Multiplicity distributions in QCD jets and jet topics

Bin Wu

Oct 15, 2025

IGFAE, Universidade de Santiago de Compostela, Galicia-Spain

"Resummation, Evolution, Factorization 2025"
University of Milan and INFN, Milan, Oct 13-17, 2025

Based on Duan, Chen, Ma, Salgado and BW, arXiv:2503.24200; arXiv:2509.06158.

MINISTERIO DE CIENCIA E INNOVACIÓN
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN
IGFAE
XUNTA DE GALICIA
USC

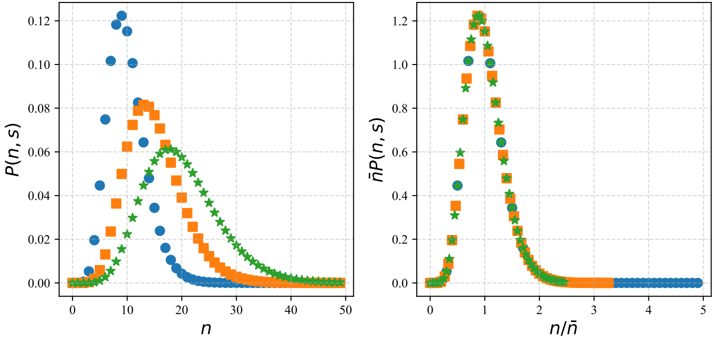
Introduction

Koba-Nielsen-Olesen (KNO) scaling

- $P(n,s)$**:** probability for producing n final-state particles in a collision at $\sqrt{s}$

$$\bar{n}P(n,s) = \Psi\left(n/\bar{n}\right)$$

$\Psi(x)$: a universal function with $\int dx\ \Psi(x) = 1$; $\bar{n}$: the mean multiplicity

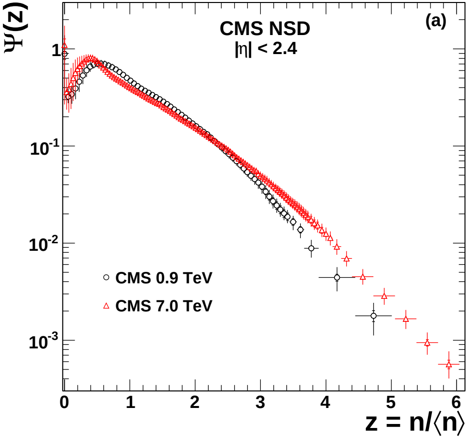


This was proposed before the discovery of QCD!

Z. Koba, H. B. Nielsen and P. Olesen, Nucl. Phys. B **40**, 317-334 (1972).

Violation of KNO scaling in *pp* collisions

- **KNO scaling is violated in *pp***



V. Khachatryan *et al.* [CMS], JHEP **01**, 079 (2011) [arXiv:1011.5531 [hep-ex]].

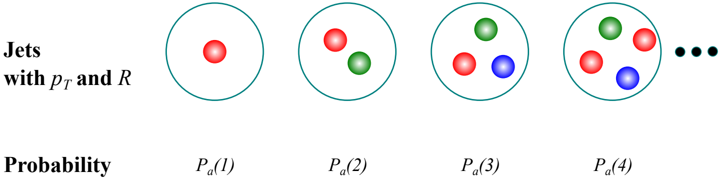
See a review: J. F. Grosse-Oetringhaus and K. Reygers, J. Phys. G **37**, 083001 (2010) [arXiv:0912.0023 [hep-ex]].

Multiplicity distributions in QCD jets

- **Earlier suggestions of KNO scaling within jets in pQCD**

Konishi, Ukawa and Veneziano, *Nucl. Phys. B* **157** (1979) 45; Bassetto, Ciafaloni and Marchesini, *Nucl. Phys. B* **163** (1980) 477.

- **Multiplicity in jets, given p_T and R, fluctuates**

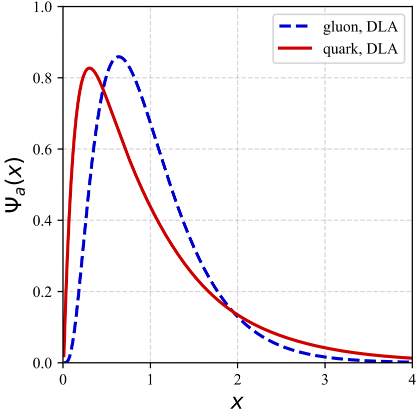


where $a = q/\bar{q}$ for quark jets and $a = g$ for gluon jets.

- **Now, let us generate Ψ to Ψ_a for jets initiated by parton a.**

KNO scaling functions for jets in DLA

- Ψ_a **has been evaluated in Double Log Approximation(DLA)**



They are different for quark and gluon jets!
In the limit $Q \to \infty$!

Dokshitzer, Fadin and Khoze, *Z. Phys. C* **18** (1983) 37; A. Bassetto, *Nucl. Phys. B* **303** (1988) 703.

Topics of this talk

1. Verify DLA KNO scaling in jets at LHC energies

   Duan, Chen, Ma, Salgado and BW,2503.24200.

2. Calculate Ψ_a by imposing energy conservation (MDLA)

   We generalize to quark jets the calculations for gluon jets in Dokshitzer, *Phys. Lett. B* **305** (1993) 295.

   See also: Dokshitzer and Webber,*JHEP* **08** (2025) 168 [2505.00652];2507.07691.

3. Describe the ATLAS results for inclusive multiplicity distributions

   ATLAS collaboration, *Phys. Rev. D* **100** (2019) 052011 [1906.09254].

4. Directly compare with $P_a(n)$ extracted using jet topics

   Metodiev and Thaler,*Phys. Rev. Lett.* **120** (2018) 241602 [1802.00008];

   Komiske, Metodiev and Thaler,*JHEP* **11** (2018) 059 [1809.01140].

KNO scaling functions in QCD jets

- **Calculations in terms of generating functions (GFs)**
  - The GF of $P_a(n)$ for a jet initiated by a parton a with scale Q

$$Z_a(u, Q) \equiv \sum_{n=0}^{\infty} u^n P_a(n, Q) \qquad \text{with } Z_a(1, Q) = 1$$

with Q the initial jet scale.

  - The GFs satisfy

$$\frac{\partial}{\partial \ln Q} Z_a(u, Q) = \int dz \frac{\alpha_s}{\pi} \hat{P}_{a\to bc}(z)[Z_b(u, zQ)Z_c(u, (1-z)Q) - Z_a(u, Q)]$$

where α_s is evaluated at the scale $z(1-z)\bar{Q}$, and

$$\hat{P}_{g\to gg}(z) = 2C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right],$$

$$\hat{P}_{g\to q\bar{q}}(z) = T_F[z^2 + (1-z)^2], \qquad \hat{P}_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$

Ellis, Stirling and Webber, *QCD and collider physics*, Cambridge University Press (2, 2011);

Dremin and Gary, *Phys. Rept.* **349** (2001) 301 [hep-ph/0004215].

- **Let us define the asymptotic KNO scaling functions:**

$$\Psi_a(n/\bar{n}_a) \equiv \lim_{Q\to\infty} [\bar{n}_a P_a(n,Q)].$$

- **Evaluation of KNO scaling functions:**

$$\Phi_a(\beta) \equiv \lim_{Q\to\infty} Z_a(e^{-\frac{\beta}{\bar{n}_a}}, Q) = \lim_{Q\to\infty} \sum_{n=0}^{\infty} \frac{e^{-\beta\frac{n}{\bar{n}_a}}}{\bar{n}_a} [\bar{n}_a P_a(n,Q)]$$

$$= \underbrace{\int_0^{\infty} dx\ \Psi_a(x) e^{-\beta x}}_{\text{Laplace transformation}} \Rightarrow \Psi_a(n/\bar{n}_a) = \int \frac{d\beta}{2\pi i} \Phi_a(\beta) e^{\beta\frac{n}{\bar{n}_a}},$$

where the integral over β runs parallel to the imaginary axis, with all singularities of Φ_a lying to the left in the complex plane.

Dokshitzer, Fadin and Khoze, *Z. Phys. C* **18** (1983) 37.

For a review: Y.L. Dokshitzer, Khoze, Mueller and Troian, *Basics of perturbative QCD*, Editions Frontieres (1991).

KNO scaling in jets within DLA

Multiplicity distributions in jets within DLA

- **To resum over double logs, it suffices:**

$$\hat{P}_{a\to ga}(z) \approx 2C_a/z, \qquad 1-z\approx 1$$

with $C_a = C_A$ for $a = g$ and $C_a = C_F$ for $a = q/\bar{q}$.

- **The evolution equation reduces to**

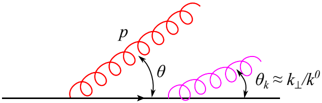
$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = Z_a(u,Q) c_a \int \frac{dz}{z}\gamma_0^2 \left[Z_g(u,zQ)-1\right]$$

with the anomalous dimension $\gamma_0 = \sqrt{\frac{2N_c\alpha_s}{\pi}}$, which can be used to resum all the double logs $\sim (\alpha_s L^2)^n$. Here,

- $c_g = 1$ for gluon jets
- $c_q = C_F/C_A$ for quark jets
- $\alpha_s = \alpha_s(zQ)$ if running is considered

In the following discussion, we take the jet scale $Q = p_T R$.

• Color coherence: angular ordering (AO)



$$zQ = k^0\theta > k_\perp \approx k^0\theta_k > Q_0,$$

with Q_0 the infrared cutoff (for parton multiplicity).

• Imposing AO:

$$Z_a(u,y) = u\exp\left\{c_a\int_0^y d\bar{y}\;(y-\bar{y})\gamma_0^2[Z_g(u,\bar{y})-1]\right\},$$

where $y \equiv \ln(Q/Q_0)$ and $\bar{y} \equiv \ln(k_\perp/Q_0)$.

Dokshitzer, Khoze, Mueller and Troian, *Basics of perturbative QCD*, Editions Frontieres (1991).

• DLA mean multiplcities:

$$\bar{n}_g = \begin{cases} \cosh(\gamma_0 y) & \text{for fixed coupling} \\ z_1\left[I_1(z_1)K_0(z_2) + K_1(z_1)I_0(z_2)\right] & \text{for running coupling} \end{cases},$$

and

$$\bar{n}_q(Q) - 1 = c_a[\bar{n}_g(Q) - 1],$$

where $z_1 \equiv A\sqrt{y+\lambda}$ and $z_2 \equiv A\sqrt{\lambda}$, with $\lambda \equiv \ln(Q_0/\Lambda)$, $A \equiv \sqrt{16N_c/\beta_0}$, and $\beta_0 = (11N_c - 2n_f)/3$.

• Within DLA, $P_a(n)$s satisfy:

$$P_a(1,Q) = \exp\left\{-c_a \int_0^y d\bar{y}\ (y-\bar{y})\gamma_0^2\right\},$$

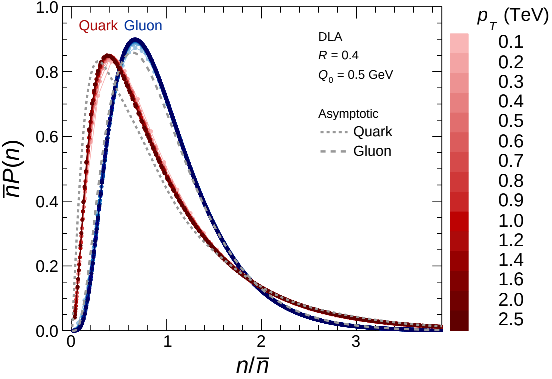
$$P_a(n+1,Q) = c_a \sum_{k=1}^{n} \frac{k}{n} P_a(n+1-k,Q) \int_0^y d\bar{y}\ (y-\bar{y})\gamma_0^2 P_g(k,\bar{y}).$$

We evaluate up to $n = 1000$ numerically to test KNO scaling.

Duan, Chen, Ma, Salgado and BW, 2503.24200.

Testing KNO scaling in DLA

- **For running coupling**



Duan, Chen, Ma, Salgado and BW, 2503.24200.

- $\bar{n}P(n)$ for both quark and gluon jets converge to some scaling functions
- Slightly deviation from the asymptotic scaling functions in

Dokshitzer, Fadin and Khoze, *Z. Phys. C* **18** (1983) 37; A. Bassetto, *Nucl. Phys. B* **303** (1988) 703.

- Their convergence depends on the choice of Q_0. $Q/Q_0 \gg 1$ is needed!

DLA results vs the ATLAS data

- **Inclusive jet multiplicity:**
  - Mixed jets: the two leading jets (anti-k_t) with a radius parameter $R = 0.4$
  - Kinematic cuts: $|\eta| < 2.1$, and $p_T^{\text{lead}}/p_T^{\text{sublead}} < 1.5$

ATLAS collaboration, *Phys. Rev. D* **100** (2019) 052011 [1906.09254].

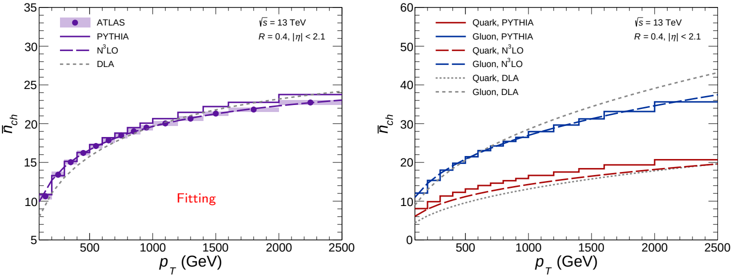
- **Theoretical framework:** LO + DLA

$$P(n) = r_q P_q(n) + r_g P_g(n),$$
$$\bar{n} = r_q \bar{n}_q + r_g \bar{n}_g,$$

with the fraction of jets initiated by parton a:

$$r_a \equiv \frac{d\sigma_a/dp_T}{d\sigma_q/dp_T + d\sigma_g/dp_T}.$$

• Mean charged-particle multiplicities



Duan, Chen, Ma, Salgado and BW,2503.24200.

- Local Hadron-Parton Duality: $\bar{n}_{ch} = K_{\rm LHPD} \times$ mean parton multiplicity

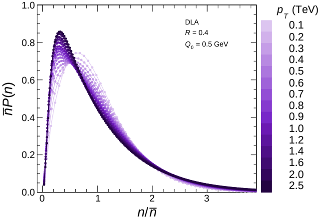
  $K_{\rm LHPD} = 0.8$ for DLA with running α_s, obtained by fitting

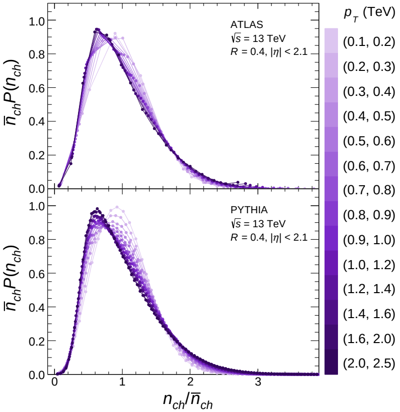
- The N^{3}LO formula is given in:

Dremin and Gary, *Phys. Lett. B* **459** (1999) 341 [hep-ph/9905477];

Capella, Dremin, Gary, Nechitailo and Tran Thanh Van, *Phys. Rev. D* **61** (2000) 074009 [hep-ph/9910226].

- **Ψ_a at DLA do NOT agree with the ATLAS data**





$$P(n) = r_q P_q(n) + r_g P_g(n) = r_q \frac{\Psi_q(x)}{\bar{n}_q} + r_g \frac{\Psi_g(x)}{\bar{n}_g},$$

by assuming KNO scaling.

But PYTHIA results are consistent with the ATLAS data

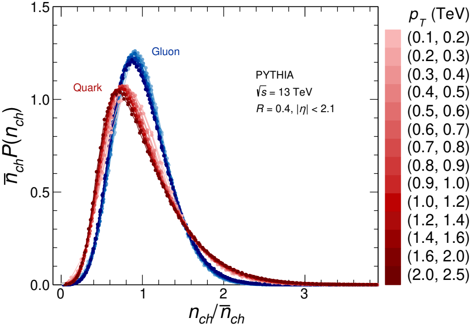
Duan, Chen, Ma, Salgado and BW,2503.24200.

See also previous observations in e^+e^-: E.D. Malaza and B.R. Webber, *Phys. Lett. B* **149** (1984) 501.

KNO scaling functions beyond DLA

KNO scaling in jets using PYTHIA

- **We extract $P_a(n)$ for quark and gluon jets using PYTHIA**



- The PYTHIA samples are generated using the A14 tune
- Quark-gluon discrimination: proximity to an initiating parton produced in a $2 \to 2$ hard scattering process.

Duan, Chen, Ma, Salgado, and BW, 2503.24200.

KNO scaling functions in MDLA

- **Modified DLA** = DLA + energy conservation

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,zQ) Z_a(u,(1-z)Q) - Z_a(u,Q) \right].$$

We generalize the calculations for gluon jets in Dokshitzer, *Phys. Lett. B* **305** (1993) 295.

- **By equaling the following two expansions**

$$\Phi_a(\beta) = \int_0^\infty dx\ \Psi_a(x) e^{-\beta x} = \sum_{k=0}^\infty \frac{(-\beta)^k}{k!} \underbrace{\int_0^\infty dx\ x^k \Psi_a(x)}_{\text{Mellin transform for } k+1} \equiv \sum_{k=0}^\infty \frac{(-\beta)^k}{k!} f_a^{(k)}$$

$$\Phi_a(\beta) = \lim_{Q\to\infty} \sum_{k=0}^\infty \frac{\left(e^{-\frac{\beta}{\bar{n}_a(Q)}} - 1\right)^k}{k!} n_a^{(k)}(Q) = \sum_{k=0}^\infty \frac{(-\beta)^k}{k!} \lim_{Q\to\infty} \frac{n_a^{(k)}(Q)}{[\bar{n}_a(Q)]^k}.$$

where the second expression corresponds to the expansion of Z_a at $u = 1$:

$$Z_a(u,Q) = \sum_{k=0}^\infty \frac{(u-1)^k}{k!} n_a^{(k)}(Q) \qquad \text{with } f_a^{(k)} \equiv \int_0^\infty dx\ x^k \Psi_a(x).$$

- By 1) using the asymptotic form $\bar{n}_a(Q) \approx c_a(Q/Q_0)^{\gamma_0}/2$;
  2) differentiating the evolution equation at $u = 1$:

$$f_g^{(m)} = \frac{\gamma_0 m}{m^2 - 1} \sum_{k=1}^{m-1} C_m^k \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} f_g^{(k)} f_g^{(q-k)},$$

$$f_q^{(m)} = \frac{c_q^{1-m}}{m^2} f_g^{(m)} + \gamma_0 \sum_{k=1}^{m-1} \frac{1}{m} C_m^k \frac{\Gamma(\gamma_0 k)\Gamma(\gamma_0(m-k)+1)}{\Gamma(\gamma_0 m + 1)} c_q^{1-k} f_g^{(k)} f_q^{(m-k)}.$$

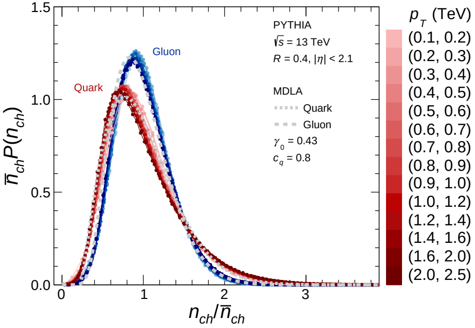
for $m \geq 2$ with $f_a^{(0)} = f_a^{(1)} = 1$.

There are two parameters:

- $\gamma_0 = \sqrt{\frac{2N_c\alpha_s}{\pi}} = 0.43$
- $c_q \approx \frac{\bar{n}_q}{\bar{n}_g} = 0.8$

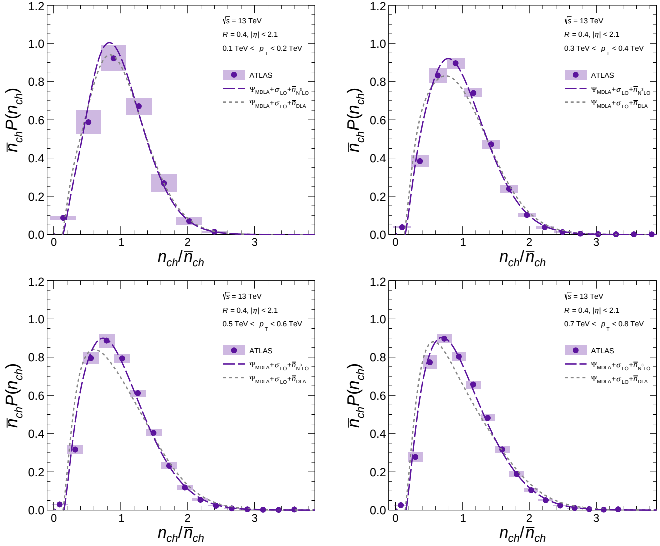
Duan, Chen, Ma, Salgado, and BW,

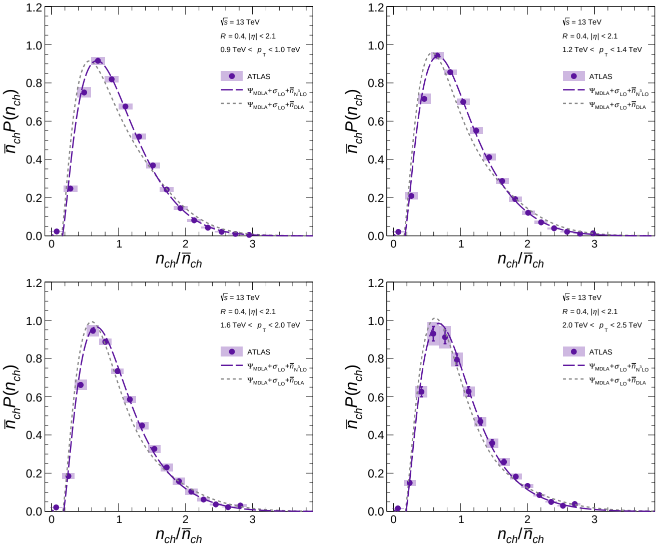
arXiv:2509.06158.



MDLA results vs the ATLAS data

- **MDLA scaling functions describe data very well!**





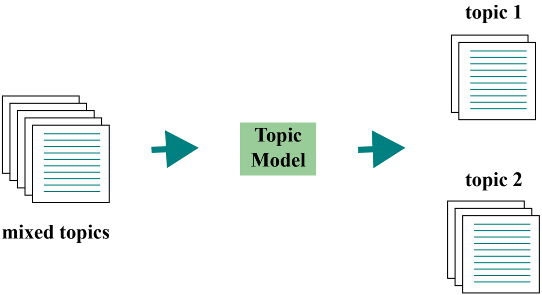
Ψ_{MDLA}: the MDLA scaling functions; σ_{LO}: the LO cross section; $\bar{n}_{DLA/N^3LO}$: the DLA/N^{3}LO mean multiplicities

Duan, Chen, Ma, Salgado, and BW, arXiv:2509.06158.

MDLA vs distributions for jet topics

Jet topics

- **Topic modeling:** a form of unsupervised machine learning



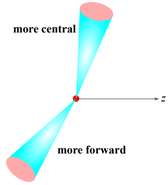
For a review: David M Blei, Communications of the ACM 55, 77–84 (2012).

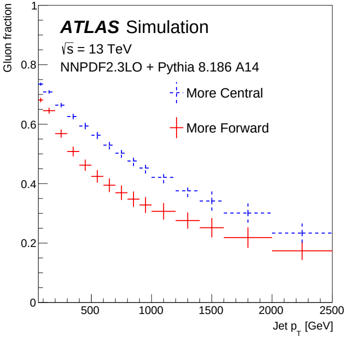
- **Topic modeling can be used to identify quark & gluon jets**

Metodiev and Thaler, *Phys. Rev. Lett.* **120** (2018) 241602 [1802.00008];

Komiske, Metodiev and Thaler, *JHEP* **11** (2018) 059 [1809.01140].

- **More central (c) and more forward (f) jet samples:**





ATLAS collaboration, *Phys. Rev. D* **100** (2019) 052011 [1906.09254].

- **Multiplicity distributions for topics 1 ($\approx q$) and 2 ($\approx g$):**

$$h_i^{T_1} = \frac{h_i^f - \min_j\{h_j^f/h_j^c\}\, h_i^c}{1 - \min_j\{h_j^f/h_j^c\}}, \qquad h_i^{T_2} = \frac{h_i^c - \min_j\{h_j^c/h_j^f\}\, h_i^f}{1 - \min_j\{h_j^c/h_j^f\}}$$

$h_i^{f/c}$: charged-particle multiplicity distributions for forward and central jets.

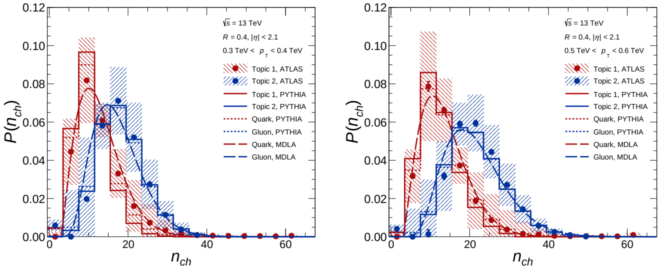
Multiplicity distributions for jet topics

- **We keep only more precise data points by imposing:**

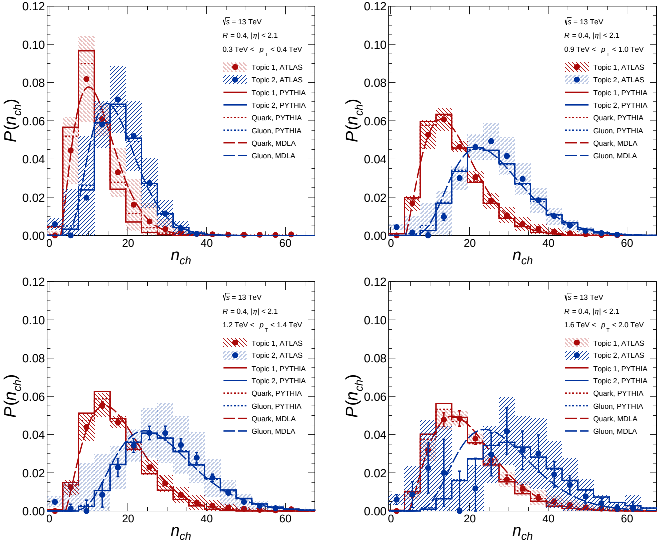
$$\sigma_{r_{a/b}}/r_{a/b} < 25\%,$$

where $r_{a/b} \equiv h_j^a/h_j^b$, and $\sigma_{r_{a/b}}$ the propagated uncertainty of this ration.

- **Comparison with MDLA and PYTHIA:**

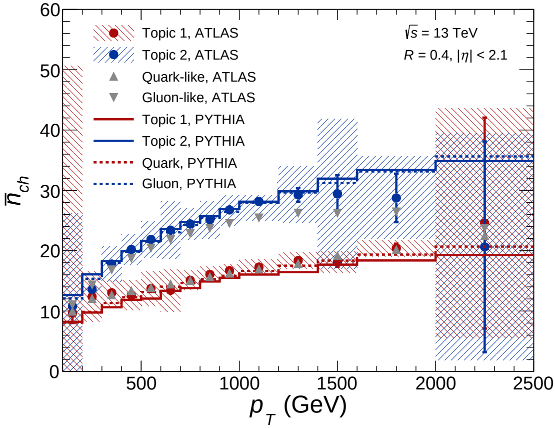


- **Comparison with MDLA and PYTHIA at higher p_T:**



Duan, Chen, Ma, Salgado, and BW, arXiv:2509.06158.

- $\bar{n}_{ch}$ **in quark (topic 1) and gluon (topic 2) jets:**



Duan, Chen, Ma, Salgado, and BW, arXiv:2509.06158.

Summary and Outlook

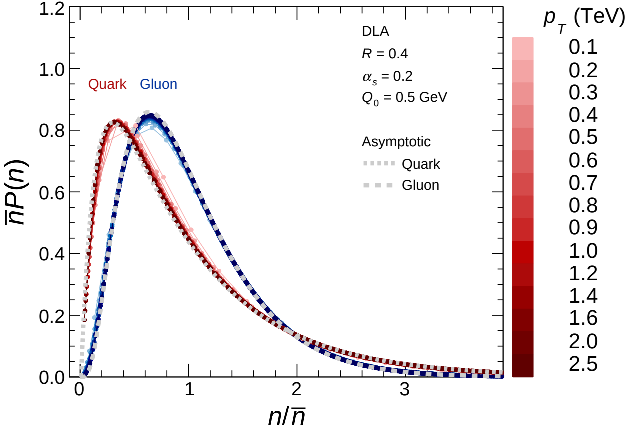
- ✓ We evaluated the MDLA KNO scaling functions for quark and gluon jets.
- ✓ With the MDLA KNO scaling functions, a very good description of inclusive charged-particle multiplicity distributions measured by ATLAS is given over a wide jet p_T range of 0.1–2.5 TeV.
- ✓ The MDLA scaling functions agree with those of quark and gluon jets extracted via jet topics within the propagated uncertainties from experimental data.
- ❑ One curiosity: Ψ_a are infrared safe although $\bar{n}_a$ and $P_a(n)$ for partons are not! It indicates that parton-level calculations should be sufficient (?).
- ❑ We did not observe the necessity of running α_s, as for e^+e^- collisions, and found some quantitative discrepancies with the results in:

Dokshitzer and Webber, 2507.07691.

Backup slides

Testing KNO scaling in DLA

- **For fixed coupling**

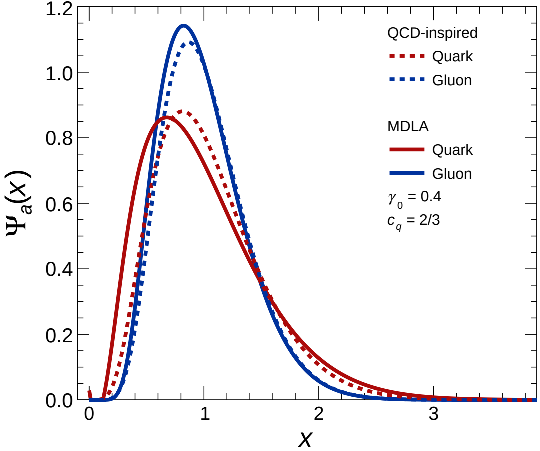


Duan, Chen, Ma, Salgado and BW, 2509.06158.

Comparison with Dokshitzer & Webber

Dokshitzer and Webber, 2507.07691.

- **Derived from the relation:** $\Phi_q(\beta) = [\Phi_g(\beta/c_q)]^{c_q}$



Duan, Chen, Ma, Salgado and BW, 2509.06158.