Resummation and zero momentum modes

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with

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"...the picture of an experimenter as one counting soft photons is not entirely realistic ... the experimenter does not count single photons but rather an unbalance of energy and momentum between the incident and emergent particles."

Bruno Touschek, Nuovo Cimento 1967

Outline

- **Bruno Touschek** formalism for Soft photon distributions in QED
- Soft photon emission in creation and disappearance of an **unstable state**
- Soft photon final state emission for electron positron scattering into multihadron or jet production
- Discrete and continuum in Touschek's formalism :

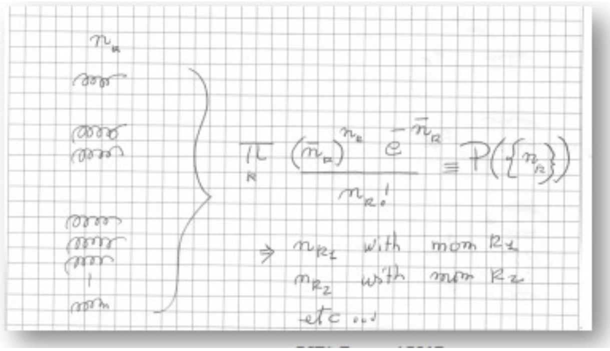
  Zero momentum mode in QED ?
- The zero momentum case in **QCD transverse momentum** distributions
- The **intrinsic** transverse momentum of Drell-Yan pairs
- The case of **the forward slope**

Bruno Touschek formalism for Soft photon distributions in QED - 1

A statistical classical approach + energy momentum conservation

$$d^4P(K) = \sum P(\{n_{\mathbf{k}}\})\delta_4(\sum_{\mathbf{k}} k n_{\mathbf{k}} - K)d^4K = \frac{d^4K}{(2\pi)^4}\int d^4x e^{iK\cdot x - h(x)}$$

$$P(\{n_{\mathbf{k}}\}) = \Pi_{\mathbf{k}}\frac{\bar{n}_{\mathbf{k}}^{n_{\mathbf{k}}}}{\bar{n}_{\mathbf{k}}!}e^{-\bar{n}_{\mathbf{k}}}$$



$$h(x) = \sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1 - e^{i(k\cdot x)}]$$

- Take the continuum limit $\quad \bar{n}(\mathbf{k}) \quad \to continuum \to \quad d^3\bar{n}(\mathbf{k}) = (2\pi)^6\frac{d^3\mathbf{k}}{\omega}|j_\perp(\omega,\mathbf{k})|^2$
- In co-variant notation and first order in **α** $\quad |\mathbf{j}_\perp|^2 = j_\mu j^\mu = \frac{\alpha}{(2\pi)^2}\sum_{i,j}\frac{(p_i\cdot p_j)\epsilon_i\epsilon_j}{(p_i\cdot k)(p_j\cdot k)}$

F. Bloch and A. Nordsieck. Note on the Radiation Field of the electron. Phys. Rev. 52, 54 59, 1937

W. Thirring and B. Touschek, A covariant formulation of the Bloch-Nordsieck method, *Phil.Mag.Ser.7* 42 (1951) 244-249

E. G. Etim, G. P., and B. Touschek. The Infra-red Radiative Corrections to Colliding Beam (Electrons and Positrons) Experiments. Nuovo Cim. B, 51:276–302, 1967.

M. Greco andG.Rossi,NuovoCimento,B., A Note on the Infrared Divergence, *Nuovo Cim.* 50 (1967) 168

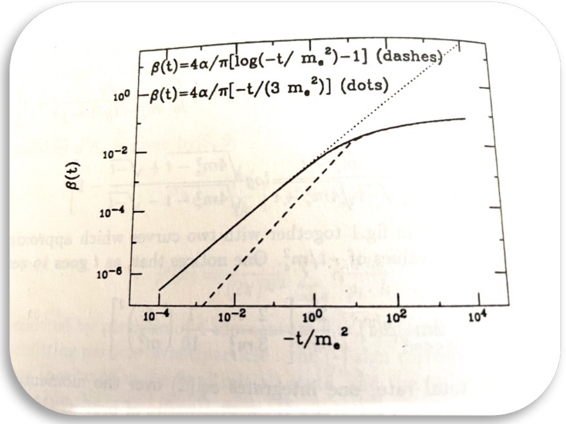
Bruno Touschek formalism for Soft photon energy distributions in QED - 2

$$d^2\sigma_{exp}(\theta,\phi) = d^4P(K)d^2\sigma_E(\theta,\phi)$$

$$\boxed{dP(\omega) = \frac{d\omega}{2\pi}\int dt e^{i\omega t - h(E,t)} = \frac{\beta_\gamma}{N_{\beta_\gamma}}\frac{d\omega}{\omega}\left(\frac{\omega}{E}\right)^{\beta_\gamma}}$$

$$p_1 + p_2 \to p_3 + p_4$$

$$\beta_\gamma = \frac{\alpha}{(2\pi)^2}\int d^2\hat{n} \sum_{polarization\ \hat{e}} \left|\sum_i \frac{(p_i\hat{e})\epsilon_i}{(\mathbf{p}_i\hat{n}) - p_{0i}}\right|^2 = \frac{\alpha}{(2\pi)^2}\int d^2\hat{n}\sum_{i,j}\frac{(p_i\cdot p_j)\epsilon_i\epsilon_j}{(p_i\cdot n)(p_j\cdot n)}$$



$$\beta_\gamma = \frac{2\alpha}{\pi}\left[-I_{12} + I_{13} + I_{14} - 2\right], \qquad I_{ij} = (2p_i{\cdot}p_j)\int_0^1 \frac{dy}{2y(1-y)[(p_i\cdot p_j) - m^2] + m^2}$$

Non rel $\rightarrow$ $\beta_\gamma = \frac{8\alpha}{3\pi}\sum_i |\epsilon_i \mathbf{v_j}|^2.$

A very special limit→

$$\left\{\begin{array}{ll} s, -u \to \infty, & I_{12}, I_{14} \approx -2\ln\frac{s}{m^2} \\ & I_{12} - I_{14} \to 0 \\ -t \to 0 & I_{13} \to \frac{-t}{m^2} \end{array}\right.$$

G.P., Reggeization of the photon in quantum electrodynamics. Phys.Lett. B, 44:109–111, 1973.

1967: the goal of electron positron colliders was to mesure vector mesons properties

$$d^2\sigma_{exp}(\theta,\phi) = d^4P(K)d^2\sigma_E(\theta,\phi)$$

no separation

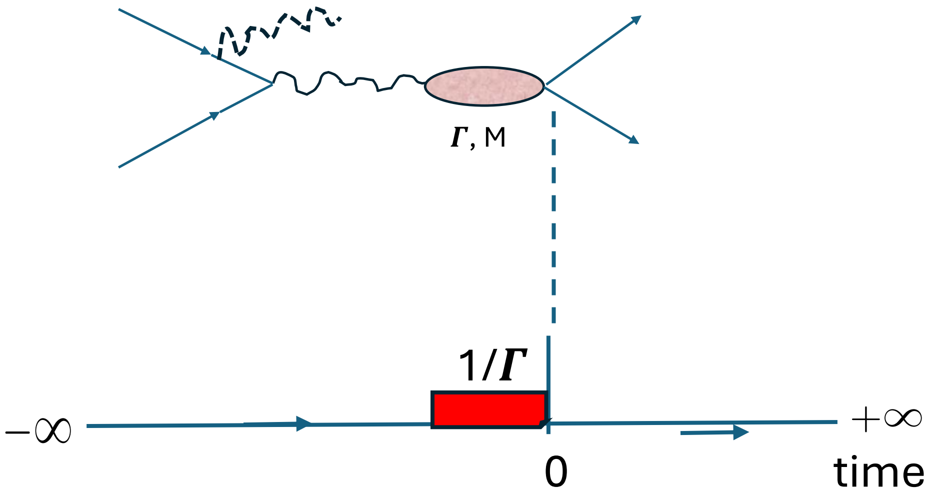
How to extend the formalism?

$$d^2\sigma(E,M,\Gamma) \propto \left(\frac{\Gamma^2}{4} + (2E-M)^2\right)^{-2}$$

Soft photon emission in creation and disappearance of an unstable state

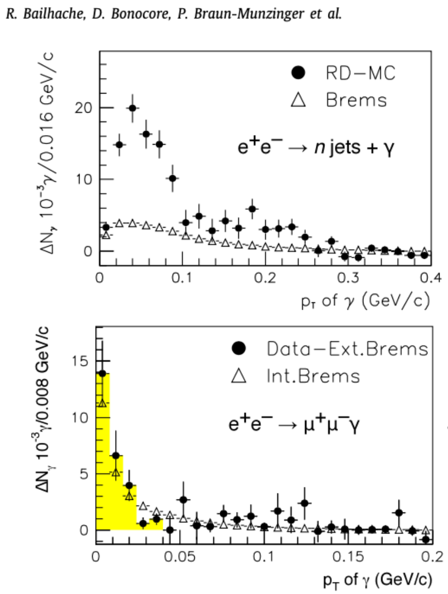
Anomalous Soft Photon effect
EMMI collaboration 2024

e+e- → soft photons +leptons



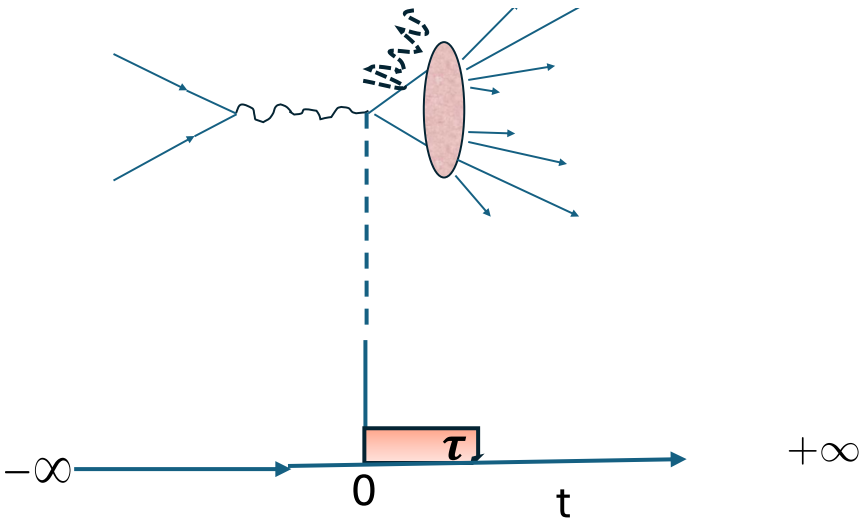
$$j_{in}(x) = [\frac{\Gamma}{2} - i(2E - M)] \int_0^\infty dt_0 \; j_{in}(x)\theta(-t - t_0)e^{-\frac{\Gamma}{2}t_0 + i(2E-M)t_0}$$

$$j_\mu^{Res}(x) = \sum_{i,f} [j_\mu^f(x)\theta(t) + S_{Res}(\Gamma, M)\int dt_0 j_\mu^i(x)\theta(-t - t_0)e^{-S_{Res}t_0}]$$



Eur.Phys.J.C57(2008)499–514

e+e- → soft photons + jets



τ : hadron formation –time?

$$j_{fin}(x) = [\frac{1}{\tau} - i(2E - \mathcal{M}_{jets})] \int_0^\infty dt_0 \; j_{fin}(x)\theta(t - t_0)e^{-\frac{1}{\tau}t_0 + i(2E-\mathcal{M}_{jets})t_0}$$

Work in progress

G. Pancheri. Infra-red radiative corrections for resonant processes. Il Nuovo Cimento A, 60:321–329, 1969.
M.Greco, G.P. and Y.N.Srivastava, Radiative Corrections for Colliding, Beam Resonances. Nucl. Phys. B, 101:234–262, 1975

Discrete and continuum extension in Touschek's formalism :
Zero momentum mode in QED ?

$$\sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}[1 - e^{i(k\cdot x)}] = \bar{n}_{\mathbf{k=0}} + \sum_{\mathbf{k}\neq 0} \bar{n}_{\mathbf{k}}[1 - e^{-i(k\cdot x)}]$$

$$\bar{n}(\mathbf{k}) \quad \to continuum \quad \to d^3\bar{n}(\mathbf{k}) = (2\pi)^6 \frac{d^3\mathbf{k}}{\omega} |j_\perp(\omega, \mathbf{k})|^2$$

$$dP(\omega) \to dP_{ZMM}(\omega) = \frac{d\omega}{2\pi} \int dt e^{i\omega t - h_0(t) + \beta_\gamma \int_0^E \frac{dk}{k}[1-e^{-ikt}]}$$

$$h_0(t) = \bar{n}_0[1 - e^{-i\omega_0 t}] = \frac{2\pi e^2}{L^3\mu^3} |\sum_i \epsilon_i \mathbf{v}_i|^2 [1 - e^{-i\mu t}] \approx i\tilde{\omega} t$$

$$dP(\omega) \to dP(\omega)_{ZMM} =$$

$$\beta_\gamma \frac{d\omega}{\omega - \tilde{\omega}} \left(\frac{\omega - \tilde{\omega}}{E}\right)^{\beta_\gamma} \Theta(\omega - \tilde{\omega})$$

$$\tilde{\omega} = \frac{2\pi e^2}{L^3\mu^3} \mu |\sum_i \epsilon_i \mathbf{v}_i|^2 \qquad n.r.$$

QED $\quad \tilde{\omega} = 0 \quad$ v.b.c

Limit depends on boundary conditions

F. Palumbo, Physical Effects of Boundary Conditions in Gauge Theories. Phys. Lett. B,132:165–168, 1983

F. Palumbo and G.P., Infrared Radiative Corrections and the Zero Momentum Mode of Abelian Gauge Theories,. Phys. Lett. B, 137:401– 403, 1984.

How about QCD?

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b} e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-\sum_{\mathbf{k}}\bar{n}_{\mathbf{k}}[1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]}$$

Semi-classical & very general

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b} e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-h_0(\mathbf{b})-\mathbf{h_{continuum}}(\mathbf{b})}$$

$$h_{continuum}(b) = \frac{8}{3\pi^2}\int^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_s(k_t^2)}{k_t^2}\ln[\frac{2q_{max}}{k_t}][1-e^{i\mathbf{k}_t\cdot\mathbf{b}}]$$

$$h_0(\mathbf{b}) = n_0[1-e^{i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}}] \approx \mathbf{n}_0[-i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}] + \mathbf{n}_0\mu^2 b^2/4 + \ldots \approx \nu_0^2 b^2/4$$

The continuum contribution to $d^2P(\mathbf{K}_\perp)$

with emission from valence quarks given by

$$h(b) = \frac{8}{3\pi^2}\int^{q_{max}} d^2\mathbf{k}_t \frac{\alpha_s(k_t)}{k_t^2}\ln(2q_{max}/k_t)[1 - e^{i\mathbf{k}_t\cdot\mathbf{b}}] \quad (23)$$

continuum

1. **Model** $\alpha_s(k_t)$ to **integrate down to zero**

2. **Split** the integral $\{0, \Lambda\}$ and $\{\Lambda, q_{max}\}$

3. $h_{continuum} \propto b^2\Lambda^2$

x (infrared with logs)+ anomalous dimensions term

$p_t^{intrinsic} \propto \Lambda^2$

Logs dependence through $q_{max}(s)$

Modelling $\alpha_s(k_t)$ in the infrared region {0,Lambda}

1. $\alpha_s(k_t \to 0) \to \propto constant$

$$\alpha_s(k_t) = \frac{12\pi}{(11N_c - 2N_f) ln[a + \frac{k_t^2}{\Lambda^2}]}$$

2. $\alpha_s(k_t \to 0) \to \propto (k_t/\Lambda)^{-2p}$

p<1 for integrability

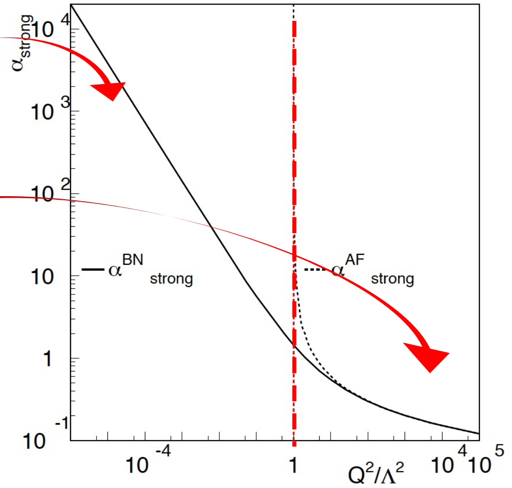
choosing 2.

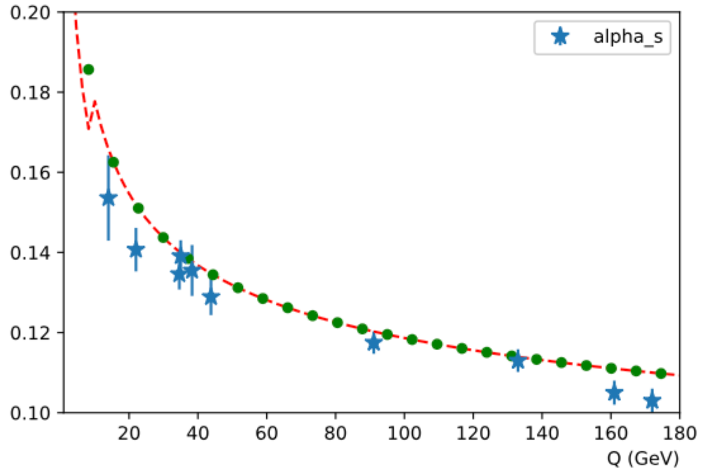
$$b_0 = \frac{11N_c - 2N_f}{12\pi}$$

$$\alpha_s(k_t)^{interpolation} = \frac{1}{\ln[1 + (\frac{k_t}{\Lambda})^{2b_0}]}$$

$$\to_{k_t \to 0} \quad (k_t/\Lambda)^{-2b_0}$$

$$\to_{k_t >> \Lambda} \quad \frac{1}{b_0 \ln(\frac{k_t}{\Lambda})^2}$$





$\Lambda = 100\ MeV$

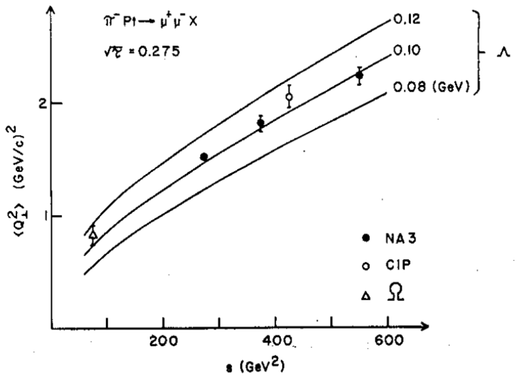
G.P., Y.N. Srivastava and O. Shekhovtsova, Ansatz about a Zero Momentum Mode in QCD and the Forward Slope in pp Elastic Scattering. Ukr. J. Phys., 69(11):879, 2024

The continuum contribution to the intrinsic transverse momentum of Drell-Yan pairs

$$h^{IR}_{continumm}(b,s) = \left(\frac{b^2\Lambda^2}{4}\right)\frac{16}{3\pi}\int_0^{\Lambda} dk_t\left(\frac{k_t}{\Lambda}\right)^{1-2b_0}\ln\left(\frac{2q_{max}(s)}{k_t}\right)$$

$$\bar{\Lambda}(b,s) = \Lambda\left\{\frac{c_F\bar{b}}{4\pi(1-p)}[\ln(2q_{max}(s)b) + \frac{1}{1-p}]\right\}^{1/2p}$$

$$< p_t^2 >_{intrinsic} = constant + \propto ln(q_{max})$$



1984 with Nakamura

PHYS. REV. D 111, 072003 (2025)

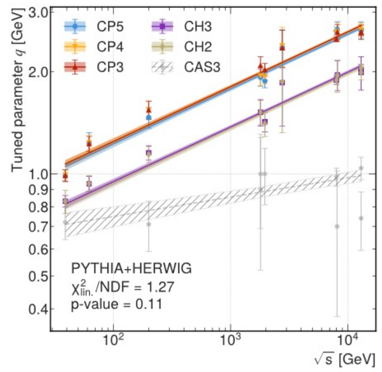
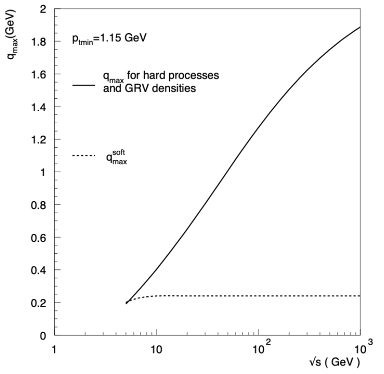
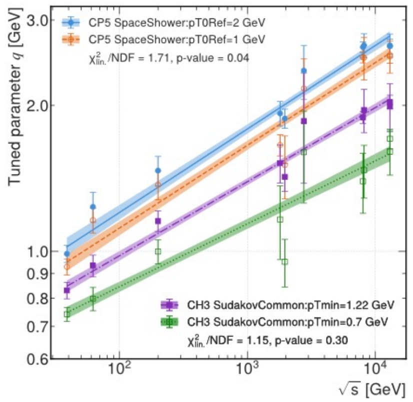


FIG. 2. Tuned parameter q values for DY measurements at different center-of-mass energies (points) for various PYTHIA and HERWIG setups (colors). The error bars on the points represent the tuning uncertainties. The tuned values are given in Appendix. For each generator setup, the function $b\sqrt{s}^a$ is fitted to the points and shown as a line, assuming the same slope a for all the settings.





Using kinematics for single soft gluon emission
From eikonal mode Phys.Rev.D72:076001,2005

CMS

A. Grau, A. R.M. Godbole, and G.P. Infrared gluons, intrinsic transverse momentum and rising total cross-sections, Nucl. Phys. B Proc. Suppl. 198, (2010)17--24
A. Grau, A. Nakamura, G.P. and Y.N. Srivastava, The Intrinsic Transverse Momentum of {Drell-Yan} Pairs, *Z.Phys.C* 21 (1984) 243
P. Chiappetta and M. Greco, Nucl. Phys.B199 (1982) 77.

The **zero momentum mode** in QCD transverse momentum distributions - 2

$$d^2P(\mathbf{K}_\perp) = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}e^{-i\mathbf{K}_\perp\cdot\mathbf{b}-h_0(\mathbf{b})-\mathbf{h_{continuum}}(\mathbf{b})}$$

$$h_0(\mathbf{b}) = n_0[1-e^{i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}}] \approx \mathbf{n}_0[-i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}] + \mathbf{n}_0\mu^2b^2/4 + \ldots \approx \nu_0^2b^2/4$$

Zero mode contribution

Cut off in b^2→ could changes the one-channel eikonal models for pp elastic scattering

G.P.,Y.. Srivastava and O. Shekhovtsova, Ansatz about a Zero Momentum Mode in QCD and the Forward Slope in pp Elastic Scattering. Ukr. J. Phys., 69(11):879, 2024

Zero modes and the case of the forward slope in p-pbar scattering

Empirical parametrization

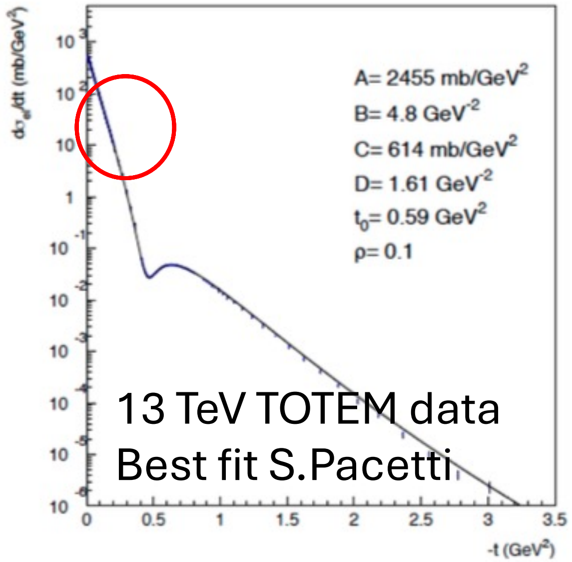
$$\mathcal{A}(s,t) = i[F_p^2(t;t_0)\sqrt{A(s)}e^{B(s)t/2} + e^{i\phi}\sqrt{C(s)}e^{D(s)t/2}]$$

Eikonal Block-Norsdieck model for total cross-section

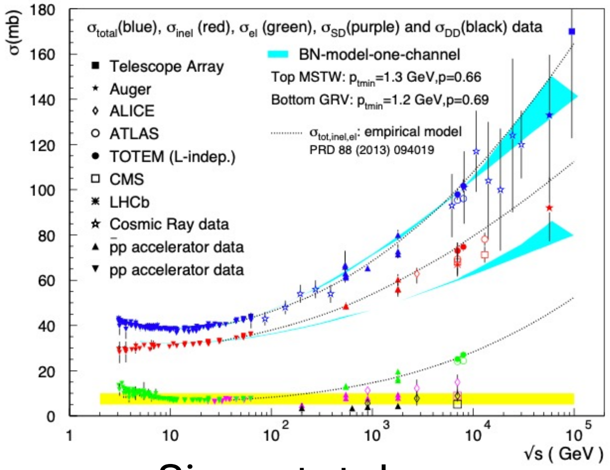
$$\sigma_{total}(s) = \int bdb[1 - e^{-\mathcal{I}m\chi(b,s)}]$$

$$Im\chi(b,s) = low\ energy\ term + A(b,s)\sigma_{mini-jet}(s)$$

$$A(b,s) = N\mathcal{F}[d^2P(\mathbf{K}_\perp] = Ne^{-h(b,s)} = \frac{e^{-h(b,s)}}{\int d^2\mathbf{b}e^{-h(b,s)}}$$



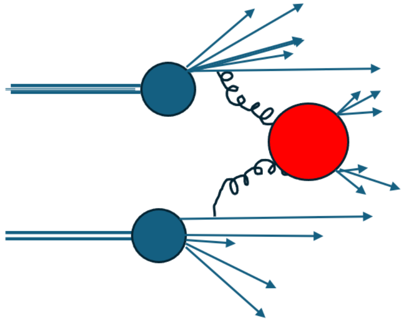
13 TeV TOTEM data
Best fit S.Pacetti



Sigma total

A.Grau, R. Godbole, G.P., Y.N. Srivastava, *Phys.Rev.D* 72 (2005) 076001

The elastic pp differential cross-section



eikonal Minijet model + Resummed Soft gluon emission from initial state quarks

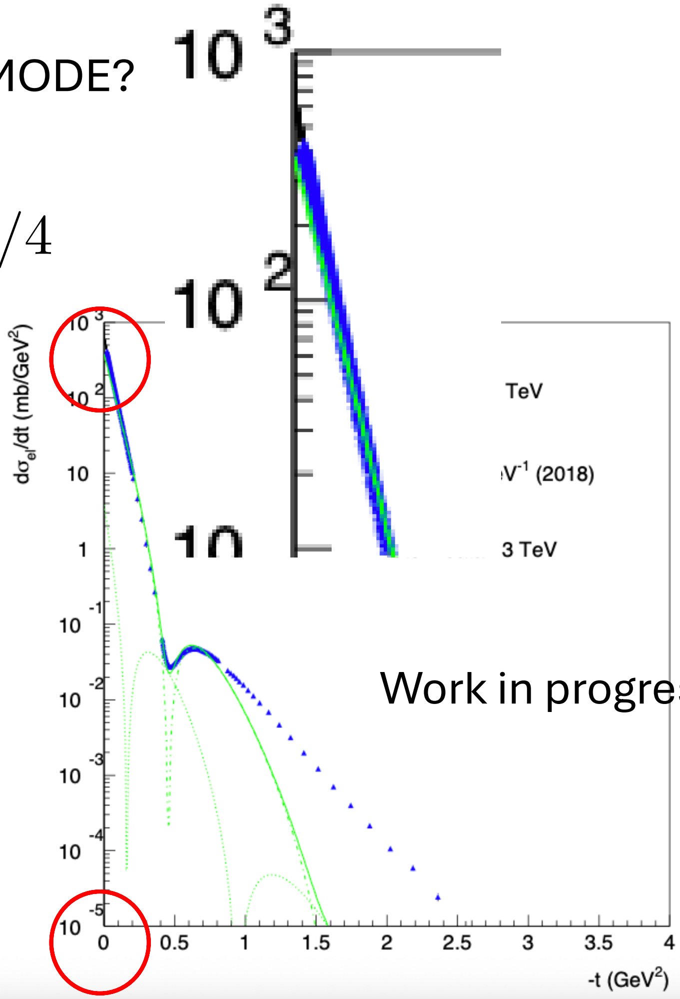
→ Bloch-Nordsieck Eikonal minijet model + ZERO MODE?

$$h_0(\mathbf{b}) = n_0[1 - e^{i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}}] \approx \mathbf{n}_0[-i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}] + \mathbf{n}_0\mu^2 b^2/4 + \ldots \approx \nu_0^2 b^2/4$$

$$\frac{d\sigma}{dt} = |\mathcal{A}s,t|^2 = |\int bdb[1 - e^{i\chi(b,s)}]J_0(b,q)|^2$$

$$Im\chi(b,s) = low\ energy\ term + A(b,s)\sigma_{mini-jet}$$

$$A(b,s) = Ne^{-h(b,s)} = \frac{e^{-h(b,s)}}{\int d^2\mathbf{b}e^{-h(b,s)}}$$



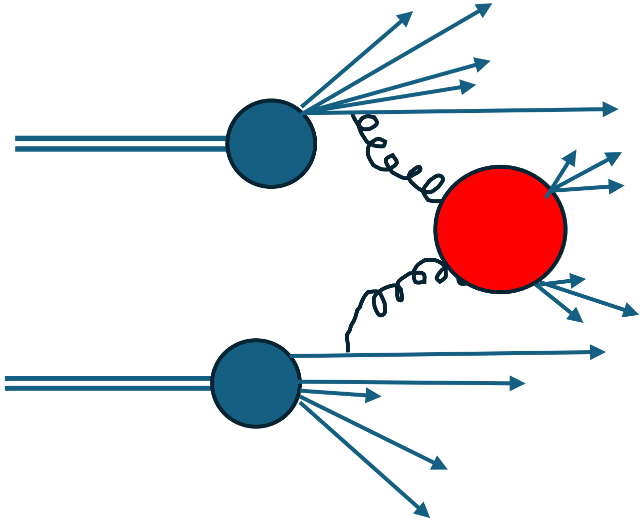
Work in progress

R.M.Godbole, A.Grau, G.P, Y.N.. Srivastava, Soft gluon radiation and energy dependence of total hadronic cross-sections. Phys. Rev. D, 72:076001, 2005.

Conclusions

- Touschek's "democratic" resummation formalism for QED can give insight to non purely QED processes such as
  - ISR corrections for resonance production in e+e-
  - FSR corrections with jets in e+e- collisions→ASP effect
- The formalism allows inspection of the transition from discrete → continuum in QED
- Extension to QCD and Inspection of the transition from discrete → continuum
  - intrinsic transverse momentum from the continuum is not sufficient → the zeromode might help →Work In Progress
  - The forward slope in eikonal modes

The elastic pp differential cross-section



eikonal
Minijet model

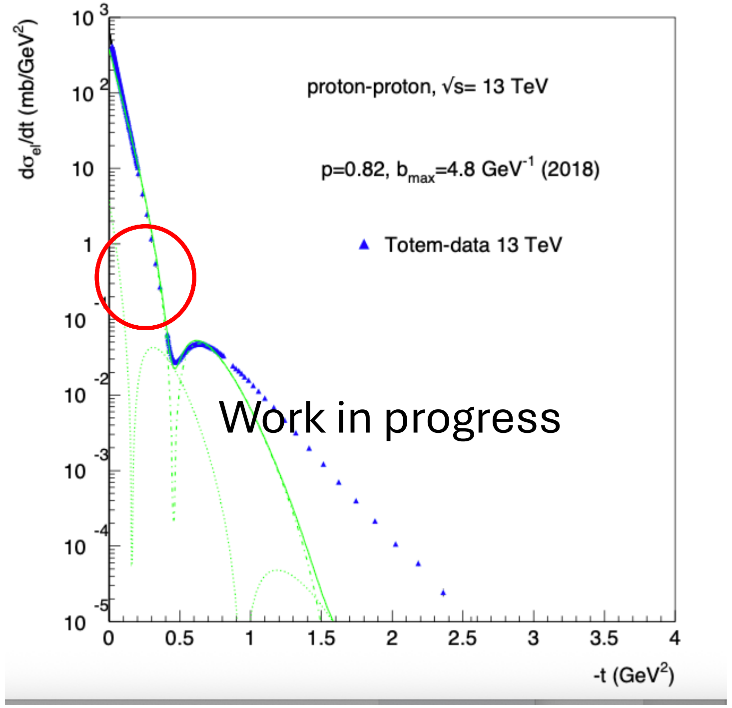
\+ Resummed Soft gluon emission from initial state quarks

Bloch-Nordsieck Eikonal minijet model

$$\frac{d\sigma}{dt} = |\mathcal{A}s,t|^2 = |\int bdb[1 - e^{i\chi(b,s)}]J_0(b,q)|^2$$

$$Im\chi(b,s) = low\ energy\ term + A(b,s)\sigma_{mini-jet}$$

$$A(b,s) = Ne^{-h(b,s)} = \frac{e^{-h(b,s)}}{\int d^2\mathbf{b}e^{-h(b,s)}}$$



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$$h_0(\mathbf{b}) = \mathbf{n}_0[1 - e^{i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}}] \approx \mathbf{n}_0[-i\mu\hat{\mathbf{k}}_t\cdot\mathbf{b}] + \mathbf{n}_0\mu^2 b^2/4 + \ldots] \approx \nu^2 b^2\ /4$$

Soft photon final state emission for electron positron scattering into multihadron or jet production

$$dP(\omega) = \frac{d\omega}{2\pi} \int dt e^{i\omega t - h(E,t)} = \frac{\beta_\gamma}{N_{\beta_\gamma}} \frac{d\omega}{\omega} \left(\frac{\omega}{E}\right)^{\beta_\gamma}$$

$$dP(\omega) \to dP_{ZMM}(\omega) = \frac{d\omega}{2\pi} \int dt e^{i\omega t - h_0(t) + \beta_\gamma \int_0^E \frac{dk}{k}[1-e^{-ikt}]}$$

G.P., Reggeization of the photon, Phys.Lett.