Lecture 3 Plan:

1) UHECR Observational Status
2) Extragalactic radiation fields
3) Cosmic ray proton interaction rates with extragalactic radiation fields
4) Cosmic ray nuclei interaction rates with extragalactic radiation fields
5) Application- what one can infer from spectral and composition information alone

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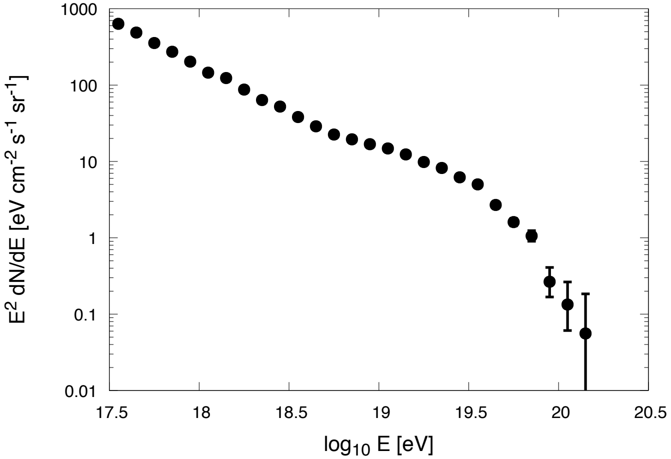
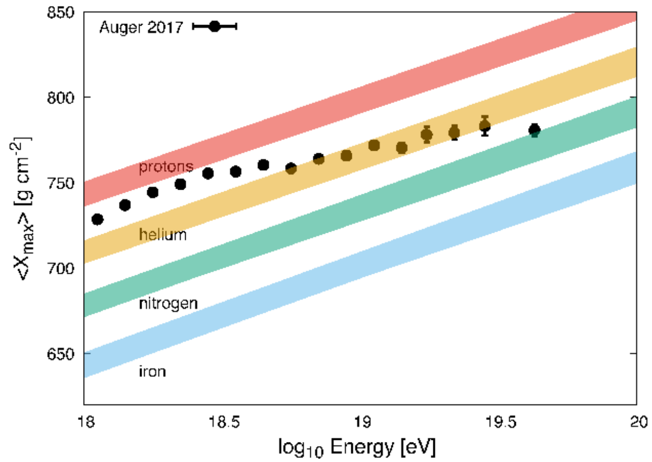
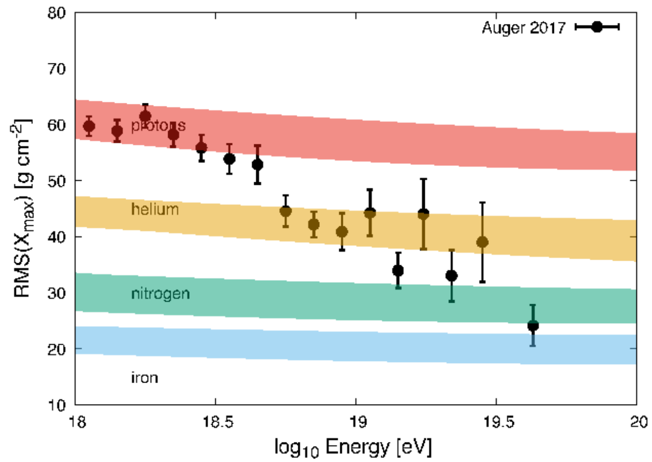
UHECR: The Observational Status

Spectrum

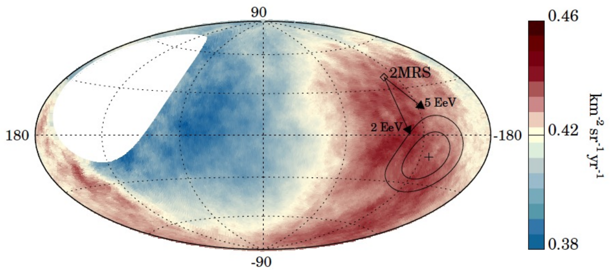


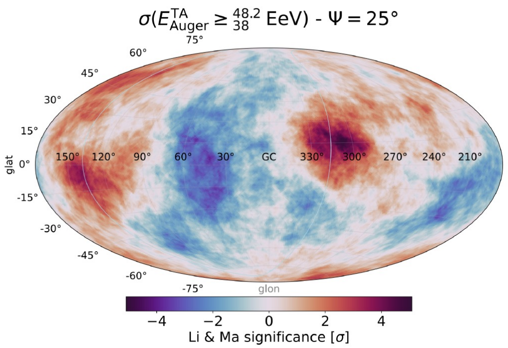
Composition





Anisotropy





Pierre Auger Collaboration. ApJ. 935 (2022)

Caccianiga et al. for the Auger and TA Collaborations. PoS (ICRC2023) 521

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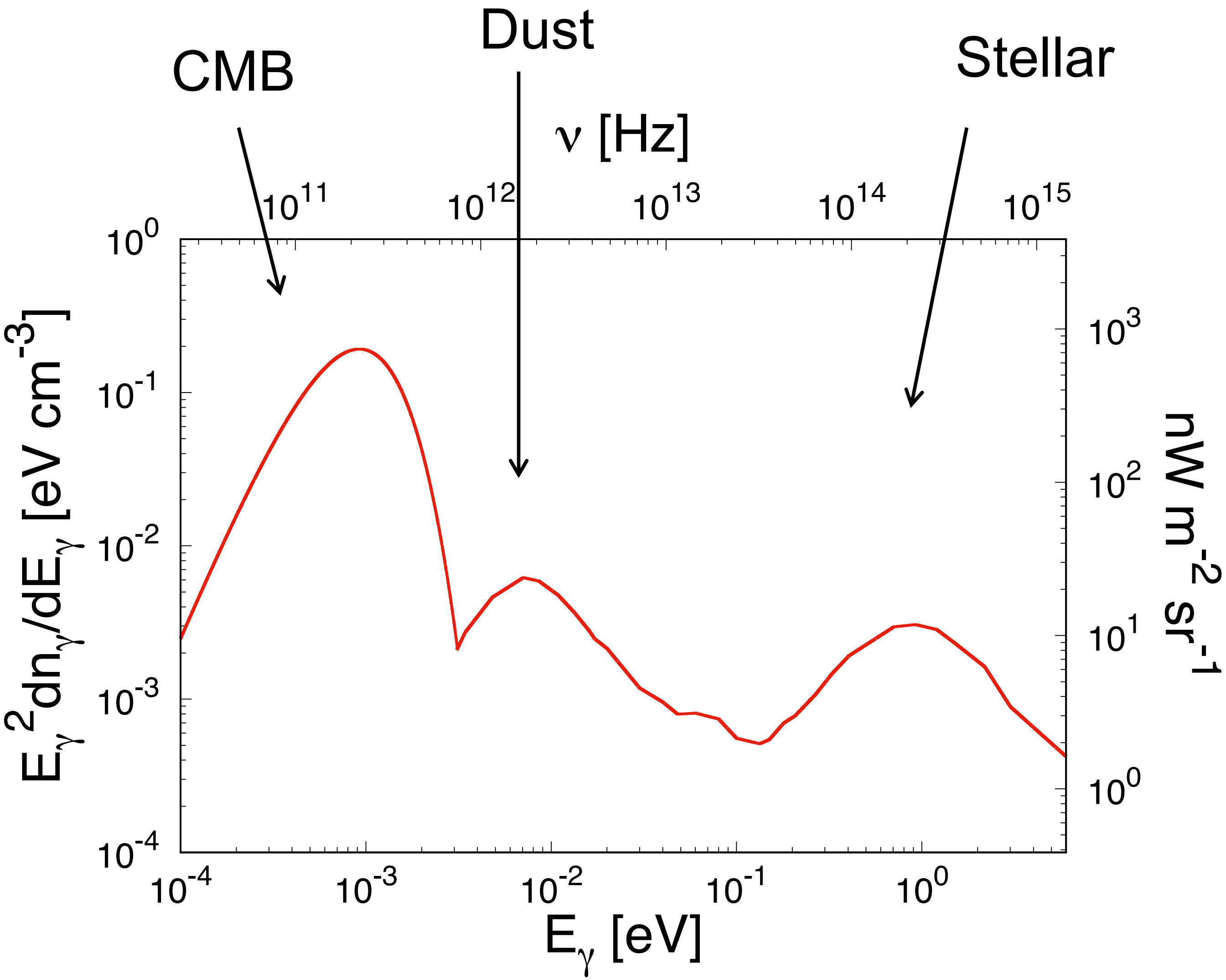
Cosmic Radiation Fields- Energy Density



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Cosmic Radiation Fields- Energy Density

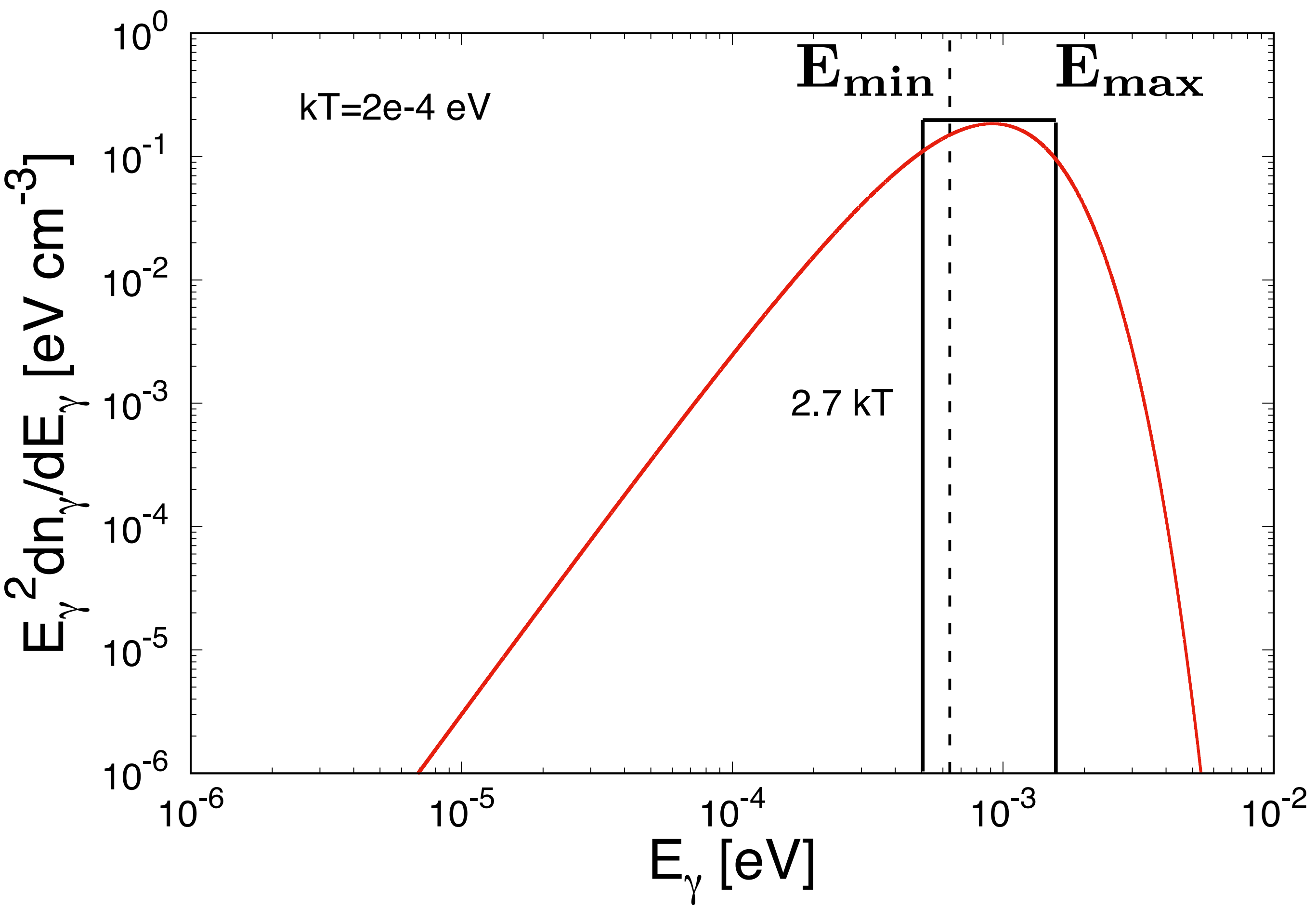
$$\mathbf{U}_\gamma = \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dE}_\gamma$$

$$= \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma^{\mathbf{2}} \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dlnE}_\gamma$$

Note- this amounts to a visual inspection version of Laplace's integral method

CMB- Energy Density

Estimating U_γ



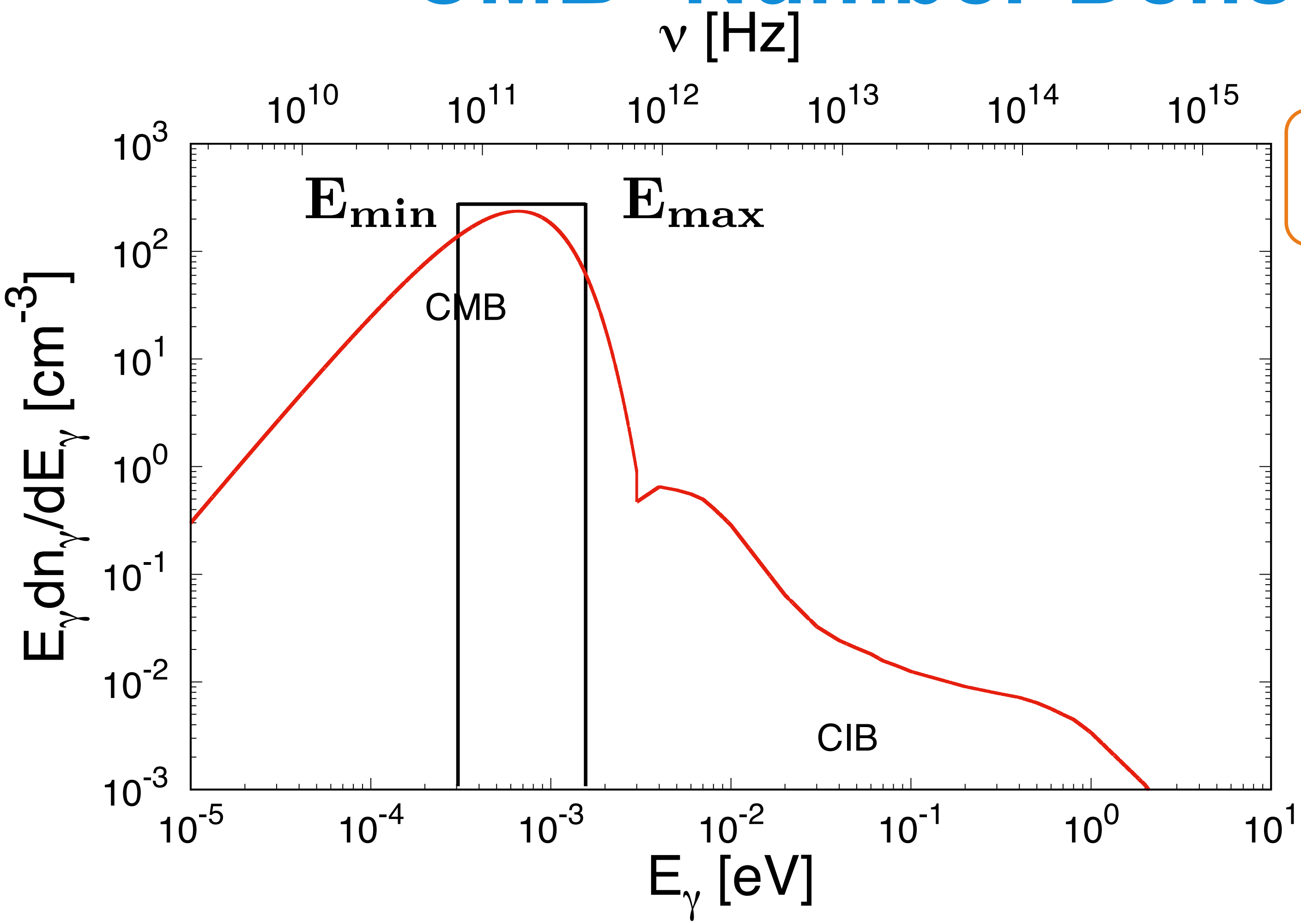
$$\mathbf{U_\gamma \approx E^2 \frac{dN}{dE}\Big|_{peak} \ln\left(\frac{E_{max}}{E_{min}}\right) \approx 0.2\ eV\ cm^{-3}}$$

Cosmic Radiation Fields- Number Density

$$\mathbf{n}_\gamma = \int_0^\infty \frac{\mathbf{dn}}{\mathbf{dE}_\gamma}\mathbf{dE}_\gamma$$

$$= \int_0^\infty \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma}\mathbf{dlnE}_\gamma$$

CMB- Number Density



Estimating $\mathbf{n}_\gamma$

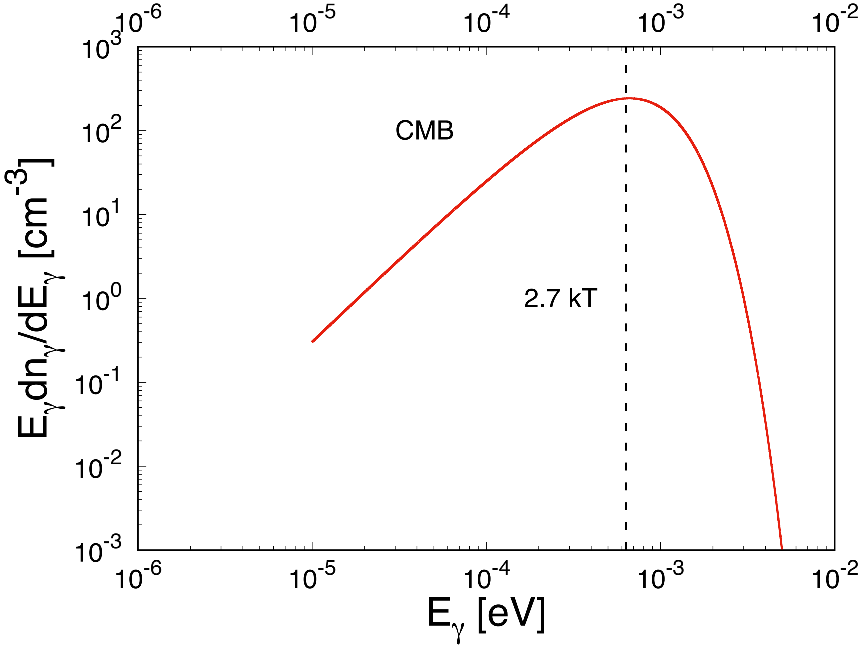
$$\mathbf{n}_\gamma \approx \mathbf{E}\frac{\mathbf{dN}}{\mathbf{dE}}\bigg|_{\mathbf{peak}} \ln\left(\frac{\mathbf{E}_{\mathbf{max}}}{\mathbf{E}_{\mathbf{min}}}\right) \approx \mathbf{400\ cm}^{-3}$$

Blackbody- Number Density

$$(hc)^3 \frac{dN}{d^3x d^3\epsilon_\gamma} = 2\frac{1}{e^{\epsilon_\gamma/kT} - 1}$$

$$n = \frac{dN}{d^3x}$$

$$\frac{dn}{d\epsilon_\gamma} = \frac{8\pi}{(hc)^3}\frac{\epsilon_\gamma^2}{e^{\epsilon_\gamma/kT} - 1}$$



$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

$$= 8\pi\frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3)$$

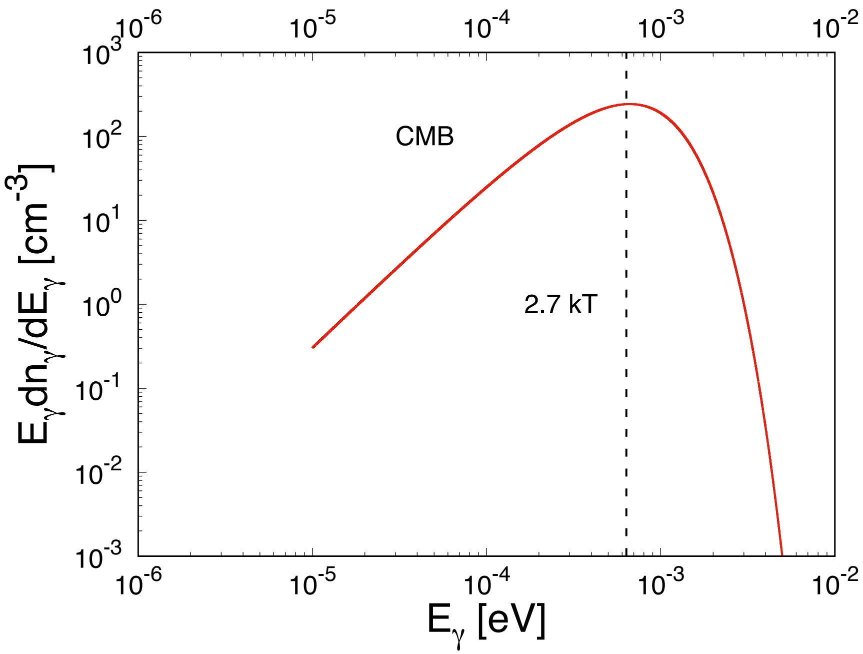
Blackbody- Number Density

$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

$$= 8\pi\frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3)$$

Start by doing the simpler problem

$$\int_0^\infty x^2 e^{-x}dx = \gamma(3)$$



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$$\int \frac{\mathbf{x^n}}{\mathbf{e^x - 1}} \mathbf{dx} = \gamma(\mathbf{n+1})\zeta(\mathbf{n+1})$$

PAUSE

Why not have a go at obtaining this result

Blackbody - Number Density

$$\mathbf{n_\gamma^{BB}} = 8\pi\frac{(\mathbf{kT})^3}{(\mathbf{hc})^3}\gamma(\mathbf{3})\zeta(\mathbf{3})$$

$$\frac{\mathbf{x^n}}{\mathbf{e^x - 1}} = \frac{\mathbf{e^{-x}x^n}}{\mathbf{1 - e^{-x}}}$$

$$= \sum_{\mathbf{m=0}}^{\infty} \mathbf{e^{-mx}e^{-x}x^n}$$

$$= \sum_{\mathbf{m=1}}^{\infty} \mathbf{e^{-mx}x^n}$$

Blackbody- Number Density

$$n_\gamma^{BB} = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3)\zeta(3)$$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \int e^{-mx} x^n dx$$

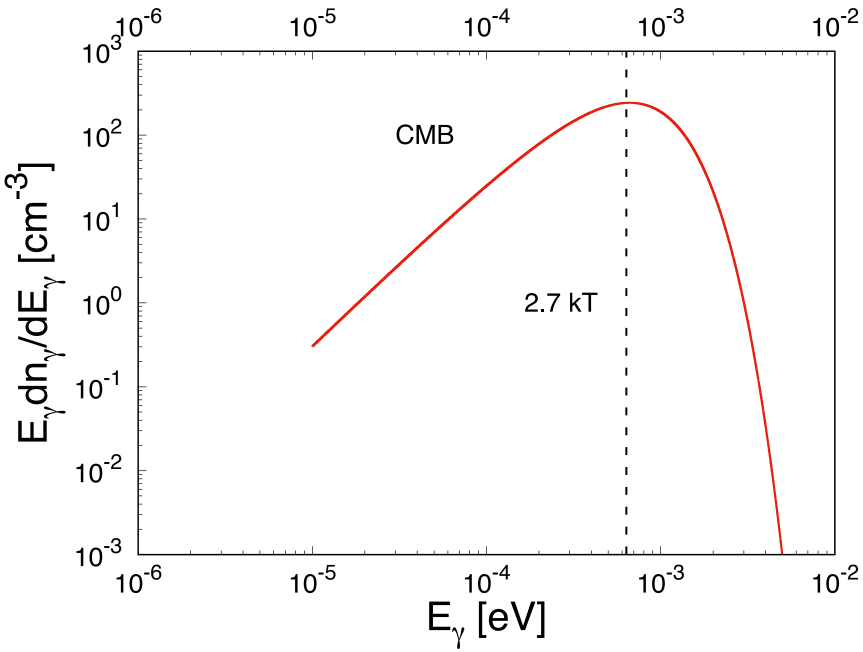
Let $y = mx$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \int e^{-y} \left(\frac{y}{m}\right)^n d\left(\frac{y}{m}\right)$$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \frac{1}{m^{n+1}} \int y^n e^{-y} dy = \gamma(n+1)\zeta(n+1)$$

CMB- Number Density

$$\frac{dn}{d\epsilon_\gamma} = \frac{8\pi}{(hc)^3}\frac{\epsilon_\gamma^2}{e^{\epsilon_\gamma/kT}-1}$$



$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3}\int_0^\infty \frac{x^2}{e^x-1}dx$$

$$\frac{8\pi(kT_{CMB})^3}{(hc)^3} \approx 170\ \mathrm{cm}^{-3}$$

$$\zeta(x) = \sum_{n=1}^{\infty}\frac{1}{n^x}$$

$$n_\gamma^{CMB} = 8\pi\frac{(kT_{CMB})^3}{(hc)^3}\gamma(3)\zeta(3) \approx 400\ \mathrm{cm}^{-3}$$

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CMB- Number Density

For a blackbody radiation field distribution, with temperature T,

$$\mathbf{n_\gamma = \int_0^\infty \frac{dn}{dE_\gamma} dE_\gamma = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3)\zeta(3) \approx 400\ cm^{-3}}$$

$$\mathbf{U_\gamma = \int_0^\infty E_\gamma \frac{dn}{dE_\gamma} dE_\gamma = 8\pi \frac{(kT)^4}{(hc)^3} \gamma(4)\zeta(4) = 0.25\ eV\ cm^{-3}}$$

$$\mathbf{\langle E_\gamma \rangle = \frac{\int_0^\infty E_\gamma \frac{dn}{dE_\gamma} dE_\gamma}{\int_0^\infty \frac{dn}{dE_\gamma} dE_\gamma} = \frac{\Gamma(4)\zeta(4)}{\Gamma(3)\zeta(3)} kT \approx 2.7\ kT}$$

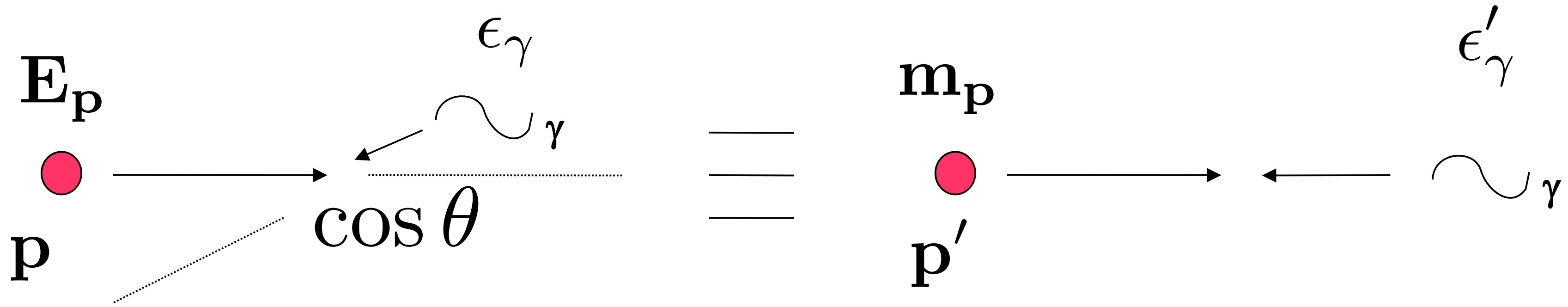
Cosmic Ray Proton Energy Losses

The Interaction Rate

radiation field ↘ cross-section ↘

$$\mathbf{R} = \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-\mathbf{1}}^{\mathbf{1}} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta)\sigma(\cos\theta)\mathbf{1} + \beta\cos\theta)$$

All values above in lab frame



The Interaction Rate

$$\mathbf{R} = \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-1}^{1} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta)\sigma(\cos\theta)(\mathbf{1} + \beta\cos\theta)$$

Since, $\epsilon'_\gamma \mathbf{m_p} = \epsilon_\gamma \mathbf{E_p}(\mathbf{1} + \beta\cos\theta)$

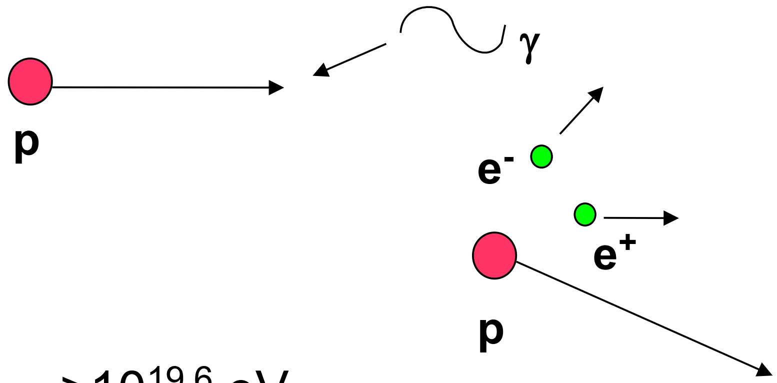
$$(\mathbf{1} + \beta\cos\theta)\mathbf{d}\cos\theta = \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma \mathbf{E_p}} \frac{\mathbf{d}(\epsilon'_\gamma \mathbf{m_p})}{\epsilon_\gamma \mathbf{E_p}}$$

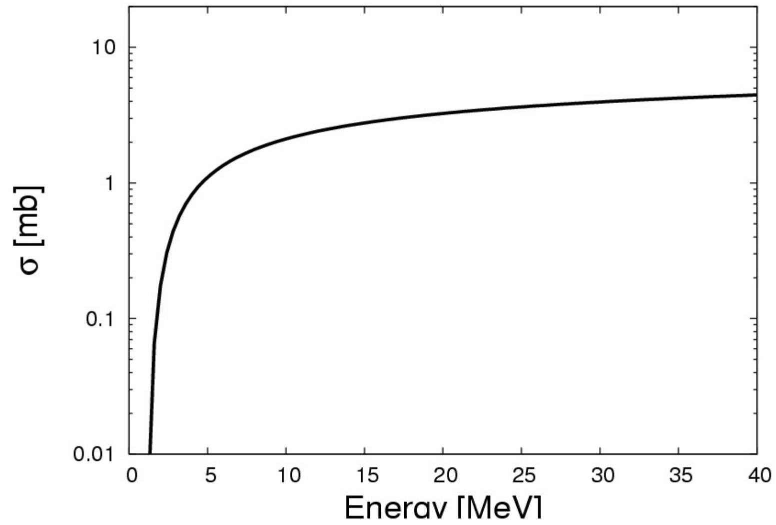
$$\mathbf{R} = \frac{\mathbf{1}}{\mathbf{2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2}\epsilon_\gamma \mathbf{E_p}} \mathbf{d}(\epsilon'_\gamma \mathbf{m_p}) \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma^{\mathbf{2}} \mathbf{E_p^2}} \sigma(\epsilon')$$

$$= \frac{\mathbf{m_p^2}}{\mathbf{2E_p^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2}\epsilon_\gamma \frac{\mathbf{E_p}}{\mathbf{m_p}}} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$

Cosmic Ray Proton Interactions

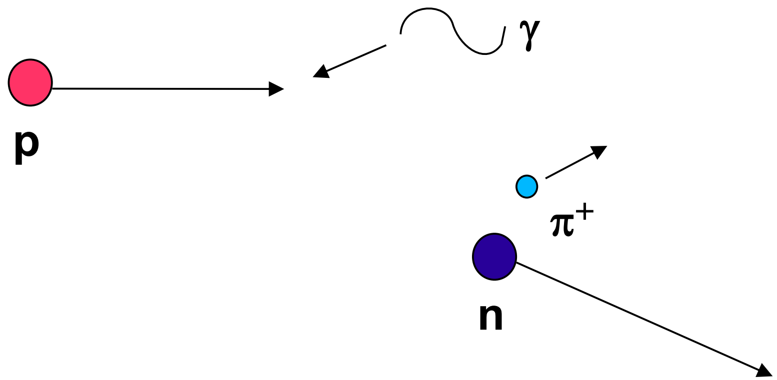
For E_{proton}<$10^{19.6}$ eV

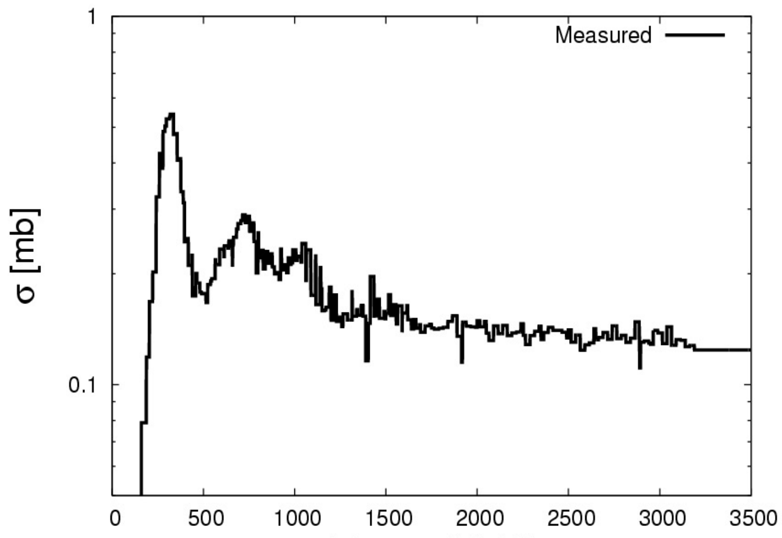




$E_{\gamma}^{th} \sim 1\text{MeV}$

For E_{proton}>$10^{19.6}$ eV





$E_{\gamma}^{th} \sim 140\text{ MeV}$

PAUSE

Why not have a go at calculating the threshold energy for proton pair creation through interaction with CMB photons

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Threshold Energy- Proton Pair Production

$$(\mathbf{E_p} + \mathbf{E}_\gamma)^2 - (\mathbf{p_p} - \mathbf{E}_\gamma)^2 = (\mathbf{m_p} + 2\mathbf{m_e})^2$$

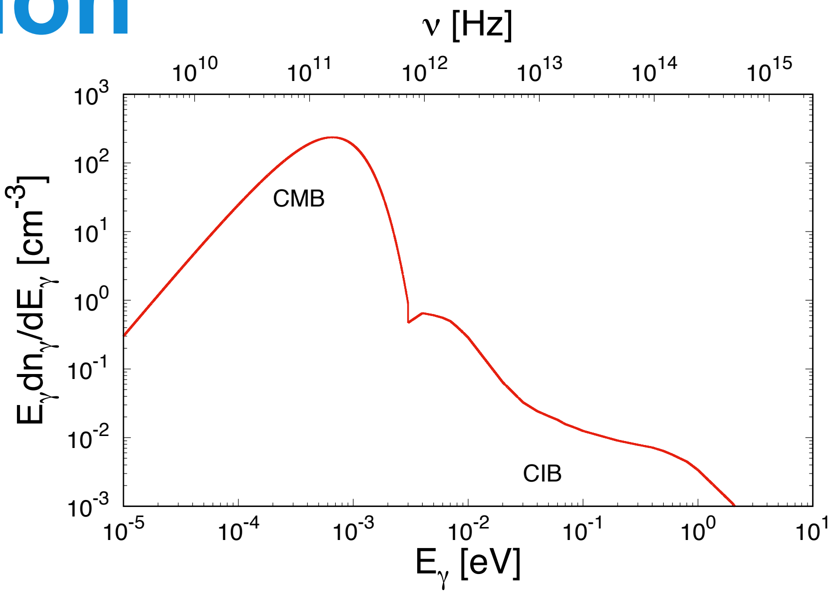
$$\mathbf{m_p^2} + 2\mathbf{E_p}\mathbf{E}_\gamma + 2\mathbf{p_p}\mathbf{E}_\gamma \approx \mathbf{m_p^2} + 4\mathbf{m_p}\mathbf{m_e}$$

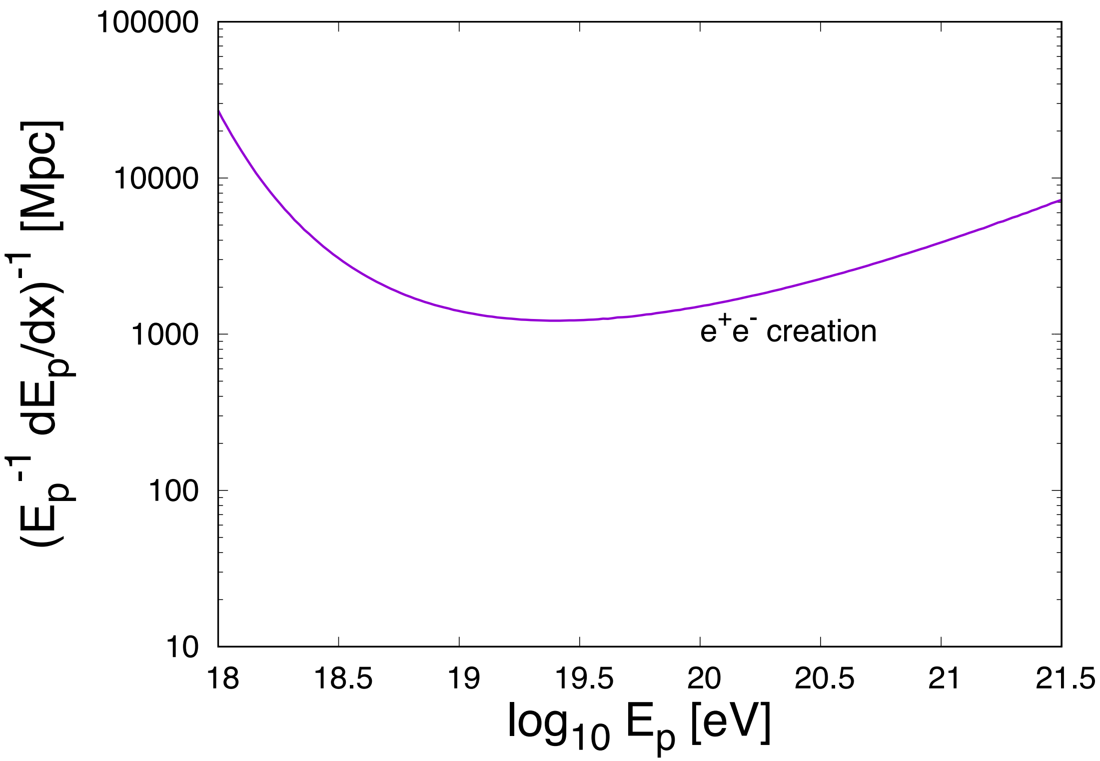
$$\boxed{\mathbf{E_p} \approx \frac{\mathbf{m_e}}{\mathbf{E}_\gamma}\mathbf{m_p}} \approx \left(\frac{0.5 \times 10^6}{6 \times 10^{-4}}\right) 0.9 \times 10^9 = 8 \times 10^{17}\ \mathbf{eV}$$

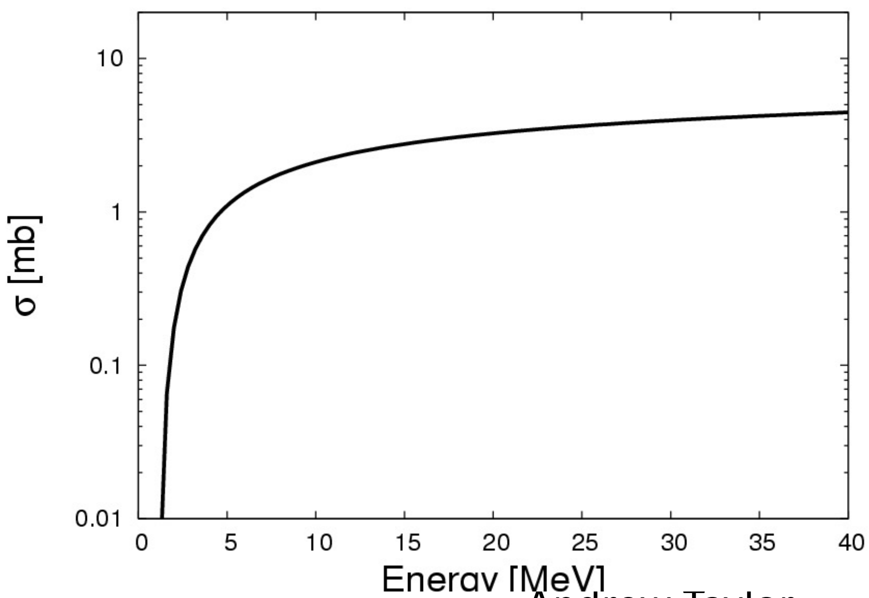
Repeat this calculation for pion production

Energy Loss Rate- Pair Production

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

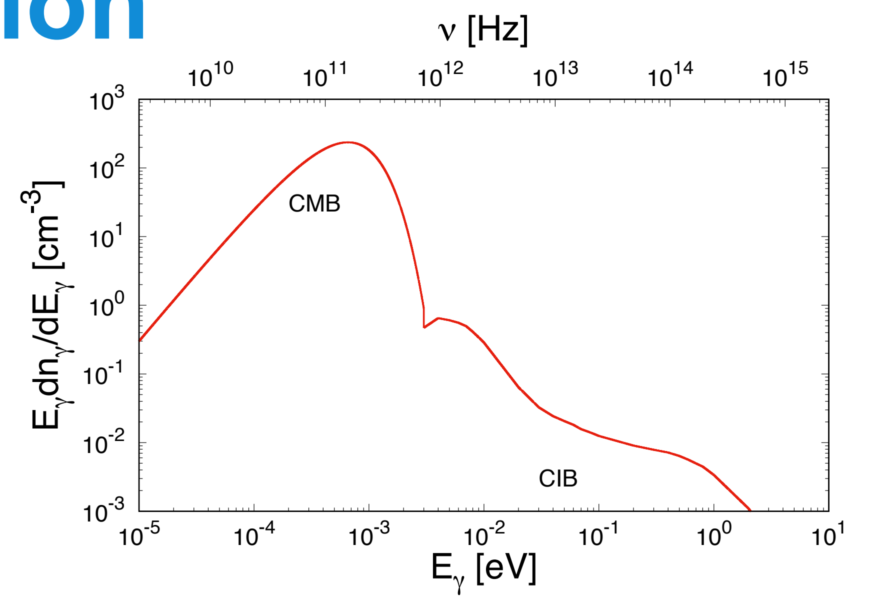


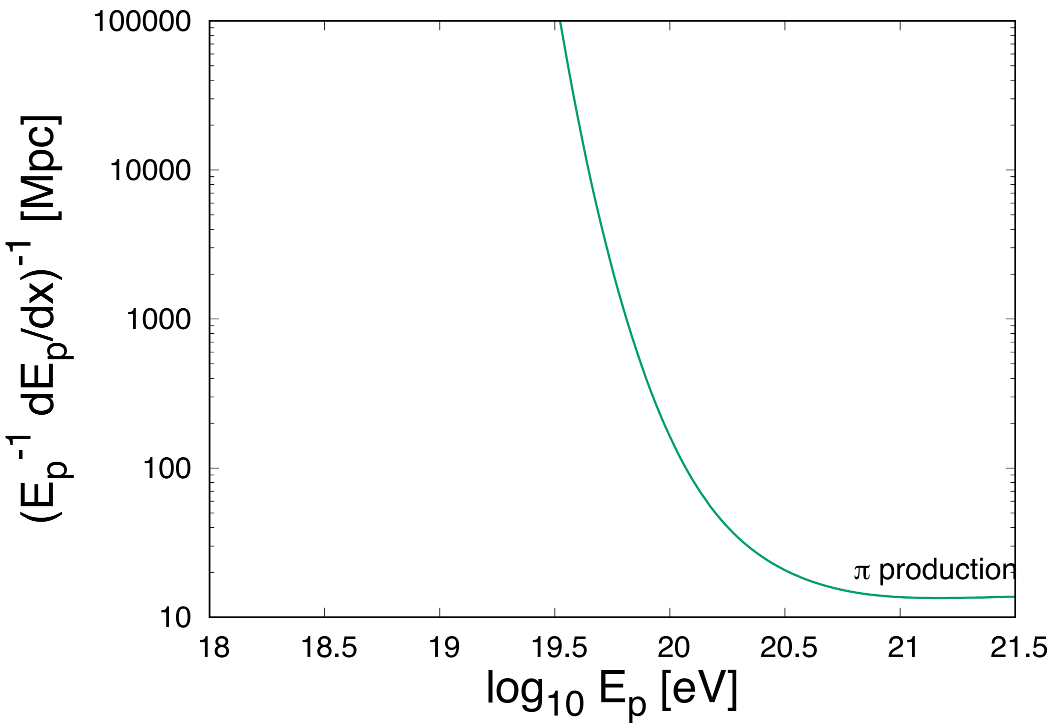




Energy Loss Rate- Pion Production

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$





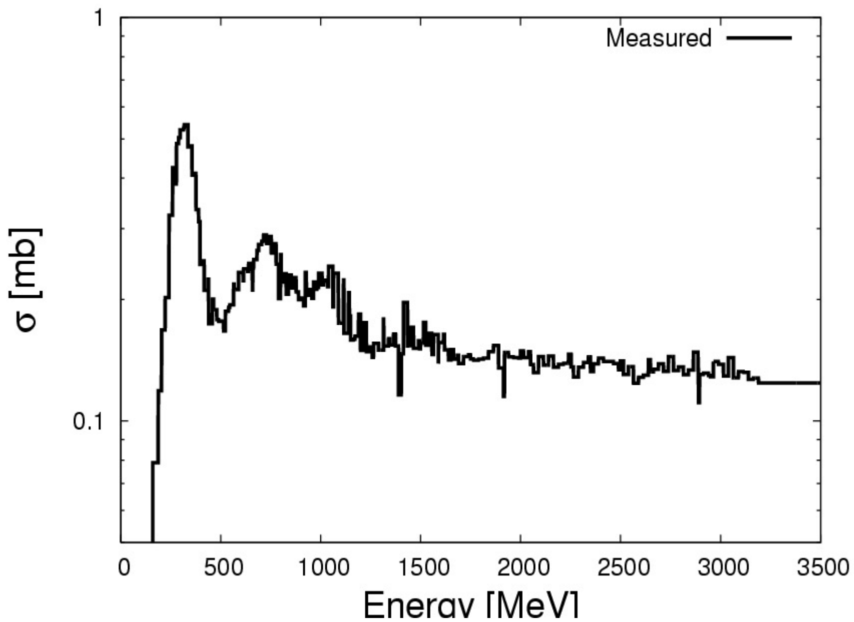


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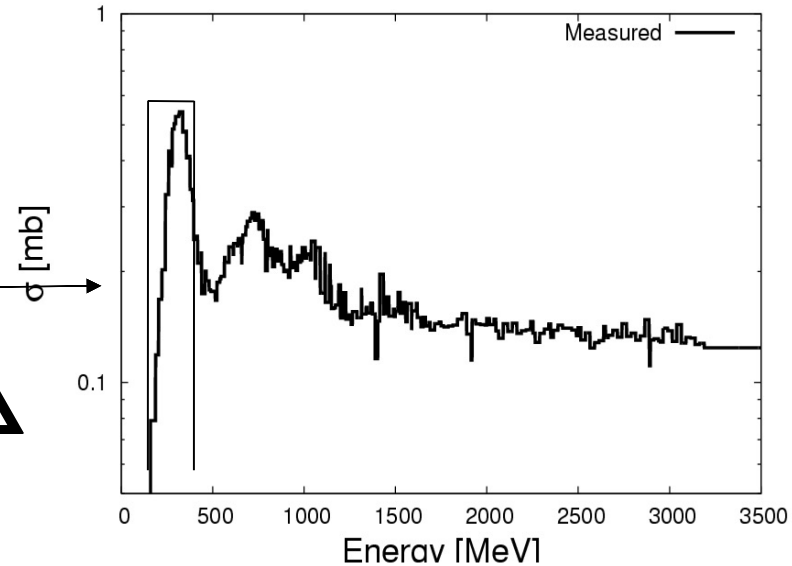
Photo-Pion Production Rate

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

Assuming the cross-section is approximately:

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \mathbf{0} \qquad \begin{array}{l} \epsilon_\gamma < \mathbf{E} - \mathbf{\Delta} \\ \epsilon_\gamma > \mathbf{E} + \mathbf{\Delta} \end{array}$$

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \sigma_{\mathbf{p}\gamma} \quad \mathbf{E} - \mathbf{\Delta} < \epsilon_\gamma < \mathbf{E} + \mathbf{\Delta}$$



Where $\sigma_{\mathbf{p}\gamma} = \mathbf{0.5\ mb}$, $\mathbf{E} = \mathbf{300\ MeV}$, $\mathbf{\Delta} = \mathbf{100\ MeV}$

$$\mathbf{R} \approx \left(\frac{\mathbf{l_0}}{\mathbf{e^{-x_1}(1 - e^{-x_1})}} \right)^{-1}$$

PAUSE

Why not have a go at obtaining this result

Photo-Pion Production Rate

$$\mathbf{R}(\mathbf{\Gamma}) \approx \sigma_0 \int_{(\mathbf{E_0}-\mathbf{\Delta_0})/2\mathbf{\Gamma}}^{(\mathbf{E_0}+\mathbf{\Delta_0})/2\mathbf{\Gamma}} \left(\frac{\epsilon^2 - \left[(\mathbf{E_0} - \mathbf{\Delta_0})/2\mathbf{\Gamma}\right]^2}{\epsilon^2} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon +$$

$$\sigma_0 \int_{(\mathbf{E_0}+\mathbf{\Delta_0})/2\mathbf{\Gamma}}^{\infty} \left(\frac{\left[(\mathbf{E_0} + \mathbf{\Delta_0})/2\mathbf{\Gamma}\right]^2 - \left[(\mathbf{E_0} - \mathbf{\Delta_0})/2\mathbf{\Gamma}\right]^2}{\epsilon^2} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon$$

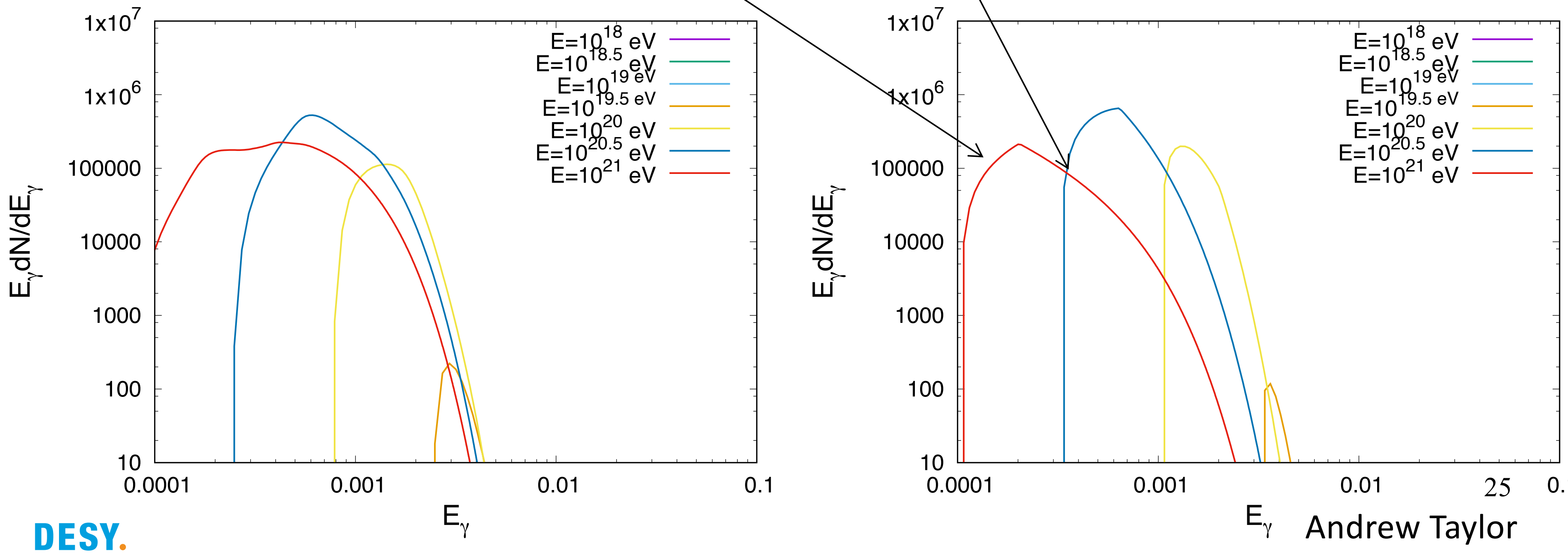


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Photo-Pion Production Rate

$$\mathbf{R(\Gamma) \approx n_0\sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0\sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{1}{l_0} [\ (\gamma_i(3, x_2(\Gamma)) - \gamma_i(3, x_1(\Gamma))) - x_1(\Gamma)^2(\gamma_i(1, x_2(\Gamma)) - \gamma_i(1, x_1(\Gamma)))+}$$

$$\mathbf{x_2(\Gamma)^2(1 - \gamma_i(1, x_2(\Gamma))) - x_1(\Gamma)^2(1 - \gamma_i(1, x_2(\Gamma)))\ ]}$$

$$\mathbf{\gamma_i(3, x) = 2 - (2 + 2x + x^2)\exp(-x) \qquad \gamma_i(1, x) = 1 - \exp(-x)}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1}(1 - e^{-x_1} + x_1(1 - 2e^{-x_1}))\right]}$$

Photo-Pion Production Rate: Blackbody Interactions

$$R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +$$

$$n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}$$

$$R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1} \left(1 - e^{-x_1} + x_1 \left(1 - 2e^{-x_1}\right)\right)\right]$$

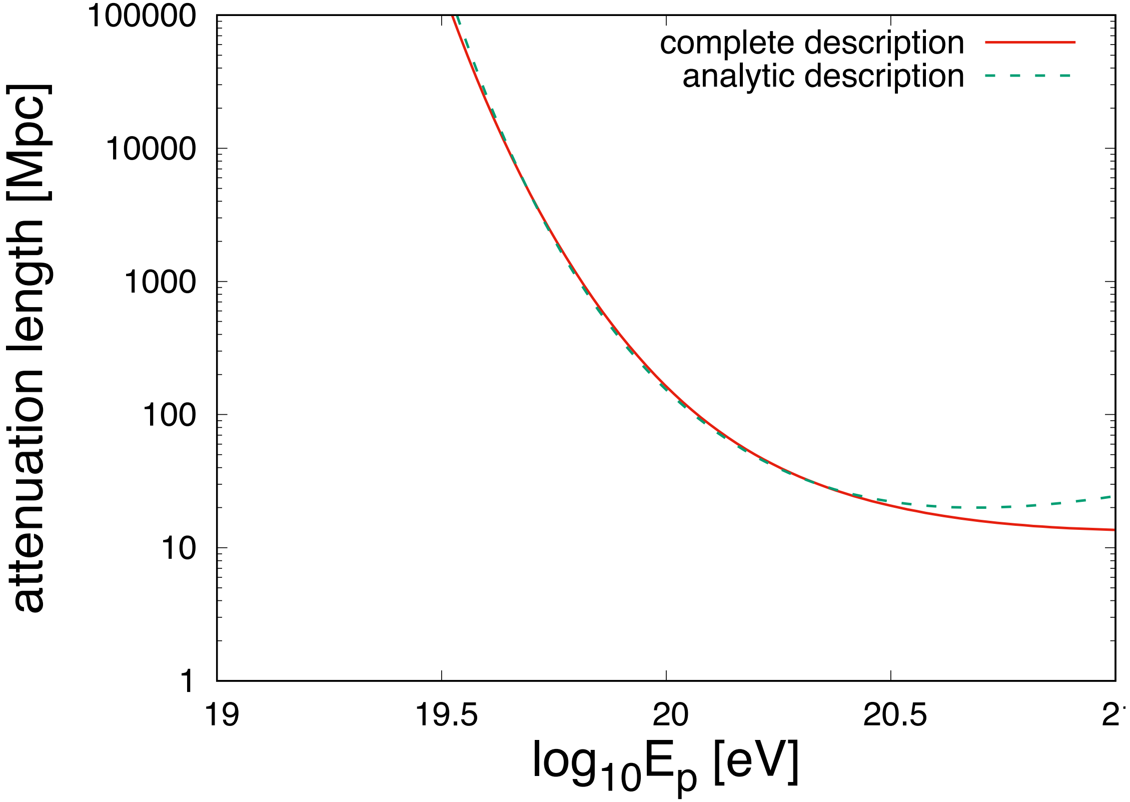
Where, $l_0 = 10 \text{ Mpc}$ $\quad x_1 = \frac{(E - \Delta) m_p}{2kT_{CMB} E_p} = \frac{10^{20.5} \text{ eV}}{E_p}$

Photo-Pion Production Rate: Blackbody Interactions

With, $\mathbf{kT_{CMB} \approx 2 \times 10^{-4}\ eV}$

$$\mathbf{R \approx 0.2\sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma}}$$

$$\mathbf{\approx \left(\frac{l_0}{e^{-x_1}(1 - e^{-x_1})} \right)^{-1}}$$



Where $\mathbf{l_0}$ is 5 Mpc and $\mathbf{x_1 = \frac{10^{20.5}\ eV}{E_p}}$

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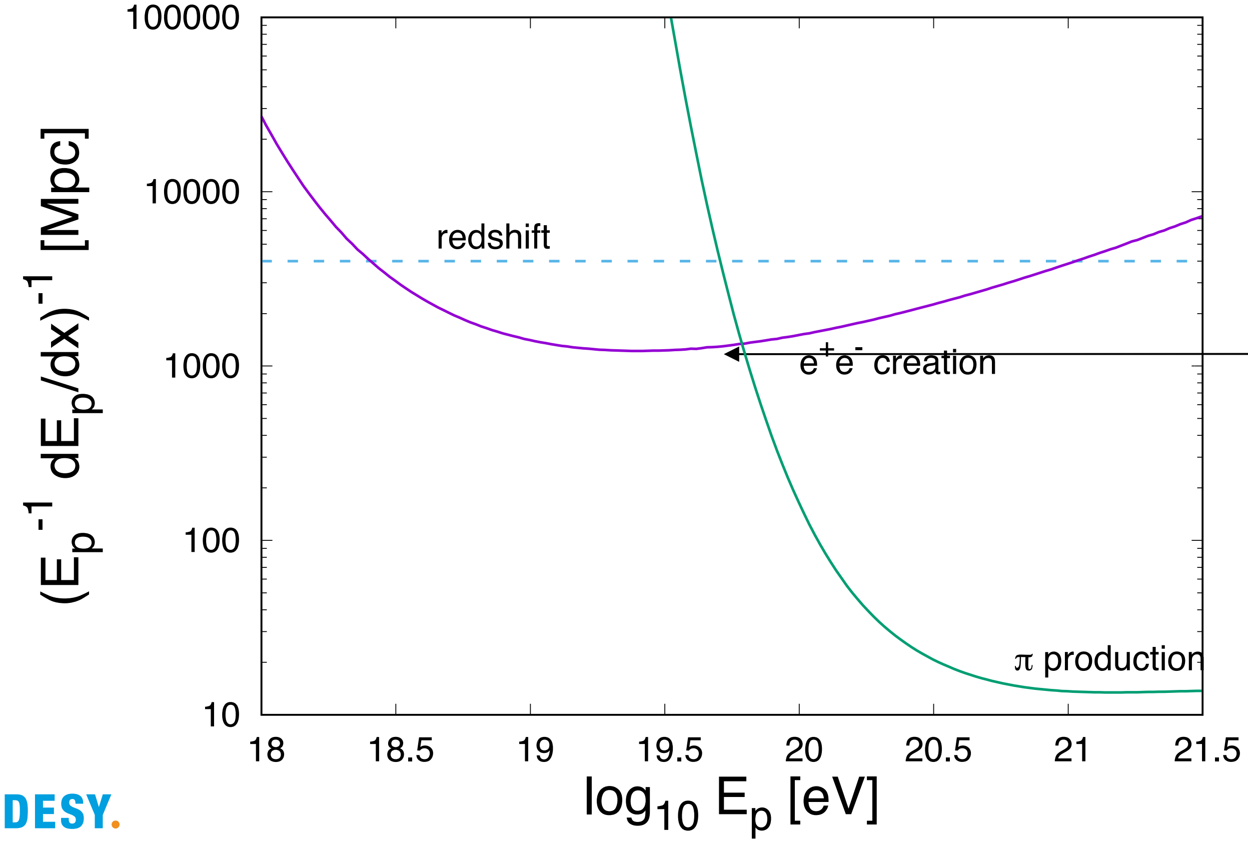
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

where K_p is the inelasticity



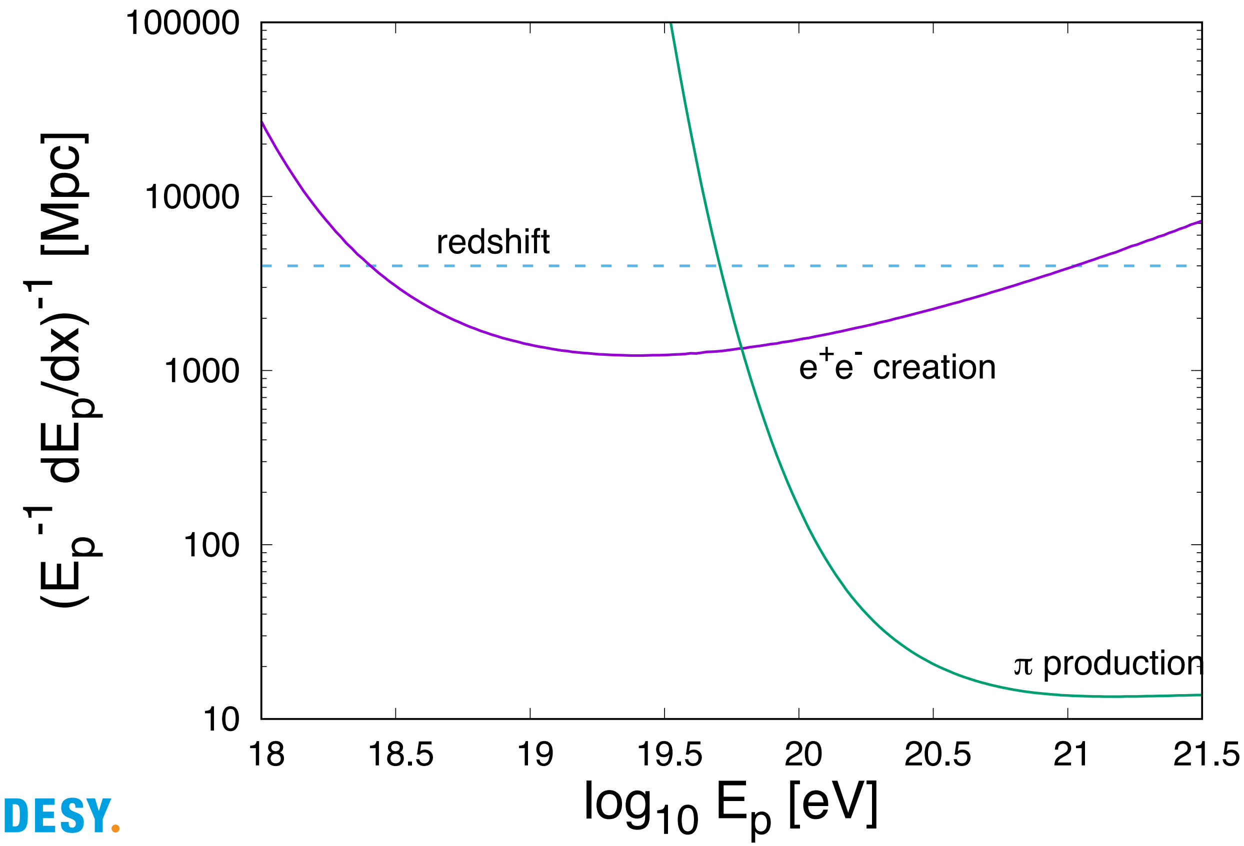
$$\approx \frac{\mathbf{m_p}}{\mathbf{m_e}} \frac{\mathbf{1}}{\mathbf{n_{CMB}} \sigma_{\mathbf{p}\gamma}}$$

Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

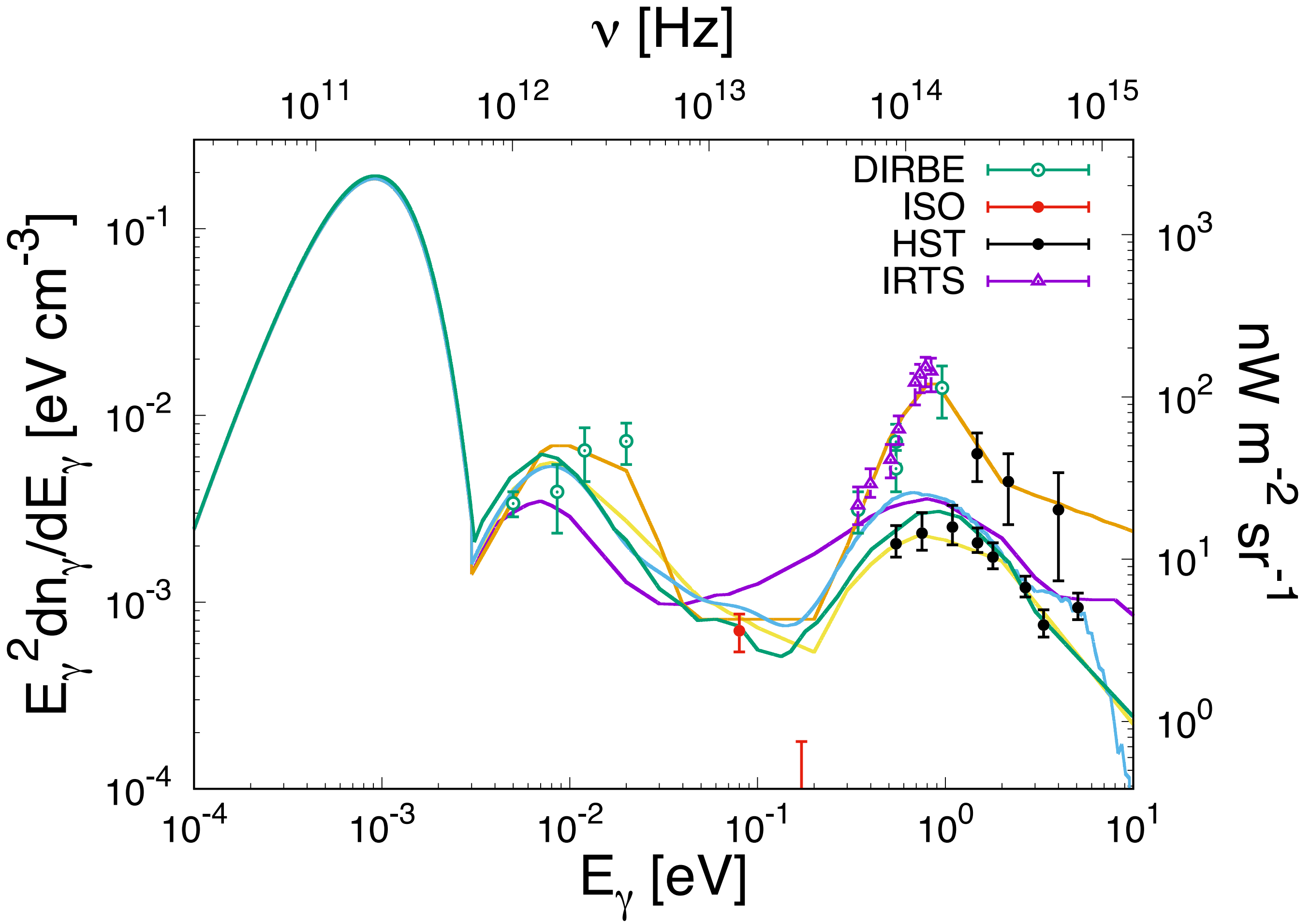
where K_p is the inelasticity



$$\approx \frac{\mathbf{m_p}}{\mathbf{m_\pi}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

EBL Radiation Field Models

The EBL isn't actually known with very great accuracy
(since it is difficult to measure directly)



This uncertainty should be propagated into theoretical calculations of interaction rates

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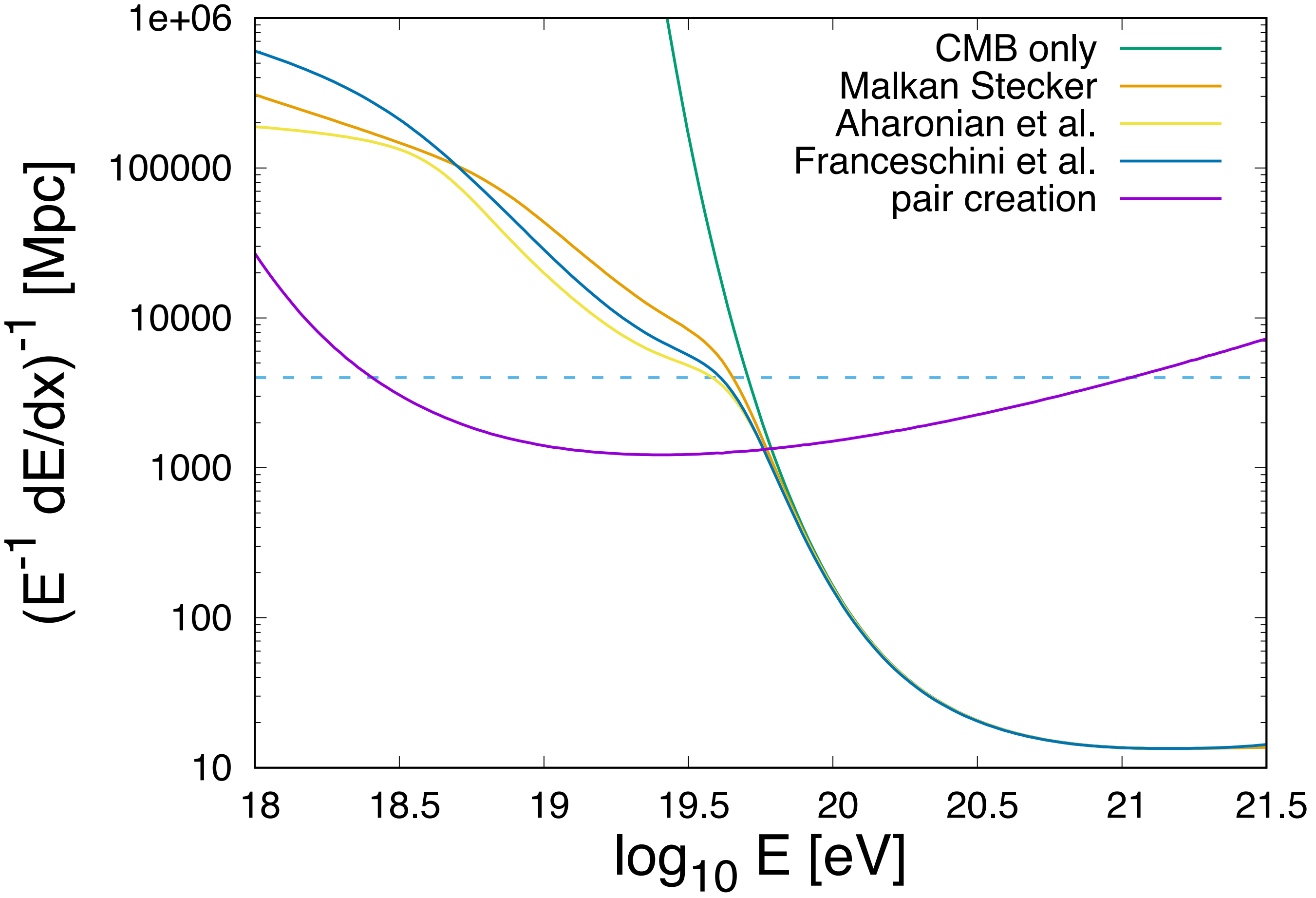
....with Different IR Backgrounds



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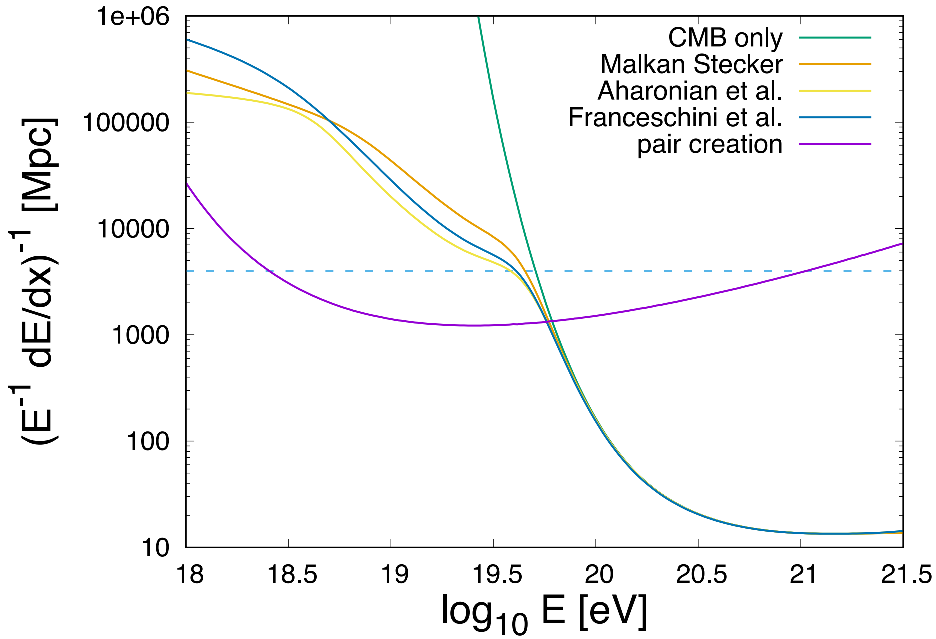
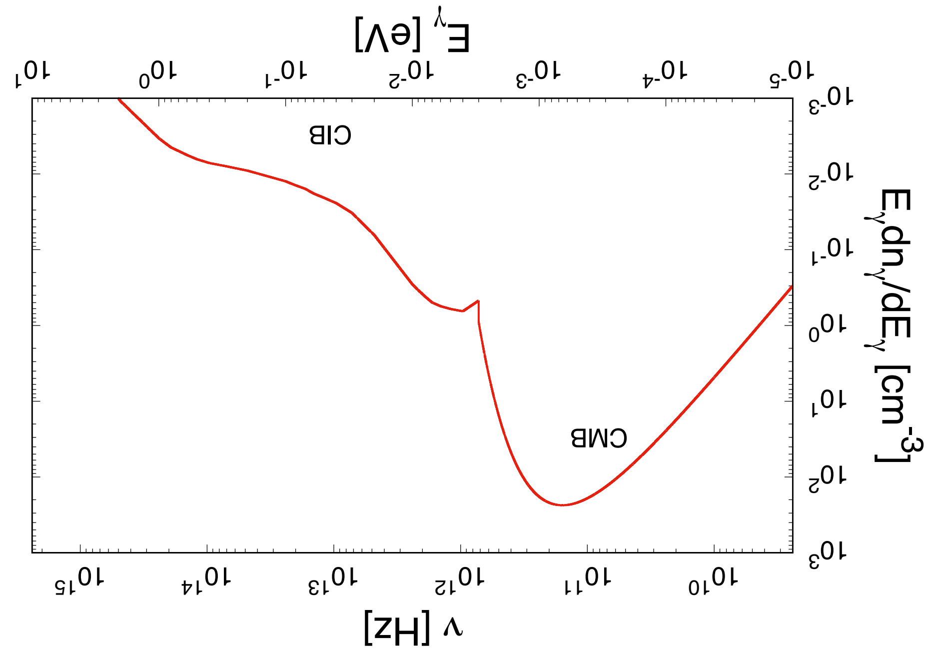
....with Different IR Backgrounds





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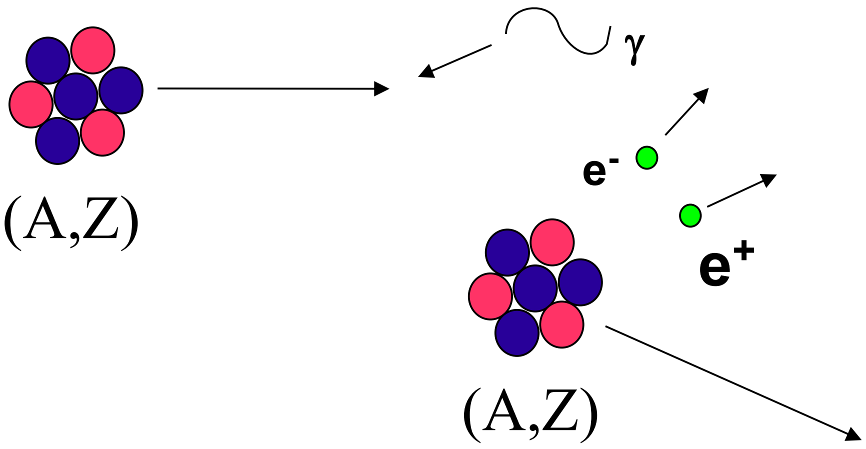
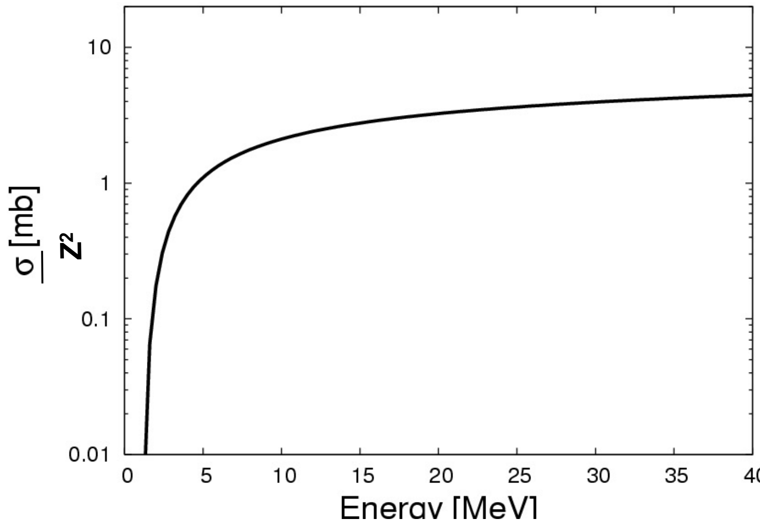
Cosmic Ray Nuclei Energy Losses

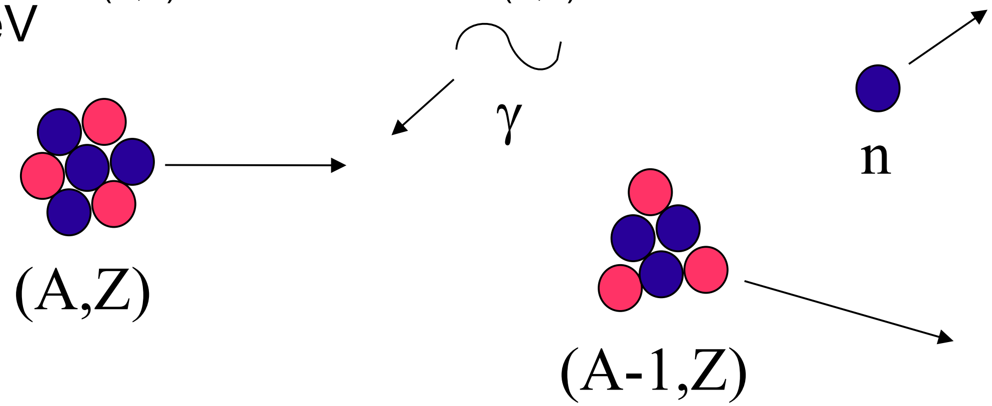
Cosmic Ray Nuclei Interactions

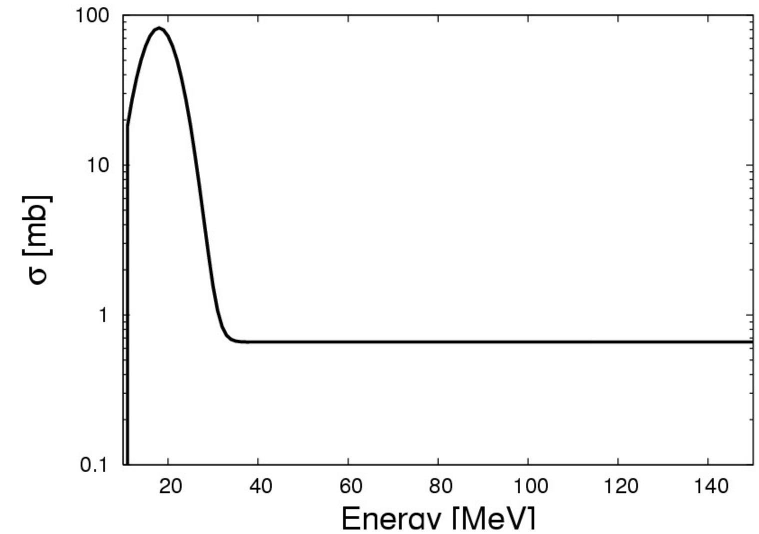
For $10^{19.7} < E_{(A,Z)} < 10^{20.2}$ eV





For $E_{(A,Z)} < 10^{19.7}$ and $E_{(A,Z)} < 10^{20.2}$ eV





Cosmic Ray Nuclei Interactions

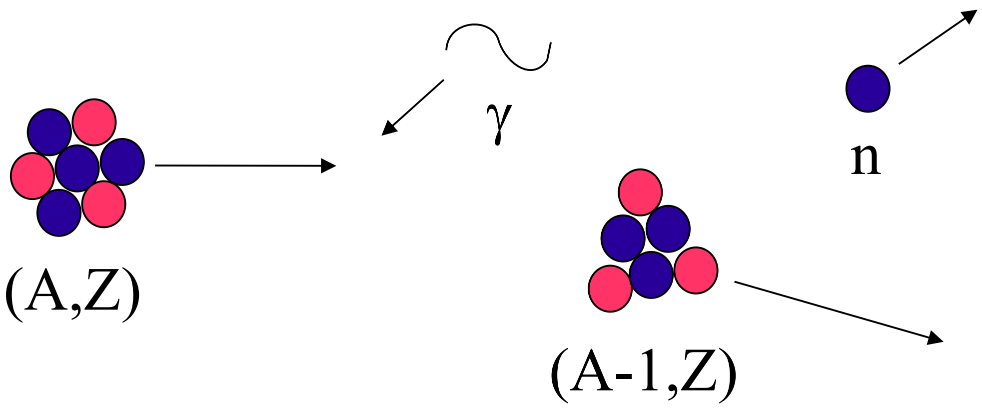


Photo-disintegration-

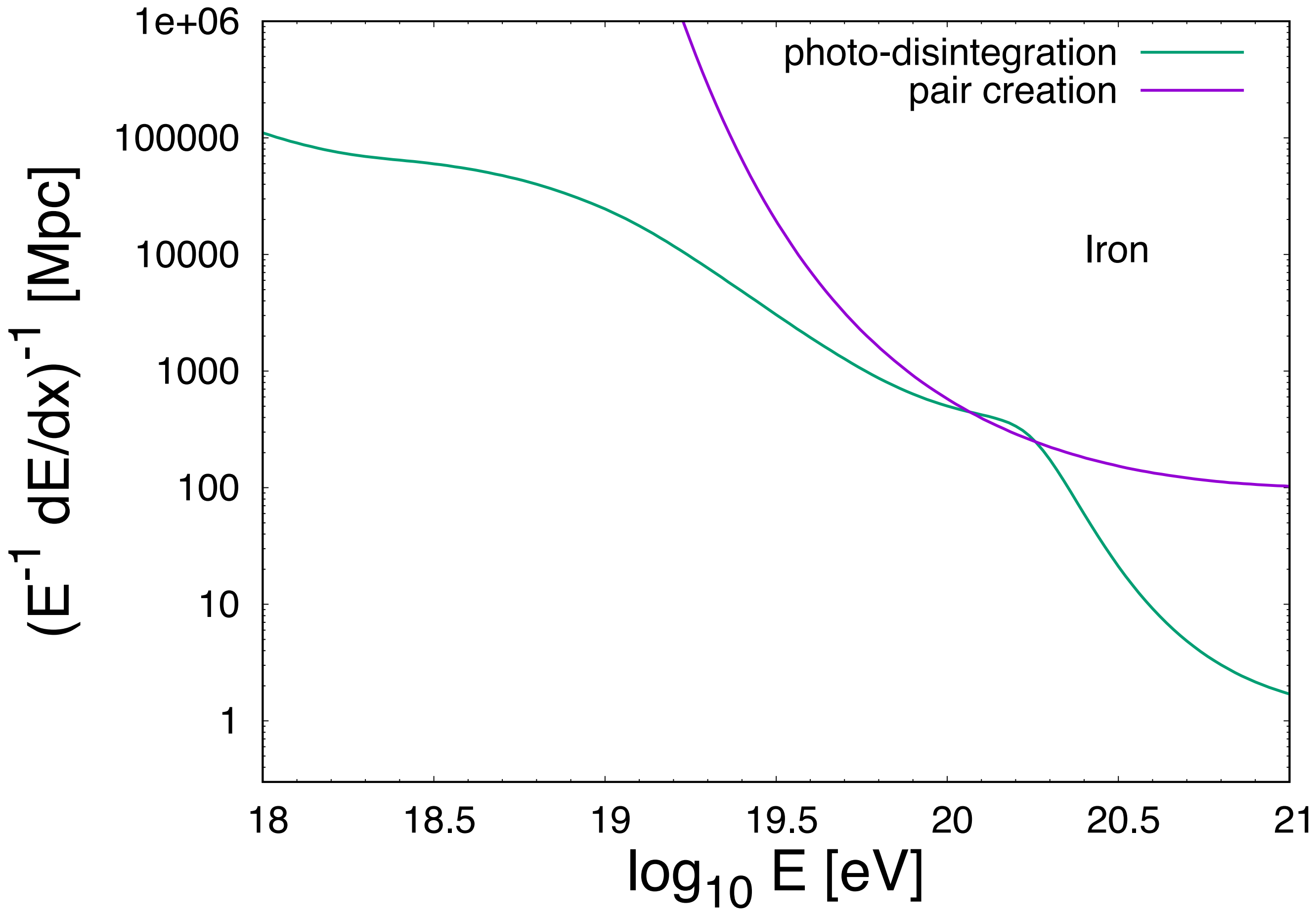
$$N_{(A,Z)} + \gamma \longrightarrow N'_{(A',Z')} + (Z-Z')p + (A-A'+Z'-Z)n, \quad E_\gamma \sim 30\text{MeV}$$

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

Energy Loss Rates due to Nuclei Interactions

$$\mathbf{R} = \frac{\mathbf{A^2 m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^\mathbf{2}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{Am_pc^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{N}\gamma}(\epsilon'_\gamma)\mathbf{K_p}$$

where R is the energy loss rate



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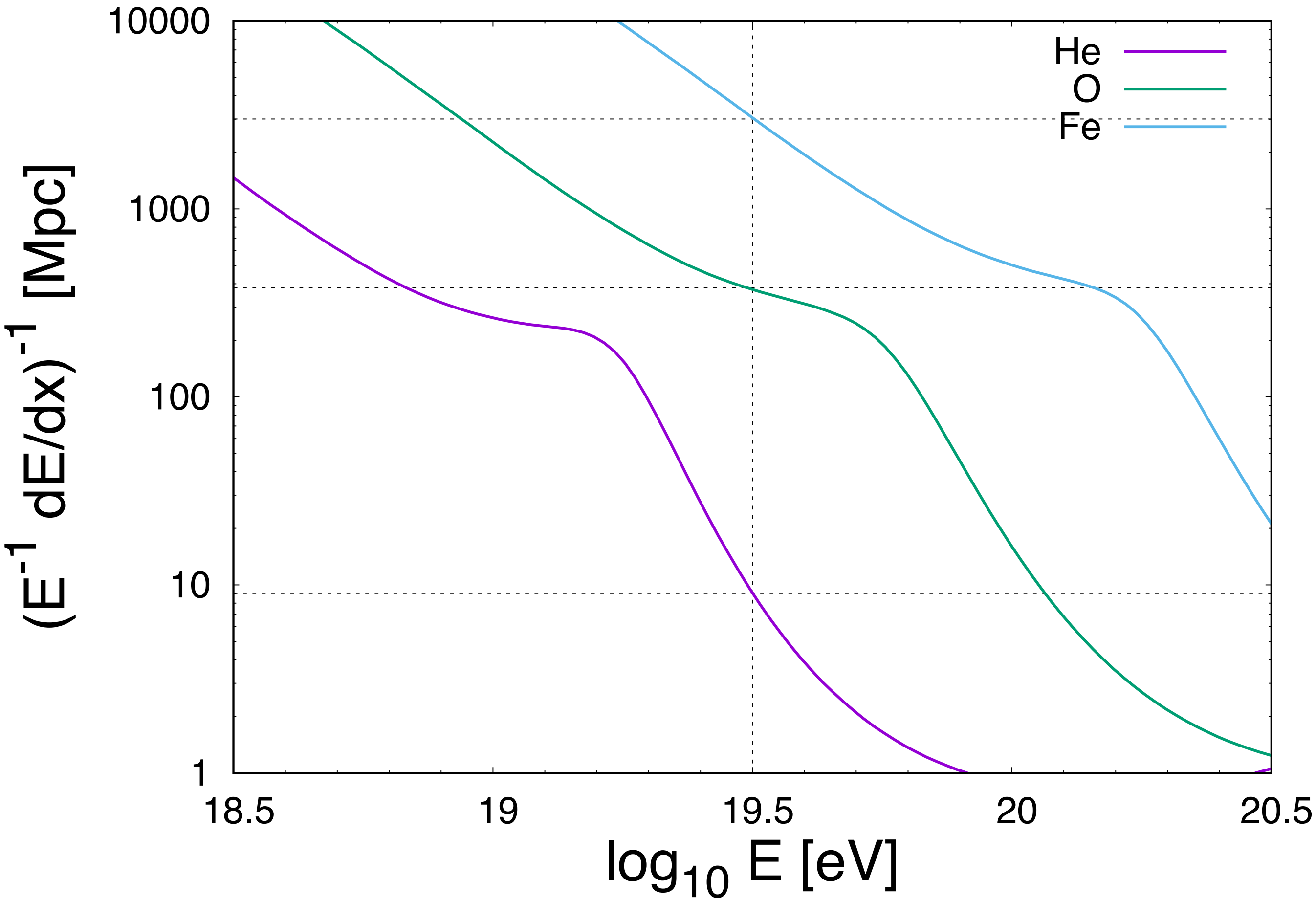
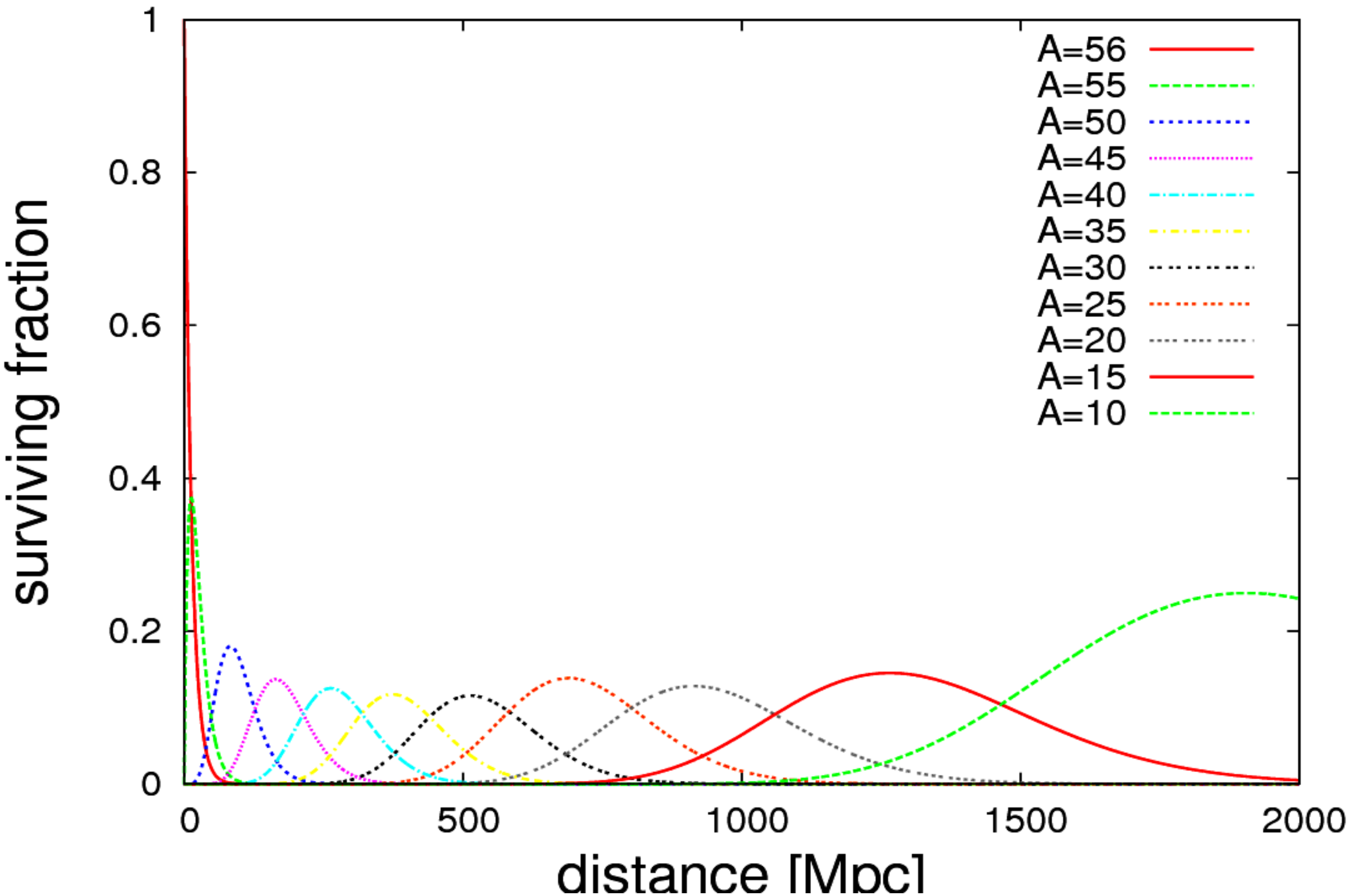
Energy Loss Rates for Different Nuclei



Cosmic Ray Disintegration During Propagation



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Cosmic Ray Spectra

Assumptions on Source Population

Spatial Distribution

$$\frac{dN}{dV_C} \propto (1+z)^n$$

$$z < z_{max}$$

$$n = -6,\ -3,\ 0,\ 3$$

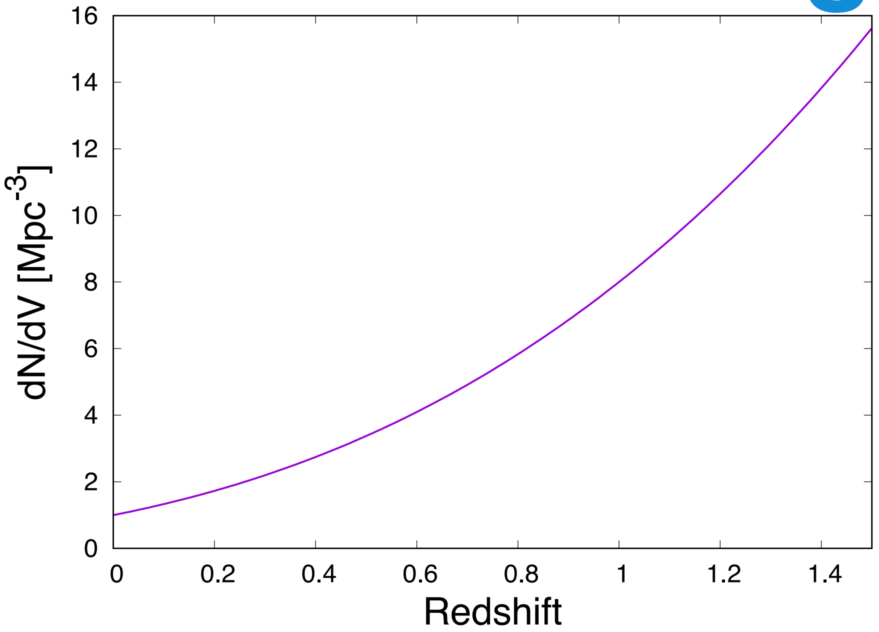
Energy Distribution

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s < (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

A Cosmological Distribution of Sources

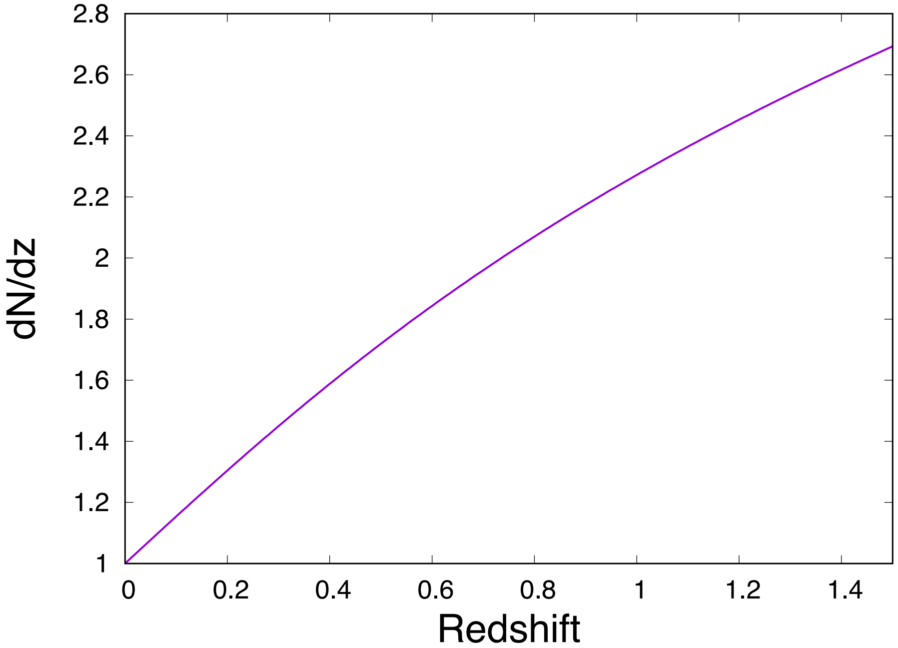


Distribution of sources in a comoving volume

$$\mathbf{dV_c = 4\pi\chi^2 d\chi}$$

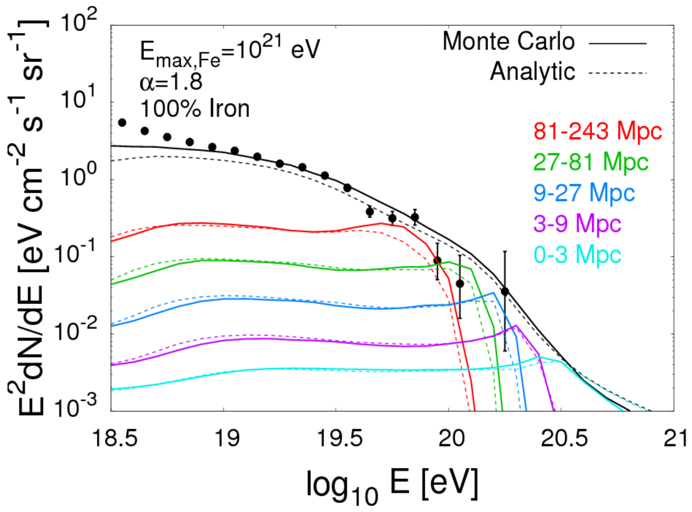
$$\mathbf{d\chi = \frac{dz}{H}}$$

$$\approx \mathbf{\frac{dz}{H_0(\Omega_M(1+z)^3 + \Omega_\Lambda)^{1/2}}}$$

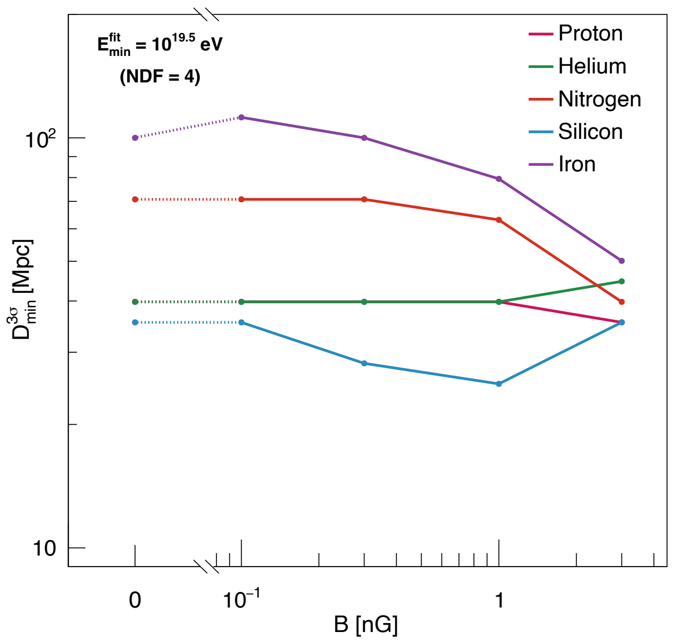


Proximity of Local Sources?

Taylor PRD, 84 105007 (2011)



Lang et PRD, 102 063012 (2020)



Note- magnetic field horizon effects are neglected here. This amounts to assuming:

$$\mathbf{d_s < (c t_H \lambda_{scat})^{1/2}}$$

ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

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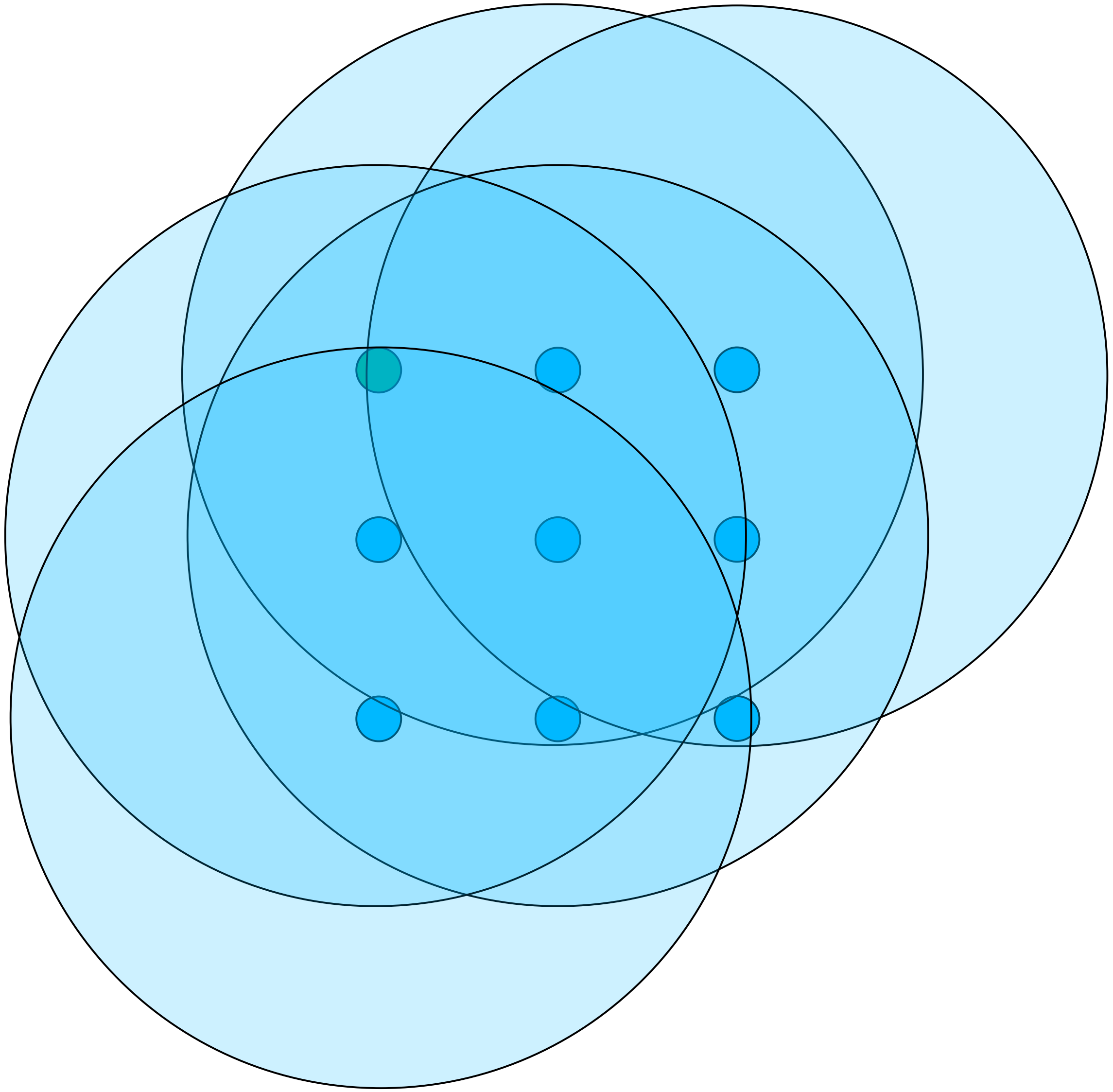
Magnetic Horizon Effect



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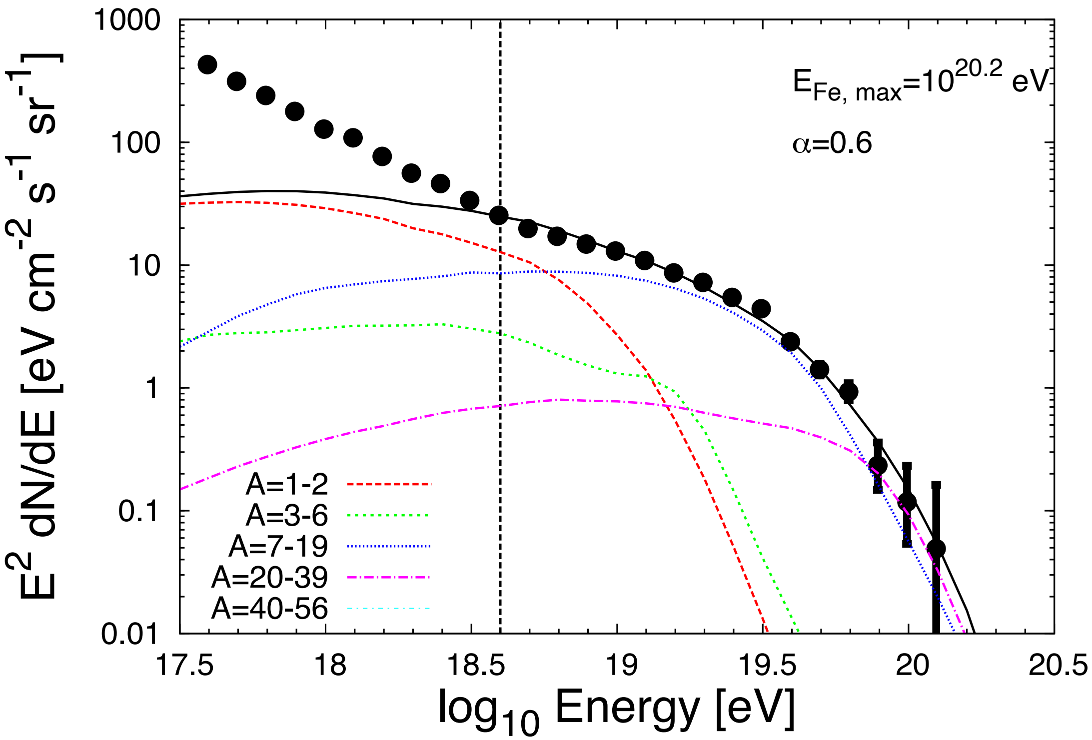
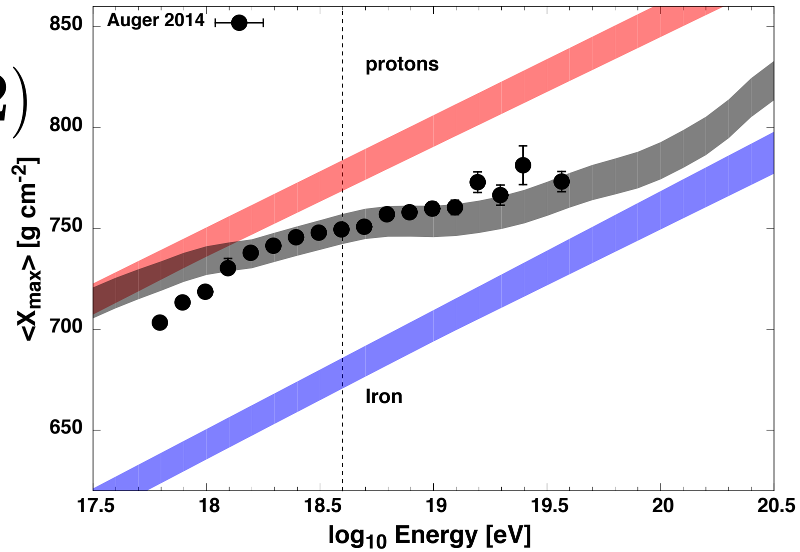
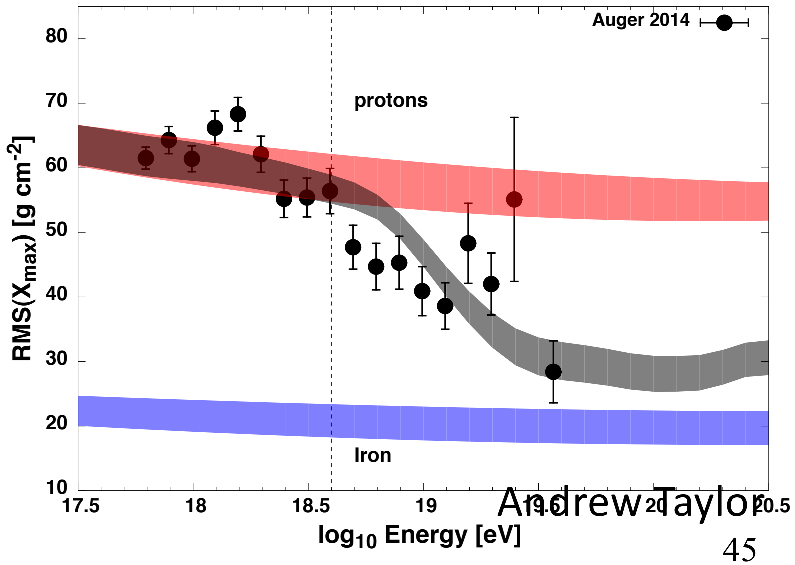
MCMC Likelihood Scan: Spectral + Composition Fits

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=3 evolution result







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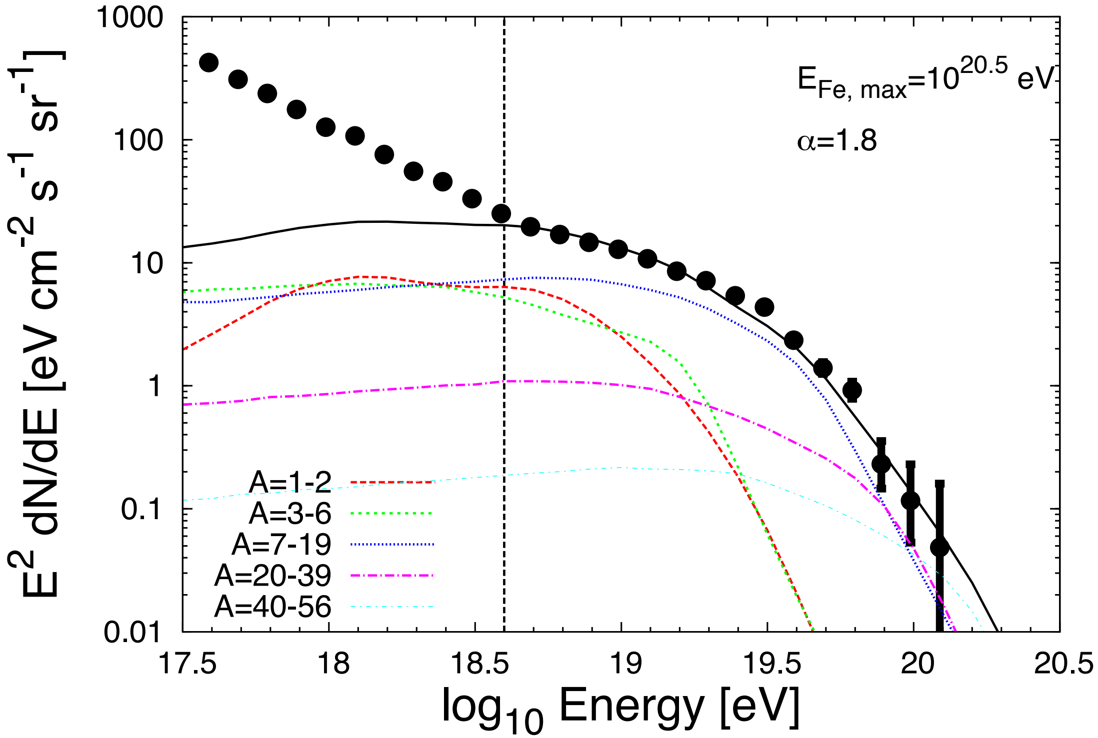
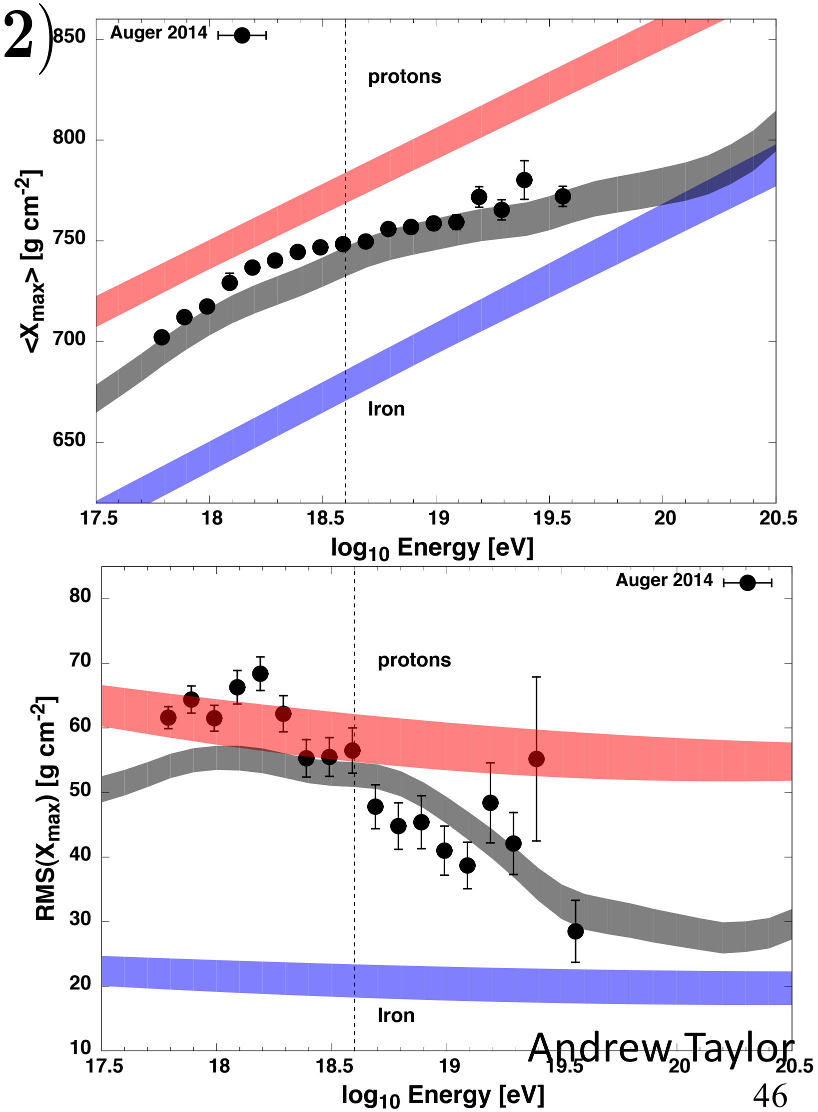
MCMC Likelihood Scan: "Soft" Spectra Solutions

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=-6 evolution result





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Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$ Best-fit Value	$n=-6$ Posterior Mean & Standard Deviation	$n=-3$ Best-fit Value	$n=-3$ Posterior Mean & Standard Deviation	$n=0$ Best-fit Value	$n=0$ Posterior Mean & Standard Deviation	$n=3$ Best-fit Value	$n=3$ Posterior Mean & Standard Deviation
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\mathrm{Fe,max}}}{\mathrm{eV}}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

note trend in index →

PAO, JCAP 04 (2017) 038

note trend in index ↓

source evolution		γ	$\log_{10}(R_{\mathrm{cut}}/\mathrm{V})$	D	$D(J)$	$D(X_{\mathrm{max}})$
$(1+z)^m$	$m=+3$	$-1.40^{+0.35}_{-0.09}$	$18.22^{+0.05}_{-0.02}$	179.1	7.5	171.7
	$m=0$	$+0.96^{+0.08}_{-0.13}$	$18.68^{+0.02}_{-0.04}$	174.3	13.2	161.1
	$m=-3$	$+1.42^{+0.06}_{-0.07}$	$18.85^{+0.04}_{-0.07}$	173.9	19.3	154.6
	$m=-6$	$+1.56^{+0.06}_{-0.07}$	18.74 ± 0.03	182.4	19.1	163.3
	$m=-12$	$+1.79 \pm 0.06$	18.73 ± 0.03	182.1	18.1	164.0
	$z \lesssim 0.02$	$+2.69 \pm 0.01$	$19.50^{+0.08}_{-0.07}$	178.6	15.3	163.3

Local source solution calls upon a more acceptable spectral index

Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$		$n=-3$		$n=0$		$n=3$	
	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\mathrm{Fe,max}}}{\mathrm{eV}}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

note trend in index →

PAO, JCAP 04 (2017) 038

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source evolution		γ	$\log_{10}(R_{\mathrm{cut}}/\mathrm{V})$	D	$D(J)$	$D(X_{\mathrm{max}})$
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$z \le 0.02$		$+2.69 \pm 0.01$	$19.50^{+0.08}_{-0.07}$	178.6	15.3	163.3

Evidence that either there aren't many such sources, or that these sources (spectrally) are copies of each other (ie. stability of solution issues)

Ehlert PRD, 107 103045 (2020)

Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$ Best-fit Value	$n=-6$ Posterior Mean & Standard Deviation	$n=-3$ Best-fit Value	$n=-3$ Posterior Mean & Standard Deviation	$n=0$ Best-fit Value	$n=0$ Posterior Mean & Standard Deviation	$n=3$ Best-fit Value	$n=3$ Posterior Mean & Standard Deviation
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\mathrm{Fe,max}}}{\mathrm{eV}}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

note trend in index →

PAO, JCAP 04 (2017) 038

note trend in index ↓

source evolution		γ	$\log_{10}(R_{\mathrm{cut}}/\mathrm{V})$	D	$D(J)$	$D(X_{\mathrm{max}})$
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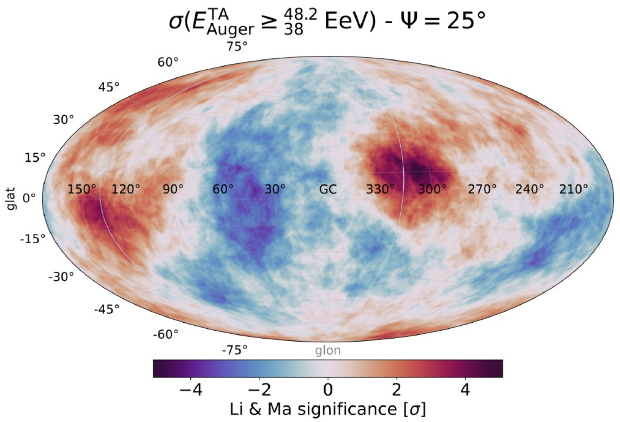

A single/few local sources solution calls upon a more acceptable spectral index/resolves the - how to square this with the anisotropy data?

Andrew Taylor

Conclusions

- The attenuation of cosmic ray protons/nuclei due to the presence of background radiation fields is reasonably well understood
- The largest limitation presently is the EBL (dust and stellar emission components)
- Despite these limitations, calculations for the propagation ultra high energy cosmic rays in these background radiation fields are predictive
- A negative evolution of sources allows for softer source injection spectra (more consistent with the Fermi acceleration model)
- The current cosmic ray data at the highest energies is suggestive that the nearest sources should be no further than a few 10s of Mpc

Threshold Energy- Proton Pion Production

$$(\mathbf{E_p} + \mathbf{E}_\gamma)^2 - (\mathbf{p_p} - \mathbf{E}_\gamma)^2 = (\mathbf{m_p} + \mathbf{m}_\pi)^2$$

$$\mathbf{m_p^2} + 2\mathbf{E_p}\mathbf{E}_\gamma + 2\mathbf{p_p}\mathbf{E}_\gamma \approx \mathbf{m_p^2} + 2\mathbf{m_p}\mathbf{m}_\pi$$

$$\mathbf{E_p} \approx \frac{\mathbf{m}_\pi}{2\mathbf{E}_\gamma}\mathbf{m_p} \approx \left(\frac{135 \times 10^6}{2 \times 6 \times 10^{-4}}\right) 0.9 \times 10^9 = 10^{20}\ \mathrm{eV}$$

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Comparison of Analytic and Monte Carlo Results