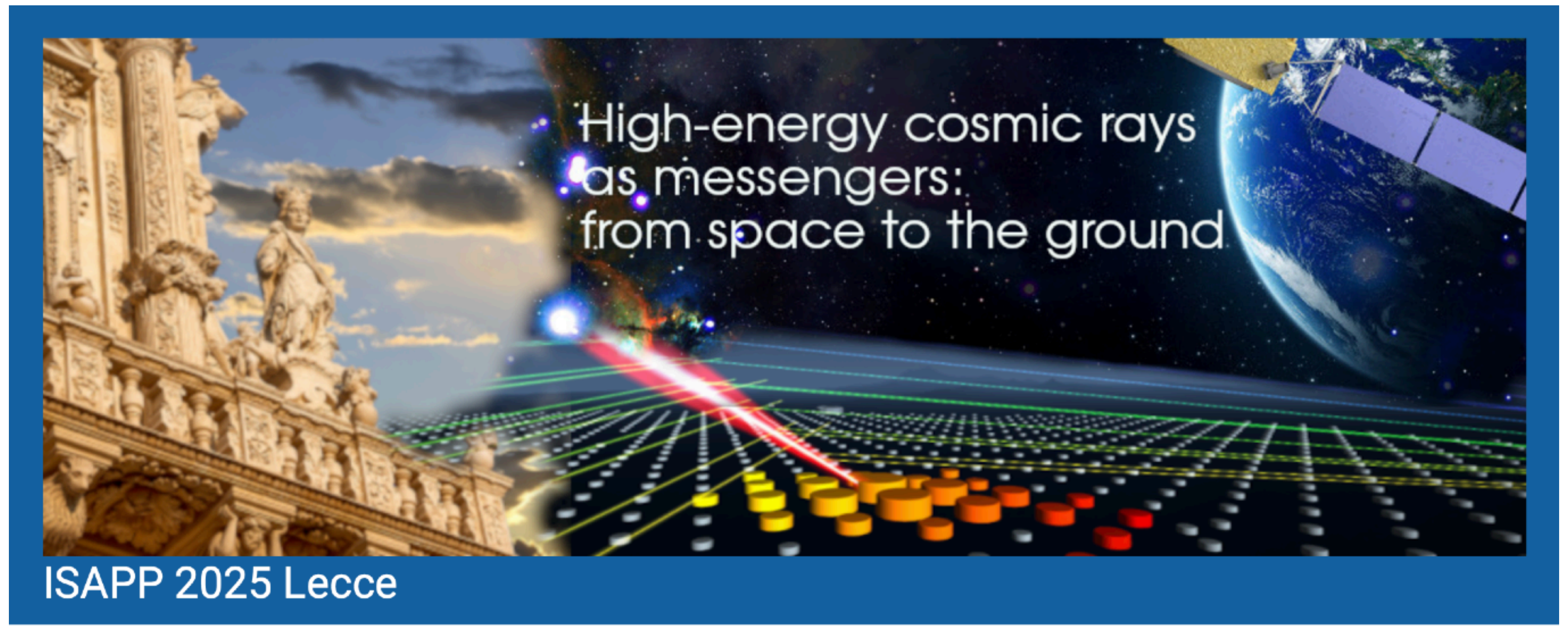
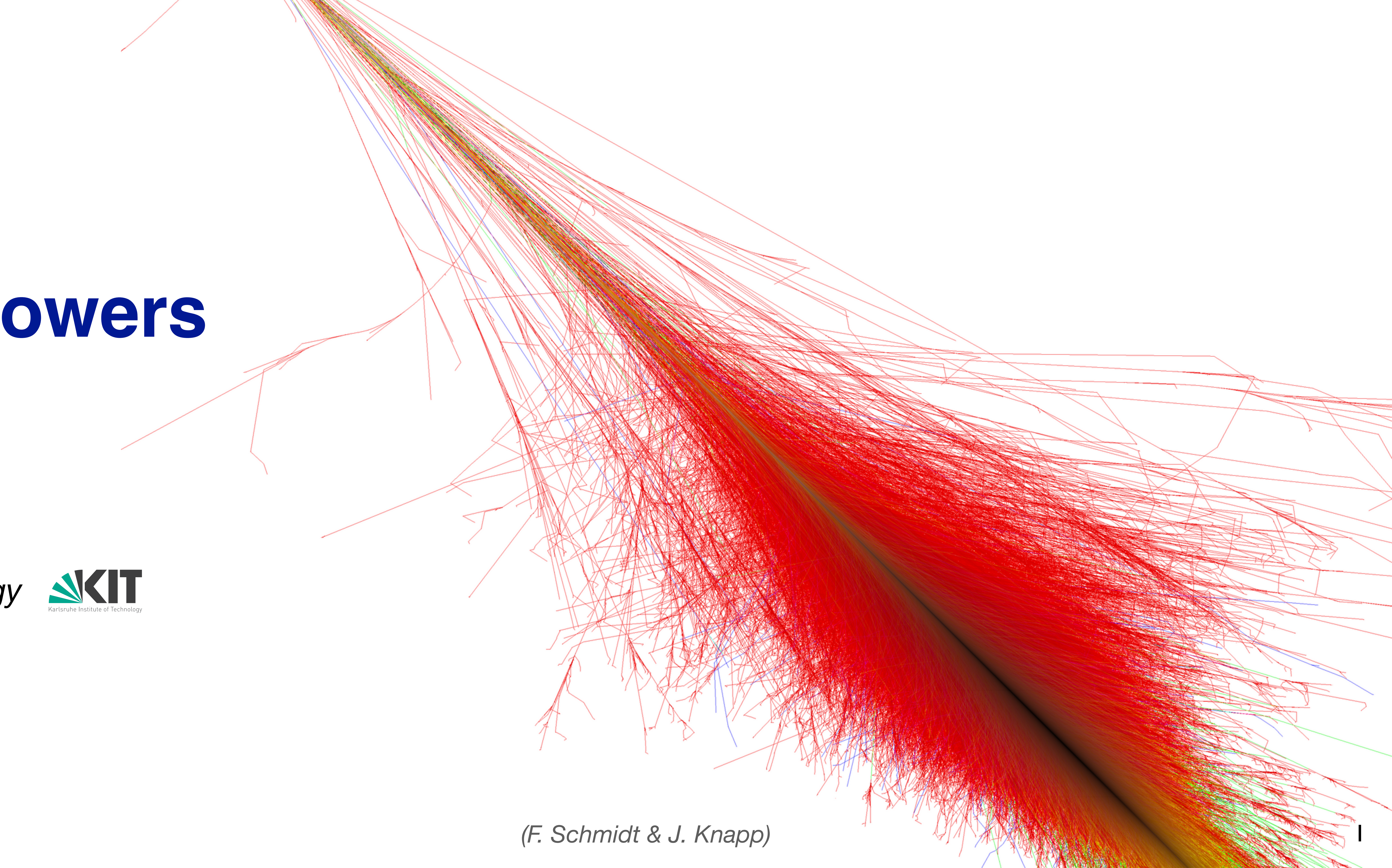
INTERNATIONAL SCHOOL ON ASTROPARTICLE PHYSICS



Physics of Air Showers

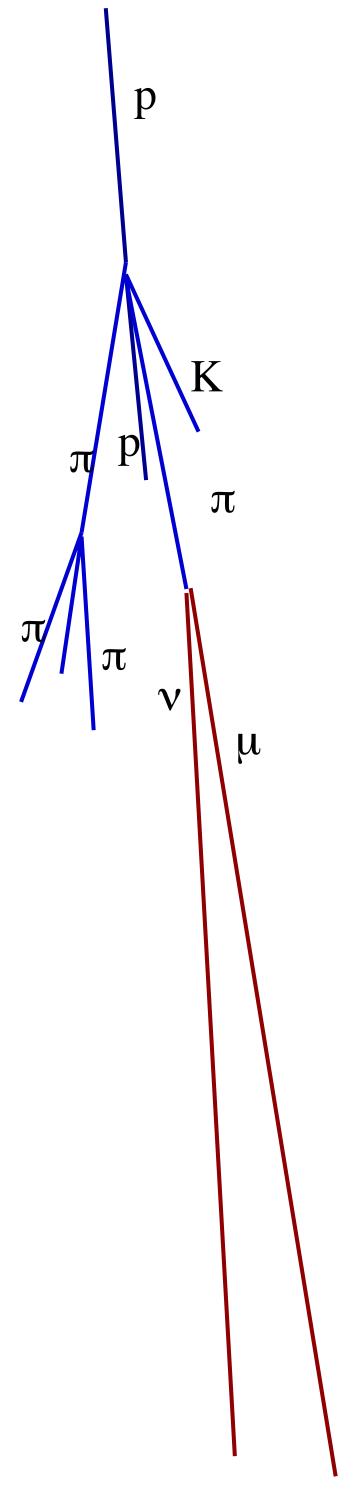
Ralph Engel

Karlsruhe Institute of Technology

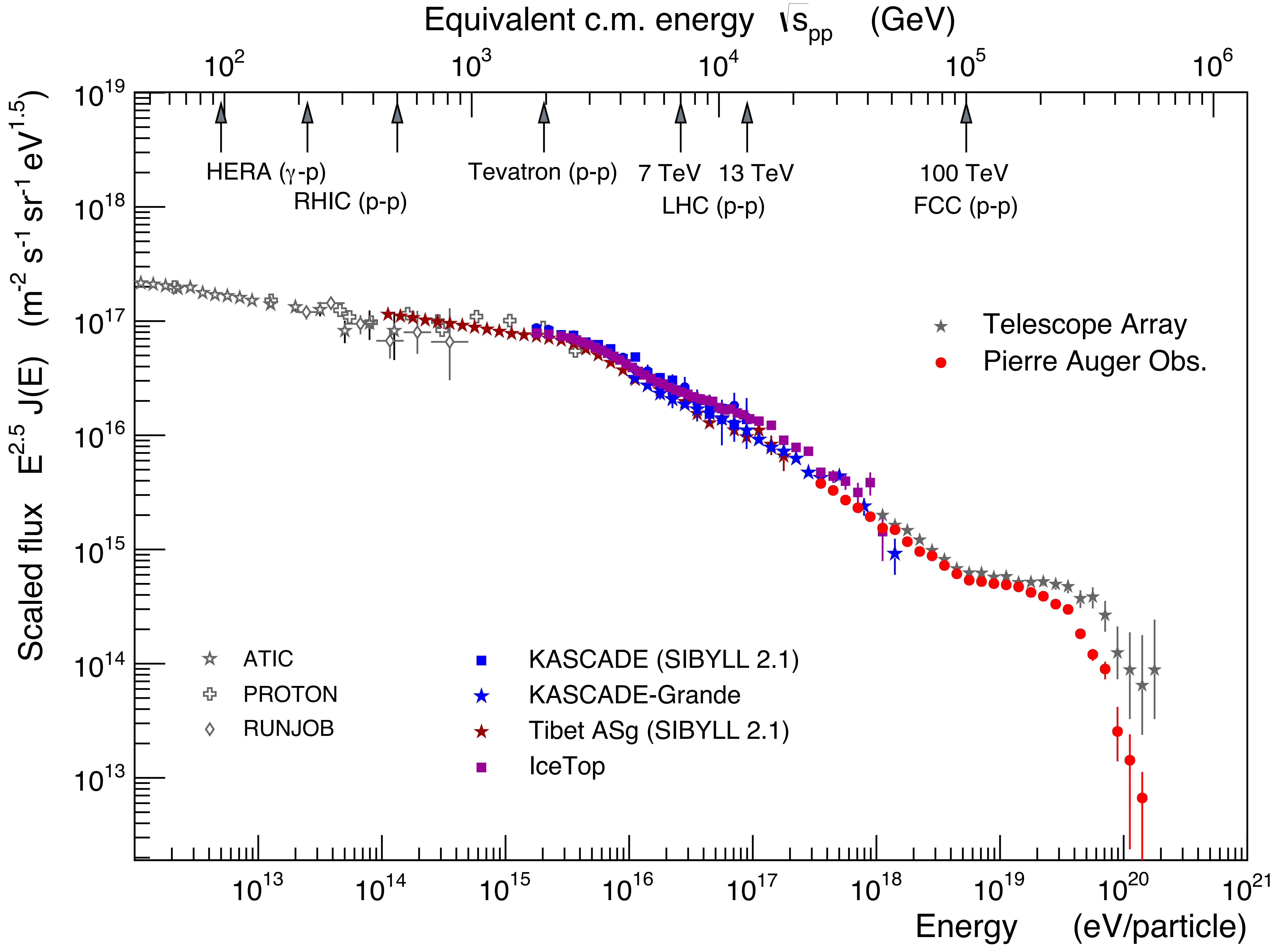


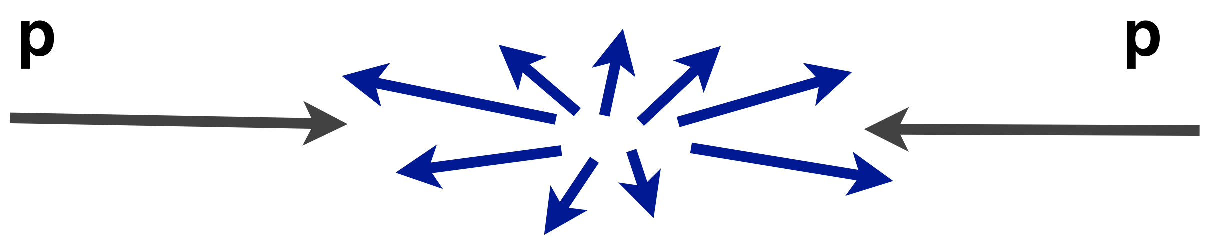
(F. Schmidt & J. Knapp)

Cosmic ray flux and interaction energies

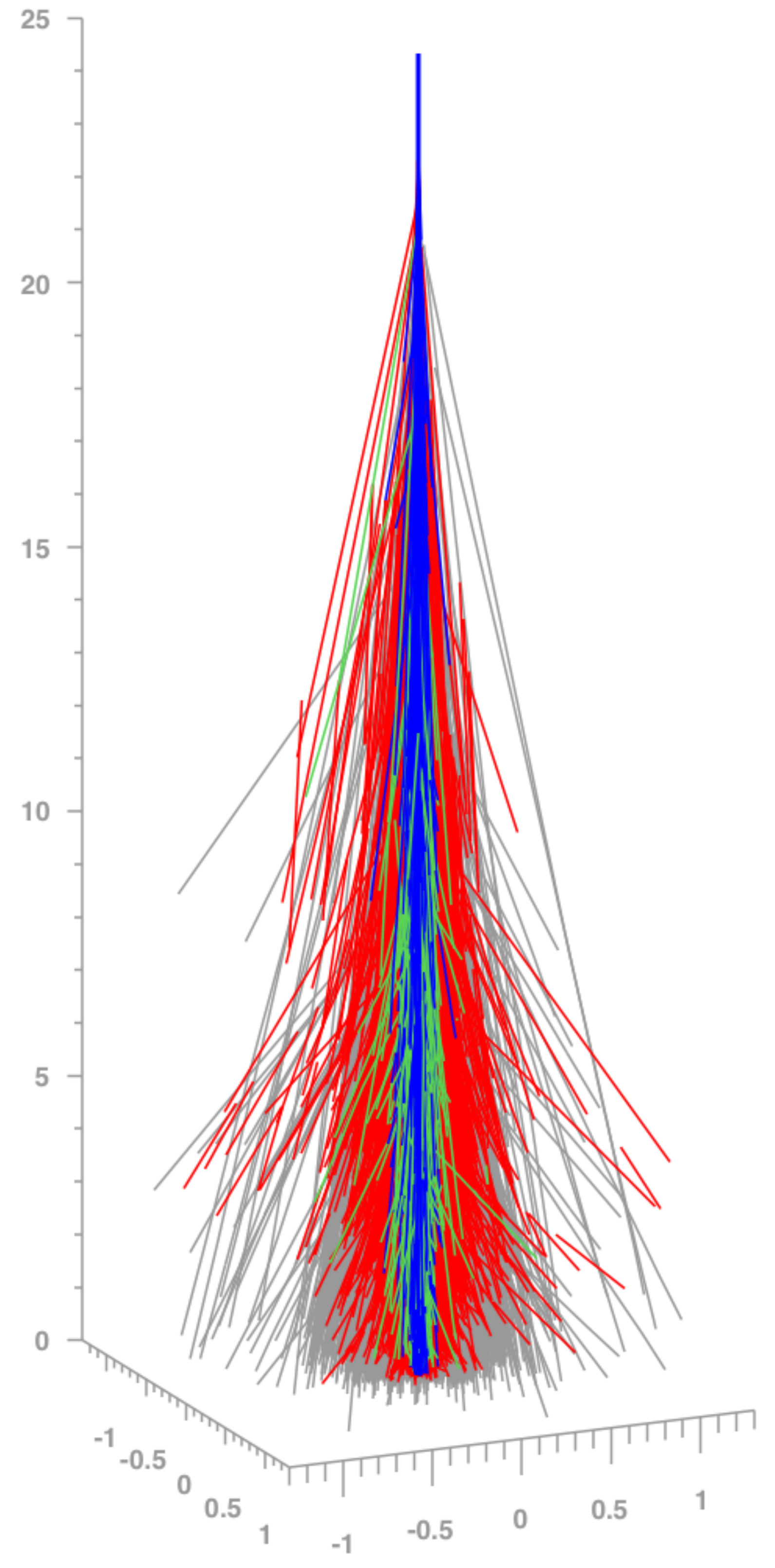


Inclusive fluxes

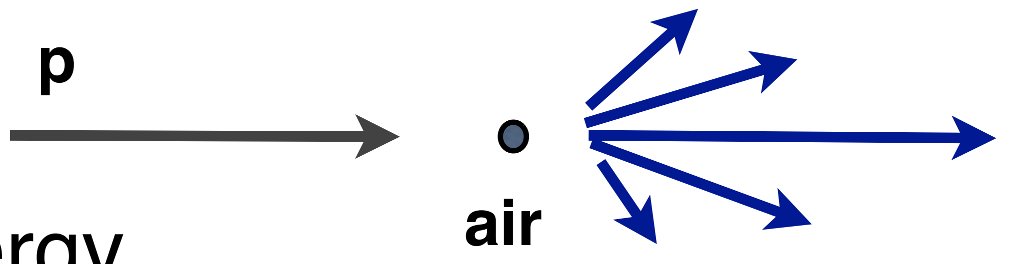




Center-of-mass energy

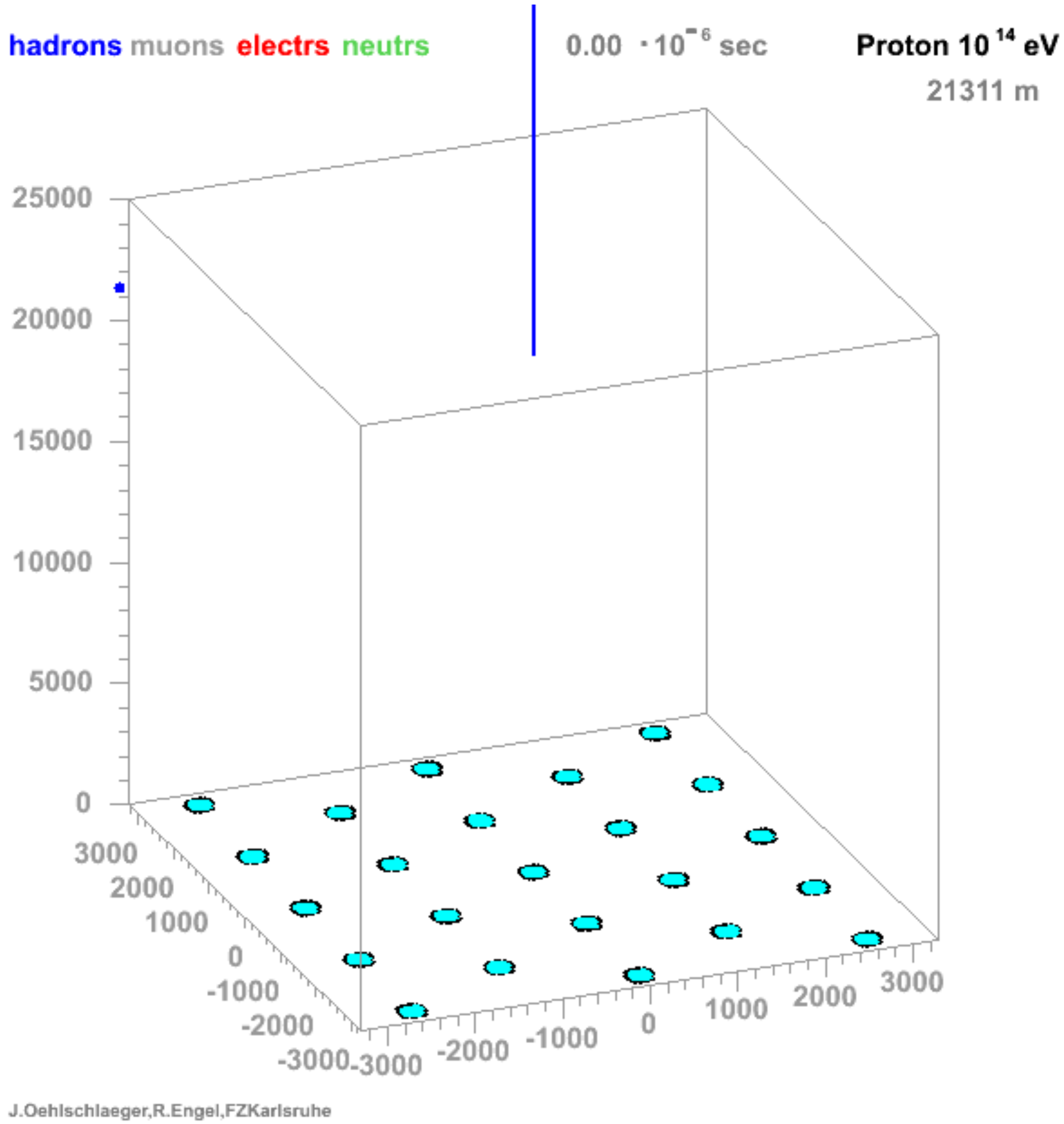


Air showers



Laboratory energy

1. Simulations

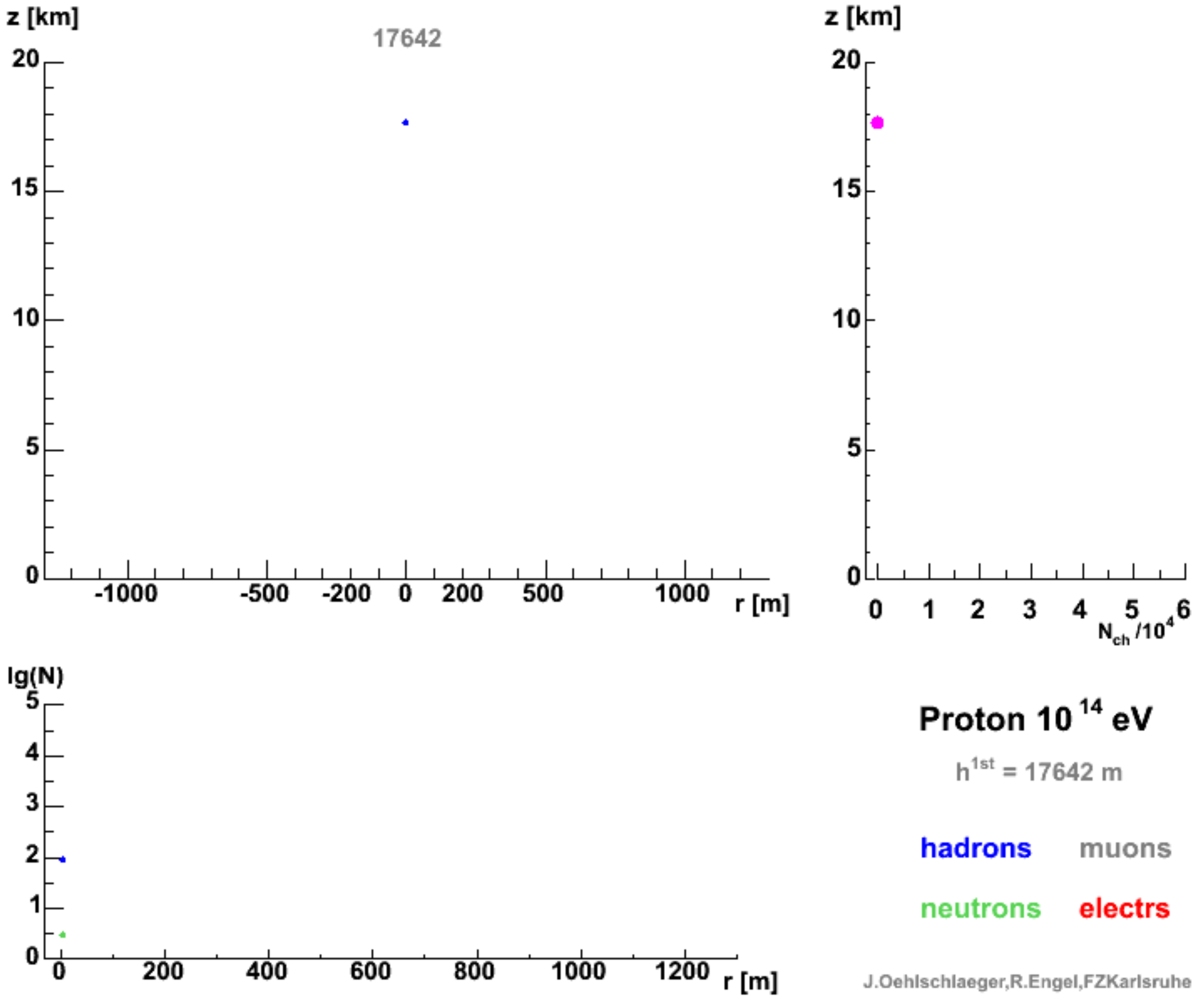


Simulation of shower development (i)

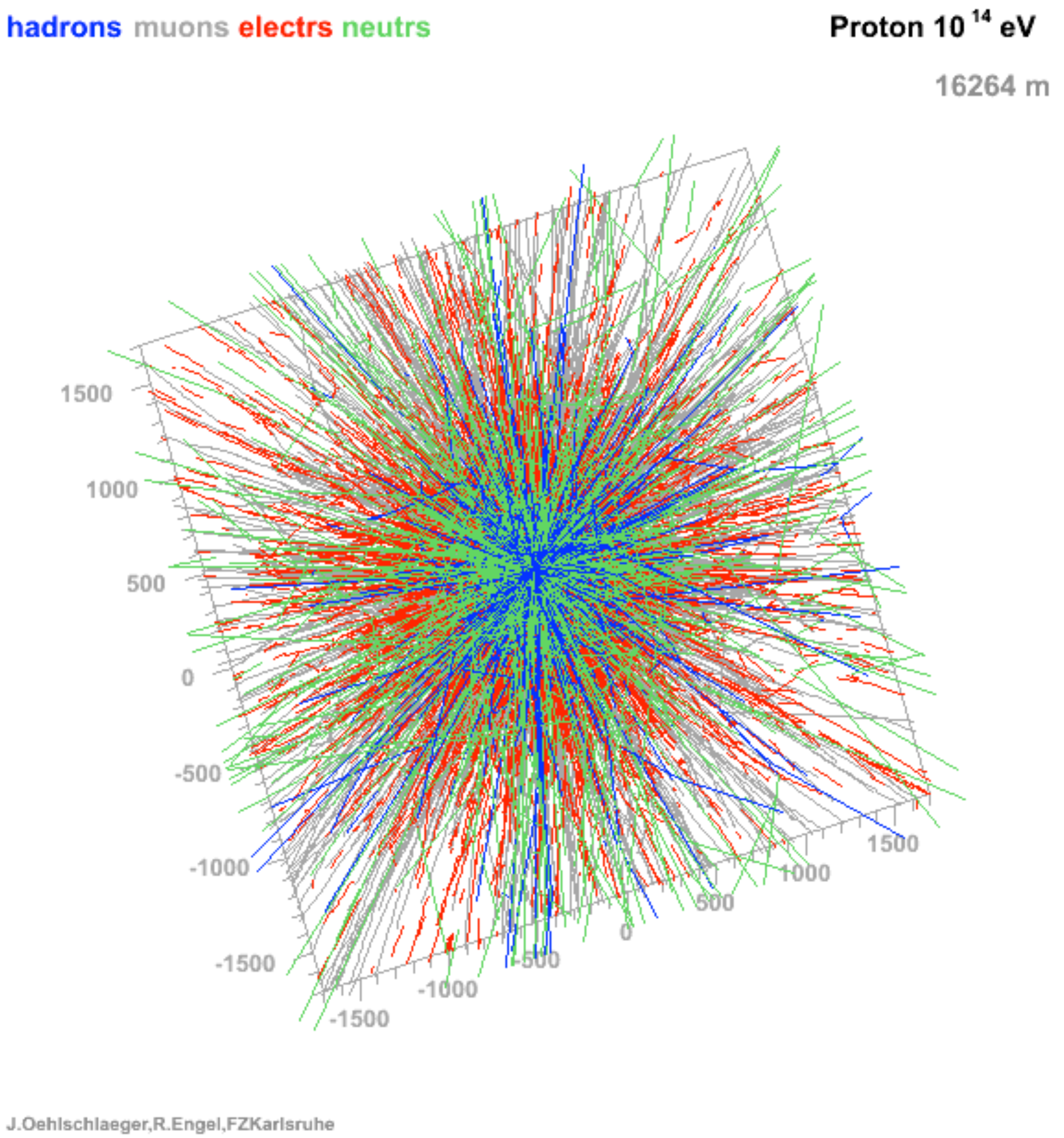
Realistic simulation with CORSIKA

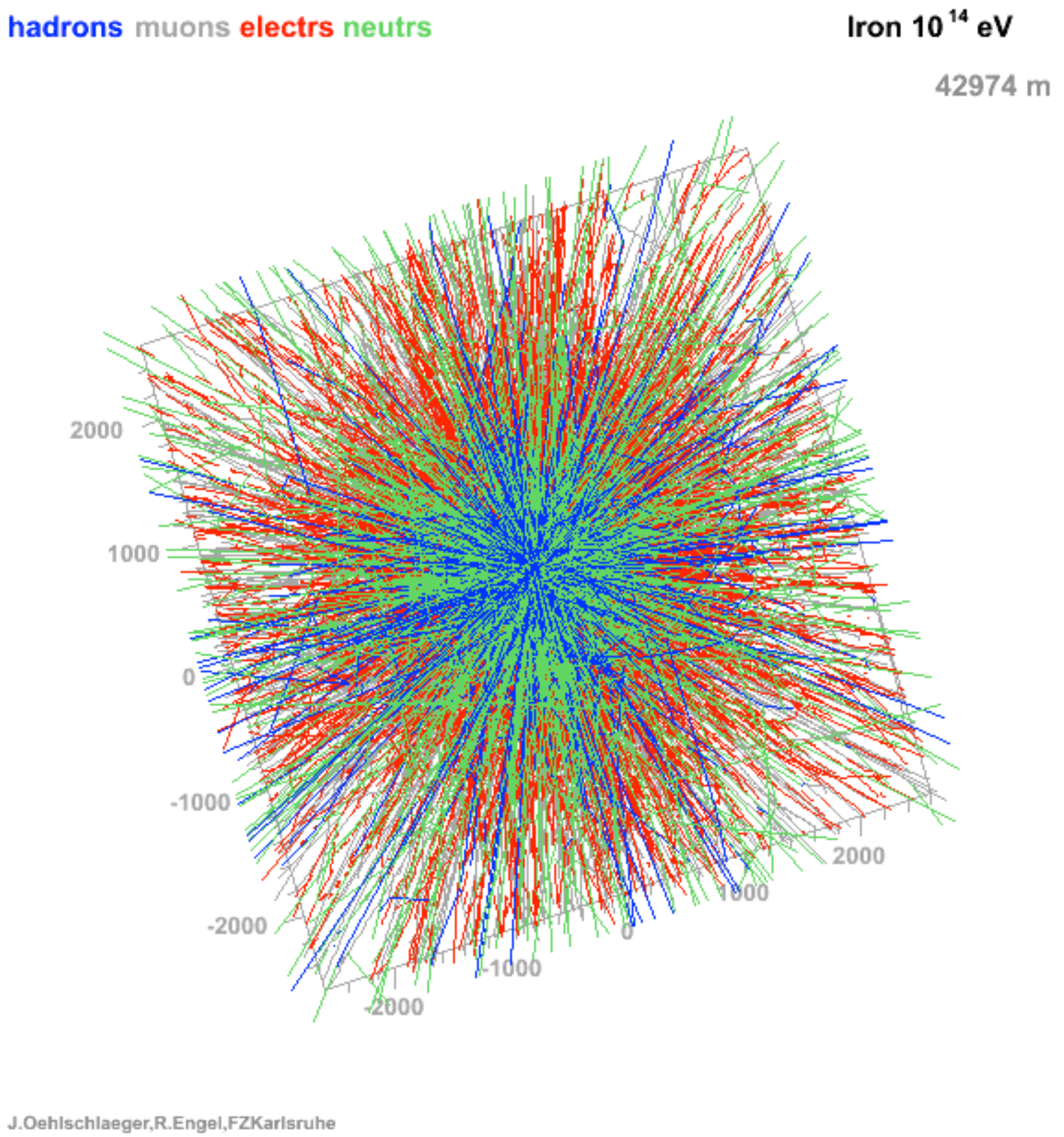
Proton shower of low energy (knee region)

Simulation of shower development (ii)

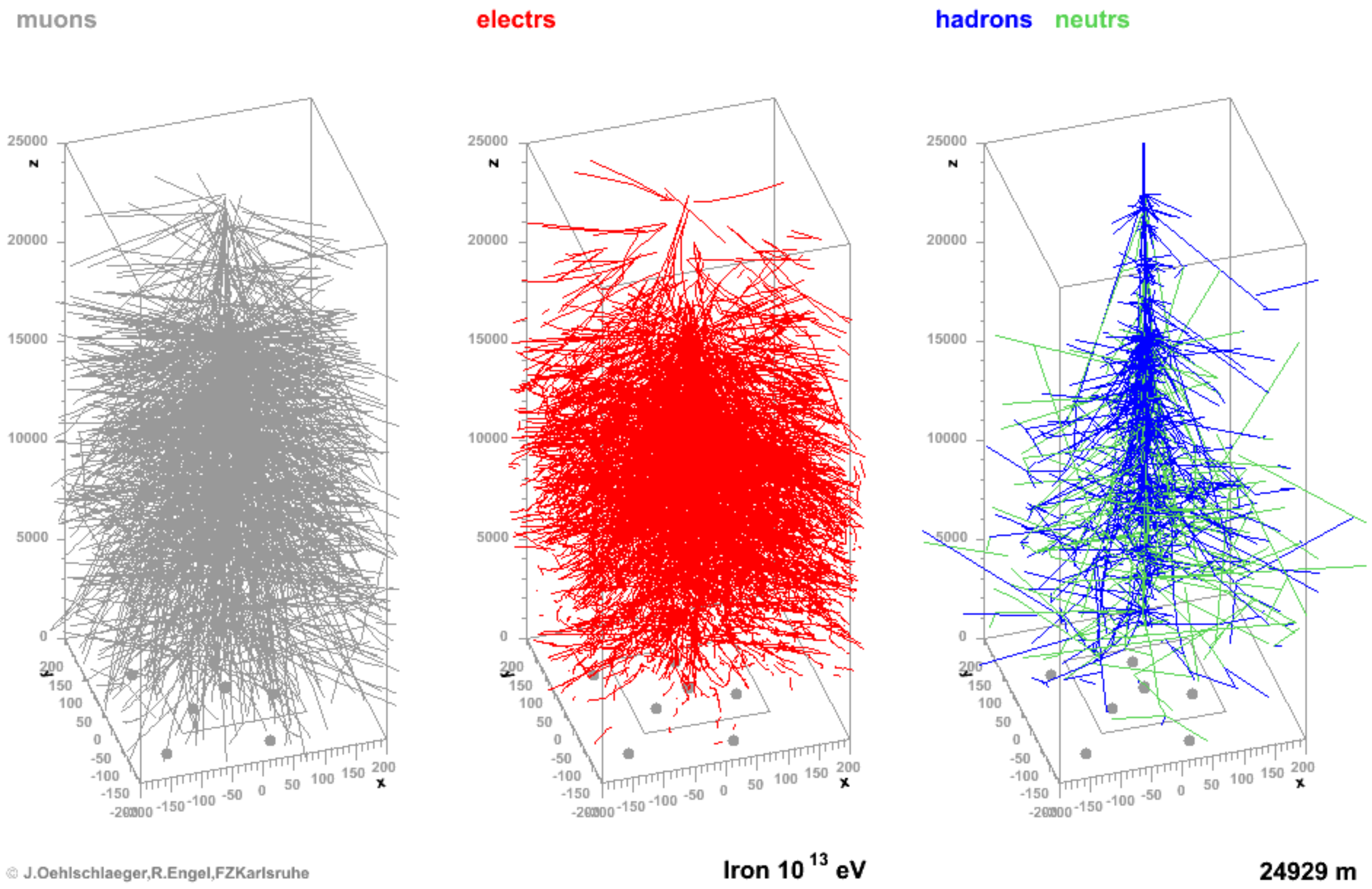


Simulation of air shower tracks (i)

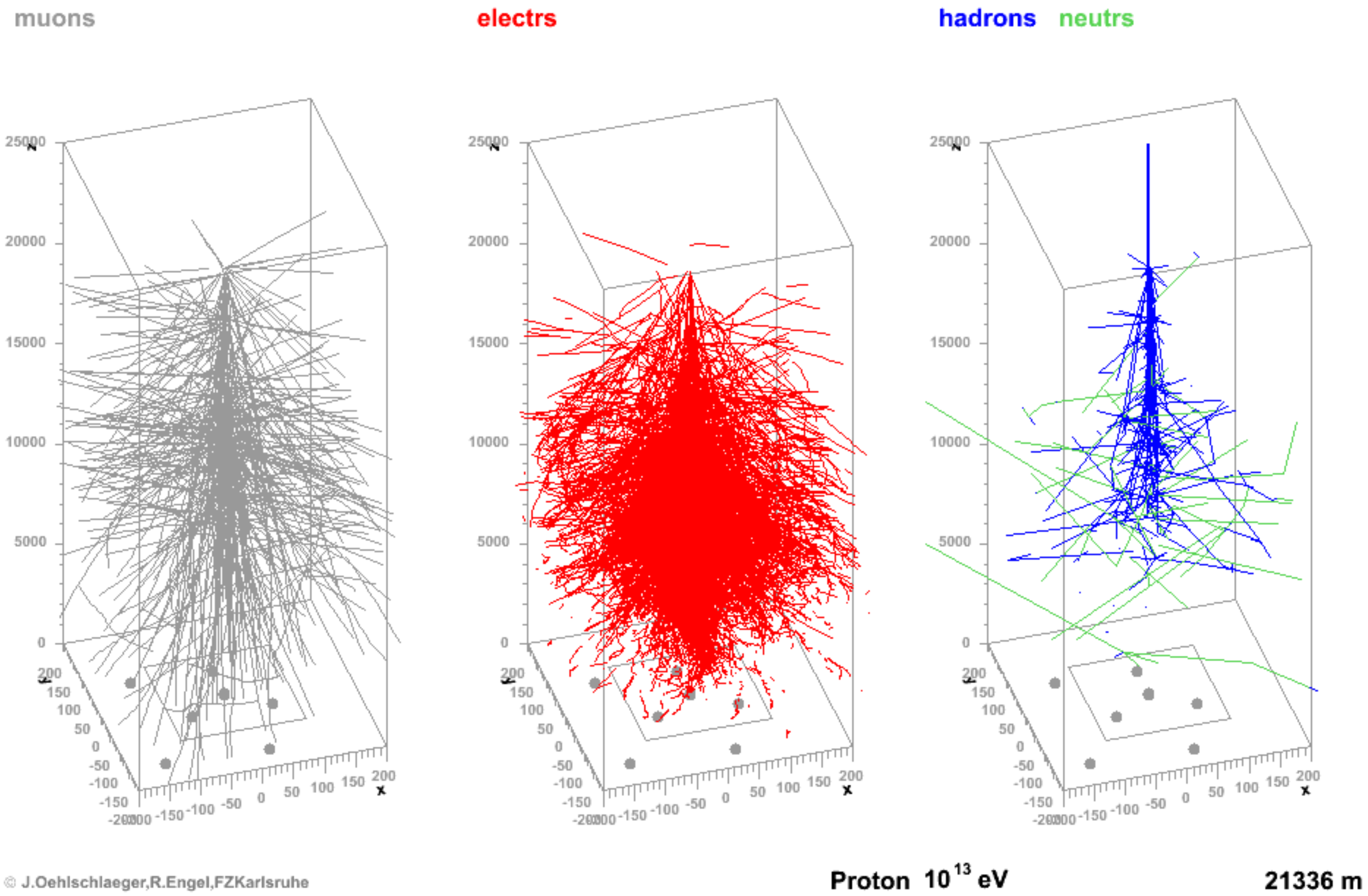




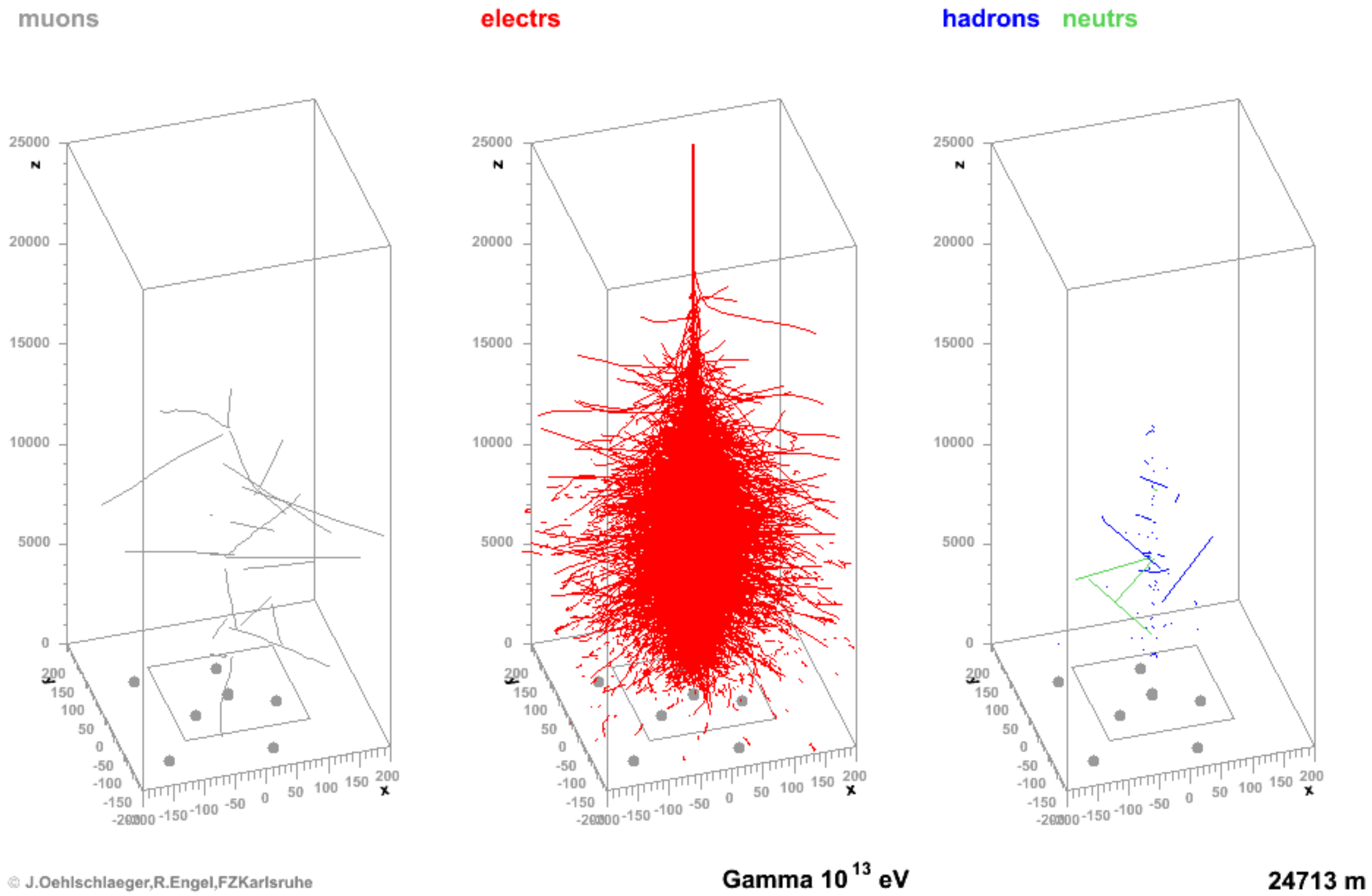
Particles of an iron shower



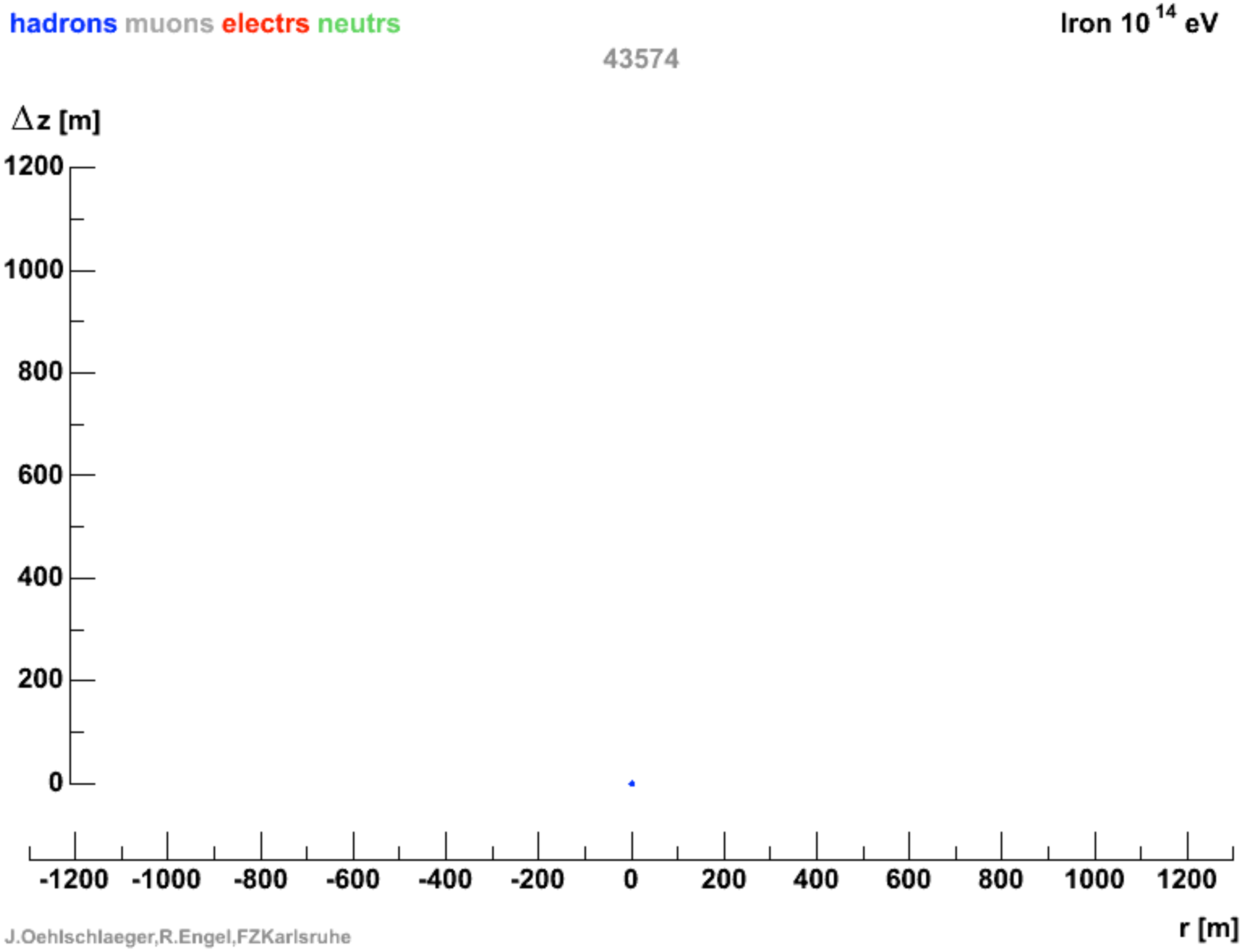
Particles of an proton shower



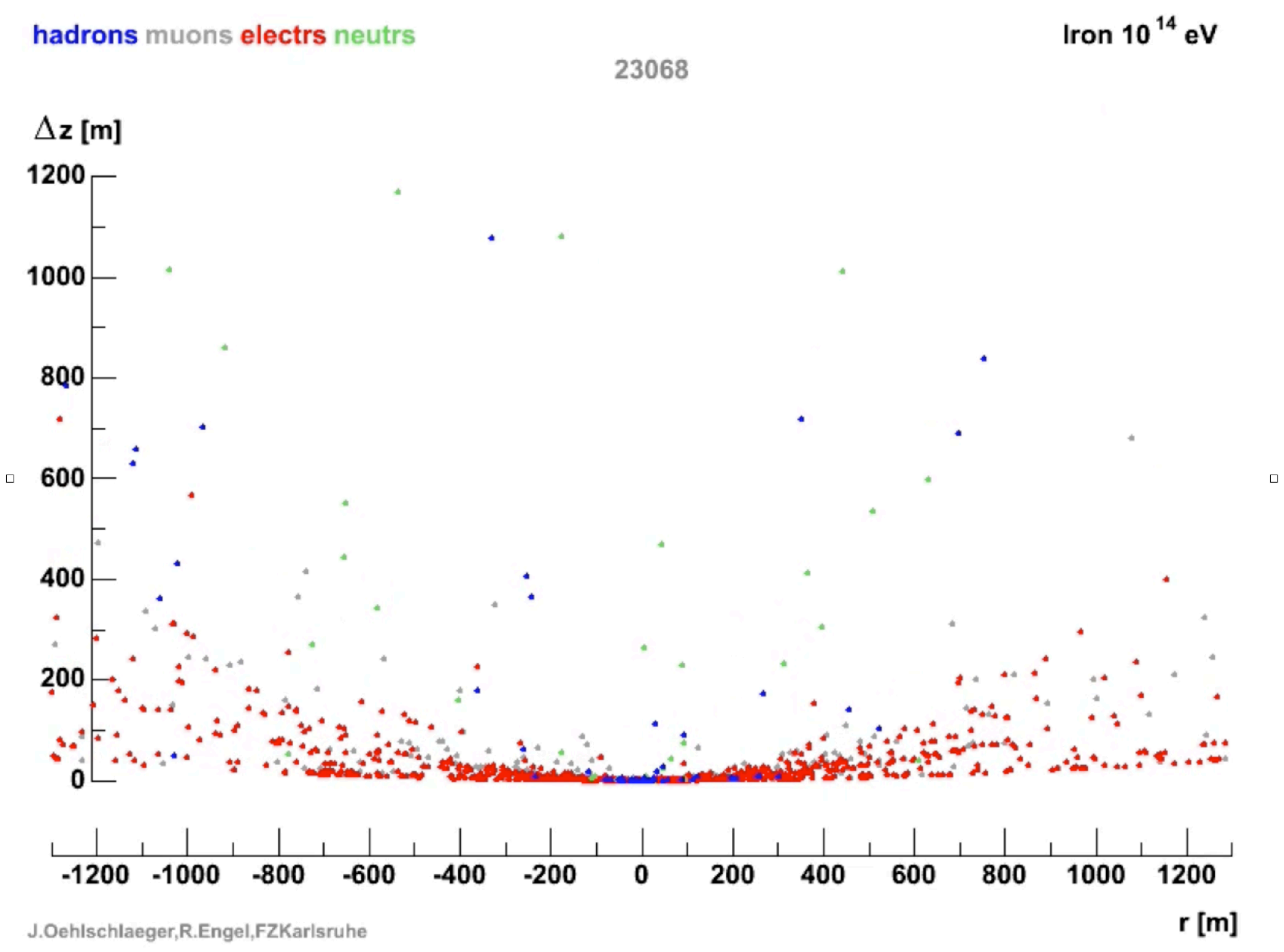
Particles of a gamma-ray shower

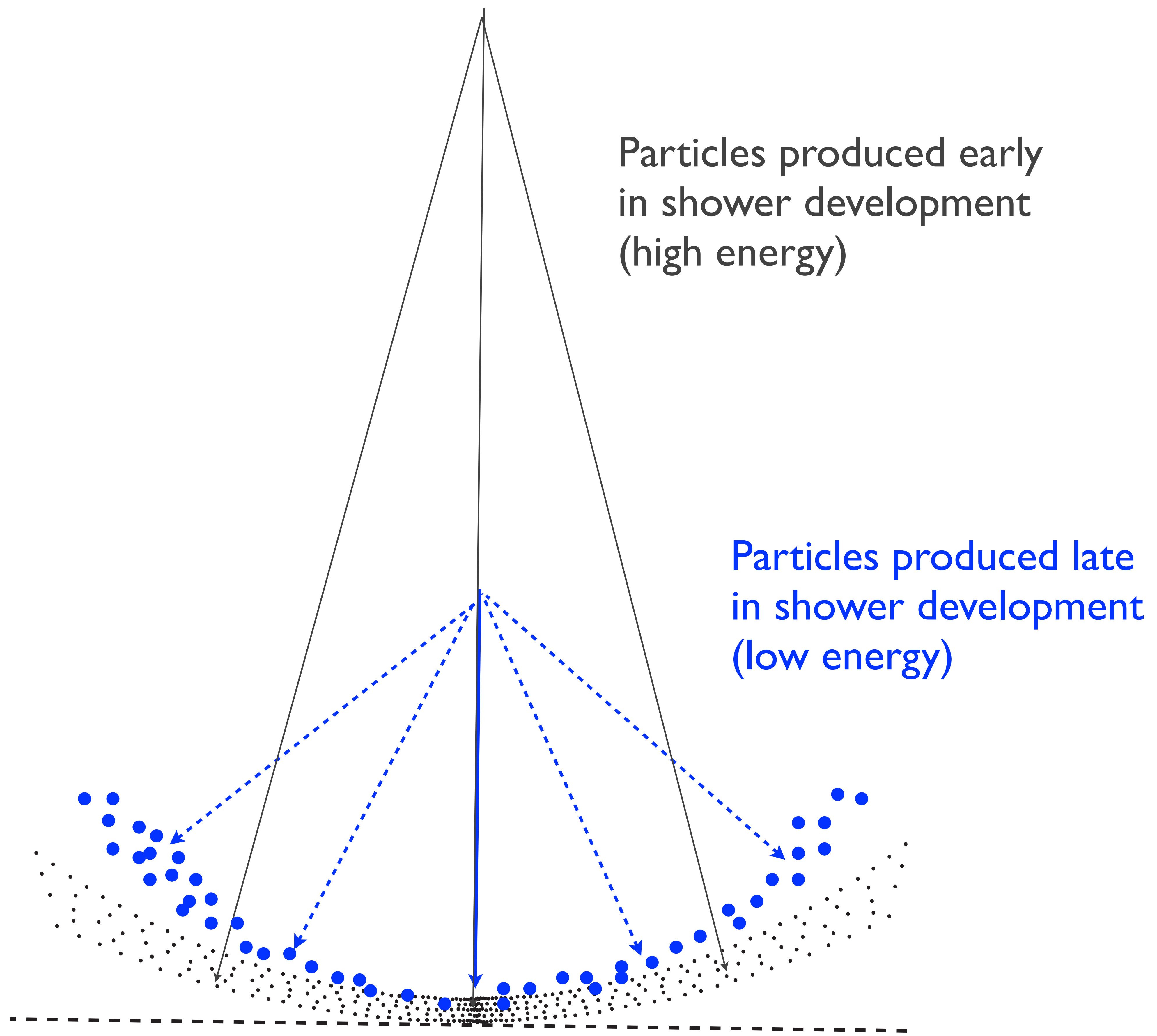


Time structure of shower disk



Time structure of shower disk



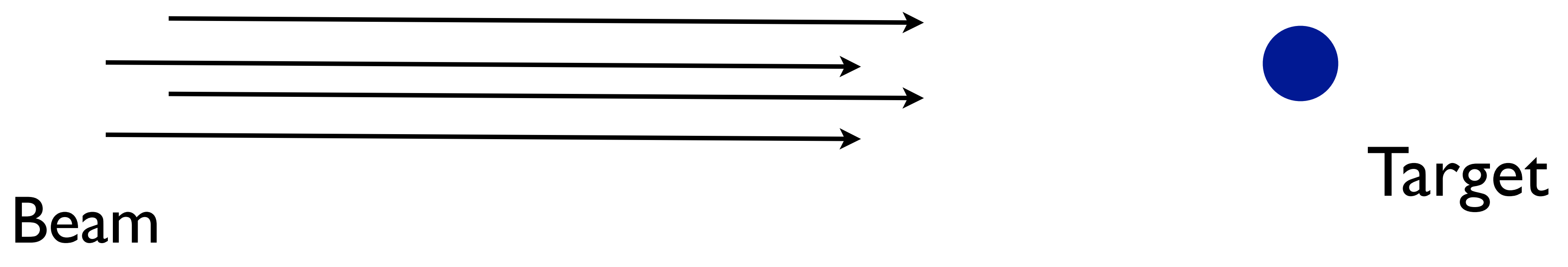


Curvature of shower front
sensitive to early muons

2. Fundamentals

Cross section, interaction rate, interaction length

$$\Phi = \frac{\mathrm{d}N_{\mathrm{beam}}}{\mathrm{d}A\ \mathrm{d}t}$$



Definition of cross section

$$\sigma = \frac{1}{\Phi}\frac{\mathrm{d}N_{\mathrm{int}}}{\mathrm{d}t}$$

(Units: 1 barn = 10^{-28} m^2
1 mb = 10^{-27} cm^2)

Flux of particles on single target

Interaction rate

Interaction length (g/cm^2)

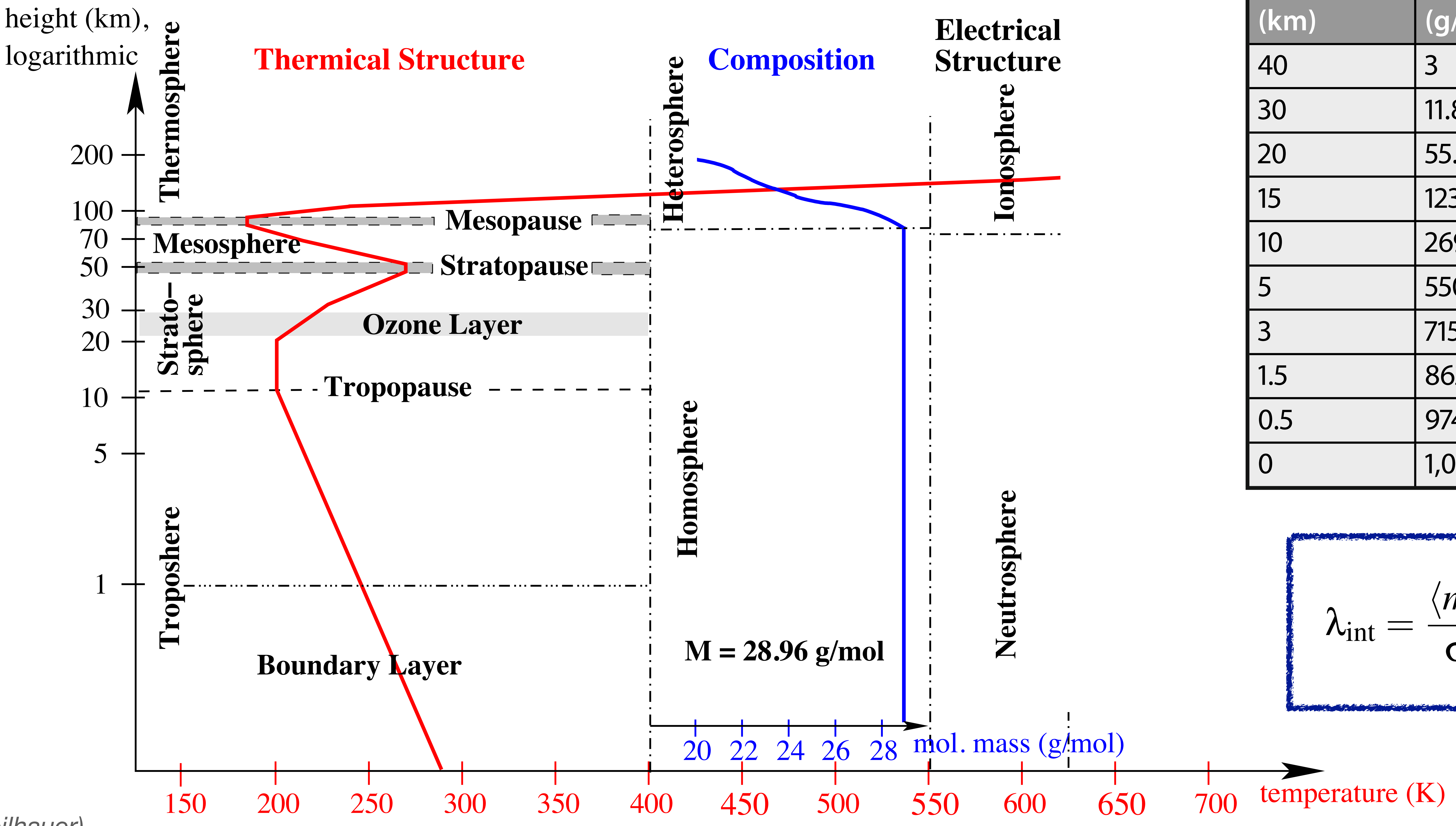
$$\lambda_{\mathrm{int}} = \frac{\langle m_{\mathrm{target}}\rangle}{\sigma}$$

$$\frac{\mathrm{d}N_{\mathrm{int}}}{\mathrm{d}t\,\mathrm{d}V} = \frac{\rho_{\mathrm{target}}}{\langle m_{\mathrm{target}}\rangle}\,\sigma\,\Phi$$

$$\mathrm{d}X = \rho_{\mathrm{target}}\,\mathrm{d}l$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}X} = -\frac{\sigma}{\langle m_{\mathrm{target}}\rangle}\Phi = -\frac{1}{\lambda_{\mathrm{int}}}\Phi$$

Molecular atmosphere of Earth



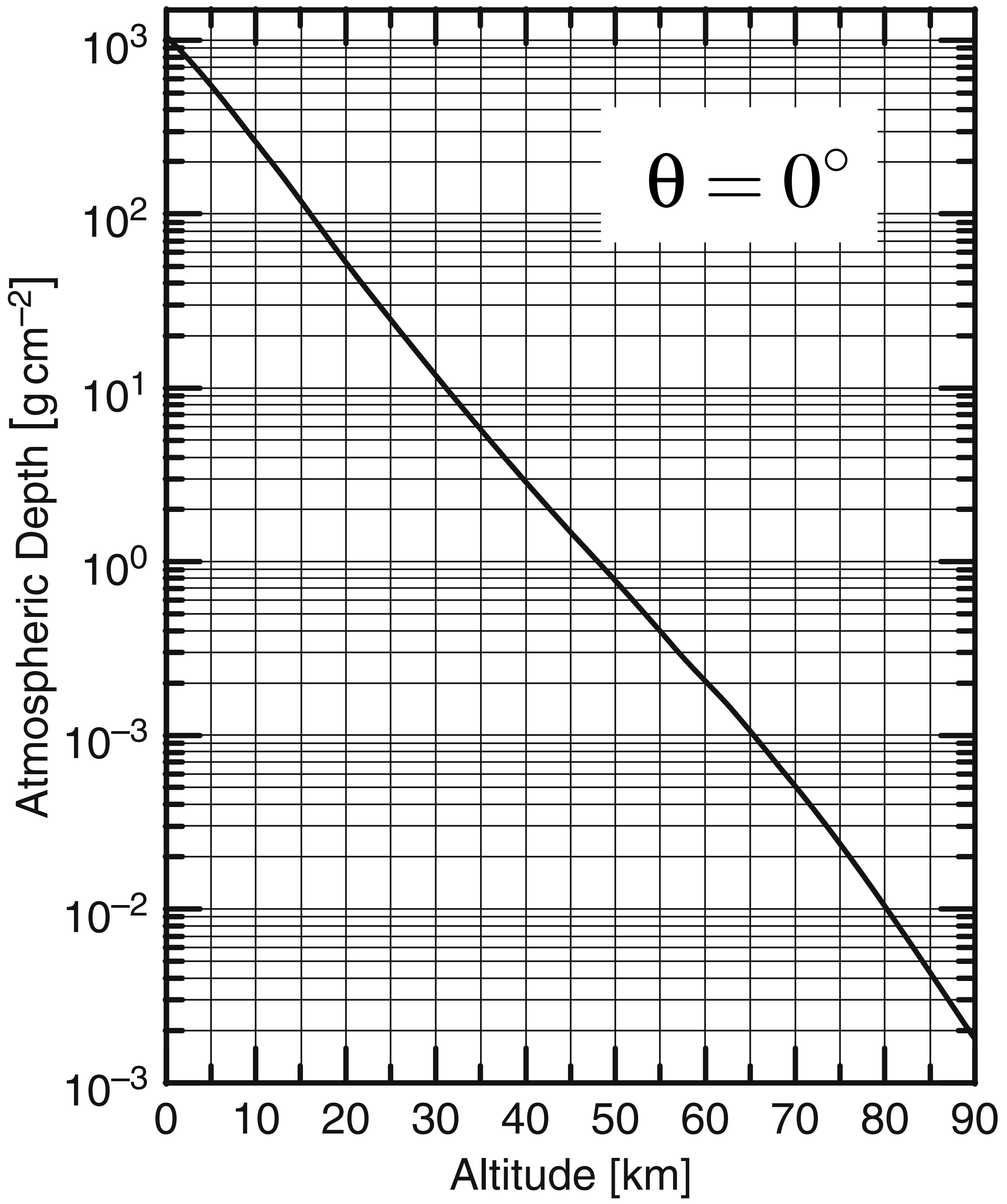
Altitude (km)	Vertical depth (g/cm^2)	Local density (10^{-3} g/cm^3)
40	3	3.8×10^{-3}
30	11.8	1.8×10^{-2}
20	55.8	8.8×10^{-2}
15	123	0.19
10	269	0.42
5	550	0.74
3	715	0.91
1.5	862	1.06
0.5	974	1.17
0	1,032	1.23

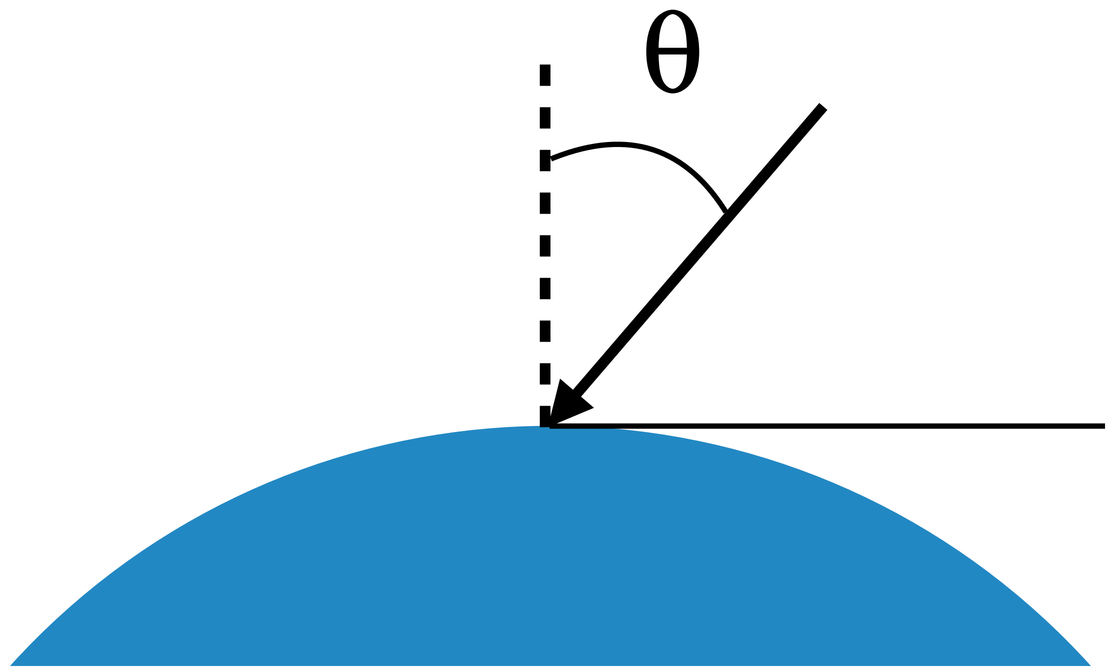
$$\lambda_{int} = \frac{\langle m_{air} \rangle}{\sigma_{int}} = \frac{24160\,\text{mb g/cm}^2}{\sigma_{int}}$$

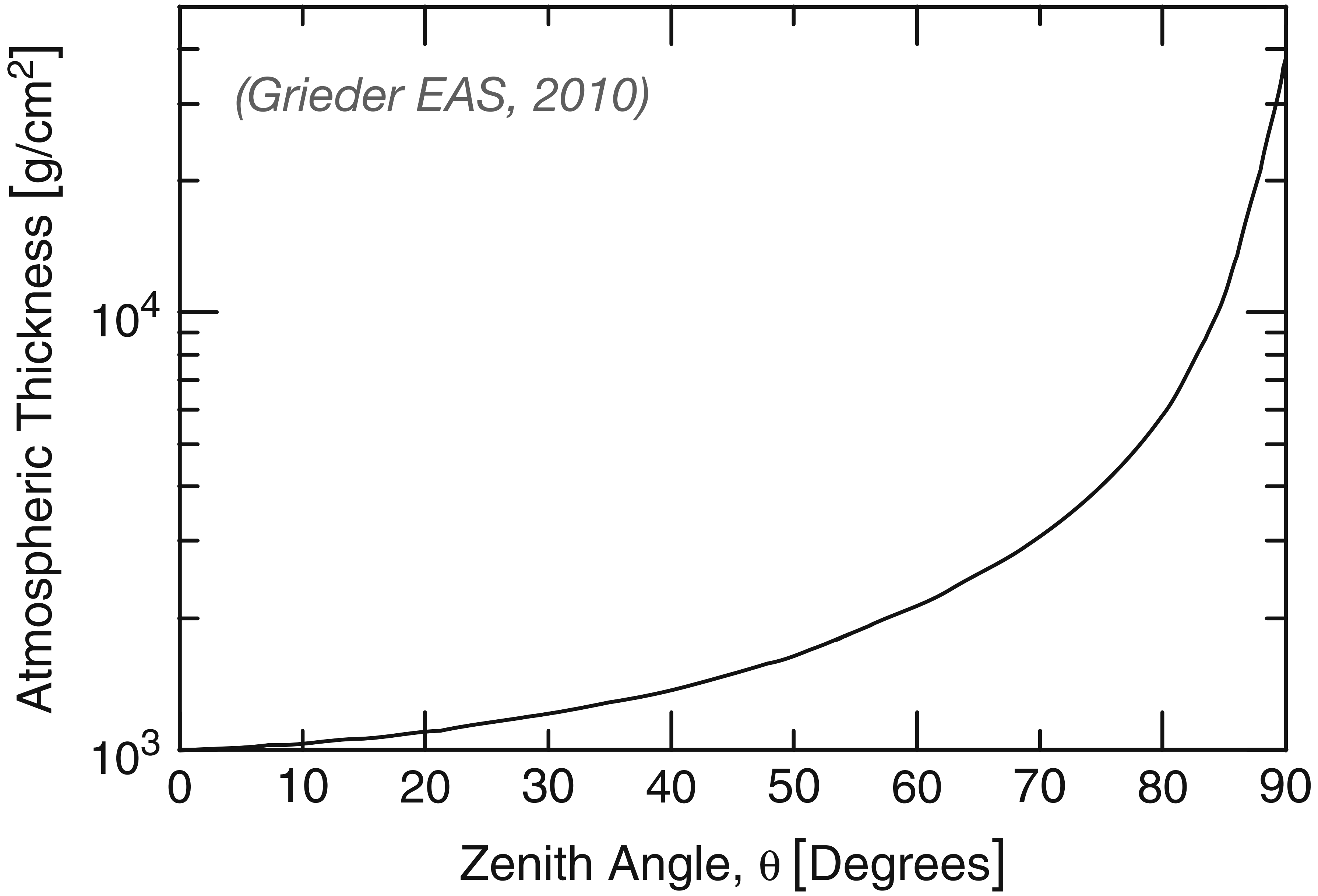
(B. Keilhauer)

The Earth's atmosphere

Altitude (km)	Local density (10^{-3} g/cm^3)
40	3.8×10^{-3}
30	1.8×10^{-2}
20	8.8×10^{-2}
15	0.19
10	0.42
5	0.74
3	0.91
1.5	1.06
0.5	1.17
0	1.23







Atmospheric **slant depth**
(integral taken along shower axis)

$$\int \rho_{\mathrm{air}}\, \mathrm{d}l = X$$

The Earth's atmosphere in numbers

altitude (km)	vertical depth (g/cm^2)	local density (10^{-3} g/cm^3)	Molière unit (m)	Cherenkov threshold (MeV)	Cherenkov angle ($^\circ$)
40	3	3.8×10^{-3}	2.4×10^4	386	0.076
30	11.8	1.8×10^{-2}	5.1×10^3	176	0.17
20	55.8	8.8×10^{-2}	1.0×10^3	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1032	1.23	76	21	1.36

$$X_v = X_0 e^{-h/h_0}$$

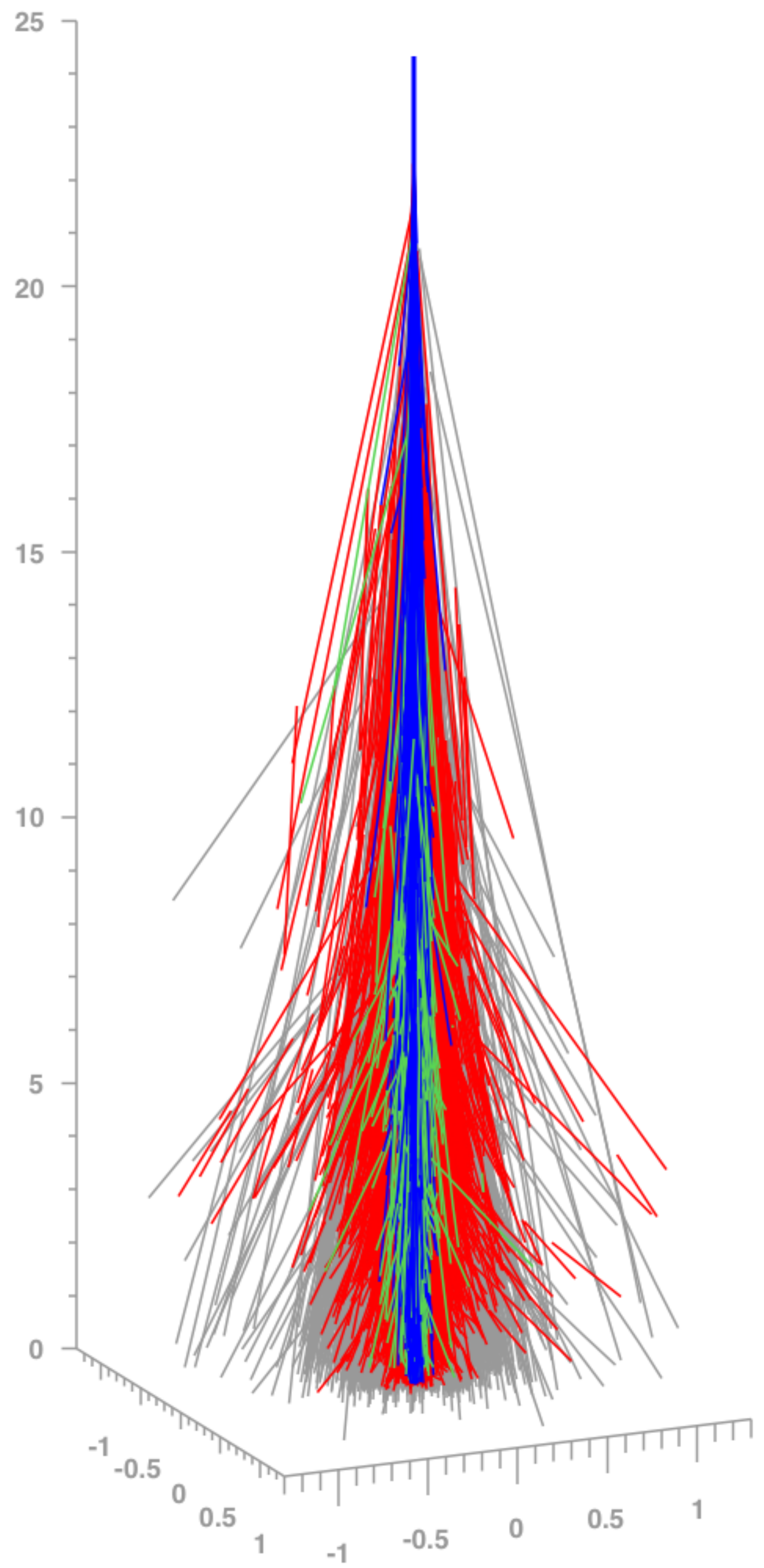
In reality the temperature and hence the scale height decrease with increasing altitude until the tropopause (12-16 km). At sea level $h_0 \cong 8.4$ km, and for $40 < X_v < 200\,\mathrm{g/cm^2}$, where production of secondary particles peaks, $h_0 \cong 6.4$ km. A useful parametrization[5] of the relation between altitude and vertical depth (due to M. Shibata) is

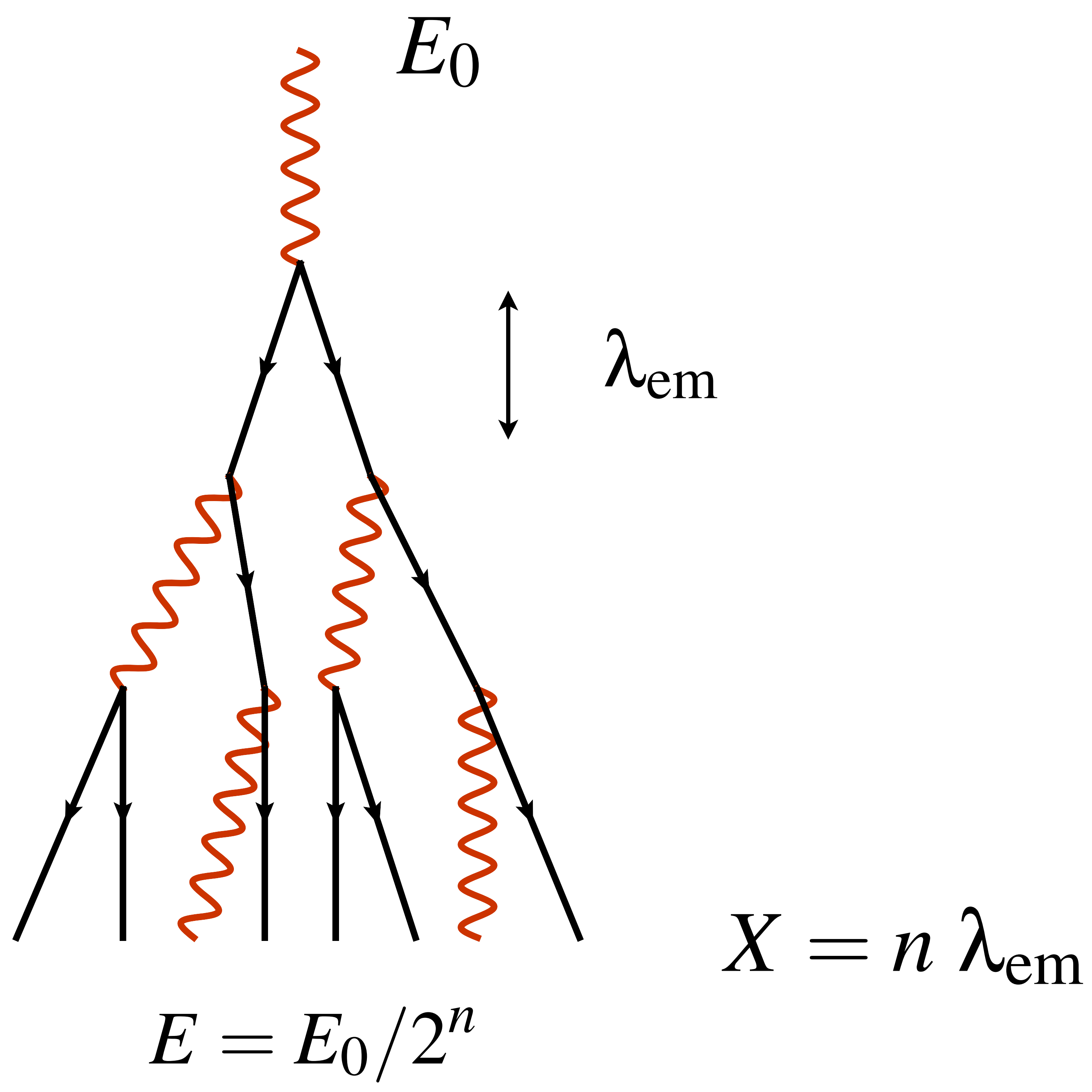
Compact parametrization of depth-altitude relation (US Standard Atmosphere)

$$h_v(\mathrm{km}) = \begin{cases} 47.05 - 6.9 \ln X_v + 0.299 \ln^2 \frac{X_v}{10}, & X_v < 25\,\mathrm{g/cm^2} \\ 45.5 - 6.34 \ln X_v, & 25 < X_v < 230\,\mathrm{g/cm^2} \\ 44.34 - 11.861 (X_v)^{0.19}, & X_v > 230\,\mathrm{g/cm^2}. \end{cases}$$

3. Electromagnetic Showers

Qualitative approach: Heitler model



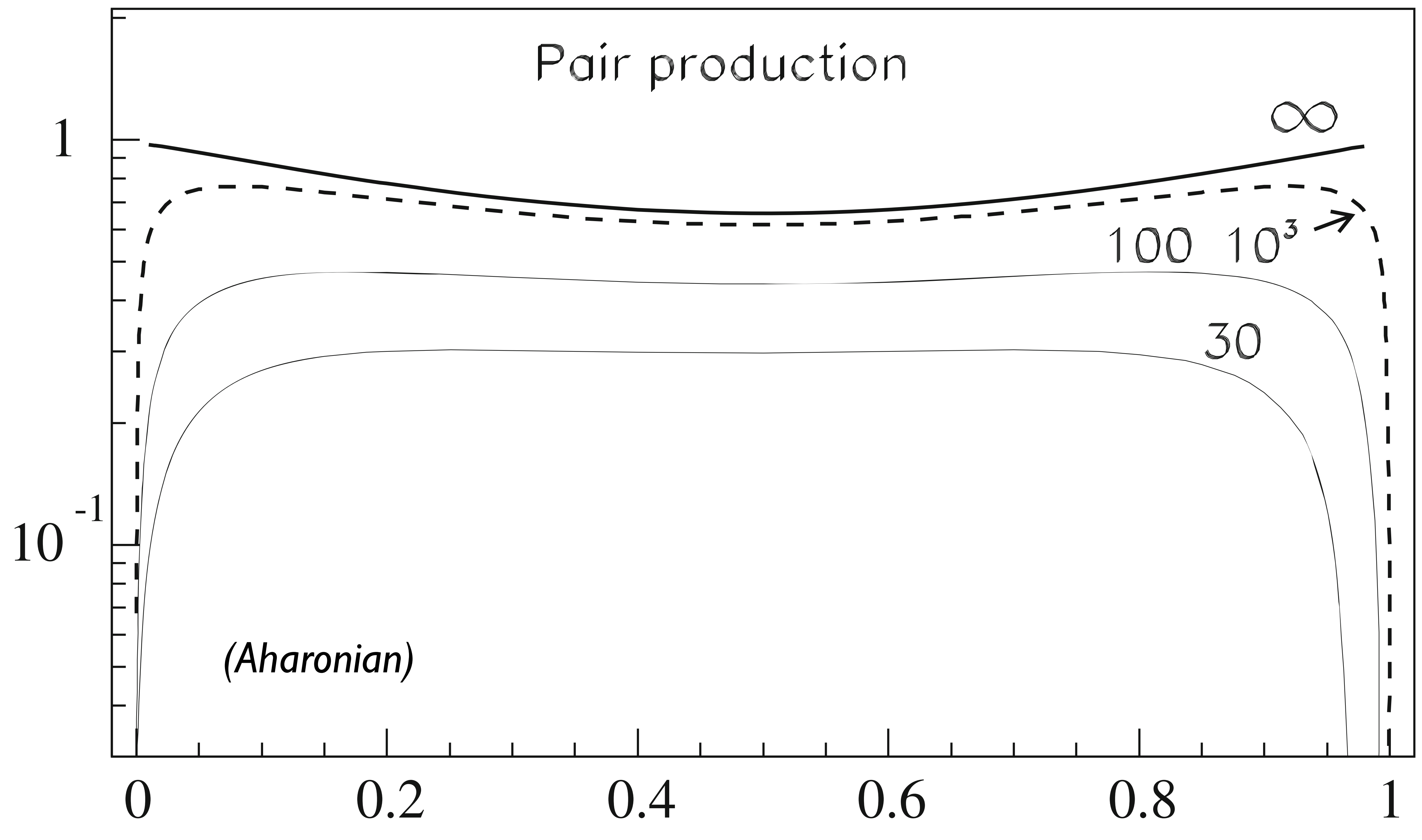


Shower maximum: $E = E_c$

$$N_{\max} = E_0/E_c$$

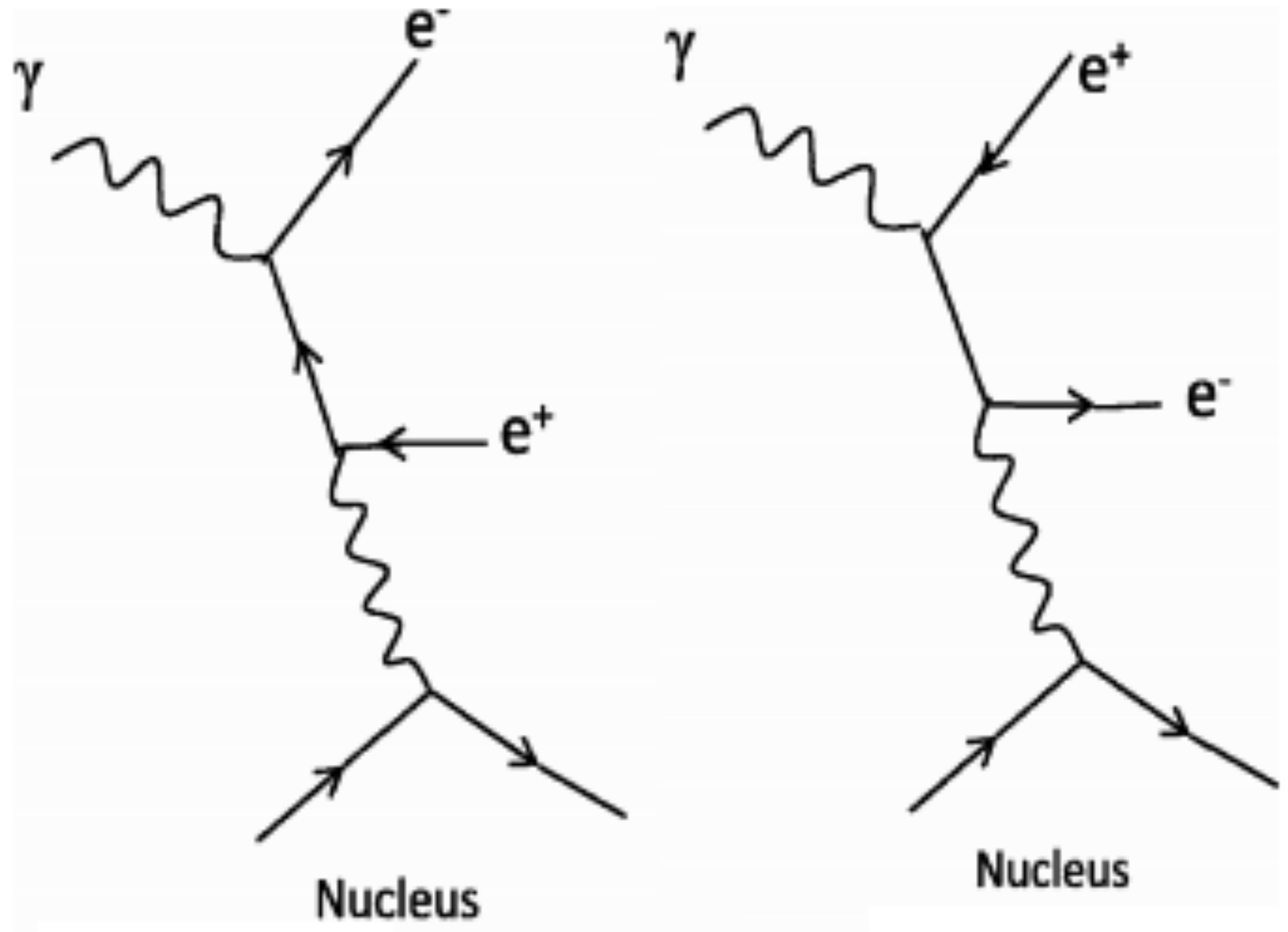
$$X_{\max} \sim \lambda_{\text{em}} \ln(E_0/E_c)$$

Bethe-Heitler pair production (i)



E_γ/m_e

$u = E_e/E_\gamma$



QED

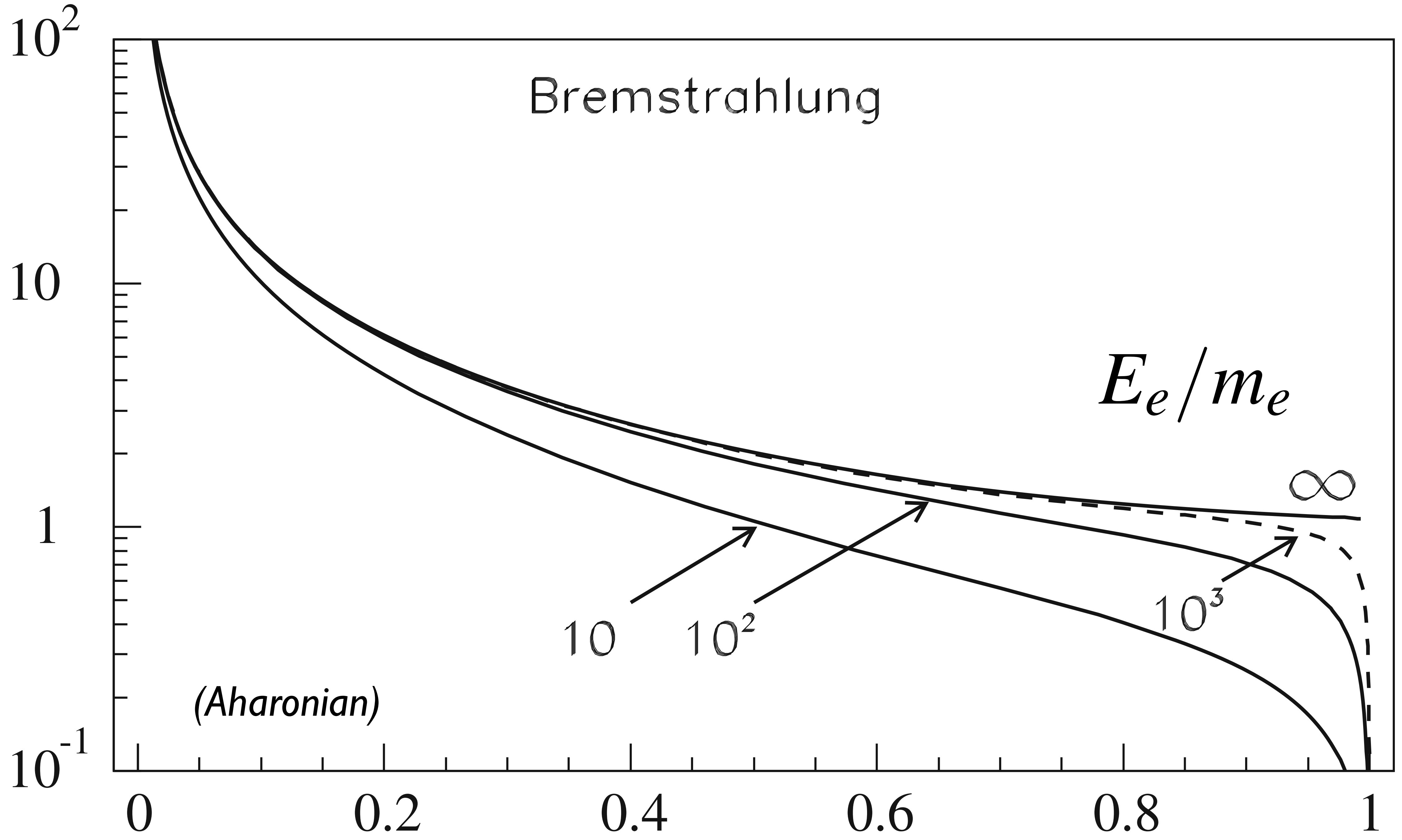
$$\frac{d\sigma_{pair}}{du} = 4\alpha_{em} r_e^2 Z(Z+1)\left\{\left[u^2+(1-u)^2+\frac{2}{3}u(1-u)\right]\ln(183Z^{-1/3})-\frac{1}{9}u(1-u)\right\}$$

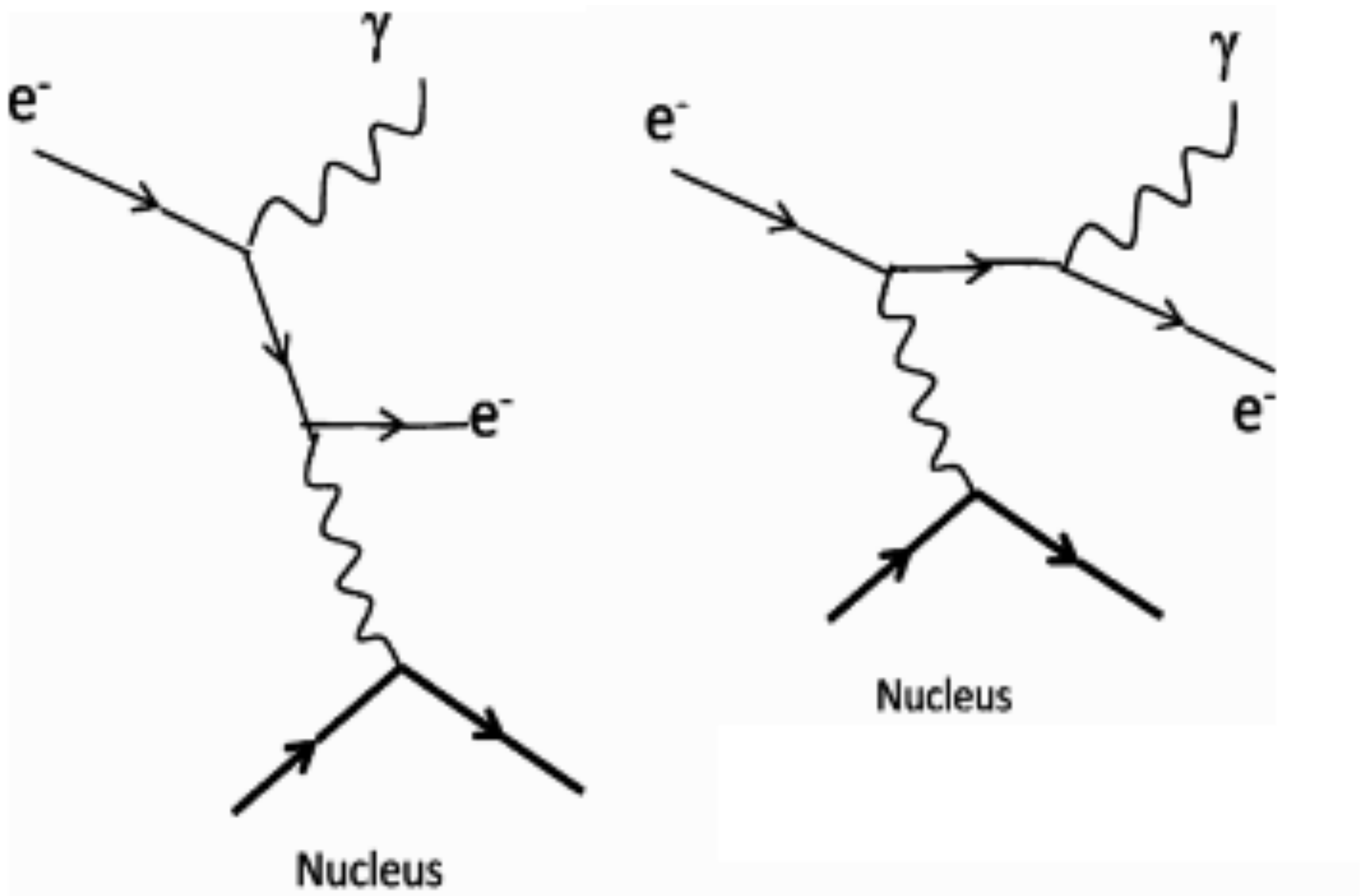
$$\sigma_{pair,tot} = \int \frac{d\sigma_{pair}}{du}\,du = 4\alpha_{em} r_e^2 Z(Z+1)\left[\frac{7}{9}\ln(183Z^{-1/3})-\frac{1}{54}\right]$$

High-energy limit

$$\sigma_{pair,tot} \sim 520\,\text{mb}$$

Electron bremsstrahlung





$$\nu = E_\gamma / E_e$$

QED

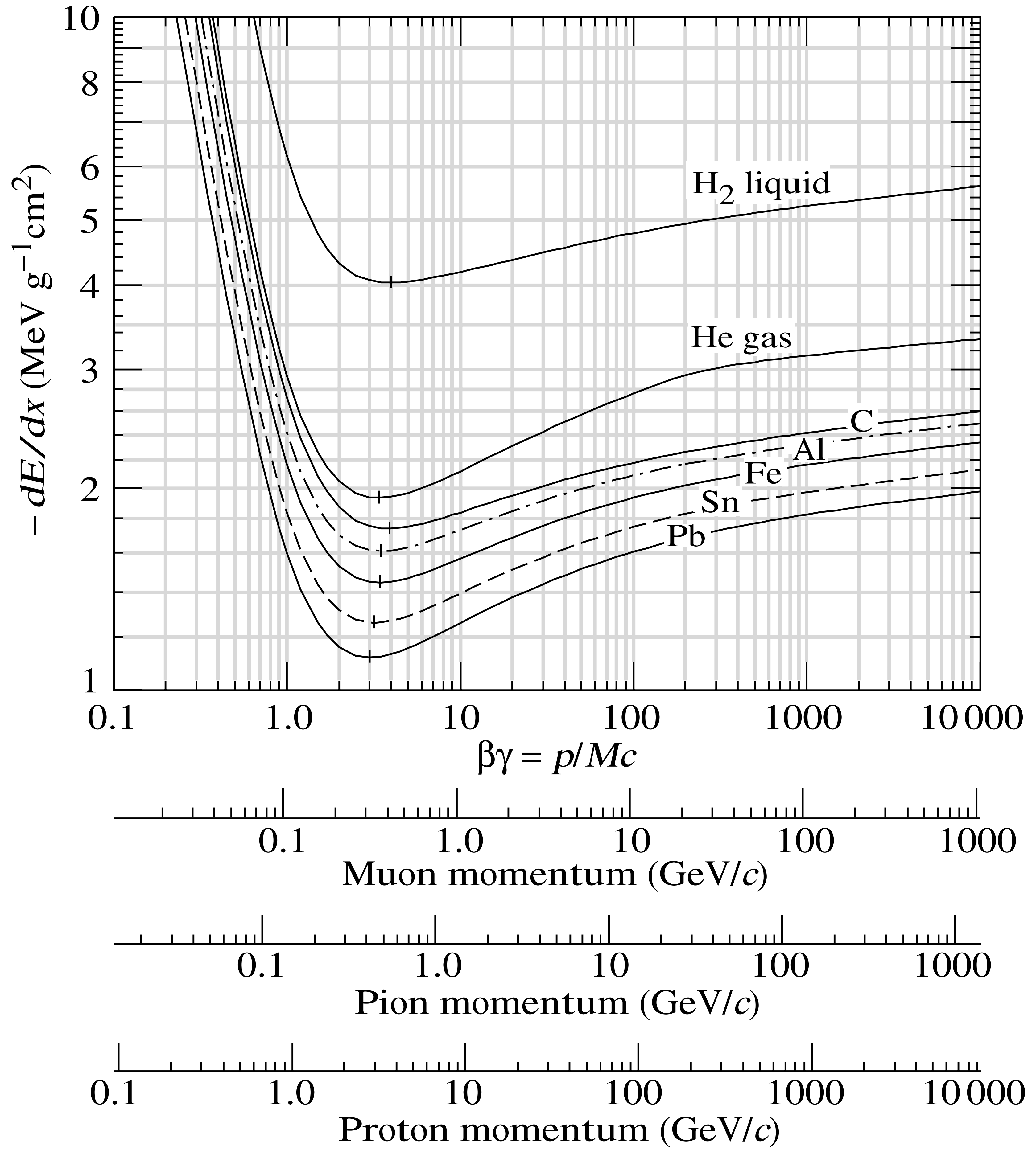
$$\frac{d\sigma_{\rm brem}}{dv} = 4\alpha_{\rm em} r_e^2 Z(Z+1)\frac{1}{v}\left\{\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\ln(183Z^{-1/3})+\frac{1}{9}(1-v)\right\}$$

$$\sigma_{\rm brem,tot} = \int \frac{d\sigma_{\rm brem}}{dv}\,dv \;\rightarrow\; \infty$$

Cross section divergent (infrared catastrophe)

Ionization energy loss of charged particles

Ionization energy loss: Bethe-Bloch formula



$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Symbol	Definition	Units or Value
α	Fine structure constant $(e^2/4\pi\epsilon_0\hbar c)$	$1/137.035\,999\,11(46)$
M	Incident particle mass	MeV$/c^2$
E	Incident part. energy γMc^2	MeV
T	Kinetic energy	MeV
m_ec^2	Electron mass $\times\ c^2$	$0.510\,998\,918(44)$ MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_ec^2$	$2.817\,940\,325(28)$ fm
N_A	Avogadro's number	$6.022\,1415(10) \times 10^{23}$ mol^{-1}
ze	Charge of incident particle	
Z	Atomic number of absorber	
A	Atomic mass of absorber	g mol^{-1}
K/A	$4\pi N_A r_e^2 m_ec^2/A$	$0.307\,075$ MeV g^{-1} cm^2 for A = 1 g mol^{-1}
I	Mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	Density effect correction to ionization energy loss	

Total energy loss of charged particles

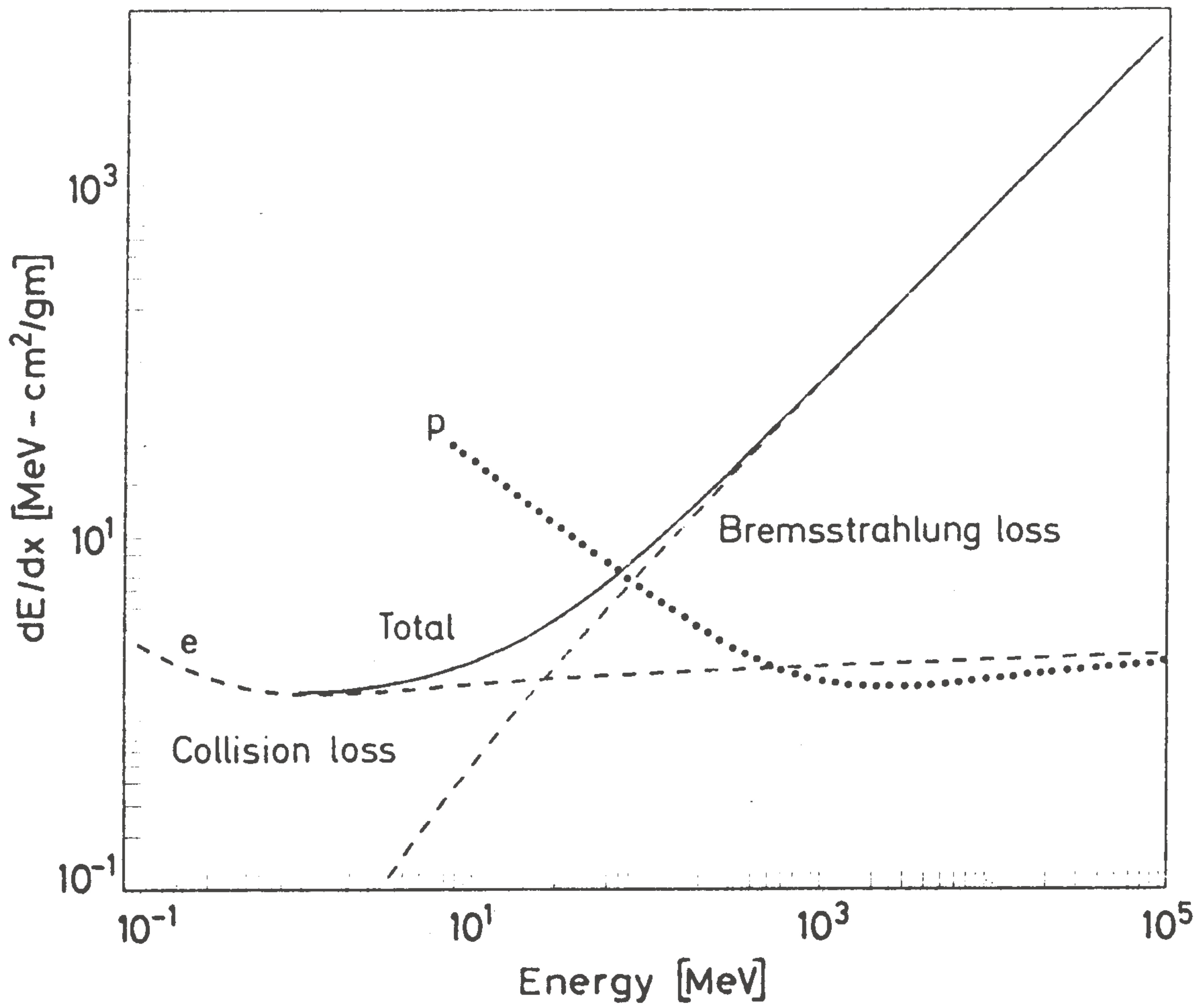
Ionization energy loss: Bethe-Bloch formula

Radiation energy loss: bremsstrahlung

$$\int v \frac{d\sigma_{brem}}{dv} dv = 4\alpha_{em} r_e^2 Z(Z+1)\left[\ln(183Z^{-1/3}) + \frac{1}{18}\right] = \frac{\langle m_{target}\rangle}{X_0}$$

Radiation length X_0

$$X_0 \sim 36\,\mathrm{g/cm^2}$$

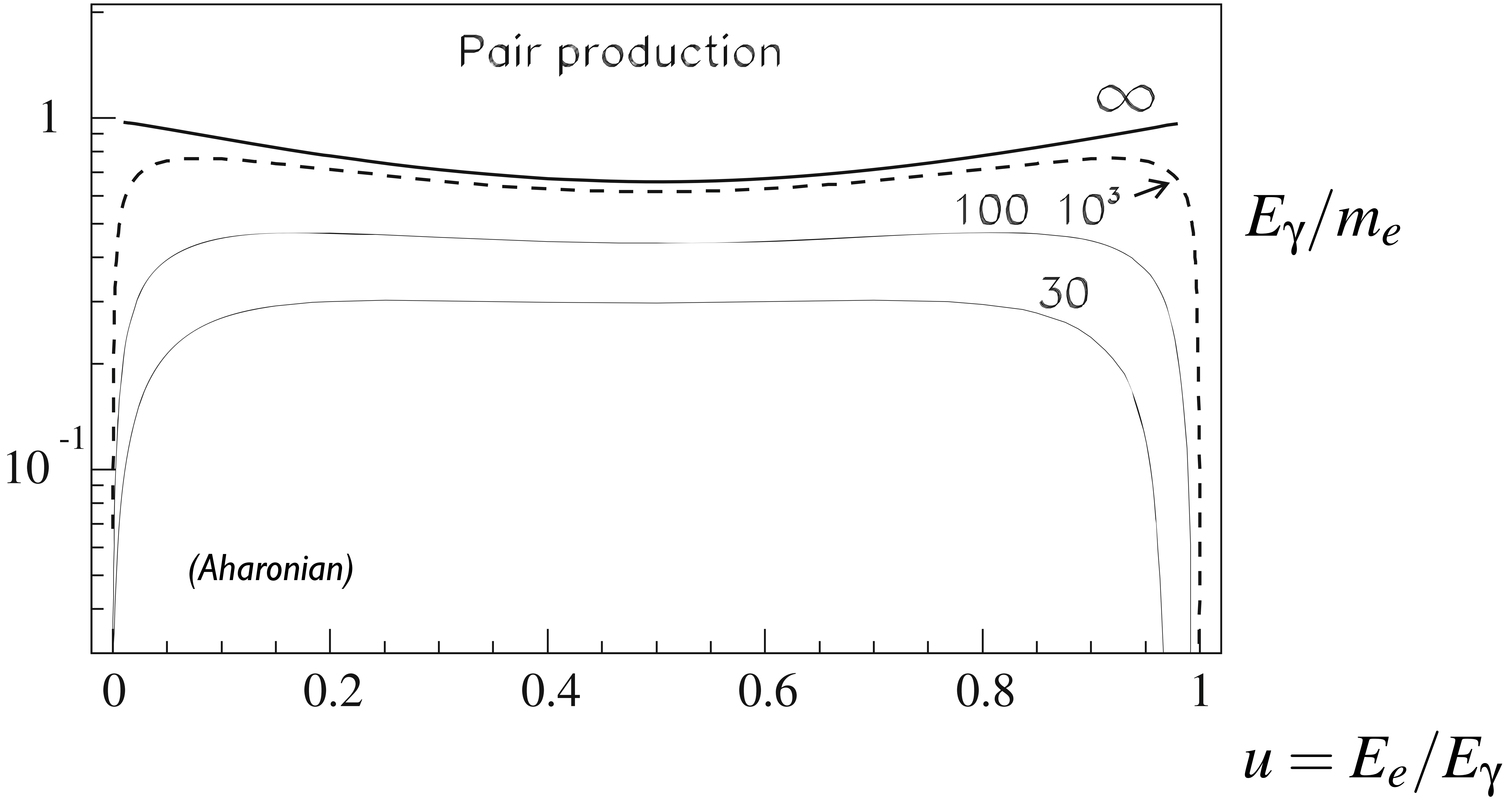


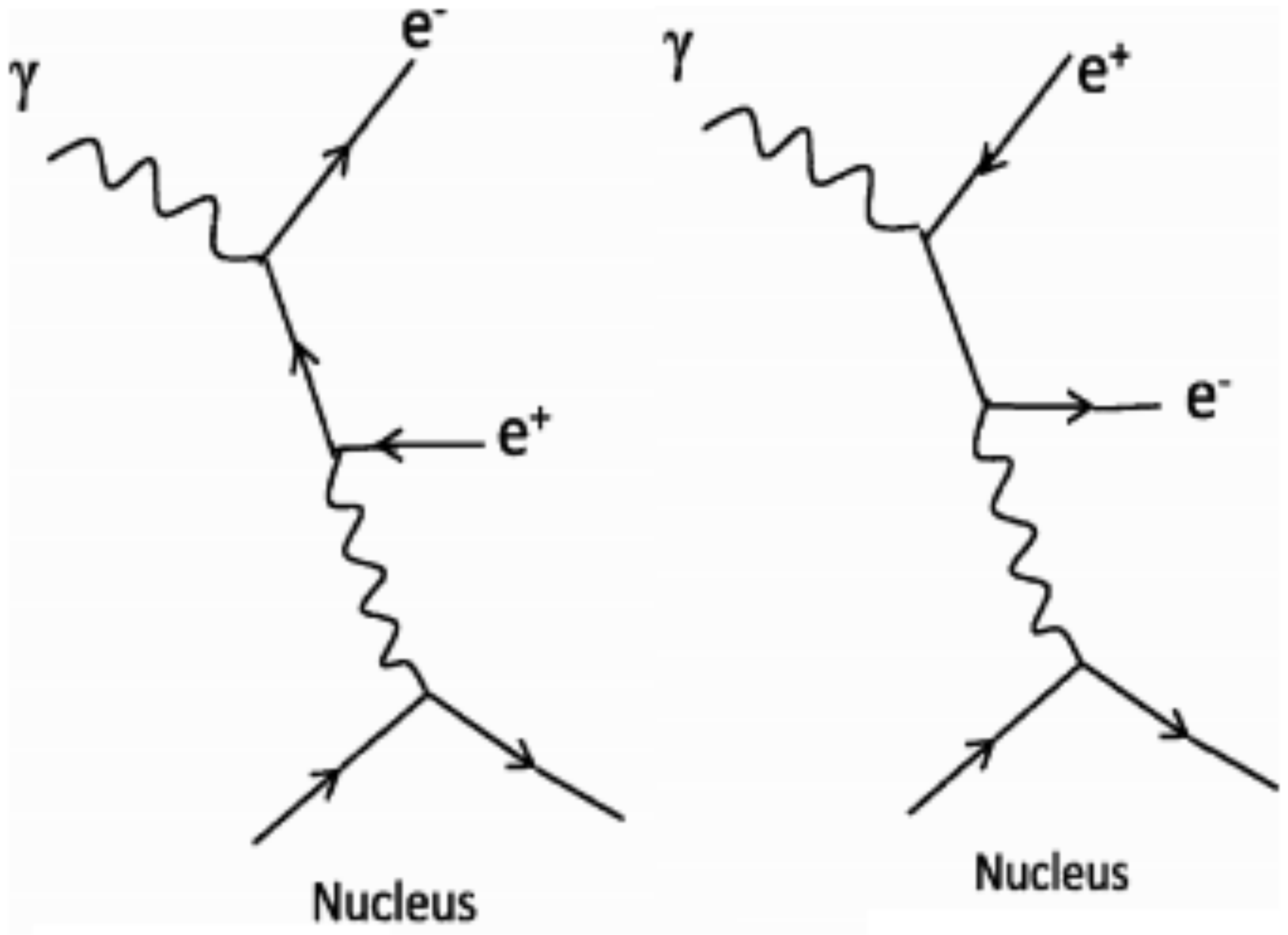
$$\frac{dE}{dX} = -\alpha(E) - \frac{E}{X_0}$$

Critical energy E_c defined as energy at which both losses are equal

$$E_c = \alpha\, X_0 \sim 85\,\mathrm{MeV}$$

Bethe-Heitler pair production (ii)





QED

High-energy limit

$$\sigma_{\text{pair,tot}} \sim 520\,\text{mb}$$

$$\lambda_{\text{pair}} = \frac{\langle m_{\text{air}} \rangle}{\sigma_{\text{pair,tot}}} = \frac{9}{7} X_0$$

Cascade equations

Energy loss of electron:

$$\frac{\mathrm{d}E}{\mathrm{d}X} = -\alpha - \frac{E}{X_0}$$

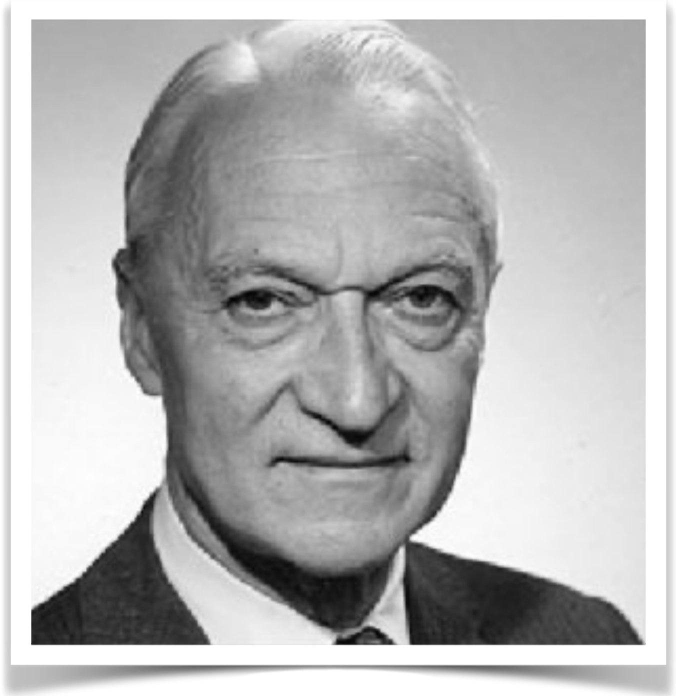
Critical energy: $E_c = \alpha\, X_0 \sim 85\,\mathrm{MeV}$

Radiation length: $X_0 \sim 36\,\mathrm{g/cm^2}$

Cascade equations

$$\frac{\mathrm{d}\Phi_e(E)}{\mathrm{d}X} = -\frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(E) + \int_E^\infty \frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(\tilde{E})P_{e\to e}(\tilde{E},E)\mathrm{d}\tilde{E}$$

$$+\int_E^\infty \frac{\sigma_\gamma}{\langle m_{\mathrm{air}}\rangle}\Phi_\gamma(\tilde{E})P_{\gamma\to e}(\tilde{E},E)\mathrm{d}\tilde{E} \; + \; \alpha\frac{\partial\Phi_e(E)}{\partial E}$$



Bruno Rossi

$$X_{\max} \approx X_0 \ln\left(\frac{E_0}{E_c}\right) \qquad N_{\max} \approx \frac{0.31}{\sqrt{\ln(E_0/E_c) - 0.33}}\frac{E_0}{E_c}$$

(Rossi & Greisen, Rev. Mod. Phys. 13 (1940) 240)

Shower age and Greisen formula

Longitudinal profile

$$N_e(X) \approx \frac{0.31}{[\ln E_0/E_c]^{1/2}} \exp\left\{ \frac{X}{X_0} \left(1 - \frac{3}{2}\ln s\right)\right\}$$

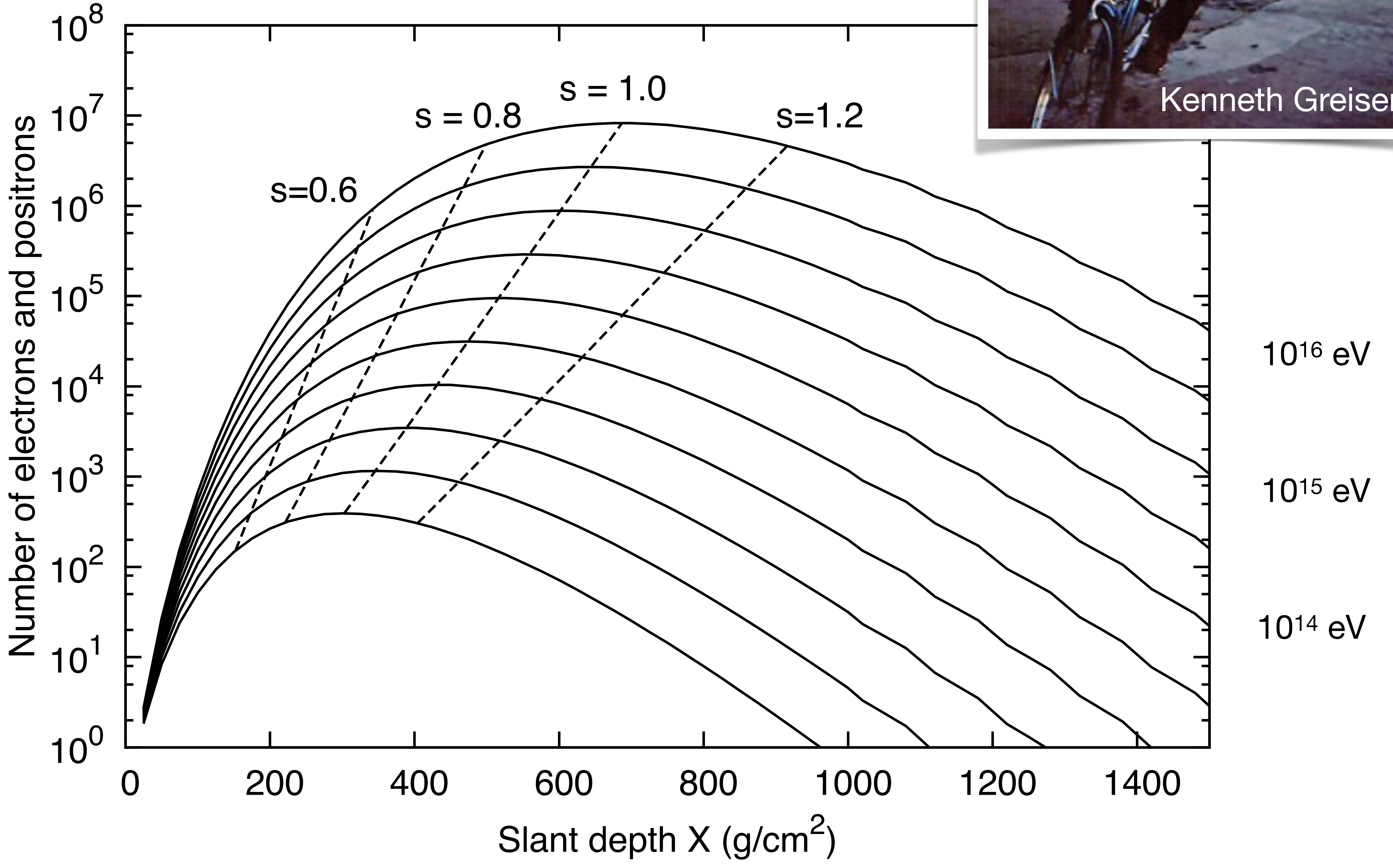
Shower age

$$s = \frac{3X}{X + 2X_{\max}}$$

Energy spectrum particles

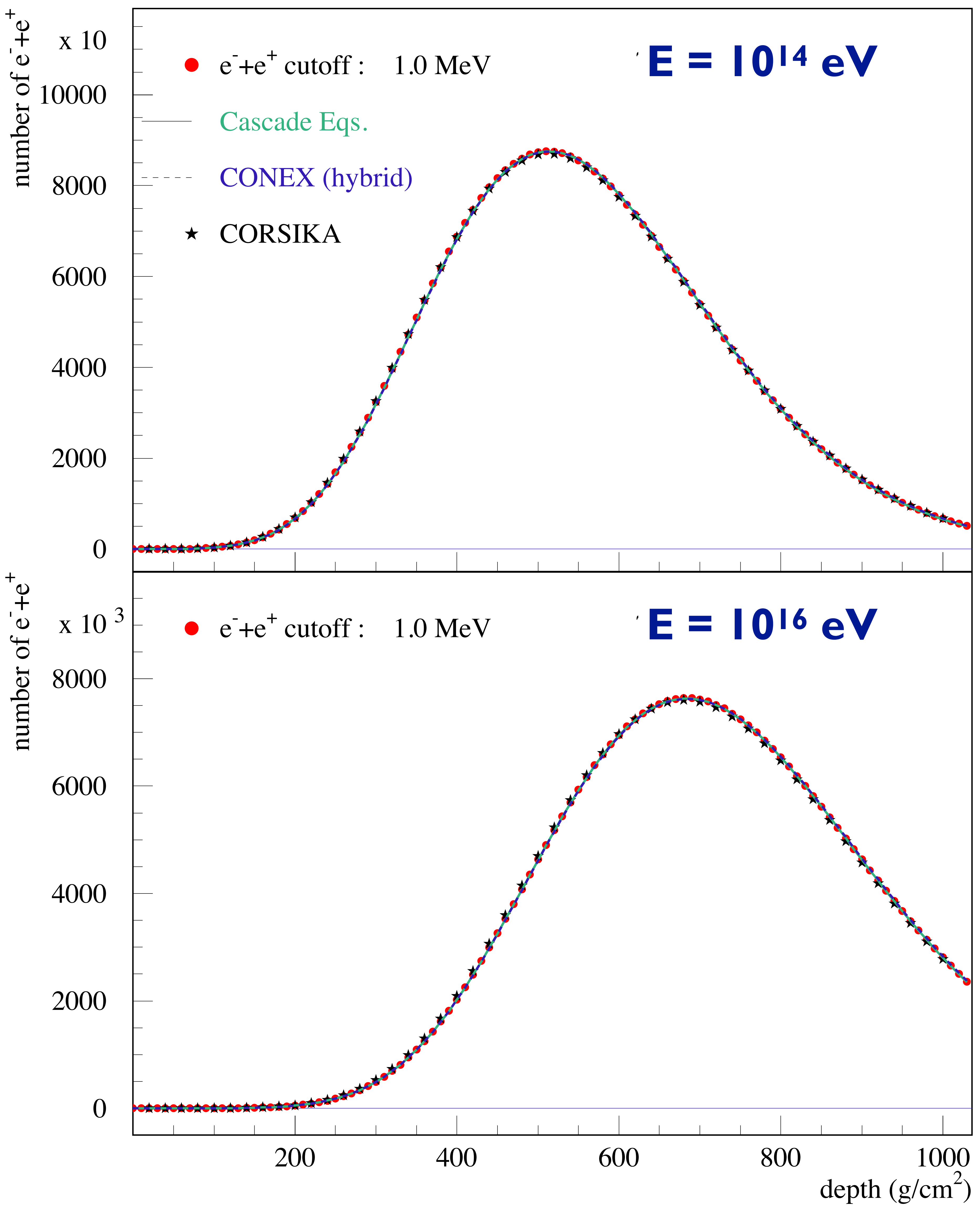
$$\frac{dN_e}{dE} \sim \frac{1}{E^{1+s}}$$

Electrons in photon-initiated shower



(Greisen 1956, see also Lipari PRD 2009)

Mean longitudinal shower profile



Calculation with cascade Eqs.

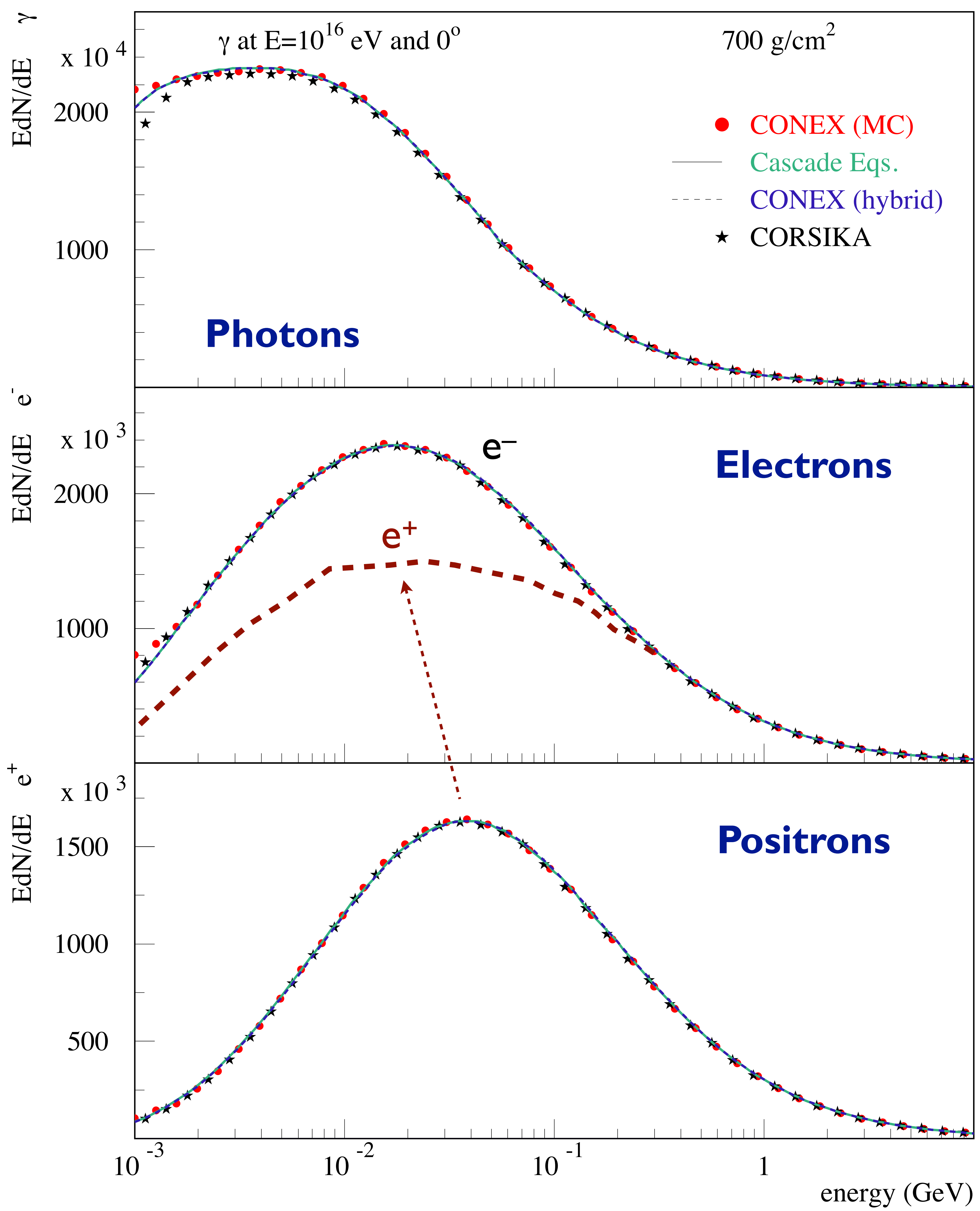
Photons
- Pair production
- Compton scattering

Electrons
- Bremsstrahlung
- Moller scattering

Positrons
- Bremsstrahlung
- Bhabha scattering

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Energy spectra of secondary particles



Number of photons divergent, energy threshold applied in calculation

- Typical energy of electrons and positrons $E_c \sim 80$ MeV
- Electron excess of 20 - 30%
- Pair production symmetric
- Excess of electrons in target

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Lateral distribution of shower particles

Coulomb scatting

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{64\pi}\frac{1}{\ln(191Z^{-1/3})}\left(\frac{E_s}{E}\right)^2\frac{1}{\sin^4\theta/2}$$

$$E_s \approx 21\,\mathrm{MeV}$$

Expectation value

$$\int \theta^2 \frac{\mathrm{d}N}{\mathrm{d}\Omega}\,\mathrm{d}\Omega$$

$$\langle\theta^2\rangle \sim \left(\frac{E_s}{E}\right)^2$$

Displacement of particle

$$r \sim \left(\frac{E_s}{E}\right)\frac{X_0}{\rho_{\mathrm{air}}}$$

$$r_1 = r_M = \left(\frac{E_s}{E_c}\right)\frac{X_0}{\rho_{\mathrm{air}}}$$

Moliere unit
(78 m at sea level)

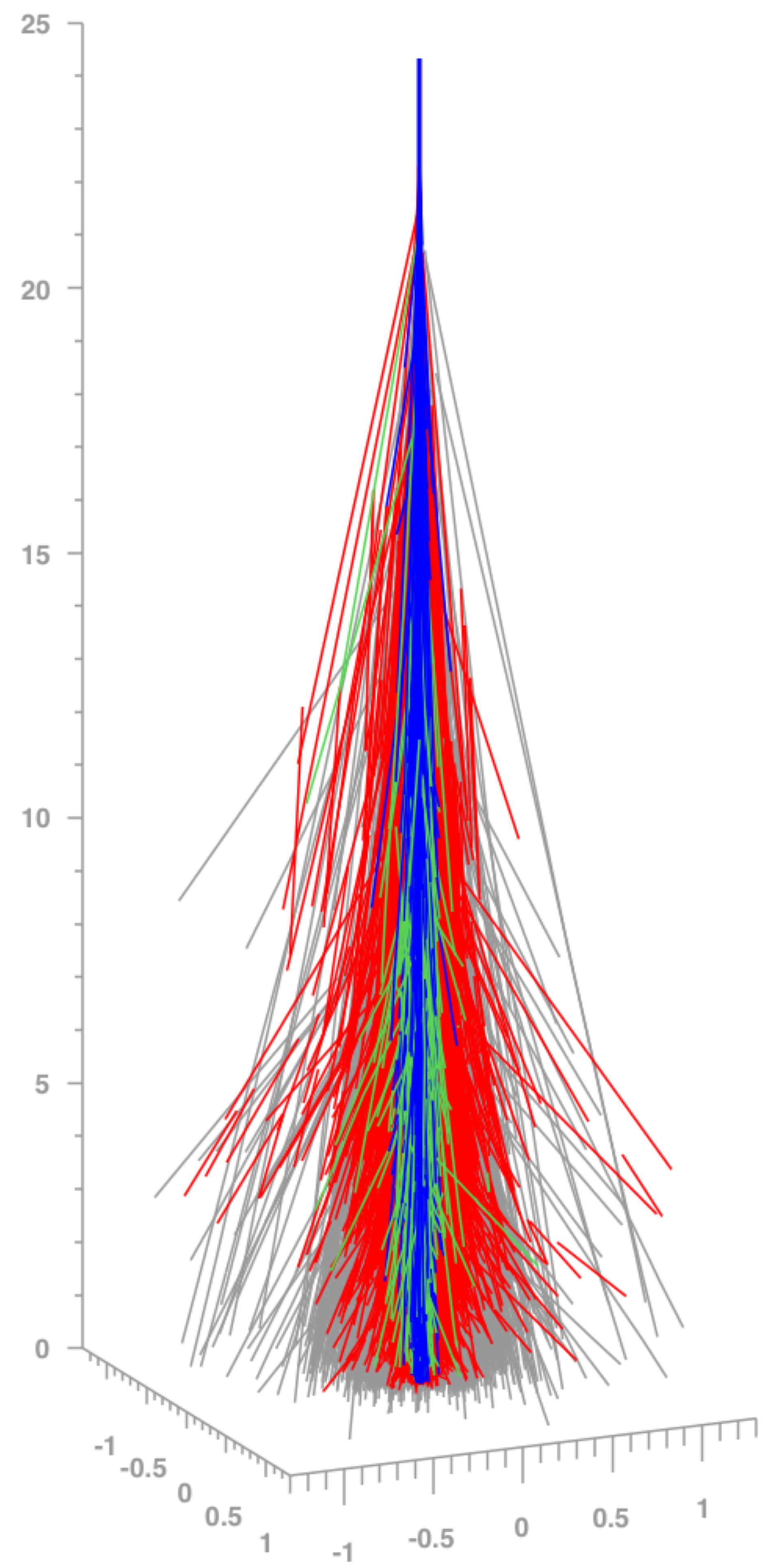
$$\frac{\mathrm{d}N_e}{\mathrm{d}E} \sim \frac{E_c}{E^{1+s}}$$

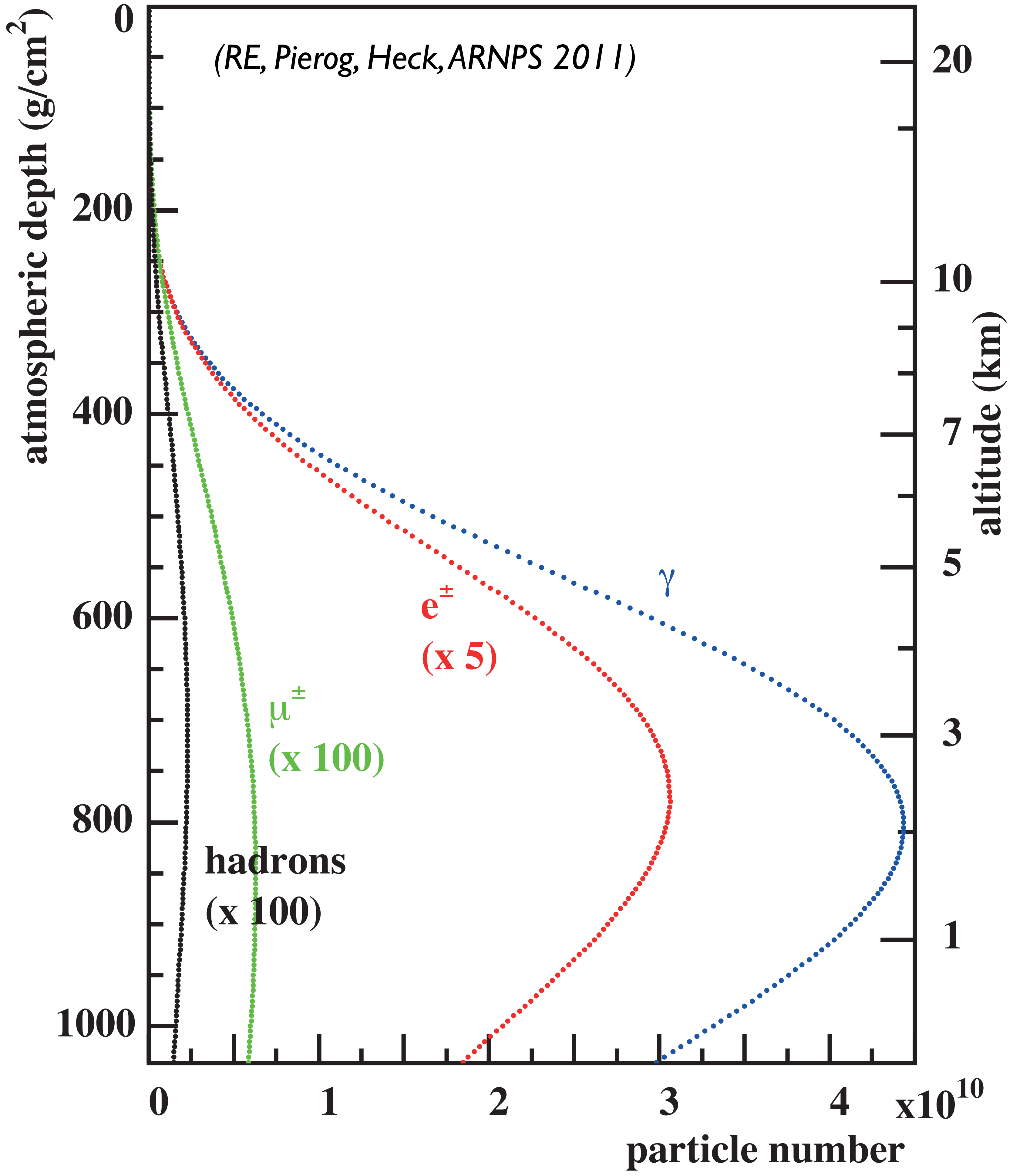
$$\frac{\mathrm{d}N_e}{r\,\mathrm{d}r} \sim \left(\frac{r}{r_1}\right)^{s-2}\left(1+\frac{r}{r_1}\right)^{s-4.5}$$

Nishimura-Kamata-Greisen (NKG)
lateral distribution function

4. Hadronic Showers

Expectation from simulations



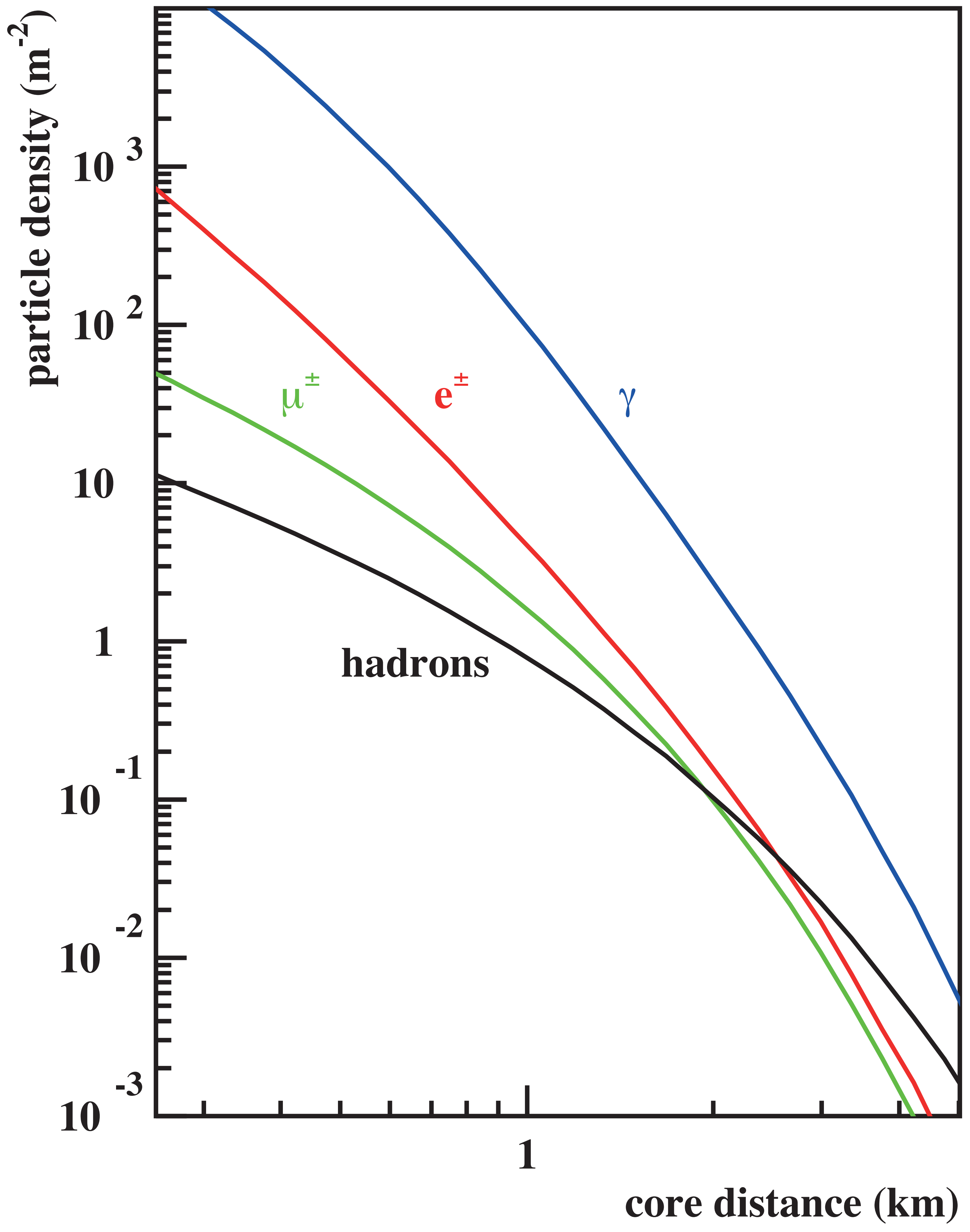


Longitudinal profile:

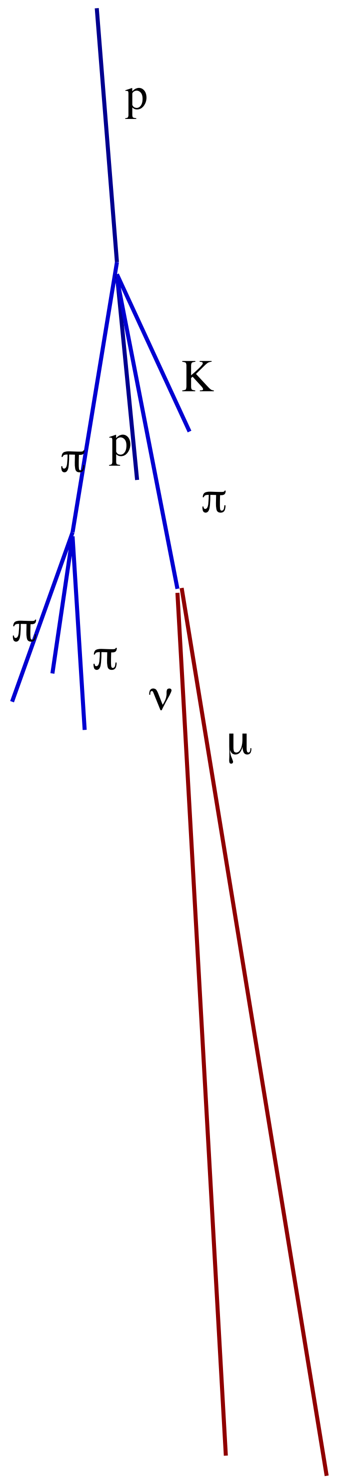
Cherenkov light
Fluorescence light
(bulk of particles measured)

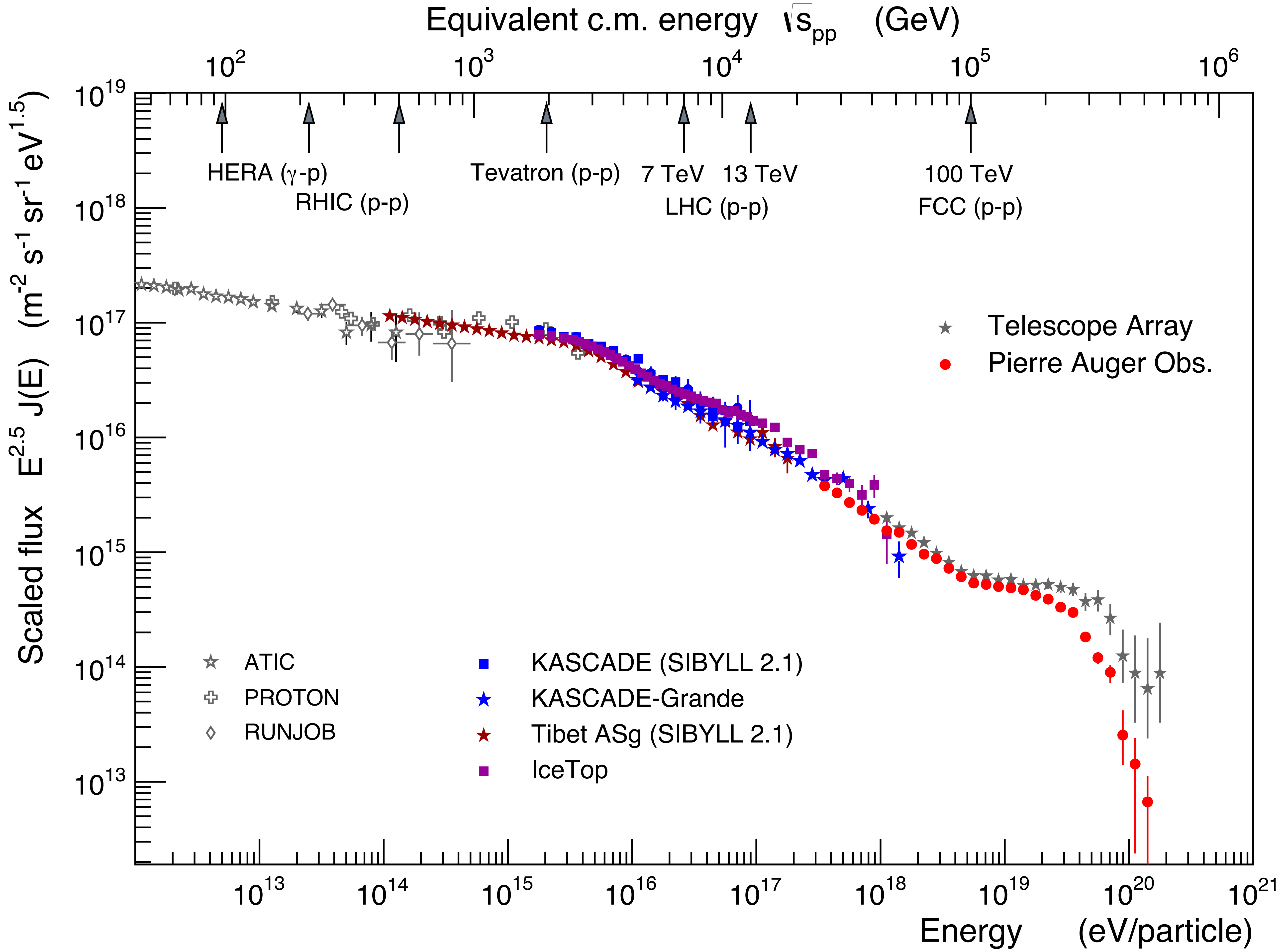
Lateral profiles:

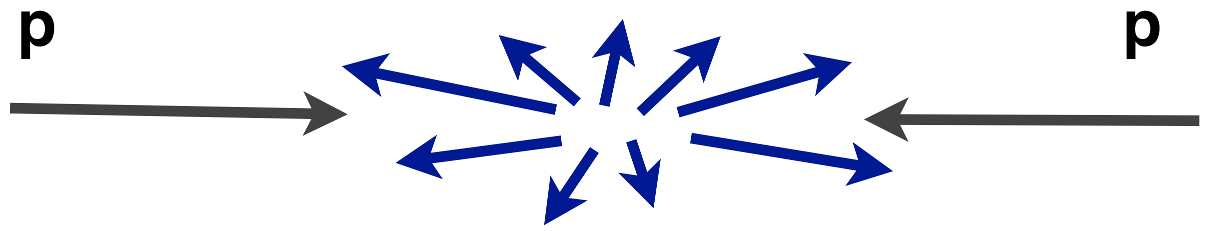
particle detectors at ground
(very small fraction of particles sampled)



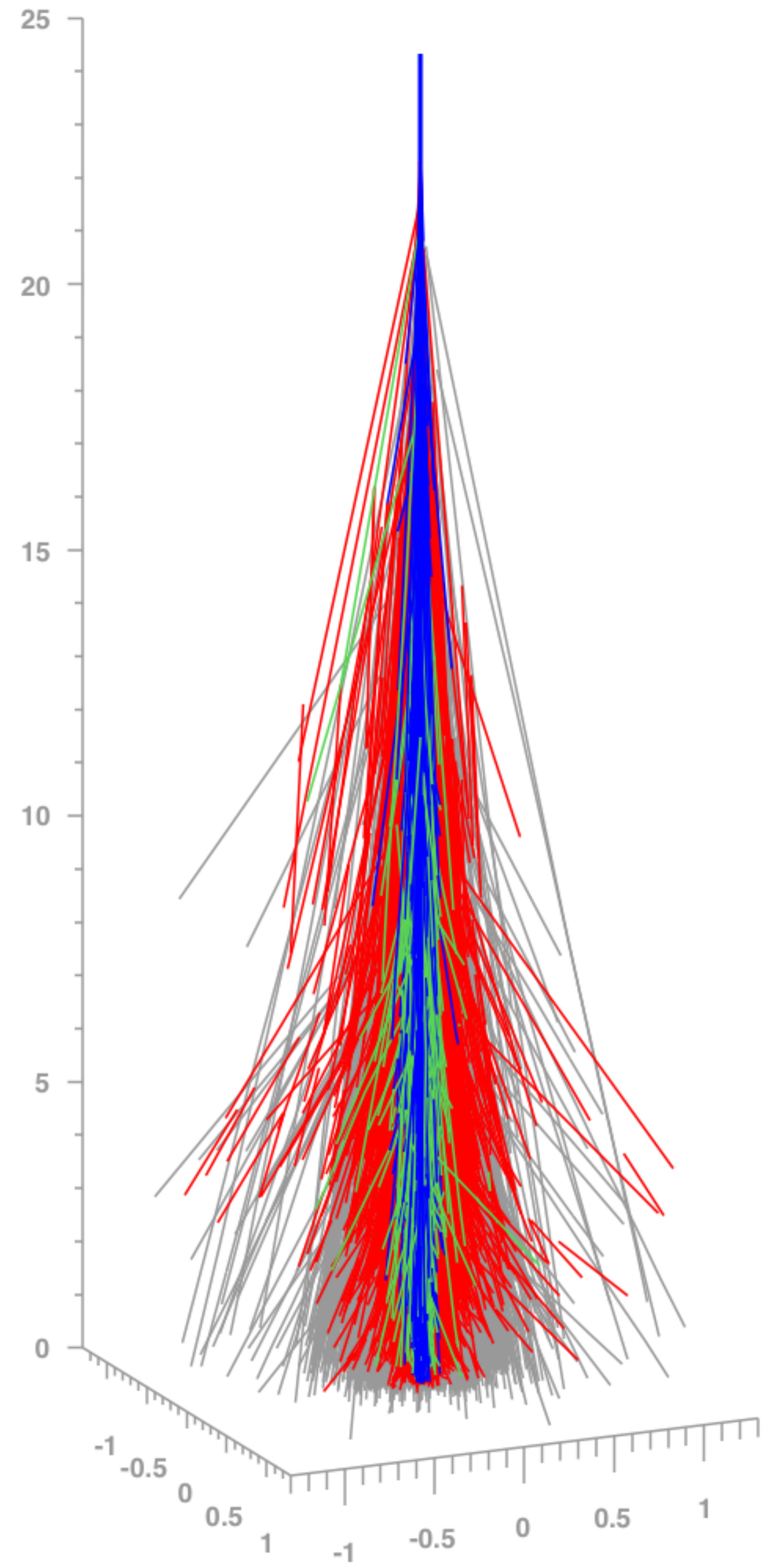
Cosmic ray flux and interaction energies



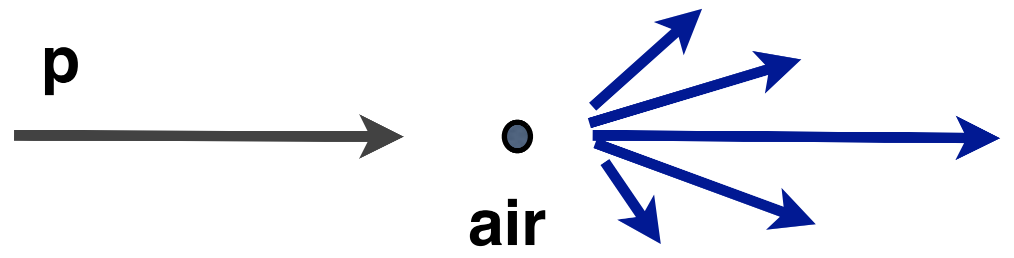




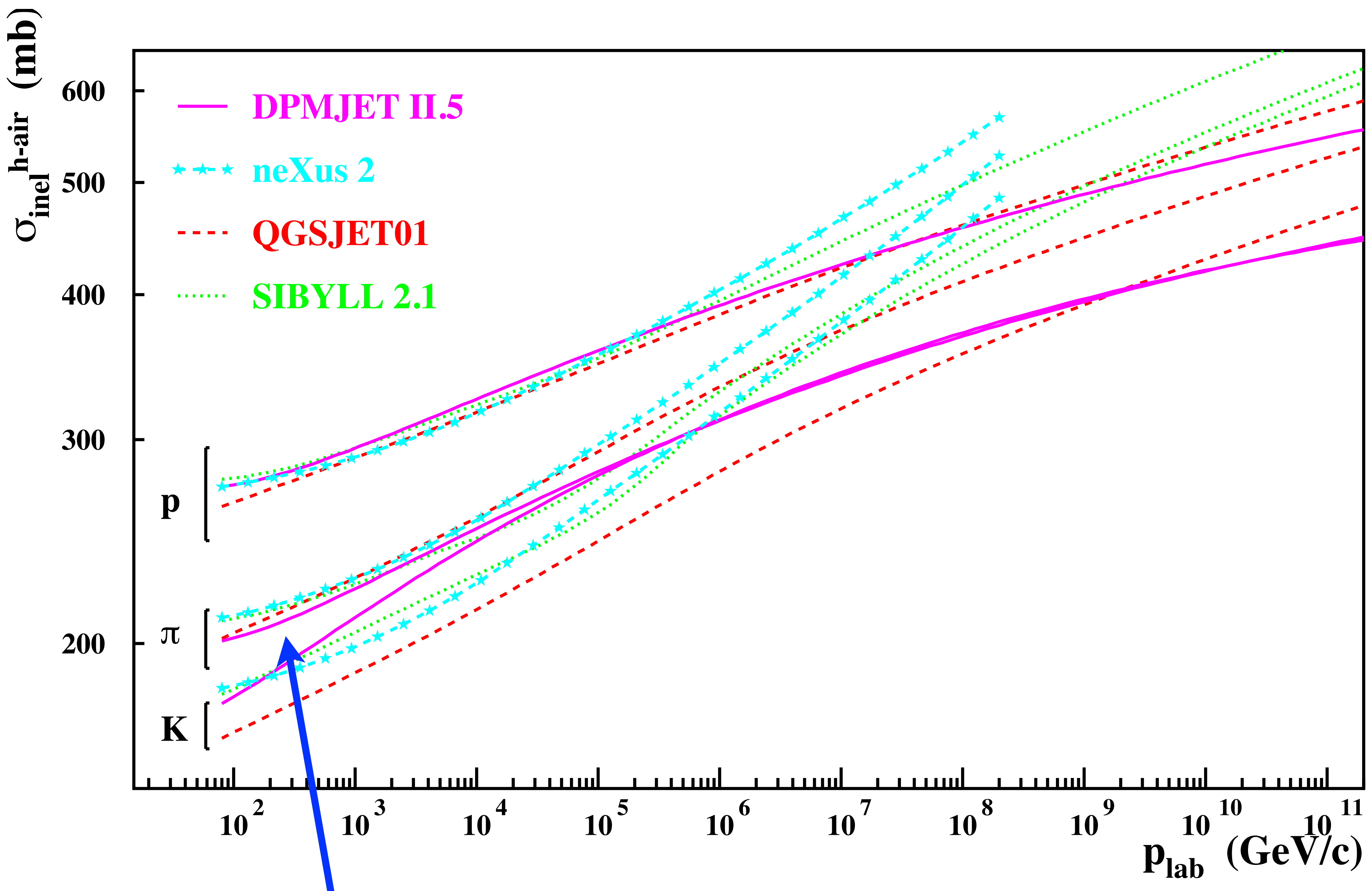
Center-of-mass energy



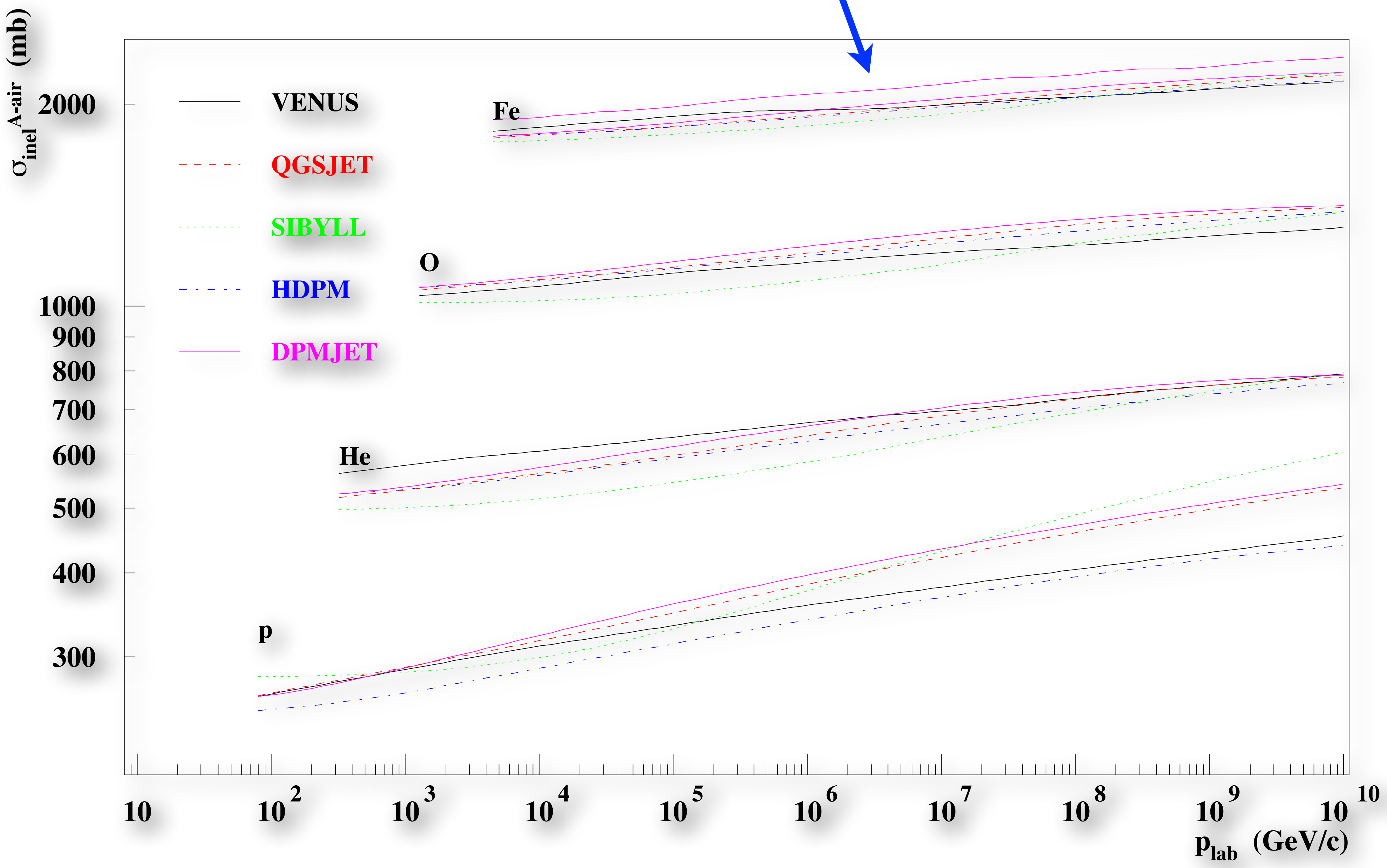
Laboratory energy



Interaction cross sections with air as target



Iron nuclei: $\lambda_{\rm int} \sim 12\ {\rm g/cm^2}$



Pions: $\lambda_{\rm int} \sim 120\ {\rm g/cm^2}$

Protons: $\lambda_{\rm int} \sim 75\ {\rm g/cm^2}$

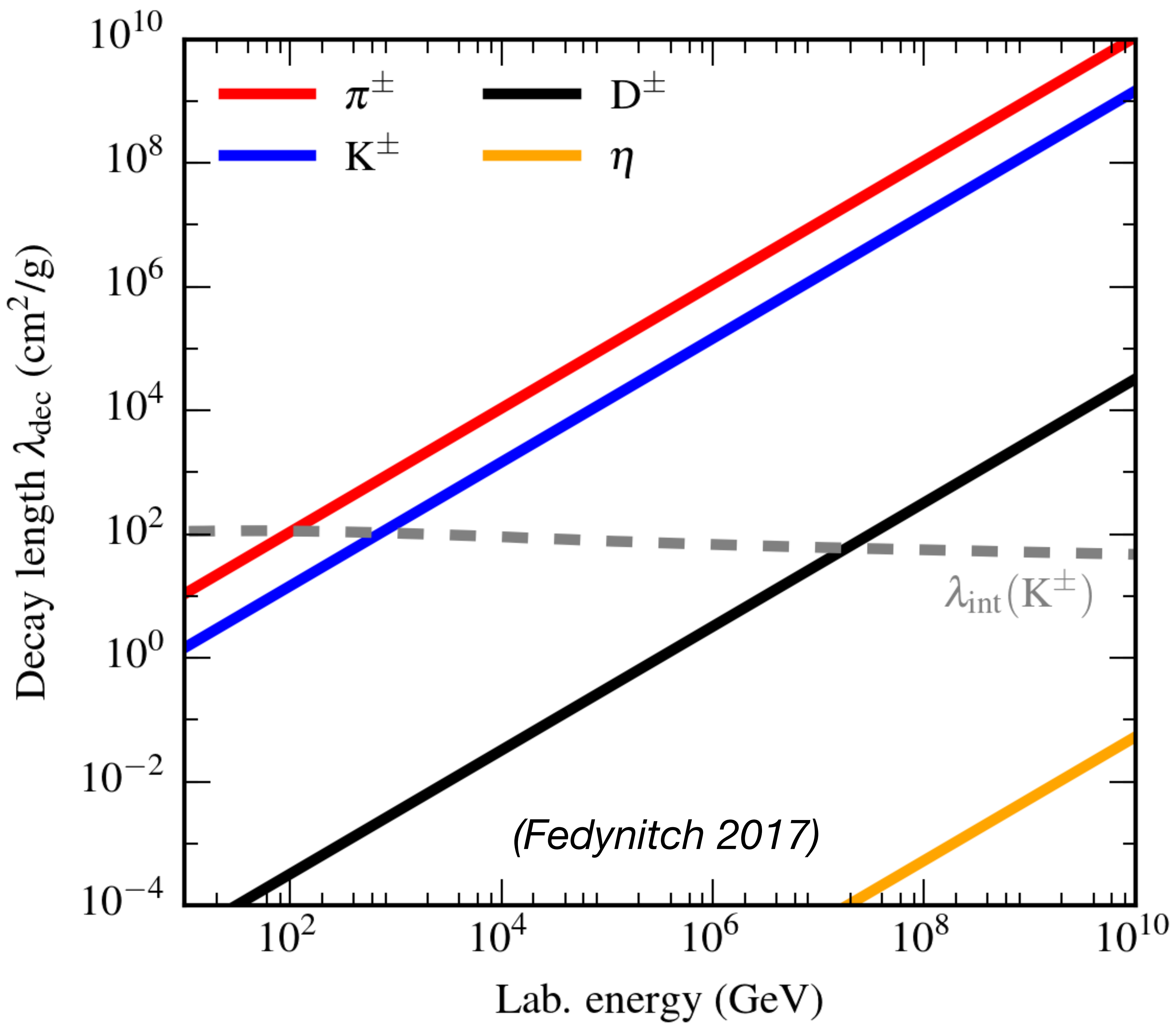
Photons: $\lambda_{\gamma,\rm pair} = \frac{9}{7} X_0 \sim 50\ {\rm g/cm^2}$

Competing processes of interaction and decay

Interaction length

$$\lambda_{\rm int} = \frac{\langle m_{\rm air}\rangle}{\sigma_{\rm int}}$$

$$\lambda_\pi \approx \lambda_K \approx 120\,{\rm g/cm^2}$$



Air at altitude of 8 km

Decay length

$$\lambda_{\rm dec} = \rho\, l_{\rm dec} \approx c\tau\rho\frac{E}{m}$$

air density

$$c\tau_{\pi^\pm} = 7.8\,{\rm m}$$

$$c\tau_{\pi^0} = 25.1\,{\rm nm}$$

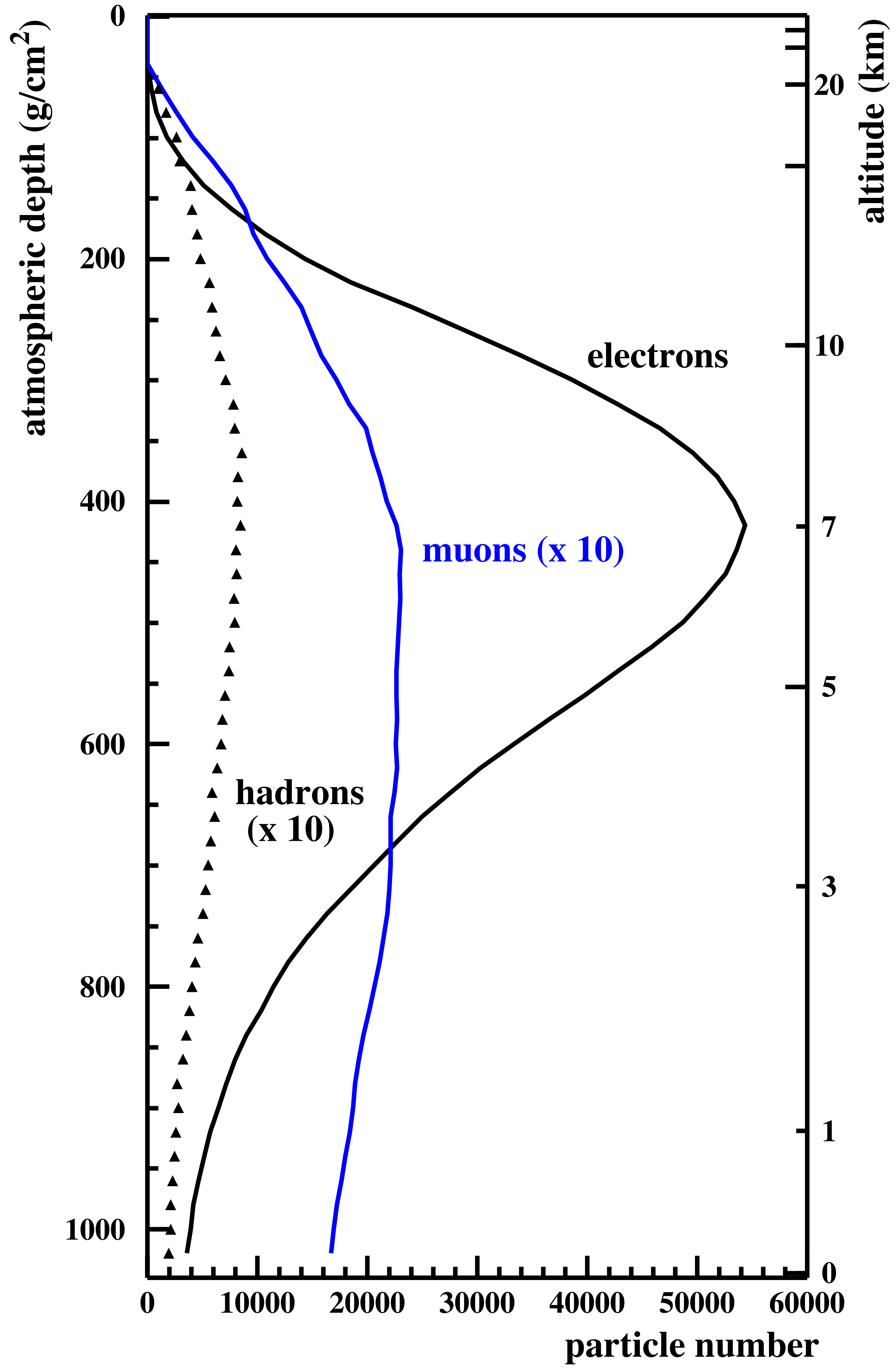
$$\pi^+ \rightarrow \mu^+\nu_\mu$$

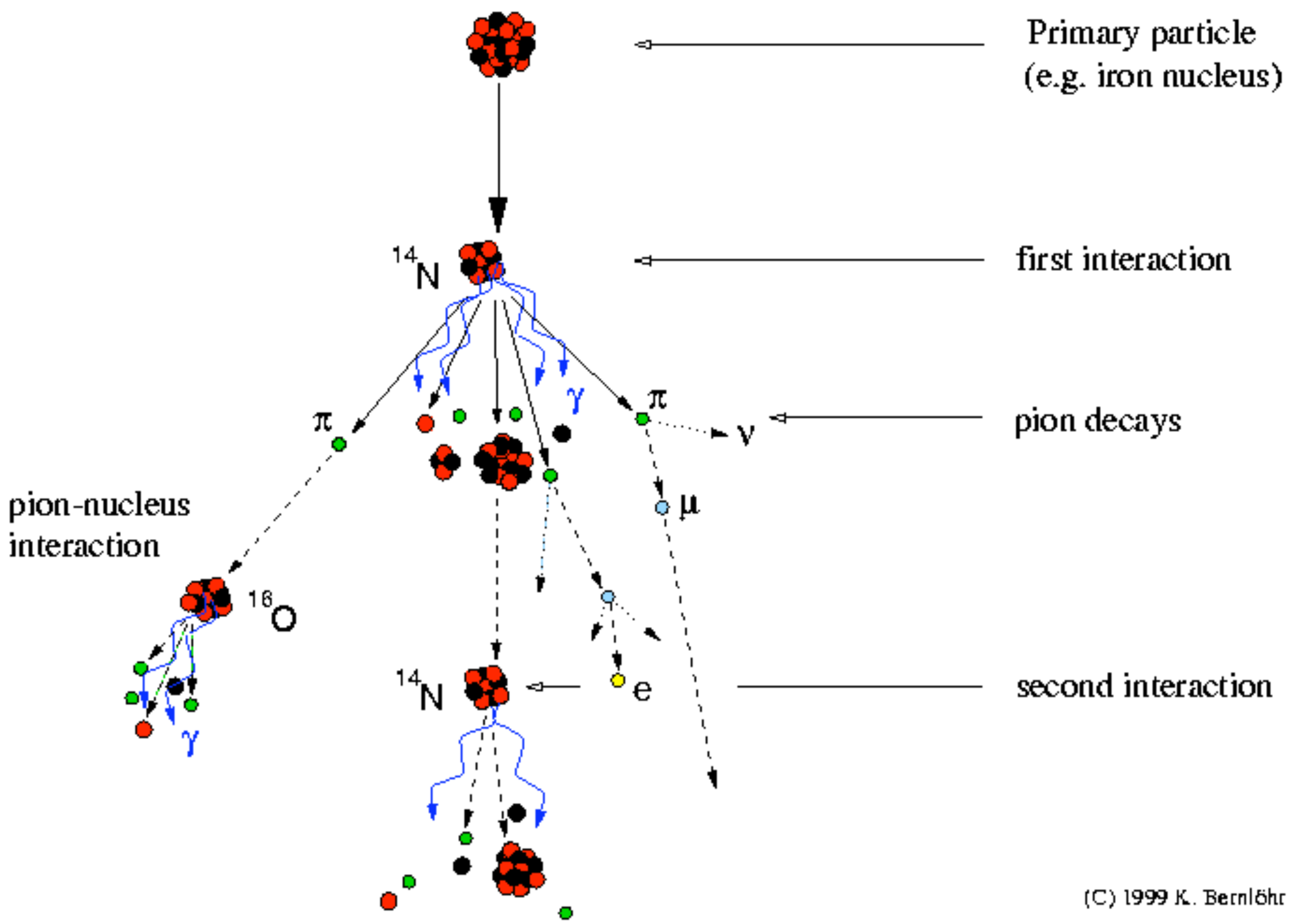
$$\pi^0 \rightarrow \gamma\gamma$$

Charged pions interact E > 30 GeV

Neutral pions always decay

Hadron-induced showers





$c\tau_{\pi^\pm} = 7.8\,\mathrm{m}$

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$$\pi^+ \longrightarrow \mu^+ \nu_\mu$$

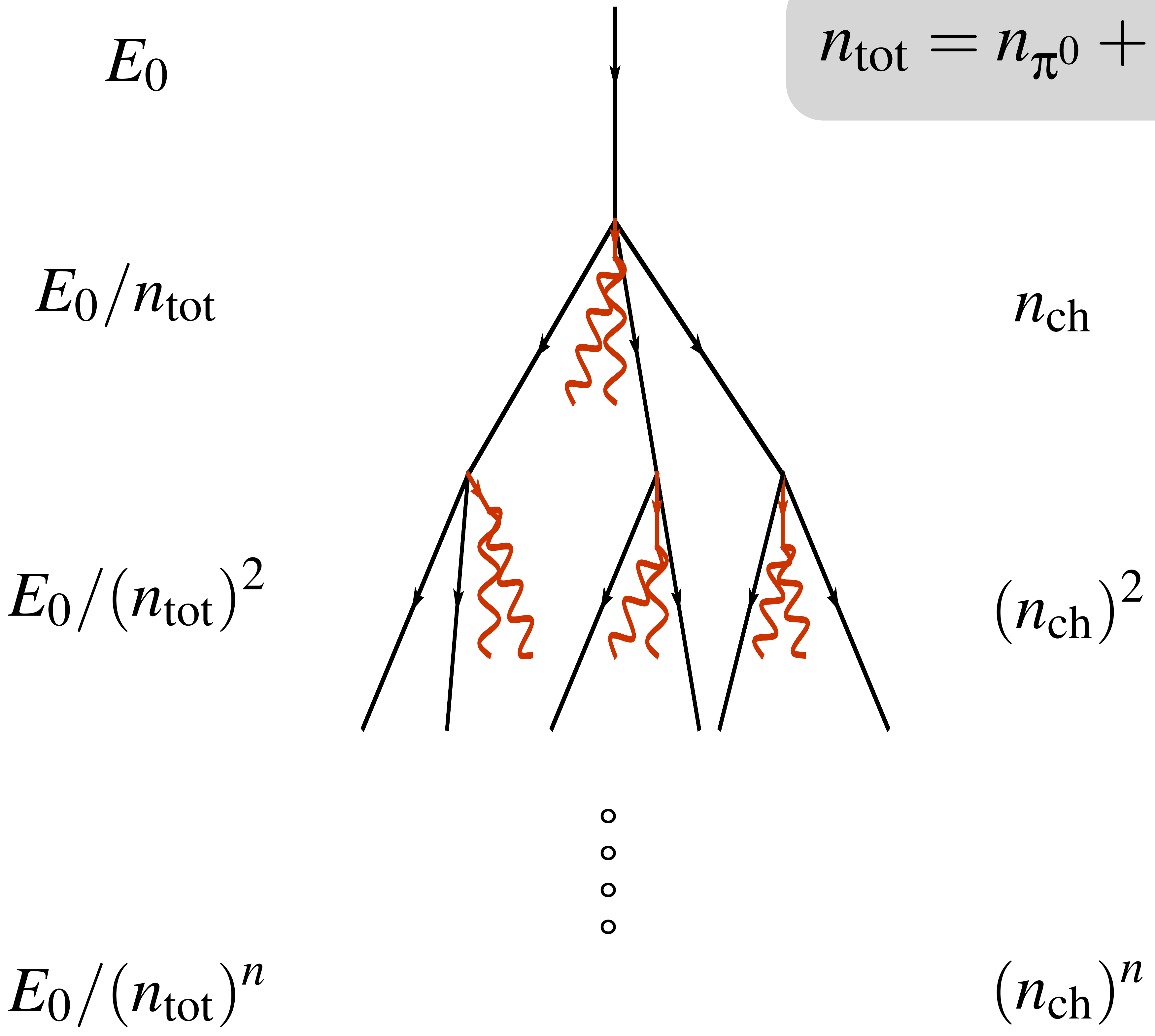
$$\pi^0 \longrightarrow \gamma\gamma$$

Charged pions interact E > 30 GeV

Neutral pions always decay

Qualitative approach: Heitler-Matthews model

$$n_{\mathrm{tot}} = n_{\pi^0} + n_{\mathrm{ch}}$$



Primary particle proton

π^0 decay immediately

$\pi^\pm$ initiate new cascades

$$N_\mu = \left(\frac{E_0}{E_{\mathrm{dec}}}\right)^\alpha$$

$$\alpha = \frac{\ln n_{\mathrm{ch}}}{\ln n_{\mathrm{tot}}} \approx 0.82 \ldots 0.95$$

Assumptions:

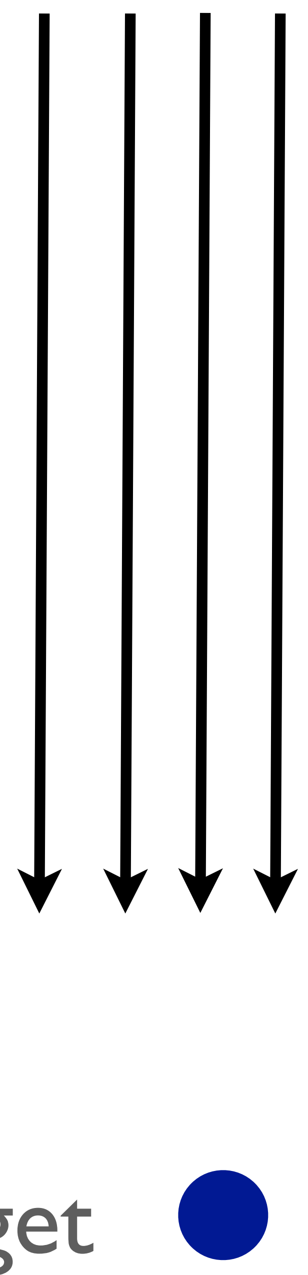
- cascade stops at $E_{\mathrm{part}} = E_{\mathrm{dec}}$
- each hadron produces one muon

(Matthews, Astropart.Phys. 22, 2005)

Superposition model

Nucleus

$$E_i = E_0/A$$



Target

Proton-induced shower

$$N_{\max} \sim E_0/E_c$$

$$X_{\max} \sim \lambda_{\text{eff}} \ln(E_0)$$

$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}}\right)^\alpha \qquad \alpha \approx 0.9$$

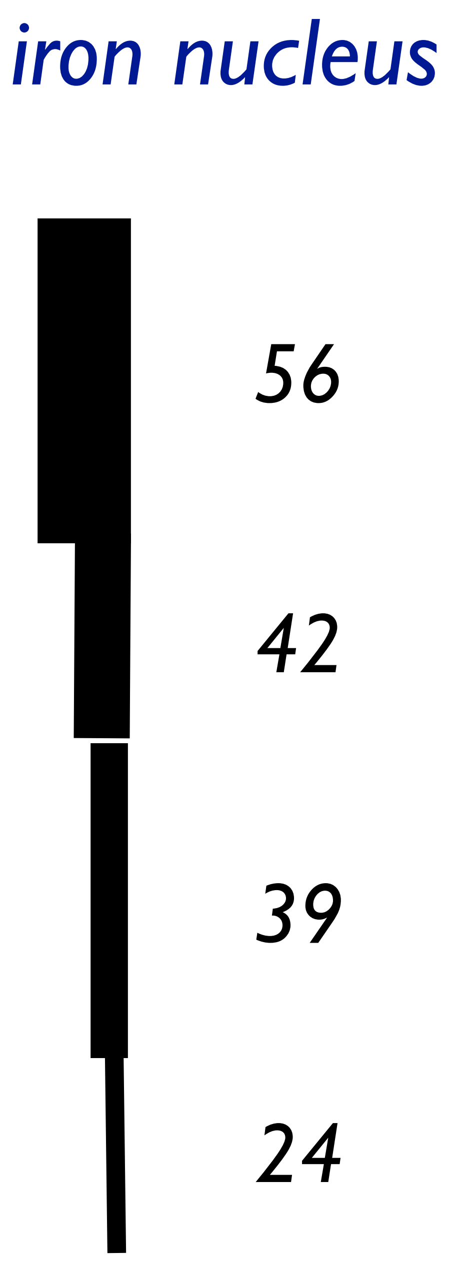
Assumption: nucleus of mass *A* and energy E_0 corresponds to *A* nucleons (protons) of energy $E_n = E_0/A$

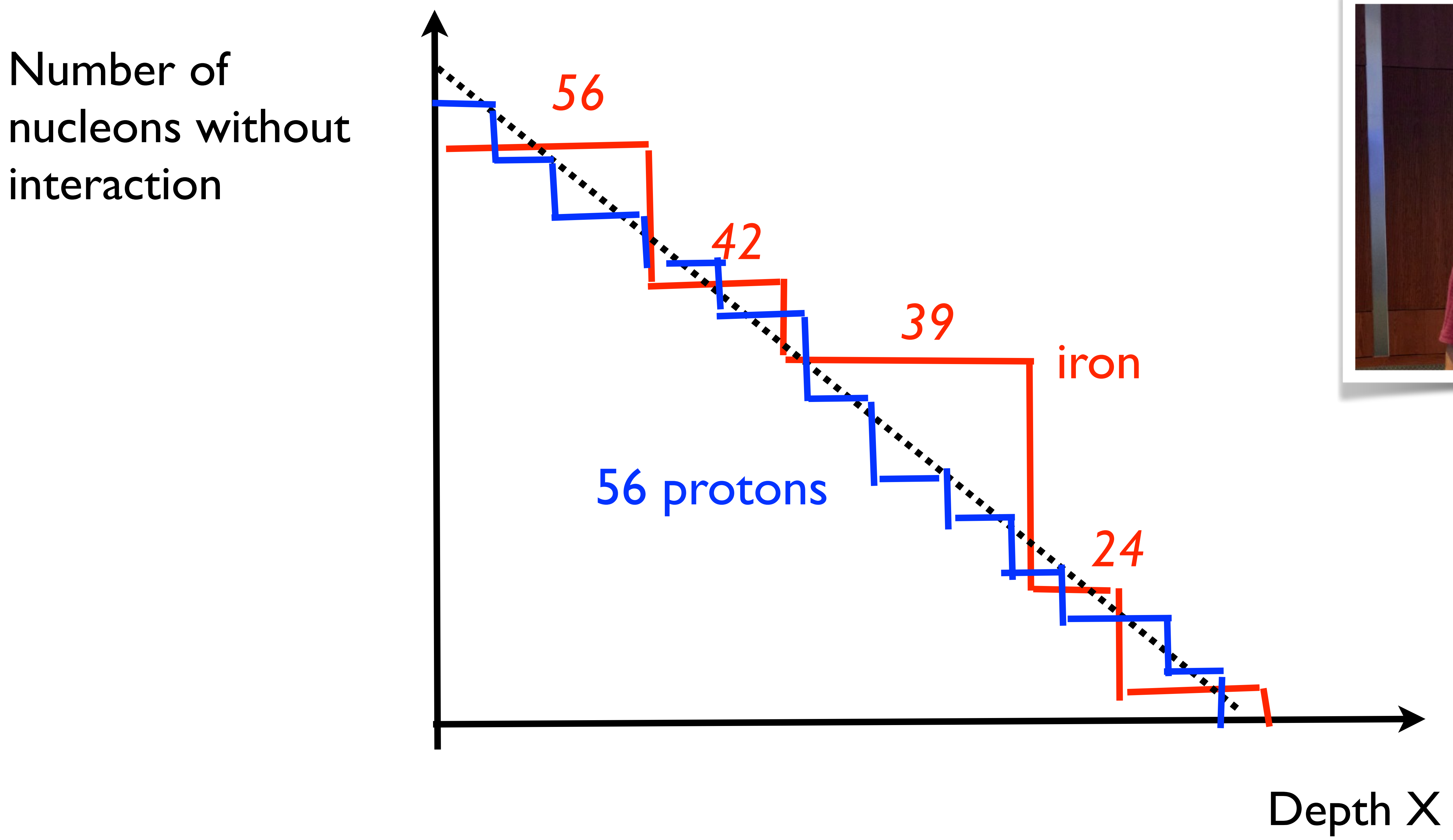
$$N_{\max}^A \sim A\left(\frac{E_0}{AE_c}\right) = N_{\max}$$

$$X_{\max}^A \sim \lambda_{\text{eff}} \ln(E_0/A)$$

$$N_\mu^A = A\left(\frac{E_0}{AE_{\text{dec}}}\right)^\alpha = A^{1-\alpha} N_\mu$$

Superposition model: correct prediction of mean values





Used in Sibyll interaction model

Jonathan ≠ Ralph

Glauber approximation (unitarity)

$$n_{\mathrm{part}} = \frac{\sigma_{\mathrm{Fe-air}}}{\sigma_{\mathrm{p-air}}}$$

Superposition and semi-superposition models applicable to inclusive (averaged) observables

(J. Engel et al. PRD D46, 1992)

Electromagnetic energy and energy transfer

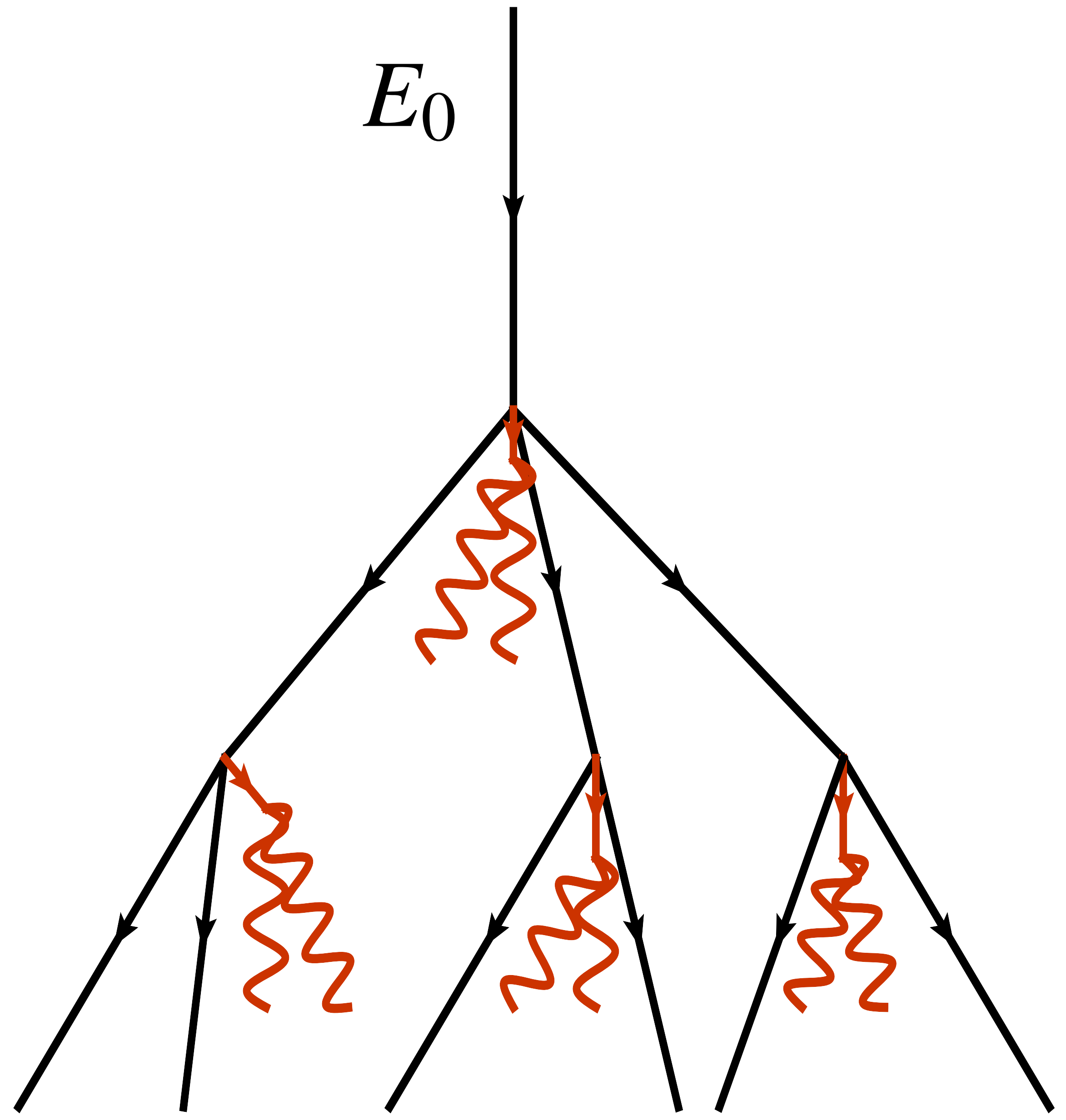
Hadronic energy

$$\frac{2}{3}E_0$$

$$\frac{2}{3}\left(\frac{2}{3}E_0\right)$$

⋮

$$E_{had} = \left(\frac{2}{3}\right)^n E_0$$



After n generations ...

$$n = 5, \quad E_{had} \sim 12\%$$
$$n = 6, \quad E_{had} \sim 8\%$$

Electromagnetic energy

$$\frac{1}{3}E_0$$

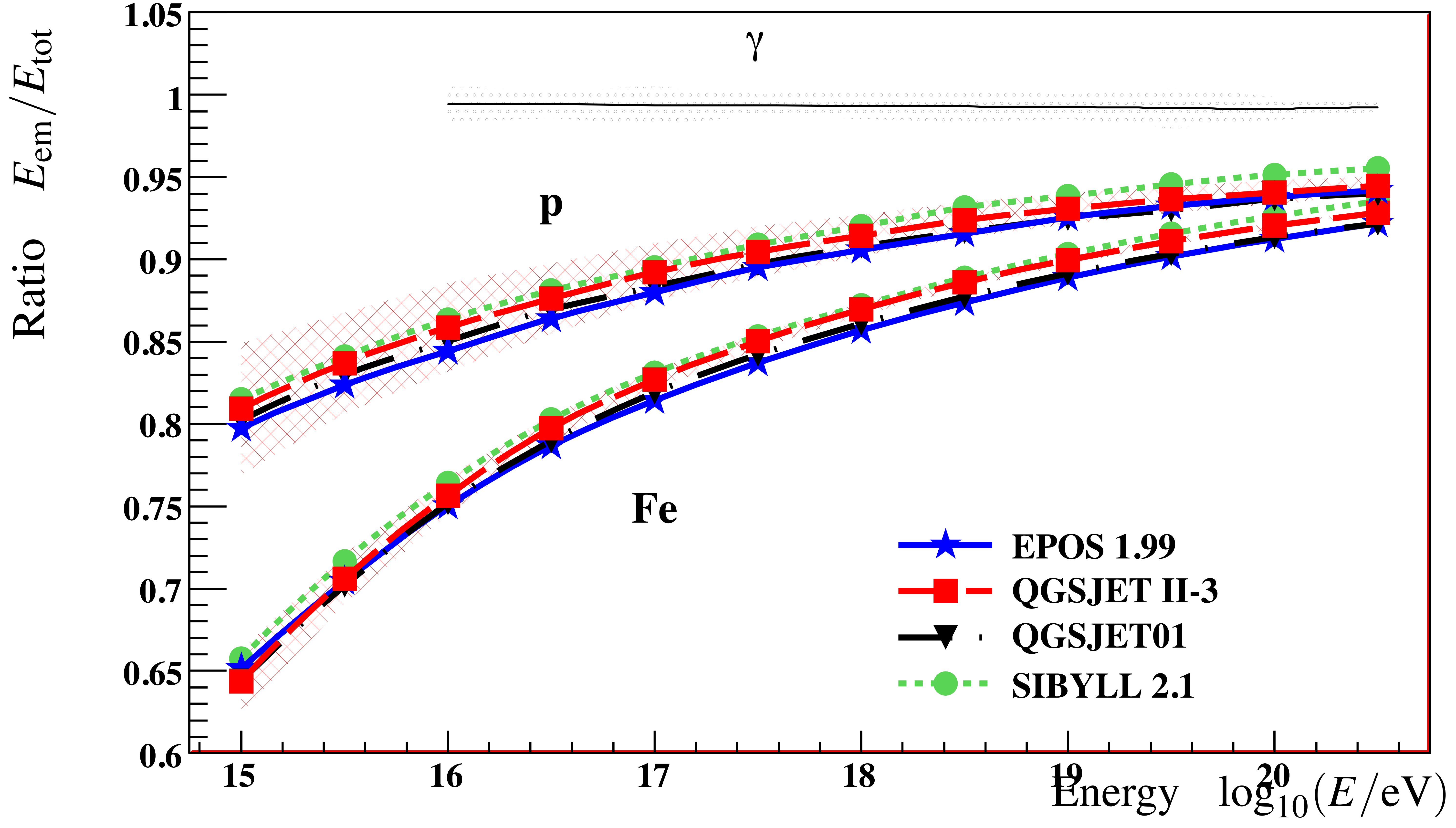
$$\frac{1}{3}E_0 + \frac{1}{3}\left(\frac{2}{3}E_0\right)$$

⋮

$$E_{em} = \left[1 - \left(\frac{2}{3}\right)^n\right] E_0$$

Energy transferred to electromagnetic component

(RE, Pierog, Heck, ARNPS 2011)



Ratio of em. to total shower energy

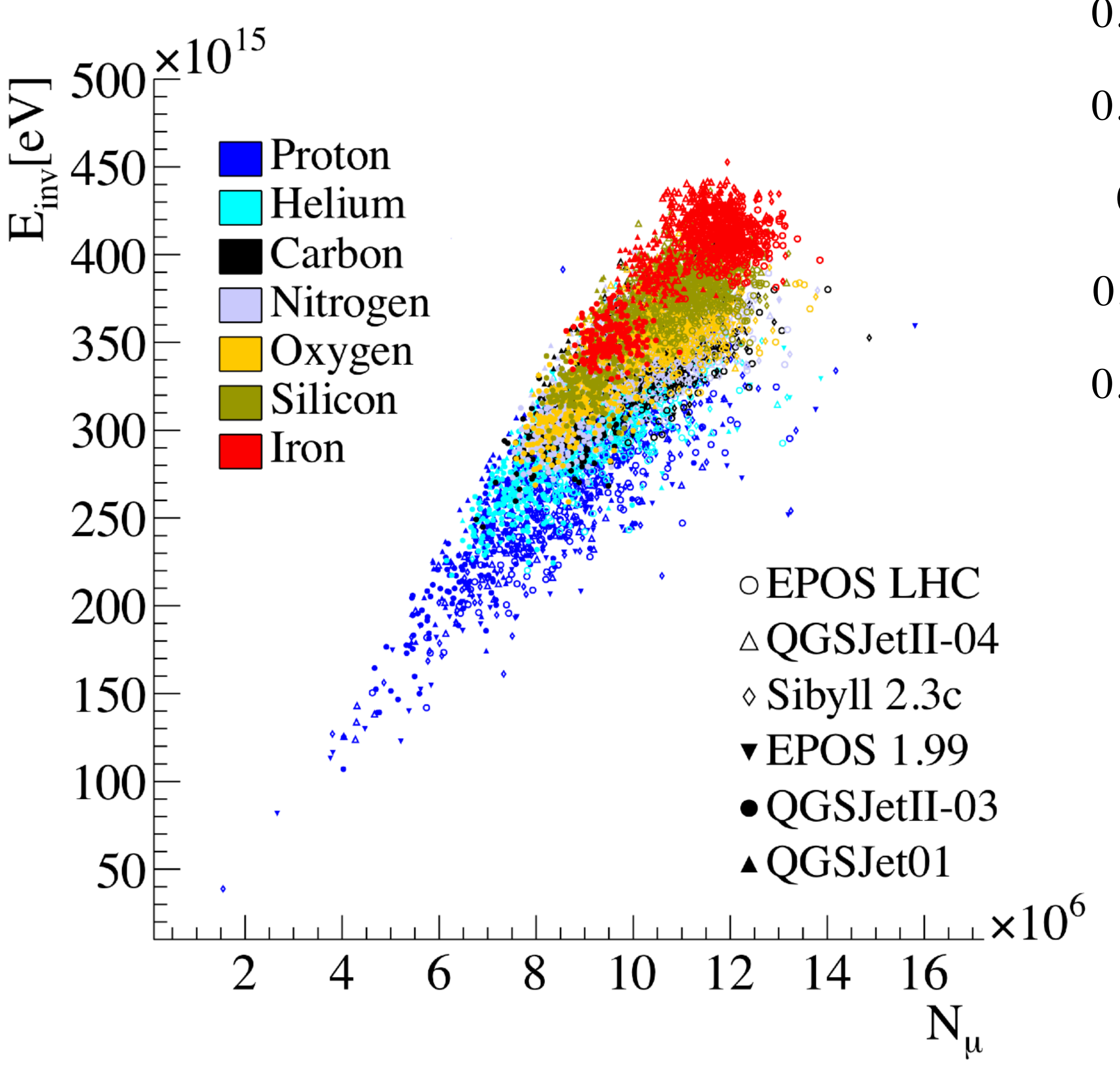
Detailed Monte Carlo simulation with CONEX

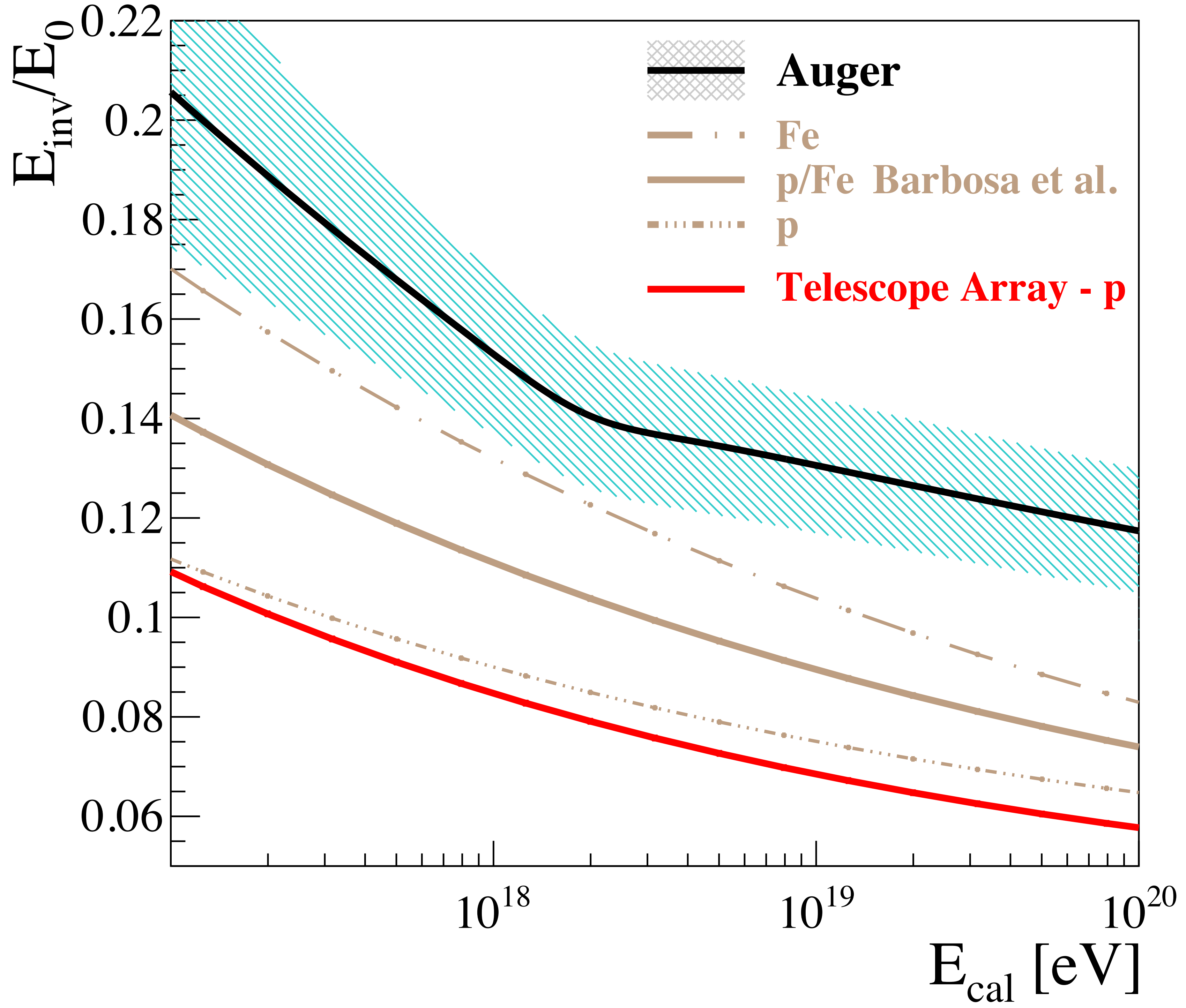
$$E_{inv} = E_{tot} - E_{em}$$

At high energy: model dependence of correction to obtain total energy small

Muons as tracers of the hadronic core

Very good correlation between muon number and invisible energy

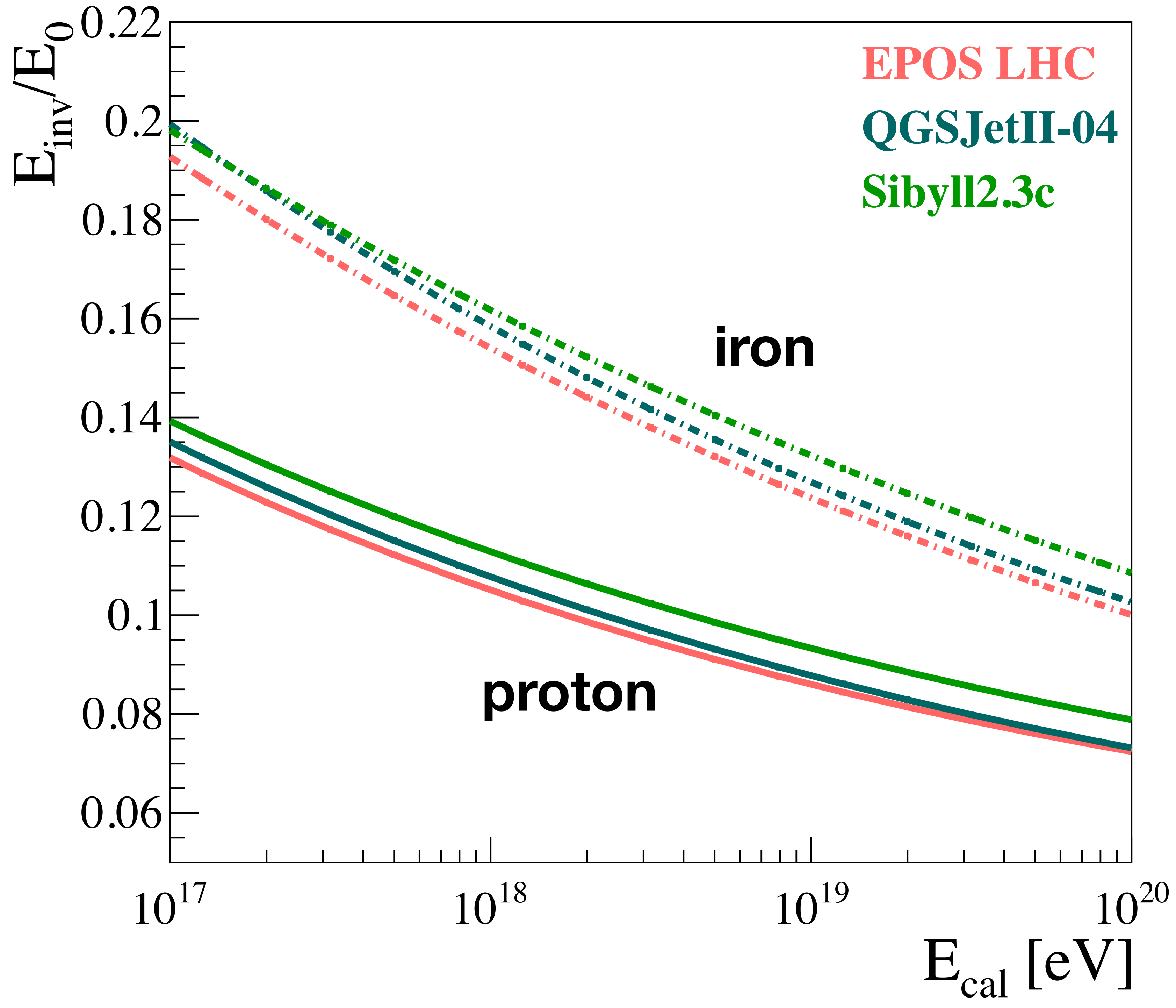




Muon-data based correction for invisible energy used in Auger

$$E_{\mathrm{inv}} = E_{\mathrm{tot}} - E_{\mathrm{em}}$$

Most recent model predictions

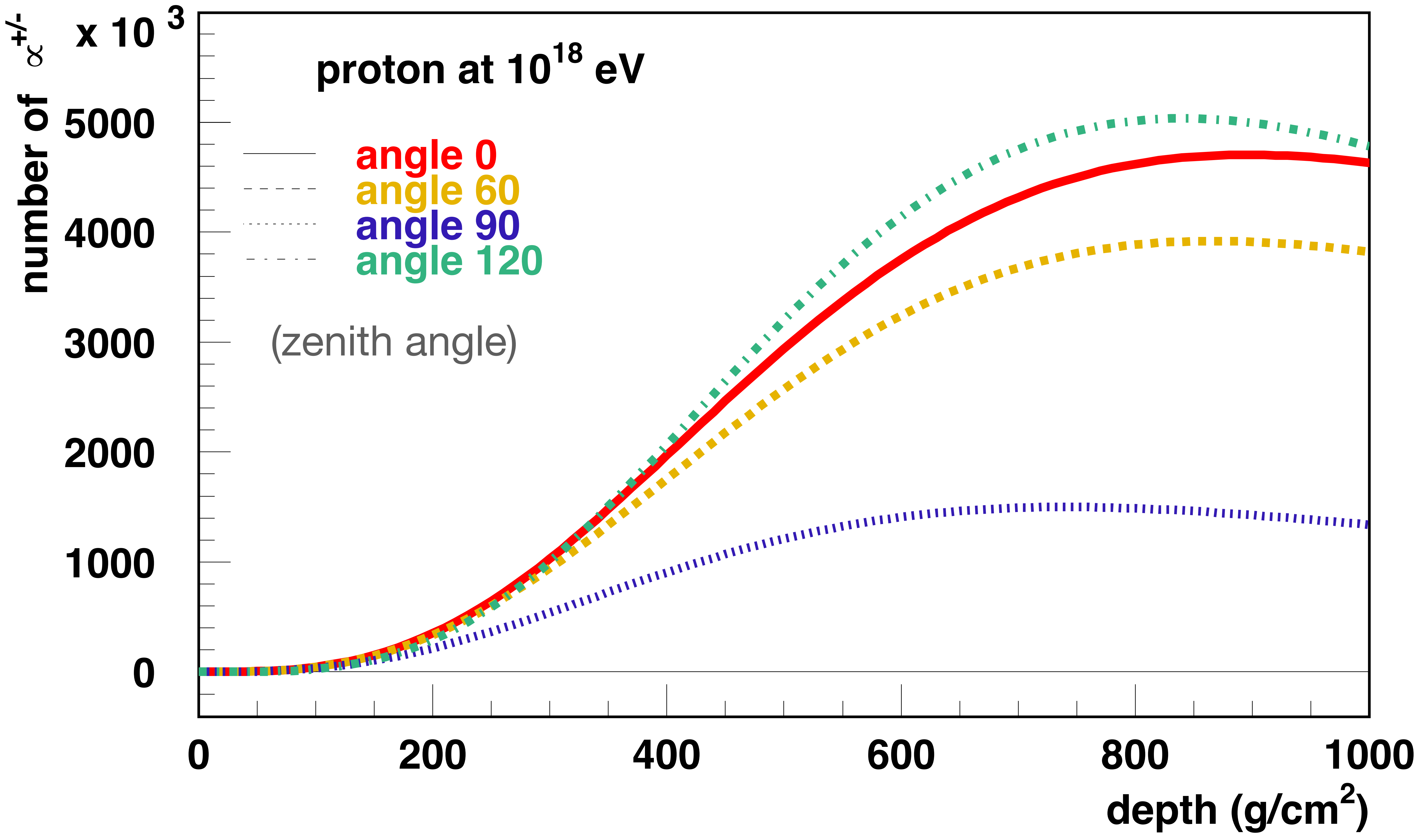


(Auger, PRD 2019)

Effect of air density (number of generations)

(Bergmann et al, APP 26, 2007)

$$N_\mu = \left(\frac{E_0}{E_{\mathrm{dec}}}\right)^{\alpha}$$



Pion decay energy depends on air density, low density corresponds to large E_{dec}

Electromagnetic showers are independent of air density, hadronic showers not

Mean depth of shower maximum

Heitler model

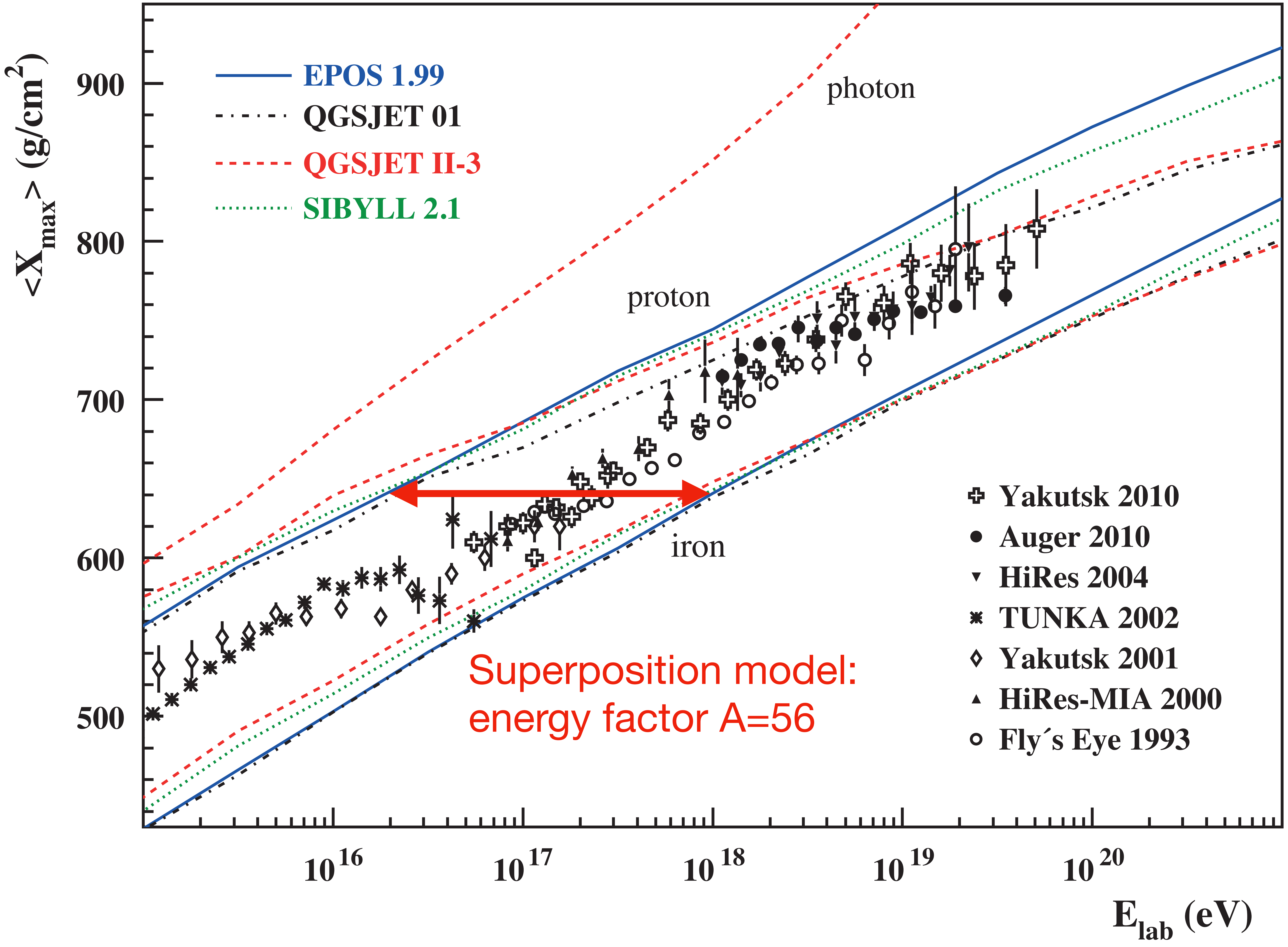
$$N_{\rm max} = E_0/E_c$$

$$X_{\rm max} \sim D_{\rm e} \ln(E_0/E_c)$$

Superposition model:

$$X^A_{\rm max} \sim D_e \ln(E_0/AE_c)$$

Note: old data and model predictions (just for clarity)



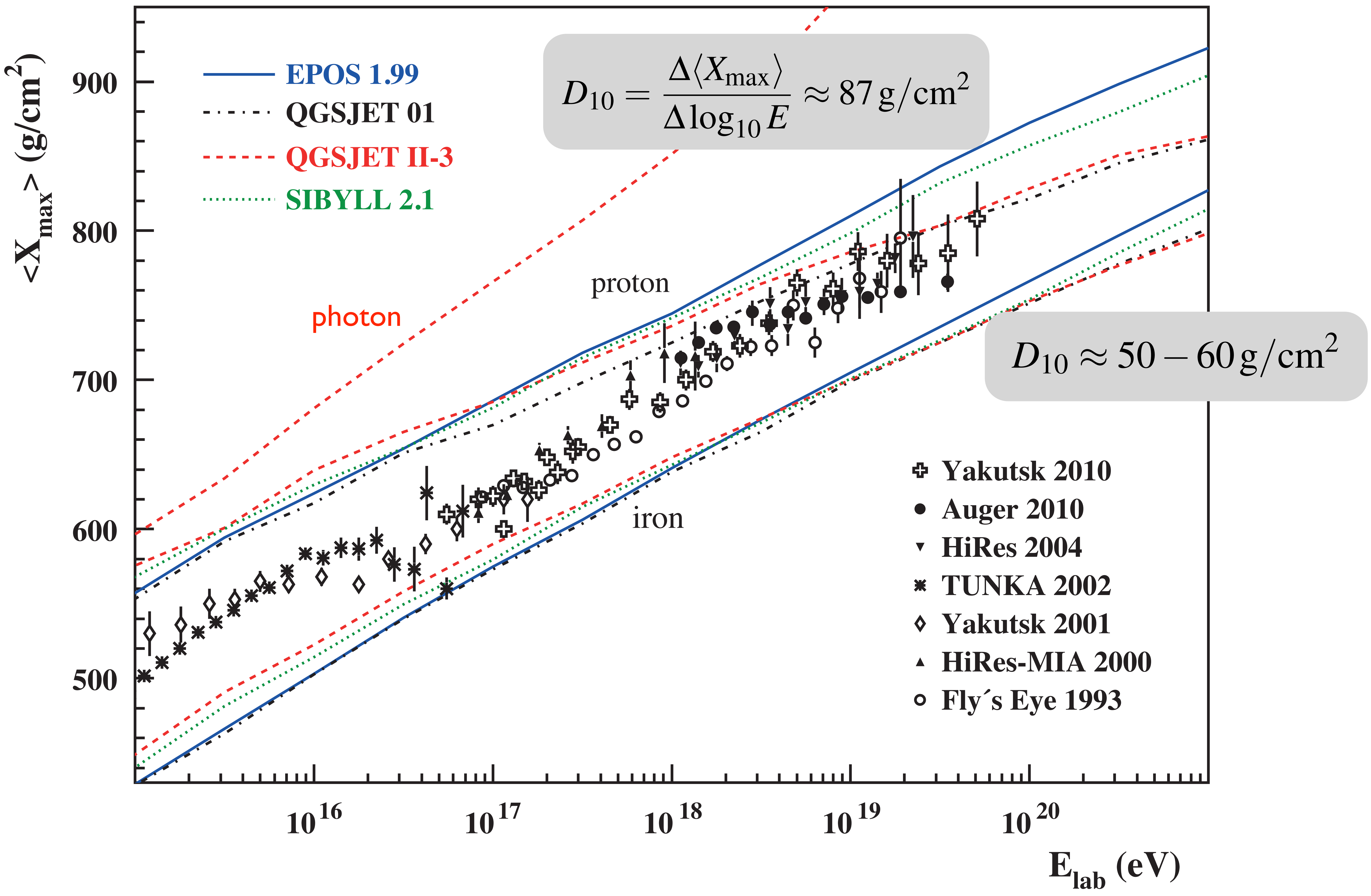
(RE, Pierog, Heck, ARNPS 2011)

Different slopes for em. and hadronic showers

$$D_{10} = \frac{\Delta\langle X_{max}\rangle}{\Delta \log_{10} E}$$

$$D_e = \frac{\Delta\langle X_{max}\rangle}{\Delta \ln E}$$

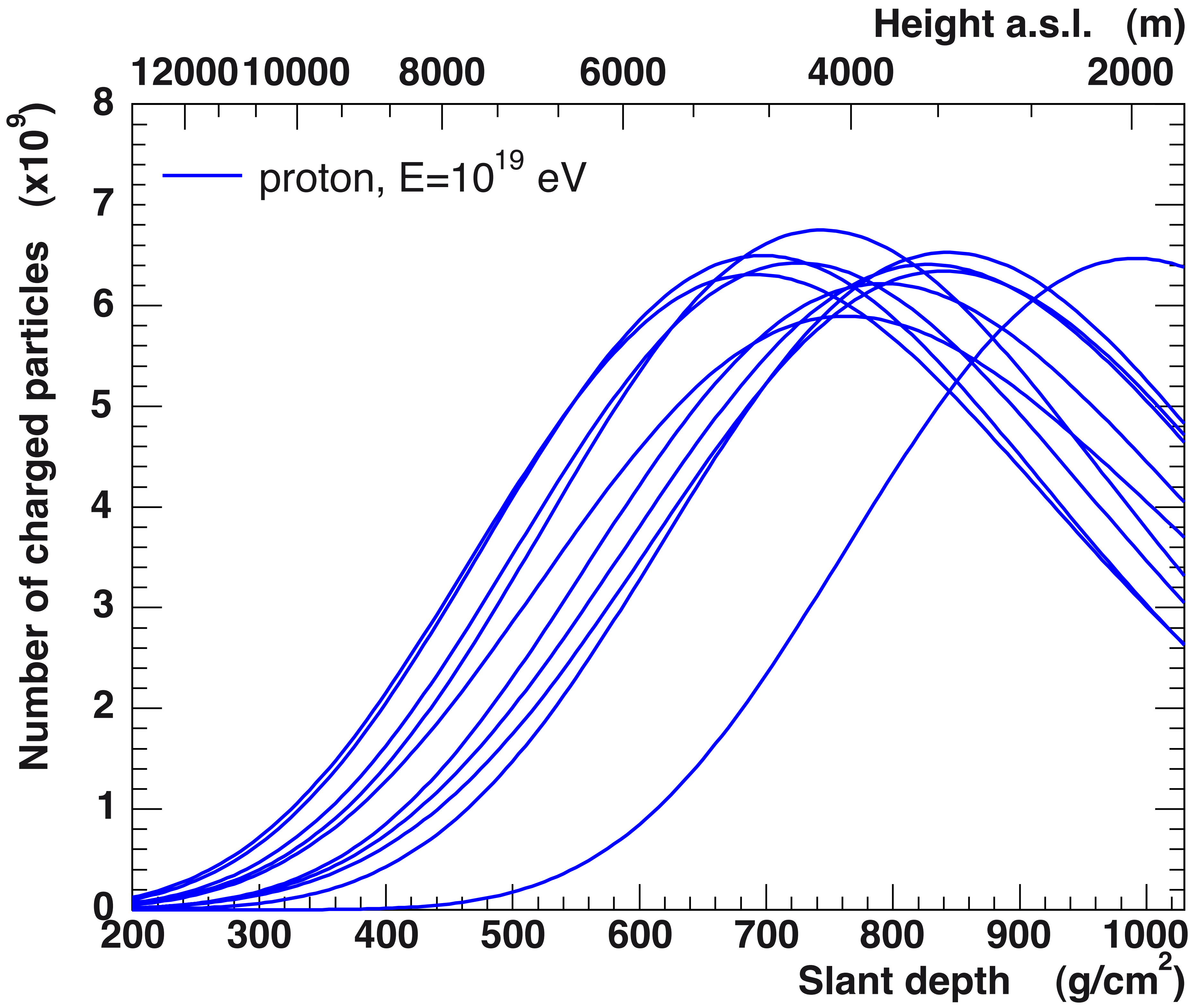
$$D_{10} = \log(10) D_e$$



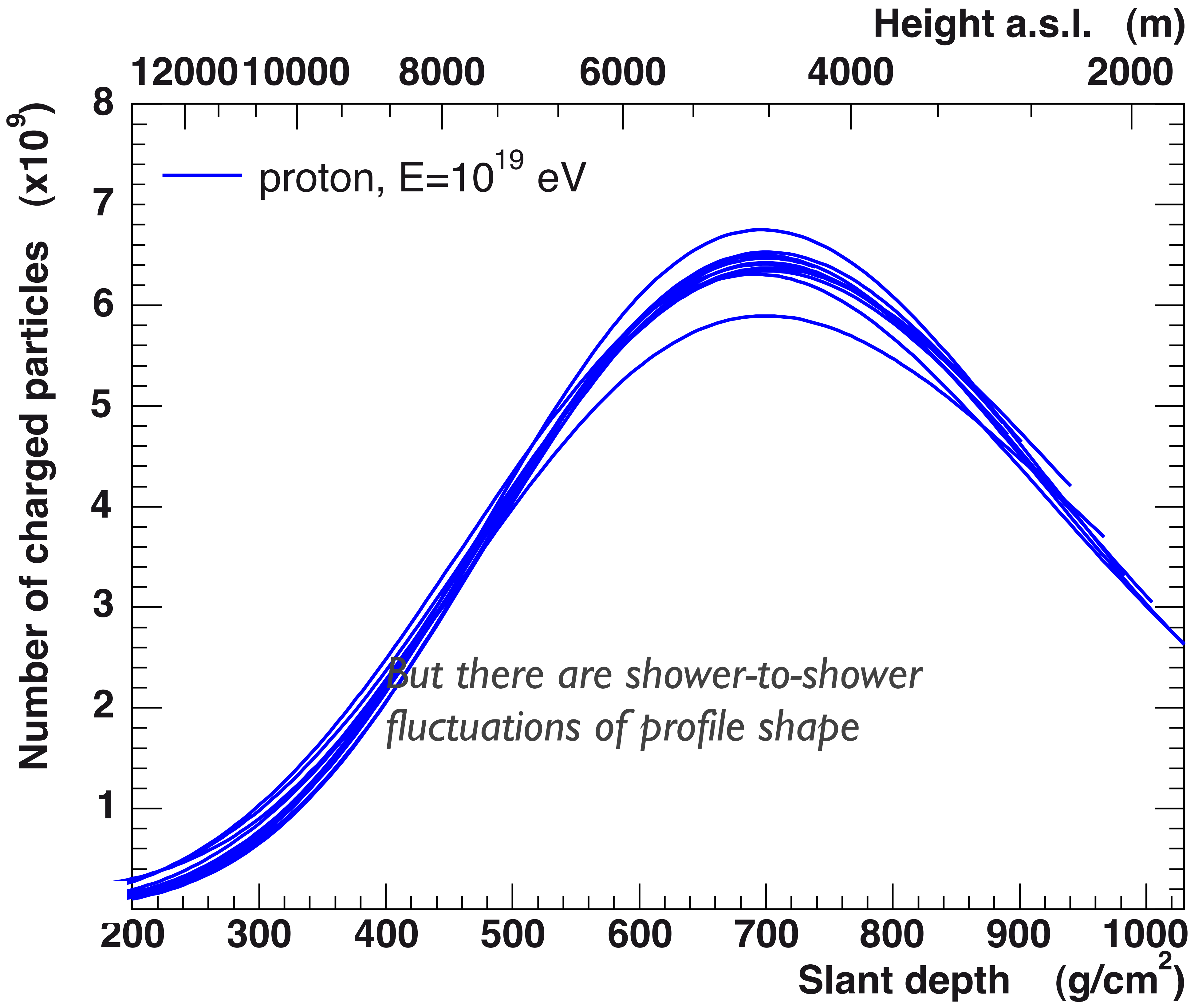
(RE, Pierog, Heck, ARNPS 2011)

Universality features of high-energy shower profiles

Simulated shower profiles

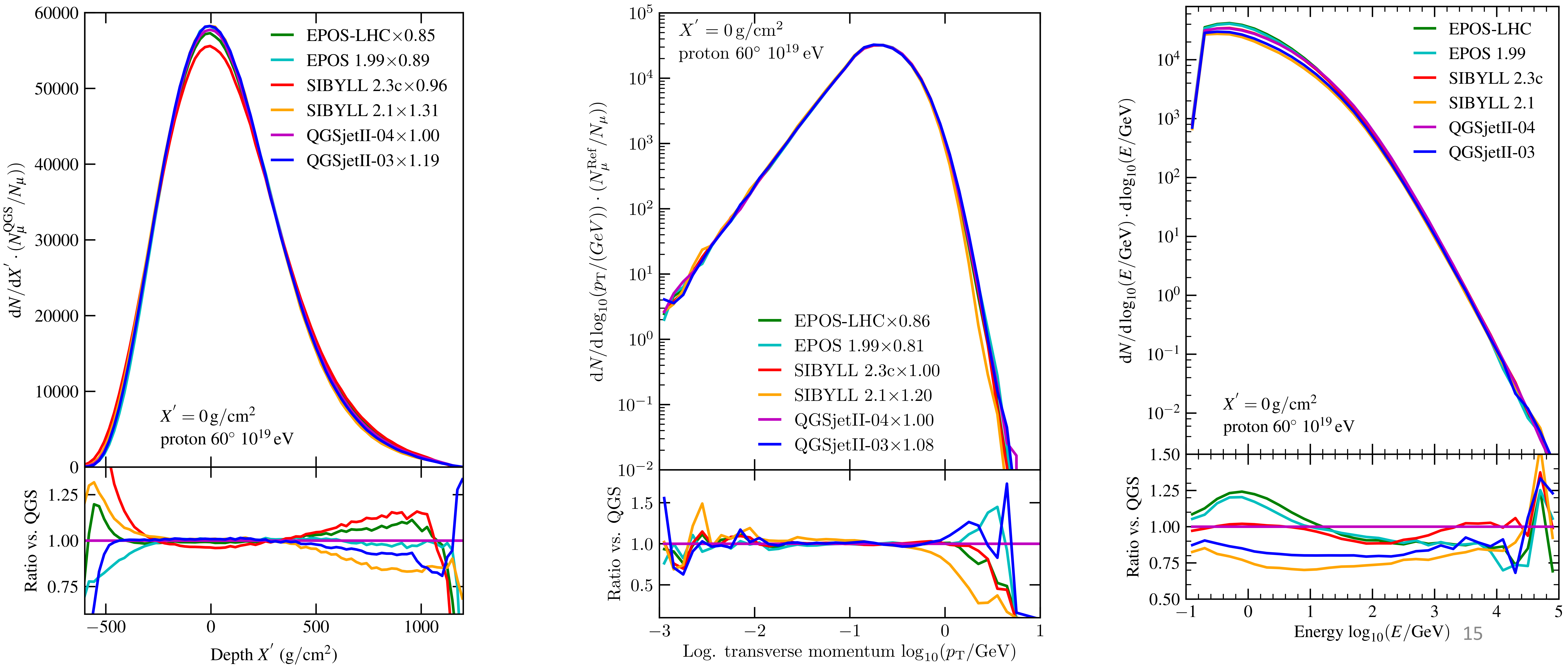


Profiles shifted in depth



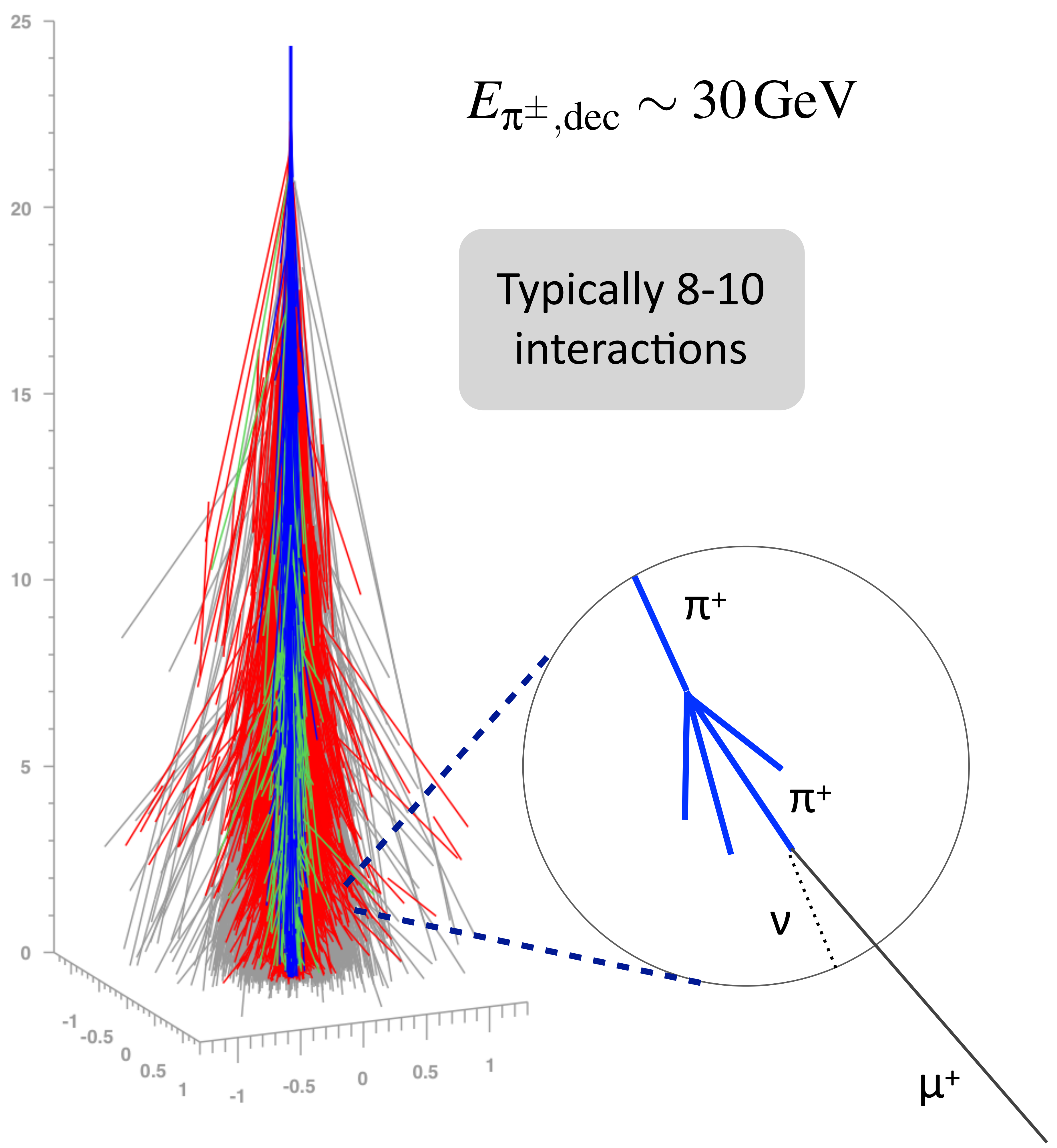
Depth of first interaction X_I and X_{max} strongly correlated, use X_{max} for analysis

Universality features of muon production



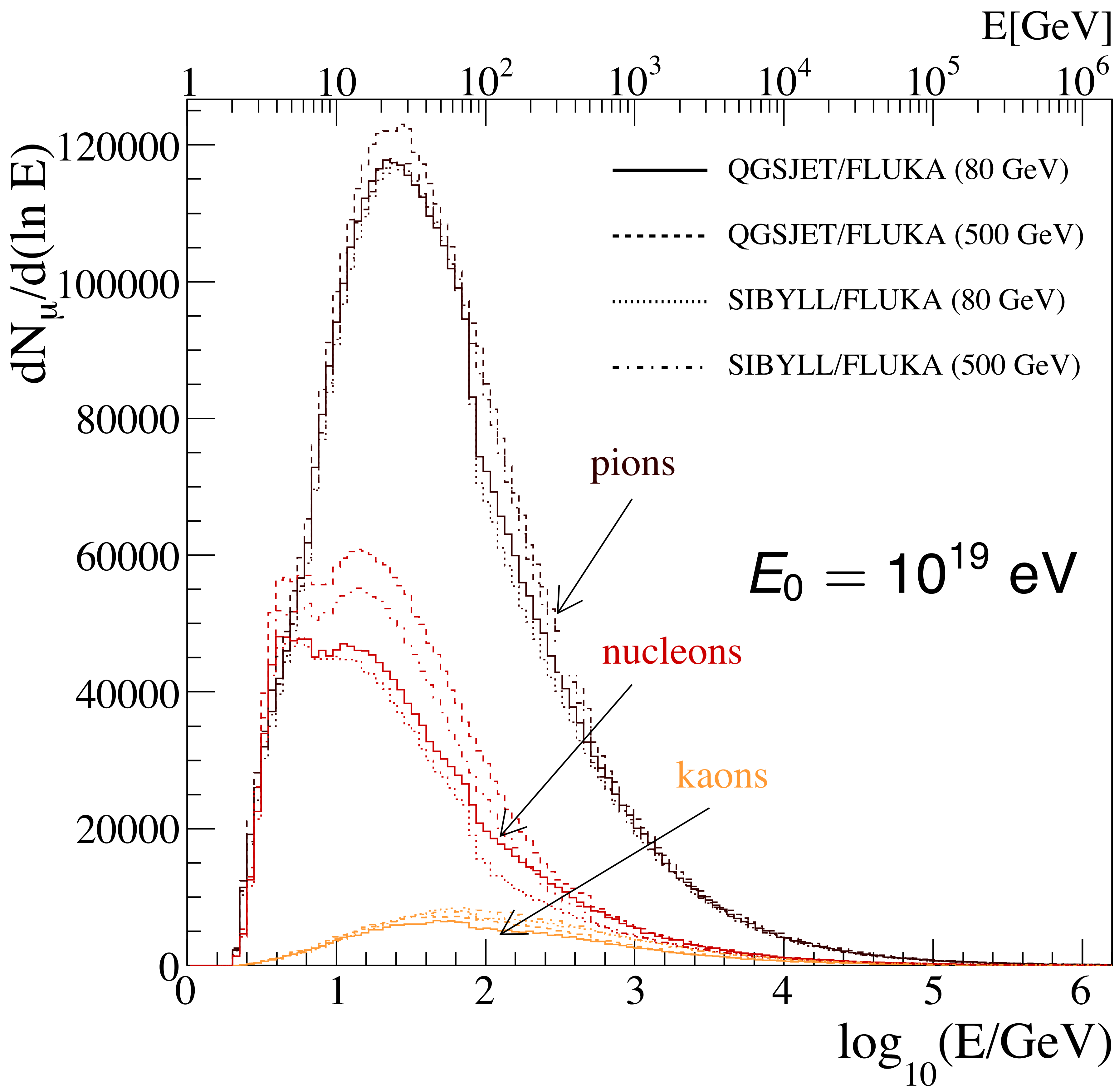
(Cazon, Epiphany Conference 2022)

Muon production at large lateral distance



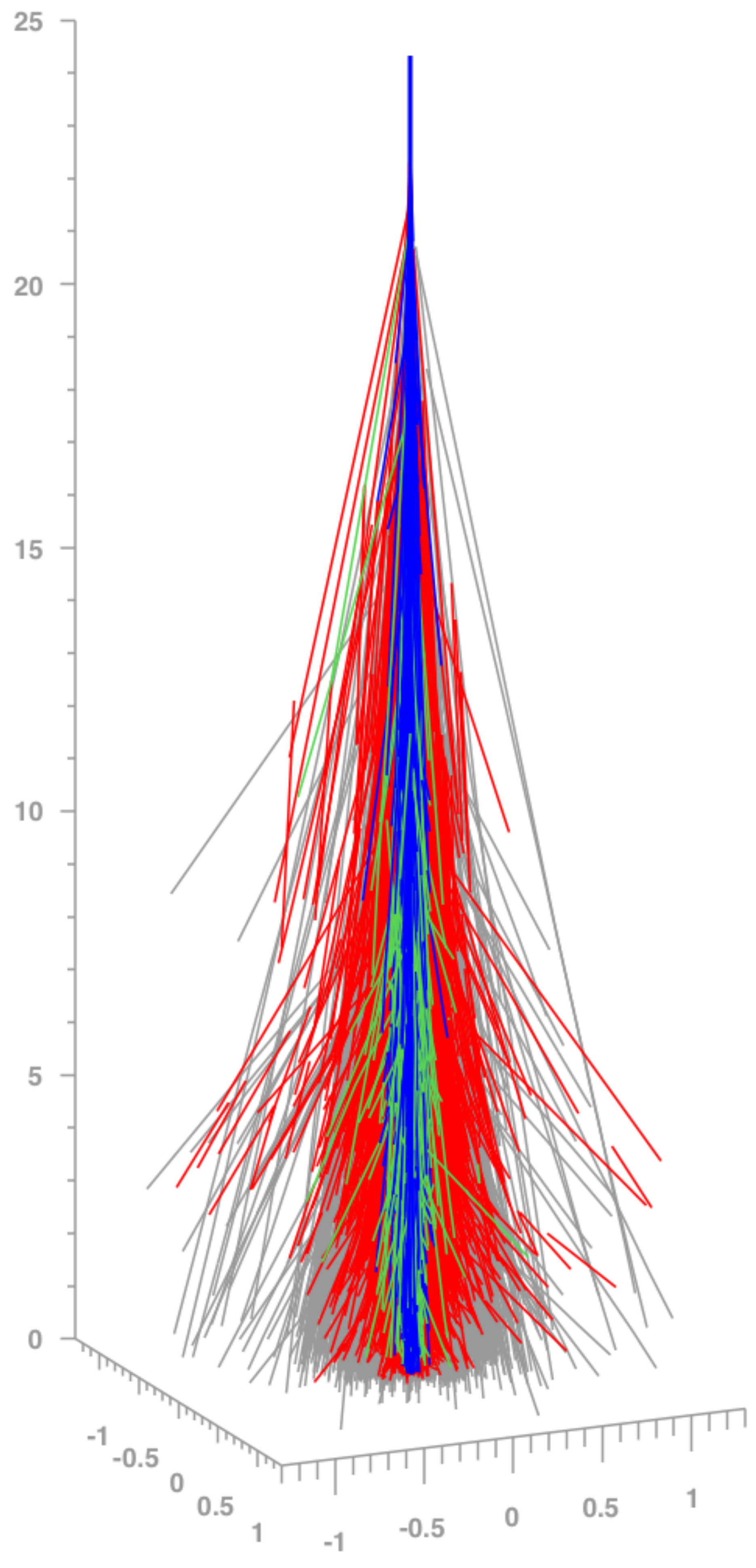
Muon observed at 1000 m from core

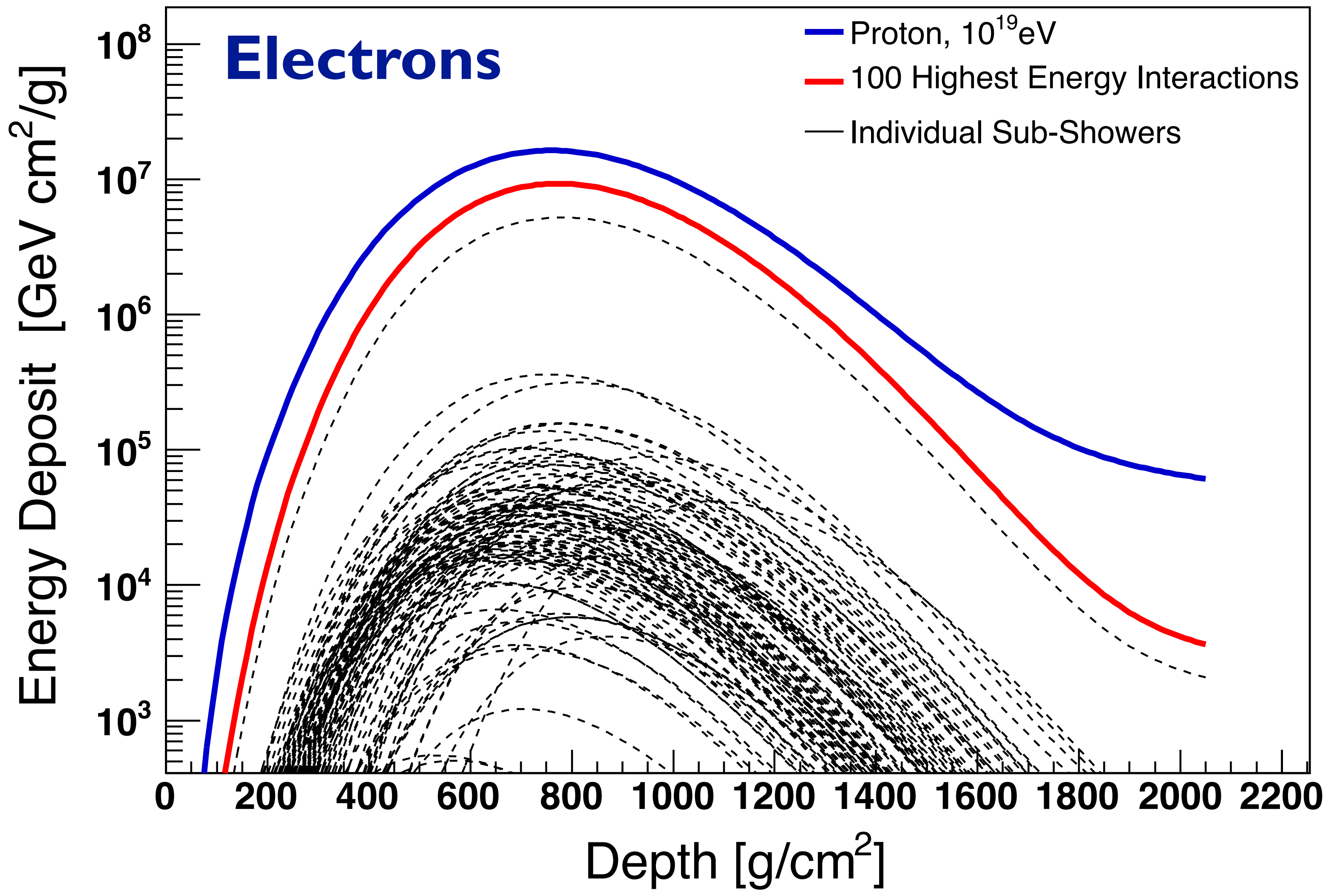
Energy distribution of **last interaction** that produced a detected muon

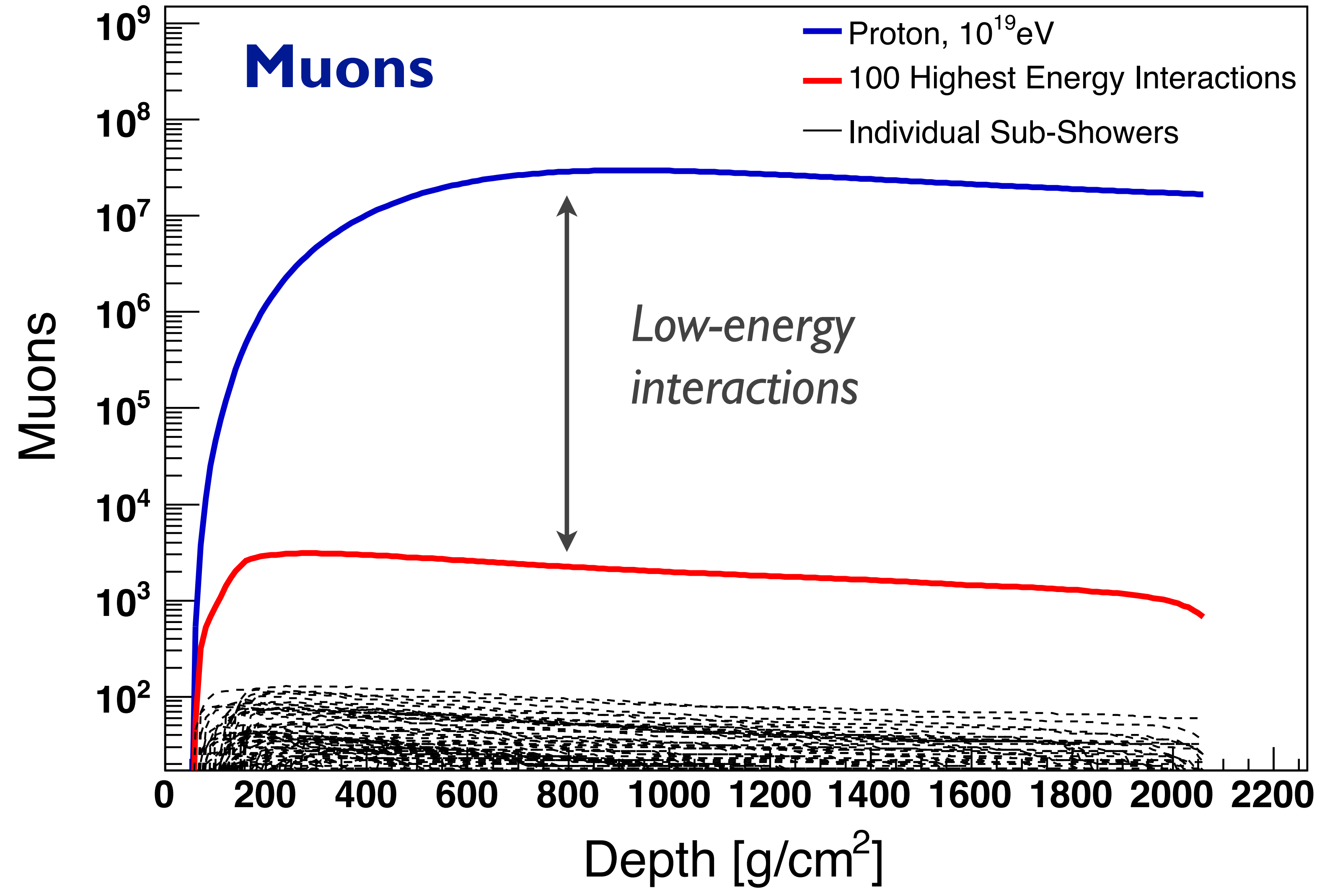


(Maris et al. ICRC 2009)

Importance of hadronic interactions at different energies







Shower particles produced in 100 interactions of highest energy

Electrons/photons:
high-energy interactions

Muons/hadrons:
low-energy interactions

Muons: 8 – 12 generations, majority of muons produced in ~30 GeV interactions

(Ulrich APS 2010)