Diffractive jets and evolution of DTMDs in coordinate and momentum space

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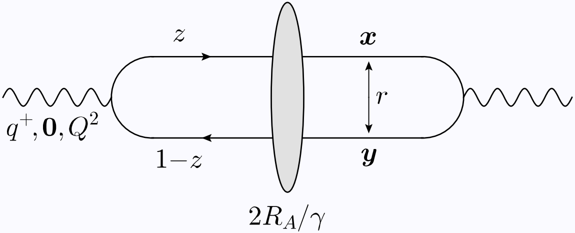
- S. Hauksson, E. Iancu, A.H. Mueller, DT, S.Y. Wei, F. Yuan: 25nn.nnnnn, 2402.14748 (JHEP), 2304.12401 (EPJC), 2207.06268 (JHEP), 2112.06353 (PRL)
- Hautmann, Hebecker, Soper, Golec-Biernat, Wüsthoff '97-'01
- Caucal, Iancu '24; Nefedov, Von Hameren '24

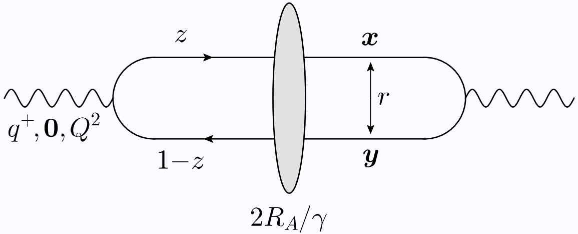
Content

- Deep inelastic scattering in the dipole picture
- "2 + 1" jets in coherent diffraction as probes of saturation
- Factorization: Gluon diffractive TMDs
- Late time emissions
- Momentum space evolution and an analytic solution
- Coordinate space evolution



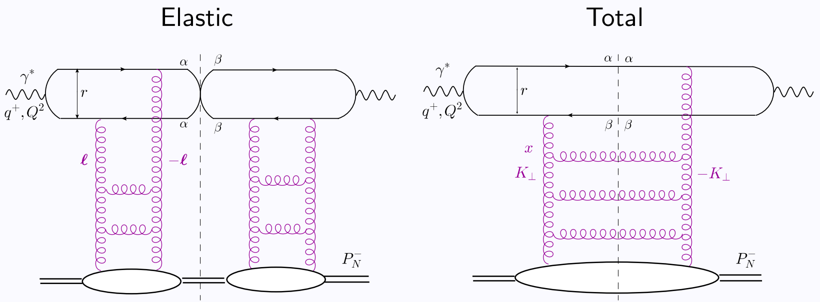
- Right moving off-shell γ^*, $q^\mu = (q^+, \mathbf{0}, -Q^2/2q^+)$
- Left moving nucleus, $p^\mu = (M_N^2/2P_N^-, \mathbf{0}, P_N^-)$ per nucleon
- Projectile lifetime $\tau_\gamma \sim 2q^+/Q^2$
- Nucleus contracted length $L \sim 2R_A M_N/P_N^- \sim A^{1/3}/P_N^-$
- $L \ll \tau_\gamma \Longleftrightarrow xA^{1/3} \ll 1$

DIS at Small-x in Dipole Picture: Factorization

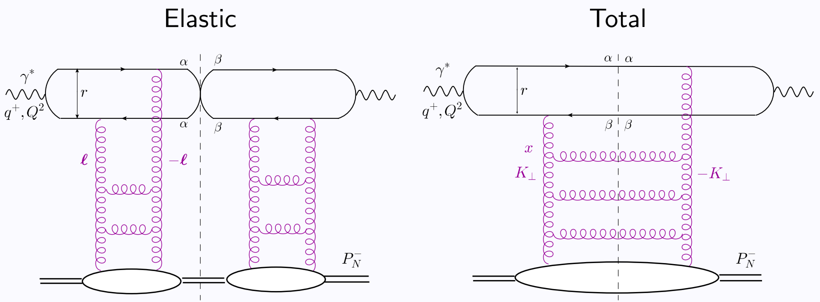


$$\sigma^{\gamma^* A}(x, Q^2) = \int \mathrm{d}^2\boldsymbol{r} \int_0^1 \mathrm{d}z \left|\Psi_{\gamma^*\to q\bar{q}}(Q^2; \boldsymbol{r}, z)\right|^2 2\pi R_A^2 T(r, x)$$

- All QCD dynamics in $T(r, x)$
- Virtuality limits large dipoles: $r \lesssim 1/\bar{Q}$, with $\bar{Q}^2 = z(1-z)Q^2$
- Saturation requires $r \gtrsim 1/Q_s$, hence $\bar{Q}^2 \lesssim Q_s^2$
- When $Q^2 \gg Q_s^2$ dominant contribution from weak scattering



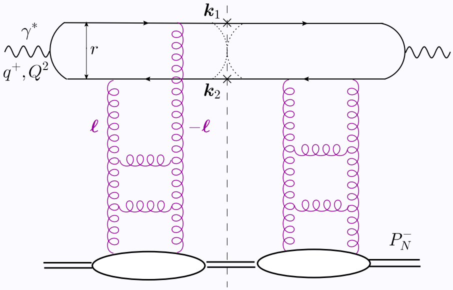
- Rapidity gap: wide angular region void of particles
- Elastic for projectile, no nuclear break-up (coherent reaction)
- Close color at amplitude level
- At least two gluons exchanged at amplitude



$$\sigma_{\rm D}^{\gamma^* A}(x,Q^2) = \int {\rm d}^2\boldsymbol{r} \int_0^1 {\rm d}z \left|\Psi_{\gamma^*\to q\bar{q}}(Q^2;\boldsymbol{r},z)\right|^2 \pi R_A^2 [T(r,x)]^2$$

- T^2 : Diffractive cross section less sensitive to small dipoles
- Even for $Q^2 \gg Q_s^2$ dipoles with $r \gtrsim 1/Q_s$ and $z \sim Q_s^2/Q^2 \ll 1$ ("aligned jets") dominate diffractive cross section

- More exclusive processes? Measured jets or hadrons?
- Hard scale sets dipole size $r \sim 1/P_\perp$, weak scattering
- Hard, symmetric, back to back $q\bar{q}$ pair:
  $k_{1\perp} \simeq k_{2\perp} \equiv P_\perp \sim Q \gg Q_s$, $z_{1,2} \sim 1/2$

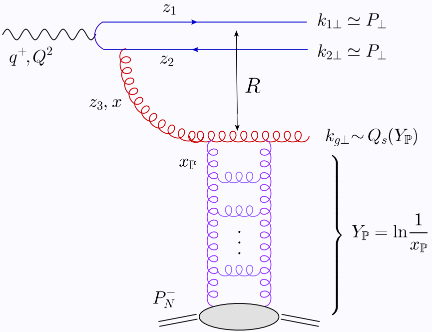


$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^* A \to q\bar{q}A}}{\mathrm{d}z_1 \mathrm{d}^2\boldsymbol{P}} \propto \pi R_A^2 \underbrace{\frac{1}{Q^2}}_{\gamma^* q\bar{q}} \underbrace{\frac{Q_s^4}{P_\perp^4}}_{T^2(r)}$$

$\sim \dfrac{1}{P_\perp^6}$ higher twist

2+1 Jets in Diffraction

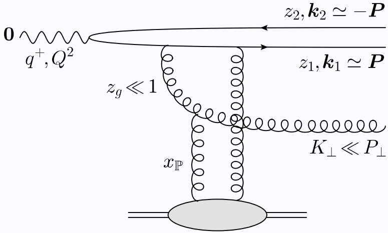
- Diffractive dijet at leading twist $1/P_\perp^4$?
- Yes, two hard jets $P_\perp \gg Q_s$ and one semi-hard $k_{g\perp} \sim Q_s \ll P_\perp$
- Third, semi-hard, jet provides dijet imbalance

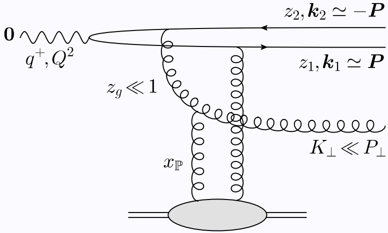


- $\mathcal{O}(\alpha_s)$ suppression
- $R \gg r$: gluon dipole
- $T_g(R, Y_{\mathbb{P}}) \simeq \mathcal{O}(1)$
- Hard factor

$$H \propto \underbrace{\frac{1}{Q^2}}_{\gamma^* q\bar{q}} \times \underbrace{r^2}_{\text{gluon emission}} \sim \frac{1}{P_\perp^4}$$

- $x_{\mathbb{P}} P_N^-$ puts trijet on-shell: $x_{\mathbb{P}} \simeq \dfrac{1}{2q^+ P_N^-}\left(\dfrac{P_\perp^2}{z_1 z_2} + \dfrac{k_{g\perp}^2}{z_g} + Q^2\right)$



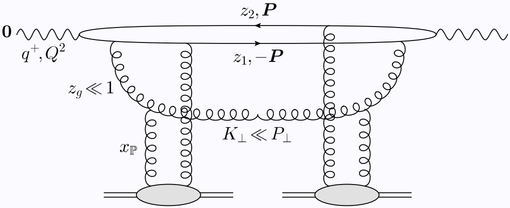


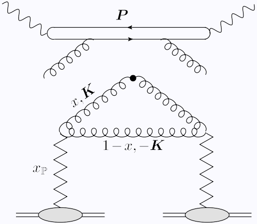
- Gluon formation time must be small enough:
  $k_g^+/k_{g\perp}^2 \lesssim q^+/Q^2 \rightsquigarrow z_g \lesssim k_{g\perp}^2/P_\perp^2 \ll 1$, gluon is soft
- Momentum space LCWF
  - Expand for $k_{g\perp} \ll P_\perp$ and $z_g \ll k_{g\perp}/P_\perp$ (no recoil)
  - Leading terms cancel $\rightsquigarrow$ Non-eikonal emission
  - Scattering is eikonal (Wilson lines), keep diffractive projection
  - Add instantaneous quark propagator graph

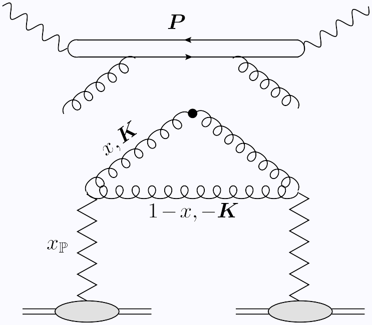
- Scales separation $\rightsquigarrow$ Factorization?
- View gluon as part of Pomeron. Variable change from ξ to x:

$$x = \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{\frac{P_\perp^2}{z_1 z_2} + Q^2}{\frac{P_\perp^2}{z_1 z_2} + \frac{k_{g\perp}^2}{z_3} + Q^2} \quad \text{or} \quad x = \beta\, \frac{x_{q\bar{q}}}{x_{\rm Bj}} \simeq \beta\, \frac{\bar{Q}^2 + P_\perp^2}{P_\perp^2}$$

- For given $x_{\rm Bj}$ and hard jets, only one of z_g, $x_{\mathbb{P}}$ and x is independent







$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma_{T,L}^* A \to q\bar{q}gA}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\, \frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

- Hard factor as in inclusive $q\bar{q}$ dijet cross section

$$H_T(z_1, z_2, Q^2, P_\perp^2) \equiv \alpha_{em}\alpha_s \Big(\sum e_f^2\Big)\, \delta_z \underbrace{(z_1^2 + z_2^2)}_{2P_{q\gamma}(\vartheta_1)} \underbrace{\frac{P_\perp^4 + \bar{Q}^4}{(P_\perp^2 + \bar{Q}^2)^4}}_{\sim 1/P_\perp^4}$$

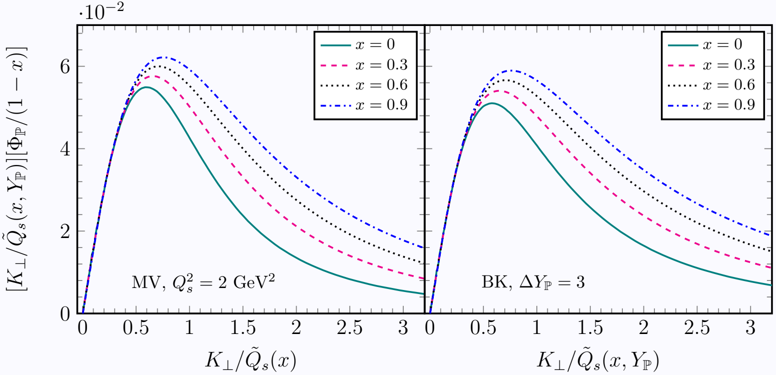
$$\frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = \underbrace{\frac{S_\perp(N_c^2-1)}{4\pi^3}}_{\text{d.o.f.}} \underbrace{\Phi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}_{\text{occupation number}}$$

- Explicit in terms of elastic amplitude $T_g(R, x_{\mathbb{P}})$

$$\Phi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2) \approx \frac{1-x}{2\pi} \begin{cases} 1 & \text{for} \quad K_\perp \ll \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x, Y_{\mathbb{P}})}{K_\perp^4} & \text{for} \quad K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Valid for large gaps: $x_{\mathbb{P}} \lesssim 10^{-2}$
- Effective saturation momentum $\tilde{Q}_s^2(x) \equiv (1-x)Q_s^2$
- Bulk of distribution at saturation $K_\perp \lesssim \tilde{Q}_s(x)$

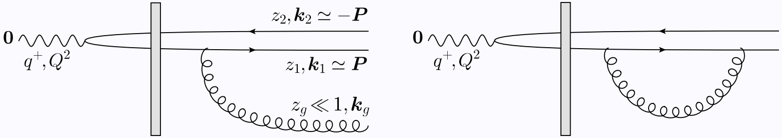
Gluon Diffractive TMD



$$\mathcal{T}_g(r) = 1 - \exp\left[-\frac{r^2 Q_A^2}{4}\,\ln\frac{4}{r^2\Lambda^2}\right] \text{ (left)} \qquad \text{plus BK (right)}$$

- Multiplied by $K_\perp$ (cf. measure $\mathrm{d}^2\boldsymbol{K}$)
- Pronounced maximum at $K_\perp \sim \tilde{Q}_s(x)$

Late time emissions



- Gluons with $\tau_g \gg \tau_\gamma \rightarrow z_g \gg k_{g\perp}^2/P_\perp^2$ do not interact
- Interested in gluons emitted at large angles
  $\theta_g \gg \theta_q \rightarrow z_g \ll k_{g\perp}/P_\perp$

$$\Delta\mathcal{F}_g(x,\boldsymbol{K}) = \frac{\alpha_s N_c}{\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2} \int_{k_{g\perp}^2/P_\perp^2}^{k_{g\perp}/P_\perp} \frac{\mathrm{d}z_g}{z_g} \Big[\mathcal{F}_g^{(0)}(x,\boldsymbol{K}+\boldsymbol{k}_g) - \mathcal{F}_g^{(0)}(x,\boldsymbol{K})\Big]$$

- If $k_{g_\perp} \ll K_\perp$, real and virtual cancel
- If $K_\perp \ll k_{g_\perp} \ll P_\perp$, real is small $\rightarrow$ uncompensated emission

$$\Delta\mathcal{F}_g(x,\boldsymbol{K},Q^2) = -\frac{\alpha_s N_c}{4\pi} \ln^2 \frac{Q^2}{K_\perp^2} \mathcal{F}_g^{(0)}(x,\boldsymbol{K},Q^2)$$

The DLA equation

Recast the problem into an evolution equation. First in DLA

$\boldsymbol{\ell} = \boldsymbol{k}_g + \boldsymbol{K}$ in the real term: momentum of t-channel gluon in "target picture"

$$\frac{\partial \mathcal{F}_g(x, K_\perp^2, Q^2)}{\partial \ln Q^2} = \frac{N_c}{2\pi}\bigg[\Theta(K_\perp, \mu_0)\,\frac{\alpha_s(K_\perp^2)}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2\,\mathcal{F}_g(x, \ell_\perp^2, Q^2) - \int_{K_\perp^2}^{Q^2}\frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\,\Theta(\ell_\perp, \mu_0)\,\alpha_s(\ell_\perp^2)\mathcal{F}_g(x, K_\perp^2, Q^2)\bigg] + \Theta(Q, \mu_0)\,\beta_0\,\frac{\alpha_s(Q^2)N_c}{\pi}\mathcal{F}_g(x, K_\perp^2, Q^2)$$

- Real "gain" term: $\ell_\perp \ll K_\perp \simeq k_{g_\perp}$
- Virtual "loss" term: $K_\perp \ll \ell_\perp$
- β_0 term: virtual corrections from RG flow of gluon DTMD
- Θ-step functions: reduce emissions of partons harder than $\mu_0 \sim Q_s$
- Solve as boundary problem at $K_\perp^2 = Q^2$ (or better a fraction of Q^2)

- Exact solution!

$$\mathcal{F}_g(x, K_\perp^2, Q^2) = \frac{1}{\pi}\, \frac{\partial xG(x, K_\perp^2)\, \exp[-\mathrm{S}(K_\perp^2, Q^2)]}{\partial K_\perp^2}$$

- Sudakov factor

$$\mathrm{S}(K_\perp^2, Q^2) = \frac{N_c}{\pi} \int_{K_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\, \Theta(\ell_\perp, \mu_0)\, \alpha_s(\ell_\perp^2) \left(\frac{1}{2} \ln \frac{Q^2}{\ell_\perp^2} - \beta_0 \right)$$
$$\equiv \mathrm{S}_d(K_\perp^2, Q^2) + \mathrm{S}_s(K_\perp^2, Q^2)$$

- Gluon DPDF "integral of motion"

$$\int_0^{Q^2} \mathrm{d}^2\boldsymbol{K}\, \mathcal{F}_g(x, K_\perp^2, Q^2) = xG(x, K_\perp^2)$$

- Obeys DGLAP, source term by "tree-level" (MV/B-JIMWLK)
- All evolutions in one analytic solution

Boundary Condition

- Boundary condition has contributions from tree-level and DGLAP

$$\mathcal{F}_0(x, K_\perp^2) = \underbrace{\mathcal{F}_g^{(0)}(x, K_\perp^2)}_{\text{tree}} + \Theta(K_\perp, \mu_0)\, \frac{\alpha_s(K_\perp^2)}{2\pi^2}\, \frac{1}{K_\perp^2} \int_x^1 \mathrm{d}\xi\, \underbrace{P_{gg}^{(+)}(\xi)}_{\text{no}\,\beta_0}\, \frac{x}{\xi} G\left(\frac{x}{\xi}, K_\perp^2\right)$$

- Rewrite solution as

$$\mathcal{F}_g(x, K_\perp^2, Q^2) = \Big[\mathcal{F}_0(x, K_\perp^2) + \Theta(K_\perp, \mu_0)\, \frac{\alpha_s(K_\perp^2) N_c}{2\pi^2}\, \ln\frac{Q^2}{K_\perp^2}\, \frac{xG(x, K_\perp^2)}{K_\perp^2}\Big] \exp\big[-\mathrm{S}(K_\perp^2, Q^2)\big]$$

- For $K_\perp \gg \mu_0$, tree $\sim 1/K_\perp^4$
- Receives $1/K_\perp^2$ contributions from both DGLAP and CSS

- Relax strong ordering in transverse momenta

$$\frac{\partial \mathcal{F}_g(x,\boldsymbol{K},Q^2)}{\partial \ln Q^2} = \frac{N_c}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{\ell}}{\pi \ell_\perp^2}\,\alpha_s(\ell_\perp^2)\Big[\mathcal{F}_g(x,\boldsymbol{K}+\boldsymbol{\ell},Q^2)-\Theta(Q-\ell_\perp)\mathcal{F}_g(x,\boldsymbol{K},Q^2)\Big] + \Theta(Q,\mu_0)\beta_0\,\frac{\alpha_s(Q^2)N_c}{\pi}\,\mathcal{F}_g(x,\boldsymbol{K},Q^2)\,,$$

- Well defined in UV and IR
- To be solved with the boundary condition at $Q^2 = K_\perp^2$

- Fourier transform $\boldsymbol{K} \leftrightarrow \boldsymbol{b}$
- Problem 1: running coupling
- Problem 2: boundary condition
- Close eyes

$$\frac{\partial \tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2)}{\partial \ln Q^2} = -\frac{N_c}{\pi}\left[\frac{1}{2}\int_{\mu_b^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\,\alpha_s(\ell_\perp^2) - \Theta(Q,\mu_0)\beta_0\alpha_s(Q^2)\right]\tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2)$$

Straightforward solution

$$\tilde{\mathcal{F}}_g(x, b_\perp^2, Q^2) = \tilde{\mathcal{F}}_0(x, \mu_b^2)\exp\big[-\mathrm{S}(\mu_b^2, Q^2)\big]$$

with $\mu_b^2 = c_0^2/b_\perp^2$ and $c_0 = 2e^{-\gamma_{\mathrm{E}}}$

⊙ If $\mu_b^2 \gg \mu_0^2$: BC is proportional to DPDF, e.g. integrate DLA solution (total derivative)

$$\tilde{\mathcal{F}}_0(x, \mu_b^2) = \frac{1}{4\pi^2}\, xG(x, \mu_b^2)$$

⊙ If $\mu_b^2 \lesssim \mu_0^2$: no DGLAP, Sudakov $K_\perp$-independent. BC proportional to FT of tree level

$$\tilde{\mathcal{F}}_0(x, \mu_b^2) = \tilde{\mathcal{F}}_g^{(0)}(x, \mu_b^2)$$

⊙ Can interpolate the two regimes, e.g.

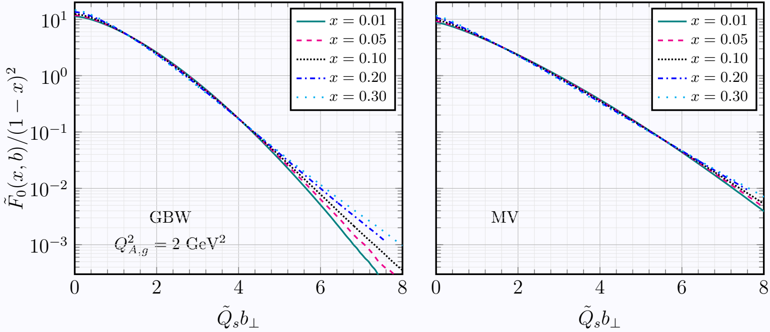
$$\tilde{\mathcal{F}}_0(x, \mu_b^2) = \frac{xG(x, \mu_b^2)}{xG^{(0)}(x, \mu_b^2)}\, \tilde{\mathcal{F}}_g^{(0)}(x, \mu_b^2)$$

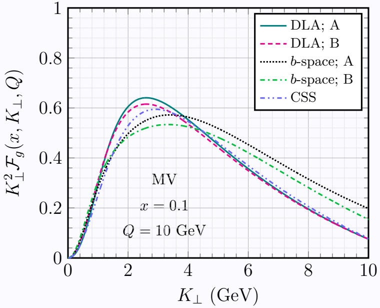
Tree level BC

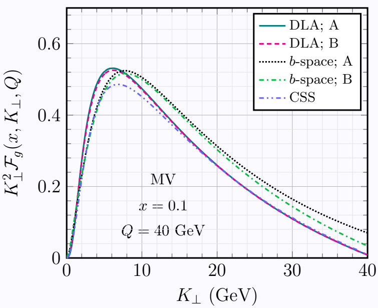
- Assume MV model. For $\mu_b \lesssim \mu_0$ dominated by $K_\perp \sim \mu_0$

$$\tilde{\mathcal{F}}_g^{(0)}(x,\mu_b^2) \sim \exp(-B)/B^2 \qquad \text{with} \qquad B = b_\perp^2 \mu_0^2/8$$

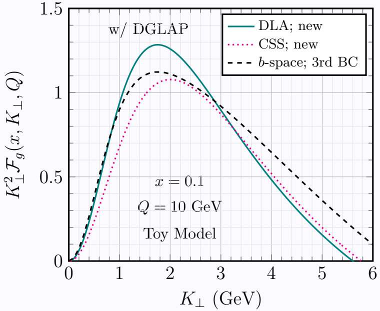
  - Exact expression in terms of E_1-function, finite $b_\perp = 0$ limit
  - This is our IR Sudakov
  - IR here refers to scale $\mu_0 \sim Q_s$

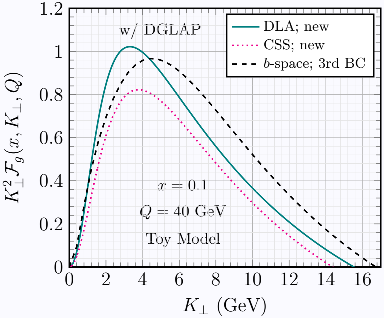






- $K^*_{\perp\text{tree}} \sim \tilde{Q}_s$ and $K^*_{\perp\text{CSS}} = Q e^{-\frac{1}{\sqrt{2\bar{\alpha}_s}}}$
- Results of 3 equations match perturbatively to order $\bar{\alpha}_s$
  Can choose the $K_\perp$-space boundary to match results to order $\bar{\alpha}_s^2$
  Cannot go beyond.





- DGLAP pushes peak to lower $K_\perp$
- Distribution negative above some $K_\perp$ for $x > 0$.

Conclusion

- Diffraction at hard momenta in γA collisions in CGC
- Diffractive hard dijet cross sections dominated by 2+1 jets due to scattering near unitarity limit
- Factorization: Diffractive gluon TMD
- CSS evolution in momentum and coordinate space
- For sufficiently large rapidity gaps and/or large nuclei gluon DTMD and DPDF calculated from "first principles"
- CGC, DGLAP and TMD evolution in the same formalism