

Lecture 4 Plan:

- 1) UHECR- the observational status**
- 2) Hydro Turbulence and Magneto-Hydro Turbulence**
- 3) Non-thermal particle transport equation in magnetic turbulence**
- 4) The extragalactic magnetic field environment**

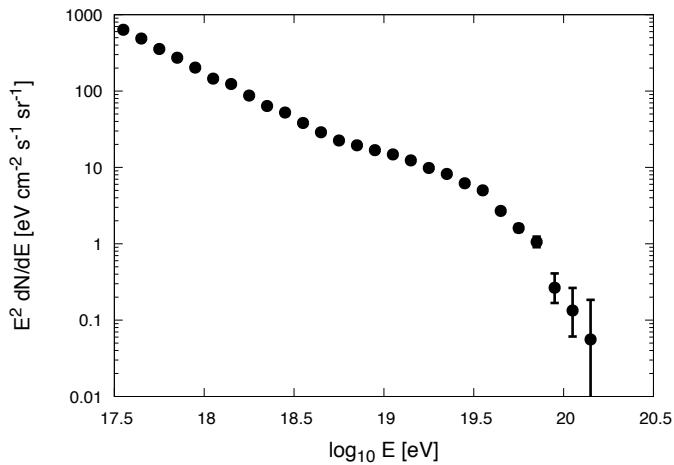
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DESY.

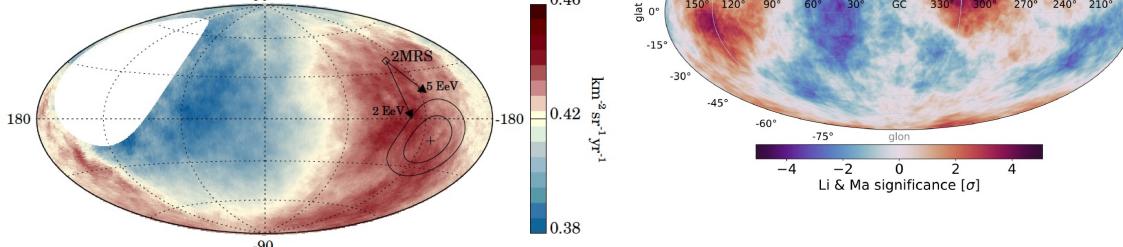
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UHECR: The Observational Status

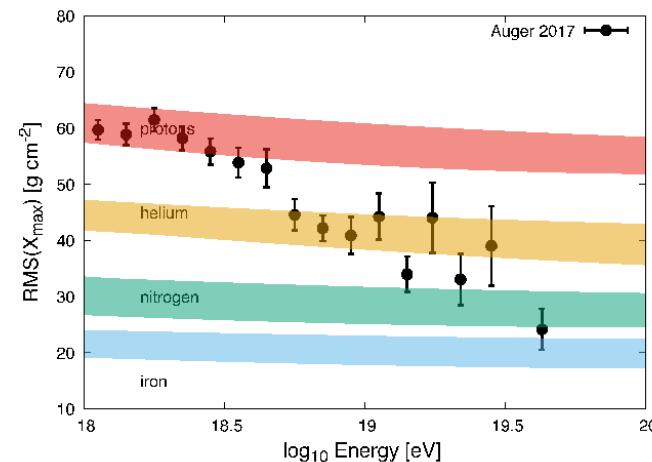
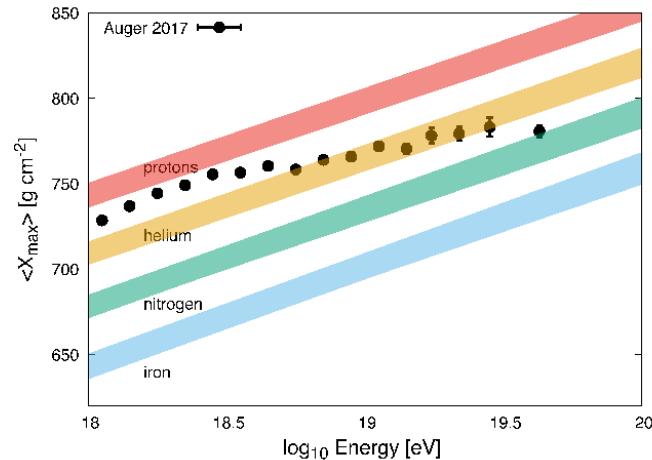
Spectrum



Anisotropy



Composition



Pierre Auger Collaboration. ApJ. 935 (2022)

Caccianiga et al. for the Auger and TA Collaborations. PoS
(ICRC2023) 521

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Hydro Turbulence

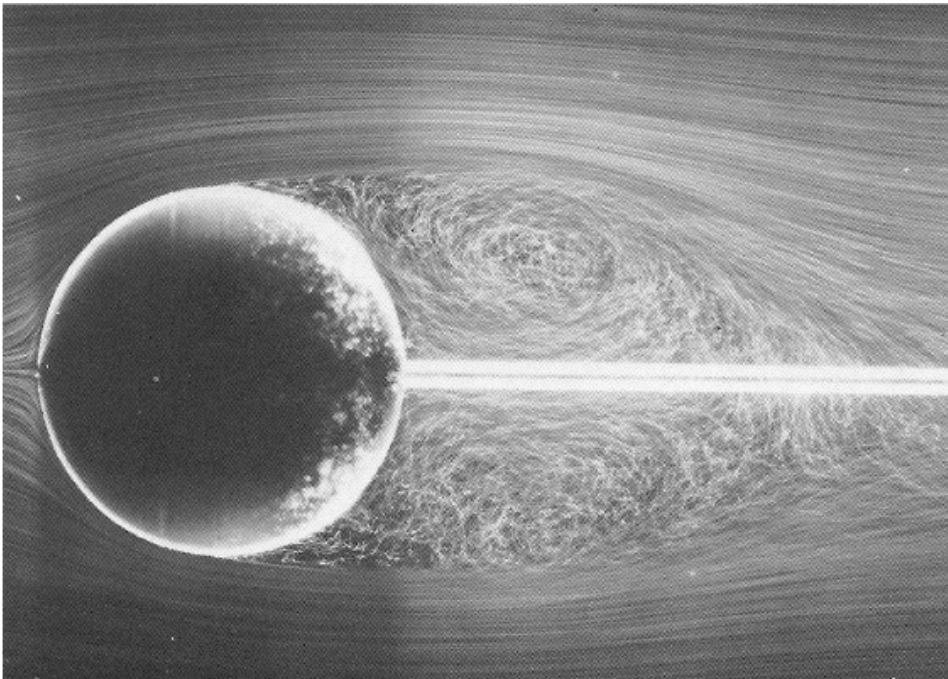
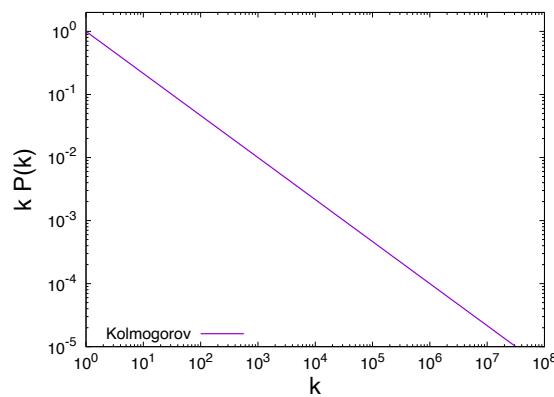


Image from University of Sydney

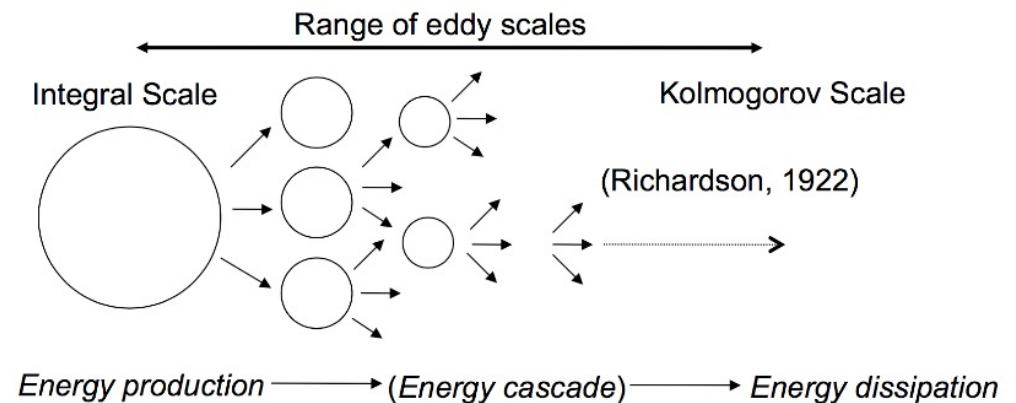


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Richardson, 1922

*"Big whorls have little whorls
That feed on their velocity;
And little whorls have lesser whorls
And so on to viscosity."*

“ ”



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Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{P} = \rho \mathbf{g}$$

Momentum flux
conservation

shorthand for $\mathbf{v} \mathbf{v}^T$

$$\mathbf{P} = p \mathbf{I} + \rho \mathbf{v} \mathbf{v}$$

Spatial part of stress energy
tensor

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{g}$$

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Magneto-Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{P} - \mathbf{P}_M) = \rho \mathbf{g}$$

Momentum flux
conservation

$$\mathbf{P} = p\mathbf{I} + \rho \mathbf{v}\mathbf{v}$$

Pressure tensor

$$\mathbf{P}_M = -\frac{\mathbf{B}^2}{8\pi}\mathbf{I} + \frac{\mathbf{B}\mathbf{B}}{4\pi}$$

Maxwell stress tensor

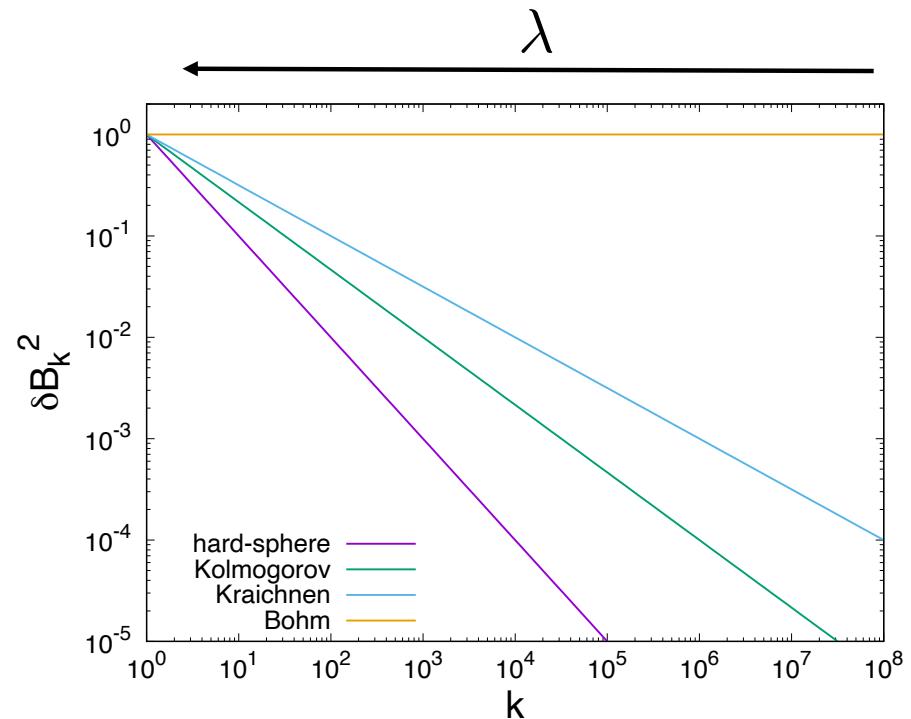
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Galactic Magneto-Hydro Turbulence

One of the key drivers is thought to be Supernova explosions

$$\delta \mathbf{B}^2 = \int \frac{d(\delta \mathbf{B}^2)}{d \ln k} d \ln k = \int \delta B_k^2 d \ln k$$
$$\delta B_k^2 = \delta B_0^2 \left(\frac{k}{k_0} \right)^{1-\alpha}$$

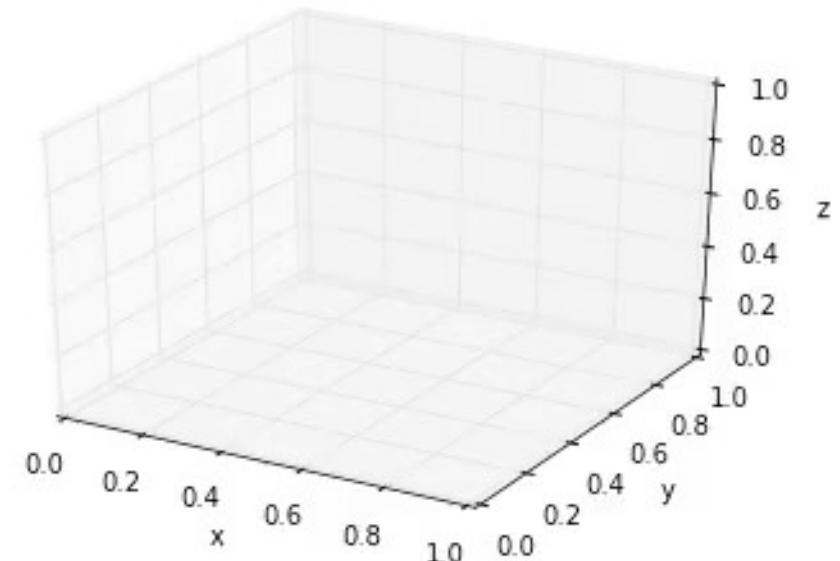
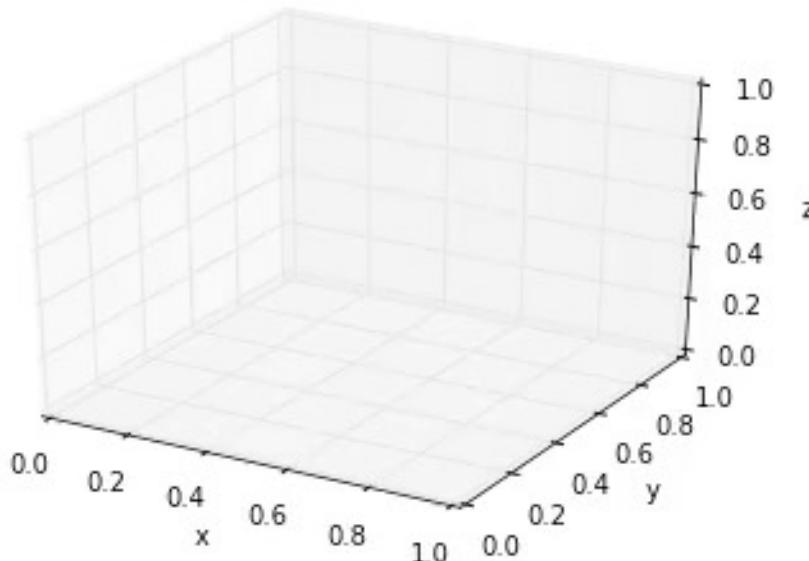


Note for MHD turbulence, the theoretically expected turbulence index is still debated

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Charged Particles in Magnetic Fields

Note- a lot of what you **may have** studied about charged particle propagation in magnetic fields **likely** assumed magnetic field variation was on much longer length scales than particle Larmor radius.



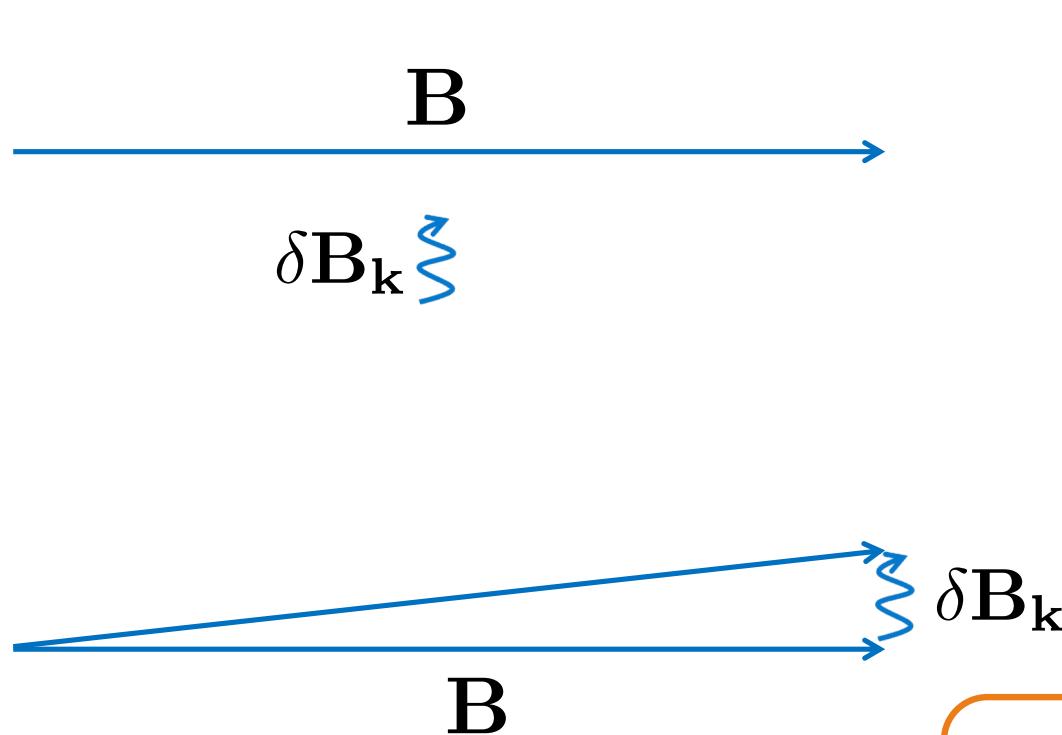
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Particle Diffusion in Magnetic Turbulence (Quasi-Linear Theory)?

The propagation of cosmic rays is dictated by the magnetic field landscape they live in.



$$\delta\theta = \frac{\delta B_k}{B}$$

$$\langle \Delta\theta^2 \rangle = N \langle \delta\theta^2 \rangle$$

$$= \left(\frac{t}{t_{\text{lar}}} \right) \langle \delta\theta^2 \rangle$$

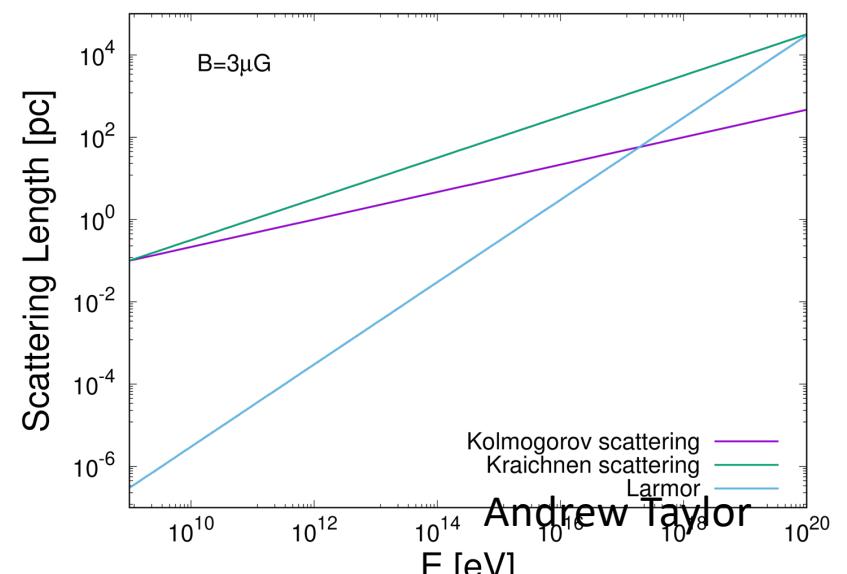
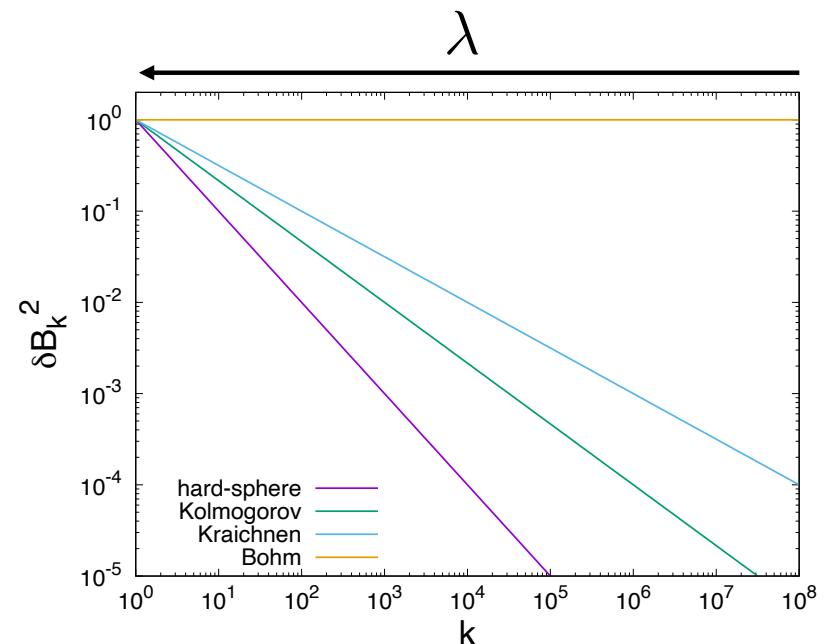
$$D_{\theta\theta} = \frac{\Delta\theta^2}{t} = \frac{1}{t_{\text{lar}}} \left(\frac{\delta B_k^2}{B^2} \right)$$

Propagation through Magnetic Fields

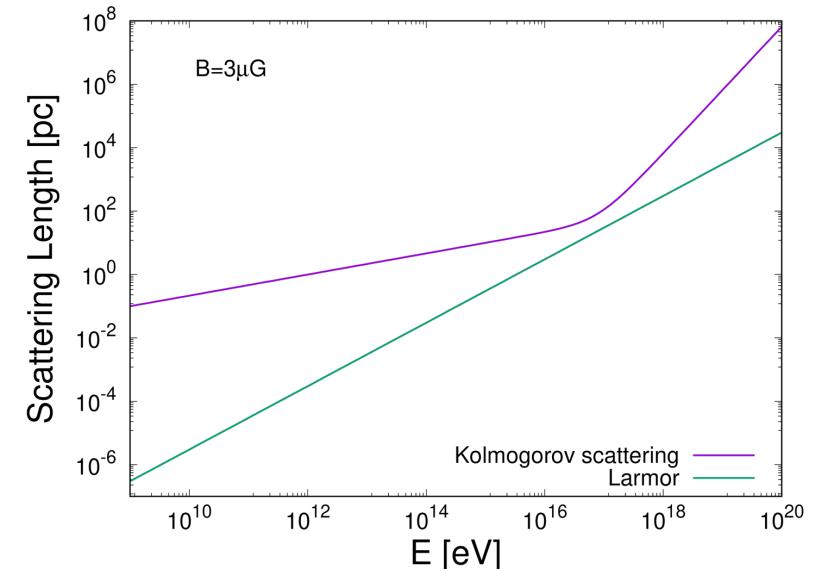
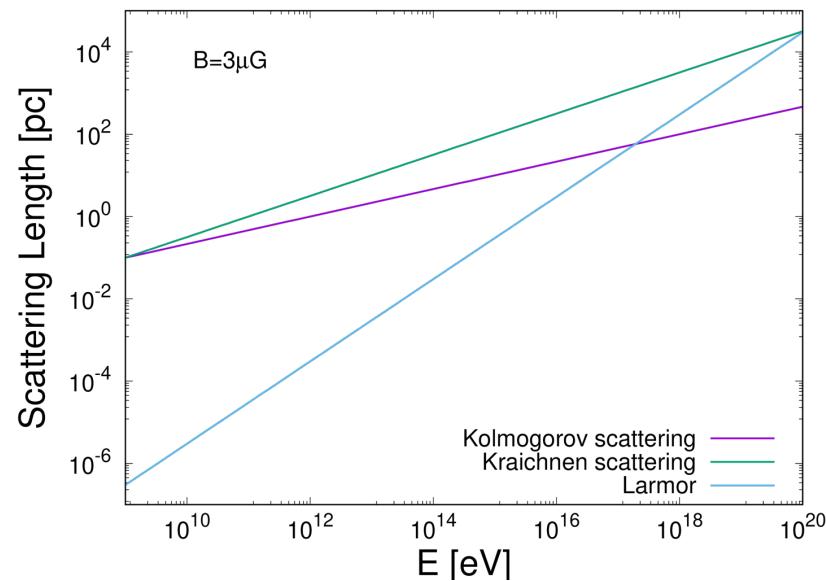
$$t_{\text{scat}} \approx \frac{1}{D_{\theta\theta}}$$

$$\frac{D_{xx}}{c} \approx l_{\text{scat}}$$

$$\frac{D_{xx}}{c} \approx R_{\text{lar}} \left(\frac{B^2}{\delta B_k^2} \right)$$



Propagation through Magnetic Fields



Transport (Continuity) Equation

$$\frac{\partial \mathbf{f}}{\partial t} + \nabla_{\mathbf{x}} \cdot \mathbf{j} = Q$$

$$\frac{\partial \mathbf{f}}{\partial t} = \nabla_{\mathbf{x}} \cdot (D_{xx} \nabla_{\mathbf{x}} \mathbf{f}) + Q$$

$$\mathbf{j} = -D_{xx} \nabla_{\mathbf{x}} \mathbf{f}$$

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Charged Particle Motion in Turbulent Magnetic Fields

$$\frac{\partial f}{\partial t} = \nabla_x \cdot (D_{xx} \nabla_x f) + Q$$

Diffusion

Source term

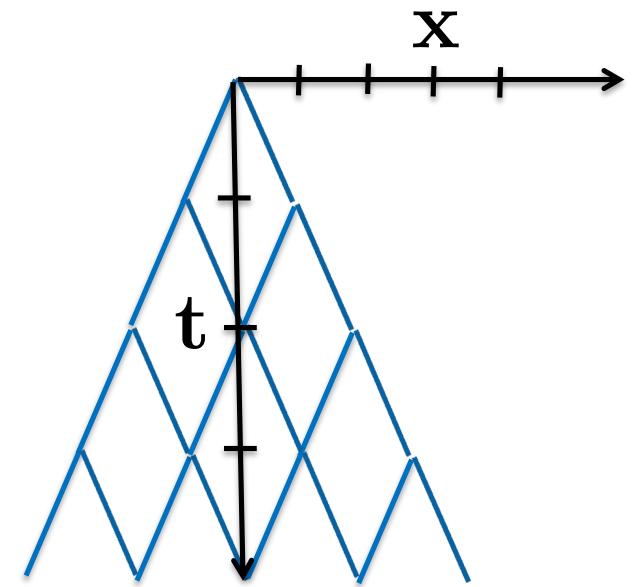
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Random Walks



$$f(x, t) = \frac{t!}{([t - x]/2)!([x + t]/2)!(2^t)}$$

At every discrete value of (x, t) , $f(x, t)$ describes the fractional population of that state

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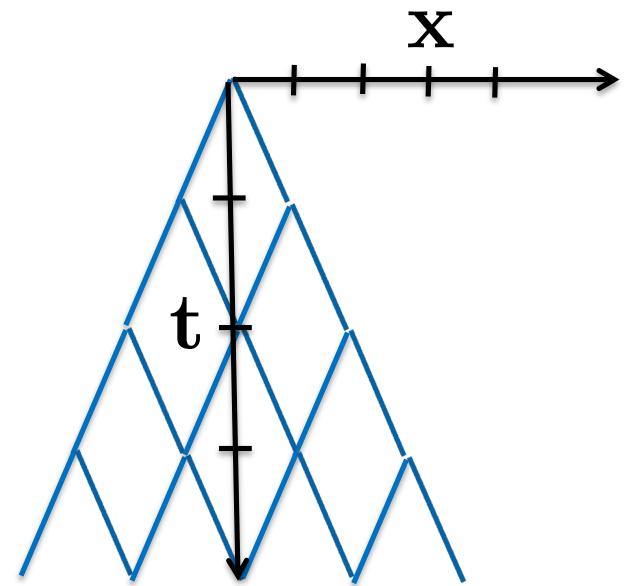
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Random Walks

$$\gamma(t+1) = t!$$

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$



$$f(x, t) = \frac{\gamma(t+1)}{[\gamma([t-x]/2 + 1)\gamma([x+t]/2 + 1)](2^t)}$$

$$f(x, t) \approx \frac{e^{-x^2/(2t)}}{(2\pi t)^{1/2}}$$

Let's have a go at demonstrating this!

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$$f(x, t) = \frac{\gamma(t+1)}{[\gamma([t-x]/2+1)\gamma([x+t]/2+1)](2^t)}$$

$$f(x, t) \approx \frac{e^{-x^2/(2t)}}{(2\pi t)^{1/2}}$$

PAUSE

Have a go at obtaining the above result



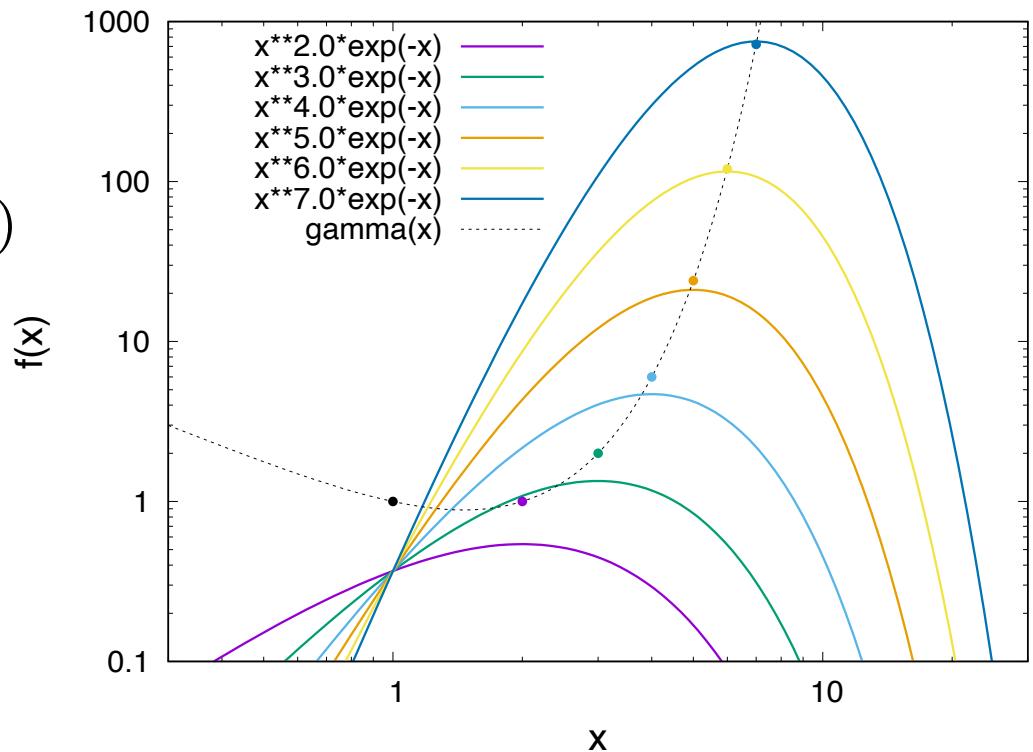
Stirling's Approximation

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$\approx x^{t+1} e^{-x} \Big|_{\text{peak}}$$

$$\approx (t+1)^{(t+1)} e^{-(t+1)}$$

Note- another example of a visual inspection version of Laplace's integral method





Binomial to Gaussian

$$n(x, t) = \frac{\gamma(t+1)}{[\gamma(\frac{[t-x]}{2} + 1)\gamma(\frac{[x+t]}{2} + 1)](2^t)}$$

$$n(x, t) \approx 2^{-t} \frac{(t+1)^{(t+1)} e^{-(t+1)}}{[(t-x)/2 + 1]^{[(t-x)/2+1]} e^{-[(t-x)/2+1]} [(t+x)/2 + 1]^{[(t+x)/2+1]} e^{-[(t+x)/2+1]}}$$

$$n(x, t) \approx e 2^{-t} \frac{(t+1)^{(t+1)}}{[(t-x)/2 + 1]^{[(t-x)/2+1]} [(t+x)/2 + 1]^{[(t+x)/2+1]}}$$



Binomial to Gaussian

$$n(x, t) \approx \frac{(1+t)^{1+t}}{(1+\frac{t}{2})^{2+t} \left(1 - \frac{x}{(2+t)}\right)^{[1+(t-x)/2]} \left(1 + \frac{x}{(2+t)}\right)^{[1+(t+x)/2]}}$$

$$n(x, t) \approx \frac{(1+t)^{1+t}}{(1+\frac{t}{2})^{2+t} \left(1 - \frac{x^2}{(2+t)^2}\right)^{(1+t/2)} \left(\frac{1+\frac{x}{(2+t)}}{1-\frac{x}{(2+t)}}\right)^{x/2}}$$

$$-\log(n(x, t)) \approx (\dots) + \frac{1}{2}(2+t) \left(-\frac{x^2}{(2+t)^2}\right) + x \frac{x}{(2+t)}$$

→ $n(x, t) \propto e^{-\frac{x^2}{2t}}$

Note- expression is in dimensionless units



Binomial to Gaussian

$$n(x, t) \approx \frac{(1+t)^{1+t}}{(1+\frac{t}{2})^{2+t} \left(1 - \frac{x}{(2+t)}\right)^{[1+(t-x)/2]} \left(1 + \frac{x}{(2+t)}\right)^{[1+(t+x)/2]}}$$

$$n(x, t) \approx \frac{(1+t)^{1+t}}{(1+\frac{t}{2})^{2+t} \left(1 - \frac{x^2}{(2+t)^2}\right)^{(1+t/2)} \left(\frac{1+\frac{x}{(2+t)}}{1-\frac{x}{(2+t)}}\right)^{x/2}}$$

$$-\log(n(x, t)) \approx (\dots) + \frac{1}{2}(2+t) \left(-\frac{x^2}{(2+t)^2}\right) + x \frac{x}{(2+t)}$$

$$\rightarrow n(x, t) \propto e^{\left(-\frac{(x/\Delta x)^2}{2(t/\Delta t)}\right)}$$

★ Steady State Distribution Around a Source of Diffusing Particles

cosmic rays diffuse in magnetic field turbulence

$$D = \frac{(\Delta x)^2}{2\Delta t}$$

$$f(r, t) \approx \frac{e^{-r^2/(4Dt)}}{(4\pi Dt)^{3/2}}$$



3D Green's function

$$\begin{aligned} F(r) &= \int_0^\infty f(r, t) dt \\ &= \frac{1}{4\pi Dr} \end{aligned}$$

Suggest you all have a go at demonstrating this!

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$$F(r) = \int_0^\infty f(r, t) dt = \frac{1}{4\pi D r}$$

PAUSE

Have a go at obtaining the above result for 3D diffusion, what happens for 1D case?

$$\frac{dN}{d^3r_{\text{tot}}} = \int_0^\infty \frac{dN(t)}{d^3r} Q dt = \int_0^\infty \frac{e^{-r^2/4Dt}}{(4\pi Dt)^{3/2}} Q dt$$

Let $x = \frac{r^2}{4Dt}$  $dt = -\frac{r^2}{4Dx^2} dx$

$$(Dt)^{3/2} = \left(\frac{r^2}{4x}\right)^{3/2}$$

$$\frac{dN}{d^3r_{\text{tot}}} = \frac{Q}{(\pi)^{3/2} 4Dr} \int_0^\infty x^{-1/2} e^{-x} dx$$

$$= \frac{Q}{4\pi Dr}$$

$$\frac{dN}{dr}_{\text{tot}} = \int_0^\infty \frac{dN(t)}{dr} Q dt = \int_0^\infty \frac{e^{-r^2/4Dt}}{(4\pi Dt)^{1/2}} Q dt$$

Let $x = \frac{r^2}{4Dt}$  $dt = -\frac{r^2}{4Dx^2} dx$

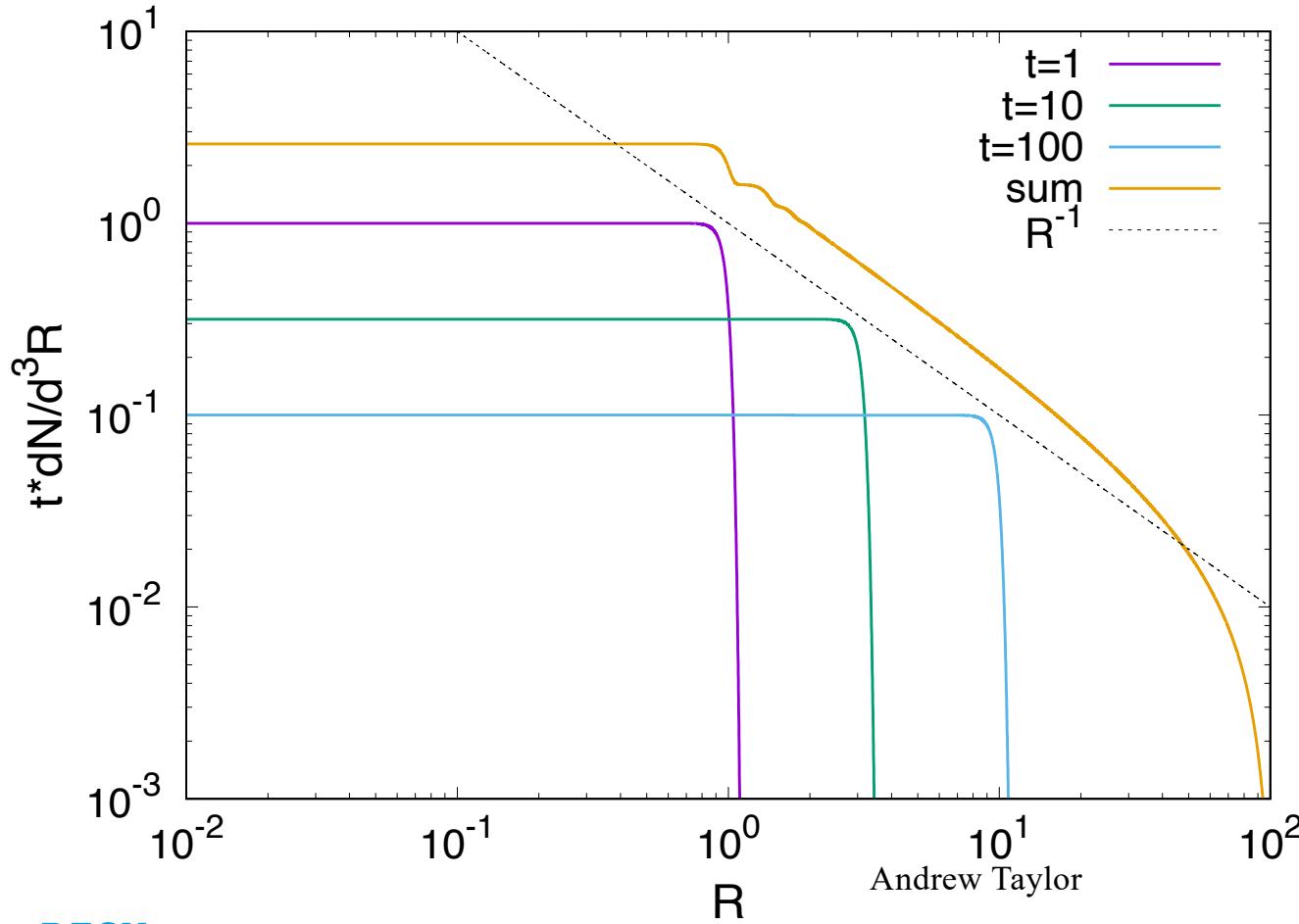
$$(Dt)^{1/2} = \left(\frac{r^2}{4x} \right)^{1/2}$$

$$\frac{dN}{dr}_{\text{tot}} = \frac{Qr}{(\pi)^{1/2} 4D} \int_0^\infty x^{-3/2} e^{-x} dx$$

The above integral does not converge!

Steady State Distribution Around a Source of Diffusing Particles

$$\int f(r, t) dt = \int t f(r, t) dlnt$$



↑
Smart to look at this

Note- yet another example of a visual inspection version of Laplace's integral method!

Spectral Effects of Magnetic Fields

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Energy Dependent Magnetic Horizon

$$l_{\text{MH}} = (D_{xx} t_H)^{1/2} = 60 \left(\frac{D_{xx}}{1 \text{ Mpc}} \right)^{1/2} \left(\frac{t_H}{4000 \text{ Mpc}} \right)^{1/2} \text{ Mpc}$$

If the diffusion coefficient, D_{xx} , is energy dependent, the magnetic horizon is also energy dependent.

Extragalactic cosmic rays cannot arrive to the Milky Way at low energies within a Hubble time!

Aloisio, R. +, ApJ 612 (2004)

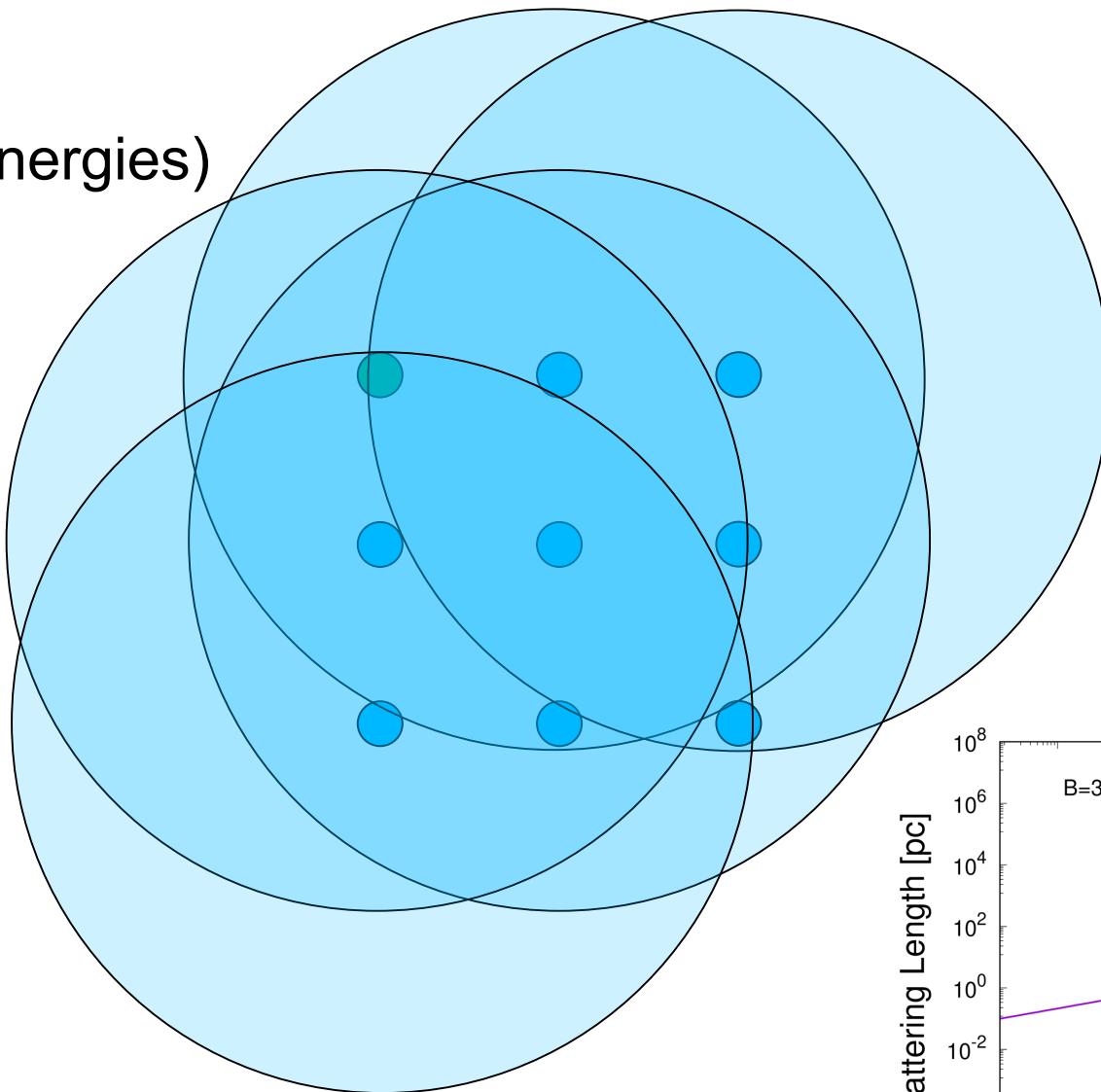
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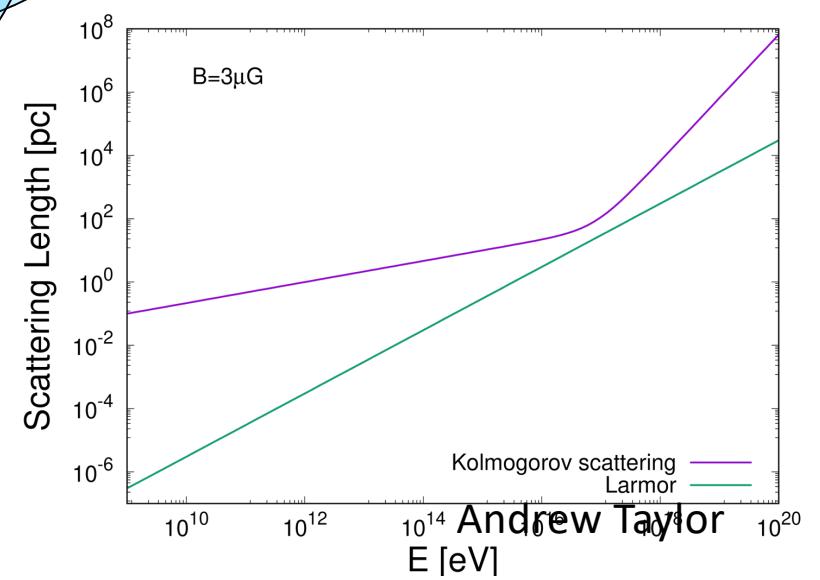
Magnetic Horizon Effect

(medium energies)



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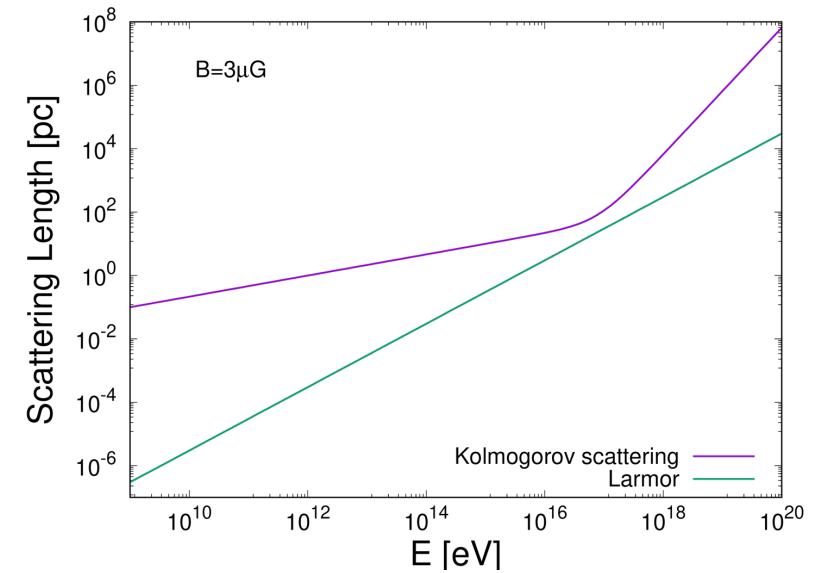
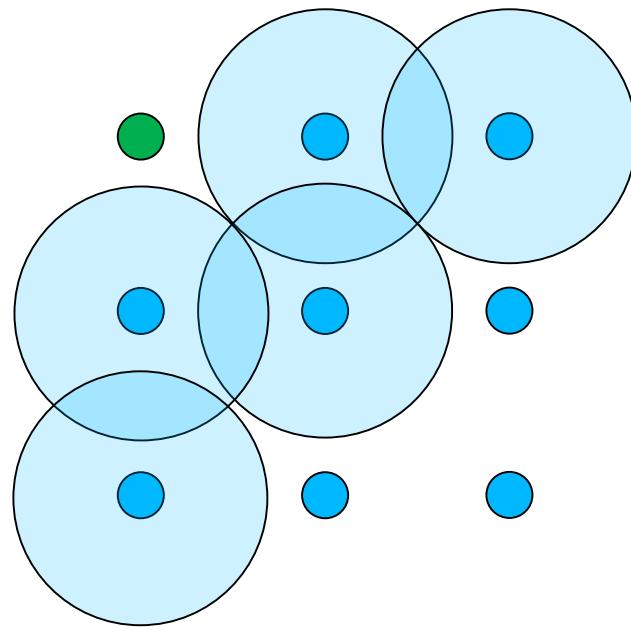
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Magnetic Horizon Effect

$$l_{\text{MH}} = (D_{xx} c t_H)^{1/2}$$

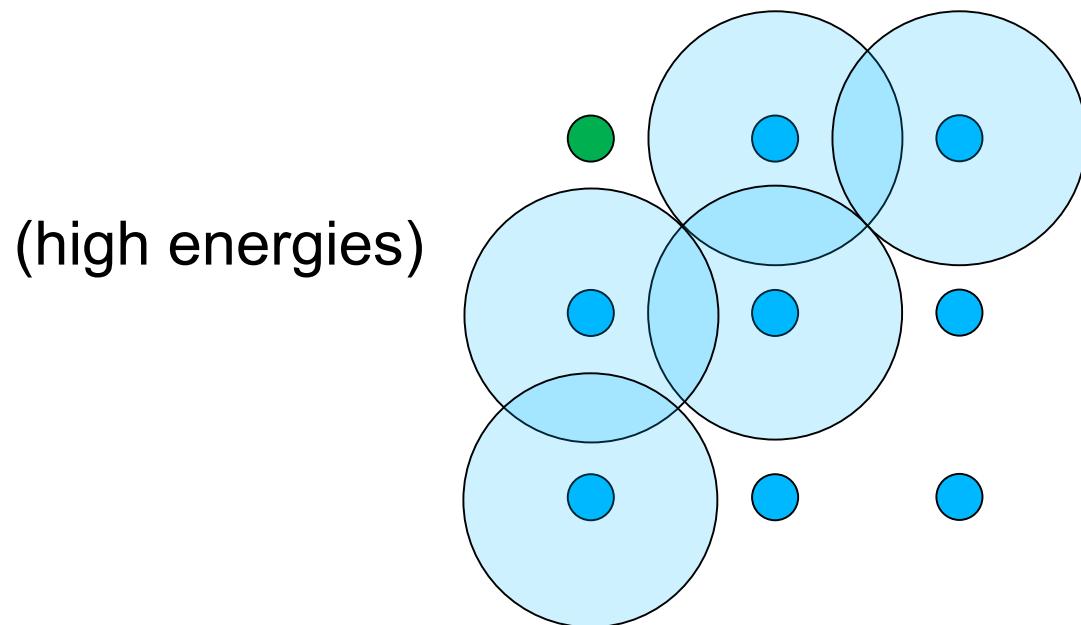
(low energies)



Once l_{MH} becomes smaller than r_s cosmic rays from the nearest sources become suppressed

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Energy Loss Horizon



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Propagation through Extragalactic Magnetic Fields

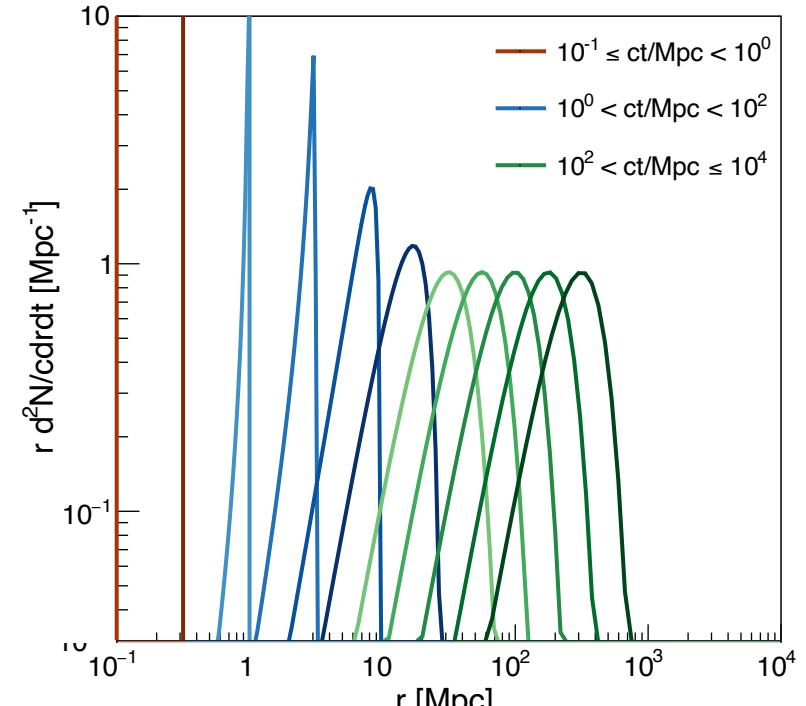
3 Phases of Propagation:

1. Ballistic
 2. Ballistic/Gaussian
 3. Gaussian
- Juttner

$$\frac{dN}{dr} = \frac{r^2 \alpha e^{(-\alpha/\sqrt{[1-(r/ct)^2])}}}{(ct)^3 K_1(\alpha)[1-(r/ct)^2]^2}$$

$$\alpha = tc^2/2D$$

Aloisio, R. +, ApJ 693 2009,



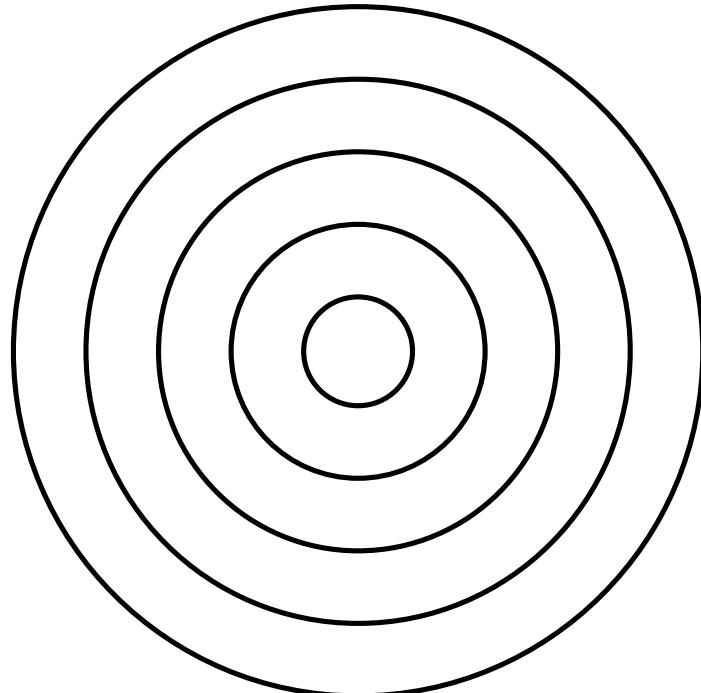
Lang, R. +, PRD 102 (2020)

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Extragalactic Magnetic Field Effects

Olbers Paradox for extragalactic cosmic rays:

- 1) Without extragalactic magnetic fields (ie. ballistic propagation)
- 2) With extragalactic magnetic fields (ie. diffusive propagation)

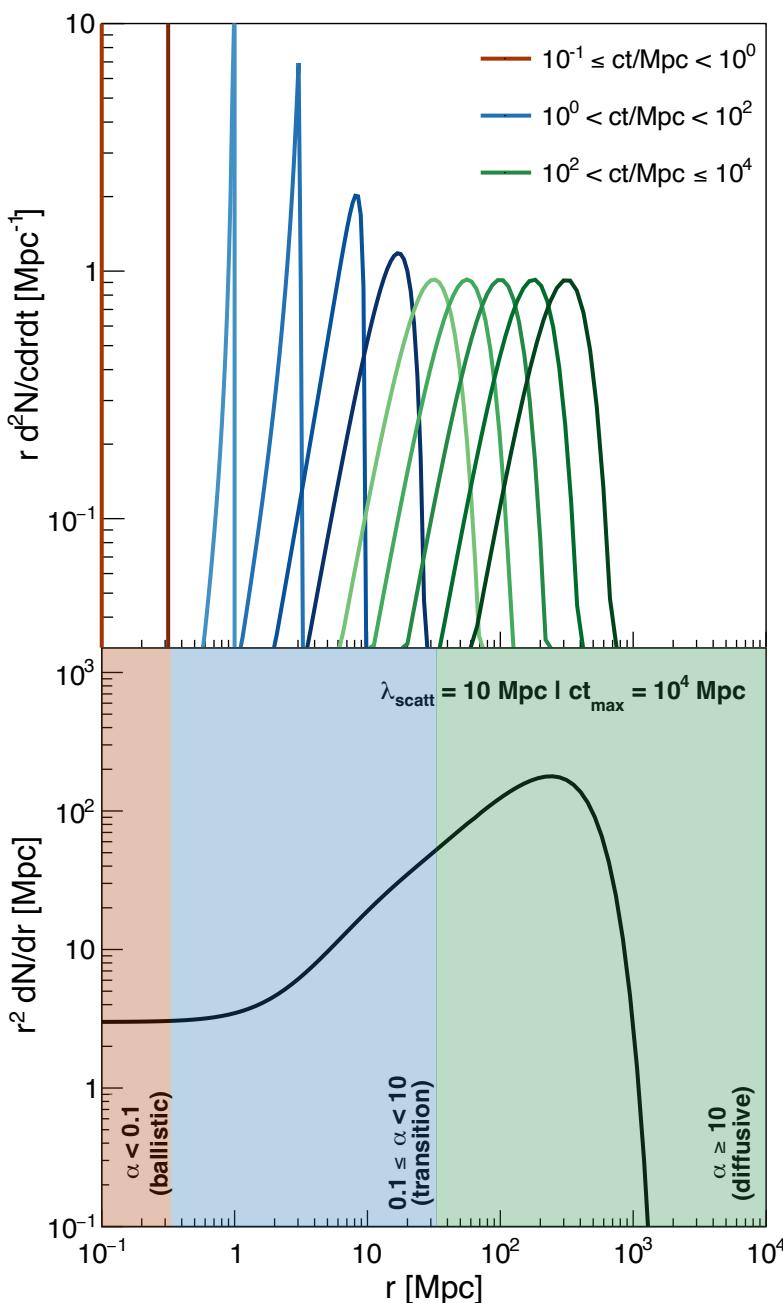


$$dF = \frac{1}{r^2} n dV$$

$$F_t = \int_0^{r_{\max}} \frac{dF}{dr} dr$$

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Magnetic Horizon Effect



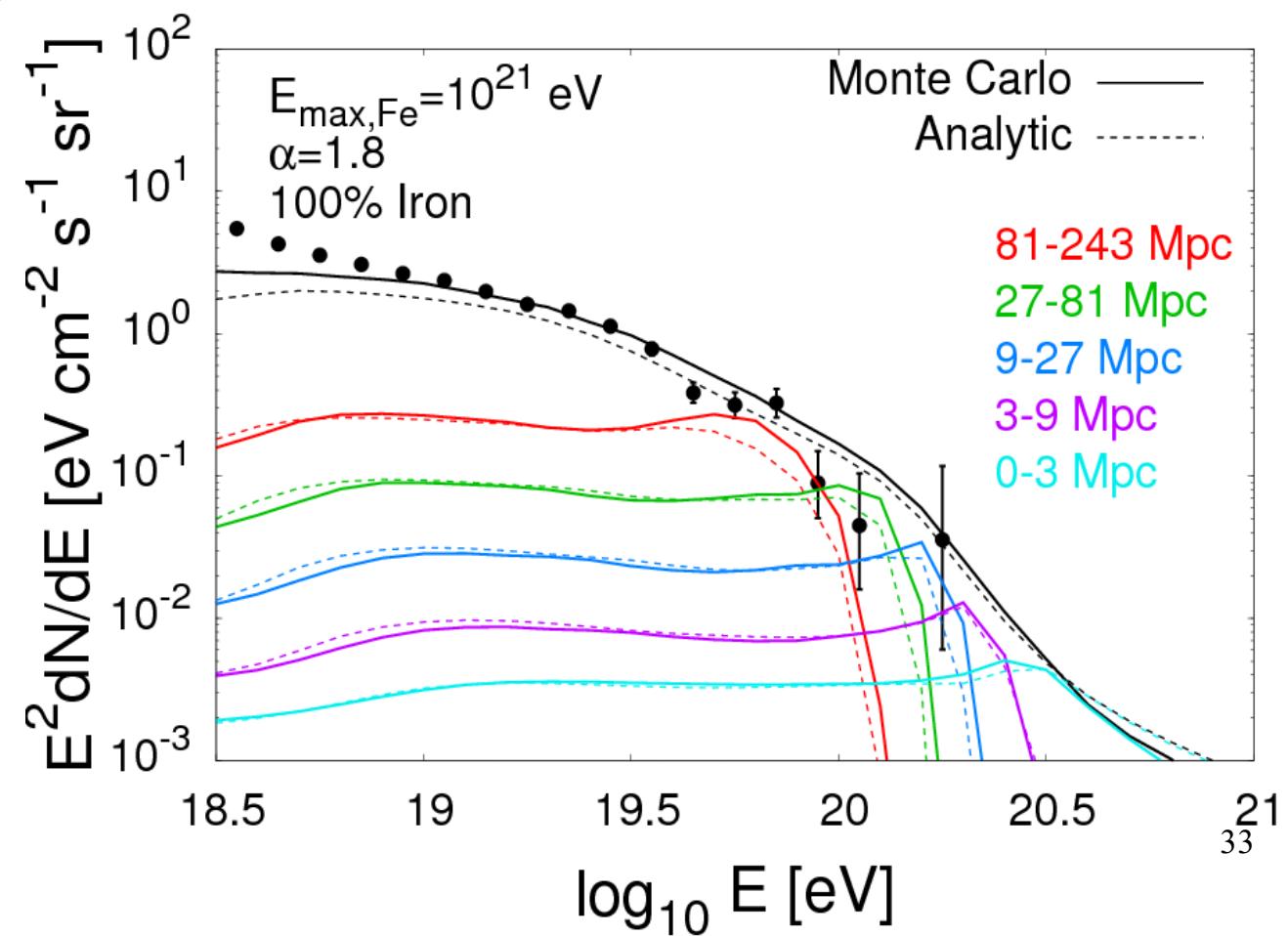
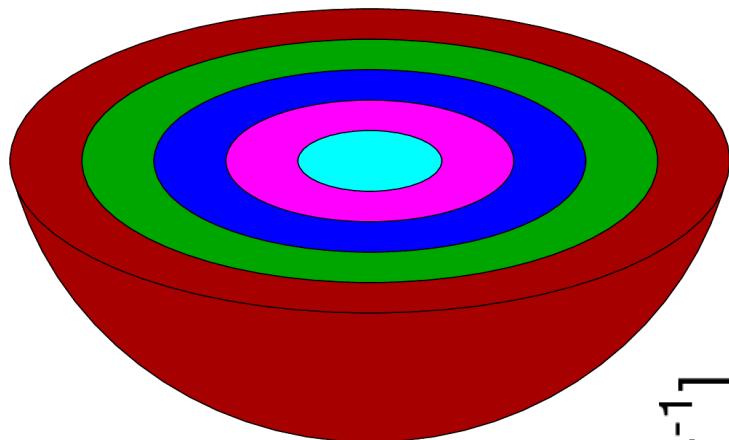
$$\mathbf{F}_t = \int_0^{r_{\max}} \frac{d\mathbf{F}}{dr} dr$$

Constant for ballistic propagation

If cosmic ray sources were continuously distributed in space, magnetic fields wouldn't alter the total cosmic ray spectrum at Earth.

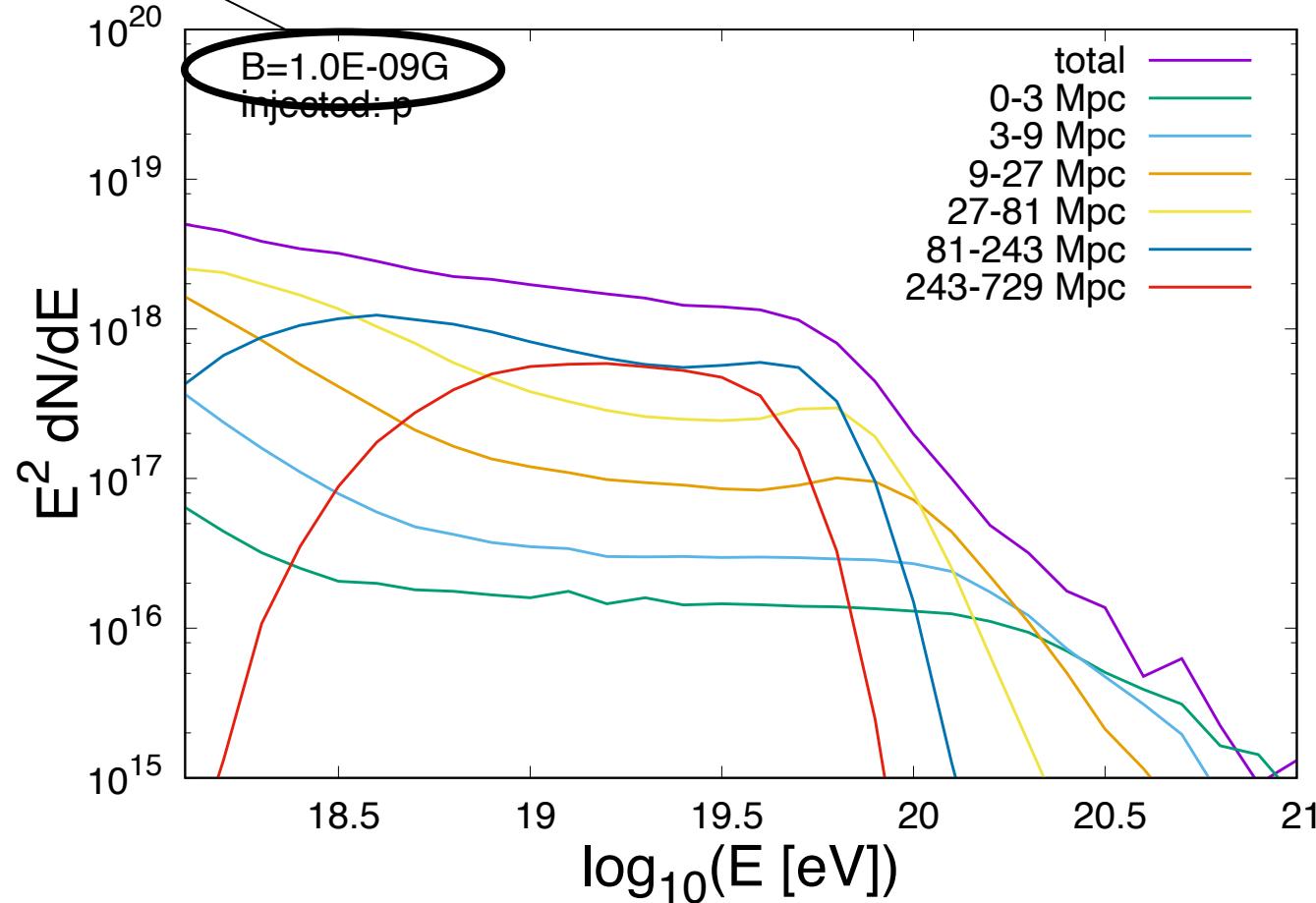
How does the discreet nature of cosmic ray sources alter this statement?

Local Scales Effect Highest Energies (logarithmic scale)



Magnetic Horizon Effect -Local Scales Also Effect Low Energies

Note strong B-field strength considered



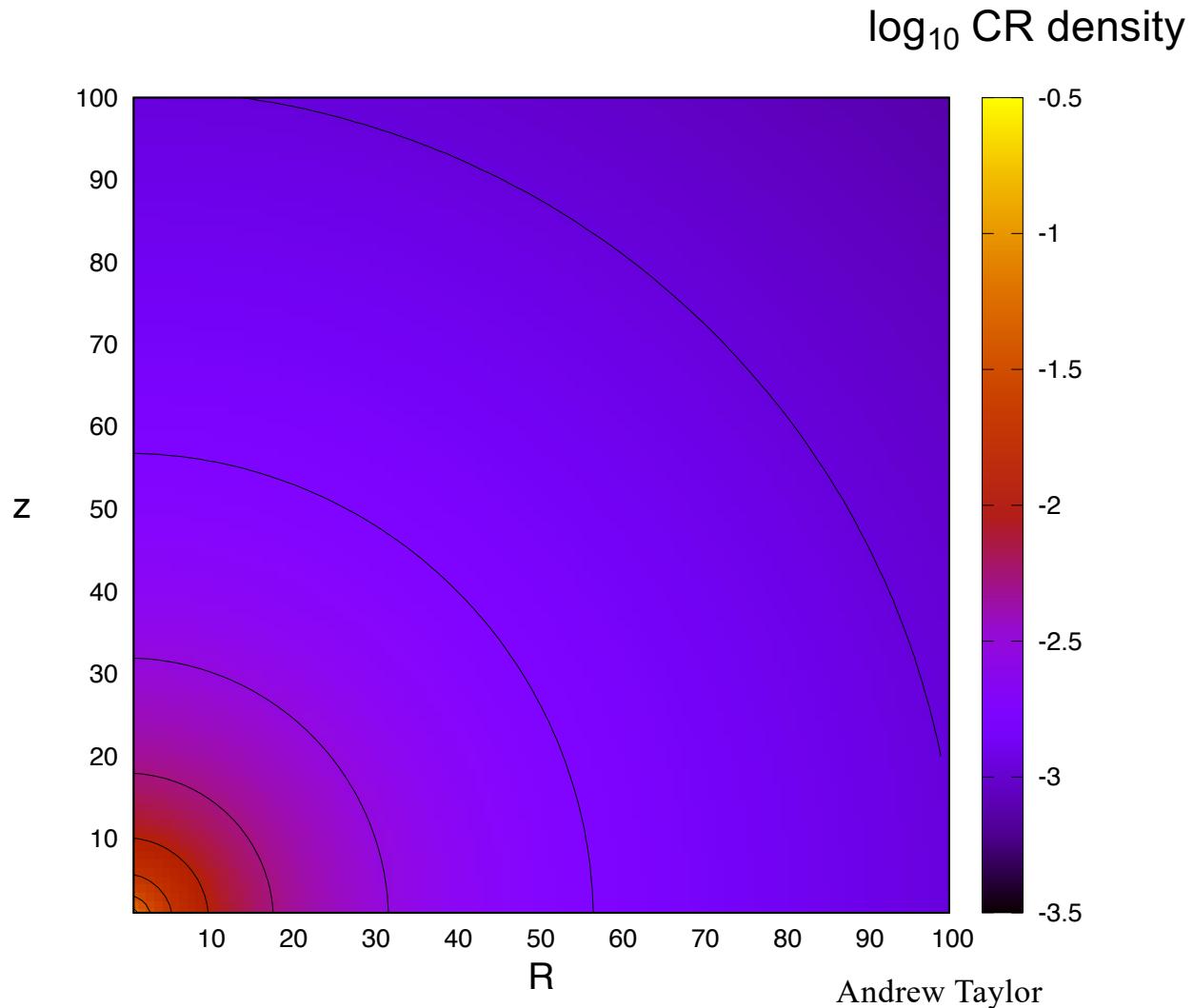
Skymap Effects of Magnetic Fields

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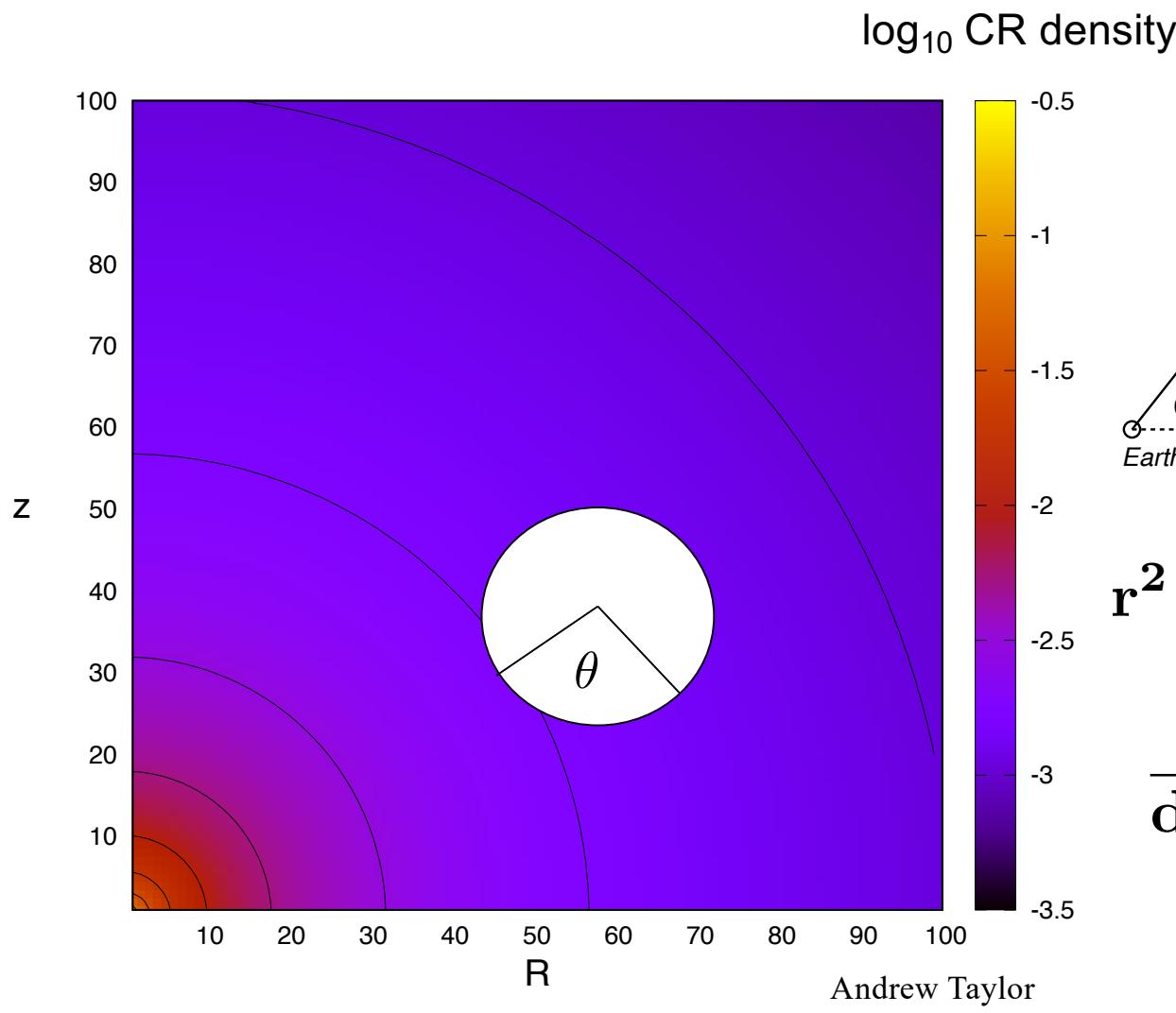
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Steady State Distribution Around a Source of Diffusing Particles

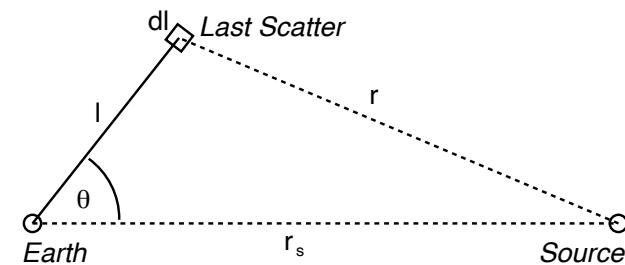


Steady State Distribution Around a Source of Diffusing Particles



Lang+, PRD 103, 063005

Dipole observed



$$r^2 = l^2 + r_s^2 - 2lr_s \cos \theta$$

$$\frac{dN}{d \cos \theta} \propto \frac{1}{r}$$

$$\propto \frac{1}{r_s} \left(1 + \frac{1}{r_s} \cos \theta \right)$$

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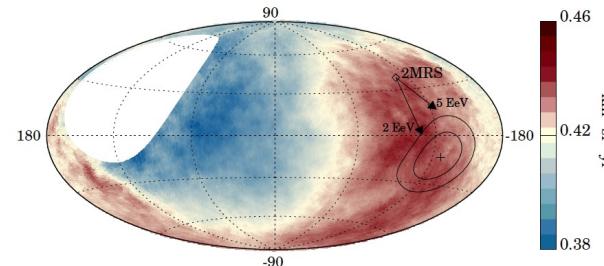
Diffusive and Ballistic Propagation of CR from Sources

$$U_{\text{CR}} = \frac{L_{\text{CR}}}{4\pi D r}$$

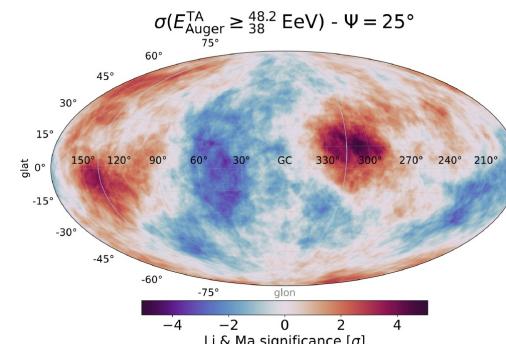
$$U_{\text{CR}} = \frac{L_{\text{CR}}}{4\pi r^2 c}$$

Dipole observed

$$\frac{dN}{d \cos \theta} \propto \left(1 + \frac{\lambda_{\text{scat}}}{r_s} \cos \theta \right)$$



Objects look like a point sources



The (Considerable) Unknowns About Extragalactic Magnetic Fields

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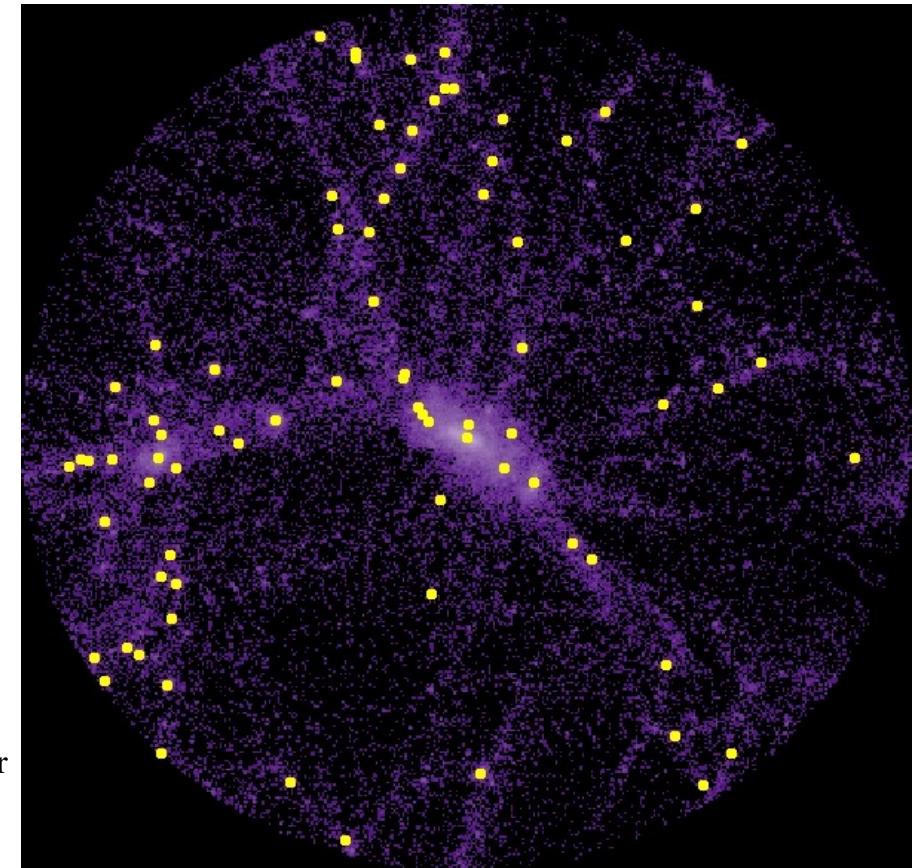
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Extragalactic Magnetic Fields

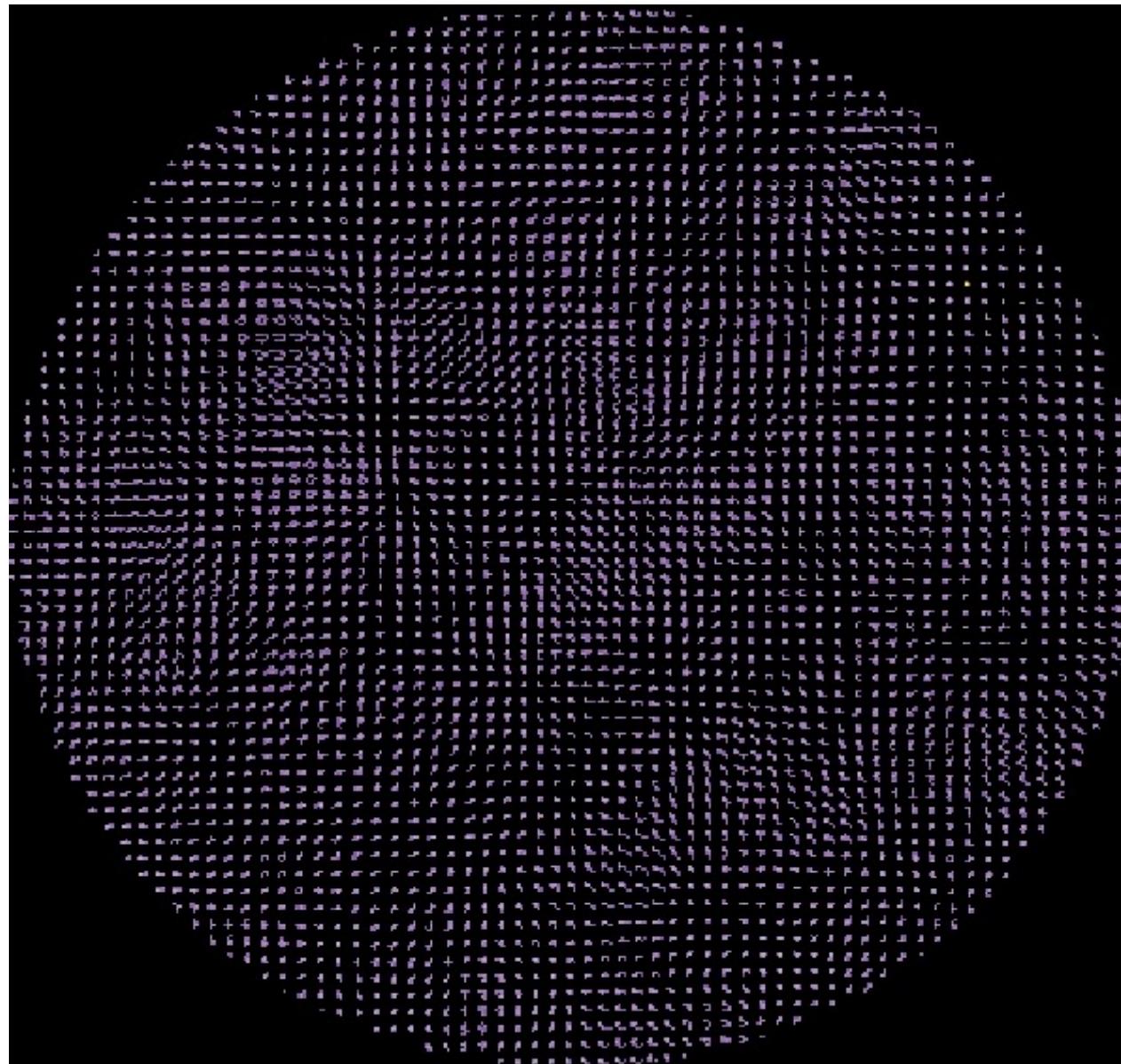
The homogeneous scale for the Universe is thought to be 100 Mpc – is possible that the magnetic field in local extragalactic space is structured (the matter is structured on these scales).

What is the EGMF structure/strength in the inhomogeneous region around the Milky Way?



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Extragalactic Magnetic Field Origin?

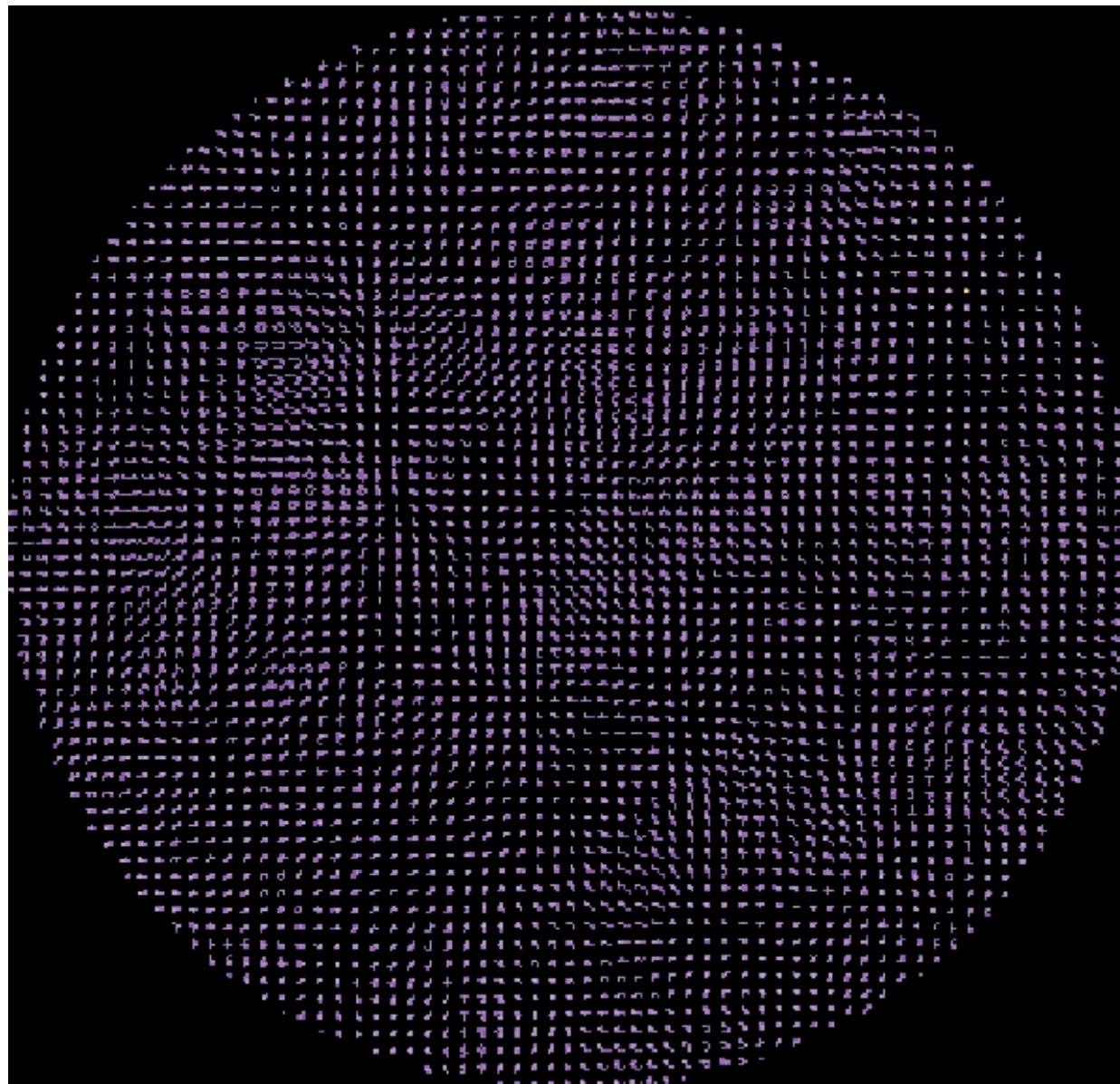


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$z = 40$

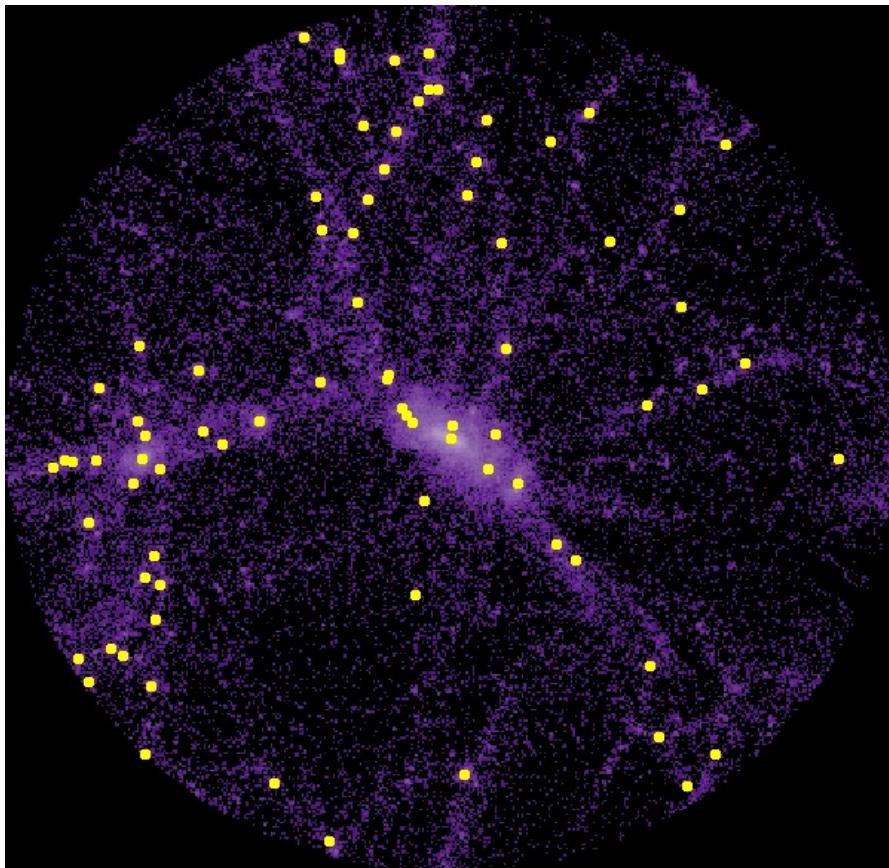
Seed B-field
strength?

Extragalactic Magnetic Field Origin?



...compression and
dynamo action lead
to $\sim \mu\text{G}$ B-field
strength growth on
galactic scales

How Do Galactic and Extragalactic Magnetic Fields Merge?



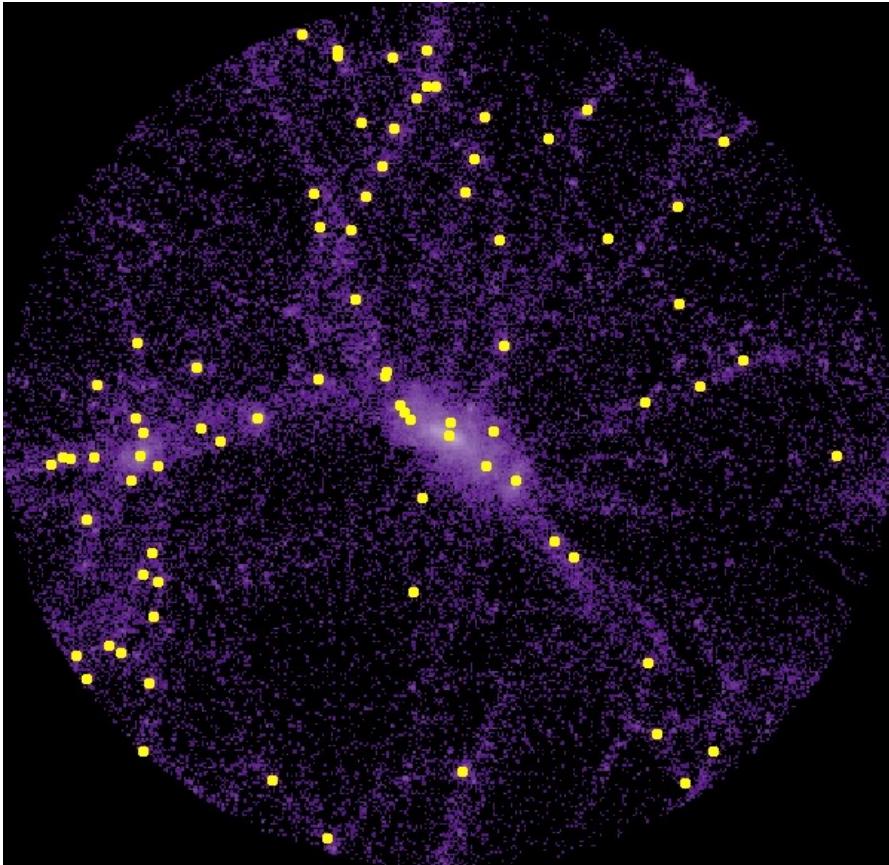
Question- do Galactic halo (out to the virial radius) or Extragalactic magnetic fields dominate the deflection of UHECR?

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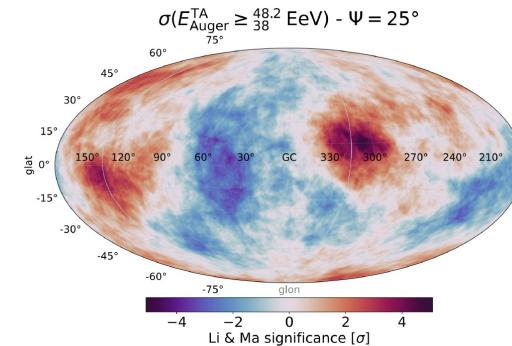
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How Do Galactic and Extragalactic Magnetic Fields Merge?



Question- do Galactic halo (out to the virial radius) or Extragalactic magnetic fields dominate the deflection of UHECR?



Perhaps the UHECR skymap can be used to determine this

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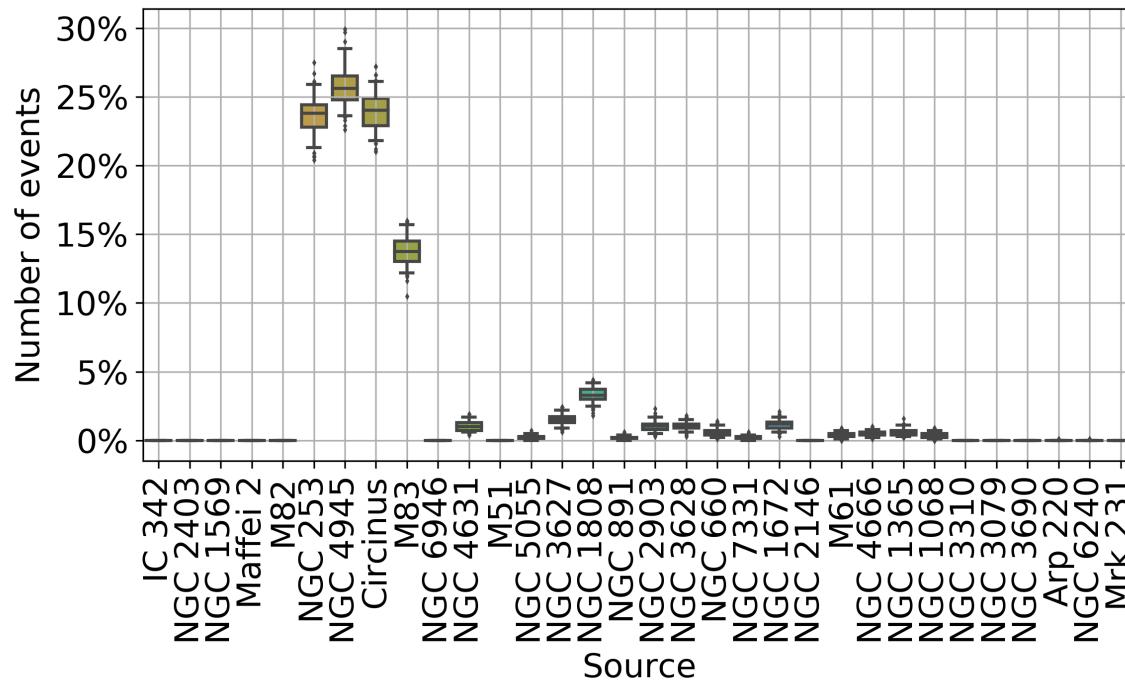
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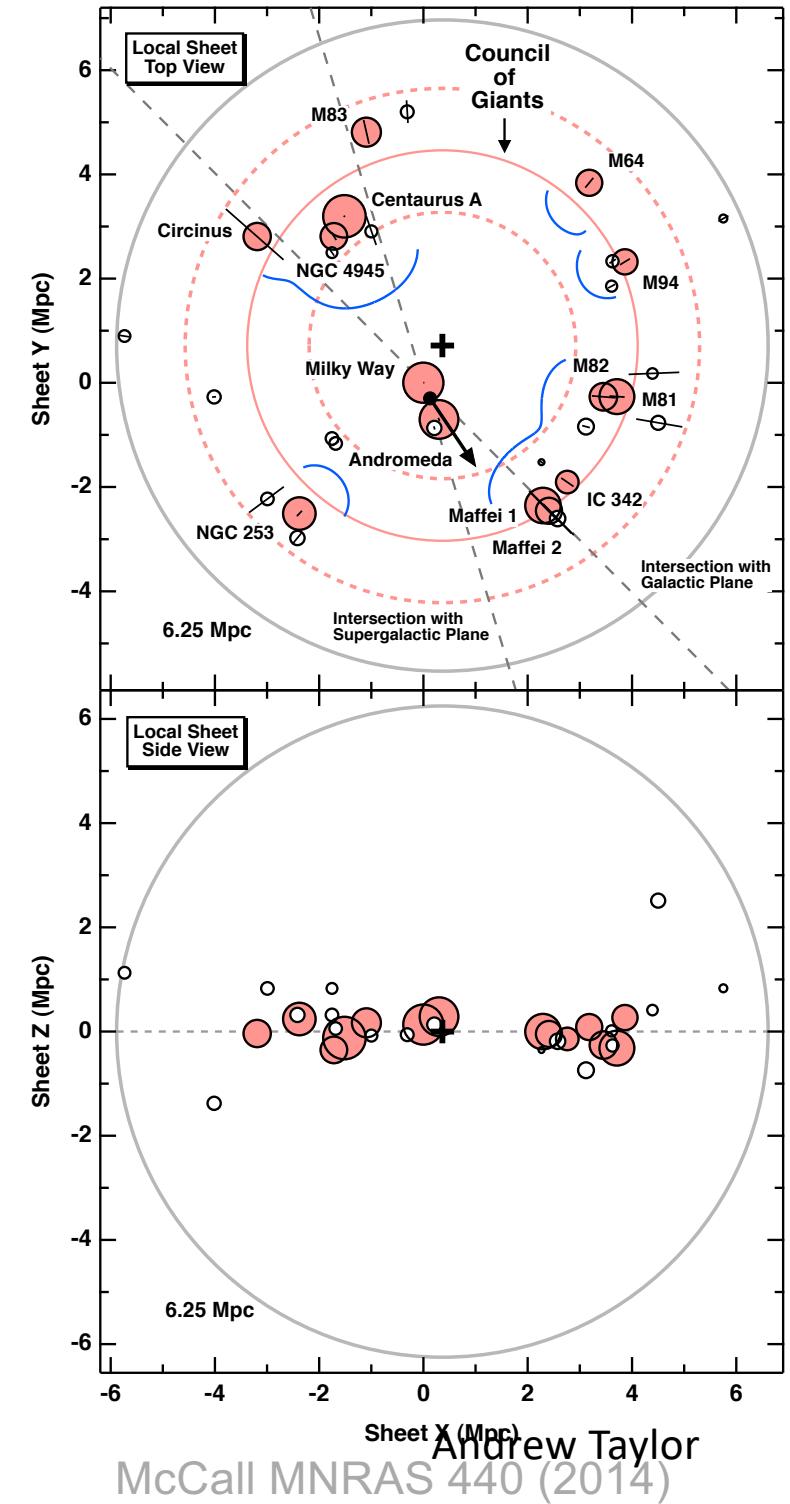
Our Local Extragalactic Neighbourhood

The local (<10 Mpc) extragalactic objects are structured, sitting in a roughly circular disk shape around the Milky Way

van Vliet MNRAS 510 (2021)



DESY. Bregman et al. ApJ 928 (2022)



Andrew Taylor
McCall MNRAS 440 (2014)

The Uniqueness of Cen A within the Council of Giants

$$t_{\text{acc}} = \eta \frac{R_{\text{lar}}}{c\beta^2}$$

$$t_{\text{esc.}} = \frac{R}{c\beta}$$

$$E_{\text{max}} = \beta e B R$$

AM Hillas (1984)

$$L_B = U_B 4\pi R^2 \beta c$$

Under the assumption of equipartition of energy between kinetic energy and magnetic field:

Lovelace et al. (1976)

$$E_{\text{max}} \lesssim \frac{Z}{\eta} (\beta L_{\text{KE}} \alpha \hbar)^{1/2} \approx 10 \frac{Z}{\eta} \left(\frac{\beta L_{\text{KE}}}{3 \times 10^{43} \text{ erg s}^{-1}} \right)^{1/2} \text{ EeV}$$

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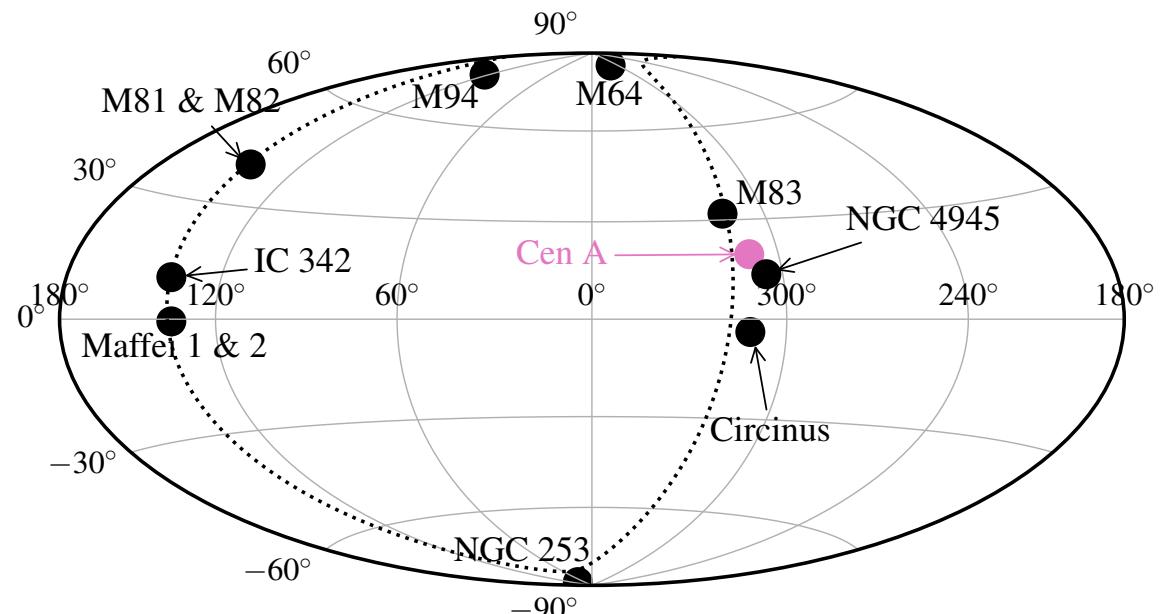
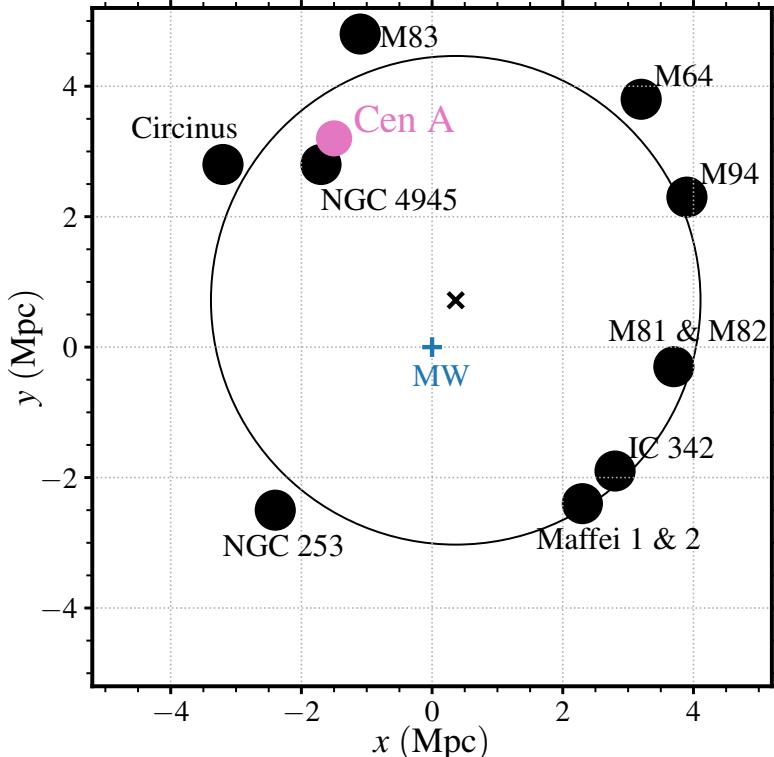
Andrew Taylor

Local Extragalactic Structure The Council of Giants

Cen A is unique within the council of giant structure are being the only object proving a kinetic luminosity capable of giving rise to multi EeV acceleration

Lovelace et al. (1976)

$$E_{\max} \lesssim \frac{Z}{\eta} (\beta L_{\text{KE}} \alpha \hbar)^{1/2} \approx 10 \frac{Z}{\eta} \left(\frac{\beta L_{\text{KE}}}{3 \times 10^{43} \text{ erg s}^{-1}} \right)^{1/2} \text{ EeV}$$



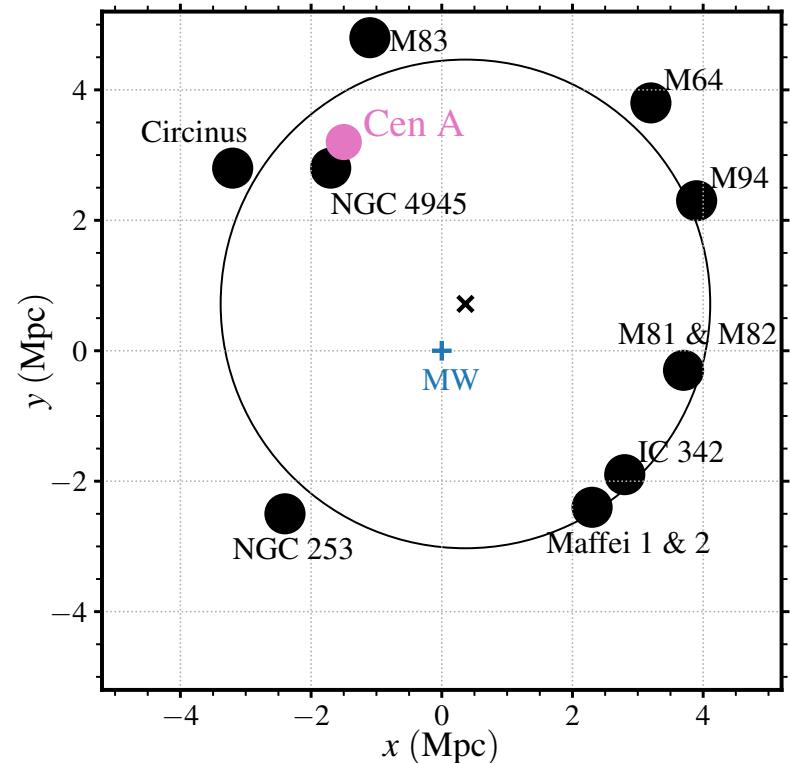
Simulation Setup

- Particles initially fill 300 kpc region surrounding Cen A (isotropic momentum distribution)
- Large angle particle scattering occurs within the virial region (< 300 kpc) of all members of the council of giant system
- Outside the virial radii of these galaxies the particle propagation is treated as ballistic
- Fundamental parameter of problem-optical depth of scattering regions

$$\tau = \frac{r_{\text{vir}}}{l_{\text{sc}}}$$

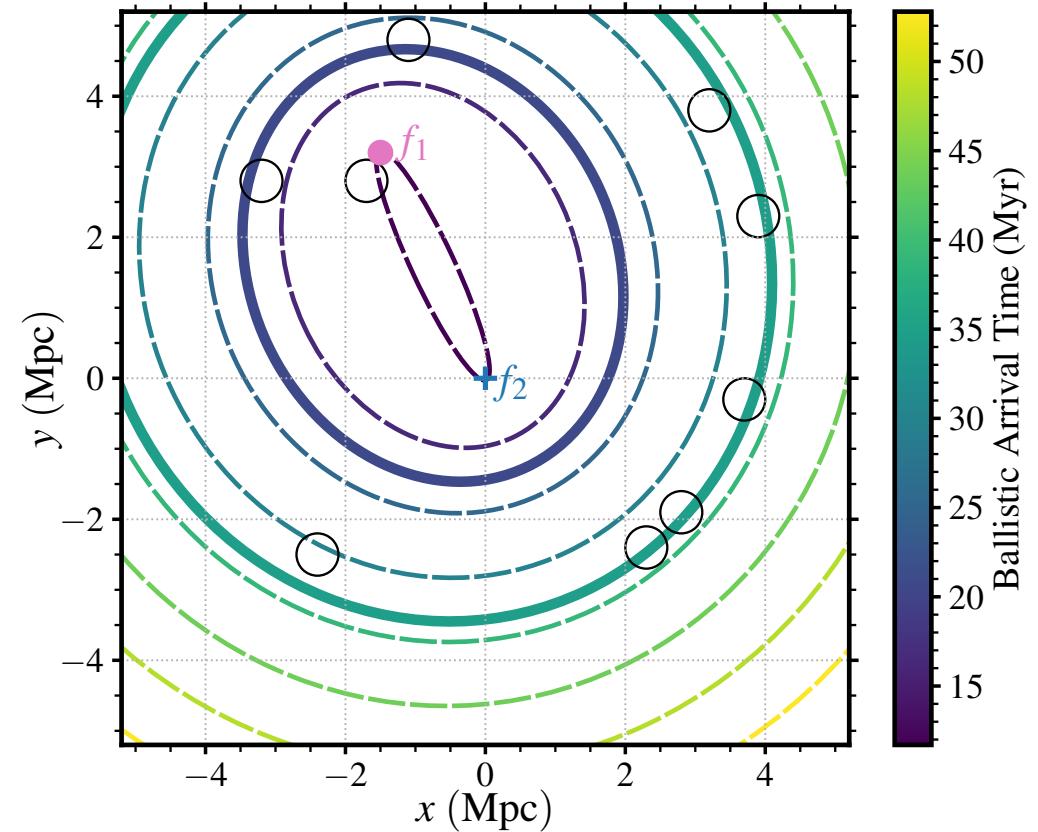
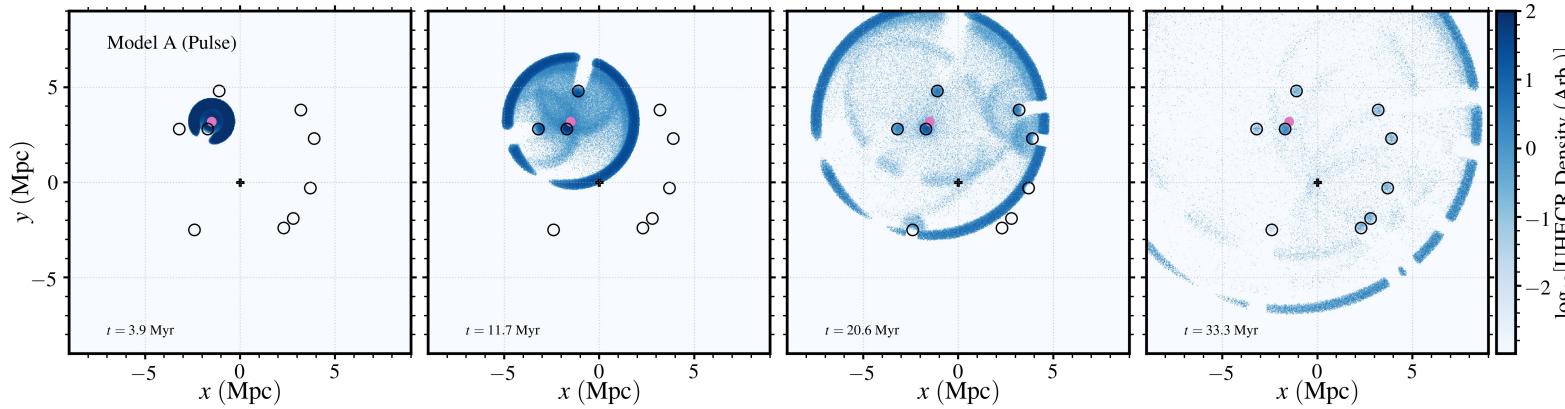
- Echo signals results are rather insensitive to optical depth of scattering regions, provided

$$\tau > 1$$

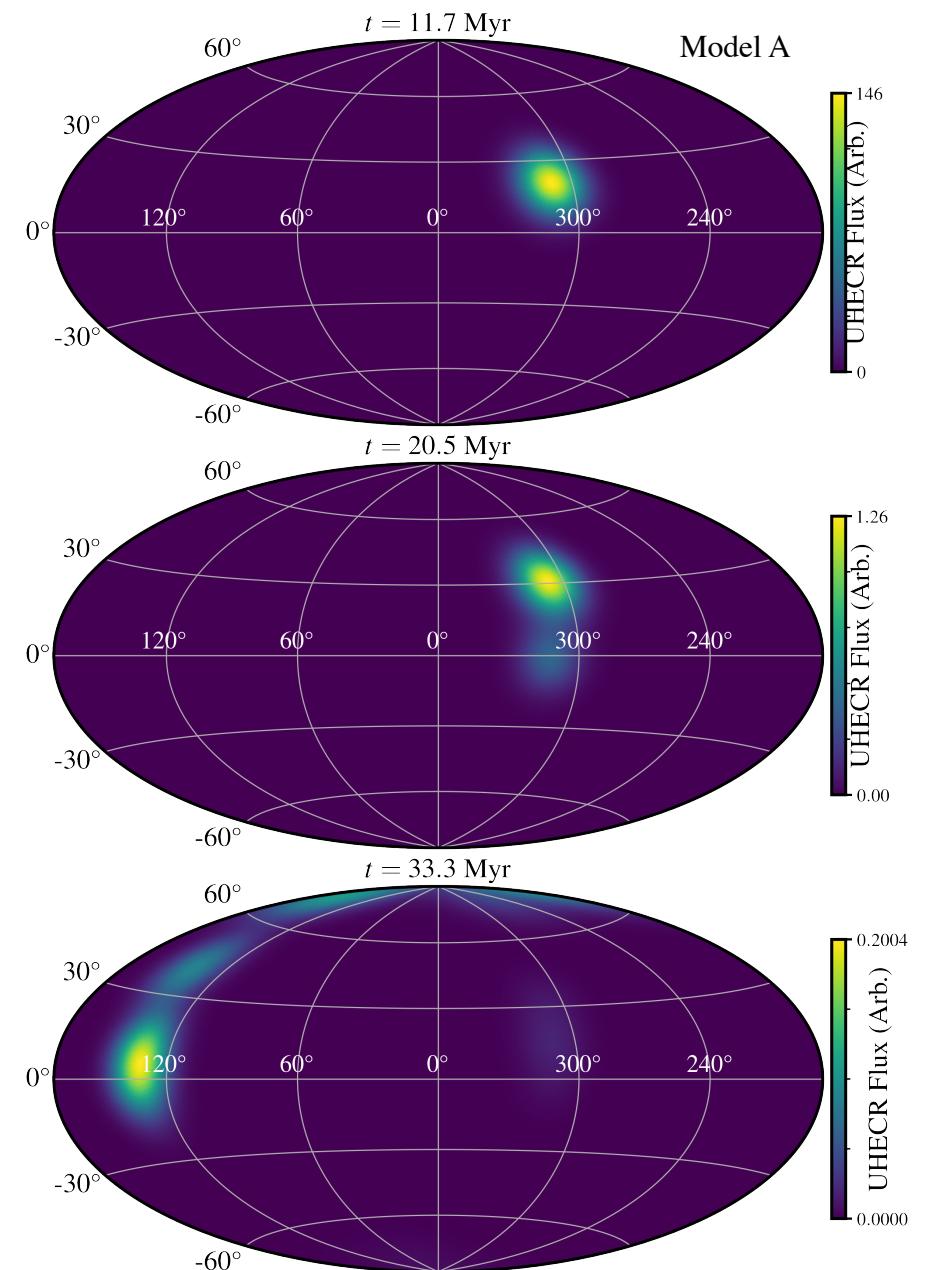
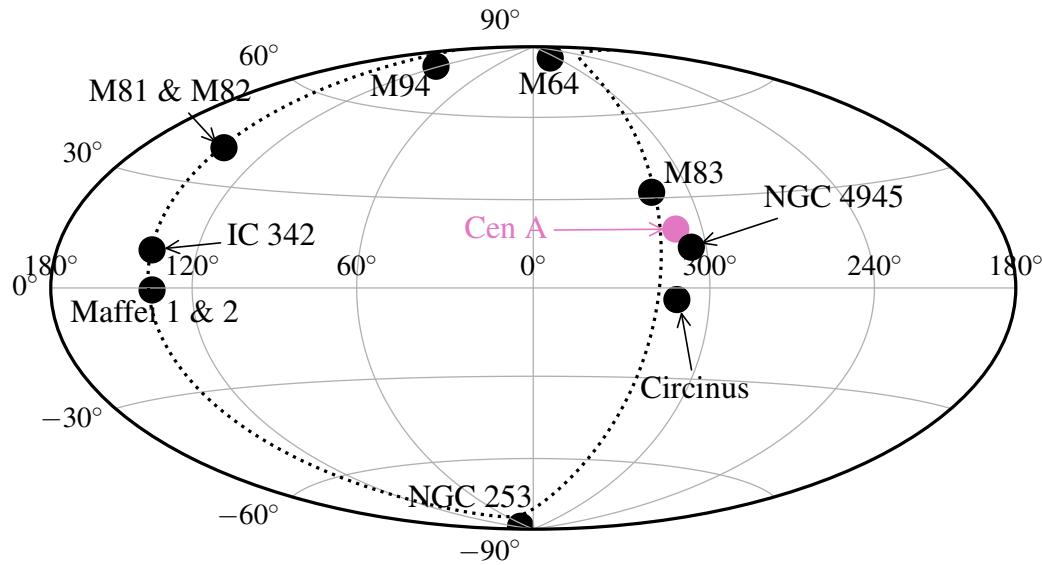


- Only He and Fe injected into the system (fragile and robust species compared to crossing time of system)
- Particles photo-disintegrate en-route in extragalactic radiation fields
- 30 EeV particles being focused on
- Deflections from MW magnetized halo intentionally left out

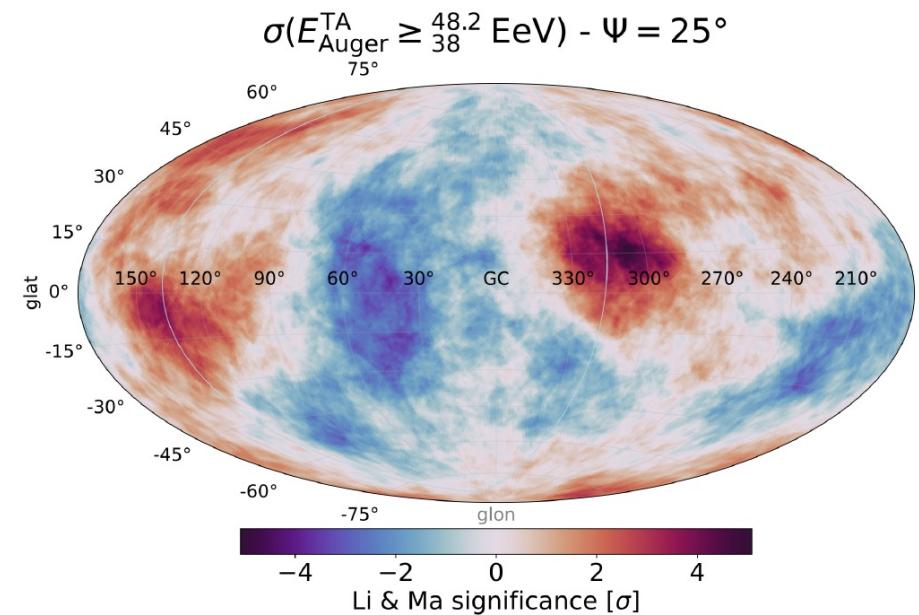
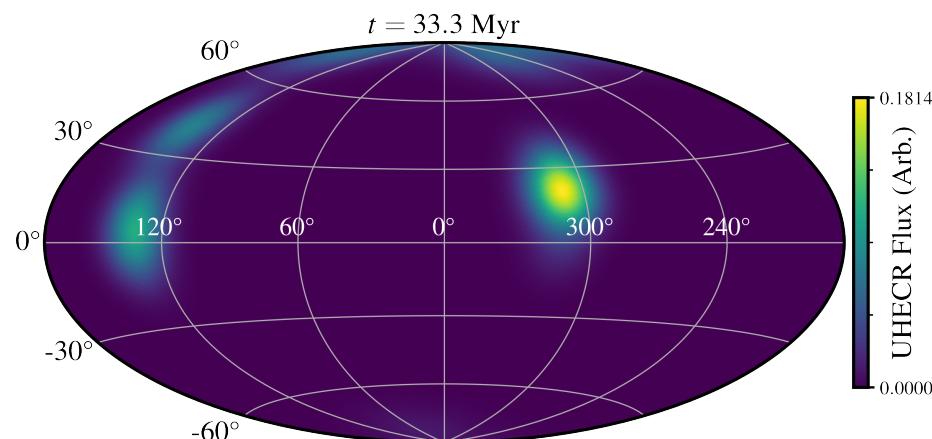
Simulations of UHECR Propagation Through the CoG Structure



Milky Way Based Observers



Is Local Magnetic Structure Imprinted on the UHECR Skymap?



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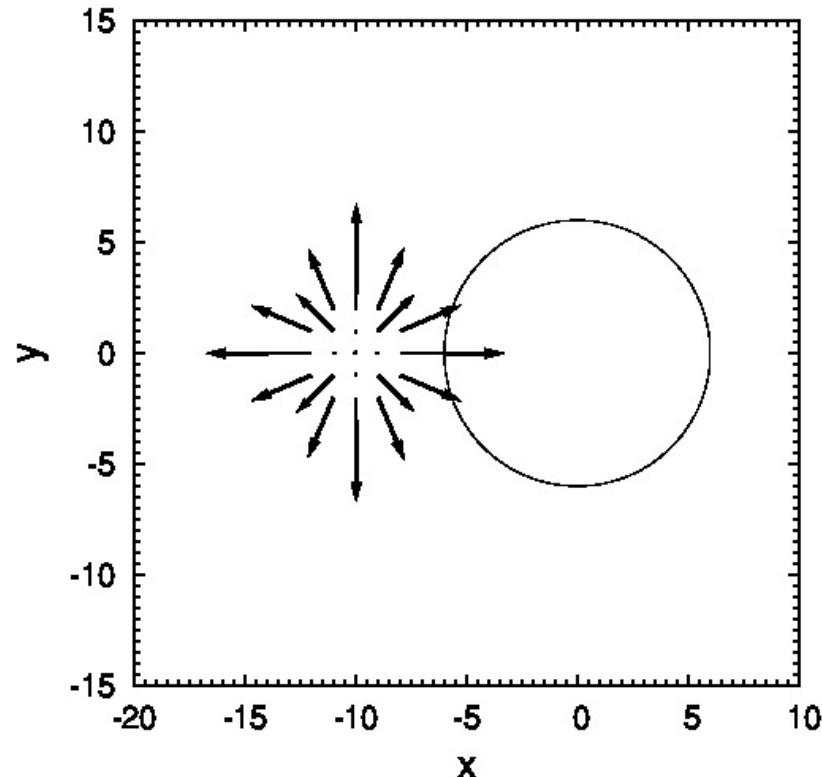
Conclusion

- Cascades in hydrodynamics and magneto-hydrodynamics lead to the formation of turbulence
- Charged particle propagation is dictated by magnetic structure, and in particular by magnetic turbulence structure
- Extragalactic magnetic fields can prevent the arrival of "low" energy cosmic rays from even the most local sources (the magnetic horizon)
- Our knowledge of the magnetic structure of the Milky Way (+ other galaxies) is particularly poor in the Galactic halo region
- The magnetic structure in our local inhomogeneous patch of the Universe is even more poorly probed
- It seems possible that the arrival of extragalactic cosmic rays can pick up an imprint of the local extragalactic magnetic field structure

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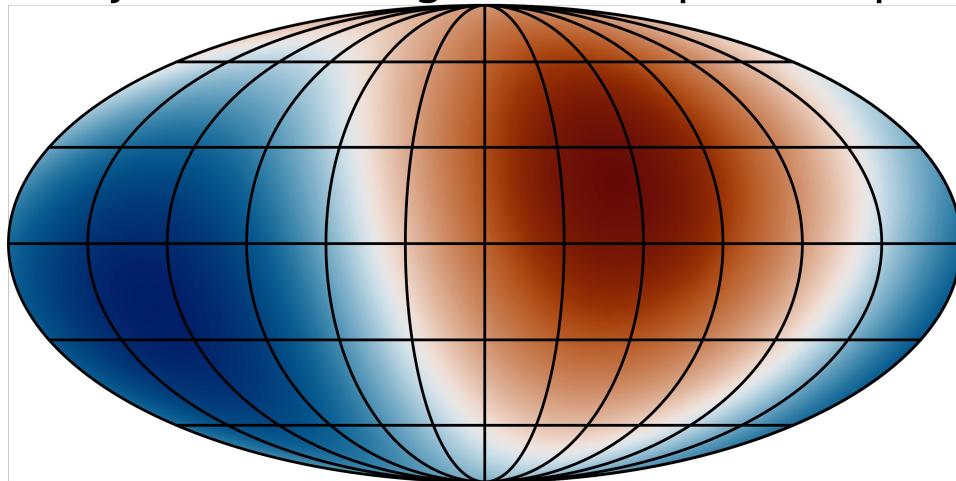
The Local Distortion of the Dipole

What Arrives in Not Necessarily What is Observed



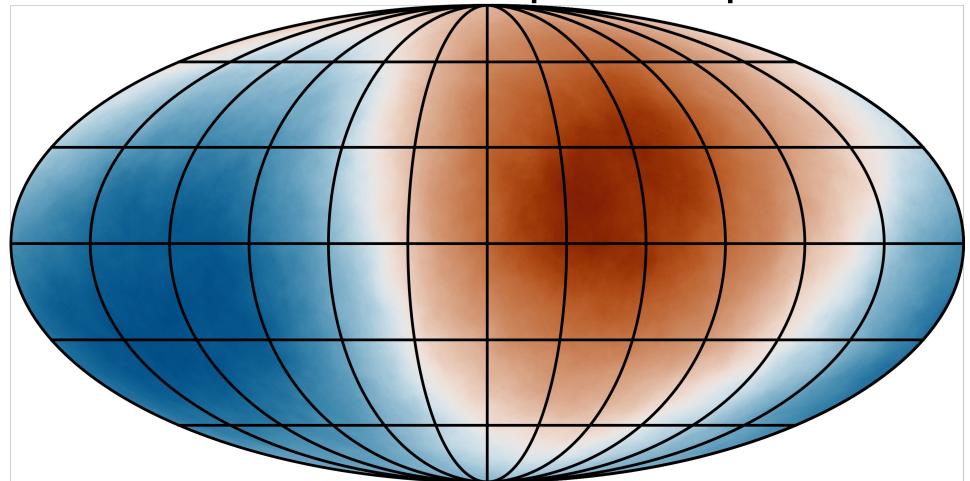
External Dipole Scenario

Injected Extragalactic Dipole Map



-0.2 0.2

Observed Dipole Map



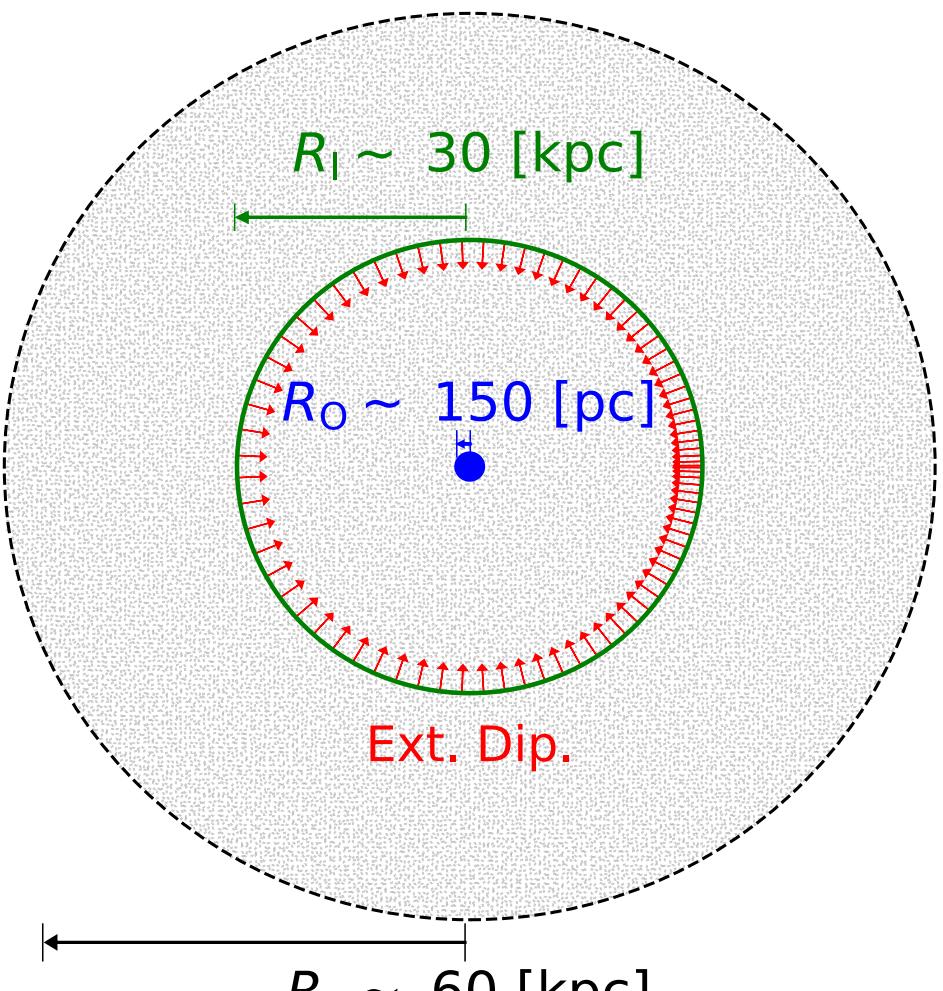
-0.02 0.02

Shaw et al. (in prep.)

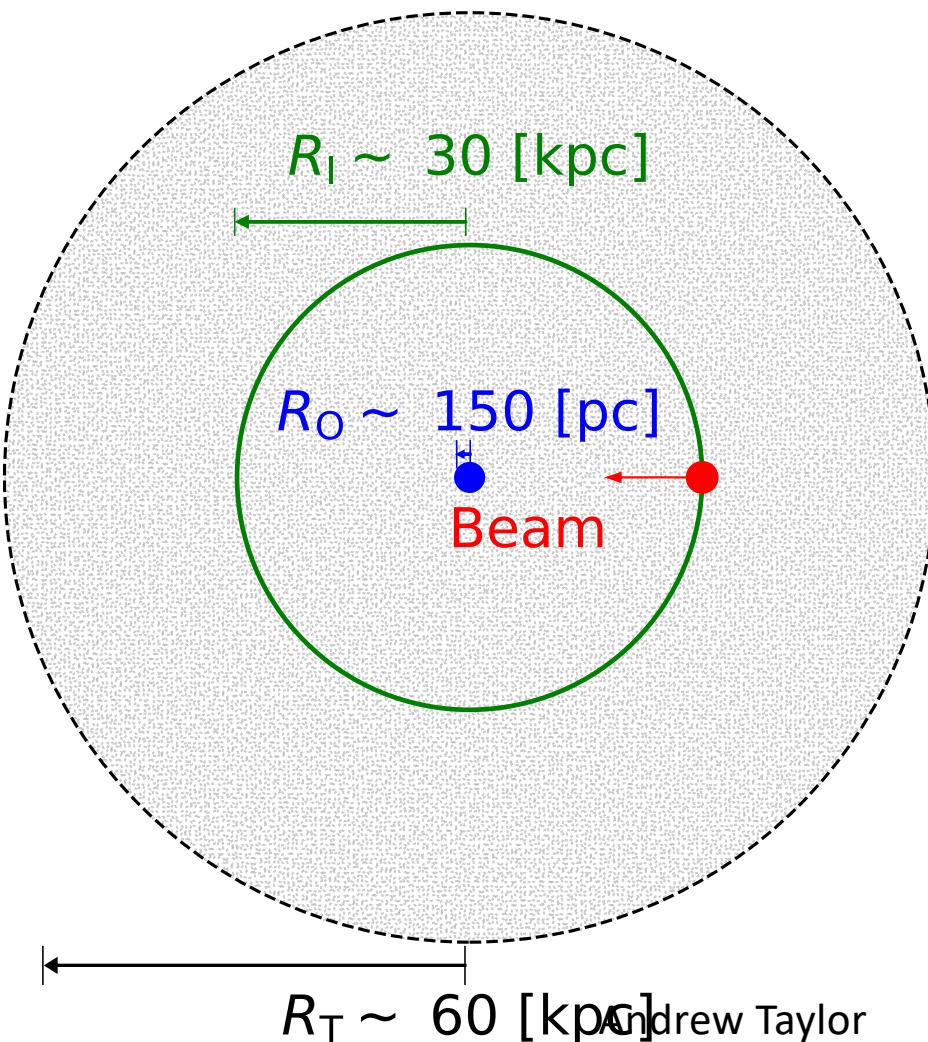
Note the suppression of the dipole amplitude caused by the presence of the Galactic halo

But What if What Arrives is Not a Dipole?

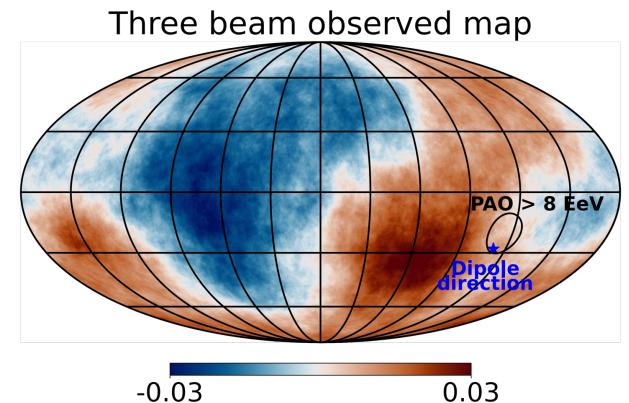
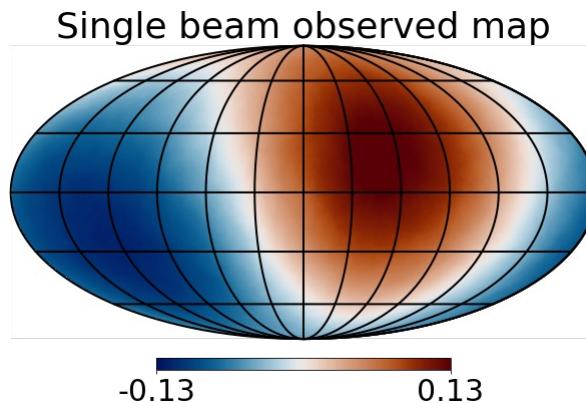
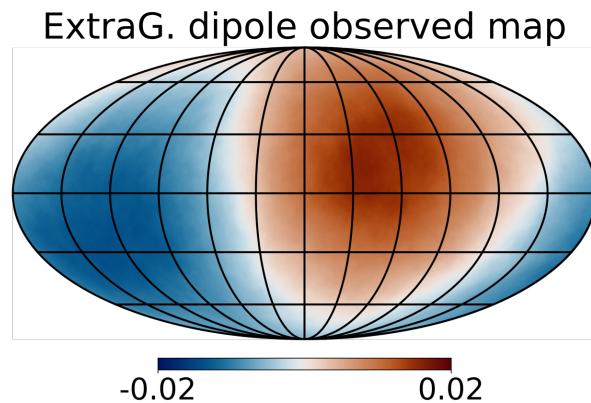
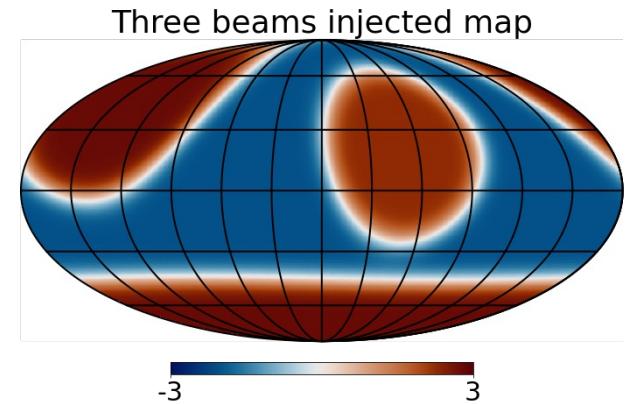
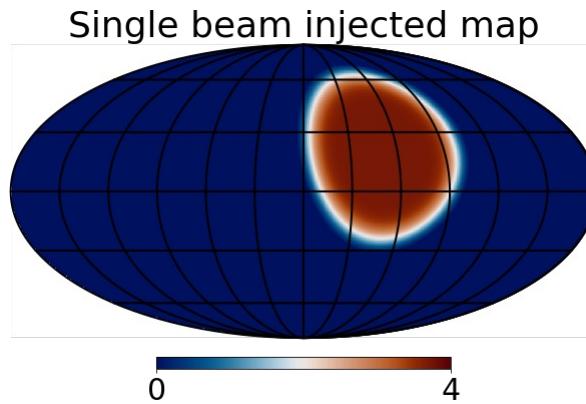
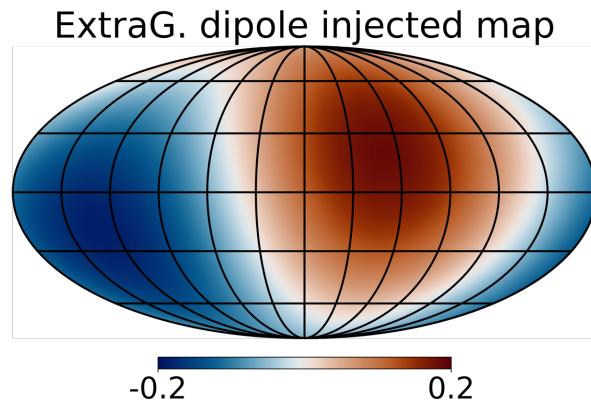
Dipole setup



Beam setup



Injected and Observed Maps

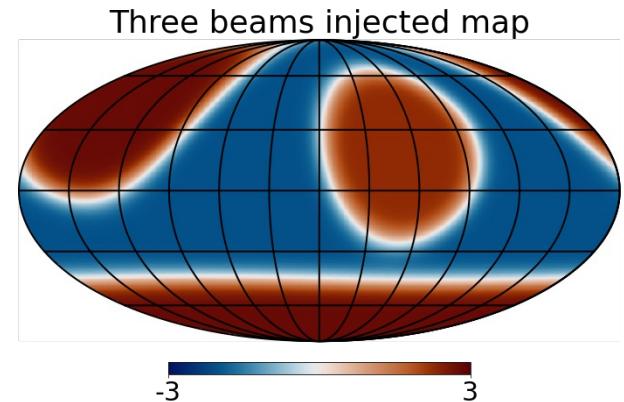
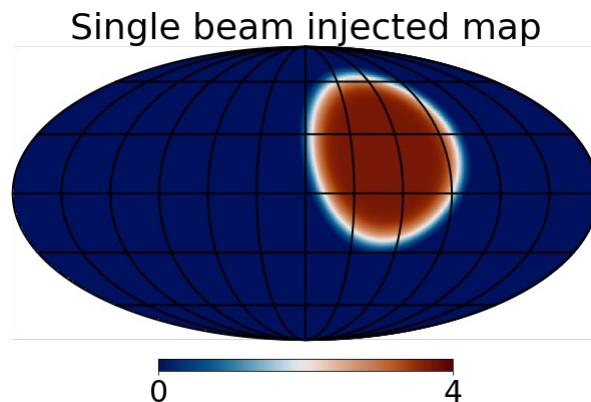
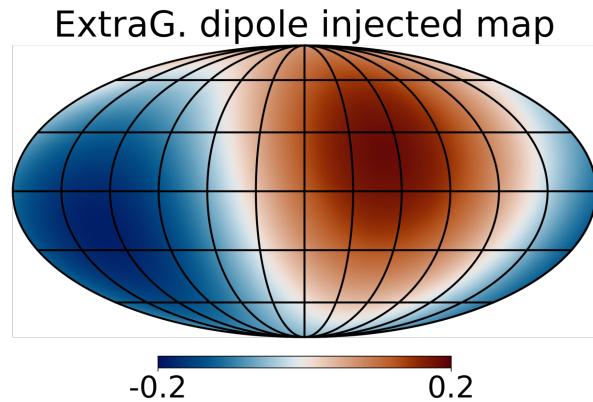


Shaw et al. (in prep.)

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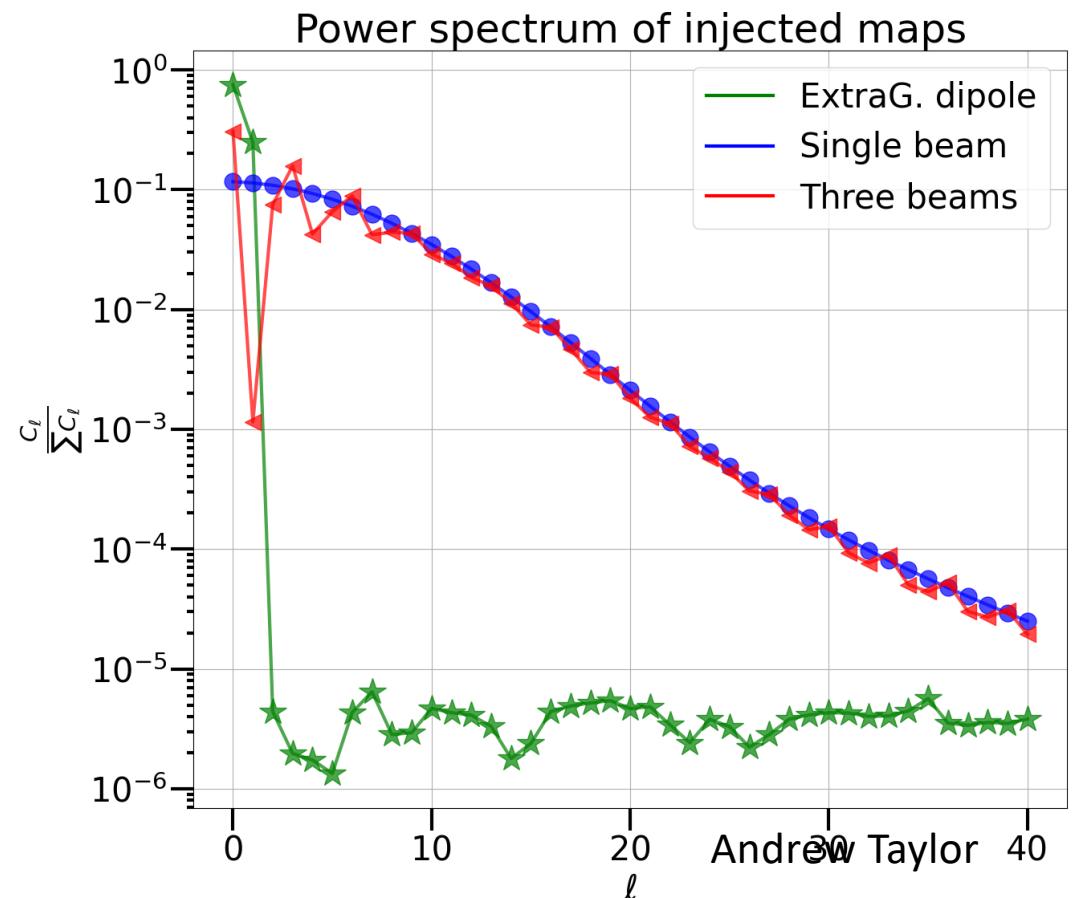
Power Spectra of Injected Maps



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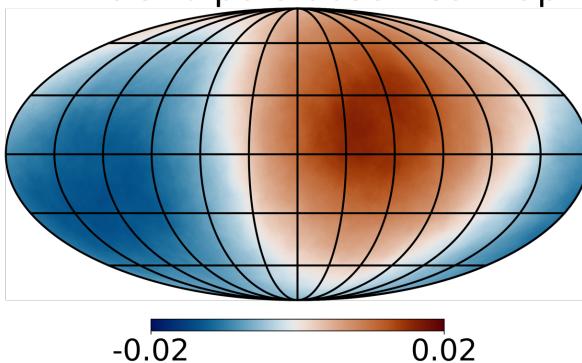
$$\Phi_l = \int_{-1}^1 d\cos\theta \frac{dN}{Nd\cos\theta} \frac{2l+1}{\sqrt{2}} P_l(\cos\theta)$$

$$C_l = \frac{\Phi_1^2 / (2l+1)}{\sum_n \Phi_n / (2n+1)}$$

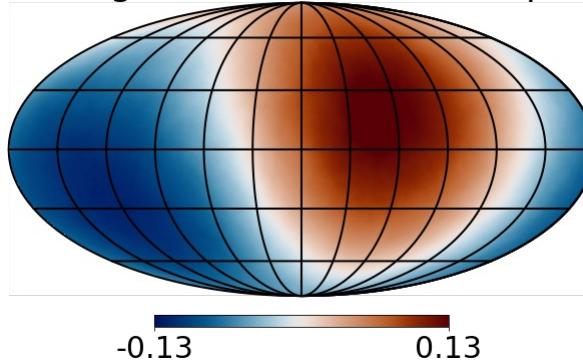


Power Spectra of Observed Maps

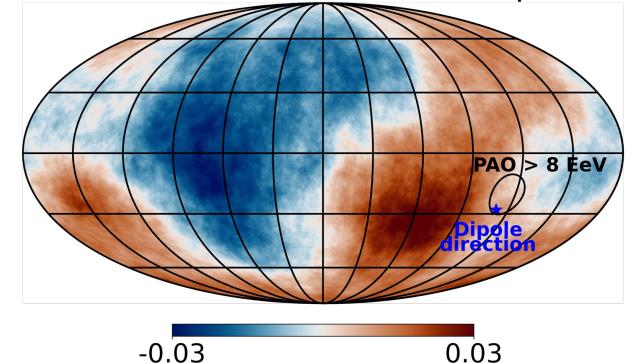
ExtraG. dipole observed map



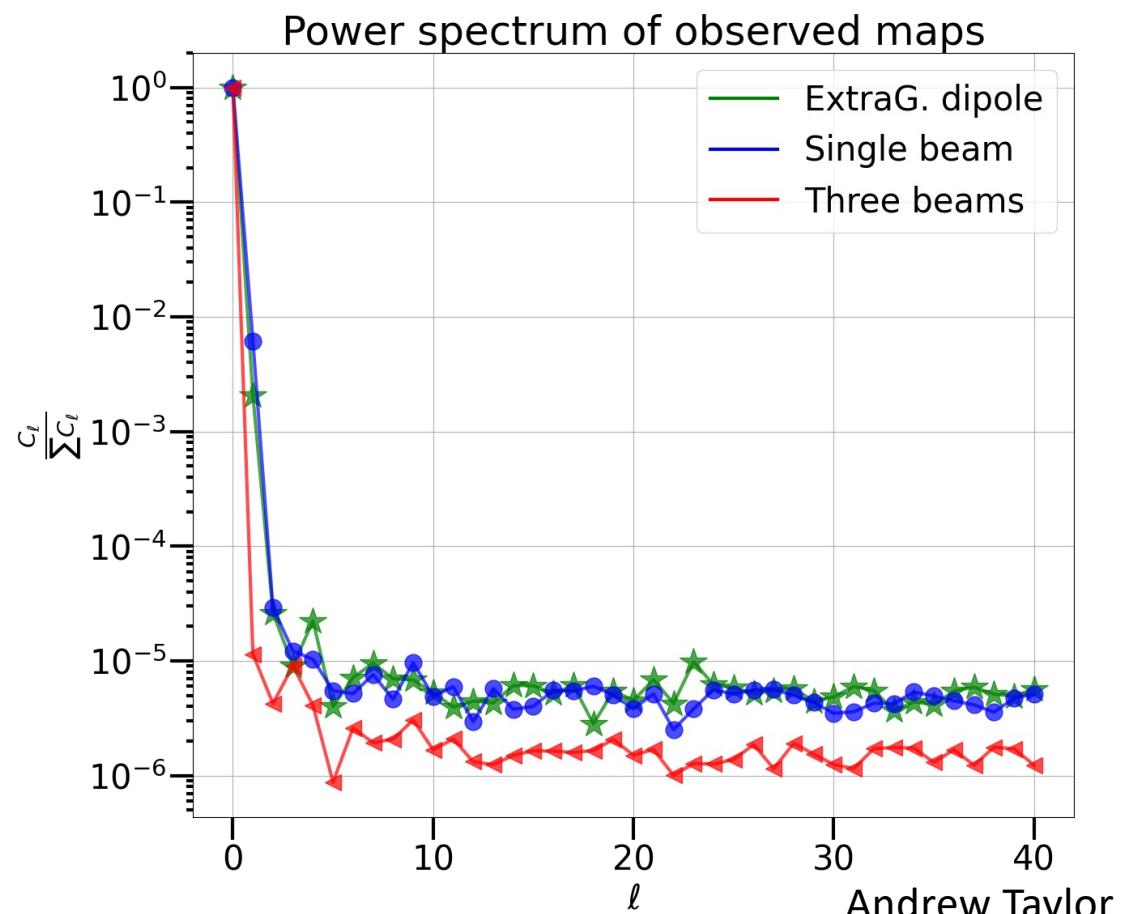
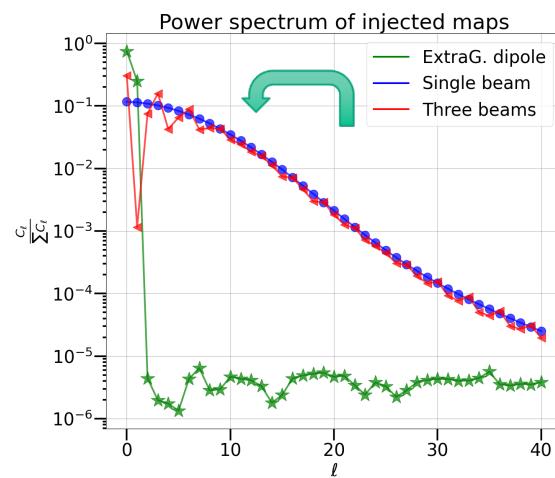
Single beam observed map



Three beam observed map

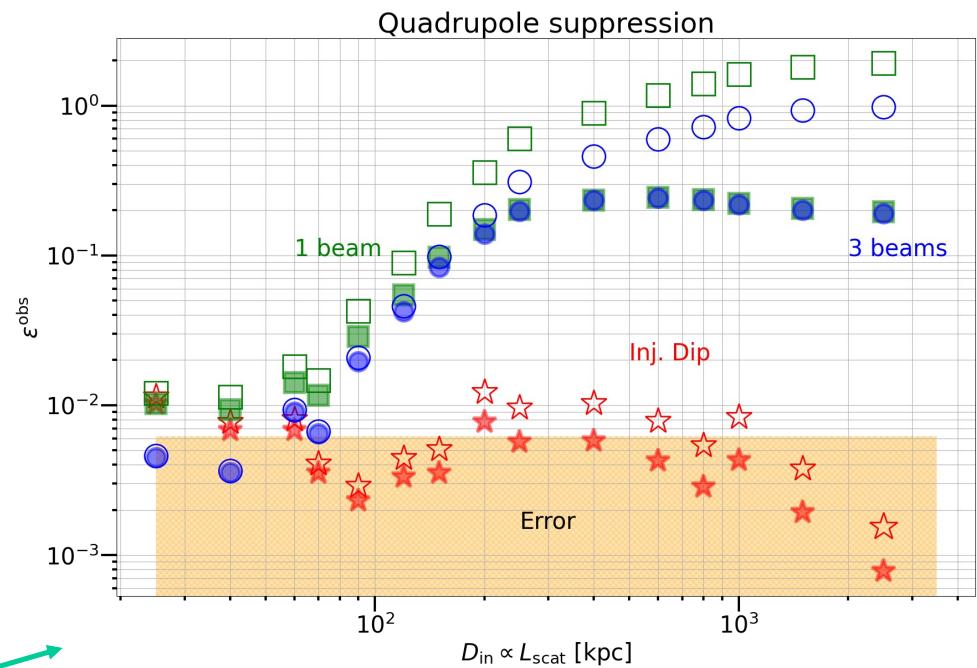
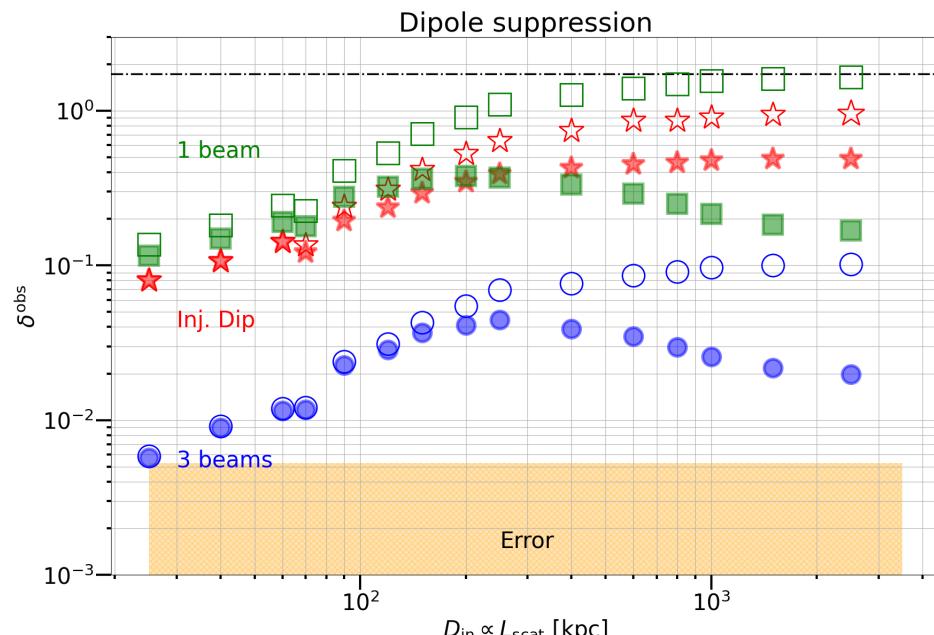


Shaw et al. (in prep.)



The Importance of Measuring the Quadrupole Moment

Shaw et al. (in prep.)



Note that the difference in growth rates of dipole and quadrupole amplitude with L_{scat}