

# Collinearly improved small- $x$ resummation of photon impact factors and $\gamma^*\gamma^*$ high energy scattering

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GA 824093

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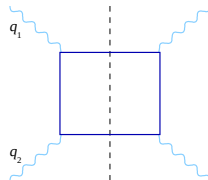
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# Outline

- BFKL approach to  $\gamma^* \gamma^*$  scattering
  - High-energy factorization formula
  - BFKL kernel and gluon Green's function
  - Impact factors
- Renormalization group improved BFKL approach
- Collinear (DGLAP) resummation of kernel and impact factors
- Results for  $\gamma^*$ -**impact factors** and **cross section**
- Outlook

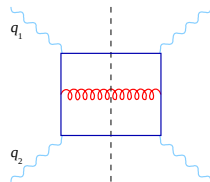
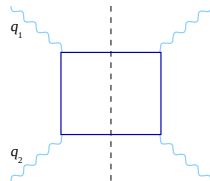
# $\gamma^* \gamma^*$ at lowest orders

- At LO, interaction is mediated by fermion lines (quark box + cross)
- Spin  $j = 1/2$  exchange  
 $\implies \sigma_0^{\text{box}} \sim s^{2(j-1)} = 1/s$
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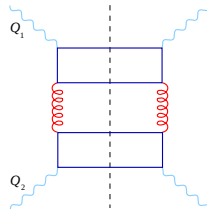


Perturbative corrections to this diagram provide logarithmic corrections

$$\sigma = \sigma_0^{\text{box}} [1 + \alpha_s \log s + (\alpha_s \log s)^2 + \dots]$$

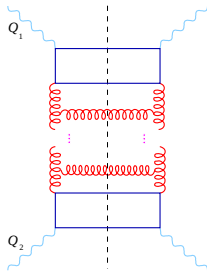
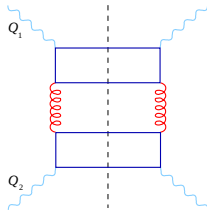
# BFKL approach to $\gamma^* \gamma^*$ scattering

- A constant cross section is obtained by exchanging a gluon, which couples to photons via quark lines.
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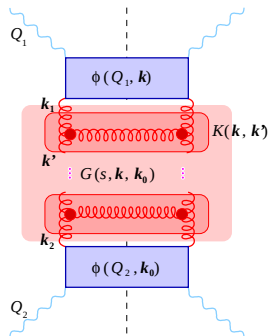
Perturbative corrections to this diagram provide logarithmic terms

$$\sigma \simeq \alpha_s^2 \sum_{n=0} c_n [\alpha_s \log(s)]^n$$

$\alpha_s \log s \sim 1 \implies$  all-order resummation

# BFKL approach for $\gamma^* \gamma^*$

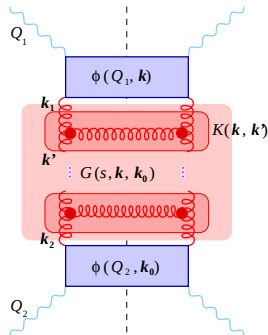
- The PT series of the leading  $\log s$  can be organized as the sum of **effective ladder-like diagrams** with gluon exchanges in the  $t$ -channel and real gluon emissions with strongly ordered rapidities.
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- **each rung** is described by the so-called **BFKL kernel**  $K(k_1, k')$ .
- Due to the extreme Lorentz contraction, the dynamics is essentially transverse
- the GGF (the blob with 4 gluon legs) obeys the BFKL equation

$$\frac{\partial}{\partial \log s} G(s, \mathbf{k}_1, \mathbf{k}_2) = \delta^2(\mathbf{k}_1 - \mathbf{k}_2) + \alpha_s \int d^2 \mathbf{k}' K(\mathbf{k}_1, \mathbf{k}') G(s, \mathbf{k}', \mathbf{k}_2)$$



# BFKL approach for $\gamma^* \gamma^*$

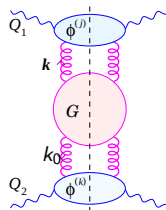
High-energy factorization formula ( $k$ -convolution)

→ diagonalization: Mellin variables  $s \leftrightarrow \omega$ ;  $Q^2, k^2 \leftrightarrow \gamma$

$\sigma(j, k)(s, Q_1, Q_2)$  ↗ photon polarizations

$$= \int d^2 k d^2 k_0 \phi^{(j)}(Q_1, k) G(s, k, k_0) \phi^{(k)}(Q_2, k_0)$$

$$= \frac{1}{Q_1 Q_2} \int \frac{d\omega}{2\pi i} \left( \frac{s}{Q_1 Q_2} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{Q_1^2}{Q_2^2} \right)^\gamma \phi^{(j)}(\gamma) G(\omega, \gamma) \phi^{(k)}(1-\gamma)$$



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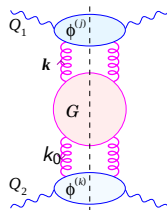
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BFKL equation and solution in LLA

$$\frac{\partial}{\partial \log s} G(s, k, k_0) = \delta^2(k - k_0) + \alpha_s \int d^2 k' K(k, k') G(s, k', k_0)$$

$$\omega \quad G(\omega, \gamma) \quad = \quad 1 \quad + \quad \alpha_s \quad \chi(\gamma) \quad G(\omega, \gamma) \quad \Rightarrow \quad G(\omega, \gamma) = \frac{1}{\omega - \alpha_s \chi(\gamma)}$$

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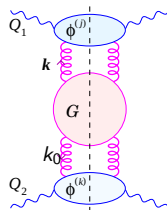
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$$\omega \quad G(\omega, \gamma) = 1 + \alpha_s \chi(\gamma) G(\omega, \gamma) \implies G(\omega, \gamma) = \frac{1}{\omega - \alpha_s \chi(\gamma)}$$

$$\chi = \chi_0 + \alpha_s \chi_1$$

$$\phi = \alpha_s \phi_0 + \alpha_s^2 \phi_1$$

are known at LO and NLO

[BFKL, Camici, Ciafaloni]

[Catani et al, Balitsky, Chirilli, Ivanov et al]

# BFKL approach for $\gamma^* \gamma^*$

LL kernel in Mellin space ( $\gamma \leftrightarrow k^2$ )

$$G(\omega, \gamma) = \frac{1}{\omega - \alpha_s \chi(\gamma)}$$
$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$$

Solution to the intercept (growth exponent for  $s \gg Q^2$ )

$$G(s, Q^2, Q^2) \sim s^{\omega_{\mathbb{P}}}, \quad \omega_{\mathbb{P}} = \alpha_s \chi(1/2) \simeq 2.7\alpha_s$$

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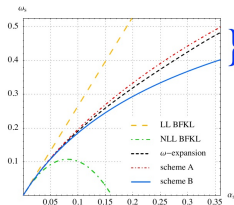
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- NLL correction s to BFKL are large and negative causing instability of the BFKL expansion
- Improvement needed [Ciafaloni,DC,Salam,Stasto]

$$\chi_0(\omega, \gamma) = 2\psi(1) - \psi(\gamma + \frac{\omega}{2}) - \psi(1 - \gamma + \frac{\omega}{2})$$

$$\omega_{\mathbb{P}} = \alpha_s \chi_0(\omega_{\mathbb{P}}, \gamma)$$

# Structure of BFKL kernel

$$K(\mathbf{k}, \mathbf{k}') = \frac{1}{k^2} \int \frac{d\gamma}{2\pi i} \left( \frac{k^2}{k'^2} \right)^\gamma \chi(\gamma)$$

$$\text{at } \mathcal{O}(\alpha_s^m) \quad \mathbf{k} \gg \mathbf{k}' \leftrightarrow \gamma \simeq 0, \quad K_m \sim \frac{1}{k^2} \log^m \frac{k^2}{k'^2} \leftrightarrow \chi_m \sim \frac{1}{\gamma^{1+m}}$$

$$\mathbf{k} \ll \mathbf{k}' \leftrightarrow \gamma \rightarrow 1$$

$$K_0(\mathbf{k}, \mathbf{k}') = \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} + \text{virt} \simeq \frac{\Theta(\mathbf{k} - \mathbf{k}')}{k^2} + \frac{\Theta(\mathbf{k}' - \mathbf{k})}{k'^2}$$

$$\chi_0(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

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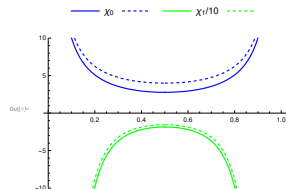
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$$\chi_0(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1 - \gamma}$$

$$\chi_1(\gamma) \simeq \frac{-1}{2\gamma^3} + \frac{A_1}{\gamma^2} + (\gamma \rightarrow 1 - \gamma)$$



# Change of energy scale and $\omega$ -shift

The presence of the cubic poles in  $\chi_1$  is due to a “wrong” choice of energy scale.  
Consider the Mellin representation of the GGF

$$G(s, \mathbf{k}_1, \mathbf{k}_2) = \frac{1}{\mathbf{k}_1 \mathbf{k}_2} \int \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right)^\gamma \frac{1}{\omega - \alpha_s \chi(\gamma)}$$

In collinear limit  $\mathbf{k}_1 \gg \mathbf{k}_2$  the “natural” energy scale  $s_0 = \mathbf{k}_1^2$ , because  $s/\mathbf{k}_1^2 \simeq 1/x_{Bj}$   
However, if we adopt the symmetric scale  $s'_0 = \mathbf{k}_1 \mathbf{k}_2$  we can rewrite

$$\left( \frac{s}{\mathbf{k}_1^2} \right)^\omega \left( \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right)^\gamma = \left( \frac{s}{\mathbf{k}_1 \mathbf{k}_2} \right)^\omega \left( \frac{\mathbf{k}_2}{\mathbf{k}_1} \right)^\omega \left( \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right)^\gamma = \left( \frac{s}{\mathbf{k}_1 \mathbf{k}_2} \right)^\omega \left( \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right)^{\gamma - \frac{\omega}{2}}$$

which amounts to an  $\omega$ -shift  $\gamma \rightarrow \gamma + \frac{\omega}{2}$  in the integrand and thus in the eig. func.  $\chi$ .

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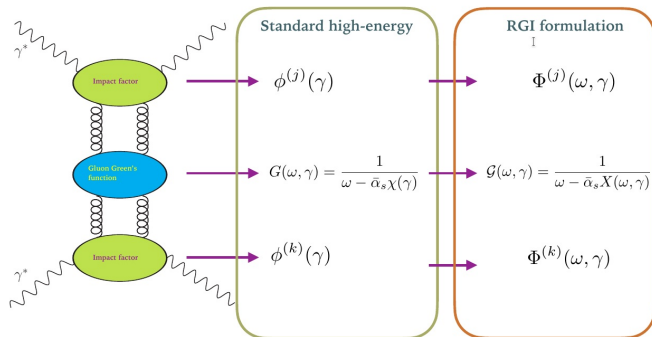
which amounts to an  $\omega$ -shift  $\gamma \rightarrow \gamma + \frac{\omega}{2}$  in the integrand and thus in the eig. func.  $\chi$ .  
The position of the  $\omega$ -pole in the factorization formula is given by

$$\omega = \alpha_s \chi(\gamma) = \mathcal{O}(\alpha_s)$$

In the collinear limit with  $s_0 = \mathbf{k}_1^2$  we have at LO  $\chi(\gamma) \simeq \frac{1}{\gamma} \implies \omega \simeq \alpha_s \frac{1}{\gamma}$   
With symmetric  $s_0 = \mathbf{k}_1 \mathbf{k}_2$  we find

$$\omega \simeq \alpha_s \frac{1}{\gamma + \frac{\omega}{2}} \simeq \alpha_s \left( \frac{1}{\gamma} - \frac{\omega}{2\gamma^2} + \mathcal{O}(\omega^2) \right) \simeq \alpha_s \left( \frac{1}{\gamma} - \frac{\alpha_s}{2\gamma^3} + \mathcal{O}(\alpha_s^2) \right)$$

# Renormalization group improvement



Consistency conditions at  $\omega = 0$       $\chi_1(\gamma) = X_1(0, \gamma) + \chi_0(\gamma) \partial_\omega X_0(0, \gamma)$

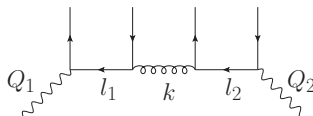
$$\begin{aligned} \phi_0^{(j)}(\gamma) \phi_1^{(k)}(1-\gamma) + \phi_1^{(j)}(\gamma) \phi_0^{(k)}(1-\gamma) &= \Phi_0^{(j)}(0, \gamma) \left[ \Phi_1^{(k)}(0, 1-\gamma) + \chi_0(1-\gamma) \partial_\omega \Phi_0^{(k)}(0, 1-\gamma) \right] \\ &\quad + \left[ \Phi_1^{(j)}(0, \gamma) + \chi_0(\gamma) \partial_\omega \Phi_0^{(j)}(0, \gamma) \right] \Phi_0^{(k)}(0, 1-\gamma) \\ &\quad + \Phi_0^{(j)}(0, \gamma) \Phi_0^{(k)}(0, 1-\gamma) \partial_\omega X_0(0, \gamma) \end{aligned}$$

# How to determine improved IF and cross section

- Compute cross section in (anti-)collinear limit, according to standard DGLAP evolution at leading  $\log Q_1^2/Q_2^2$ , so as to determine the  $\gamma$ -pole structure of the cross section in Mellin space
- Factorize such cross section in RGI impact factors and GGF. This determines their (anti-)collinear poles.
- Check: expand impact factors in  $\omega \sim \alpha_s$  so as to reproduce spurious and physical poles of standard BFKL impact factors
- Complete the RGI impact factor with the “regular” remainder of the BFKL ones
- Use RGI factorization formula to compute the cross section

# Transverse impact factor

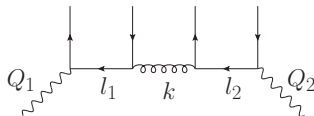
Consider LO cross section for transverse virtual photons with one gluon exchange



$$\begin{aligned}
 \sigma_0^{(TT)}(s, Q_1 \gg Q_2) &\simeq \frac{4\pi\alpha^2}{Q_1^2} F_T(x_{Bj}, Q_1^2; Q_2^2), \quad x_{Bj} \simeq Q_1^2/s \\
 &= \int_{x_{Bj}}^1 \frac{dz_1}{z_1} \frac{dz_k}{z_k} \frac{dz_2}{z_2} \Theta(z_1 < z_k < z_2) \delta\left(1 - \frac{x_{Bj}}{z_1}\right) P_{qg}\left(\frac{z_1}{z_k}\right) P_{gq}\left(\frac{z_k}{z_2}\right) P_{q\gamma}(z_2) \\
 &\times \int_{Q_2^2}^{Q_1^2} \frac{dl_1^2}{l_1^2} \frac{dk^2}{k^2} \frac{dl_2^2}{l_2^2} \Theta(l_2^2 < k^2 < l_1^2) \alpha \left( \sum_q e_q^2 \right) \frac{\alpha_s(l_1^2)}{2\pi} \frac{\alpha_s(k^2)}{2\pi} \frac{\alpha}{2\pi} \left( \sum_q e_q^2 \right)
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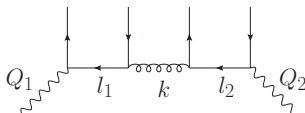
$$\begin{aligned}\sigma_0^{(TT)}(s, Q_1 \gg Q_2) &\simeq \frac{4\pi\alpha^2}{Q_1^2} F_T(x_{Bj}, Q_1^2; Q_2^2), \quad x_{Bj} \simeq Q_1^2/s \\ &= \int_{x_{Bj}}^1 \frac{dz_1}{z_1} \frac{dz_k}{z_k} \frac{dz_2}{z_2} \Theta(z_1 < z_k < z_2) \delta\left(1 - \frac{x_{Bj}}{z_1}\right) P_{qg}\left(\frac{z_1}{z_k}\right) P_{gq}\left(\frac{z_k}{z_2}\right) P_{q\gamma}(z_2) \\ &\times \int_{Q_2^2}^{Q_1^2} \frac{dl_1^2}{l_1^2} \frac{dk^2}{k^2} \frac{dl_2^2}{l_2^2} \Theta(l_2^2 < k^2 < l_1^2) \alpha \left( \sum_q e_q^2 \right) \frac{\alpha_s(l_1^2)}{2\pi} \frac{\alpha_s(k^2)}{2\pi} \frac{\alpha}{2\pi} \left( \sum_q e_q^2 \right)\end{aligned}$$

Mellin transform  $x_{Bj} \rightarrow \omega$ ,  $Q_1^2/Q_2^2 \rightarrow \gamma$  (with fixed coupling)

$$\sigma_0^{(TT)}(\omega, \gamma) \simeq \alpha \left( \sum_{q \in A} e_q^2 \right) \frac{1}{\gamma} \cdot \frac{\alpha_s}{2\pi} \frac{P_{qg}(\omega)}{\gamma} \cdot \frac{\alpha_s}{2\pi} \frac{P_{gq}(\omega)}{\gamma} \cdot \frac{\alpha}{2\pi} \left( \sum_{q \in B} e_q^2 \right) \frac{P_{q\gamma}(\omega)}{\gamma}$$

# Transverse impact factor

Use exact LO DGLAP anomalous dimensions



$$P_{qq}(\omega) = C_F \omega A_{qq}(\omega)$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega} \left[ 1 + \omega A_{gq}(\omega) \right]$$

$$P_{qg}(\omega) = \frac{2}{3} T_R \left[ 1 + \omega A_{qg}(\omega) \right]$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega} \left[ 1 + \omega A_{gg}(\omega) \right]$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R} P_{qg}(\omega)$$

$$A_{qq}(0) = \frac{5}{4} - \frac{\pi^2}{3}$$

$$A_{gq}(0) = -\frac{3}{4}$$

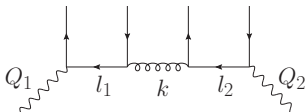
$$A_{qg}(0) = -\frac{13}{12}$$

$$A_{gg}(0) = -\frac{11}{6} + \bar{b}$$

$$\bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

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$$P_{q\gamma}(\omega) = \frac{N_c}{T_R} P_{qg}(\omega)$$

$$\bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

$$\sigma_0^{(TT)}(\omega, \gamma) = \Phi_0^{(T)} G_0 \Phi_0^{(T)}$$

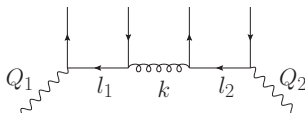
$$\simeq \left[ \alpha \alpha_s \left( \sum_q e_q^2 \right) 2P_{qg}(\omega) \sqrt{2(N_c^2 - 1)} \left( \frac{1}{(\gamma)^2} \right) \right]^2 \times \frac{1}{\omega}$$

Double poles of **BNP** impact factors with exact kinematics

$G_0$

# Transverse impact factor

Use exact LO DGLAP anomalous dimensions



$$P_{qq}(\omega) = C_F \omega A_{qq}(\omega)$$

$$A_{qq}(0) = \frac{5}{4} - \frac{\pi^2}{3}$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega} \left[ 1 + \omega A_{gq}(\omega) \right]$$

$$A_{gq}(0) = -\frac{3}{4}$$

$$P_{qg}(\omega) = \frac{2}{3} T_R \left[ 1 + \omega A_{qg}(\omega) \right]$$

$$A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega} \left[ 1 + \omega A_{gg}(\omega) \right]$$

$$A_{gg}(0) = -\frac{11}{6} + \bar{b}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R} P_{qg}(\omega)$$

$$\bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

$$\sigma_0^{(TT)}(\omega, \gamma) = \Phi_0^{(T)} G_0 \Phi_0^{(T)}$$

$$\simeq \left[ \alpha \alpha_s \left( \sum_q e_q^2 \right) 2P_{qg}(\omega) \sqrt{2(N_c^2 - 1)} \left( \frac{1}{(\gamma + \frac{\omega}{2})^2} + \frac{1}{(1 + \frac{\omega}{2} - \gamma)^2} \right) \right]^2 \times \frac{1}{\omega}$$

Double poles of **BNP impact factors with exact kinematics**

$G_0$

Possible choice of RGI:  $\Phi^{(T)}(\omega, \gamma) = \Phi^{(T, \text{BNP})}(\omega, \gamma)$

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Compatibility between BFKL and RGI factorization formula yields

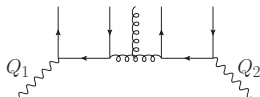
$$\phi_1(\gamma) + \phi_1(1-\gamma) = \Phi_1(0, \gamma) + \Phi_1(0, 1-\gamma) \quad \rightarrow \text{physical poles} \\ + \chi_0(\gamma) [\partial_\omega \Phi_0(0, \gamma) + \partial_\omega \Phi_0(0, 1-\gamma)] + \phi_0(\gamma) \partial_\omega \chi_0(0, \gamma)$$

For the highest  $\gamma \rightarrow 0$  poles (quartic in this case)

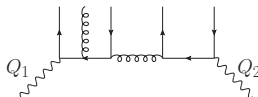
$$\phi_0(\gamma) \left[ -\frac{5}{2} \frac{1}{\gamma^2} \right] = \frac{1}{\gamma} \left[ \phi_0(\gamma) \frac{-1}{\gamma} \times 2 \right] + \phi_0(\gamma) \frac{-1}{2\gamma^2}$$

# Transverse impact factor at NLO

Collinear analysis at  $\mathcal{O}(\alpha_s^3)$  provides the remaining **physical cubic poles**



(a)

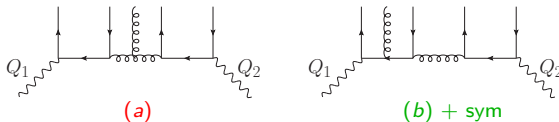


(b) + sym

$$\sigma_1^{(TT)}(\omega, \gamma) = \sigma_0^{(TT)}(\omega, \gamma) \times \left[ \frac{\alpha_s}{2\pi} \frac{P_{gg}}{\gamma} + 2 \frac{\alpha_s}{2\pi} \frac{P_{qq}}{\gamma} - \frac{\alpha_s b_0}{\gamma} + \mathcal{O}(\gamma^0) \right]$$

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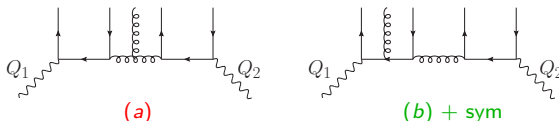
(a) Can attribute it to GGF

(b) Can attribute it to the impact factor

- Running coupling term: can attribute it to either or both

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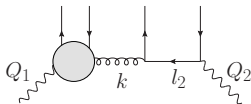
(b) **Can attribute it to the impact factor**

- Running coupling term: can attribute it to either or both
- Determine the pole structure of  $\Phi_1^{(T)}(\omega, \gamma)$  (reproducing cubic poles of  $\phi_1$ )
- Add “regular” part of  $\phi_1$  according to compatibility condition

$$\Phi_1(0, \gamma) = \frac{1}{2} [\phi_1(\gamma) + \phi_1(1 - \gamma) - \phi_0(\gamma) \partial_\omega X_0(0, \gamma) - \chi_0(\gamma) (\partial_\omega \Phi_0(0, \gamma) + \partial_\omega \Phi_0(0, 1 - \gamma))]$$

# Longitudinal impact factor

LO cross section for longitudinal ( $Q_1$ ) against transverse ( $Q_2$ ) virtual photons with one gluon exchange

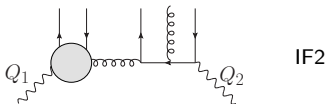
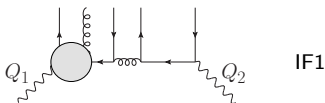
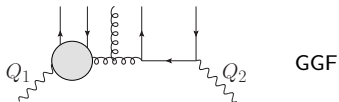


Collinear analysis reproduce simple poles of BNP impact factors with exact kinematics

$$\Phi^{(L)}(\omega, \gamma) = \Phi^{(L, \text{BNP})}$$

Note:  $\phi_0^{(L)} \sim 1/\gamma$  while  $\phi_0^{(T)} \sim 1/\gamma^2$

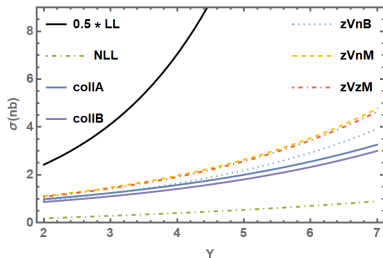
NLO cross section in collinear limit



Second diagram provides a  $C_F$  term which is absent in BFKL impact factor (stems from  $C_{L,q}$  coeff. function)

# Numerical results for $\gamma^* - \gamma^*$ cross section

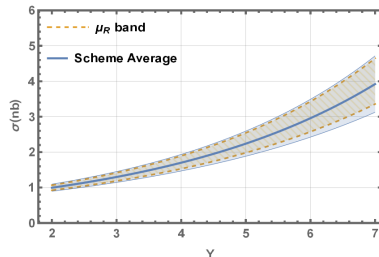
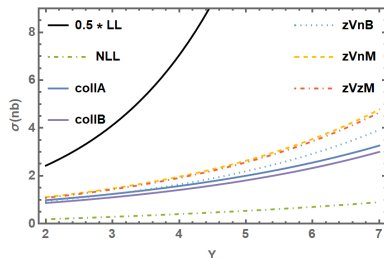
We compare BFKL results with RGI ones in various schemes for  $d\sigma/dY$   
 $Y = \log(s/Q^2)$ ,  $Q^2 = 17 \text{ GeV}^2$ ,  $N_f = 4$  active flavours



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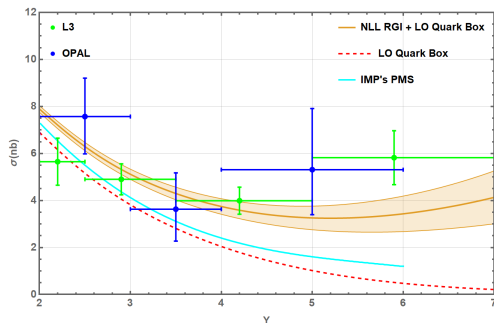
- Spread in resummation scheme and renormalization scale choice are of the same order

# Numerical results for $\gamma^* - \gamma^*$ cross section

We sum the “quark-box” and the RGI BFKL contributions

We compare with previous estimates [*Ivanov et al*]

and with LEP data: L3 ( $Q^2 = 16 \text{ GeV}^2$ ) and OPAL ( $Q^2 = 17.9 \text{ GeV}^2$ )



- Quark box dominates at small  $Y$ , BFKL with RGI dominates at large  $Y$
- RGI results provide a fair description of data, compatible within uncertainties

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- **NLO DGLAP improvement**
- **NLO quark box** could slightly modify the result at low  $Y$  though not the large energy behavior