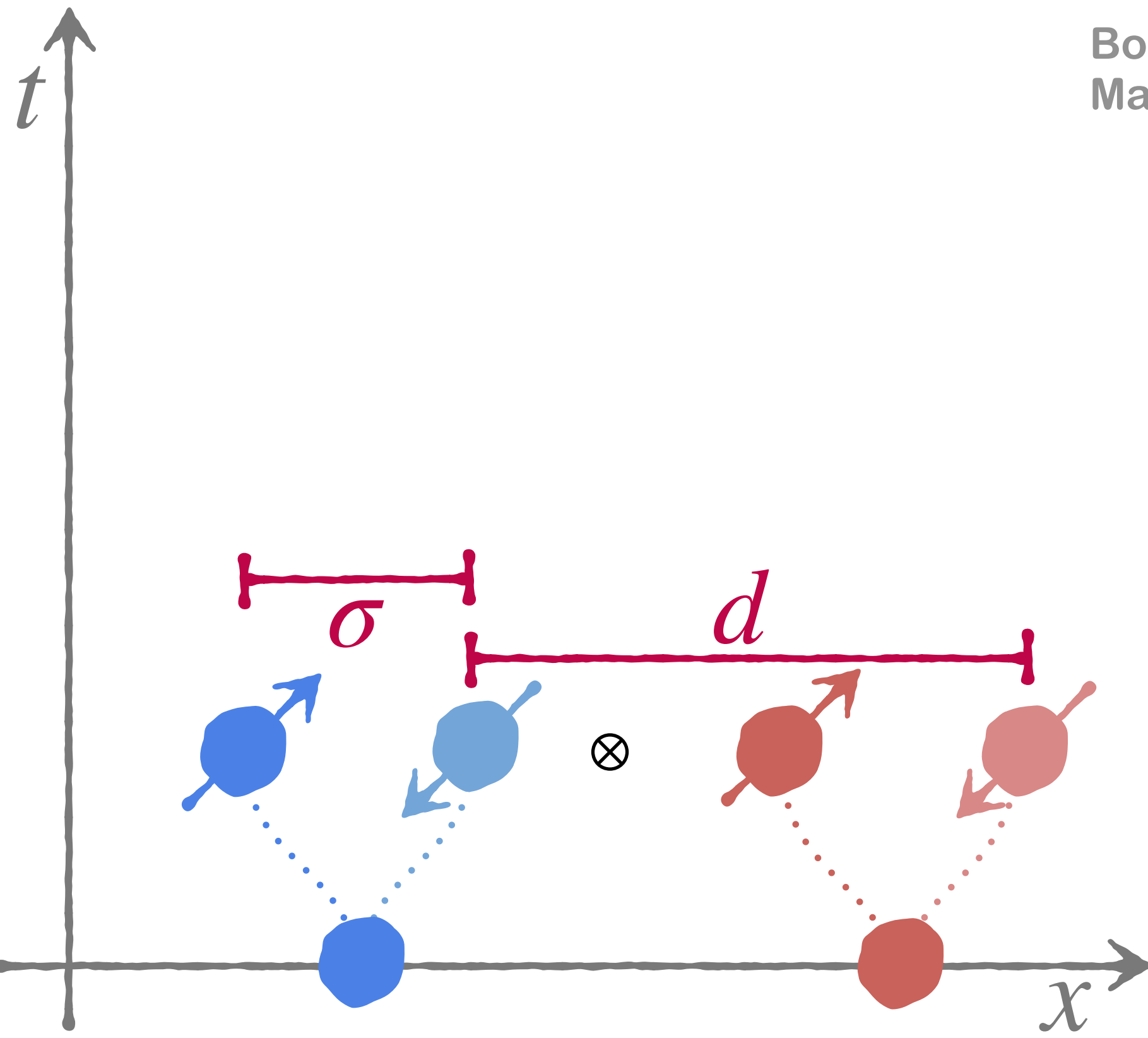
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LOCC: Bennett et al. PRA (1995)



# GRAVITATIONALLY INDUCED ENTANGLEMENT

Bose et al. PRL (2017)  
Marletto, Vedral PRL (2017)

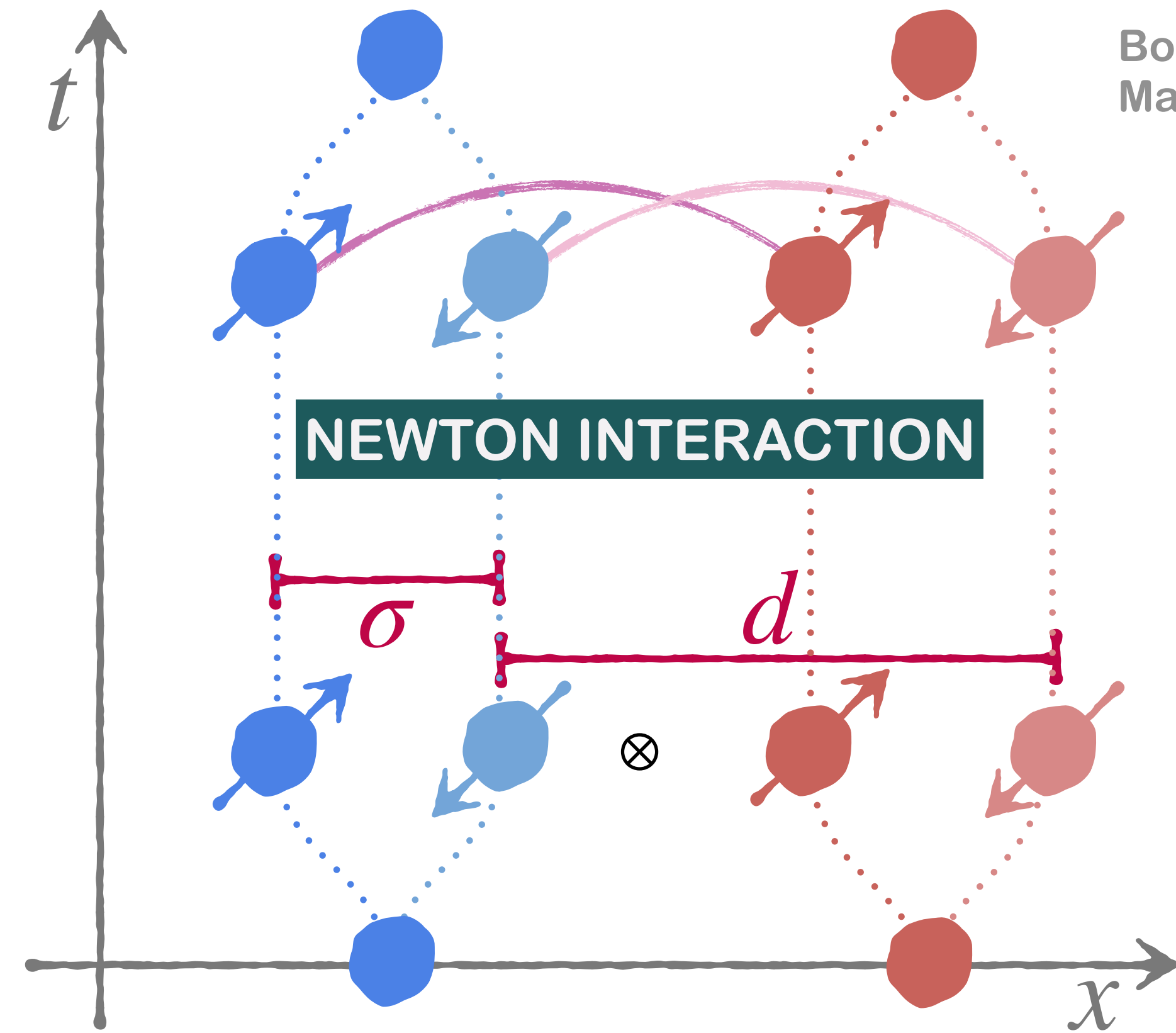


**PRODUCT STATE**  $\sim (|x_1\rangle_A + |x_2\rangle_A) \otimes (|x_1 + d\rangle_B + |x_2 + d\rangle_B)$

LOCC: Bennett et al. PRA (1995)

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Bose et al. PRL (2017)  
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PRODUCT STATE

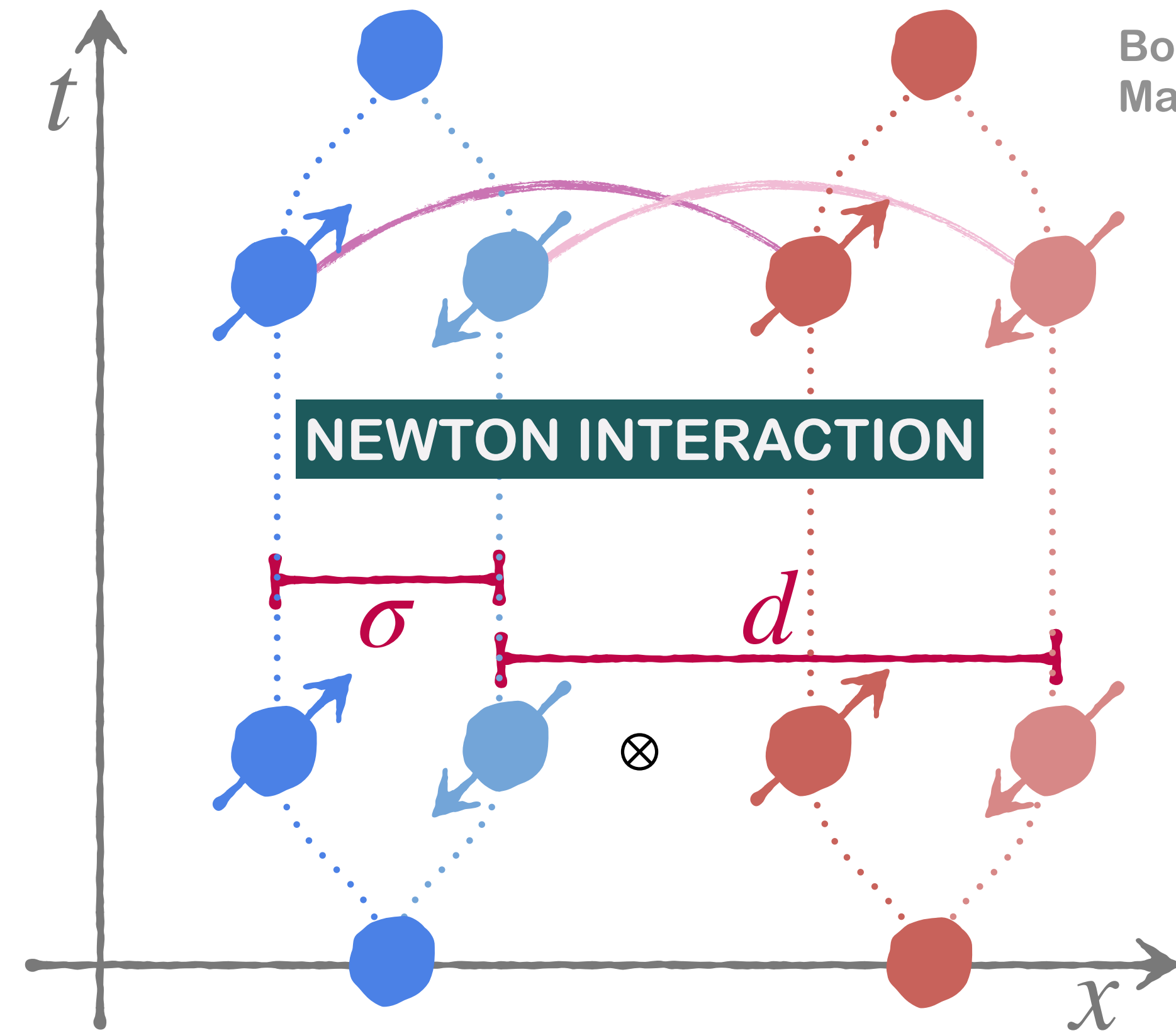
$$\hat{H}_I = \hat{V}_N = -G \frac{m_A m_B}{|\hat{x}_A - \hat{x}_B|}$$

$$\sim (|x_1\rangle_A + |x_2\rangle_A) \otimes (|x_1 + d\rangle_B + |x_2 + d\rangle_B)$$

LOCC: Bennett et al. PRA (1995)

# GRAVITATIONALLY INDUCED ENTANGLEMENT

Bose et al. PRL (2017)  
Marletto, Vedral PRL (2017)



ENTANGLED STATE

Nonclassical channel  
(LOCC theorem)

PRODUCT STATE

$$\sim \sum_{i,j=1,2} e^{i\phi_{ij}} |x_i\rangle_A |x_j + d\rangle_B$$

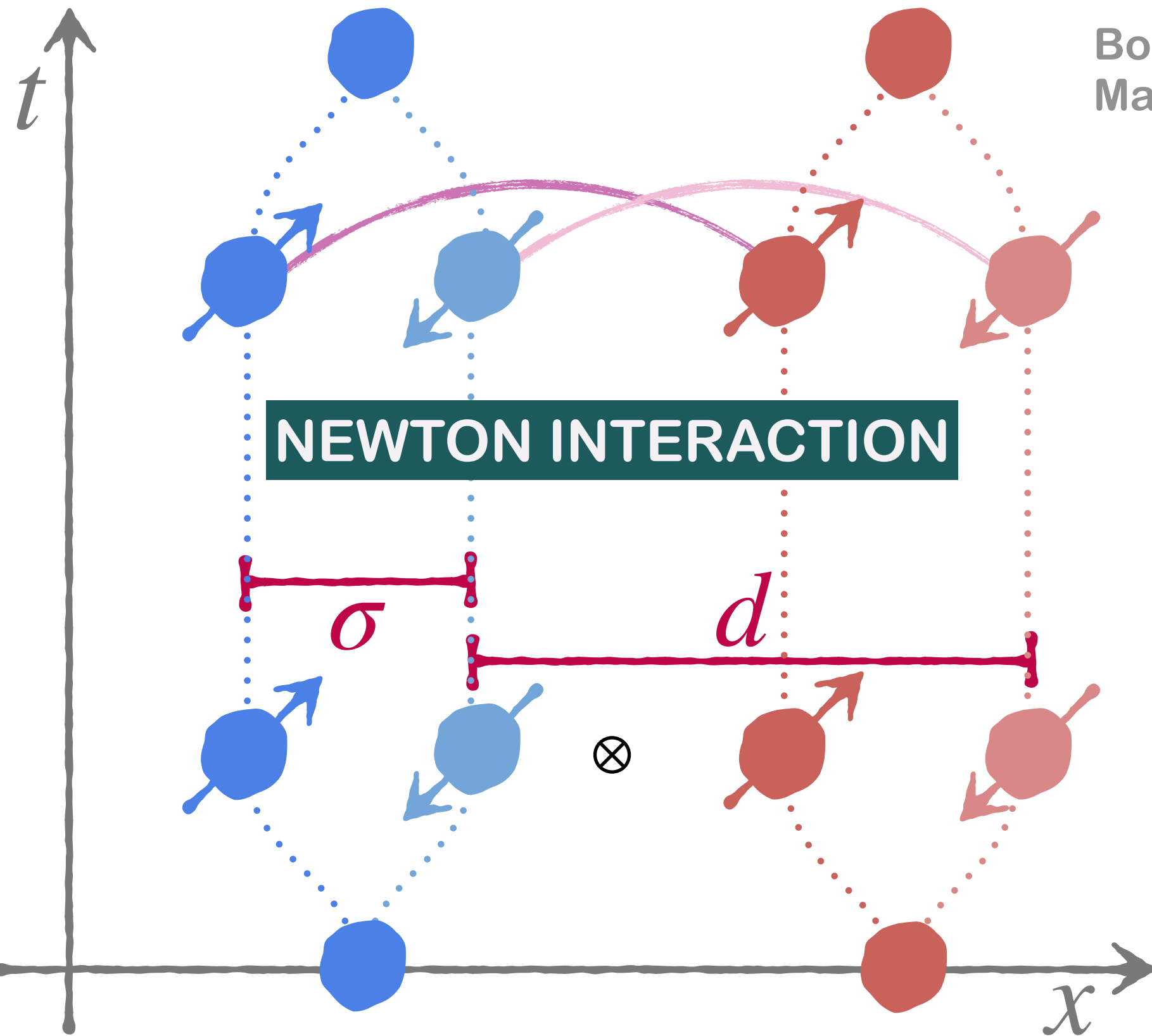
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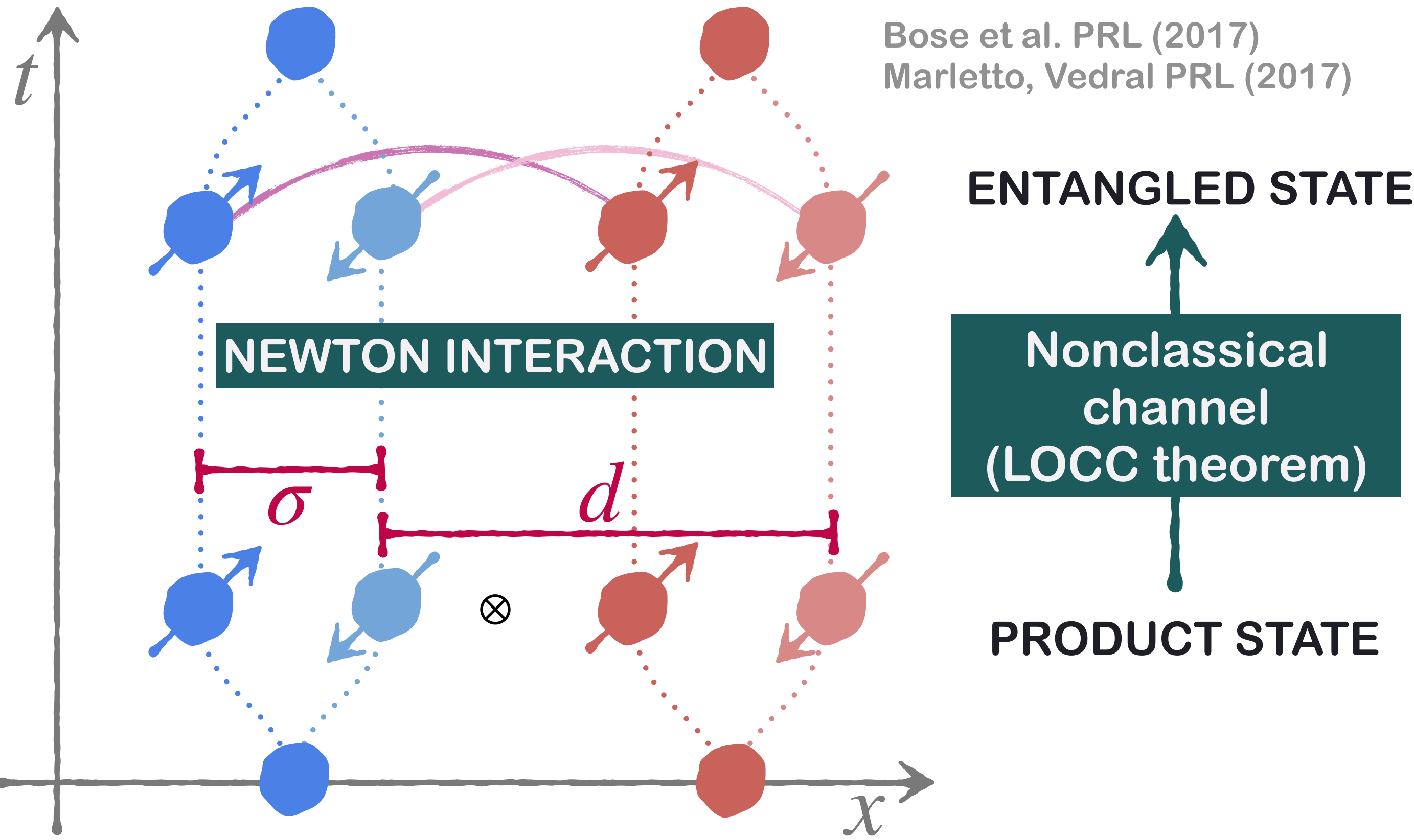
PRODUCT STATE

$$\sim (|x_1\rangle_A + |x_2\rangle_A) \otimes (|x_1 + d\rangle_B + |x_2 + d\rangle_B)$$

**Many people contributed!** E.g. Anastopoulos, Aspelmeyer, Barker, Belenchia, Bengyat, Bhatar, Blencowe, Bose, Brukner, Carney, Castro-Ruiz, Chen, Christodoulou, Cooper, Di Biagio, Galley, Geraci, Hackermüller, Howl, Hu, Huggett, Iyer, Kent, Kim, Krisnanda, Lami, Linneman, Liu, Mahesh, Marletto, Marshman, Martín-Martínez, Mazumdar, Milburn, Morley, Müller, Mummery, Naik, Pal, Paterek, Paternostro, Pedernales, Perche, Pitalúa-García, Plenio, Qvarfort, Rovelli, Schneider, Schut, Selby, Serafini, Sillanpää, Tam, Taylor, Toros, Ulbricht, Vedral, Wald, Yant...

LOCC: Bennett et al. PRA (1995)

# GRAVITATIONALLY INDUCED ENTANGLEMENT



Bose et al. PRL (2017)  
Marletto, Vedral PRL (2017)

**NEWTON INTERACTION**

**ENTANGLED STATE**

**Nonclassical channel  
(LOCC theorem)**

**PRODUCT STATE**

# GRAVITATIONALLY INDUCED ENTANGLEMENT

Bose et al. PRL (2017)  
Marletto, Vedral PRL (2017)

## ENTANGLEMENT RATE

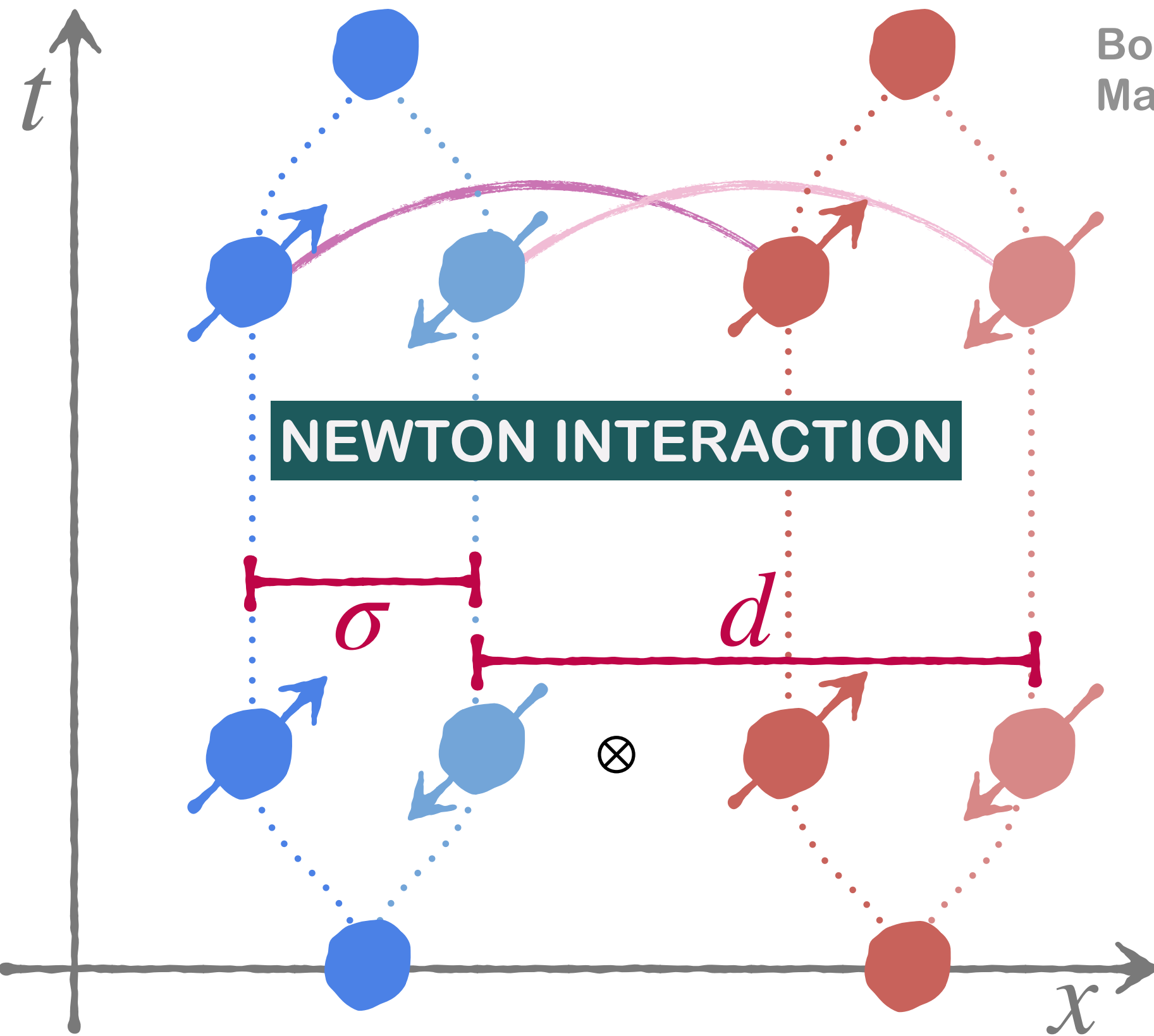
$$\Gamma_{ent} = \frac{d}{dt} \Delta\phi = \frac{G m^2 \sigma^2}{\hbar d^3}$$

$$\begin{aligned} m &\approx 10^{-5} \text{ g} \\ d &\approx 100 \mu\text{m} \\ \sigma &\approx 1 \text{ nm} \end{aligned}$$

ENTANGLED STATE

Nonclassical  
channel  
(LOCC theorem)

PRODUCT STATE



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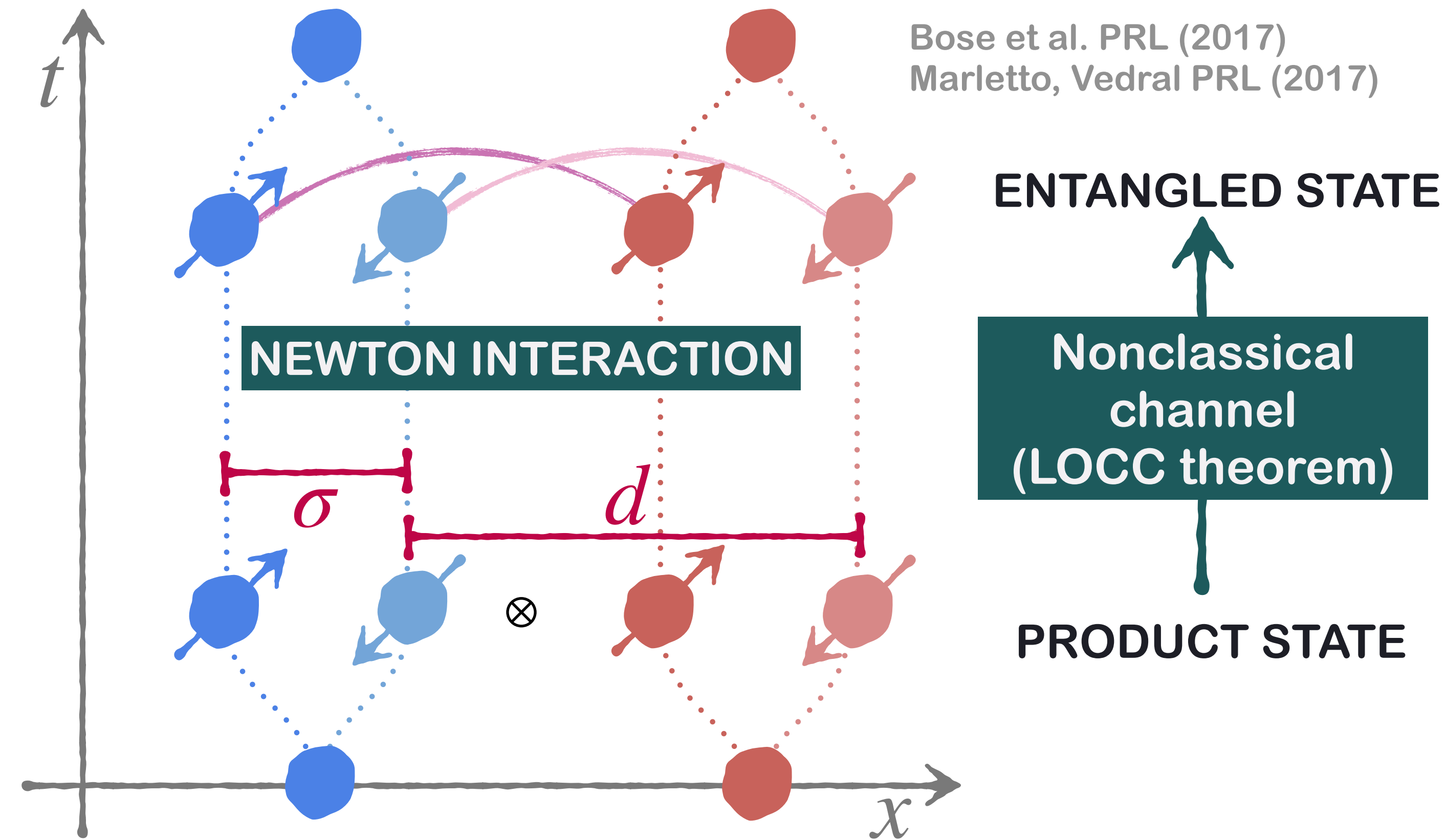
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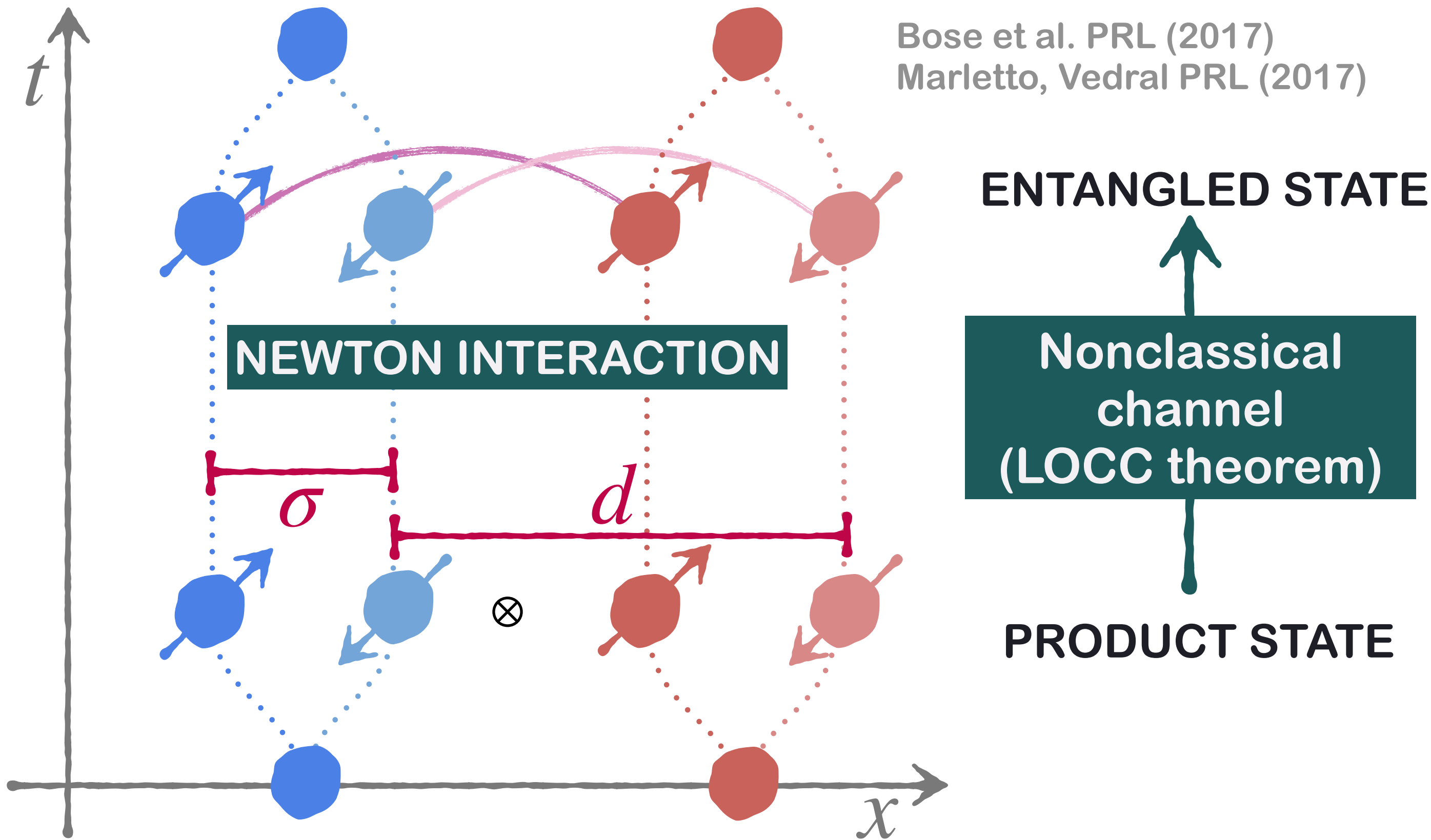
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$$\tau_{ent} = \Gamma_{ent}^{-1} \approx 0.1 \text{ s}$$



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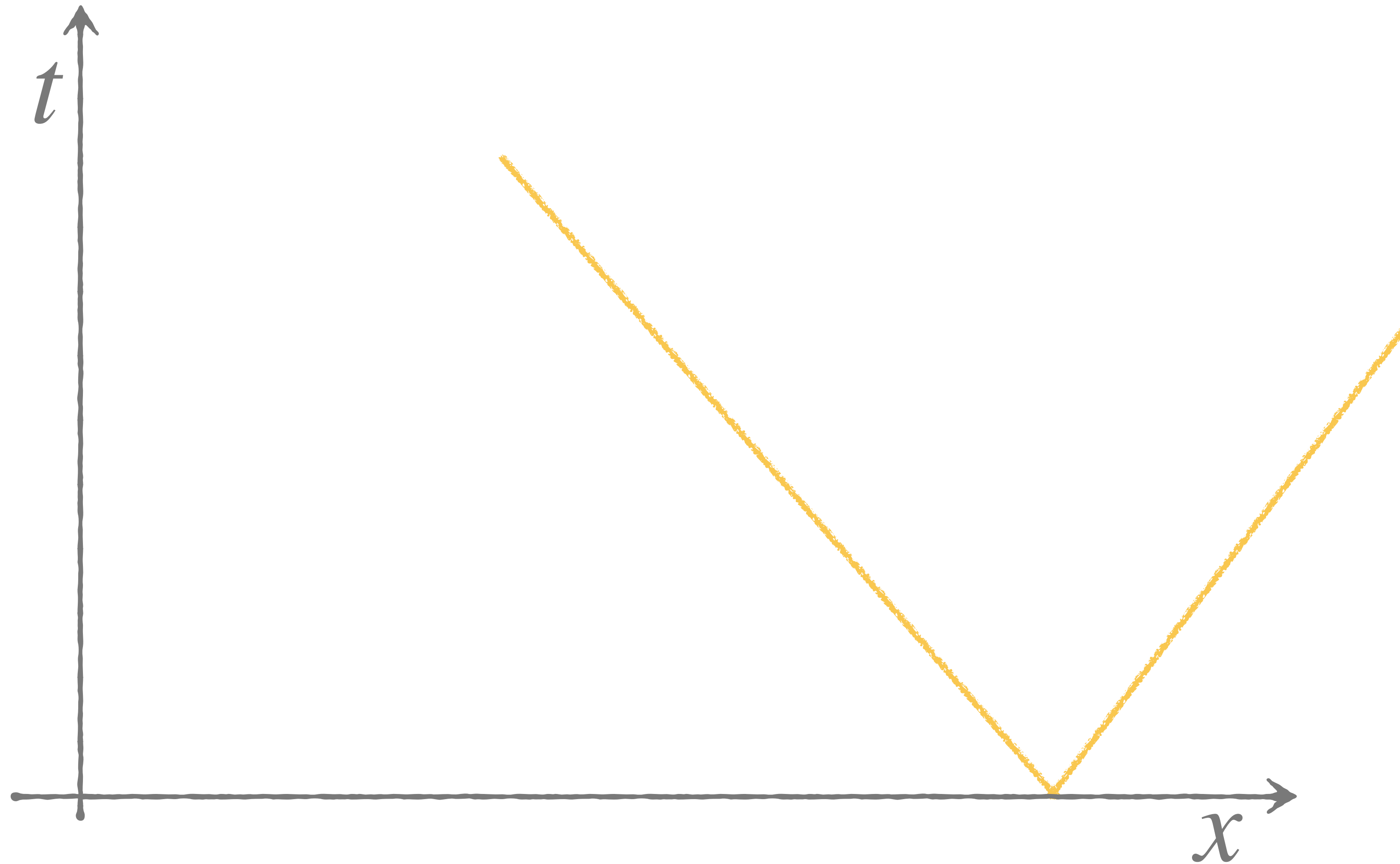
**QUESTION/OBJECTION:**  
You only used the Newton potential  
to generate entanglement!



# NEWTON POTENTIAL AS QUANTUM INFORMATION CARRIER

Along the lines of Baym, Ozawa (2009)  
and Mari, De Palma, Giovannetti (2016)  
For QED

Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner, Aspelmeyer, PRD (2018)  
2019 **First prize Essay** of the Gravity Research Foundation

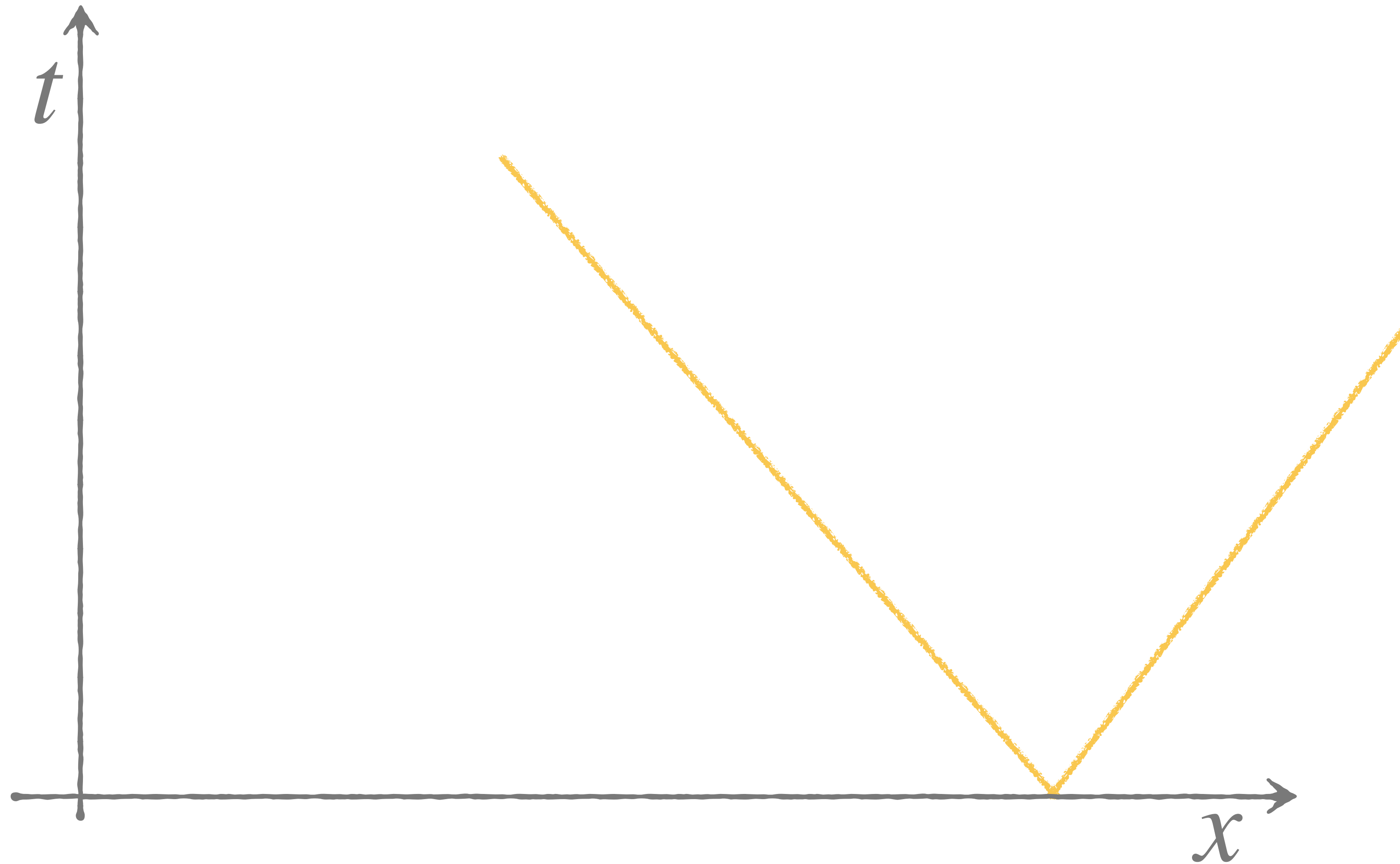


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G is a field (GR)  
No interaction at a distance  
Linearized quantum gravity

Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner, Aspelmeyer, PRD (2018)  
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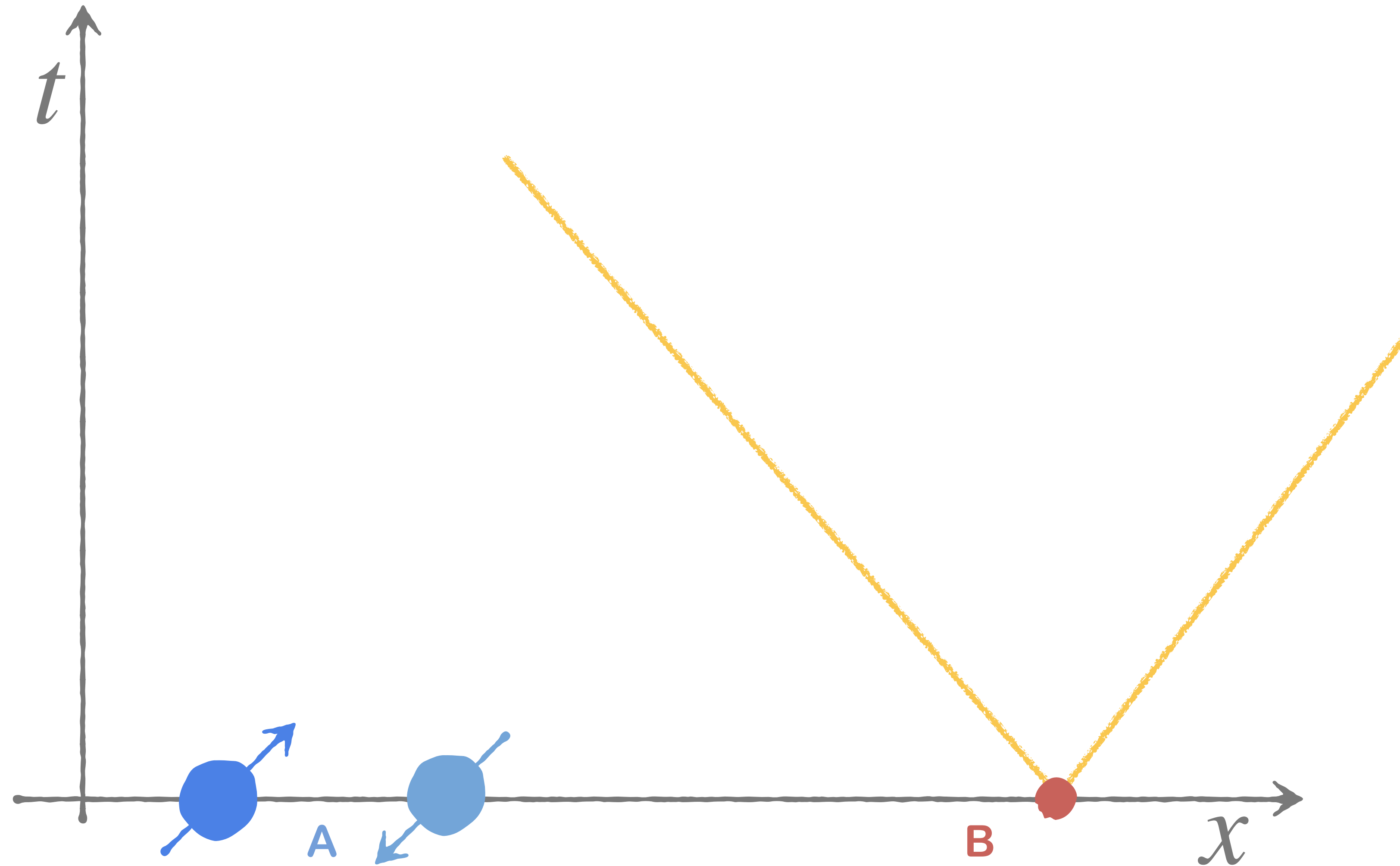
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$$\frac{1}{\sqrt{2}} (|L\rangle_A |\alpha_L\rangle_G + |R\rangle_A |\alpha_R\rangle_G) |\psi_0\rangle_B$$



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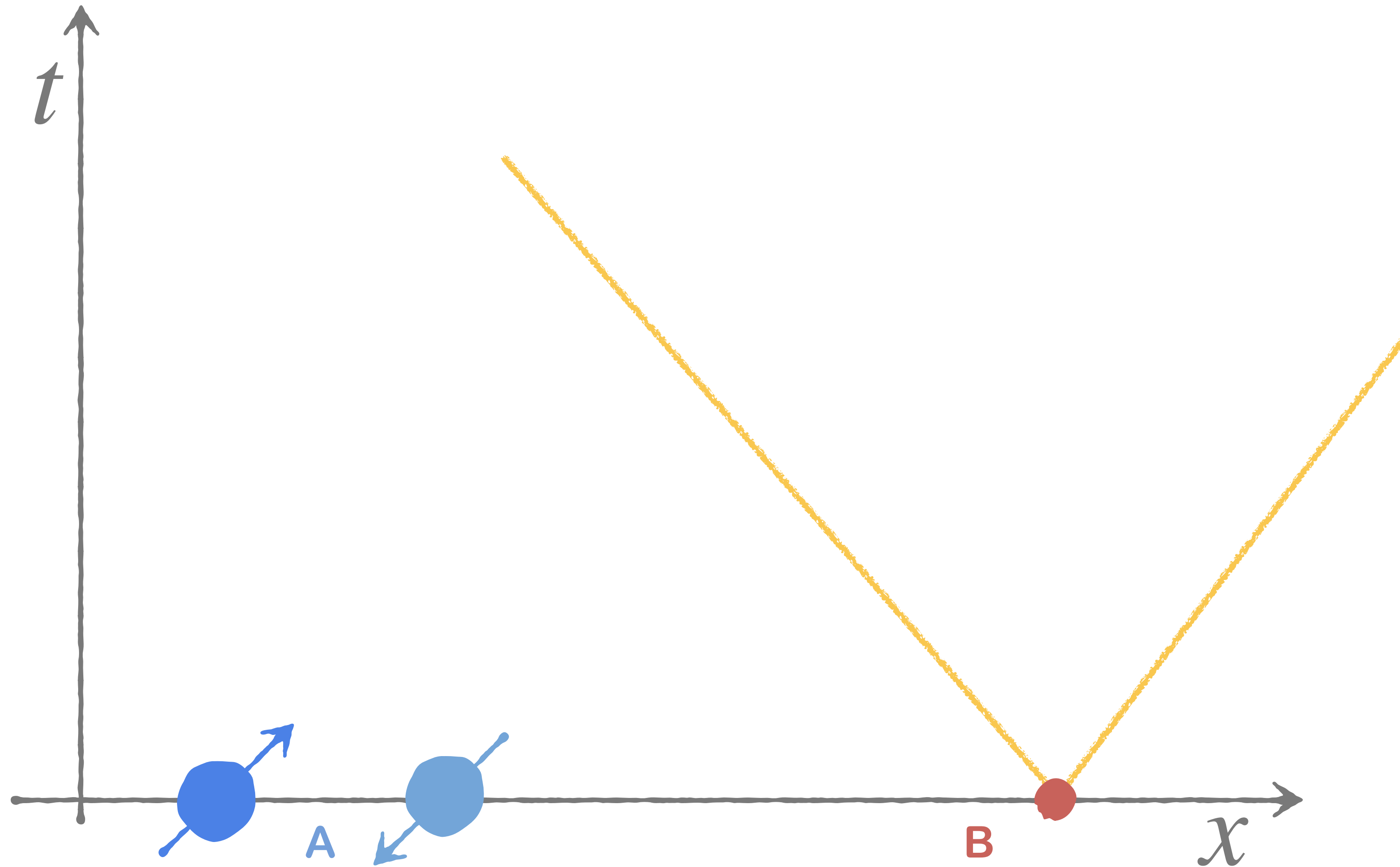
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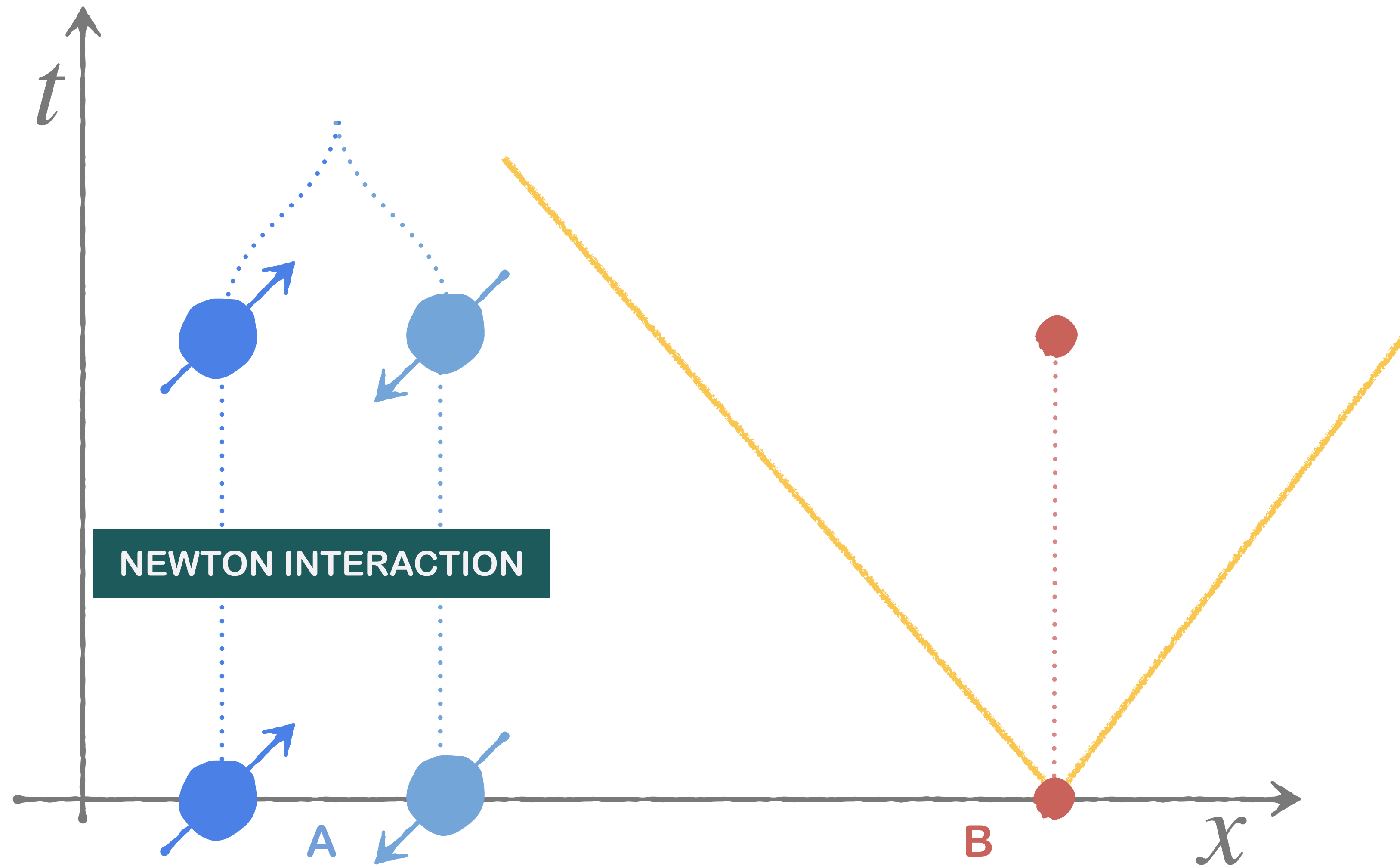
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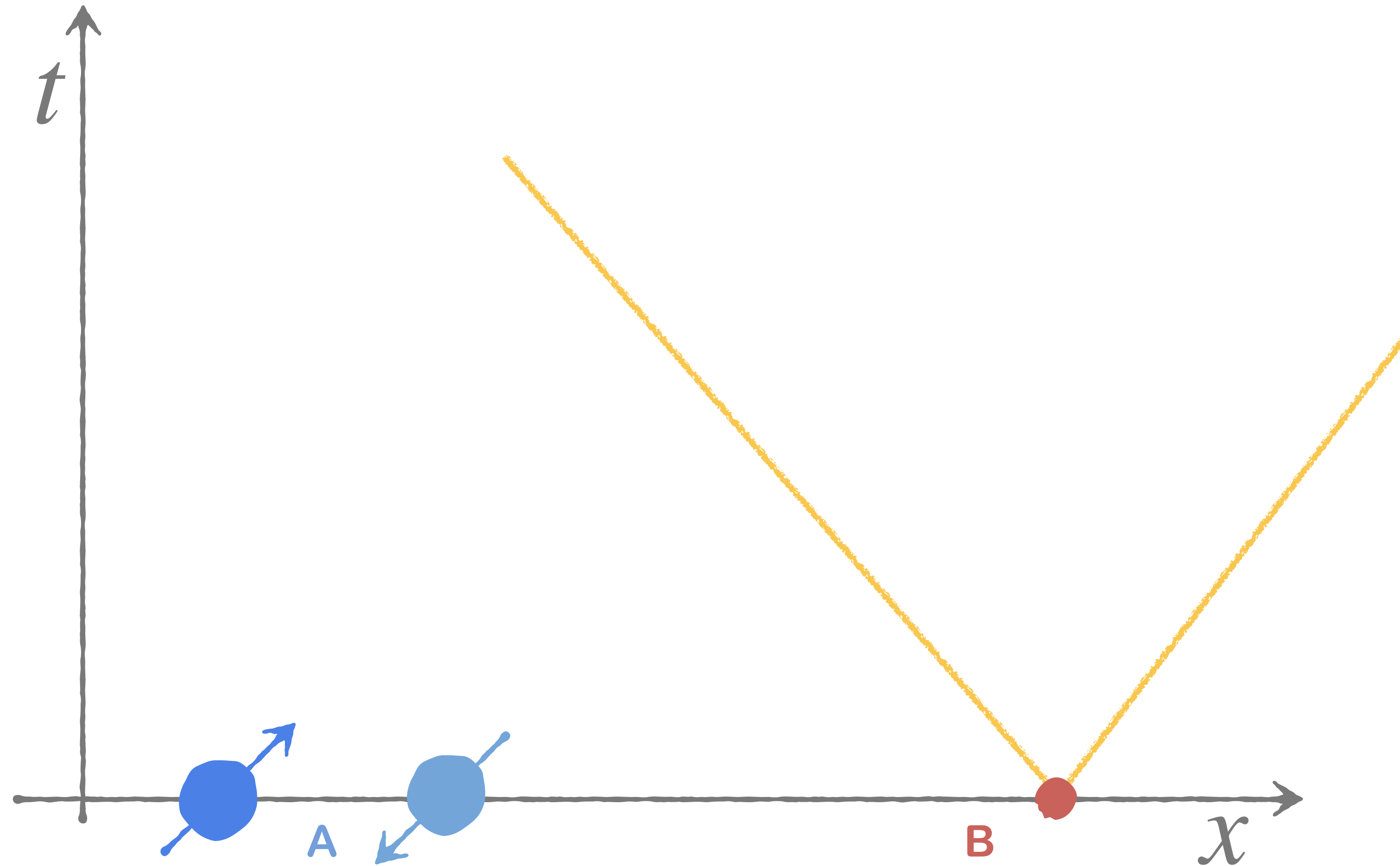
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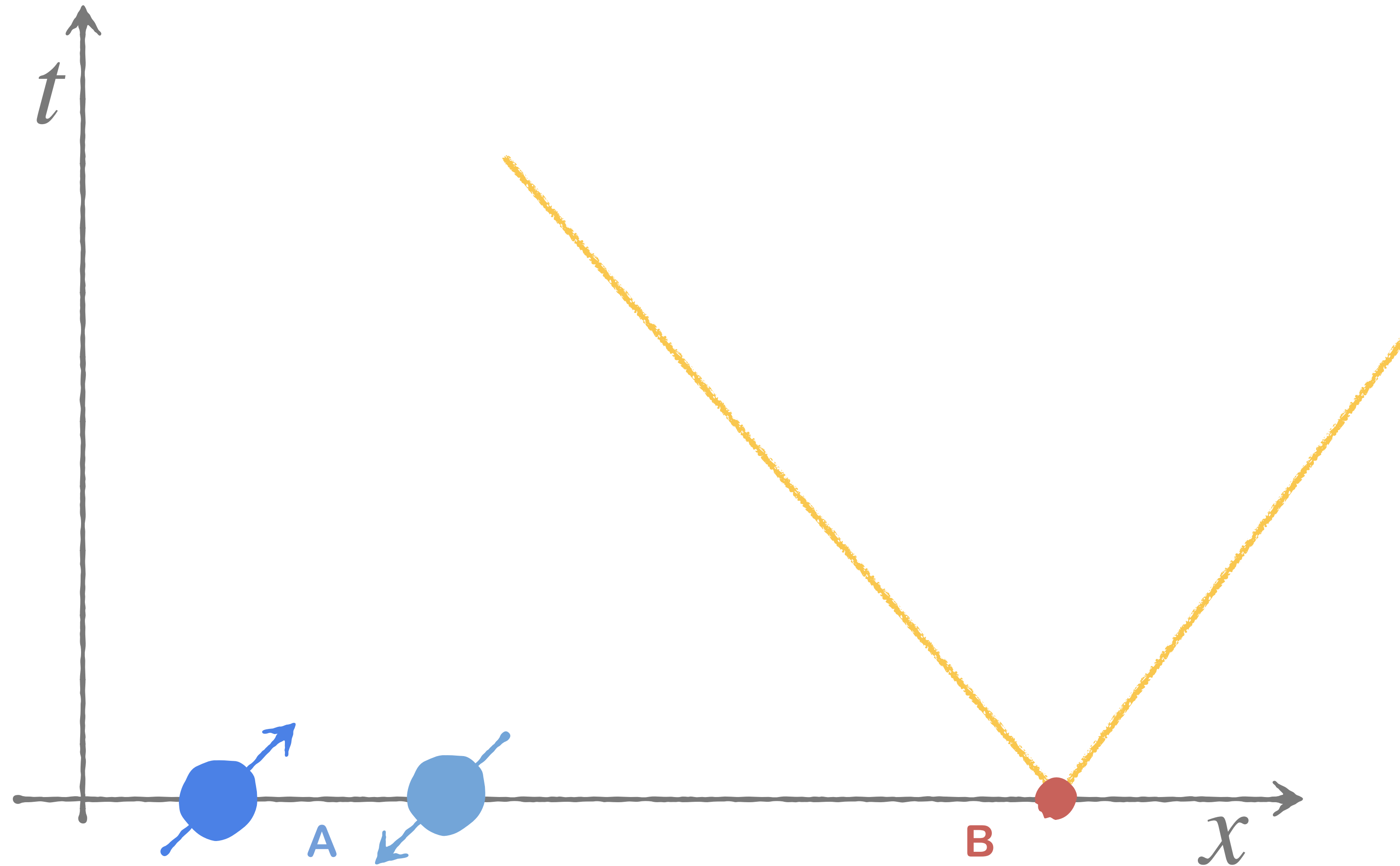
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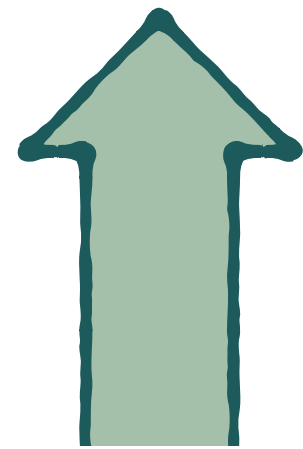
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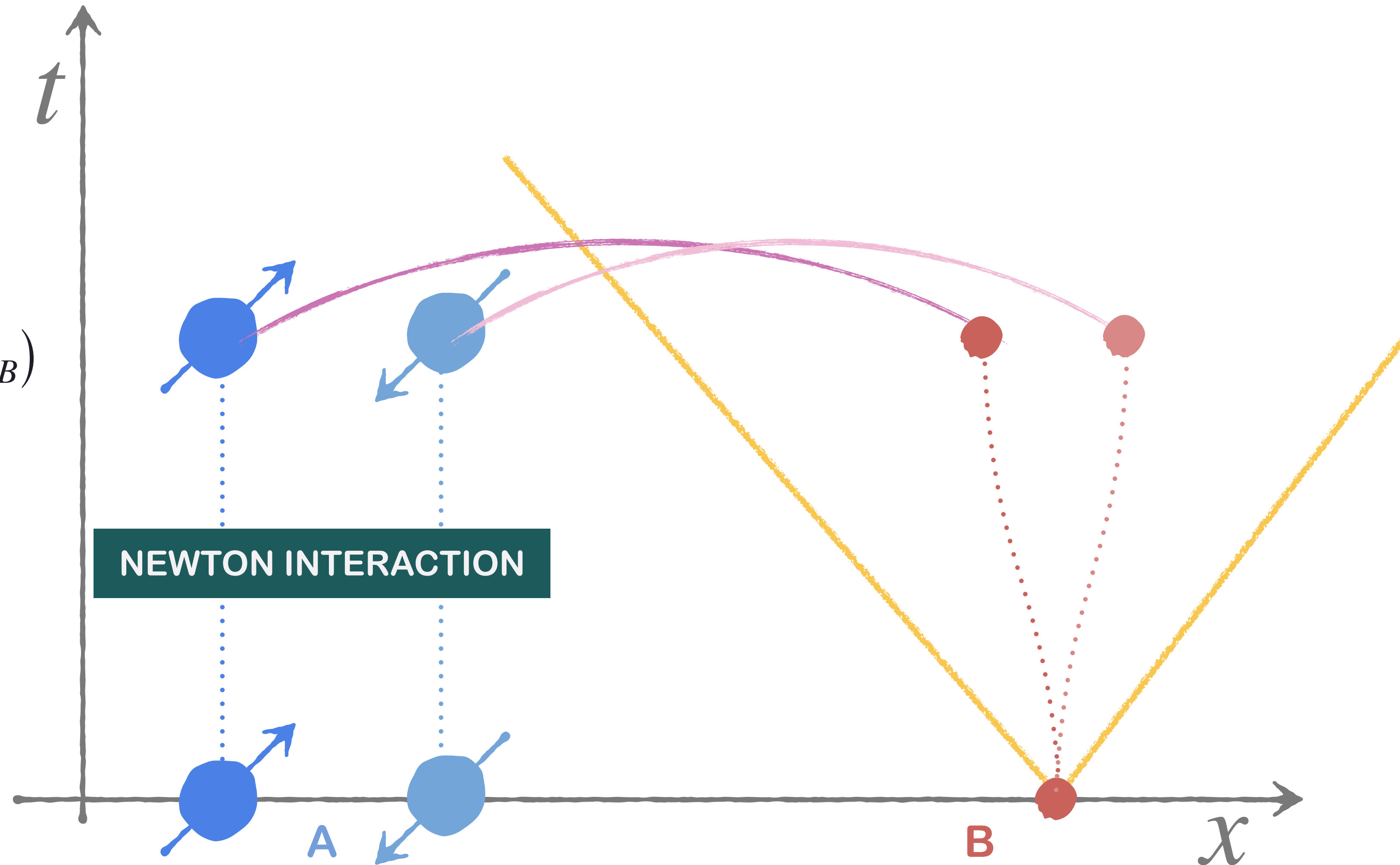
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B releases the trap

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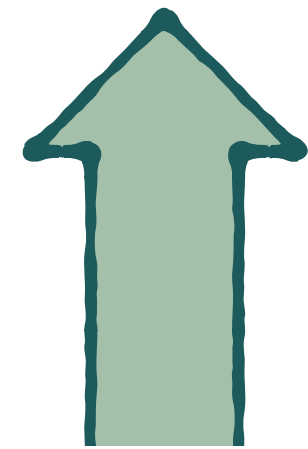


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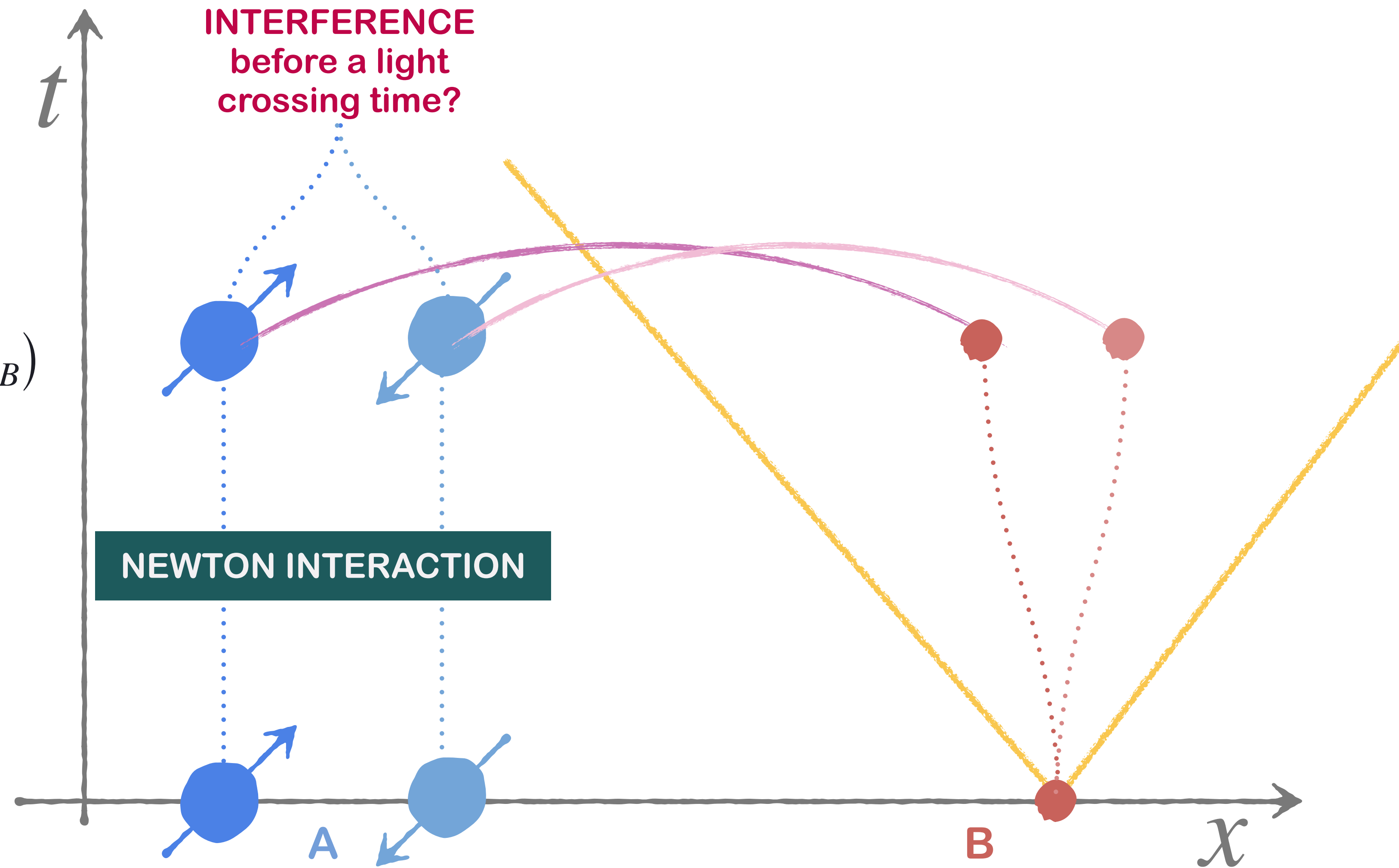
$$\frac{1}{\sqrt{2}} (|L\rangle_A |\alpha_L\rangle_G |L\rangle_B + |R\rangle_A |\alpha_R\rangle_G |R\rangle_B)$$



B releases the trap

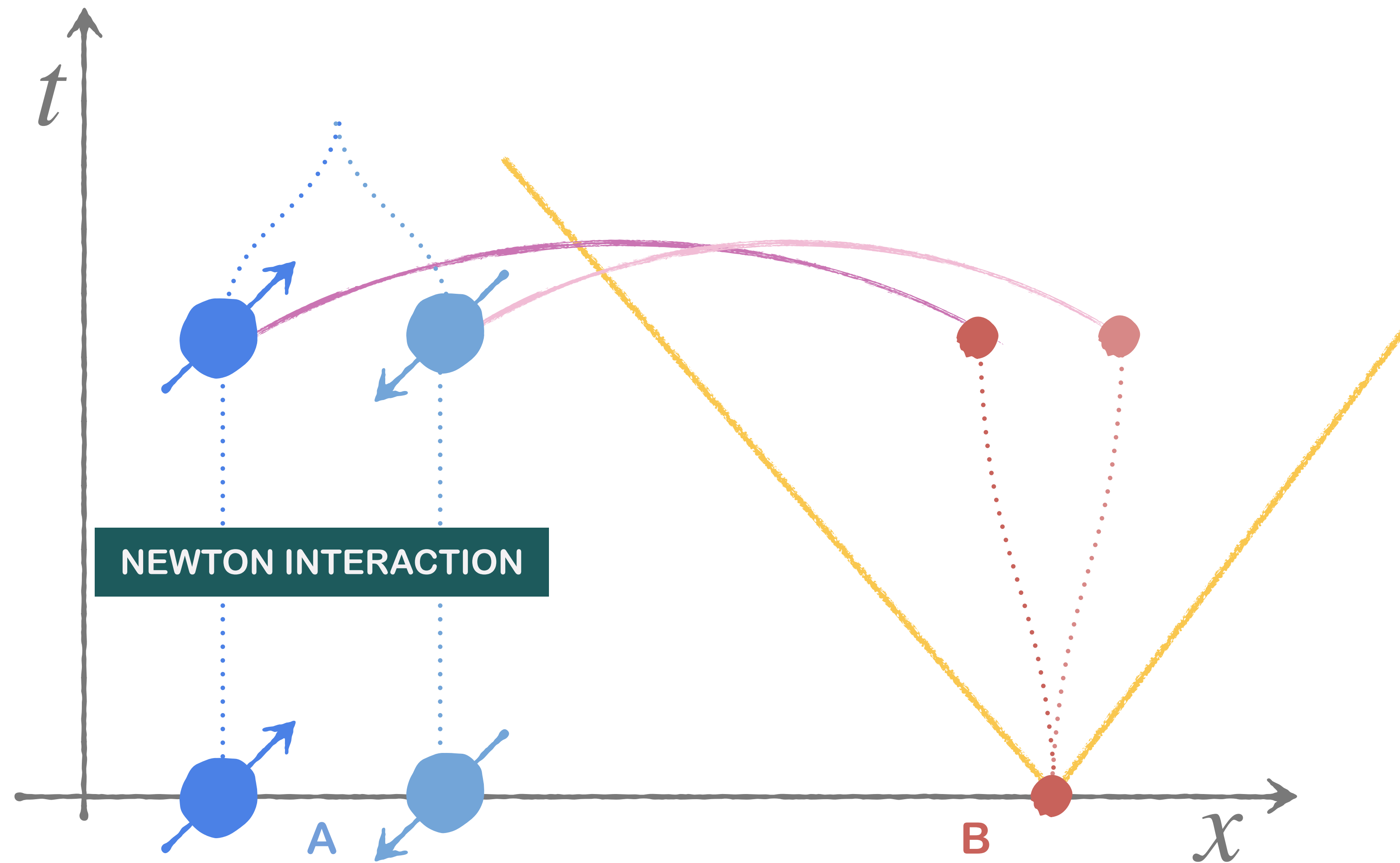
$$\frac{1}{\sqrt{2}} (|L\rangle_A |\alpha_L\rangle_G + |R\rangle_A |\alpha_R\rangle_G) |\psi_0\rangle_B$$

Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner, Aspelmeyer, PRD (2018)  
2019 **First prize Essay** of the Gravity Research Foundation



# NEWTON POTENTIAL AS QUANTUM INFORMATION CARRIER

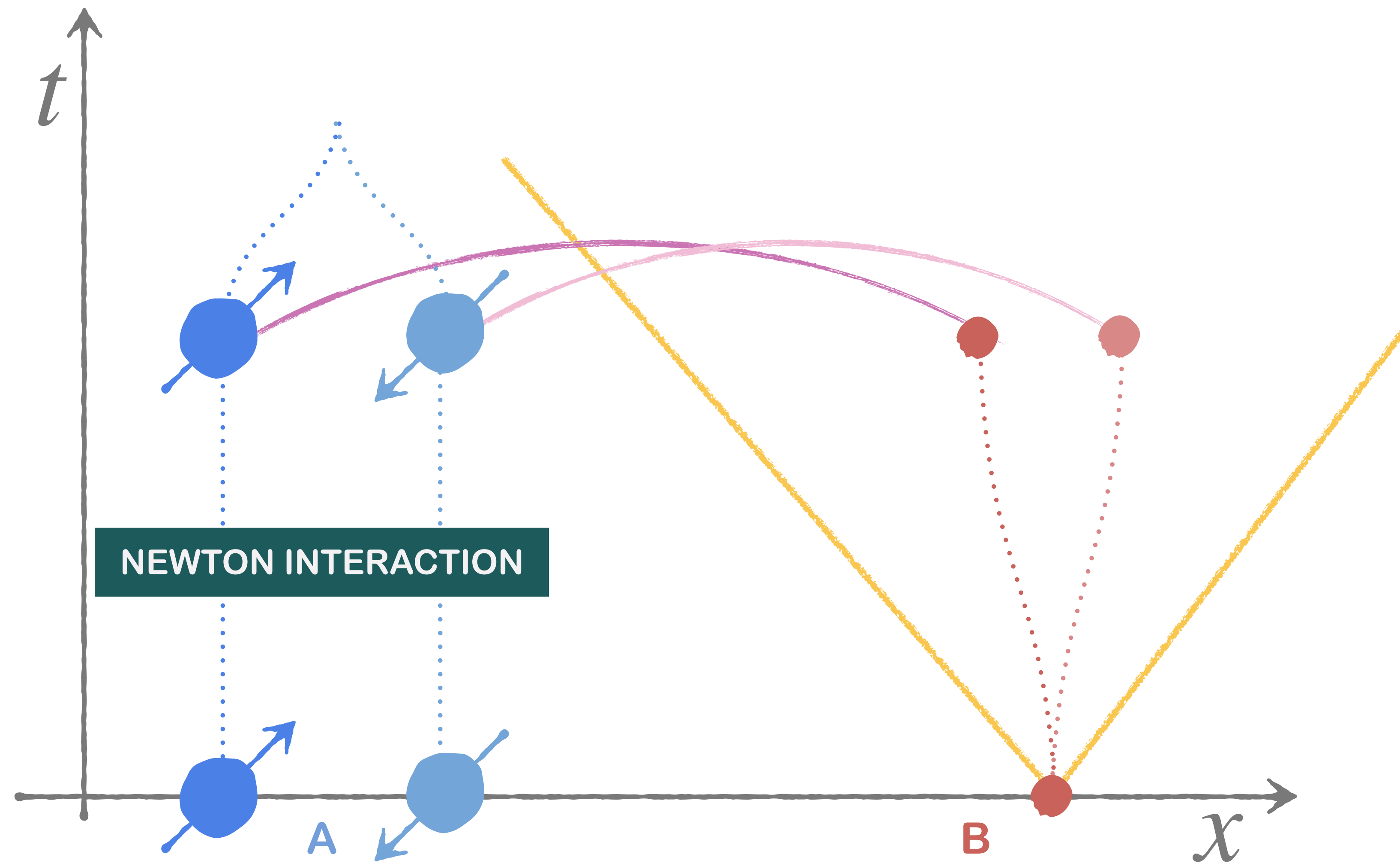
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## TAKE-HOME MESSAGE



# NEWTON POTENTIAL AS QUANTUM INFORMATION CARRIER

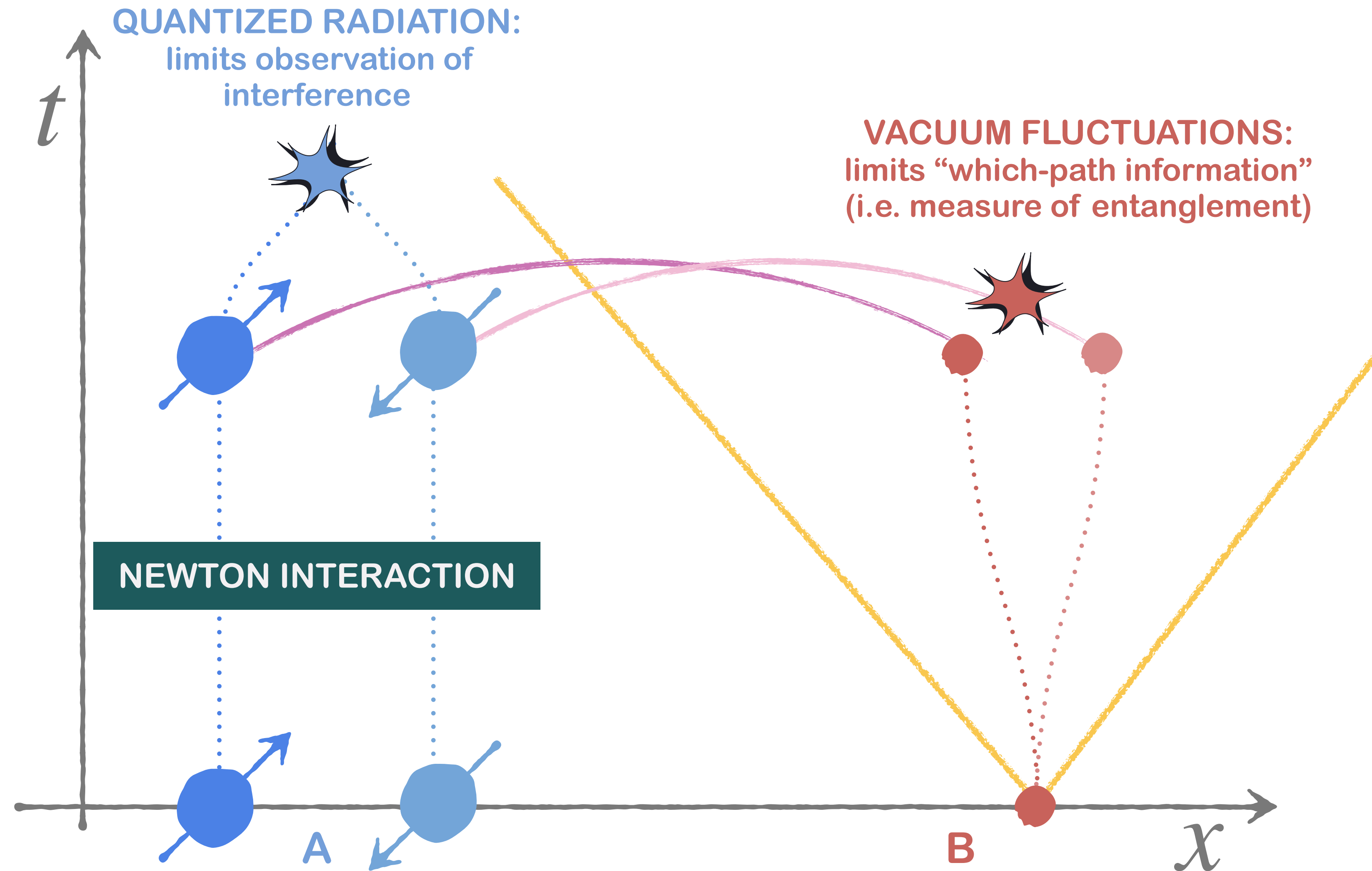
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Quantum properties of the gravitational field

**QUANTIZED RADIATION**  
**VACUUM FLUCTUATIONS**

are essential to obtain a consistent description of the experiment



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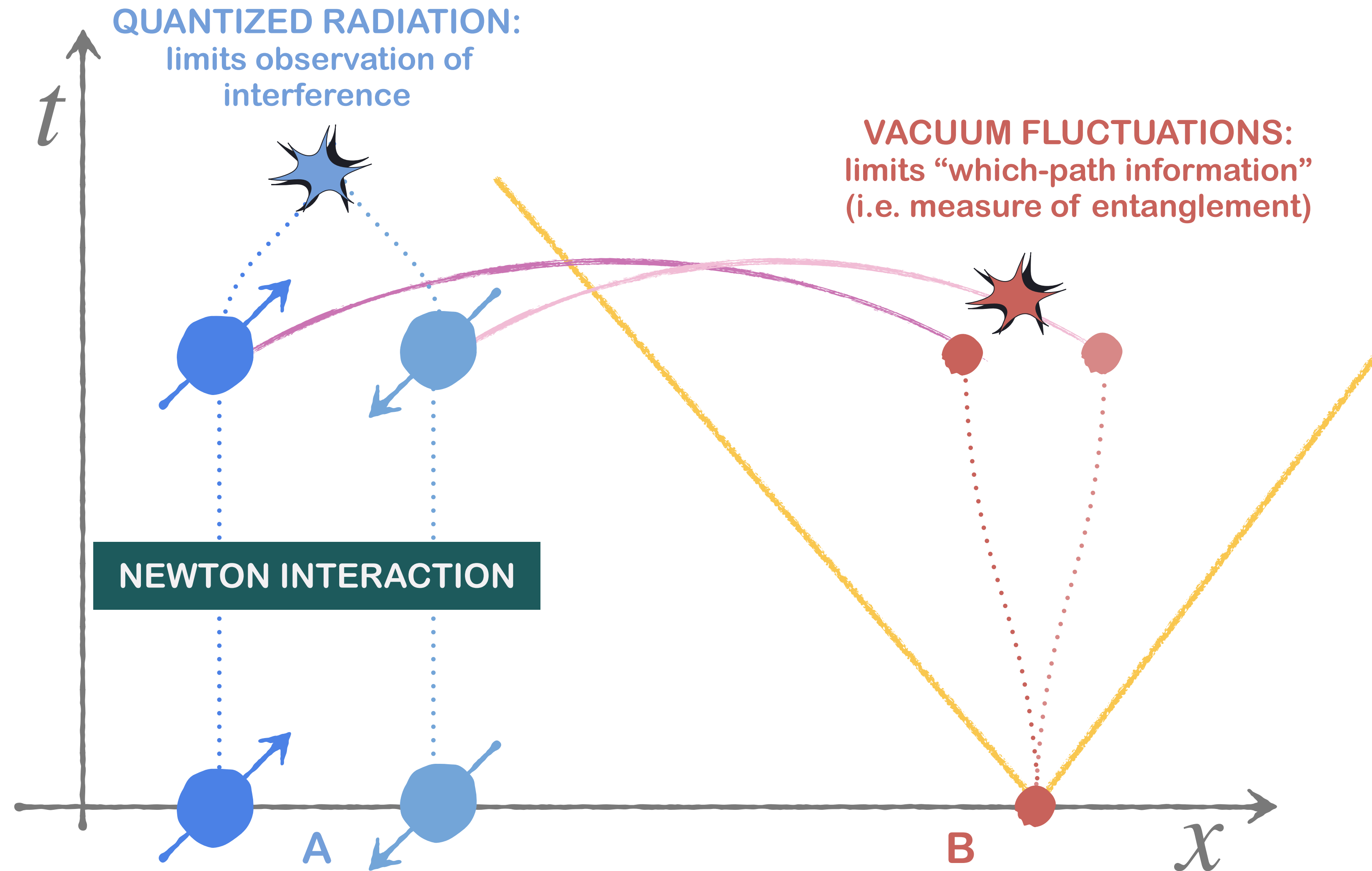
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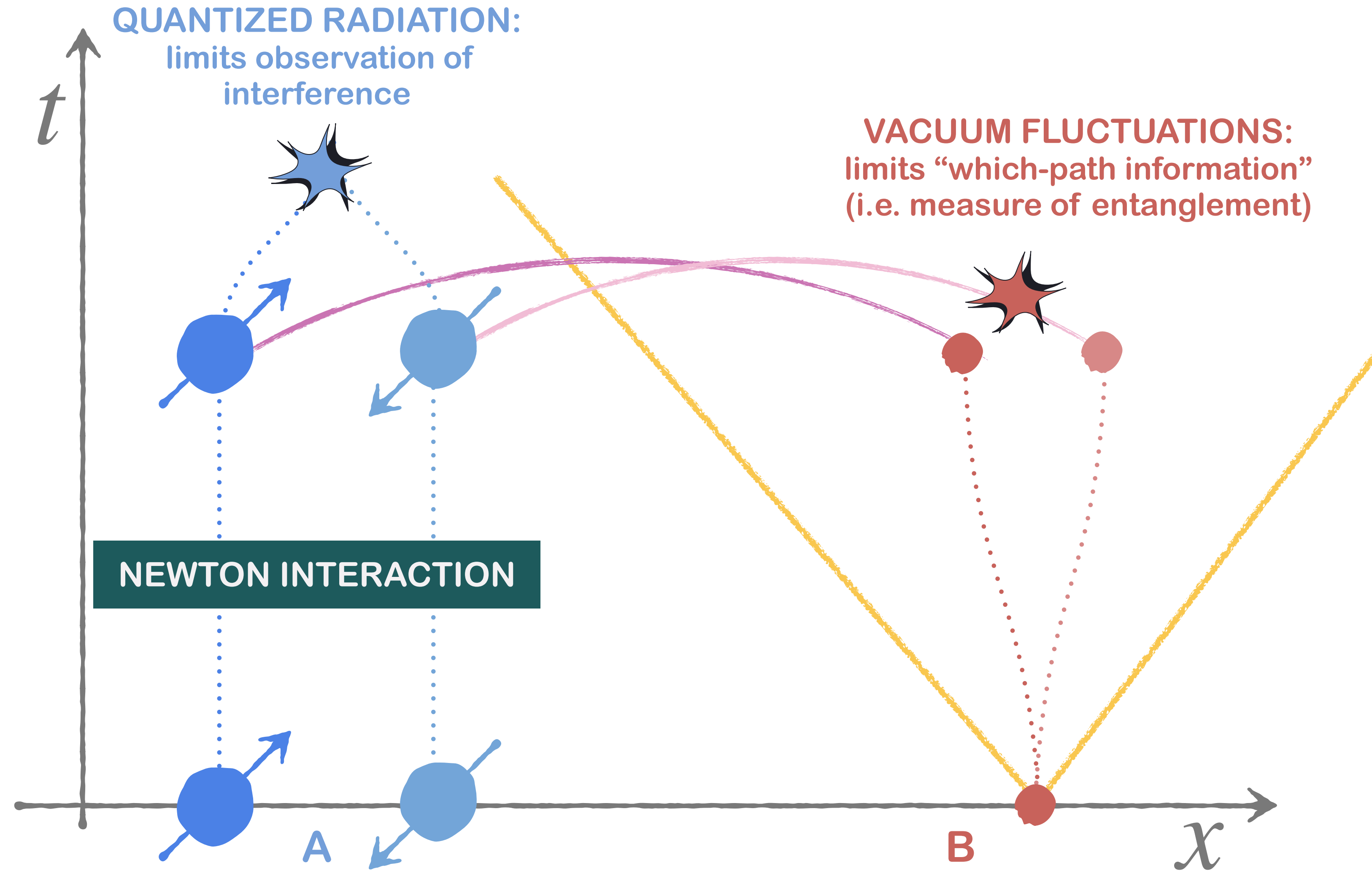
**QUANTIZED RADIATION**  
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are essential to obtain a consistent description of the experiment

**ARGUMENT:**  
Newtonian potential has a  
quantum information content

If instead we want to keep a classical description of gravity, we need to drastically modify our basic principles.

See also Danielson,  
Satishchandran, Wald PRD (2022)



# IMPLICATIONS OF ENTANGLEMENT VIA THE NEWTON POTENTIAL

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# IMPLICATIONS OF ENTANGLEMENT VIA THE NEWTON POTENTIAL

EMBEDDING INTO GR

GENERAL  
RELATIVITY

+

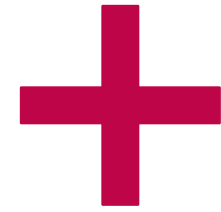
QUANTUM  
MECHANICS



# IMPLICATIONS OF ENTANGLEMENT VIA THE NEWTON POTENTIAL

## EMBEDDING INTO GR

GENERAL  
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QUANTUM  
MECHANICS

1. Classical GR + QM does not generate entanglement  
**THEORY-INDEPENDENT NO-GO THEOREM** ✓

Galley, **F.G.**, Selby Quantum (2022)

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## EXPERIMENT

GENERAL  
RELATIVITY

+

QUANTUM  
MECHANICS

limit to

NEWTON  
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limit to

NEWTON  
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Newton potential is compatible with the  
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limit to

NEWTON POTENTIAL

+

QUANTUM MECHANICS

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**NEW RESULT: MORE GENERAL EFFECT THAN NEWTON POTENTIAL IN TABLE-TOP EXPERIMENTS** ✓

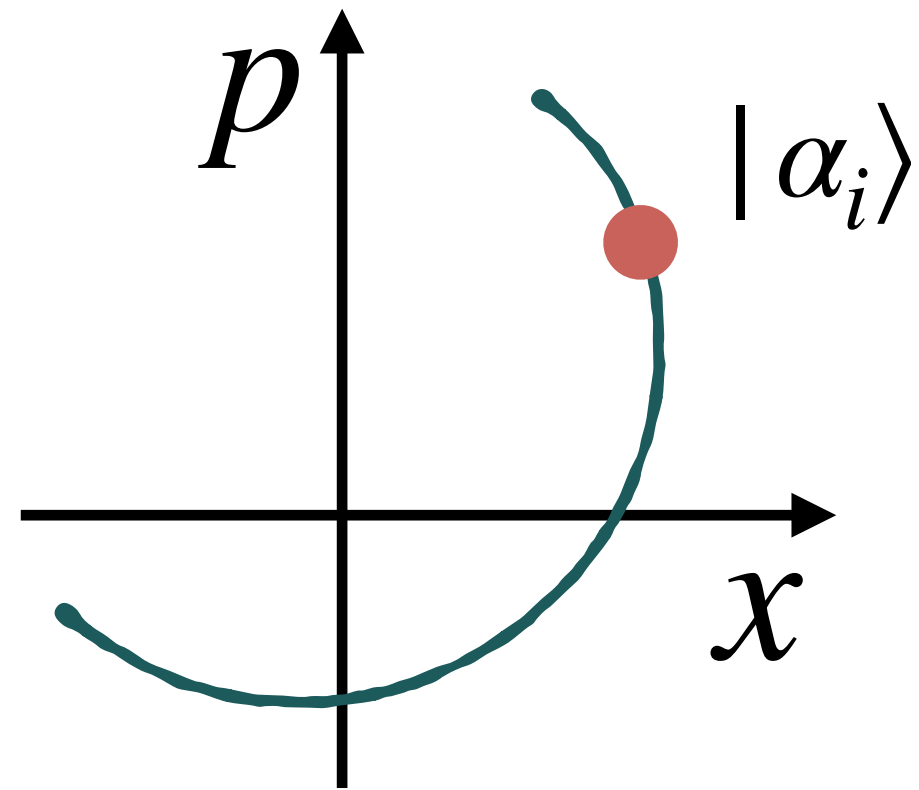
Chen, F.G., 2402.10288 (2024)

# THE INTUITIVE IDEA: GENERAL QUANTUM STATES OF THE SOURCE

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## SUPERPOSITION OF LOCALISED GAUSSIANS

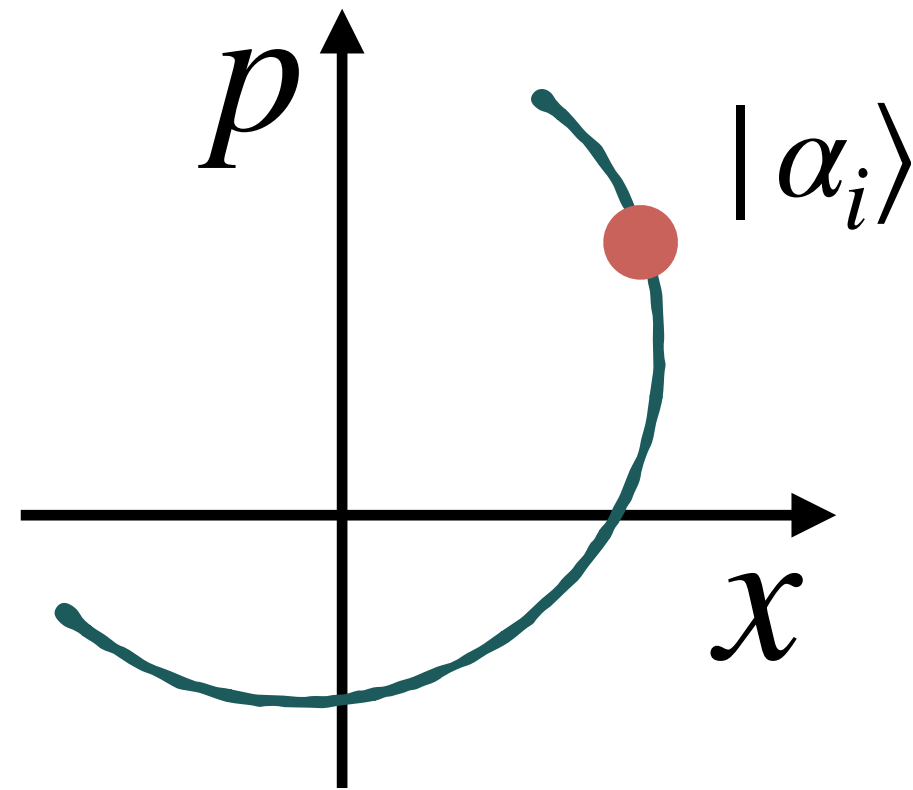
$$\frac{1}{\sqrt{2}} (|\alpha_1\rangle + |\alpha_2\rangle)$$



Arbitrary localisation  
in  $x$  and  $p$

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Arbitrary localisation  
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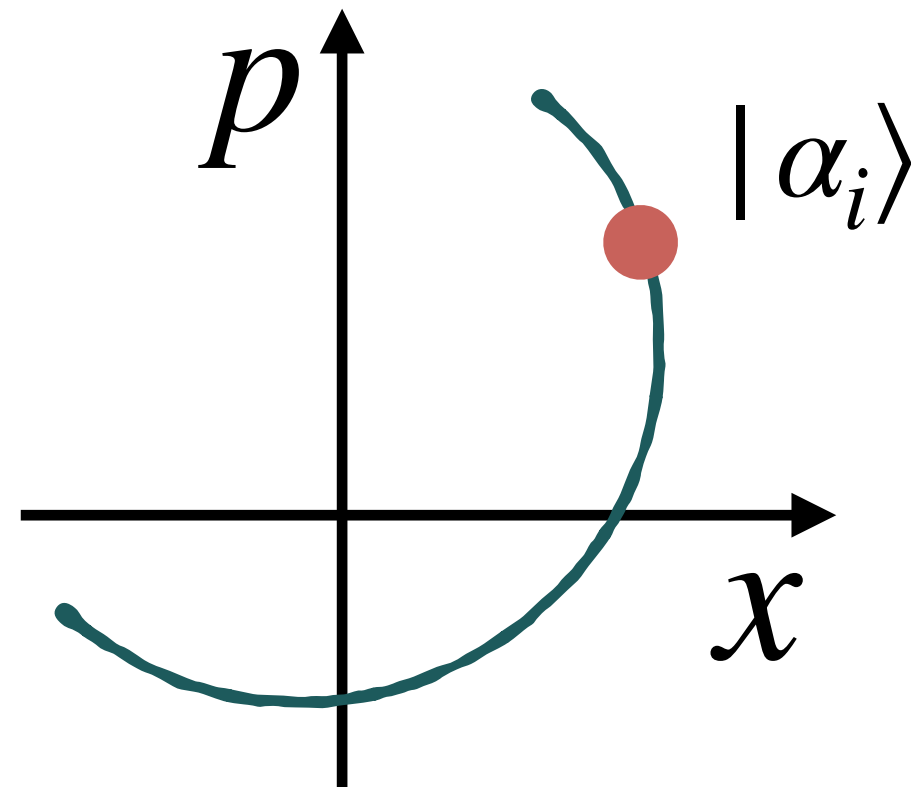
*Effectively a classical source*



# THE INTUITIVE IDEA: GENERAL QUANTUM STATES OF THE SOURCE

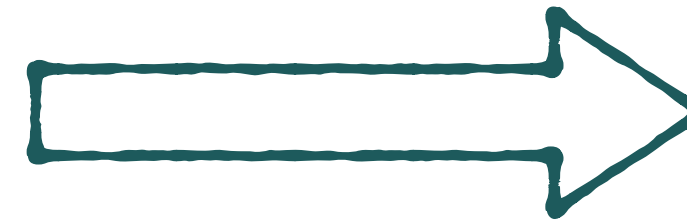
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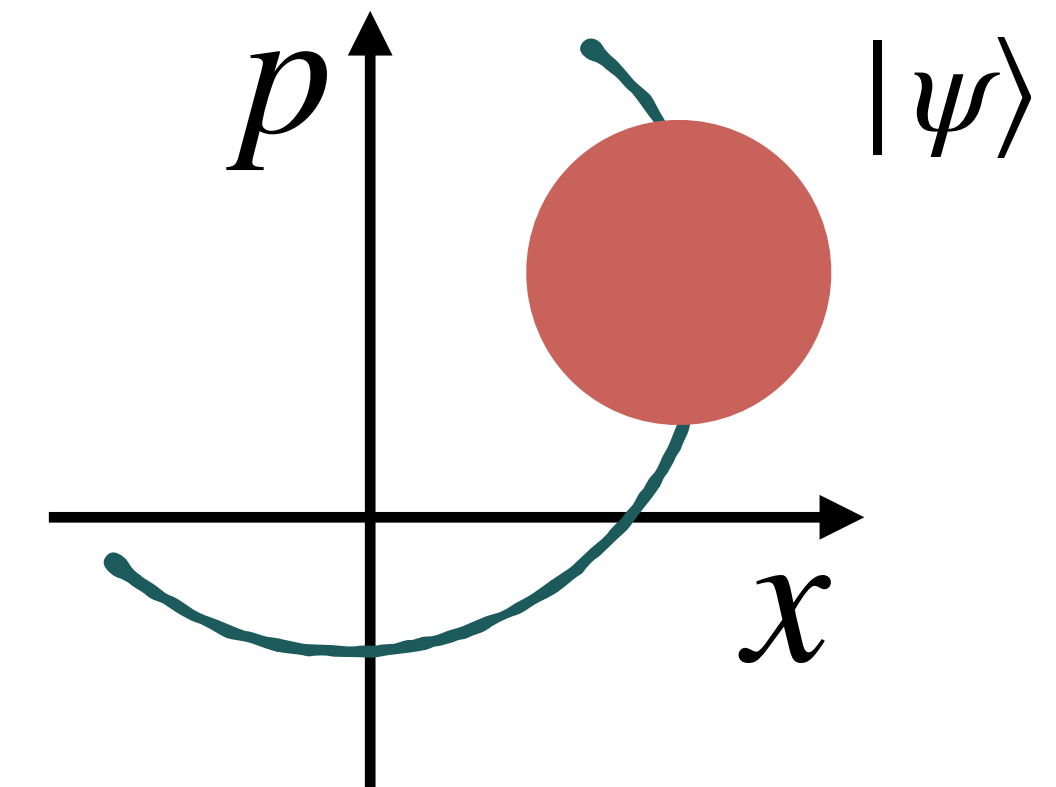


Arbitrary localisation  
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*Effectively a classical source*



## GENERAL QUANTUM STATE



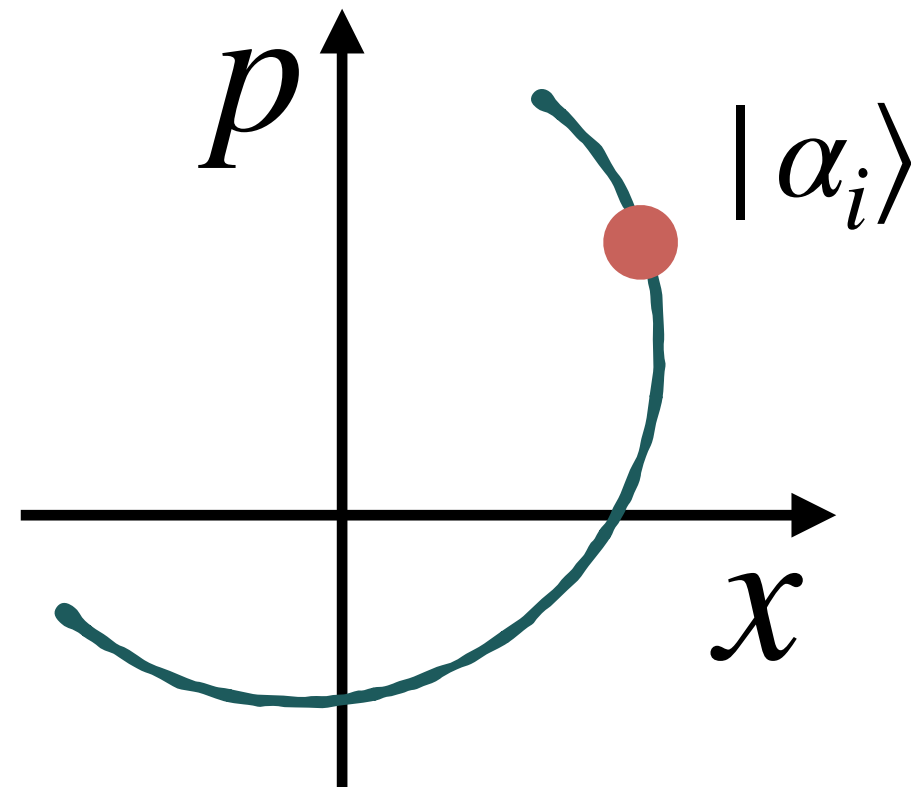
Delocalisation can be  
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$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

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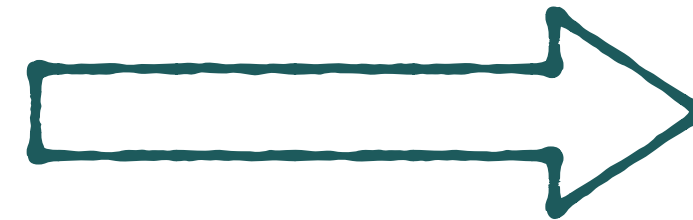
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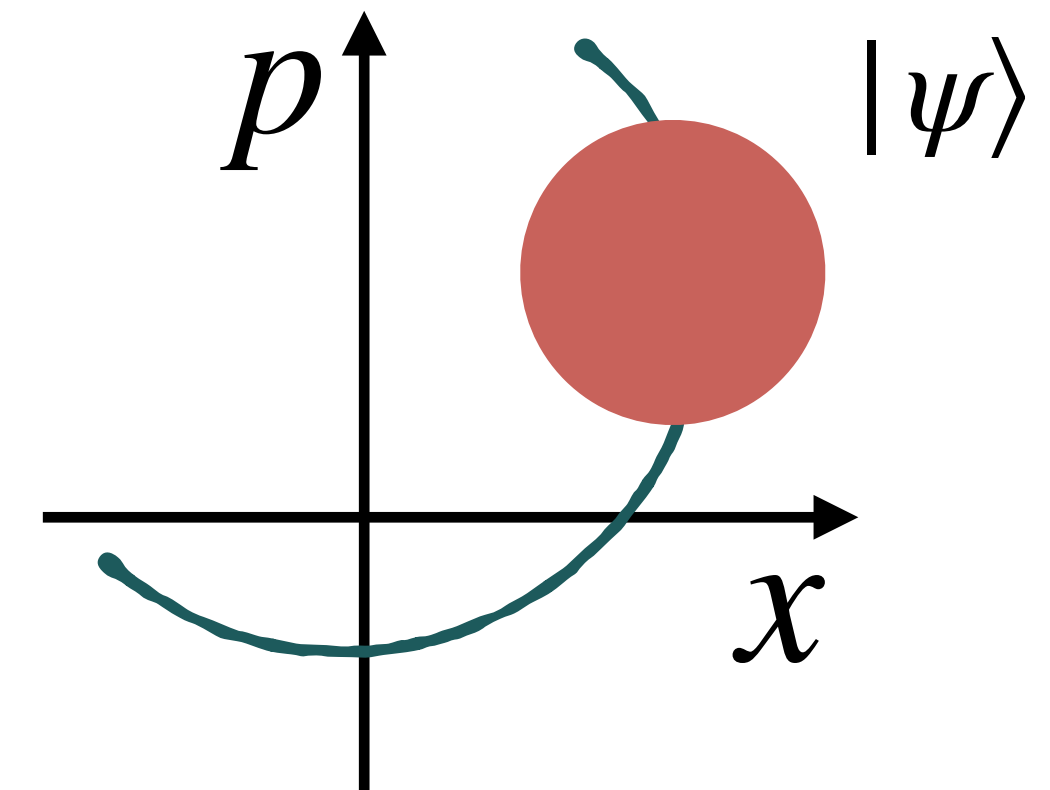


Arbitrary localisation  
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*Effectively a classical source*



## GENERAL QUANTUM STATE



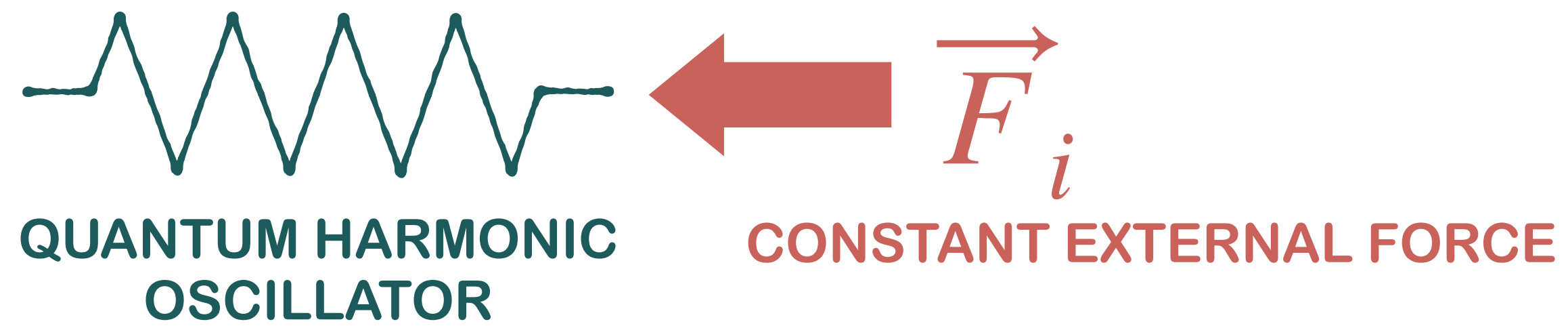
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*Not a classical source!*

# METHODOLOGY: ANALOGY WITH THE HARMONIC OSCILLATOR

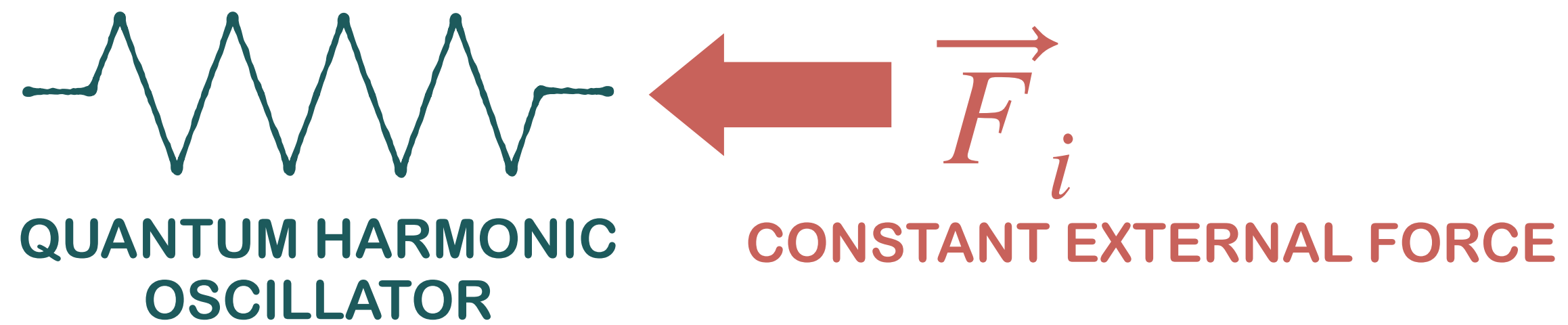
Chen, Giacomini, Rovelli Quantum (2023)



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 - m\gamma\hat{x}$$

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Chen, Giacomini, Rovelli Quantum (2023)



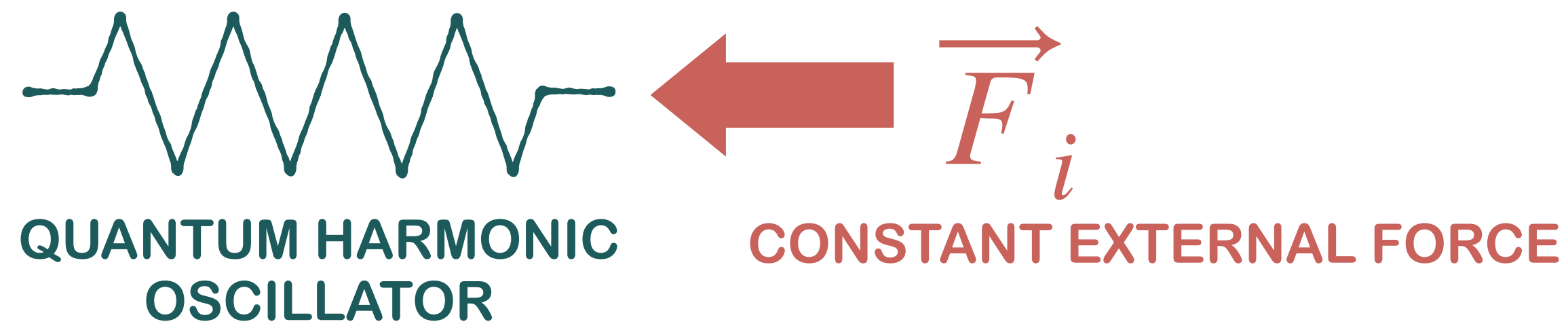
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Particular solution of classical EoM

$$x_\gamma = \frac{\gamma}{\omega^2}$$

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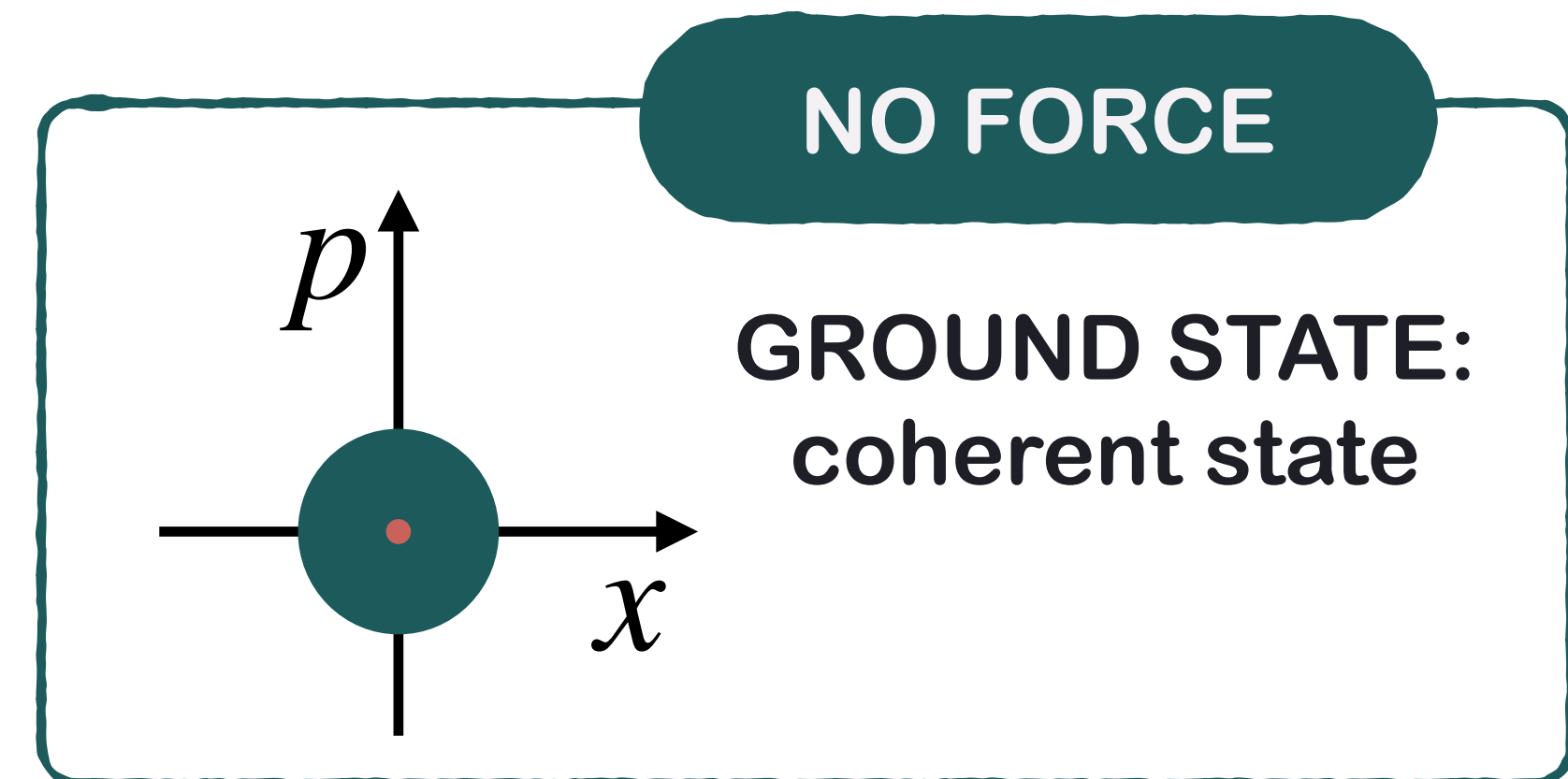
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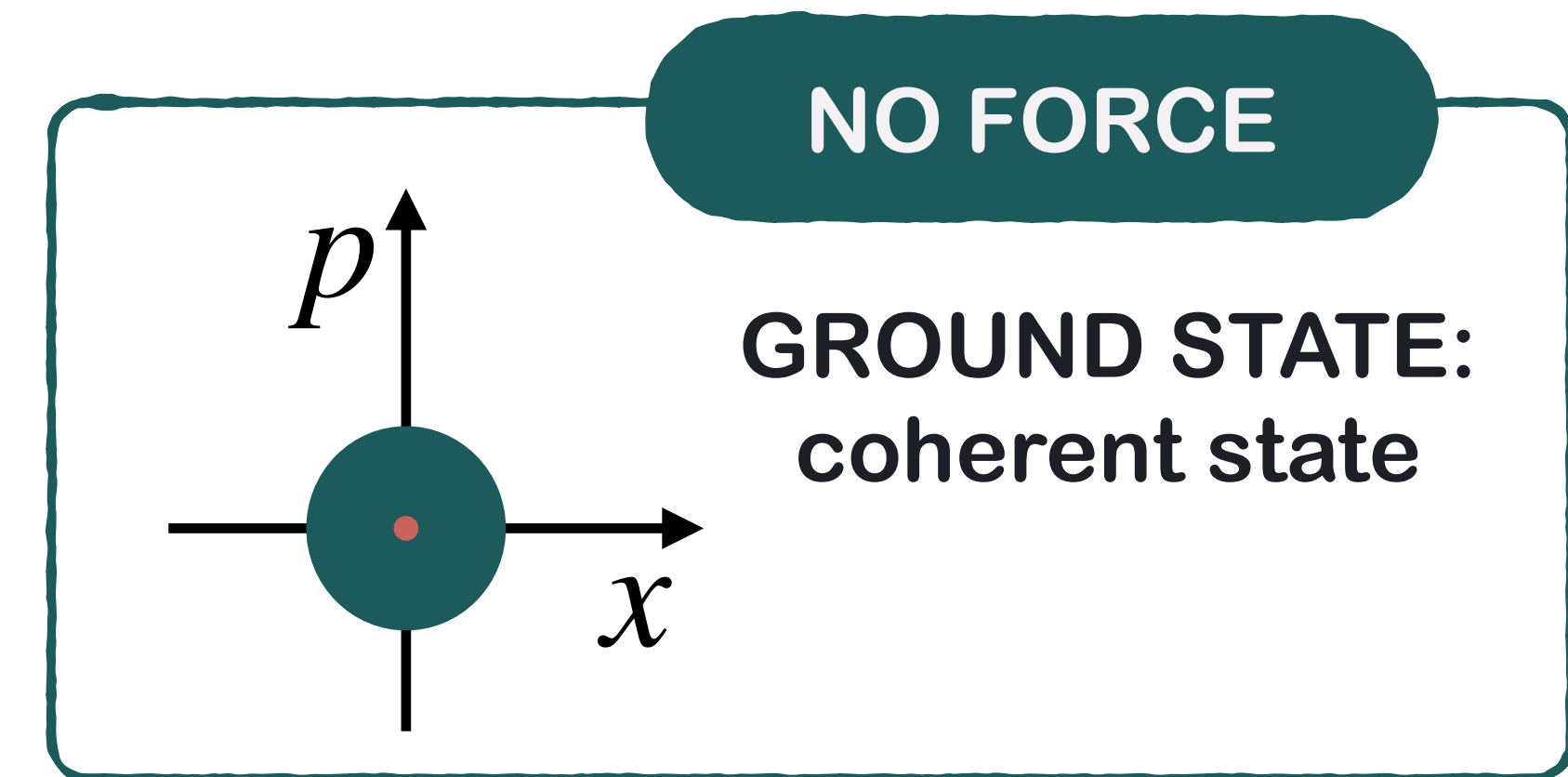
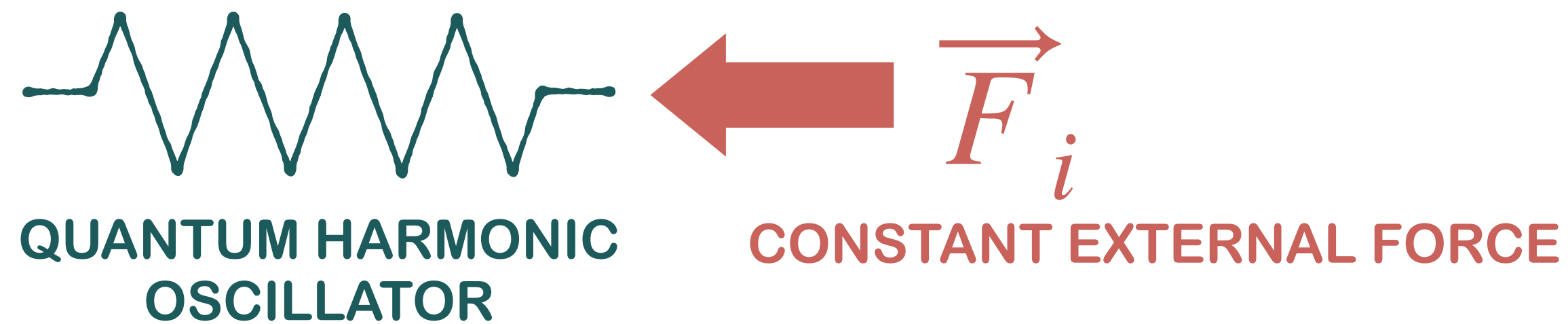
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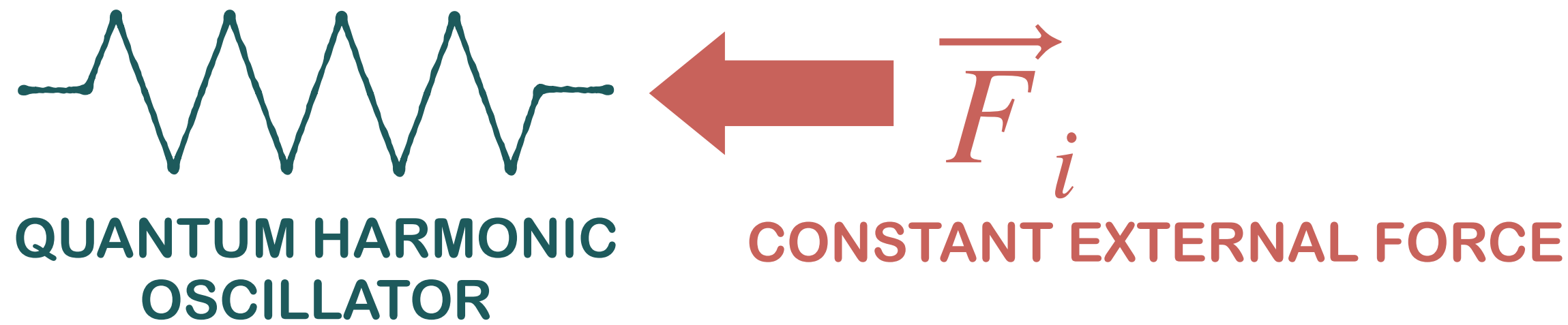
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Change of coordinates  $\hat{x}' = \hat{x} - x_\gamma$

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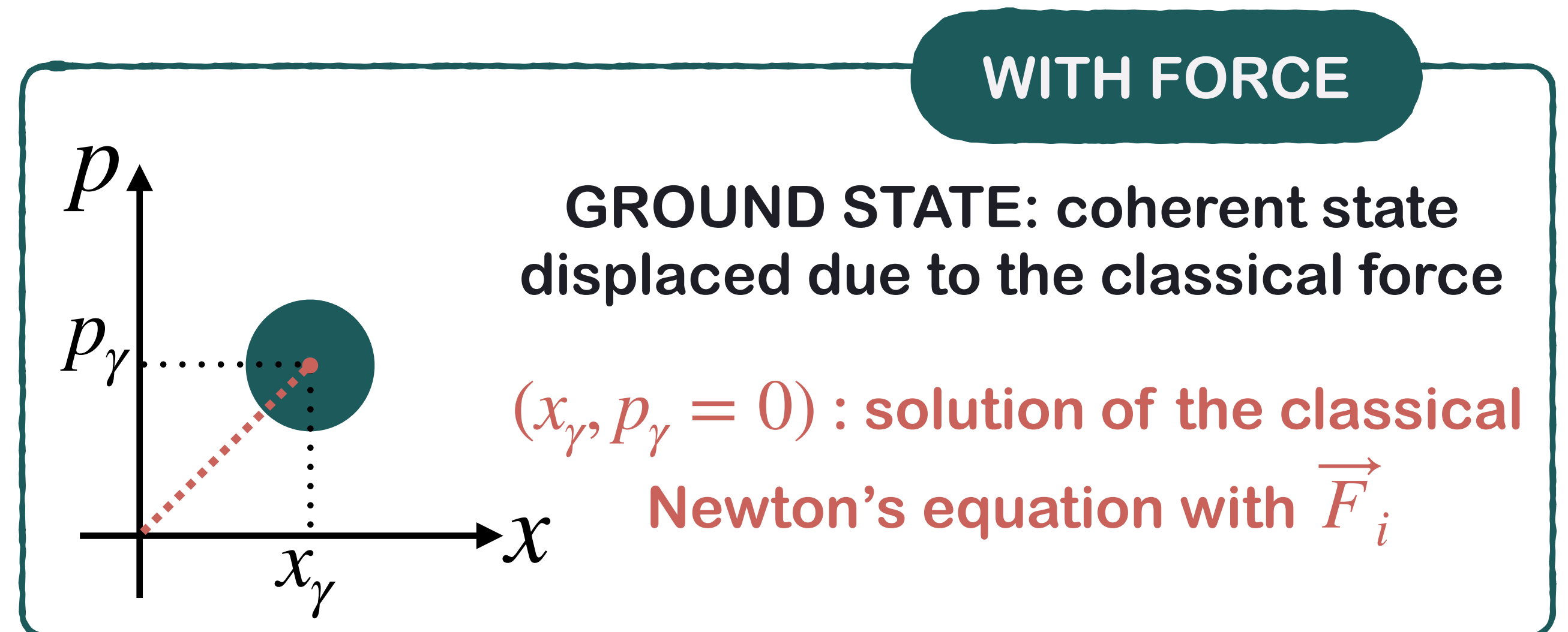
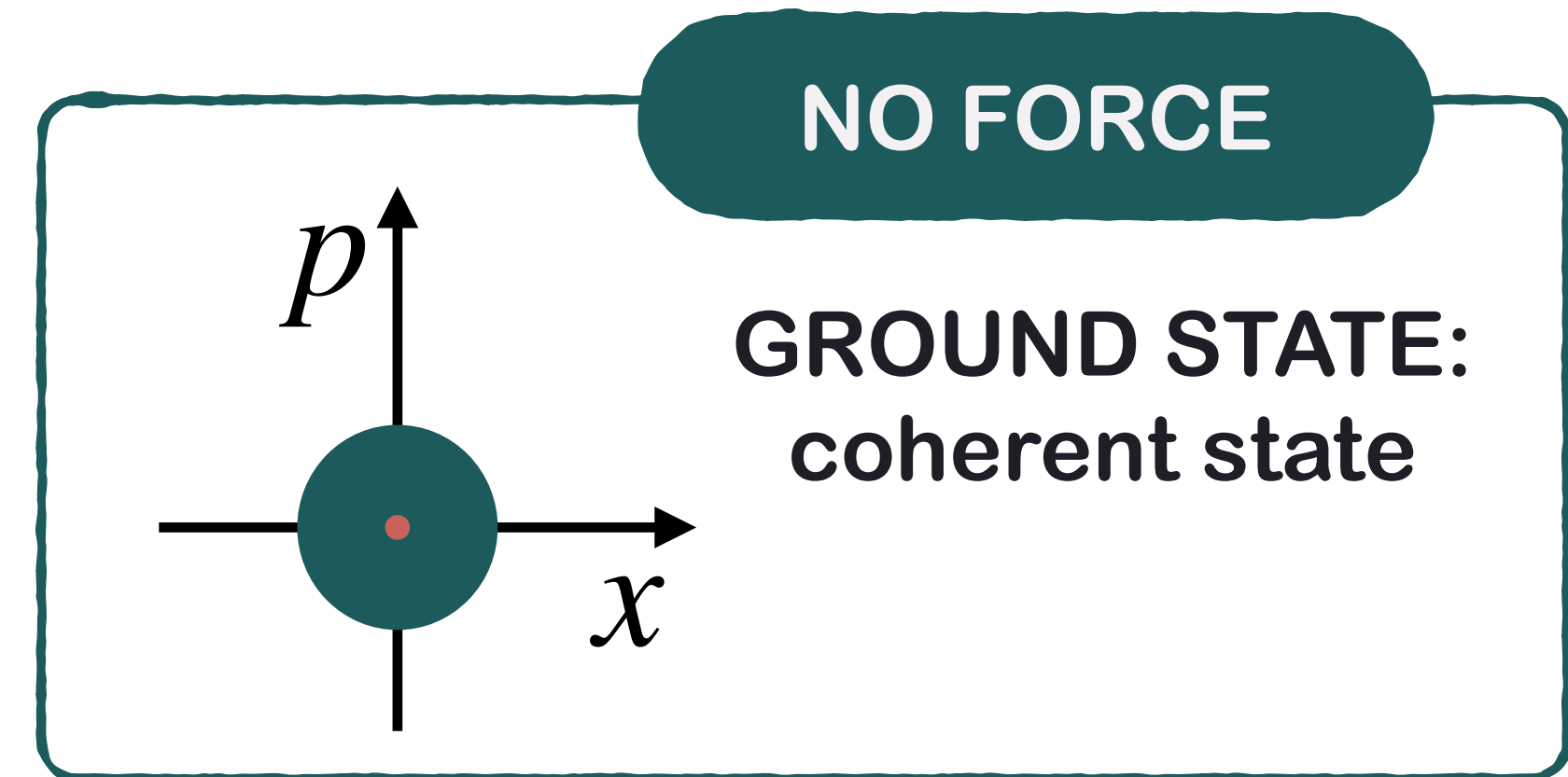
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# METHODOLOGY: ANALOGY WITH THE HARMONIC OSCILLATOR

Chen, Giacomini, Rovelli Quantum (2023)



QUANTUM HARMONIC OSCILLATOR

(gravity)

$$\hat{x} \rightarrow \hat{h}_{ij}; \quad \hat{p} \rightarrow \hat{\pi}_{ij}$$

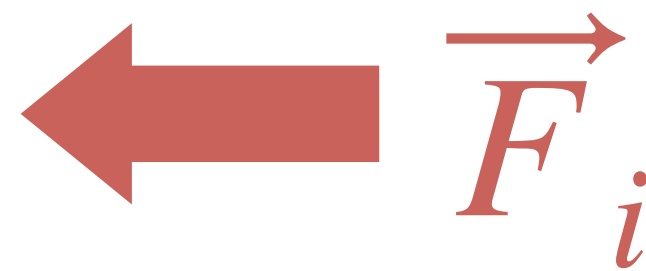
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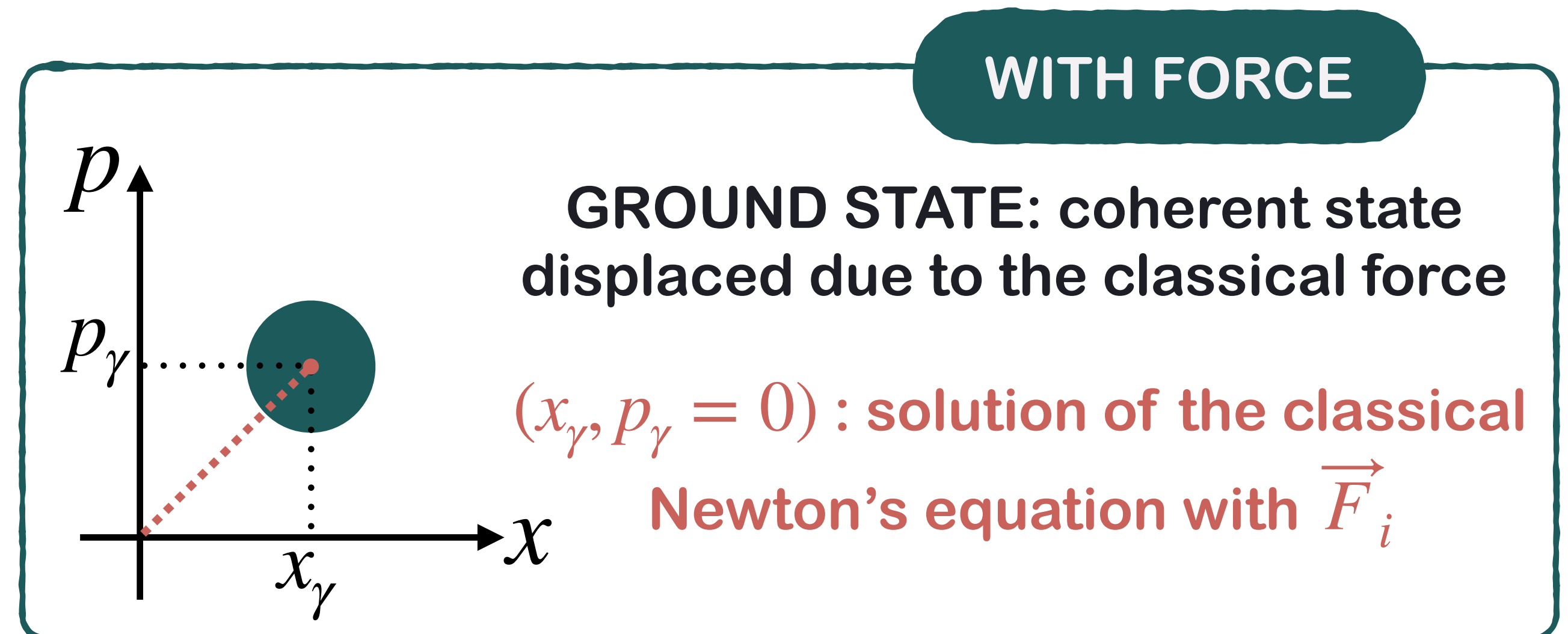
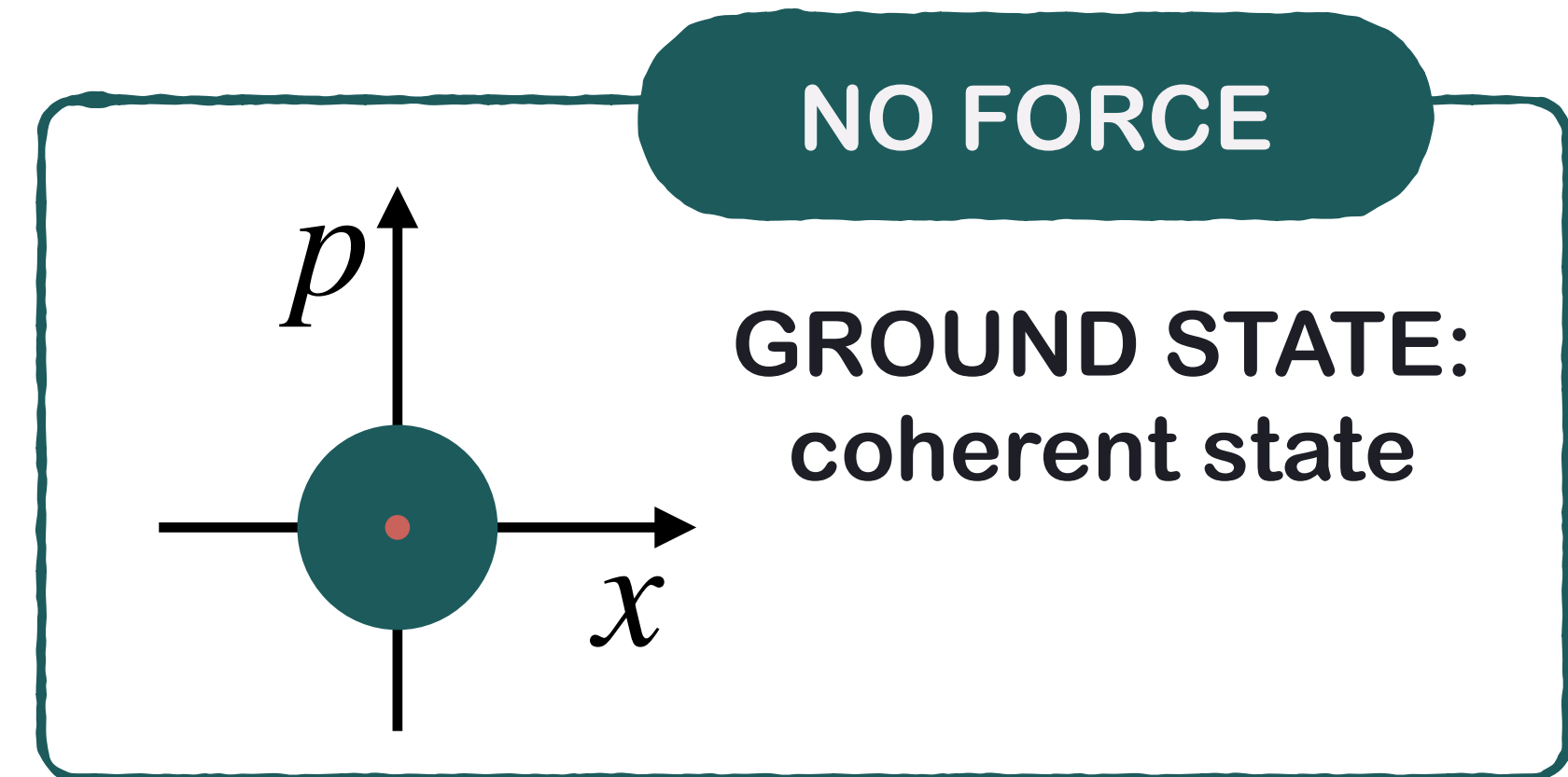
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CONSTANT EXTERNAL FORCE

eigenvalue of  $T_{00}(x - x_i)$





# METHODOLOGY - GRAVITY AS A (PERTURBATIVE) QUANTUM FIELD

---

Chen, **F.G.**, 2402.10288 (2024)

**linearized quantum gravity**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} \rightarrow \hat{h}_{\mu\nu}$$

$$\hat{H}_G(\hat{h}_{ij}, \hat{\pi}_{ij}) |\Psi\rangle_{G+S} = E_0(x) |\Psi\rangle_{G+S}$$

(ground state)      Free gravity Hamiltonian

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Chen, **F.G.**, 2402.10288 (2024)

$$\hat{H}_G(\hat{h}_{ij}, \hat{\pi}_{ij}) |\Psi\rangle_{G+S} = E_0(x) |\Psi\rangle_{G+S} \quad \text{Free gravity Hamiltonian}$$

(ground state)

$$[\partial_i \partial^i \hat{h}^T(x) + \hat{T}_{00}(x)] |\Psi\rangle_{G+S} = 0$$

$$\partial_j \hat{\pi}^{ij}(x) |\Psi\rangle_{G+S} = 0$$

**GAUGE**

Scalar constraint: Gauss law

Vector constraint:  
transversality condition

**linearized quantum gravity**

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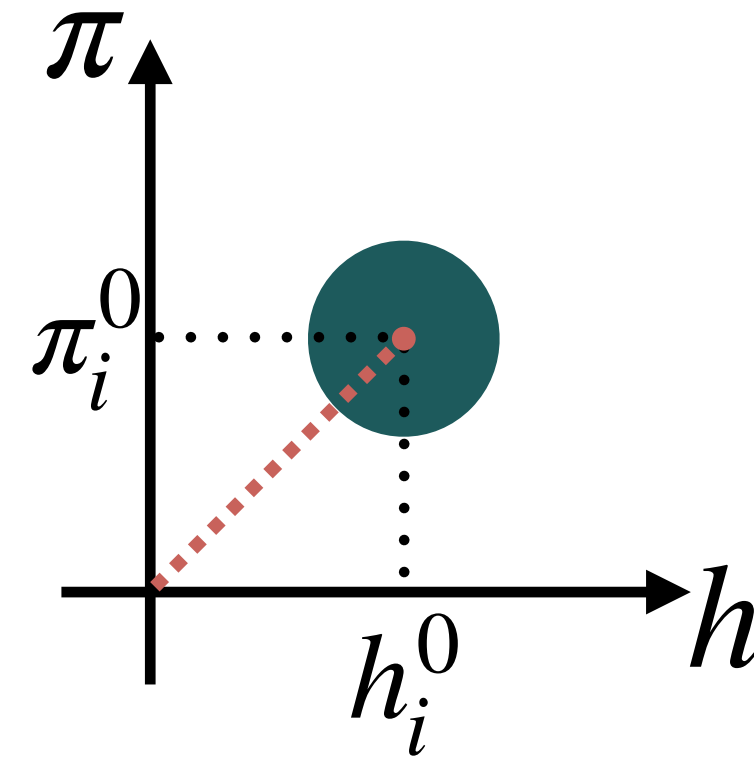
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**GROUND STATE:** coherent state displaced by the eigenvalue solution

- Localised source: Newton potential
- Delocalised source: general function

# THE QUANTUM STATE OF GRAVITY OF A GENERAL QUANTUM SOURCE

Chen, Giacomini, 2402.10288 (2024)



Interaction  
Hamiltonian

$$\hat{H}_I = -\frac{1}{2} \int d^3x \hat{h}_{\mu\nu}(\vec{x}) [\hat{T}_A^{\mu\nu}(\vec{x}) + \hat{T}_B^{\mu\nu}(\vec{x})]$$

... zero in the temporal gauge ( $h^{0\mu} = 0$ )!

Constraint changes the gravitational energy

$$\hat{C} = \partial_i \partial^i \hat{h}^T(x) + \hat{T}_{00}^A(x) + \hat{T}_{00}^B(x)$$

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Entangling phase:

$$\Delta\phi = -\frac{G}{c^4 \hbar} \int d^3x d^3y \frac{E_A(\vec{x}) E_B(\vec{y})}{|\vec{x} - \vec{y}|}$$



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Chen, Giacomini, 2402.10288 (2024)



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depend on the matter distribution

# COMPARISON TO CLASSICAL GRAVITY

Chen, Giacomini, 2402.10288 (2024)

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## LIMIT TO THE NEWTON POTENTIAL

Quantum state: coherent  
semiclassical state

$$|\psi\rangle \approx |\alpha(x_i, p_i)\rangle$$

Classical mass density

$$E^Z(\vec{x}) = m_Z c^2 \delta(\vec{x} - \vec{x}_i)$$

$$\Delta\phi = -\frac{G}{\hbar} \frac{m_A m_B}{|\vec{x}_A - \vec{x}_B|}$$

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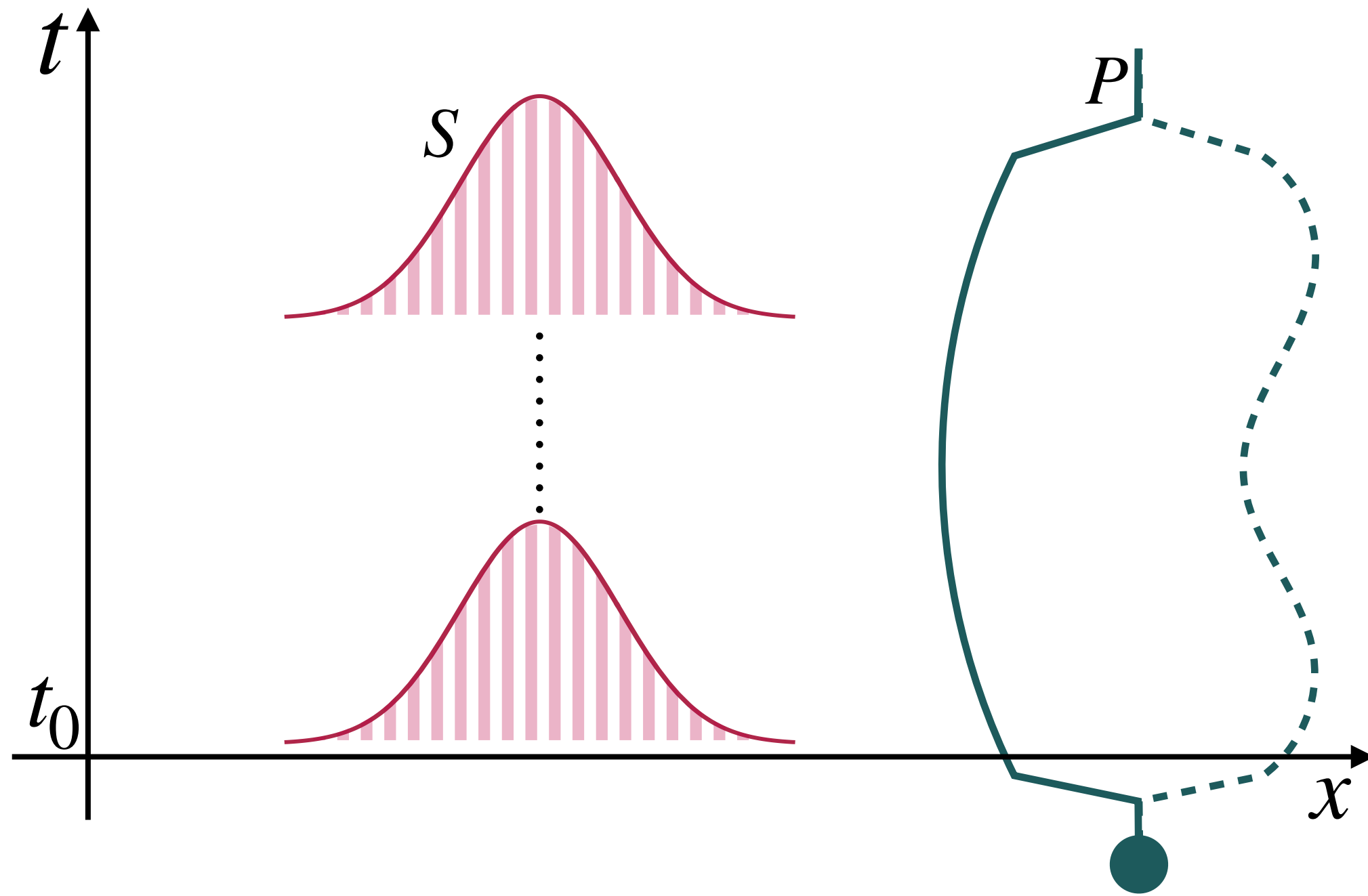
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## CANNOT BE REPRODUCED WITH:

1. Newton potential
2. Schrödinger-Newton equation
3. Classical-quantum coupling (Semiclassical gravity)

# QUANTUM COMMUTATOR OF THE GRAVITATIONAL FIELD

Chen, Giacomini, 2402.10288 (2024)

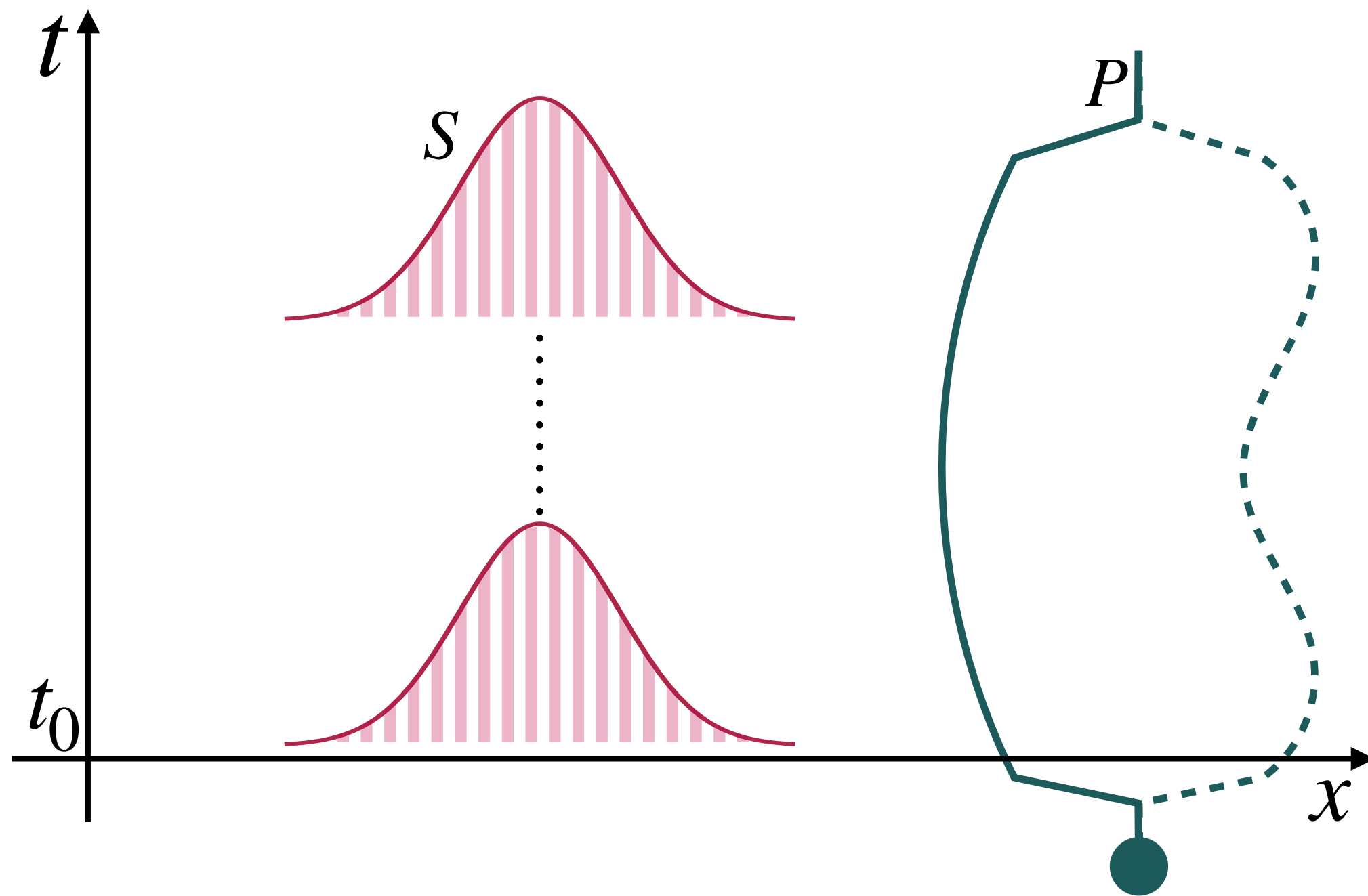


Total Hamiltonian

$$\hat{H}_{tot} = \hat{H}_S + \hat{H}_P + \hat{H}_G + \hat{H}_I$$

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Chen, Giacomini, 2402.10288 (2024)



Total Hamiltonian

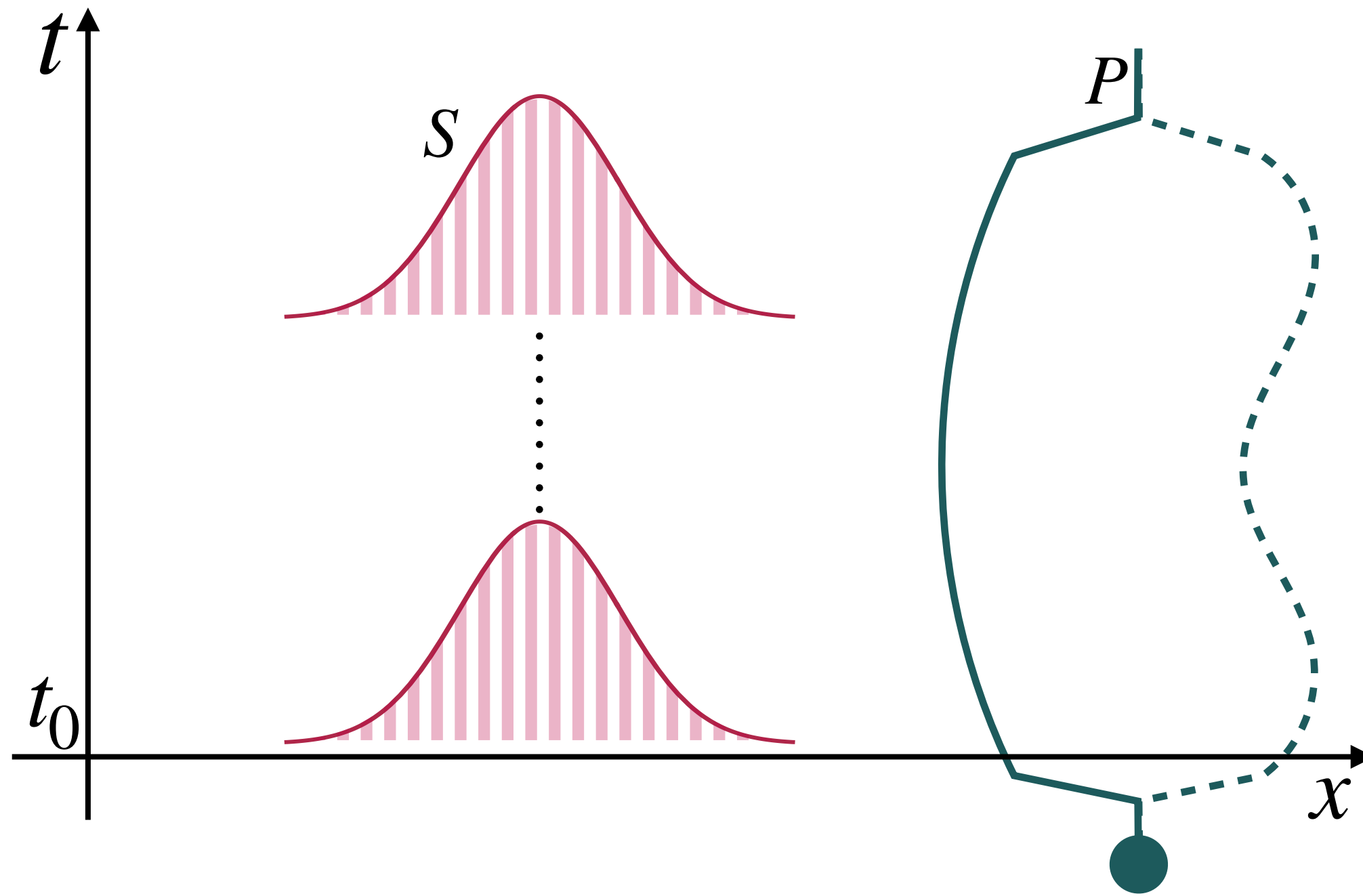
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Chen, Giacomini, 2402.10288 (2024)



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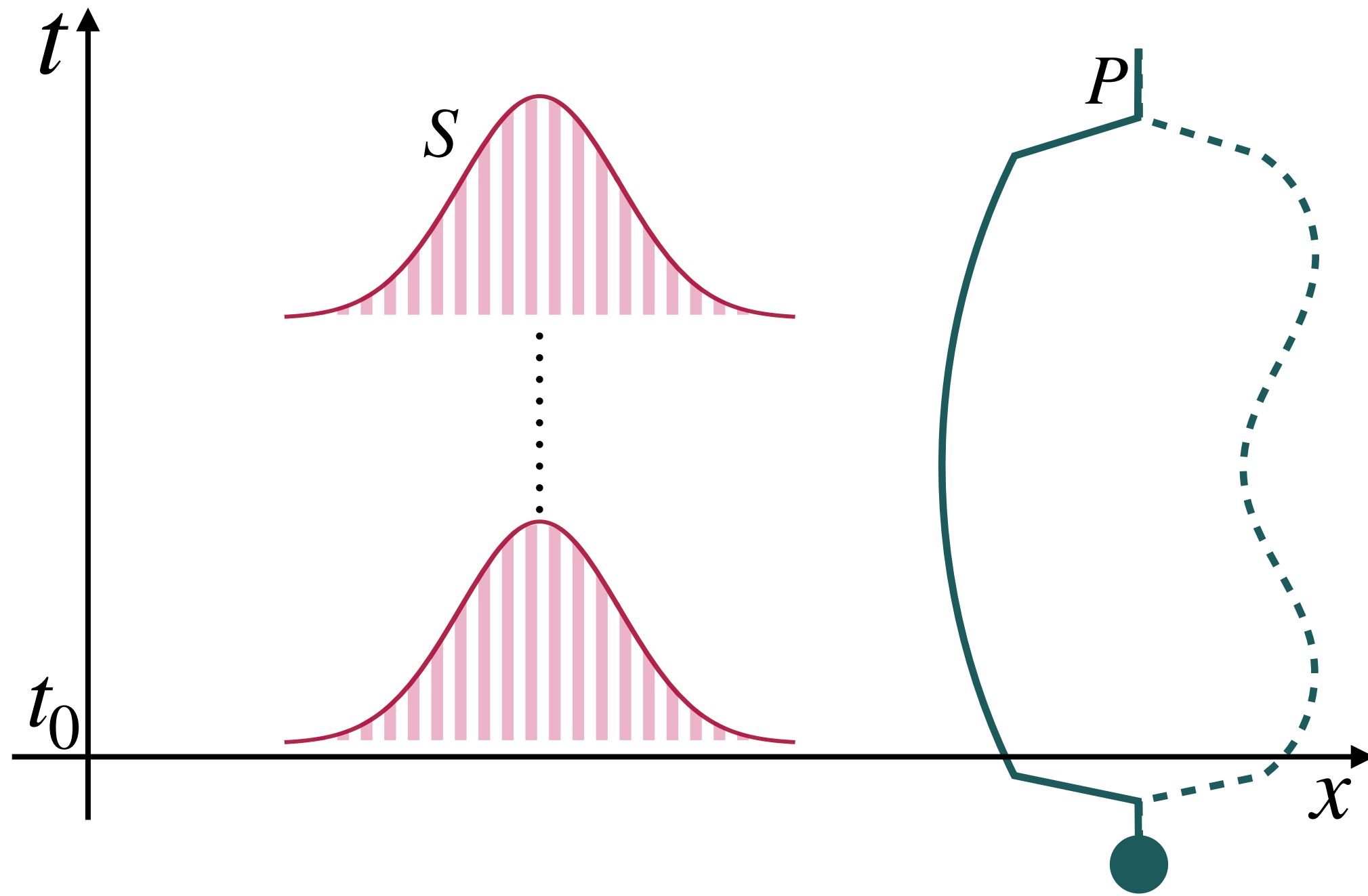
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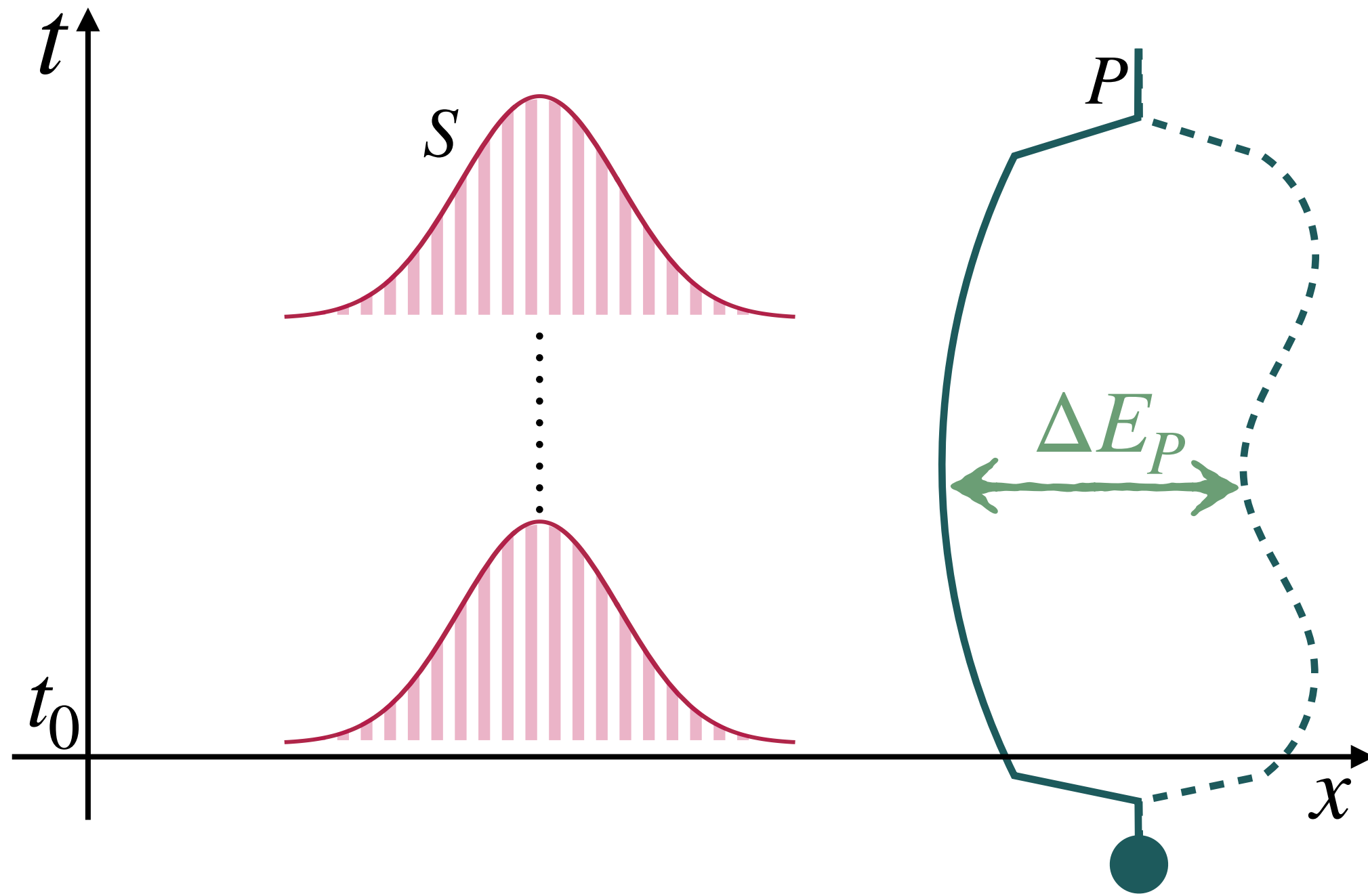
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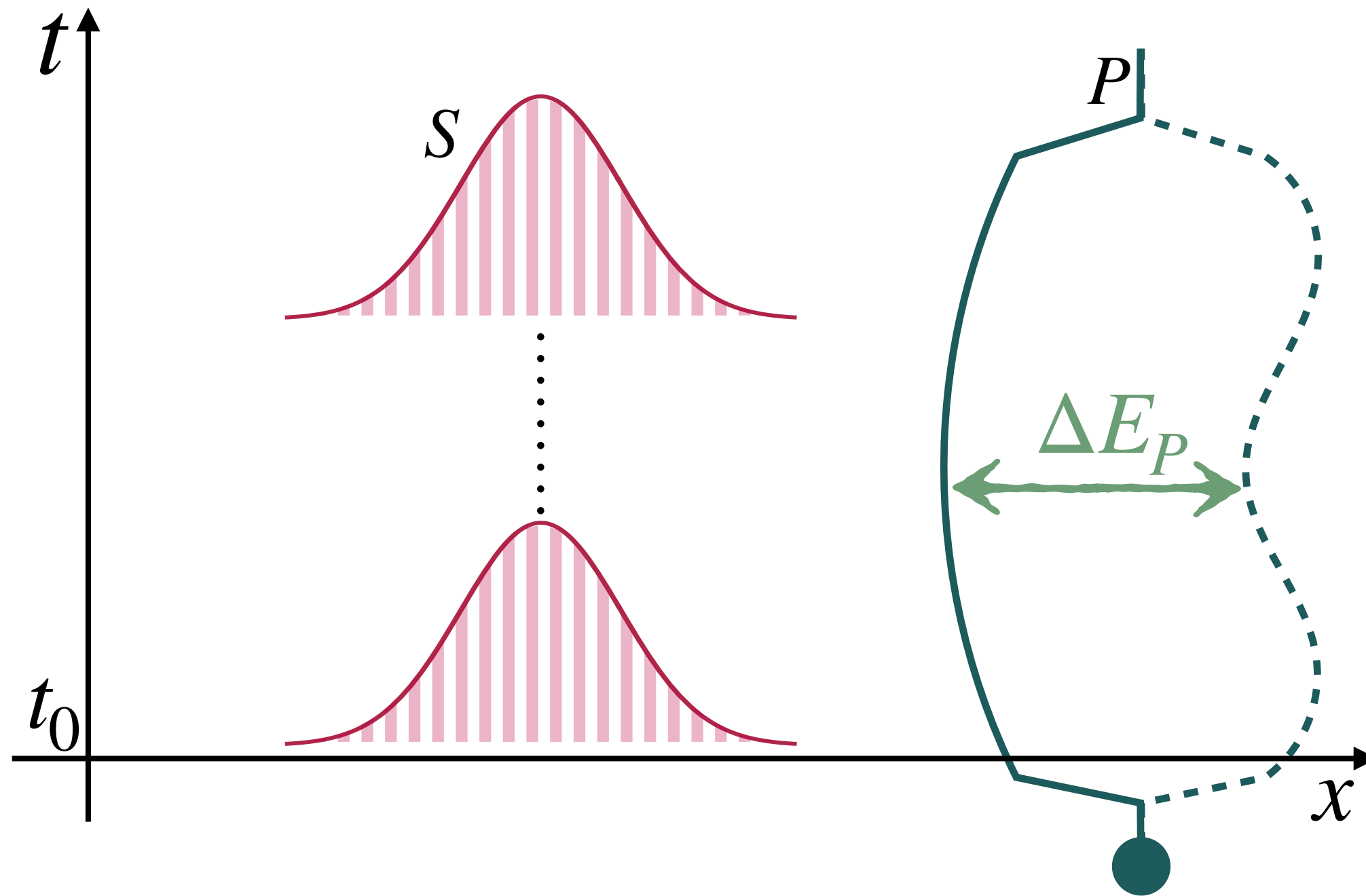
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Stronger indication that gravity is a quantum field

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Two effects beyond the Newton potential, but same order in the coupling, when we consider a static quantum source of gravity in a delocalized state:

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**OPEN QUESTIONS:** concrete implementation and estimates



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# BACK-UP SLIDES

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# WHAT IS THE QUANTUM STATE OF GRAVITY ASSOCIATED TO A QUANTUM SOURCE?

Chen, Giacomini, Rovelli Quantum (2023)

Then the quantum state of the Coulomb/Newton field is the ground state  $|h_i^0\rangle_G$  of the Hamiltonian with the charge/mass in the quantum state  $|\Phi_i\rangle$

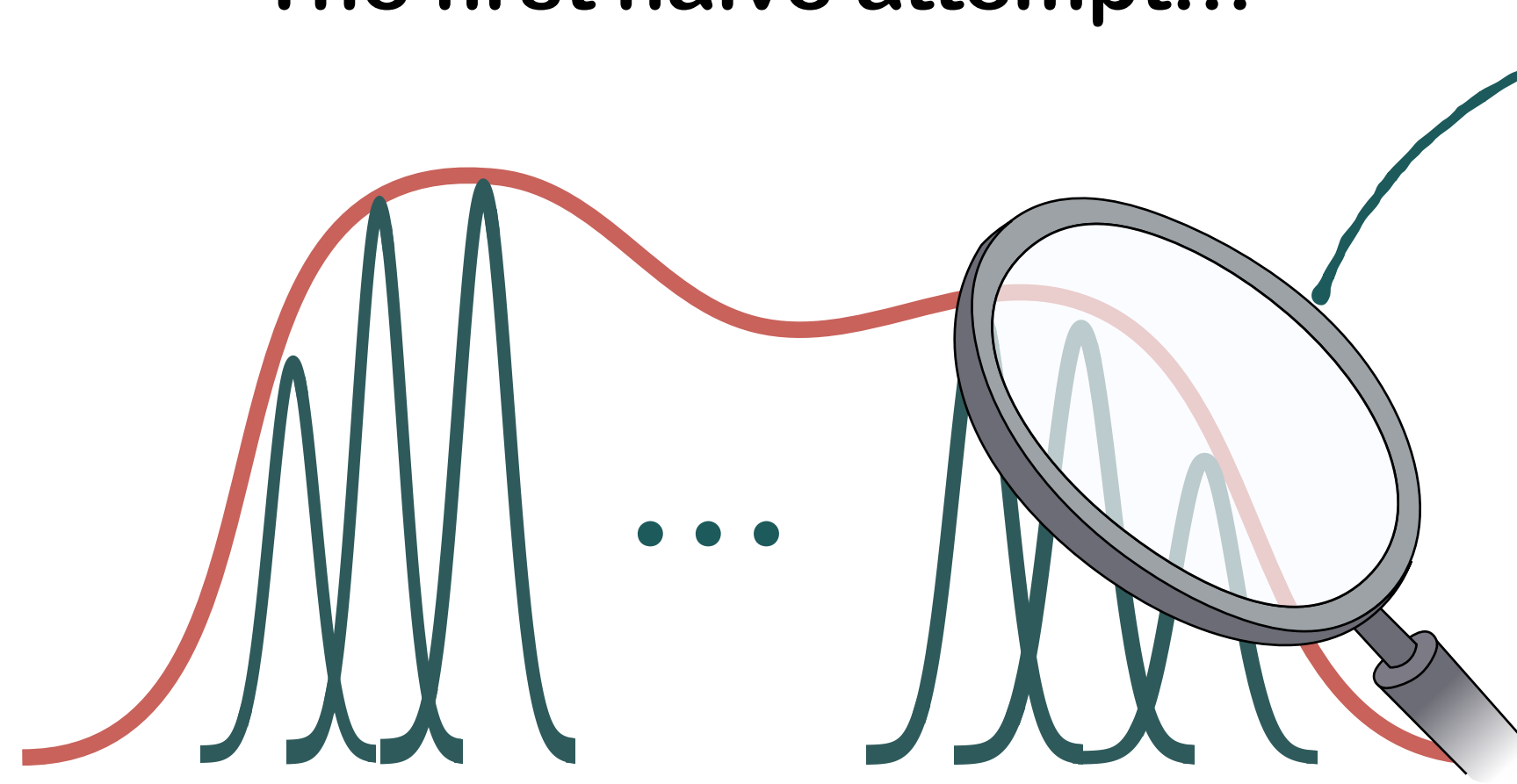
	Electromagnetism	Linearized Gravity
Temporal gauge	$A_0 = 0$	$h_{0\mu} = 0$
Canonical variables	$\{A_i(\vec{x}), E_j(\vec{x}')\}$	$\{h_{ij}(\vec{x}), \pi^{kl}(\vec{x}')\}$
No. of constraints	1	4
Similar constraints (without matter)	Gauss law in $A$ basis $\partial_j \frac{\delta}{\delta A_j(\vec{x})} \Psi[A] = 0$	Vector constraint in $h$ basis $\partial_i \frac{\delta}{\delta h_{ij}(\vec{x})} \Psi[h_{ij}] = 0$
Similar constraints (with matter)	Gauss law in $E$ basis with charge $\nabla \cdot E = \Delta\phi = \rho$	Scalar constraints $\Delta h^T = -\rho$
Vacuum state	Gaussian of transverse mode	Gaussian of transverse mode with zero trace
The d.o.f activated with a static source	Longitudinal mode $A_L$	Trace of transverse mode $h_T$

$$|\Psi\rangle_{G+M} = \sum_i \alpha_i |\Phi_i, h_i^0\rangle_{MG}$$

# THE QUANTUM STATE OF GRAVITY OF A GENERAL QUANTUM SOURCE

Chen, Giacomini, 2402.10288 (2024)

The first naive attempt...



$\langle \alpha_x | \alpha_{x+\epsilon} \rangle \neq 0$   
 $\langle \alpha_x, h_\alpha | \alpha_{x+\epsilon}, h_\alpha \rangle = 0 \quad \times$

The shift by the classical solution makes the states perfectly distinguishable

$$|\psi\rangle = \int d\mu(\alpha) \psi(\alpha) |\alpha\rangle$$

$$\langle \Delta x \rangle_\alpha \ll \text{Exp. resol.}$$

$$\langle \Delta p \rangle_\alpha \ll \text{Exp. resol.}$$

...still the Newton potential!

$$|\psi\rangle = \int d\mu(E) \psi(E) |E\rangle$$

$$\hat{T}_{00}(x) |E\rangle = E(\vec{x}) |E\rangle \quad E(\vec{x}) \neq mc^2 \delta(\vec{x} - \vec{x}_i)$$

$$\langle \psi, h_\psi | \phi, h_\phi \rangle = \langle \psi | \phi \rangle \quad \checkmark$$