A parton shower consistent with parton densities at LO and NLO: PDF2ISR

Resummation, Evolution & Factorisation workshop, Milan

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Plan for today

1. Introduction
2. Parton Branching method
3. PDF2ISR
4. Conclusion

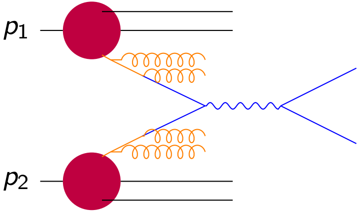
Introduction

Introduction

Collinear factorisation theorem successful for most observables

$$\sigma \propto f_1 \otimes f_2 \otimes \hat{\sigma}$$

If two scales scales are involved, e.g.: DY pT $\rightarrow$ $\alpha_s^n() \log^m(Q^2/p_T^2)$ need to be resummed



These can be resumed via TMDs, SCET or Parton Showers

Parton Showers

Parton showers make use of DGLAP equation to resum:

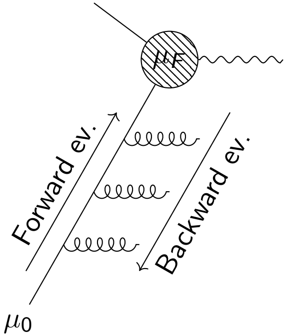
- In terms of real splitting functions and Sudakov form factor

$$\Delta_{ib} = \exp - \sum_b \int_{\mu_{i-1}}^{\mu} \frac{d^2\mu'}{\mu'} \int_{z_{min}}^{z_{max}} dz\ P_{ib}^R(z, \alpha_s(\mu^2))$$

- We can study exclusive individual effects of each emission

For the initial state shower $\rightarrow$ Backard evolution:

$$\Delta_{ib}^{BW} = \exp - \sum_b \int_{\mu_{i-1}}^{\mu} \frac{d^2\mu'}{\mu'} \int_{z_{min}}^{z_{max}} dz\ P_{ib}^R(z, \alpha_s(\mu^2)) \frac{f_i(x_i, \mu')}{f_b(x_b, \mu')}$$



Parton Showers

Most common parton showers: Herwig, Pythia, Sherpa... only resum at LL

- PanScales NLL resummation and NNLL in some observables

However, the relation between the PDF used in the calculation and the parton shower are overlooked:

- The emission phase space
- Ordering of the emissions
- QCD order

In this work we implement the Parton-Branching method into Pythia8:

- PB method allows for extraction of PDFs and TMDs

Parton Branching method

Forward evolution

$$\mu^2 \frac{\partial(xf_a(x,\mu))}{\partial\mu^2} = \sum_b \int_x^1 dz\ P_{ab}(z,\alpha_s)\ \frac{x}{z} f_b(\frac{x}{z},\mu^2)$$

To solve the DGLAP evolution equations iteratively, we rewrite the evolution equations in terms of the Sudakov form factor:

$$\Delta^s_{ab} = \exp - \sum_b \int_{\mu_0}^{\mu} \frac{d^2\mu'}{\mu'} \int_{z_{min}}^{z_{max}} dz\ zP^R_{ab}(z,\alpha_s(\mu^2))$$

which represents the probability of no emission

$$f_a(x,\mu) = \Delta_a(\mu) f(x,\mu) + \sum_b \int_{\mu_0}^{\mu} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_{ab}(\mu')}{\Delta_{ab}(\mu)} \int_x^{z_{max}} dz\ zP^R_{ab}(z)$$

To avoid the (1/(1-z)) pole for $z \to 1$ we introduce $z_{max} = 1 - \epsilon\ : \epsilon \ll 1$

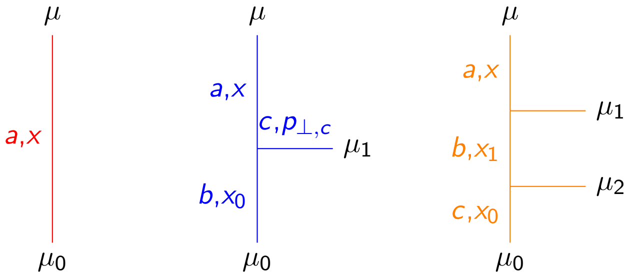
However, we do not have any information on the transverse momentum

The Parton Branching method

In the Parton Branching (PB) method we go further:

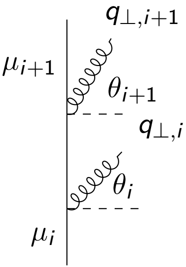
- At every splitting we compute the momentum of the emitted and propagating partons
- Generating a TMD with its corresponding evolution equation

$$\widetilde{A}_a(x,\mu^2,k_\perp) = \widetilde{A}(x,\mu_0^2,k_\perp)\Delta_a(\mu^2) + \int_{\mu_0}^{\mu} \frac{d^2\mu_{1\perp}}{\pi\mu_1^2}\frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \times \sum_b \int_x^{z_{max}} dz_1\, P_{ab}^R(\mu_1^2,z_1)\widetilde{A}_b\left(\frac{x}{z_1},\mu_0^2,k_\perp\right)\Delta_b(\mu_1^2) + \ldots$$



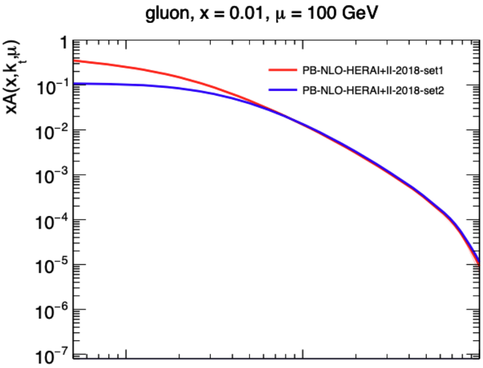
Parton branching and angular ordering

In the PB method the emissions follow the angular ordering condition: $\theta_i < \theta_{i+1}$



PB-set1	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$\alpha_s(q^2_{\perp,i})$
PB-set2	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$\alpha_s(\mu_i^2)$

Each TMD will have its corresponding PDF



(A. Bermudez et al., 2019)

Backwards evolution in PS

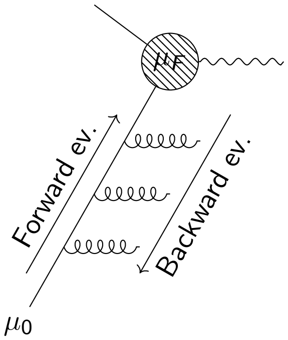
In Initial State Radiation (ISR), in the PS follows a backwards evolution (BE), from the hard scale, μ_f, down to the starting scale μ_0:

$$\sigma = \int dx_i dx_j f_i(x_i, \mu_f) f_j(x_j, \mu_f) \hat{\sigma}_{ij \to X}(x_i, x_j, \mu_f, ...)$$

$$\Delta_{ib}^{FW} = \exp - \sum_b \int_{\mu_{i-1}}^{\mu} \frac{d^2\mu'}{\mu'} \int_{z_{min}}^{z_{max}} dz\ P_{ib}^R(z, \alpha_s(\mu^2))$$

$$\downarrow$$

$$\Delta_{ib}^{BW} = \exp - \sum_b \int_{\mu_{i-1}}^{\mu} \frac{d^2\mu'}{\mu'} \int_{z_{min}}^{z_{max}} dz\ P_{ib}^R(z, \alpha_s(\mu^2)) \frac{f_i(x_i, \mu')}{f_b(x_b, \mu')}$$



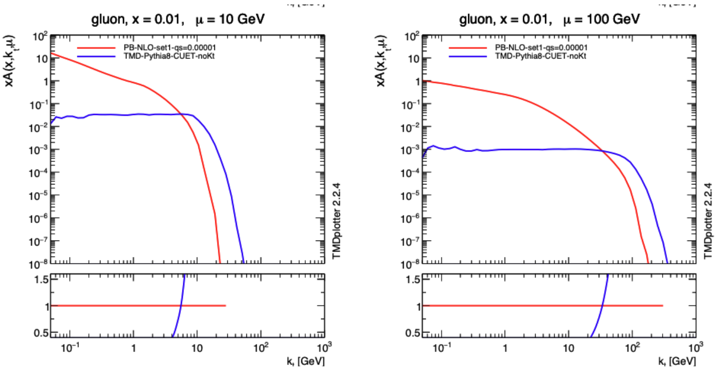
PDF2ISR

Comparison between TMDs and Parton Showers: PS2TMD

Method to extract TMDs from PS ISR (H. Jung, S. Steel, S. Taheri Monfared, Y. Zhou, 2021):

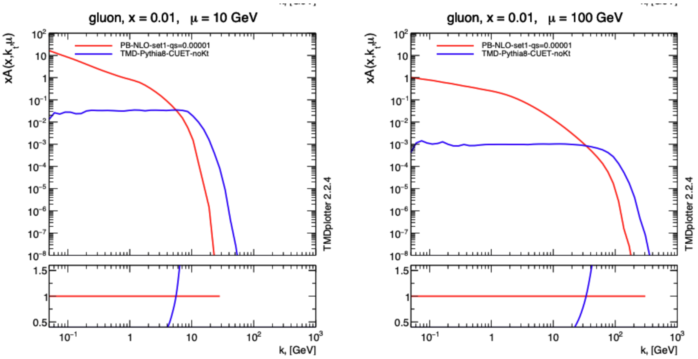
- Introduce PDF in a PS $\rightarrow$ get a TMD

Can we introduce PB-PDF into Pythia8 and get the correct PB-TMD?



From Pyhtia8 to PDF2ISR

	Pythia8	PB method
Ordering	$\mu_i^2 = q_{\perp,i}^2 = (1-z)Q^2$	$q_{\perp,i}^2 = (1-z)^2\mu_i^2$
Phase space	$0 < z < z_{dyn}^{P8}$	$0 < z < 1 - 10^5$
Splitting Functions	LO	NLO
α_s treatment	PDG param.	QCDNum num.



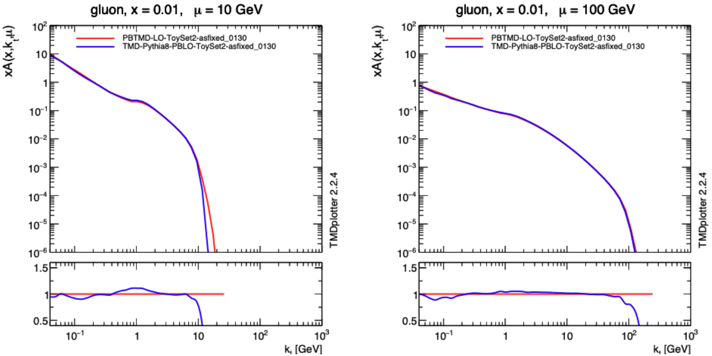
From Pyhtia8 to PDF2ISR: phase space

New PB-PDF/TMDs are fitted

	Pythia8	PB method
Ordering	$\mu_i^2 = q_{\perp,i}^2 = (1-z)Q^2$	$q_{\perp,i}^2 = (1-z)^2\mu_i^2$
Phase space	$0 < z < z_{dyn}^{P8}$	$0 < z < 1 - 10^5$
Splitting Functions	LO	NLO
α_s treatment	PDG param.	QCDNum num.

$\rightarrow$

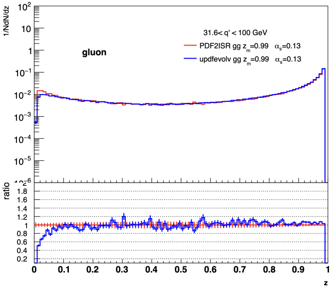
	Pythia8 (PDF2ISR)	PB method
Ordering	$q_{\perp,i}^2 = (1-z)^2\mu_i^2$	$q_{\perp,i}^2 = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	LO	LO
α_s treatment	$\alpha_s^{fixed} = 0.130$	$\alpha_s^{fixed} = 0.130$

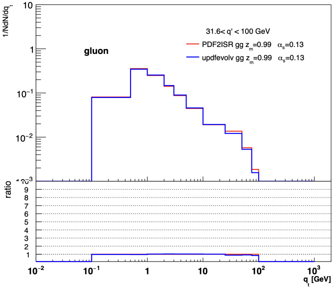


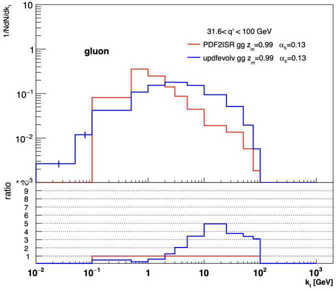
Forward and backward evolution comparison

	Pythia8 (PDF2ISR)	PB method
Ordering	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$q^2_{\perp,i} = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	LO	LO
α_s treatment	$\alpha_s^{fixed} = 0.130$	$\alpha_s^{fixed} = 0.130$

- Good agreement in $q_\perp$ and z
- $k_T = \sum_i q_{\perp,i}$, large discrepancy:
  - BW ev.: $q_\perp$ defined in the parton-parton CM-frame $\rightarrow$ boost to the general CM frame





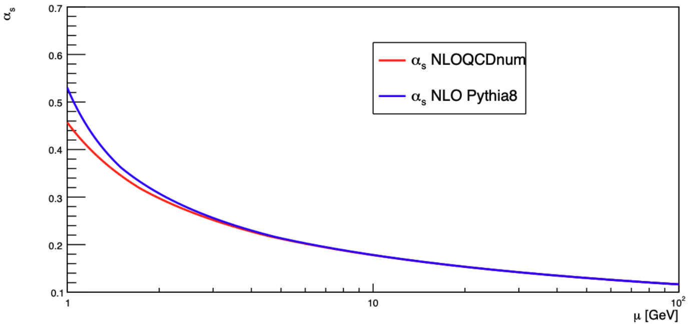


From Pyhtia8 to PDF2ISR: α_s treatment

	Pythia8	PB method
Ordering	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$q^2_{\perp,i} = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	LO	NLO
α_s treatment	PDG param.	QCDNum num.

$\rightarrow$

	Pythia8 (PDF2ISR)	PB method
Ordering	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$q^2_{\perp,i} = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	LO	NLO
α_s treatment	QCDNum num.	QCDNum num.



Implementation of NLO splitting functions: veto agorithm

(S. Mrenna and P. Skands, 2016) (L. Lönnblad, 2013)

NLO splitting functions negative $\rightarrow$ No probabilistic interpretation

LO splitting functions for emission generation

$$\downarrow$$

Reweighting to NLO: $r_{NLO} = \dfrac{P^{NLO}(z)\ \alpha_s^{NLO}(\mathrm{k_T})}{P^{NLO}(z)\ \alpha_{so}}$

$$\swarrow \quad \searrow$$

$r_{NLO} < 0.5$ | $r_{NLO} > 0.5$

Accept emission | Reject emission

$\downarrow$ | $\downarrow$

$w_{ev} = w_{ev} \times r$ | $w_{ev} = w_{ev} \times (2-r)$

From Pyhtia8 to PDF2ISR: LO $\rightarrow$ NLO

We use PB-set1 PDF/TMDs

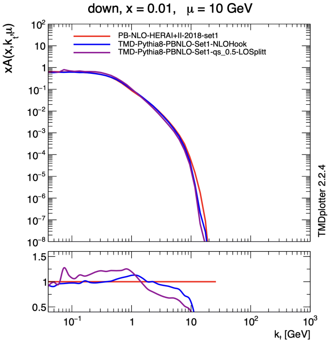
$\uparrow$

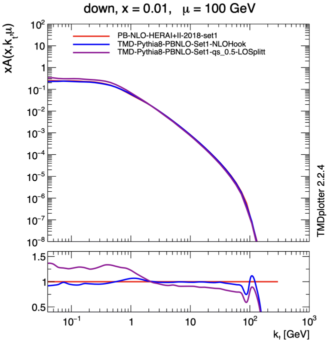
	Pythia8	PB method
Ordering	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$q^2_{\perp,i} = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	LO	NLO
α_s treatment	QCDNum num.	QCDNum num.

$\rightarrow$

	Pythia8 (PDF2ISR)	PB method
Ordering	$q^2_{\perp,i} = (1-z)^2\mu_i^2$	$q^2_{\perp,i} = (1-z)^2\mu_i^2$
Phase space	$0 < z < 1 - 10^5$	$0 < z < 1 - 10^5$
Splitting Functions	NLO[1] / LO	NLO
α_s treatment	QCDNum num.	QCDNum num.

[1](S. Mrenna and P. Skands, 2016) (L. Lönnblad, 2013)
Extra veto aglorithm using NLO/LO ratios





Conclusion

Conclusion

We have constructed a ISR consistent with the collinear distribution at LO and NLO:

- Correct treatment of kinemactics
- Implementation of NLO splitting functions via veto algorithm

This method is universal:

- Can be applied to any Parton Shower and any PDF

Once the details of the PDF evolution are known

- Can be extended to NNLO

Resummation accuracy:

- Naively we could think we resum up to partial NNLL (see Ola's talk yesterday)
- The discrepancy at large k_T, however, can spoil this accouracy

Backup

Emission probability and resolvability

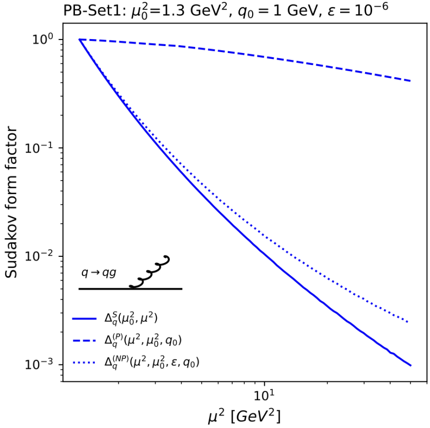
In PS a resolution parameter is introduced, z_{dyn}:

- $z < z_{dyn}$: resolvable radiation
- $z_{max} > z > z_{dyn}$: non-resolvable radiation

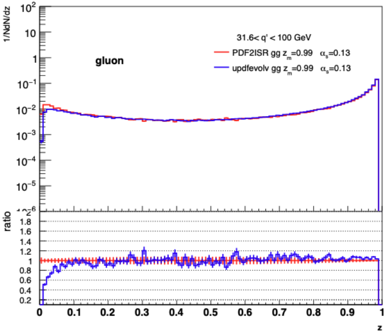
Using the angular ordering condition, $z_{dyn} = 1 - q_0/\mu$:

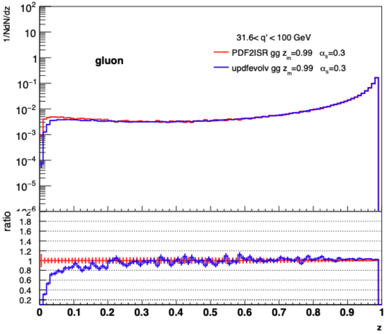
$$\Delta^s(\mu_0, \mu) = \Delta_{obs}(\mu_0, \mu, q_0)\Delta_{non-obs}(\mu_0, \mu, q_0, \epsilon)$$

For the evolution in PS $\Delta_{non-obs}$ is not taken into account, leaving non-resolvable emissions out

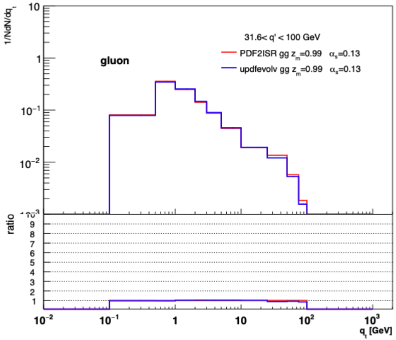


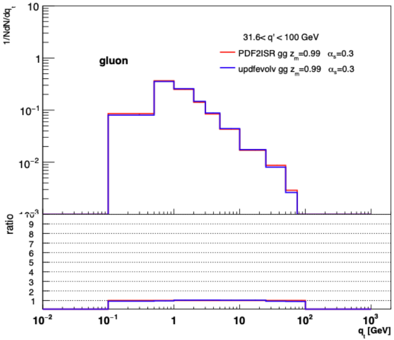
Forward Backward evolution comparison: z



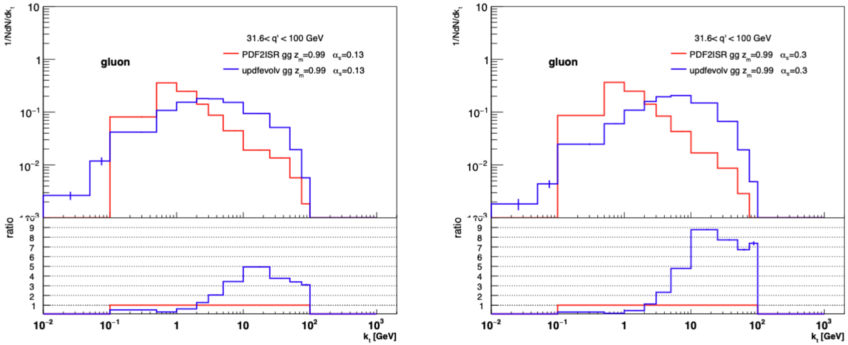


Forward Backward evolution comparison: qt

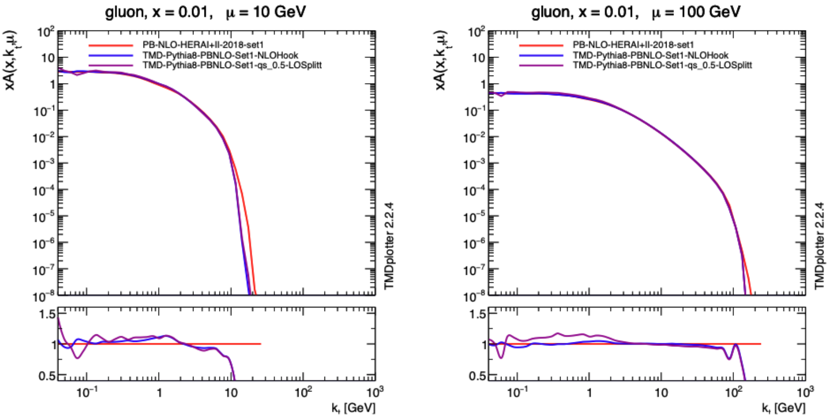




Forward Backward evolution comparison: kt



NLO/LO gluon



gluon, x = 0.01, μ = 10 GeV
gluon, x = 0.01, μ = 100 GeV
PB-NLO-HERAI+II-2018-set1
TMD-Pythia8-PBNLO-Set1-NLOHook
TMD-Pythia8-PBNLO-Set1-qs_0.5-LOSplitt
xA(x,k_t,μ)
k_t [GeV]
TMDplotter 2.2.4