RadISH with TMD non-perturbative effect

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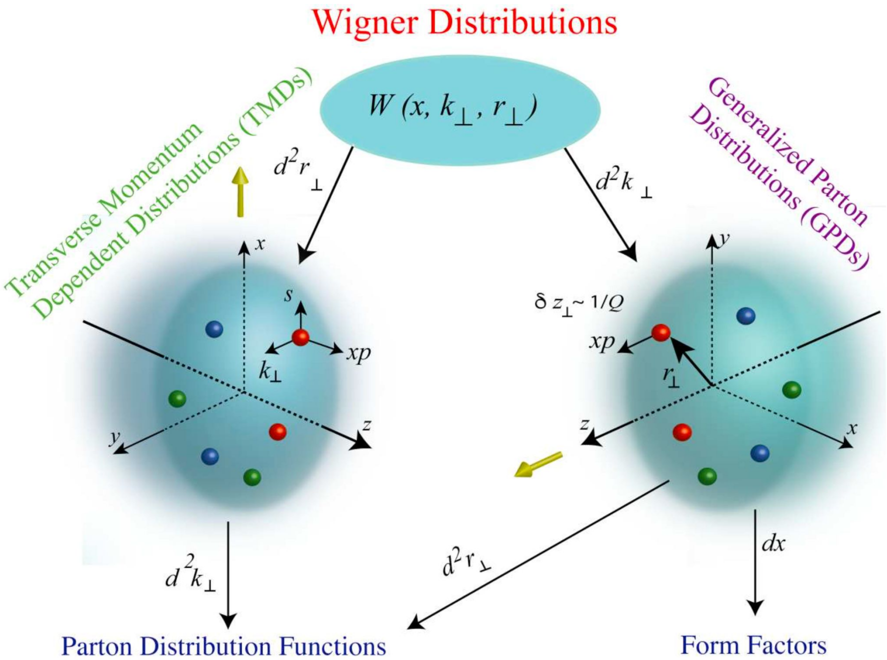
"ALDO PONTREMOLI"

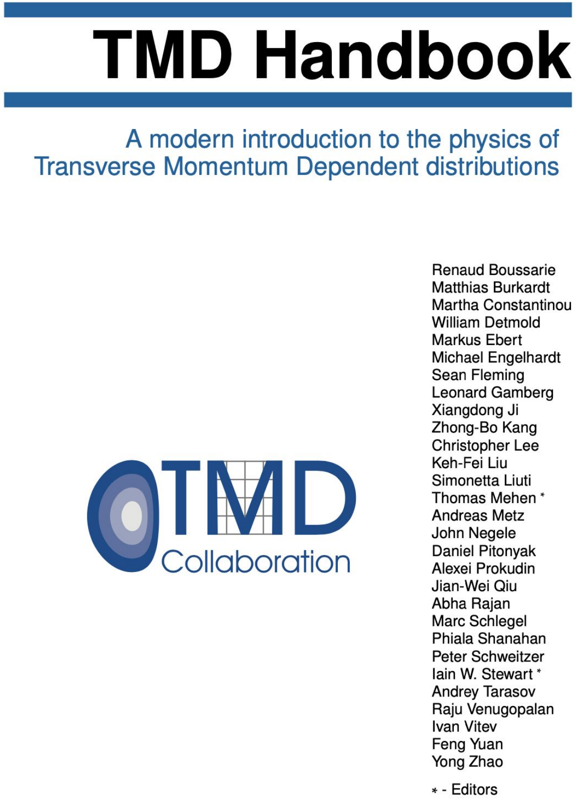
INFN

REF, Oct 14, 2025

Structures in the transverse direction

- TMD (transverse momentum dependent) distributions
- GPDs (generalized parton distributions)




TMD Handbook

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration

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Martha Constantinou
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* - Editors

TMD sensitive processes

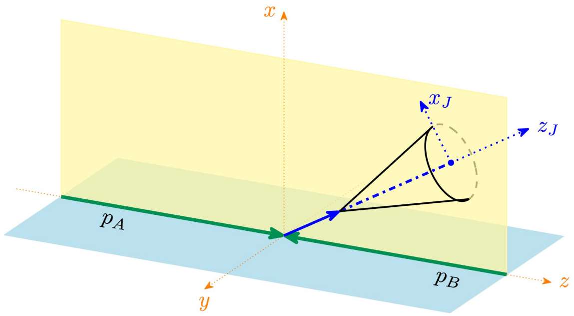
Quark sensitive:

- Drell-Yan
- Semi-inclusive deep inelastic scattering (SIDIS)
- e^+e^- annihilation

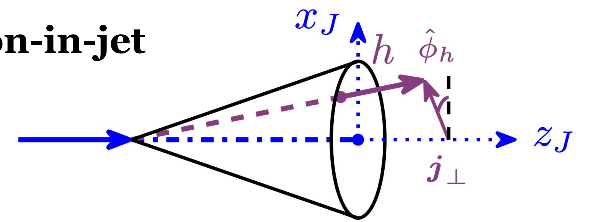
Gluon sensitive:

- Higgs production in hadronic collisions
- Quarkonium production (*e.g.*, η_c, η_b) in hadronic collisions

jets

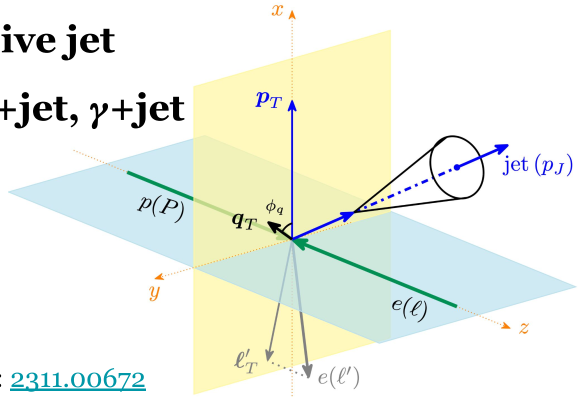


hadron-in-jet



exclusive jet

***e.g.*, *e*+jet, *γ*+jet**



Kang, Xing, Zhao & Zhou: [2311.00672](2311.00672)

TMD sensitive processes

Quark sensitive:

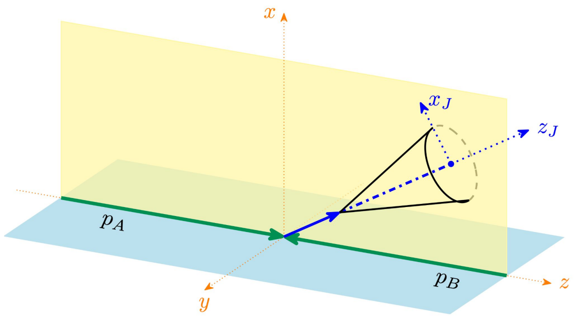
- Drell-Yan
- Semi-inclusive deep inelastic scattering (SIDIS)
- e^+e^- annihilation

Global fits (unpolarized)

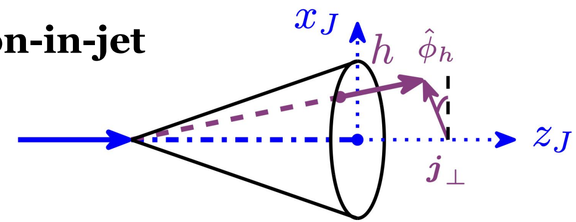
Gluon sensitive:

- Higgs production in hadronic collisions
- Quarkonium production (*e.g.*, η_c, η_b) in hadronic collisions

jets



hadron-in-jet



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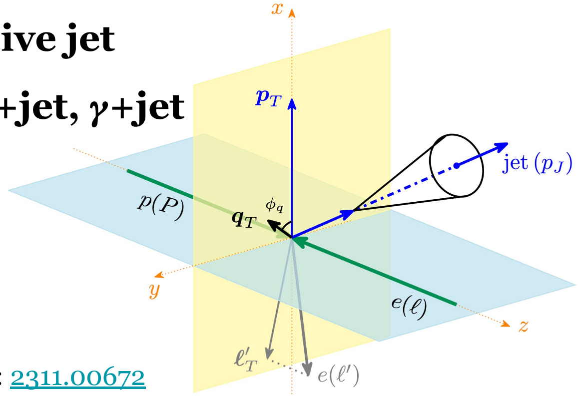


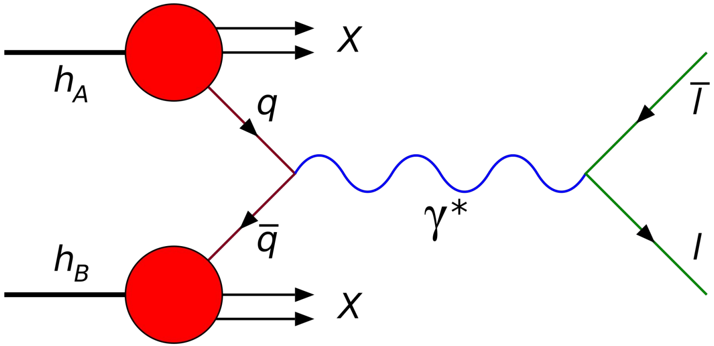
Kang, Xing, Zhao & Zhou: 2311.00672

TMD factorization (Drell-Yan)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_T^2} = \sum_q H_{q\bar{q}}^{\mathrm{DY}}(Q,\mu)\tilde{f}_{1,q}\left(x_a, \boldsymbol{k}_{a,T}, \mu, \frac{\zeta}{\nu^2}\right)\tilde{f}_{1,\bar{q}}\left(x_b, \boldsymbol{k}_{b,T}, \mu, \frac{\zeta}{\nu^2}\right)\delta^{(2)}\left(\boldsymbol{q}_T - \boldsymbol{k}_{a,T} - \boldsymbol{k}_{b,T}\right)$$

- Observed transverse momentum $\boldsymbol{q}_T$: sensitive to parton transverse momentum
- Parton transverse momentum:
  - radiative (perturbative)
  - intrinsic (non-perturbative, NP)
- Renormalization: regulates divergences

$$\boxed{pp \to \gamma/Z \to \ell + \bar{\ell} + X}$$



Wikipedia

QCD evolution of TMD PDFs

$$F_i\left(x,b,\mu,\zeta\right) = F_i\left(x,b,\mu_0,\zeta_0\right)$$

TMD distribution at initial scale

$$\times \exp\left(\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \boxed{\gamma_F\left(\alpha_s\left(\mu'\right), \frac{\zeta}{\mu'^2}\right)}\right)$$

evolution in μ

Calculable in pQCD

$$\times \left(\frac{\zeta}{\zeta_0}\right)^{-\boxed{D(b\mu_0,\alpha_s(\mu_0))}+\boxed{g_K(b,\lambda)}}$$

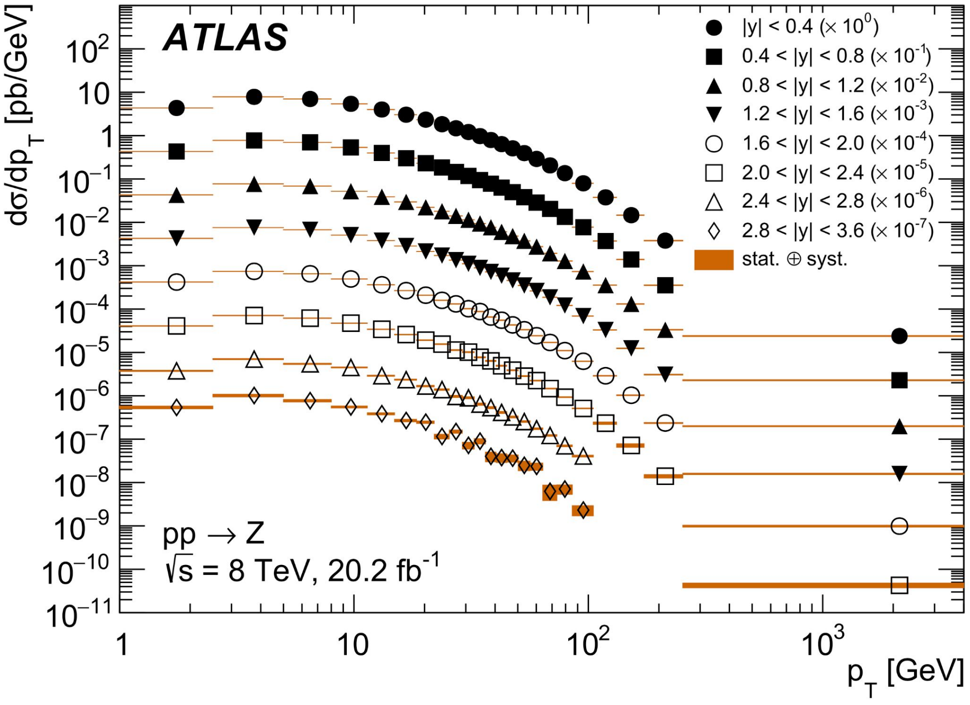
evolution in ζ

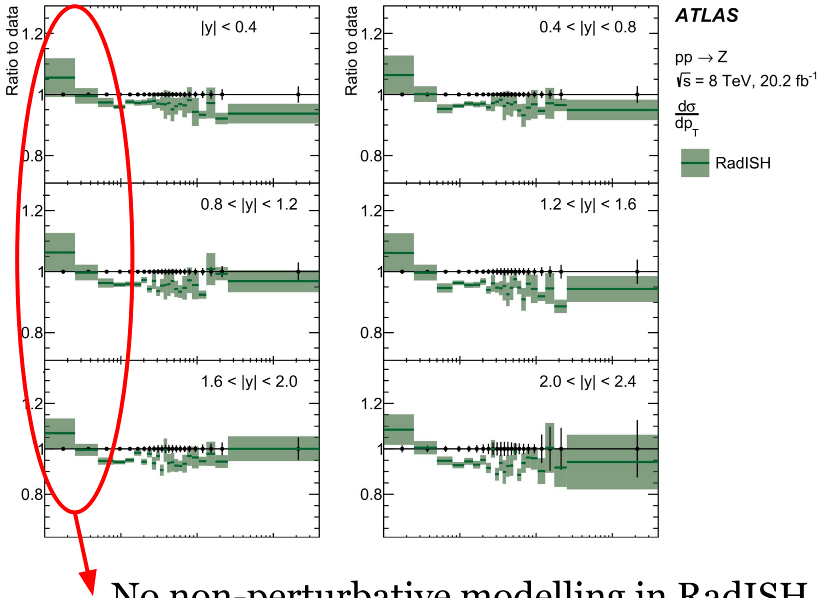
Non-perturbative correction at large b

See also: book of J.C., 2003.07453 and many other references

$$F_i\left(x,b,\mu_0,\zeta_0\right) = \sum_j \boxed{C_{i\leftarrow j}\left(x,b,\mu_0\right)} \otimes f_{j/p}\left(x,\mu_0\right) \boxed{F_{\mathrm{NP}}\left(b,\lambda\right)}$$

High precision DY measurement at ATLAS





No non-perturbative modelling in RadISH

$$\times \left(\frac{\zeta}{\zeta_0}\right)^{-D(b\mu_0,\alpha_s(\mu_0))+\boxed{g_K(b,\lambda)}}$$

ATLAS: 2309.09318

RadISH in a nutshell

- RadISH performs resummation in direct space, with control on formal accuracy
- Formula can be evaluated with Monte Carlo methods
- Dependence on ϵ vanishes
- Result formally equivalent to b-space formulation (1705.09127)

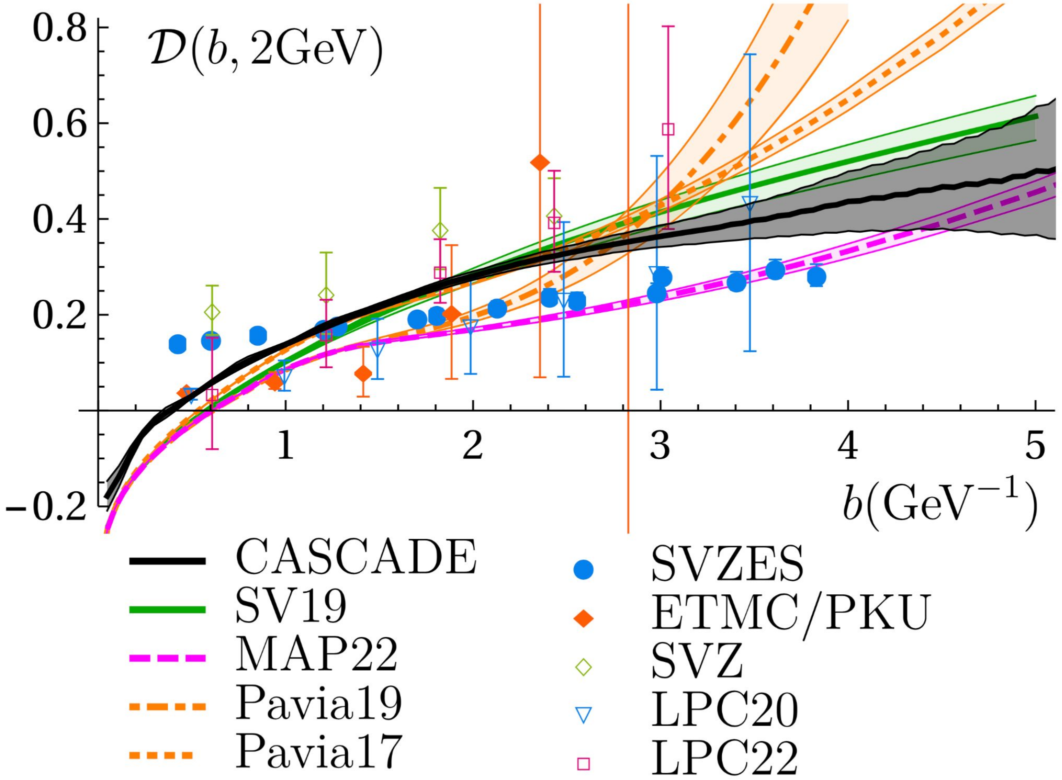
$$\nu_i \equiv \frac{k_{t,i}}{M}, \qquad \zeta_i \equiv \frac{\nu_i}{\nu_1}$$

$$\Sigma(v) = \sigma^{(0)} \int \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon k_{t1})} \sum_{\ell_1=1,2} R'_{\ell_1}(k_{t1}) \times$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1,2} R'_{\ell_i}(\zeta_i k_{t1})\, \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

Behaviour of NP kernels

Collins-Soper (CS) kernel fitted and calculated on the lattice

- Fits parameterized in b-space
- Large-b behaviour is log-like



Martinez & Vladimirov: 2206.01105

Implement NP kernel in k_T space

With inspiration from hep-ph/9504219, we devise the modelling for the NP kernel:

Separates pert. and NP regions

Coupling at k = 0, to be determined via fits

$$R_{\mathrm{NP}} = \int_0^{\boxed{\mu_I}} \frac{\mathrm{d}k}{k}[1 - J_0(bk)] \left[\frac{A_1}{2\pi}\boxed{\alpha_s^{\mathrm{s}}(0)}\ln(M/k) + \frac{B_1}{2\pi}\boxed{\alpha_s^{\mathrm{c}}(0)}\right]$$

$$- \int_0^{\mu_I} \frac{\mathrm{d}k}{k}[1 - J_0(bk)] \left[\frac{A_1}{2\pi}\boxed{\alpha_s^{\mathrm{s,pert}}(k)}\ln(M/k) + \frac{B_1}{2\pi}\boxed{\alpha_s^{\mathrm{c,pert}}(k)}\right]$$

Bessel function can imply NP behaviour at large *b*

Effective coupling of soft origin

Effective coupling of hard-collinear origin

Implement NP kernel in k_T space

Demand the NP part to contribution only below μ_I, and cancel its perturbative part with R':

$$R'(k) = \boxed{\Theta(M-k)\Theta(k-\mu_I)} \left[\frac{A_1}{2\pi}\alpha_s^{\mathrm{s,pert}}(k)\ln(M/k) + \frac{B_1}{2\pi}\alpha_s^{\mathrm{c,pert}}(k)\right]$$

R' contributes only at $k > \mu_I$

R_{NP}' contributes at $k < \mu_I$

$$R'_{\mathrm{NP}}(k) = \boxed{\Theta(\mu_I - k)} \left[\frac{A_1}{2\pi}\alpha_s^{\mathrm{s}}(0)\ln(M/k) + \frac{B_1}{2\pi}\alpha_s^{\mathrm{c}}(0)\right]$$

$$R_{\mathrm{NP}} = \int_0^M \frac{\mathrm{d}k}{k}[1 - J_0(bk)]R'_{\mathrm{NP}}$$

Match with NP kernel in literature

$$R_{\mathrm{NP}}^{(1)} = \frac{b^2\mu_I^2}{8}\left[\boxed{\frac{A_1}{2\pi}\alpha_s^{\mathrm{s}}(0)\ln(M/\mu_I)} + \boxed{\frac{A_1}{2\pi}\alpha_s^{\mathrm{s}}(0)\frac{1}{2}} + \boxed{\frac{B_1}{2\pi}\alpha_s^{\mathrm{c}}(0)}\right]$$

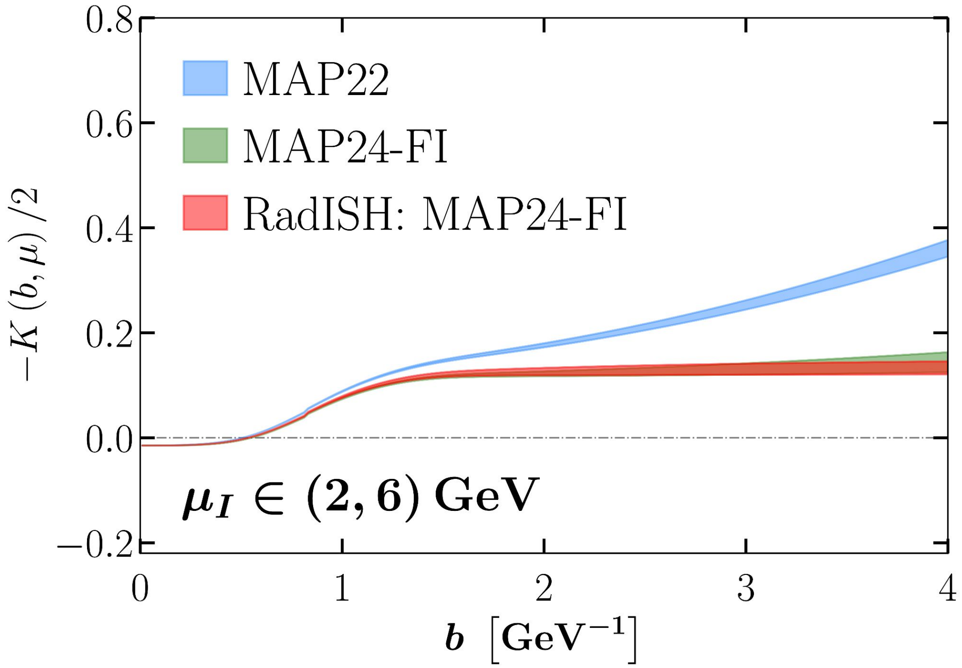
$$R_{\mathrm{NP}}^{\mathrm{lit}} = \frac{1}{2}b^2 g_2^2 \ln(M/Q_0) = \boxed{\frac{1}{2}b^2 g_2^2 \ln(M/\mu_I)} + \boxed{\frac{1}{2}b^2 g_2^2 \ln(\mu_I/Q_0)}$$

MAP: 2206.07598

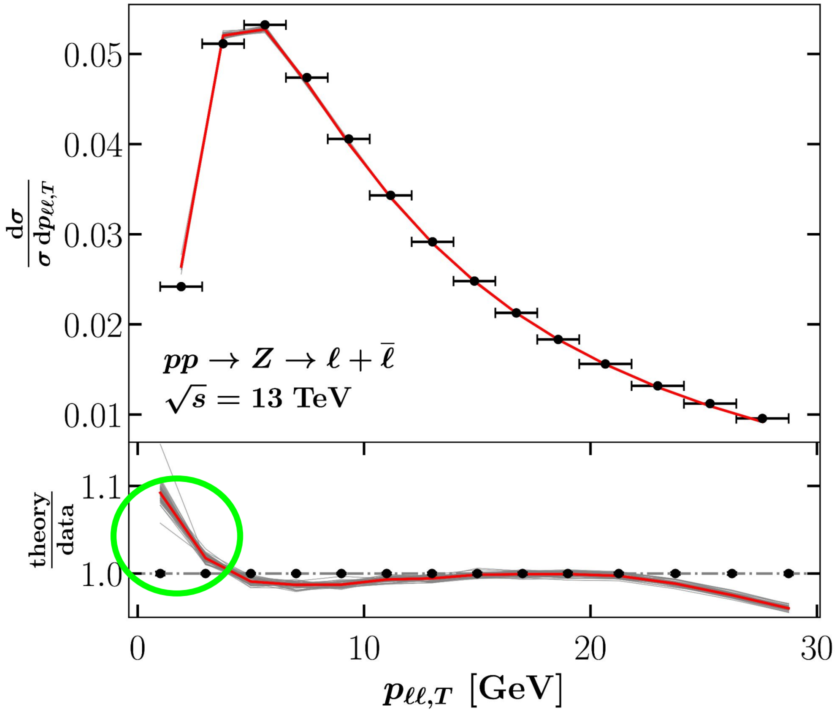
Implement NP kernel in k_T space

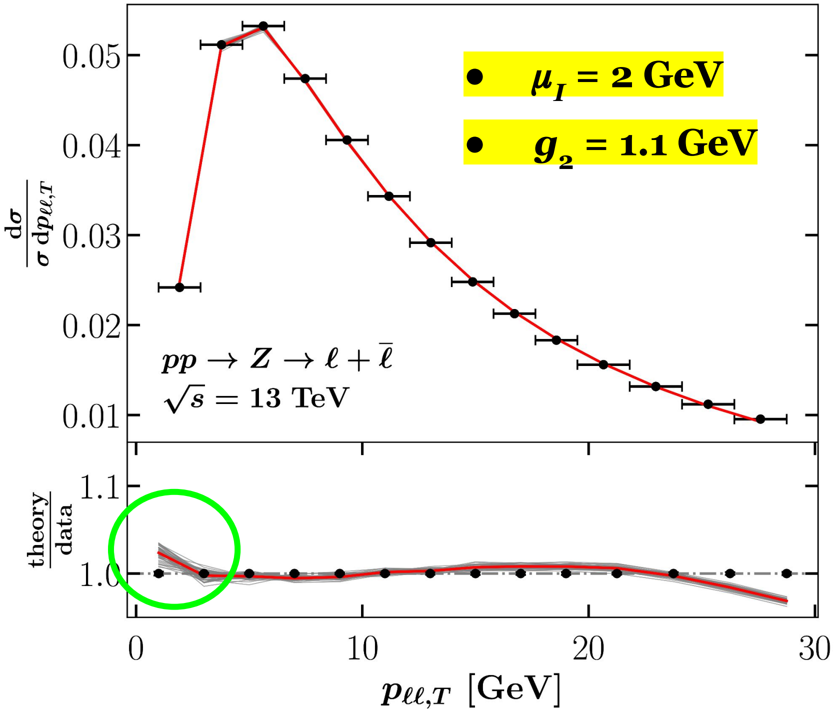
$$R_{\rm NP}^{(1)} = \frac{b^2\mu_I^2}{8}\left[\frac{A_1}{2\pi}\alpha_s^{\rm s}(0)\ln(M/\mu_I) + \frac{A_1}{2\pi}\alpha_s^{\rm s}(0)\frac{1}{2} + \frac{B_1}{2\pi}\alpha_s^{\rm c}(0)\right]$$

- Excellent match is seen between the CS kernels from MAP and RadISH implementation
- In practice μ_I and g_2 can be adjusted
- g_2 value taken from MAP [2405.13833]()



Phenomenology at 13 TeV

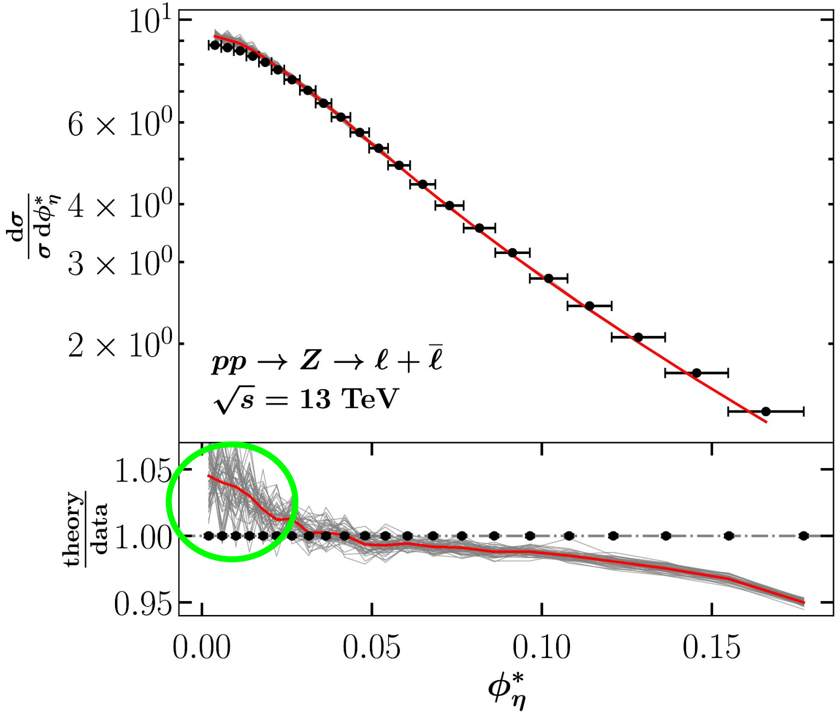


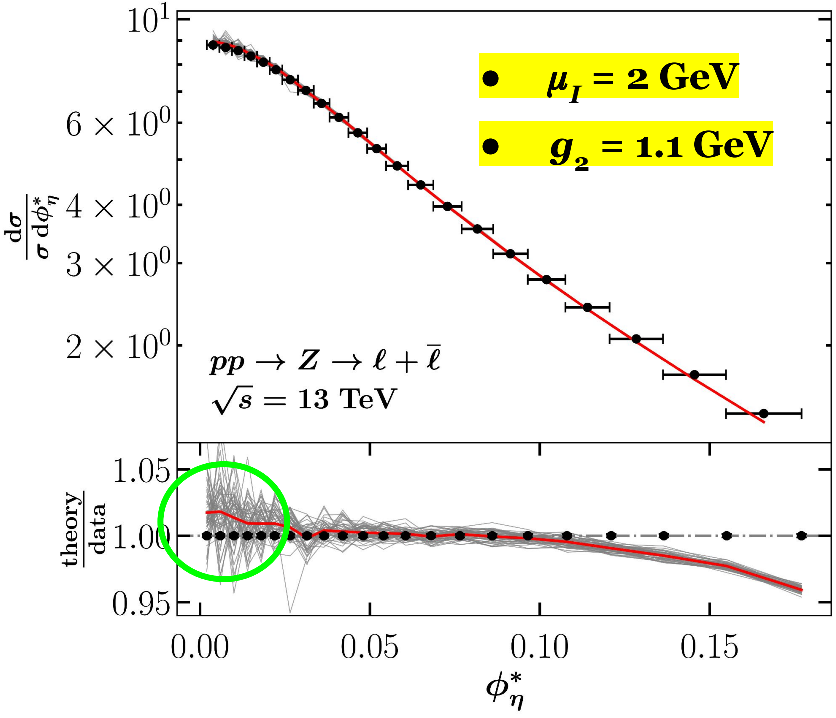


Data from 1912.02844

~7% improvement!

Phenomenology at 13 TeV



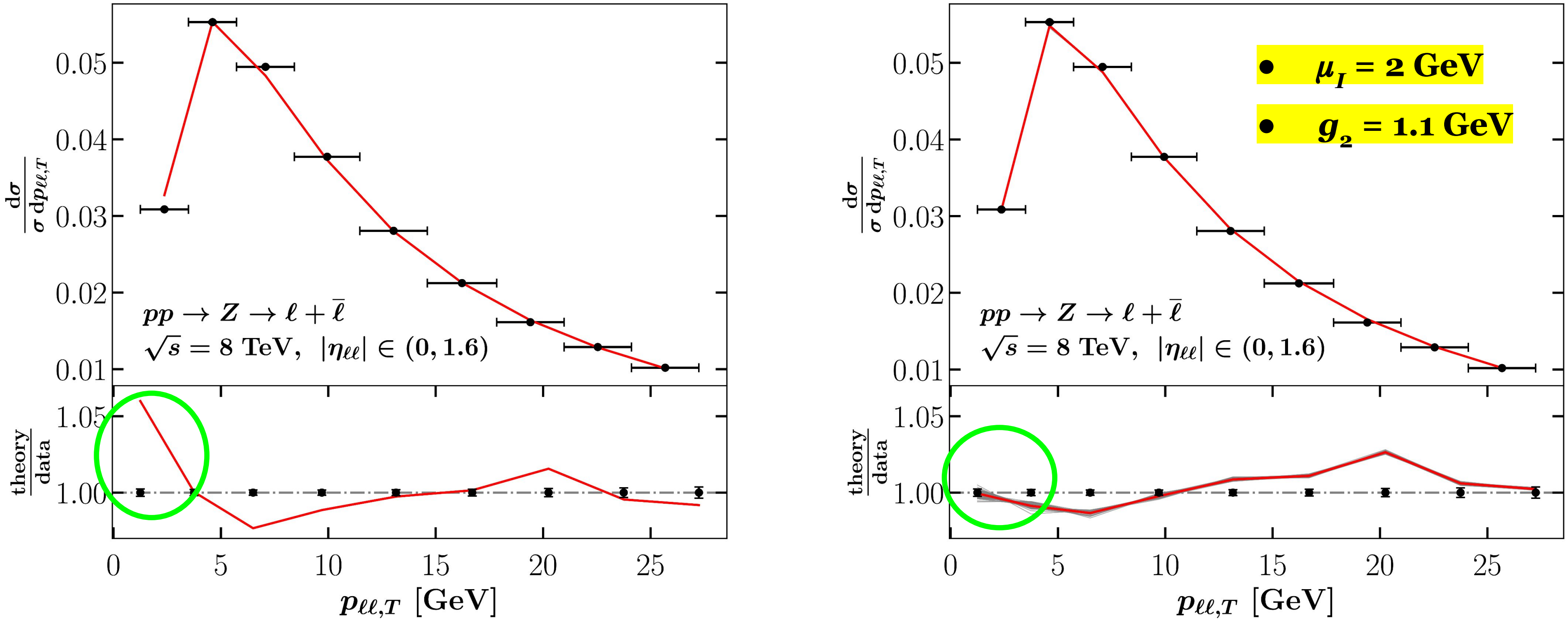


Data from [1912.02844]

~2% improvement!

$$\phi^*_\eta \equiv \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\left(\theta^*_\eta\right)$$

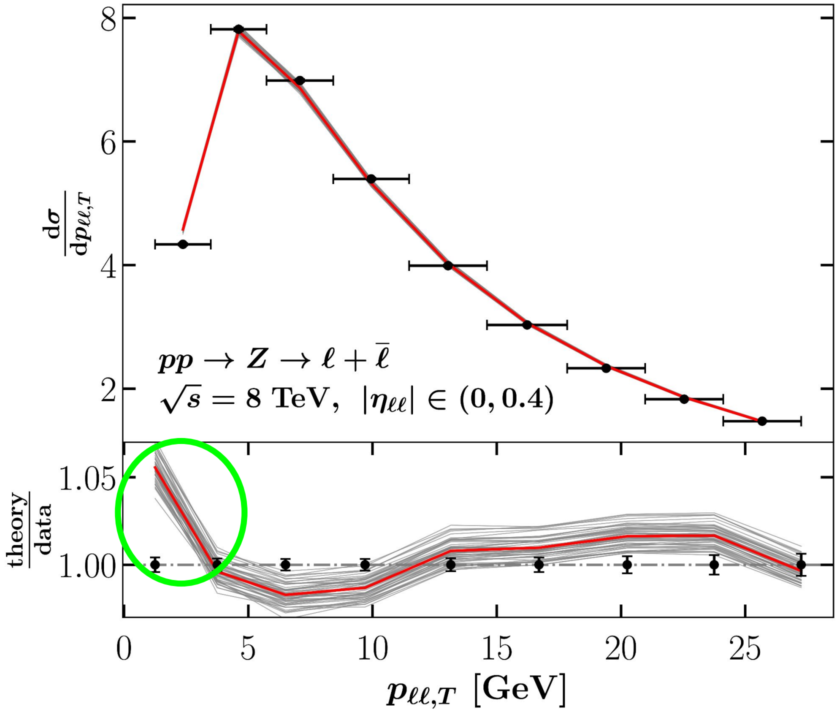
Phenomenology at 8 TeV

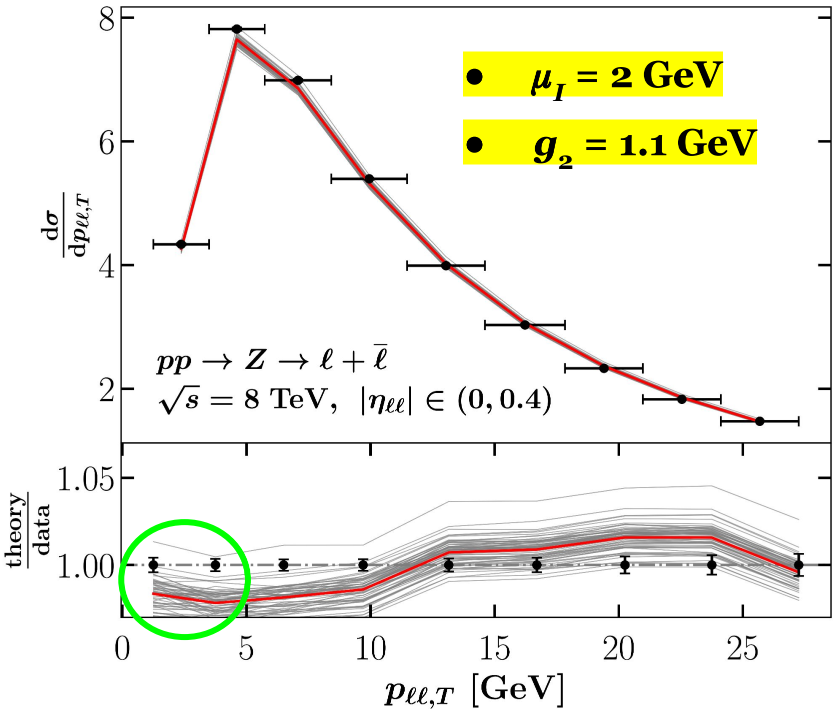


Data from 2309.09318

~5% improvement!

Phenomenology at 8 TeV





Data from 2309.09318

~5% improvement!

Summary and outlook

- We have implemented non-perturbative (NP) CS kernel in RadISH calculation
  - Improved description is achieved across different collision energies and observables with the same set of parameters
- We will further include scale variance
- We will study the impact of NP effect in extraction of physical constants: W mass
- We need to include of intrinsic transverse momentum effects

Thank you for your attention!