

$t\bar{t}$ production near threshold at the LHC

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REF 2025, Univ. di Milano, 15/10/2025



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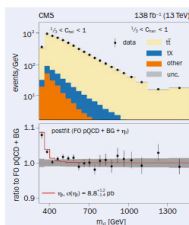
NEWS ANALYSIS

STRONG INTERACTIONS

CMS observes top-antitop excess

CERN's Large Hadron Collider continues to deliver surprises. While searching for additional Higgs bosons, the CMS collaboration may have instead uncovered evidence for the smallest composite particle yet observed in nature – a "quasi-bound" hadron made up of the most massive and shortest-lived fundamental particle known to science and its antimatter counterpart. The findings, which do not yet constitute a discovery claim and could also be susceptible to other explanations, were uploaded to the preprint archive on 28 March, following careful deliberation within the community.

Almost all of the Standard Model's shortcomings motivate the search for additional Higgs bosons. Their properties are usually assumed to be simple. Much as the 125 GeV Higgs boson discovered in 2012 appears to interact with each fundamental fermion with a strength proportional to the fermion's mass, theories postulating additional Higgs bosons generally expect them to couple more strongly to heavier quarks. This puts the singularly massive top quark at centre stage. If an additional Higgs boson



Threshold excess The invariant mass spectrum of top quark-antiquark pairs observed by the CMS experiment in certain domains of the reconstructed spin-correlation observables $c_{\mu\mu}$

quarks. Charmonium (ψ) mesons were discovered at Brookhaven National Laboratory in 1974. The November Revolution of quantum chromodynamics (bottom-antibottom quarks) was discovered at Fermilab in 1975. Heavy quarks move more slowly compared to the speed of light, making the strong interaction mediated by a static potential. At the scale of the separation between the quarks are far apart, the potential is proportional to their separation. At the scale of the self-interacting gluon elongating flux tube, the force of attraction. At the scale of the potential is due to individual gluons and in form, and inversely separation, leading to a force of attraction. This is where compact quarks formed, in a near perfect position, wherein position are bound by the Bohr radii of the charmonium and bottomonium approximately 0.3 fm.

The LHC produces top-antitop pairs at a rate of the order of tens per seconds, and has now accumulated nearly a billion of such events.

In the May-June issue of the CERNCOURIER an article regarding a CMS publication (arXiv:2503.22382, March 28 2025) has appeared.

From the CERNCOURIER, statement by John Ellis:

*“The signal reported by CMS, if confirmed, could be due either to a quasi-bound top–antitop meson, commonly called ‘**toponium**’, or possibly an elementary spin-zero boson such as appears in models with additional Higgs bosons, or conceivably even a combination of the two,” says theorist John Ellis of King’s College London.*

*“The mass of the lowest-lying toponium state can be calculated quite accurately in QCD, and is expected to lie just below the nominal top–antitop threshold. However, this threshold is smeared out by the short lifetime of the top quark, as well as the mass resolution of an LHC detector, so **toponium would appear spread out as a broad excess of events in the final states with leptons and jets that generally appear in top decays.**”*

Later confirmed by ATLAS: ATLAS-CONF-2025-008, (7 July 2025).

Threshold production of $t\bar{t}$ and spin correlations

- ▶ Focus upon the decay $t \rightarrow e^+ \nu b$.
- ▶ The positron direction coincides with the direction of the top spin.
- ▶ Threshold production is S -wave dominated. Thus the angular momentum of the $t\bar{t}$ pair is determined exclusively by the spins, and can be zero (singlet) or one (triplet).

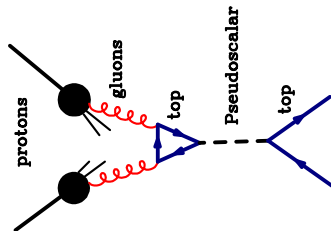
At LHC and for threshold production the singlet is dominant*

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The singlet state is a maximally entangled state.

(*) Gluon fusion is dominant at the LHC, and the Landau-Yang theorem, that forbids an angular momentum 1 for pairs of photons, also holds at tree-level for $gg \rightarrow Q\bar{Q}$.

The interest in the study of spin correlations in $t\bar{t}$ pairs is also motivated by the search of pseudoscalar that couple to $t\bar{t}$ pairs:



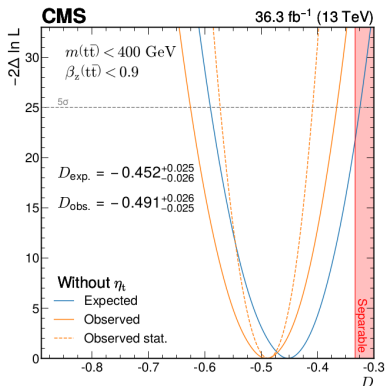
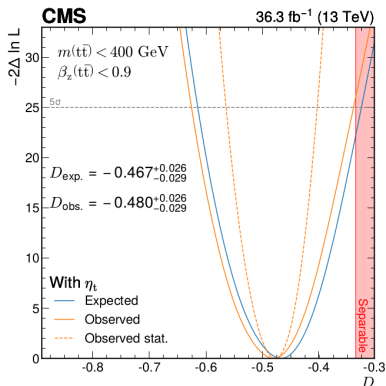
Search for heavy pseudoscalar and scalar bosons decaying to a top quark pair in proton-proton collisions at $\sqrt{S} = 13$ TeV
CMS Collaboration, (Jul 7, 2025), e-Print:2507.05119 [hep-ex].

Spin correlations in $t\bar{t}$ at LHC

(November 2023)

ATLAS: "Observation of quantum entanglement with top quarks ..."

$D = -0.537 \pm 0.002(\text{stat.}) \pm 0.019(\text{syst.})$ for $340\text{GeV} < m_{t\bar{t}} < 380\text{GeV}$.



$D < -1/3$ signals the presence of quantum correlations;

$D_{\text{exp.}}$ stands for the theoretical result.

Spin correlations in $t\bar{t}$ at LHC

In the dileptonic $t\bar{t}$ events one defines the observable

$$c_{\text{hel}} = \vec{\ell}_+ \cdot \vec{\ell}_- .$$

where the lepton directions $\vec{\ell}_{\pm}$ are defined starting from a frame in the $t\bar{t}$ CM system, that is boosted in turn to the t and $\bar{t}$ rest frame.

For a (non-relativistic) $t\bar{t}$ pair in a spin singlet state, by simple Quantum Mechanics we find that c_{hel} is distributed as

$$\frac{1}{\sigma} \frac{d\sigma}{dc_{\text{hel}}} = \frac{1 + c_{\text{hel}}}{2}$$

(observe that it vanishes when the top spins are **aligned**)

We also obtain immediately

$$\langle c_{\text{hel}} \rangle = \frac{1}{3}$$

and introduces the quantity $D = -3\langle c_{\text{hel}} \rangle$, that equals -1 for a pure spin singlet state. ($D < -1/3$ signals quantum entanglement.)

- ▶ To ease the tension, theoretical models obtained by adding an η_t bound state to the SM production mechanism have been proposed by [\[Maltoni,Severy,Vryonidou,2024\]](#)
- ▶ Some authors are improving MC generator by including full treatment of threshold-enhanced contributions to $t\bar{t}$ production in the non-relativistic approximation [\[Fuks,Hagiwara,Ma,Zheng,2024\]](#).
- ▶ Susequent CMS and ATLAS results on a Pseudoscalar excess near threshold seem to support these approaches ...

Lots of attention in the literature:

2003.02280 Entanglement and quantum tomography with top quarks at the LHC
2102.11883 Testing Bell Inequalities at the LHC with Top-Quark Pairs
2110.10112 Quantum tops at the LHC: from entanglement to Bell inequalities
2203.05582 Quantum information with top quarks in QCD
2203.05619 Quantum SMEFT tomography: Top quark pair production at the LHC
2205.00542 Improved tests of entanglement and Bell inequalities with LHC tops
2208.11723 Constraining new physics in entangled two-qubit systems: top-quark ...
2210.09330 Quantum entanglement and top spin correlations in SMEFT
2305.07075 Entanglement and Bell inequalities with boosted $t\bar{t}$
2307.06991 Postdecay quantum entanglement in top pair production
2401.08751 Quantum detection of new physics in top-quark pair production ...
2401.10988 A closer look at post-decay $t\bar{t}$ entanglement
2407.01672 Optimizing entanglement and Bell inequality violation in top antitop ...
2402.07972 Quantum entanglement and Bell inequality violation at colliders
2403.14757 New physics in spin entanglement

entanglement sells ...

Radiative corrections and threshold enhancement

- At Born level, the $t\bar{t}$ pair can be produced in a color singlet or octet state, in a ratio of 2 to 5 near threshold. Coulomb exchange yields enhanced corrections near threshold, proportional to the corresponding color factor: C_F for singlet, $-1/(2N_C)$ for the octet. The singlet prevails:

$$\left[2 \times \frac{4}{3} + 5 \times \left(-\frac{1}{6} \right) \right] = \left[\frac{8}{3} - \frac{5}{6} \right] > 0$$

- Corrections of order $(a/v)^n$, where $a = \alpha_s \times \text{color factor}$, arise at all orders in perturbation theory.
- Since gg at threshold leads to a spin singlet state, the α_s/v correction leads to an enhancement of the correlation.
- Event generators for $t\bar{t}$ production accurate at NLO order do exist, so they should fairly represent these a/v corrections. Generators also exist at NNLO order, that include also $(a/v)^2$ corrections. No NNNLO generator yet ...

Threshold enhancement: Bound States

The $t\bar{t}$ pair near threshold can form a bound Coulombic system.

Bound Coulombic systems have the following characteristics:

- ▶ The velocity of the quark in the CM system is of order a_s : $v \approx a_s$. $(a_s/v)^k$ terms are all of the same order, and must be resummed.
- ▶ The quark momentum is $\approx a_s m$; the size of the state is $\approx 1/(ma_s)$.
- ▶ The relevant scale for the coupling constant is ma_s ; so we must have $ma_s(\mu) \approx \mu$ that leads to $a_s(\mu) \approx 0.16$.
- ▶ The time to sweep an orbit is $\text{size}/v \approx 1/(ma_s^2)$.
- ▶ The typical bound state energy is of order ma_s^2 .
- ▶ The wave function at the origin is of order $1/\text{size}^3 \approx m^3 a_s^3$.
- ▶ Time to sweep an orbit (or the inverse of the binding energy) dangerously close to the top lifetime. No narrow bound state! But “lucky” top pairs that live much longer than their average lifetime will be able to make a few orbits, so, given enough resolution, a little “bump” should be visible in the cross section.

Threshold enhancement: Bound States

Examine now the contribution of a bound state to the cross section:

- ▶ Must be proportional to

$$\sigma(E) \propto \delta(E - 2m) |\psi(0)|^2 \alpha_s^2$$

where α_s^2 accounts for the hard production.

- ▶ By dimensional reasoning

$$\sigma(E) \approx \frac{1}{m^4} \delta(E - 2m) |\psi(0)|^2 \alpha_s^2$$

- ▶ Estimating $|\psi(0)|^2 \approx 1/r_b^3 \approx (a_s m)^3$ we get

$$\sigma(E) \approx \frac{1}{m} \delta(E - 2m) \alpha_s^2 a_s^3$$

corresponding to an NNNLO correction.

Threshold enhancement

The integral of the cross section near threshold up to NNLO goes like

$$\int dE \sigma(E) \propto \int m dv^2 \frac{\alpha_s^2 v}{m^2} \left(1, \frac{a_s}{v}, \frac{a_s^2}{v^2} \right) \approx \frac{\alpha_s^2 v_{\text{cut}}^3}{m} \left(1, \frac{a_s}{v_{\text{cut}}}, \frac{a_s^2}{v_{\text{cut}}^2} \right)$$

At higher orders a_s/v singularities only come with even powers, so $(a_s/v)^3$ is missing. However a bound state contributes to the integrated cross section as

$$\int dE \frac{\alpha_s^2 a_s^3}{m^2} \delta(E - 2m) \approx \frac{\alpha_s^2 a_s^3}{m^2} = \frac{\alpha_s^2 v_{\text{cut}}^3}{m^2} \left(\frac{a_s^3}{v_{\text{cut}}^3} \right)$$

in line with the NLO and NNLO contributions;

But this term cannot arise in perturbation theory, since it vanishes for negative α_s (bound states exist only for positive a_s).

What is going on? What about higher orders? The next term is $(a_s/v)^4$, and is not integrable ... Do we still get an $(a_s/v_{\text{cut}})^4$ contribution to the cross section?

Motivation of our work

- ▶ The experiments do not measure exactly the distribution of $m_{t\bar{t}}$, but rather a smeared distribution, since the top mass resolution is of the order of 15 GeV. In fact the experimental results are quoted for mass bins that go from threshold (340 GeV) up to 360, 380, 400 GeV.
- ▶ Smearing, by the uncertainty principle, means that the result is sensitive to times of the order of the inverse bin size. Bound state formation takes place in longer times. So, part of the cross section goes into bound state formation, part in open production, but the experimental result should be insensitive to the exact proportion of the two contributions.

In order to better understand what goes on we considered a simple quantum mechanical example: the single particle in a delta-function potential in one dimension.

Toy model for the production of a bound state

Schrödinger equation

$$-\frac{1}{2m} \frac{d^2}{dx^2} \psi - \lambda \delta(x) \psi = E \psi.$$

Eigenstates ($k > 0$):

$$\psi_0(x) = \theta(\lambda) \sqrt{m\lambda} [e^{\lambda mx} \theta(-x) + e^{-\lambda mx} \theta(x)], \quad E_0 = -\frac{m\lambda^2}{2};$$

$$\psi_k(x) = \sqrt{\frac{2}{L(1 + \lambda^2/v_k^2)}} \left[\cos(kx) - \frac{x}{|x|} \frac{\lambda}{v_k} \sin(kx) \right], \quad v_k = \frac{k}{m}; E_k = \frac{k^2}{2m}$$

$$\hat{\psi}_k(x) = \sqrt{\frac{2}{L}} \sin(kx) \quad (\text{decoupled, irrelevant})$$

where L is the dimension of the system.

Holds independently of the sign of λ .

Density of states:

$$\rho(E) = \theta(\lambda)\lambda m \delta\left(E + \frac{1}{2}m\lambda^2\right) + \frac{1}{\pi} \frac{m}{k_E} \frac{1}{1 + \lambda^2/v_E^2}$$

proportional to the probability that starting from a state concentrated at $x = 0$ one reaches a final state of energy E ($k_E = \sqrt{2mE^2}$, $v_E = k_E/m$).

This expression does not seem to have a power expansion in the coupling λ . The first term contains $\theta(\lambda)$. The second one has a divergent expansion for $v_E < \lambda$.

Expanding in powers of λ for $E > 0$ we get

$$\rho(E) = \frac{1}{\pi} \frac{1}{v_E} \left[1 - \frac{\lambda^2}{v_E^2} + \dots \right]$$

that has singularities in powers of λ/v , non integrable in the energy.

Consider the integral of the density of states from the threshold up to a given value E' . We get

$$\begin{aligned}\int^{E'} dE \rho(E) &= \theta(\lambda) m \lambda + \frac{1}{\pi} \left[k' - m \lambda \arctan \frac{k'}{m \lambda} \right] \\ &= \frac{1}{\pi} k' + m \left[\lambda \theta(\lambda) - \frac{|\lambda|}{2} \right] + \frac{m \lambda^2}{\pi} \frac{m}{k'} + \dots\end{aligned}$$

- We see that **the non-analytic term arising from the bound state** combines with a **non-analytic term arising from the continuum** and form an analytic term.

$$\left[\lambda \theta(\lambda) - \frac{|\lambda|}{2} \right] = \left[\frac{\lambda + |\lambda|}{2} - \frac{|\lambda|}{2} \right] = \frac{\lambda}{2}.$$

- The term of order λ^2 in the expansion of ρ has yielded a finite term of order $1/\nu'$ term. i.e. we should have interpreted it as a distribution.

Full result for the toy model

With a little more work we get

$$\begin{aligned}\rho(E) &= \theta(\lambda)\lambda m\delta\left(E + \frac{1}{2}m\lambda^2\right) + \frac{1}{\pi} \frac{m}{k_E} \frac{1}{1 + \lambda^2/v_E^2} \\ &\Rightarrow \frac{\lambda m}{2} \delta\left(E + \frac{1}{2}m\lambda^2\right) + \frac{1}{\pi} \frac{m}{k_E} \left(\frac{1}{1 + \lambda^2/v_E^2}\right)_+\end{aligned}$$

where the $+$ sign indicates that the expression should be Taylor-expanded, and each term should be regularized according to the analytic regularization prescription.

Also the δ function can be Taylor-expanded in terms its derivatives. Thus ρ has a well-defined perturbative expansion with coefficients that are distributions, and its product with functions that are regular near threshold yields integrable functions.

The $t\bar{t}$ case

Is equivalent to the Hydrogen atom problem. It leads to

$$\rho_l(\mathcal{E}) = \frac{\theta(a_l)m^3 a_l^3}{8\pi} \sum_{n=1}^{\infty} \frac{1}{n^3} \delta(\mathcal{E} - E_{l,n}) + \frac{(m)^{3/2}}{4\pi^2} \sqrt{\mathcal{E}} F(\pi a_l v^{-1}),$$
$$E_{l,n} = -\frac{m a_l^2}{4n^2}, \quad F(z) = \frac{z}{1 - \exp(-z)}$$

where $\mathcal{E} = E - 2m$, $l = 1(8)$ for color singlet (octet), $a_1 = C_F \alpha_s$,
 $a_8 = -\alpha_s/(2N_C)$.

The factor $F(\pi a_l v^{-1})$ is known as Sommerfeld factor. Reasoning as in the toy model example, we reach the same conclusions

$$\rho_l(\mathcal{E}) \rightarrow \frac{m^3 a_l^3}{16\pi} \sum_{n=1}^{\infty} \frac{1}{n^3} \delta(\mathcal{E} - E_{l,n}) + \frac{(m)^{3/2}}{4\pi^2} \sqrt{\mathcal{E}} F^+(\pi a_l v^{-1}),$$

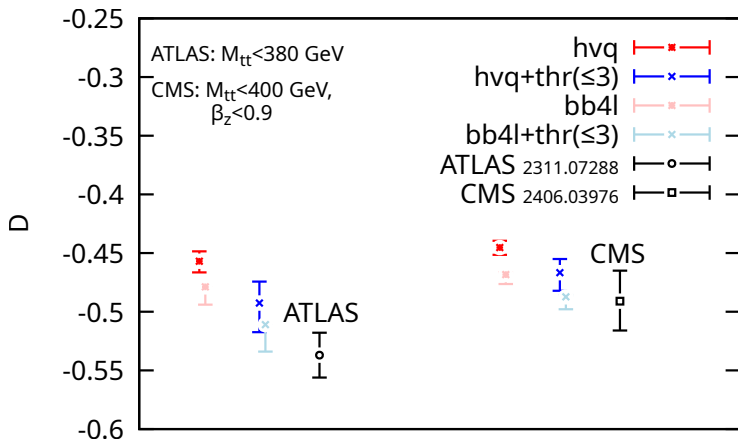
where F^+ represents the Taylor expansion of F , with the coefficients interpreted as (analytically regulated) distributions. If we also expand $\delta(\mathcal{E} - E_{l,n})$, we get the full perturbative expansion of ρ .

Keeping only the first three perturbative orders we get

$$\rho_I(\mathcal{E}) \rightarrow \frac{(m)^2}{4\pi^2} \left(v + \frac{\pi a_I}{2} + \frac{\pi^2 a_I^2}{12v} + \frac{\pi \zeta(3) a_I^3}{4} m \delta(\mathcal{E}) \right)$$

Among state of the art calculations used for this process, some include the first correction (NLO calculations), some up to the second one (NNLO), none includes the third term, which is however very small.

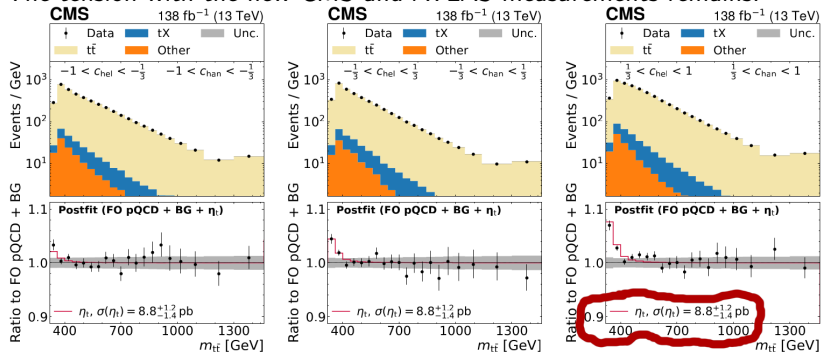
Comparison with data



- There is also a dependence upon the generator one uses... to be better understood.
- The tension with data disappears.

The pseudoscalar excess

The tension with the new CMS and ATLAS measurements remains.



what is the meaning of that 8.8pb?

(As I understand it)

- ▶ A sample is generated using POWHEG-hvq.
- ▶ The sample is reweighted so that the $m_{t\bar{t}}$ distribution and an angular variable as obtained by MATRIX (NNL).
- ▶ An additional component of the cross section, corresponding to the production of an η_t is added to the cross section
- ▶ The normalization of the η_t component is fitted to the data, yielding the 8.8pb.

If we compute the threshold enhanced contributions to the cross section in the first bin $m_{t\bar{t}} < 360$ GeV we get:

- ▶ 3.36pb from the a/v term, after subtracting what is already included at NLO.
- ▶ 4.43pb from the $(a/v)^2$ term
- ▶ 1.56pb from the delta term
- ▶ $(1.56\text{pb})/2$ is the contribution of all bound states.

The total is 9pb. But CMS should have already included at least a good part of the 3.36pb (correction to the a/v) term, and 4.43pb from the $(a/v)^2$ term, since MATRIX includes that.

Conclusions

- ▶ We have re-re-learned a lesson about why NOT to include bound state effects in inclusive cross sections.
- ▶ The comparison with data for the pseudoscalar excess has many obscure points.
- ▶ We eagerly wait for unfolded cross sections from CMS and ATLAS. We believe this is needed in order to really understand if the excess is compatible with the Standard Model.
- ▶ There are also other possible explanations of the excess, due to limitations in current MC generators (not discussed in this talk for brevity).

Previous literature

It turns out that our findings were not new ...

While completing our work, we learned from Beneke that he had already worked on this issue, in the framework of threshold resummation for top production at hadron colliders. His work was inspired by a paper of Kirill Melnikov on the calculation of $g - 2$, as also Kirill pointed out to us.

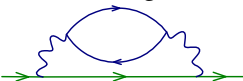
Interesting enough, Kirill also rediscovered this issue by himself, but he was informed by the referee that in fact the solution of the problem dates back to a paper by Michail Braun (father of Volodia) in 1968. For some reasons, in spite of these previous references, people seem to stumble on it every so often ...

Previous literature

Melnikov, Vainshtein, Voloshin 2014

“Remarks on the effect of bound states and threshold in $g-2$ ”

In the very high-order computation of the $g-2$ for the electron, diagrams with

photon self energy come up  and some authors (Mishima, 2013, Fael, Passera, 2014) could not resist the temptation to add positronium contributions to the electron blob (it enters at order α^5).

Melnikov et al show that you should not do that.

In the same paper they criticise in passing work by Kniehl, Kuhn and Stuart, 1988, Kniehl, Sirlin, 1993 and Yndurain, 1994, that advocated adding the toponium bound state contribution in top loops affecting Electroweak precision calculations (again, you should not).

At the beginning of Melnikov's paper:

These questions were analyzed long ago in Ref. [Braun]. However, that paper is, perhaps, not as well-known as it should be.

Other Instances of the problem

Eides, 2014

"Recent ideas on the calculation of lepton anomalous magnetic moments"

Same motivation and conclusions as in Melnikov's paper.

Interesting to quote:

Nothing we will say below is new and not contained in [Braun], but the claims made in [Mishima, Fael] show that the old arguments deserve to be repeated.

Other Instances of the problem

[Beneke,2006](#)

Same as our result for the case of the totally inclusive cross section, in the framework of the resummation of threshold effects. Inspired by the previous work of Melnikov.

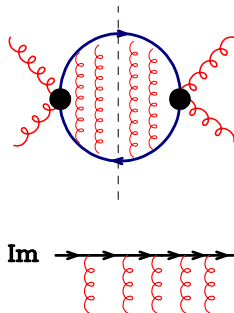
Backup

THE ISSUE

Consider the squared amplitude for the production of a $t\bar{t}$ pair near threshold.

- ▶ Small black blob size: $1/m$.
- ▶ Dominant corrections near threshold: Coulomb exchange, leading to $(\alpha_s/v)^n$ singularities, where v is the top velocity in the $t\bar{t}$ rest frame.

The corresponding non-relativistic reduced problem corresponds to the imaginary part of the forward Resolvent $R(E, 0, 0)$.



A brief reminder: the solution of the quantum mechanical problem is encoded in the Green's function G , that satisfies the Schrödinger equation

$$\left[i \frac{\partial}{\partial t} - H \right] G(t, \vec{x}, 0, \vec{x}_0) = i \delta(t) \delta^3(\vec{x} - \vec{x}_0)$$

where G gives the amplitude for the (reduced) particle to be created at point $\vec{x}_0$ at time 0 and to be found at point $\vec{x}$ at time t .

Its Fourier transform, assuming that G vanishes for negative t satisfies

$$(E + i\epsilon - H)R(E, \vec{x}, \vec{x}_0) = \delta^3(\vec{x} - \vec{x}_0),$$

where R (the Resolvent) is the time Fourier transform of G .

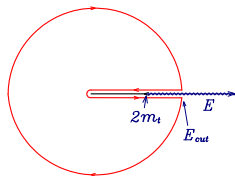
The imaginary part of the forward resolvent is the total cross section (by the optical theorem), that in our case is equal to the differential cross section, since near threshold only the s wave contribution counts.

The resolvent has the operator expression

$$R = \frac{1}{E - H}$$

that makes it clear that R is analytic in the complex plane except for points on the spectrum of H .

By analyticity its integral from the threshold up to a cut E_{cut} is equal to a **contour integral on a circle of radius E_{cut}** . This should also account for contributions arising from bound states below the $2m$ threshold.



The Resolvent

$$R(E) = \frac{1}{E - H},$$

is defined for complex E . Since we know the spectrum, we can write

$$R(E, x_1, x_2) = \frac{\psi_0(x_1) \psi_0(x_2)}{E - E_0} + \left[\sum_k = \frac{L}{2\pi} \int dk \right] \frac{\psi_k(x_1) \psi_k(x_2)}{E - E_k},$$

and obtain immediately the forward resolvent

$$R(E, 0, 0) = \theta(\lambda) \frac{k_0}{E - E_0} + \frac{1}{\pi} \int dk \frac{1}{1 + \xi_k^2} \frac{1}{E - E_k}.$$

We have $\text{Im}(R(E, 0, 0)) = 2\pi\rho(E)$, where ρ is the spectral density

$$\begin{aligned}\rho(E) &= \theta(\lambda)k_0\delta(E_0 - E) + \frac{1}{\pi} \int dk \frac{1}{1 + \xi_k^2} \delta(E_k - E) \\ &= \theta(\lambda)k_0\delta(E - E_0) + \frac{1}{\pi} \frac{m}{k_E} \frac{1}{1 + \xi_k^2}\end{aligned}$$

that is proportional to the cross section for producing a final state of energy E , and $k_E = \sqrt{2mE^2}$.

Expanding in powers of λ for $E > 0$ we obtain

$$\rho(E) = \frac{1}{\pi} \frac{m}{k_E} - \frac{\lambda^2}{\pi} \left(\frac{m}{k_E} \right)^3 + \dots$$

showing the well-known λ/v singularities.

This naive perturbative expansion of ρ is not integrable in dE at threshold ... (and furthermore it is blind to bound states)

Using directly perturbation theory:

$$\begin{aligned}R(E) &= \frac{1}{H_0 + V - E} = \frac{1}{H_0 - E} - \frac{1}{H_0 - E} V \frac{1}{H_0 - E} + \dots \\R(E, 0, 0) &= \sum_k \frac{|\psi_k^{(0)}(0)|^2}{E_k - E} - \lambda \sum_k \sum_{k'} \frac{|\psi_k^{(0)}(0)|^2 |\psi_{k'}^{(0)}(0)|^2}{(E_k - E)(E_{k'} - E)} \\&= \sum_k \frac{|\psi_k^{(0)}(0)|^2}{E_k - E} - \lambda \left(\sum_k \frac{|\psi_k^{(0)}(0)|^2}{(E_k - E)} \right)^2 \\&= \sqrt{-\frac{m}{2E}} + \lambda \frac{m}{2E}\end{aligned}$$

Its imaginary part above the real axis is

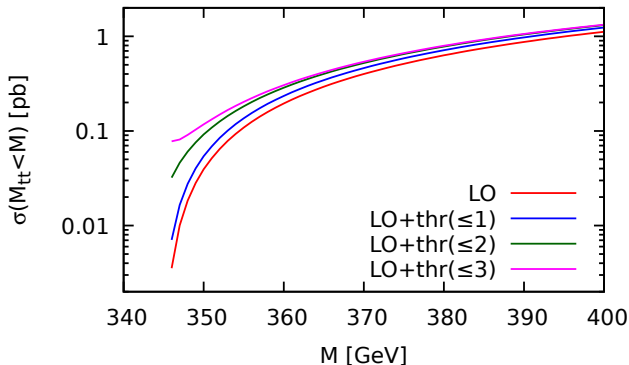
$$R(E + i\epsilon, 0, 0) = i\theta(E)\sqrt{\frac{m}{2E}} + i\pi\frac{\lambda m}{2}\delta(E)$$

leading to

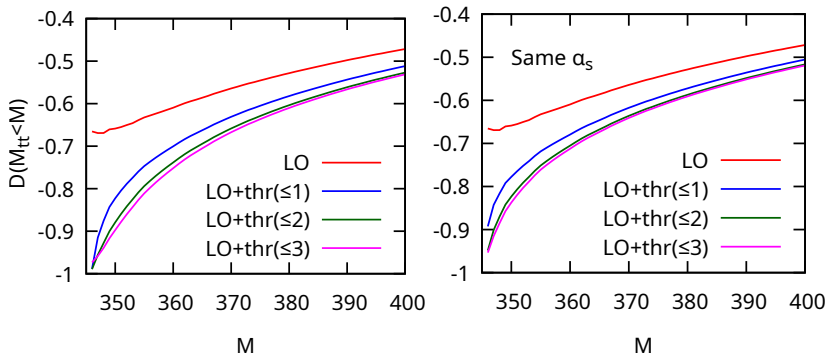
$$\begin{aligned}\rho(E) &= \theta(E)\frac{m}{\pi k} + \frac{\lambda m}{2}\delta(E). \\ \int_{-\infty}^E dE' \rho(E') &= \frac{k_E}{\pi} + \frac{\lambda m}{2}\end{aligned}$$

We consider the $t\bar{t}$ production cross section as a function of a cut M on the invariant mass of the pair. To begin with, we show the results obtained at the Born level, when including the α_s/v term, when including both the α_s/v and $(\alpha_s/v)^2$ term, and when including also the δ term.

Care should be used in the choice of the scale at which α_s is evaluated. The Born term **always includes to powers of α_s evaluated at a hard scale of order m** . The powers of α_s associated with the threshold corrections should instead be evaluated at a **scale of the order of the top momentum in the $t\bar{t}$ rest frame when $m_{t\bar{t}} = M$** , according to the analyticity argument presented earlier.



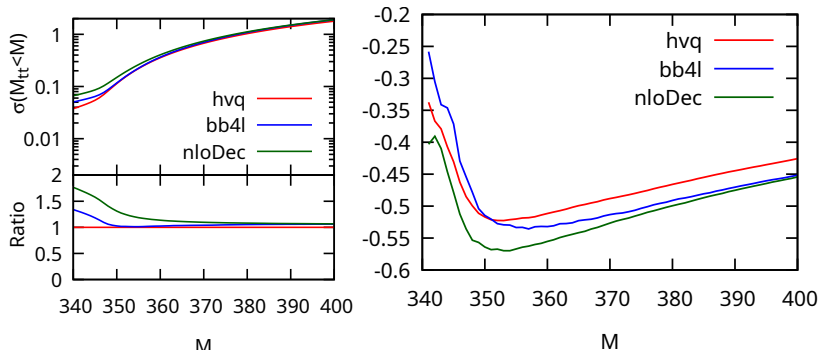
The cross section as a function of the mass cut shows the strong enhancement when we approach the threshold, that increases with the order of the included threshold corrections. As we approach realistic cuts (i.e. 380 GeV) the inclusion of the third order term gives a tiny contribution.



We see a large enhancement of the correlation (the D parameter turning more negative) as we add threshold corrections up to first, second and third order.

On the right plot, the coupling in the threshold correction has been kept the same as in the hard process, which is what happens in typical NLO+PS and NNLO+PS generator. We see that this choice underestimates the threshold corrections.

Nominal Monte Carlo results



Prediction for the cross section and for the D observable as a function of an invariant mass cut. The generators are:

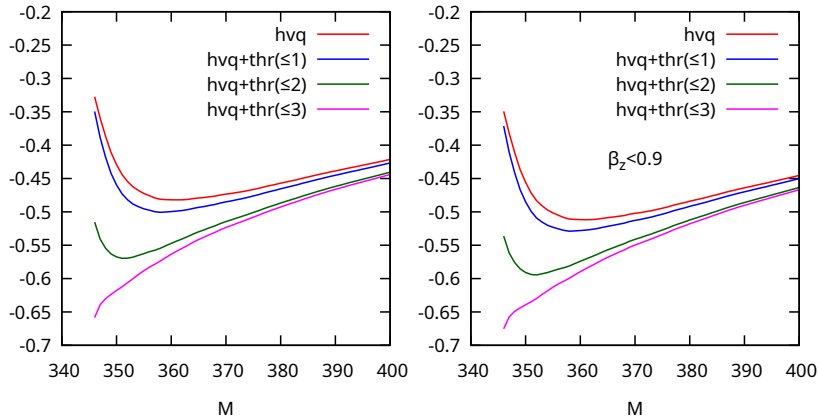
- POWHEG-hvq [Frixione, Ridolfi, P.N., 2007](#)
- POWHEG-ttb_NLO_dec, [Campbell, Ellis, Re, PN](#)
- POWHEG-b_bbar_4l [Ježo, Lindert, Oleari, Pozzorini, P.N., 2016](#)

- ▶ We find important differences depending upon which generator we are using.
- ▶ The generator differs in the way they implement spin correlation in decays, with **h_vq being the least accurate and bb4l the most accurate**
- ▶ h_vq uses its own implementation of the algorithm of [Frixione etal, 2007](#) (also implemented in the MadSpin code [Artoisenet etal,2013](#)).
- ▶ Using instead h_vq+MadSpin the “*D*” plot is very close to the ttbNLOdec one.
- ▶ There is a strong temptation to privilege the bb4l code. However, we have been unable to pinpoint the causes of the differences, so that more studies are needed in this direction.

PREDICTIONS

- ▶ Our best prediction is obtained using an NLO+PS generator (hvv or bb4l), and adding to it the threshold corrections that are not already present at the NLO level.
- ▶ The second and third order corrections can be added without problems.
- ▶ The first order correction can be added as long as we subtract what the NLO calculation already provides, i.e. a term $\sigma_I(-a_I/v)$, but with the coupling a_I evaluated at the hard scale of the generator.

Predictions



- ▶ On the right plot: β_Z is the longitudinal velocity of the $t\bar{t}$ system (cut used by CMS to reduce the $q\bar{q}$ contribution).
- ▶ The first order correction somewhat reduced because of the overcounting subtraction.
- ▶ First and second order terms much larger than the third order term, which includes bound state effects.