Extended fracture functions in the Regge limit

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with Farid Salazar (based on arXiv:2502.02634)

Goals and outline

This talk, in a nutshell:

Discuss particle production in the target fragmentation region in DIS at small x.

- Emergence of new building block in TMD factorised cross-section at small x:
  $\Rightarrow$ **extended (or generalized) fracture functions**.

- First calculation of NLO leading twist longitudinal structure function for SIDIS at small x.

Single inclusive particle production in DIS

- Single-inclusive jet/hadron production in DIS

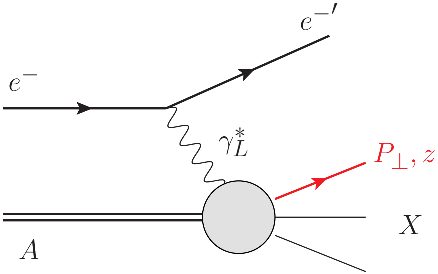
$$\frac{\mathrm{d}\sigma^{\gamma^*+A\to j/h+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}z} = \sigma_0^{\mathrm{DIS}} \times [F_{UU,T} + \varepsilon F_{UU,L}]$$

with $\boldsymbol{P}_\perp$ measured in the Breit/dipole frame.

- Longitudinally polarised virtual photon.

- Hadronic activity in the target hemisphere of the Breit frame in the limit

$$s \gg Q^2 \gg P_\perp^2$$



$$z = \frac{k\cdot P}{P\cdot q} = \frac{k^+}{q^+}, \quad \xi = \frac{k\cdot q}{P\cdot q} \simeq \frac{k^-}{P^-}$$

$$\sigma_0^{\mathrm{DIS}} = \frac{4\pi\alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}Q^4}\left(1-y+\frac{y^2}{2}\right)$$

Fracture functions

- The formalism to describe particle production in the TFR is that of fracture functions $\sim$ "conditional" PDF.

  Trentadue, Veneziano, PLB 323 (1994)

- They *"tell us about the structure function of a given target hadron once it has fragmented into another given final state hadron".*

$$M^i_{A,h}(x, \xi_h; Q^2)\,, \quad \xi_h = \frac{k_h^-}{P^-}$$

- Framework **extended** to account for the TMD of the measured hadron: Trentadue, Veneziano, NPB 519 (1998)

$$M^i_{A,h}(x, \xi; Q^2) \to M^i_{A,h}(x, \xi_h, \boldsymbol{P}_\perp; Q^2) \quad \text{for } \boldsymbol{P}_\perp^2 \ll Q^2$$

Physics Letters B 323 (1994) 201–211
North-Holland

PHYSICS LETTERS B

Fracture functions.
An improved description of inclusive hard processes in QCD

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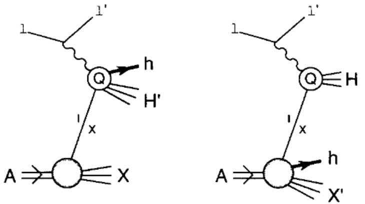
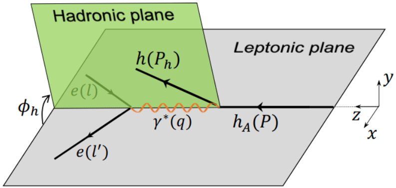
Editor R Gatto

We propose to describe semi-inclusive hard processes in perturbative QCD by means of "*fracture*" functions, hybrids between structure and fragmentation functions We argue that fracture functions factorize correctly and evolve in Q^2 in a predictable way As an application, we discuss how information on fracture functions obtained at HERA can be used to compute inclusive processes at future hadron colliders



(a) (b)

Factorization theorem with extended fracture functions in SIDIS

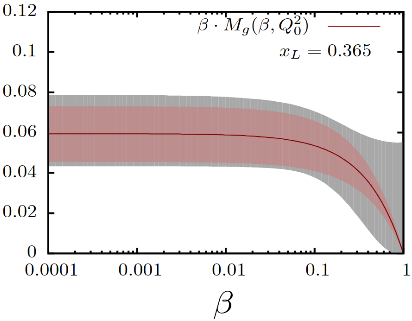


$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2}{x_B y Q^2}\Big\{A(y)F_{UU,T} + E(y)F_{UU,L} + B(y)F_{UU}^{\cos\phi_h}\cos\phi_h + E(y)F_{UU}^{\cos 2\phi_h}\cos 2\phi_h$$
$$+ \lambda_e D(y)F_{LU}^{\sin\phi_h}\sin\phi_h + S_L\Big[B(y)F_{UL}^{\sin\phi_h}\sin\phi_h + E(y)F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\Big] + \lambda_e S_L\Big[C(y)F_{LL} + D(y)F_{LL}^{\cos\phi_h}\cos\phi_h\Big]$$
$$+ |\vec{S}_\perp|\Big[\Big(A(y)F_{UT,T}^{\sin(\phi_h-\phi_S)} + E(y)F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)\sin(\phi_h-\phi_S) + E(y)F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)$$
$$+ B(y)F_{UT}^{\sin\phi_S}\sin\phi_S + B(y)F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S) + E(y)F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\Big]$$
$$+ \lambda_e|\vec{S}_\perp|\Big[D(y)F_{LT}^{\cos\phi_S}\cos\phi_S + C(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) + D(y)F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}.$$

- 18 structure functions

Anselmino, Barone, Kotzinian, 1102.4214, Chen, Ma, Tong, 2308.11251

- Extracted from HERA data with forward neutrons: de Florian, Sassot, PRD 56 (1997)



$\beta \to \frac{x}{1-\xi}$, $x_L \to \xi$, $Q_0^2 = 1$ GeV^2

Fig. from Ceccopieri, EPJC 74 (2019)

Operator definition of extended fracture functions

Connection with diffractive PDFs

- Operator definition: extended FrF = diffractive PDF ?

$$M^q_{A,h}(x, \xi_h, P_{h\perp}) \equiv \int \frac{\mathrm{d}\eta^+}{2\xi(2\pi)^4} e^{-ixP^-\eta^+} \sum_X \langle A|\bar{\psi}(\eta^+)\gamma^-|P_h X\rangle\langle X P_h|\psi(0)|A\rangle$$

(gauge links omitted)
See e.g. Berera, Soper, PRD 53 (1996), Hautmann, Kunszt, Soper, PRL 81 (1998)

- For low $|t| \ll 1/R_A^2$, if $h = A$ is the recoiled nucleon: FrF reduces to the diffractive PDFs, with the more standard variables $t = -P_{h\perp}^2/\xi_h$ and $x_{\mathbb{P}} = 1 - \xi_h$.

- **Here, inclusive case**: the target is typically broken apart ($|t| \gg 1/R_A^2$) and the hadron which is measured in the TFR comes from a parton freed by the collision which eventually hadronizes.

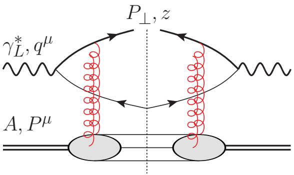
- In particular, no rapidity gap is requested in the process.

Two possible strategies

- We want to compute $M_{A,h}^{q/g}(x, \xi_h, P_{h\perp})$ in the limit $x \ll 1$.
- Two strategies:
  - $\Rightarrow$ start from the operator definition of extended FrF and compute at small x,
  - $\Rightarrow$ compute directly SIDIS for general kinematics and isolate the LP component coming from TFR hadronic activity.
- We follow the second approach: first compute the leading power in $x_{\rm Bj}$ for general final state kinematics, and then expand in powers of $P_\perp^2/Q^2$ and isolate the LP contribution.

LO for SIDIS at small x

- Color dipole picture + CGC effective field theory. (See talks by Edmond and Dionysis before.)



- Building blocks are Wilson line operators, such as the dipole correlator

$$D_F(x, \boldsymbol{r}_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}[V(\boldsymbol{0}_\perp)V^\dagger(\boldsymbol{r}_\perp)]\right\rangle_x \qquad V(\boldsymbol{x}_\perp) = \mathcal{P}e^{ig\int_{-\infty}^{\infty}\mathrm{d}x^+ A^-(x^+,\boldsymbol{x}_\perp)}$$

- At LO in α_s and up to $\mathcal{O}(x)$ corrections,

$$F^{\mathrm{LO}}_{UU,L}(x, Q^2, \boldsymbol{P}_\perp) = \frac{N_c e_i^2}{2\pi^4}\int \mathrm{d}^2\boldsymbol{q}_\perp \mathcal{D}_F(x, \boldsymbol{q}_\perp)\int_0^1 \mathrm{d}z \left|\frac{\bar{Q}^2}{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2 + \bar{Q}^2} - \frac{\bar{Q}^2}{P_\perp^2 + \bar{Q}^2}\right|^2$$

with $\bar{Q}^2 = z(1-z)Q^2$.

Twist expansion for LO SIDIS

- Now, let's consider the limit $Q^2 \gg \boldsymbol{P}_\perp^2$.

- Limit controlled by aligned jets, i.e. $1 - z \sim P_\perp^2/Q^2 \ll 1$ or $z \sim P_\perp^2/Q^2 \ll 1$:

$$F_{UU,L}^{\mathrm{LO}} = \frac{N_c P_\perp^2}{\pi^4 Q^2} e_i^2 \int \mathrm{d}^2\boldsymbol{q}_\perp \mathcal{D}_F(x, \boldsymbol{q}_\perp) \left[\frac{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}{P_\perp^2} + 1 - \frac{2(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}{P_\perp^2 - (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2} \ln\left(\frac{P_\perp^2}{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2} \right) \right]$$

- Longitudinally polarised γ^*: power suppressed. $\Rightarrow$ no leading twist contribution to $F_{UU,L}$ at LO at small x.

- Similar result at moderate x: $F_{UU,L}$ (and F_L) vanishes at LO (Callan-Gross relation).

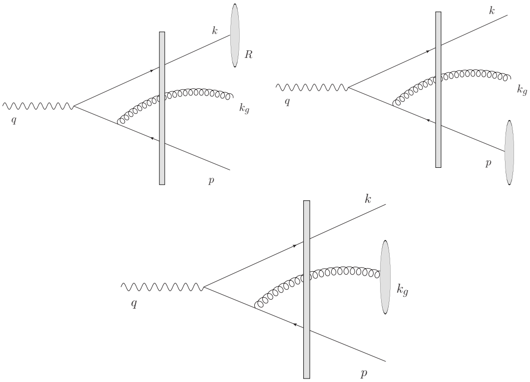
NLO calculation of SIDIS at small x

- Let's extend this analysis to NLO at small x.

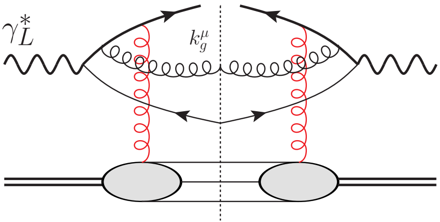
- We follow the same steps:
  - start from inclusive dijet at NLO in the CGC.
    PC, Salazar, Venugopalan, JHEP 2021 (11), 1-108, Bergabo, Jalilian-Marian, PRD 106 (2022), Iancu, Mullian, JHEP 07 (2023) 121,...
  - integrate out the unmeasured jets to get SIDIS at NLO in the CGC.
  - extract the leading power in the $P_\perp/Q$

From inclusive dijet to SIDIS

- Integrated out one of the two or three jets.
  PC, Ferrand, Salazar, 2401.01934
- $q \leftrightarrow \bar{q}$ tagged jet contributions can be related by C symmetry.
  $\Rightarrow 2\mathfrak{Re}$ in the final formula.
- Careful: in real corrections, tagged jet can be the gluon one.
- Similar calculation for single inclusive hadron production in Bergabo, Jalilian-Marian JHEP 01, 095 (2023) & arXiv:2401.06259



Isolating the leading power term (1/2)



- A bit technical. Consider for instance the contribution from diagram $\mathrm{R1}\times\mathrm{R1}^*$:

$$F_{UU,L}^{\mathrm{R1}\times\mathrm{R1}^*} = \frac{\alpha_s N_c}{8\pi^8}\int_{\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i\boldsymbol{P}_\perp\cdot\boldsymbol{r}_{xx'}}\int_0^1 \mathrm{d}z\int_0^{1-z}\frac{\mathrm{d}z_g}{z_g}\left(1-\frac{z_g}{1-z}\right)^2\bar{Q}^4 K_0(QX_R)K_0(QX'_R)$$
$$\times\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zx'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zx'}^2}\left(1+\frac{z_g}{z}+\frac{z_g^2}{2z^2}\right)\left\{\frac{N_c}{2}\left\langle D_{xx'}-D_{xz}D_{zy}-D_{yz}D_{zx'}+1\right\rangle_x+\frac{1}{2N_c}(\boldsymbol{z}_\perp\to\boldsymbol{y}_\perp)\right\},$$

- $X_R^2 = z(1-z-z_g)\boldsymbol{r}_{xy}^2 + zz_g\boldsymbol{r}_{zx}^2 + (1-z-z_g)z_g\boldsymbol{r}_{zy}^2$ effective size of the $q\bar{q}g$ system.

Isolating the leading power term (2/2)

- Aligned jets $\Rightarrow 1-z \sim P_\perp^2/Q^2 \ll 1$ or $z \sim P_\perp^2/Q^2 \ll 1$, yet no phase space for $1-z \ll 1$ because of longitudinal momentum conservation.
- $Q \gg P_\perp \Rightarrow r_{zy} \ll r_{xy}$. In this limit the color structure reduces to the LO one:

$$\begin{aligned}\Xi &\approx C_F \left\langle D_{xx'} - D_{xy} - D_{yx'} + 1 \right\rangle_x \,, \\ &= C_F \int_{\boldsymbol{q}_\perp} \left(e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{xy}} - 1\right)\left(e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{x'y}} - 1\right) \mathcal{D}_F(x, \boldsymbol{q}_\perp)\,.\end{aligned}$$

- In the end:

$$F_{UU,L}^{\mathrm{R1}\times\mathrm{R1}^*} = \frac{\alpha_s C_F N_c}{2\pi^5} \int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}z_g \; z_g(1-z_g)^2 \int_{\boldsymbol{B}_\perp, \boldsymbol{q}_\perp} \mathcal{D}_F(x, \boldsymbol{q}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{l}_{3\perp}}{(2\pi)} \mathcal{H}^{j}_{\mathrm{R1}\times\mathrm{R1}^*} \mathcal{H}^{j*}_{\mathrm{R1}\times\mathrm{R1}^*}\,.$$

which can be computed analytically.

Final leading twist result for $F_{UU,L}(x, Q^2, P_\perp)$

- Gathering the quark and gluon tagged jet contributions, we get

$$F^{\mathrm{LP}}_{UU,L}(x, \boldsymbol{P}_\perp) = \frac{\alpha_s C_F}{2\pi} \sum_{i=q,\bar{q}} e_i^2 \; x\mathcal{F}_i(x, \boldsymbol{P}_\perp) + \frac{\alpha_s}{3\pi} \sum_{i=q} e_i^2 \; x\mathcal{F}_g(x, \boldsymbol{P}_\perp) \, .$$

with

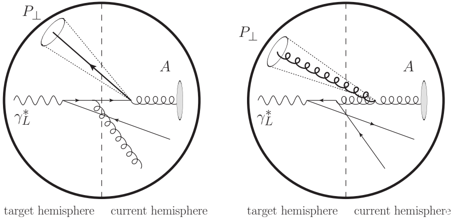
$$x\mathcal{F}_q(x, \boldsymbol{P}_\perp) = \frac{N_c}{4\pi^4} \int_{\boldsymbol{B}_\perp, \boldsymbol{q}_\perp} \mathcal{D}_F(x, \boldsymbol{q}_\perp) \left[1 - \frac{\boldsymbol{P}_\perp \cdot (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)}{P_\perp^2 - (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2} \ln\left(\frac{P_\perp^2}{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}\right)\right] \, .$$

and

$$x\mathcal{F}_g(x, \boldsymbol{P}_\perp) = \frac{N_c^2 - 1}{4\pi^4} \int_{\boldsymbol{B}_\perp, \boldsymbol{q}_\perp} \mathcal{D}_A(x, \boldsymbol{q}_\perp) \left\{ \ln\left(\frac{1}{x}\right) \frac{q_\perp^2}{P_\perp^2} - \frac{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2 - q_\perp^2}{2P_\perp^2} \ln\left(\frac{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}{P_\perp^2}\right) \right.$$
$$\left. - 1 + \left(1 - \frac{2(\boldsymbol{P}_\perp \cdot (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp))^2}{P_\perp^2 (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}\right) \frac{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}{P_\perp^2 - (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2} \ln\left(\frac{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}{P_\perp^2}\right) \right\} \, .$$

Particle production in the TFR at small x

- TMD factorisation theorem for SIDIS at moderate x stipulates that $F_{UU,L} = \mathcal{O}(P_\perp^2/Q^2)$.
- Implicit assumption of the theorem: the hadron is produced in the curent fragmentation region (γ^* hemisphere).
- Our calculation tells us that $z \sim P_\perp^2/Q^2$.
  $\Rightarrow \eta = \ln(zQ/P_\perp) \sim \ln(P_\perp/Q) < 0$ in Breit frame.
- Particle produced in the **target fragmentation hemisphere**: no contradiction!



Validation: F_L at NLO at small x

- Based on these results, one can easily extract the NLO correction to F_L at leading twist:

$$F_L^{\rm NLO}(x,Q^2) = \frac{\alpha_s C_F}{2\pi} \sum_{i=q,\bar{q}} e_i^2 xq(x,Q^2) + \frac{\alpha_s}{3\pi} \sum_{i=q} e_i^2 \, x\tilde{g}(x,Q^2)$$

with

$$xq(x,Q^2) \equiv \int^{Q^2} \mathrm{d}^2\boldsymbol{P}_\perp x\mathcal{F}_q(x,\boldsymbol{P}_\perp) \sim \alpha_s \ln(Q^2/\Lambda^2) P_{qg} \otimes xg(x,Q^2)$$

$$x\tilde{g}(x,Q^2) = xg(x,Q^2) + \int^{Q^2} \mathrm{d}^2\boldsymbol{P}_\perp x\mathcal{F}_g(x,\boldsymbol{P}_\perp) \sim xg(x,Q^2) + \alpha_s \ln(Q^2/\Lambda^2) P_{gg} \otimes xg(x,Q^2)$$

- Agrees with the $x \to 0$ limit of F_L at one loop in collinear factorization.

Altarelli, Martinelli (PLB 76, 89, 1978) . ✓ consistency check!

$$F_L(x,Q^2) = \frac{\alpha_s}{2\pi} x^2 \int_x^1 \frac{\mathrm{d}z}{z^3} \left[2C_F \sum_{i=q,\bar{q}} e_i^2 zq_i(z,Q^2) + 4 \left(\sum_{i=q} e_i^2 \right) \left(1 - \frac{x}{z}\right) zg(z,Q^2) \right]$$

Extended quark fracture function at small x

- So far, ξ (or z) integrated out. To actually connect with FrF, let's work differentially in ξ:

$$\frac{\mathrm{d}\sigma^{\gamma_L^*+A\to h+X}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}\xi} = \varepsilon\sigma_0^{\mathrm{DIS}} \times \left[\frac{\alpha_s C_F}{2\pi} M_{A,q}^q(x,\xi,\boldsymbol{P}_\perp) + \frac{\alpha_s}{3\pi} M_{A,g}^g(x,\xi,\boldsymbol{P}_\perp)\right]$$

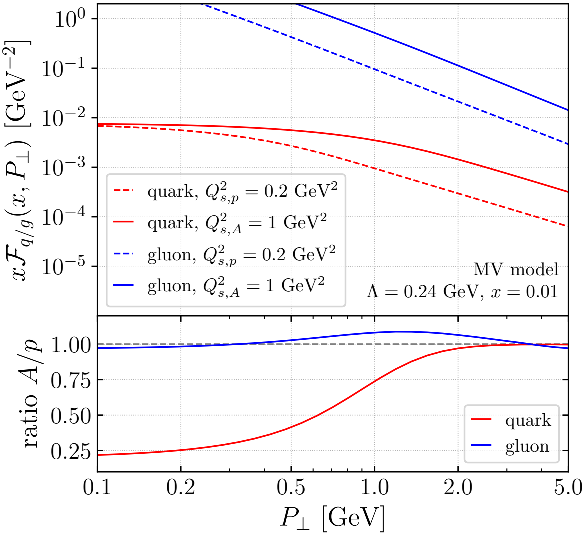
with the quark TMD FrF for $x,\xi \ll 1$ given by

$$M_{A,q}^q(x,\xi,\boldsymbol{P}_\perp) \equiv \frac{N_c}{8\pi^4}\frac{x}{(x+\xi)^2}\int_{\boldsymbol{b}_\perp,\boldsymbol{q}_\perp} \mathcal{D}_F\left(x+\xi,\boldsymbol{q}_\perp\right)$$
$$\times\left\{1+\frac{(x+\xi)^2\boldsymbol{P}_\perp^2(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2}{[\xi(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2+x\boldsymbol{P}_\perp^2]^2} - \frac{2(\xi+x)[\boldsymbol{P}_\perp\cdot(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)]}{[\xi(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2+x\boldsymbol{P}_\perp^2]}\right\}$$

- Limit $\boldsymbol{P}_\perp \gg Q_s$ agrees with collinear calculation. Chen, Ma, Tong, JHEP 11 (2021) 038
- Similar result for the gluon FrF.

Numerical results in the McLerran-Venugopalan model

- Show the ξ-integrated FrF.
- MV model to evaluate the dipole correlator.
- $\boldsymbol{P}_\perp$ dependence of the quark FrF identical to that of the quark TMD at this order.
- Both FrF have perturbative $\sim 1/P_\perp^2$ tail for $P_\perp \gg Q_s$.
- Quark FF $\sim N_c/(8\pi^4)$ for $P_\perp \ll Q_s$.
- Gluon FrF displays a $1/P_\perp^2$ tail even for $P_\perp \ll Q_s$.



Summary and outlook

- First NLO calculation of the TMD longitudinal structure function at small x and $P_\perp^2 \ll Q^2$.
- Leading twist factorisation in terms of *extended fracture functions*.
- CGC provide explicit expressions for these extended FrF in terms of Wilson line operators.
- More detailed pheno for the EIC and application to NEC are on-going (see back-up slides).
- Other interesting prospects: investigate TMD extended FrF in inclusive or diffractive jet+hadron production with one jet in the CFR and one hadron in the TFR.
- On the more conceptual side: obtain the quantum evolution of these objects, in particular BK/BFKL as $x \to 0$, as well as the Q^2 dependence which should be given by DGLAP (Hauksson, Iancu, Mueller, Triantafyllopoulos, Wei, JHEP 06 (2024) 180)

Thank you for your attention!

Back-up slides

Connection with Nucleon Energy Correlators (NEC)

- NEC (Liu, Zhu, 2209.02080) are used as probe of saturation in DIS through the quark channel,

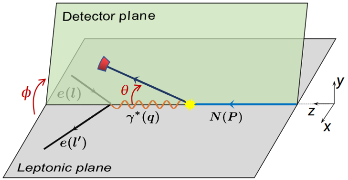
$$\Sigma(Q^2, x, \theta) = \sum_i \int \mathrm{d}\sigma(x, Q^2, p_i) \frac{E_i}{E_h} \delta(\theta - \theta_i)$$
$$= \sigma_{\mathrm{DIS}} \otimes f_{i,EEC}(x, \theta)$$

- NEC and FrF are related by a sum rule

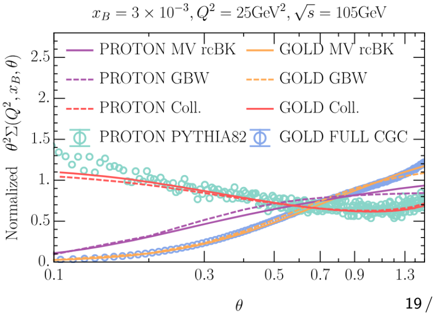
Chen, Ma, Tong, 2406.08559

$$f_{i,EEC}(x, \theta) = \sum_h \int_0^{1-x} \mathrm{d}\xi \xi \frac{\boldsymbol{P}_\perp^2}{2\theta^2} M^i_{A,h}(x, \xi, \boldsymbol{P}_\perp) \,|_{\boldsymbol{P}_\perp = \xi P - \theta \hat{\boldsymbol{n}}}$$

- At NLO, the gluon contribution enters as well, and we predict a dramatic change in the scaling at small angles as compared to the quark case $\to$



Liu, Liu, Pan, Yuan, Zhu, 2301.01788



Twist expansion of F_L

- Not completely obvious though: the LO calculation at small x matches the $x \to 0$ limit of F_L at NLO in the collinear limit.
- For this reason, F_L as obtained by integrating $F_{UU,L}$ has a leading twist contribution!

$$F_L(x, Q^2) = \int \mathrm{d}^2 \boldsymbol{P}_\perp F_{UU,L}(x, Q^2, \boldsymbol{P}_\perp)$$
$$= \frac{\alpha_s}{3\pi} xg(x, Q^2) + \mathcal{O}(1/Q^2)$$

with

$$xg(x, Q^2) \equiv \int^{Q^2} \mathrm{d}^2 \boldsymbol{q}_\perp \int_{\boldsymbol{B}_\perp} \frac{N_c}{2\pi^2 \alpha_s} q_\perp^2 \mathcal{D}_F(x, \boldsymbol{q}_\perp)$$

- Leading twist contribution to F_L at small x coming from symmetric jet configurations ($z \sim 1/2$).

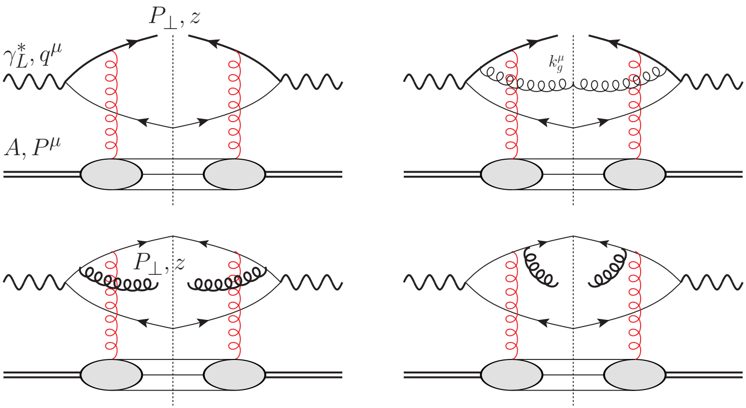
$P_\perp \gg Q_s$ limit of quark extended FF

- Let's consider the limit $P_\perp \gg Q_s$ of $M^q_{A,h}(x,\xi,\boldsymbol{P}_\perp)$:

$$M^q_{A,q}(x,\xi,\boldsymbol{P}_\perp) \equiv \frac{N_c}{8\pi^4}\frac{x}{(x+\xi)^2}\int_{\boldsymbol{b}_\perp,\boldsymbol{q}_\perp}\mathcal{D}_F\left(x+\xi,\boldsymbol{q}_\perp\right)\frac{q_\perp^2}{P_\perp^2}\frac{x^2+\xi^2}{(x+\xi)^2}$$

$$= \frac{\alpha_s}{4\pi^2}\frac{1}{P_\perp^2}\frac{x(x^2+\xi^2)}{(x+\xi)^3}g(x+\xi,P_\perp^2)$$

NLO diagrams contributing at leading twist



- The top left graph is LO and vanishes at leading power.
- For gluon-tagged jet, include also $q\bar{q}$ exchange diagrams and interferences between gluon emission before and after the shockwave.