Andrea Simonelli

Reggeon exchange off the lightcone

Resummation, Evolution, Factorization 2025

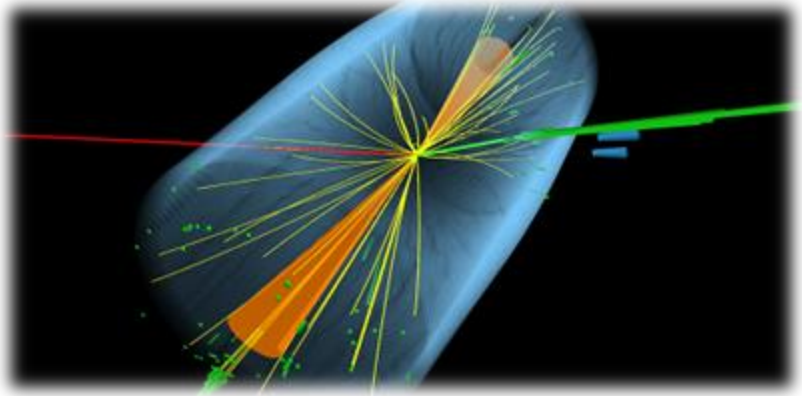
INFN Istituto Nazionale di Fisica Nucleare Sezione di Roma

Introduction

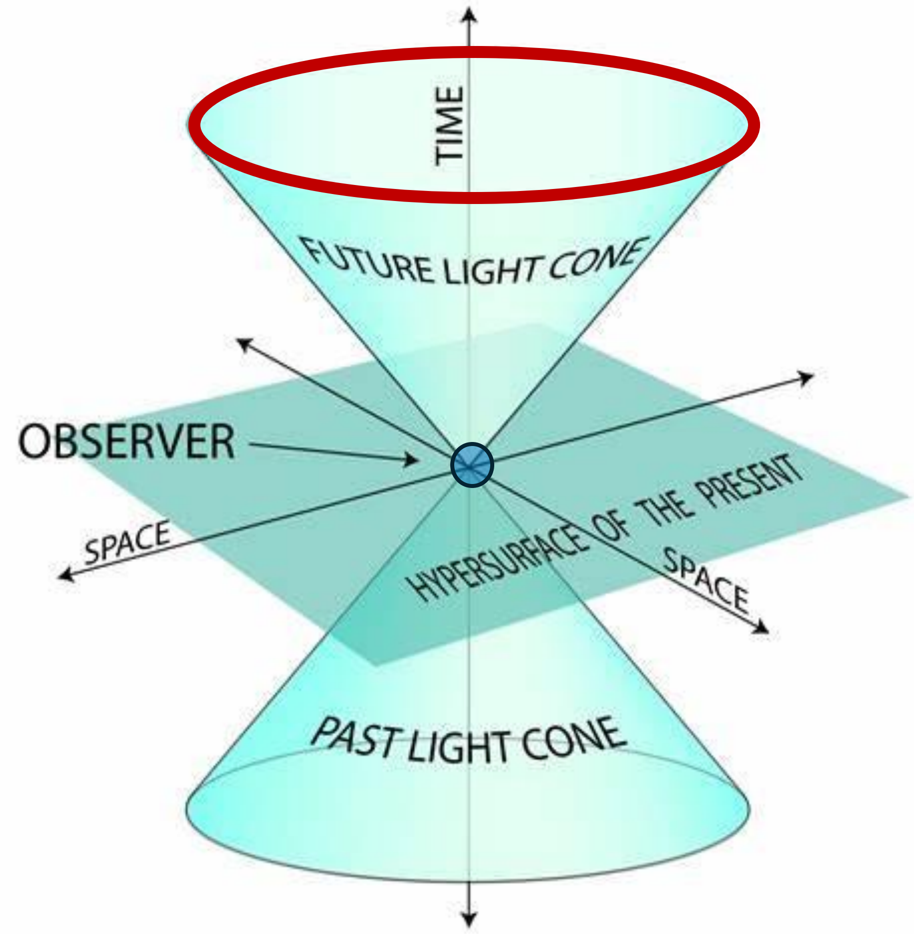
A broader meaning of universality

Factorization

Take a (hadronic) process…



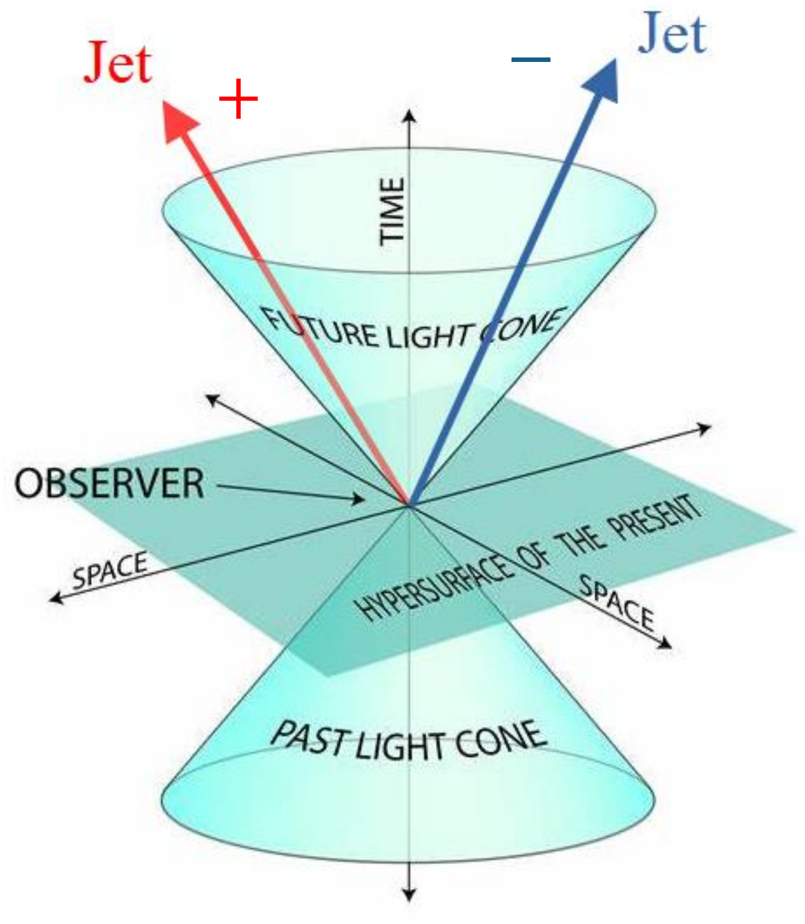
…it has a well-defined underlying spacetime structure fixed by its kinematics



In order to make predictions, short-distance contributions have to be **factorized** from long-distance contributions

Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

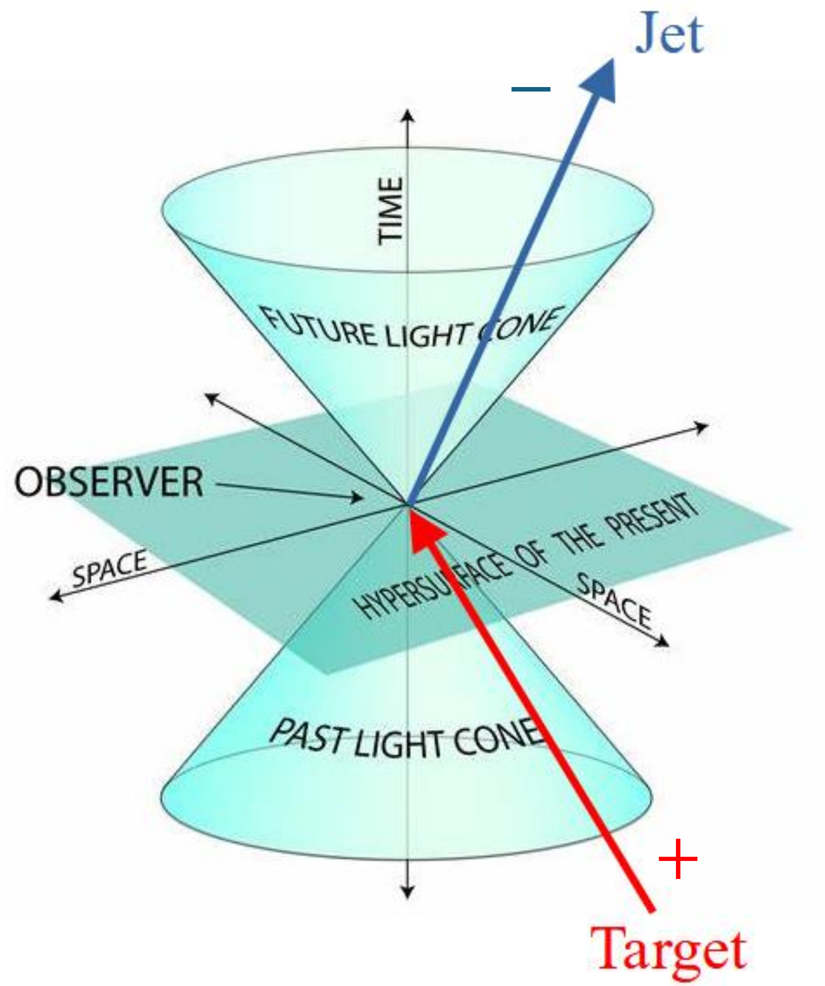
- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….



Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

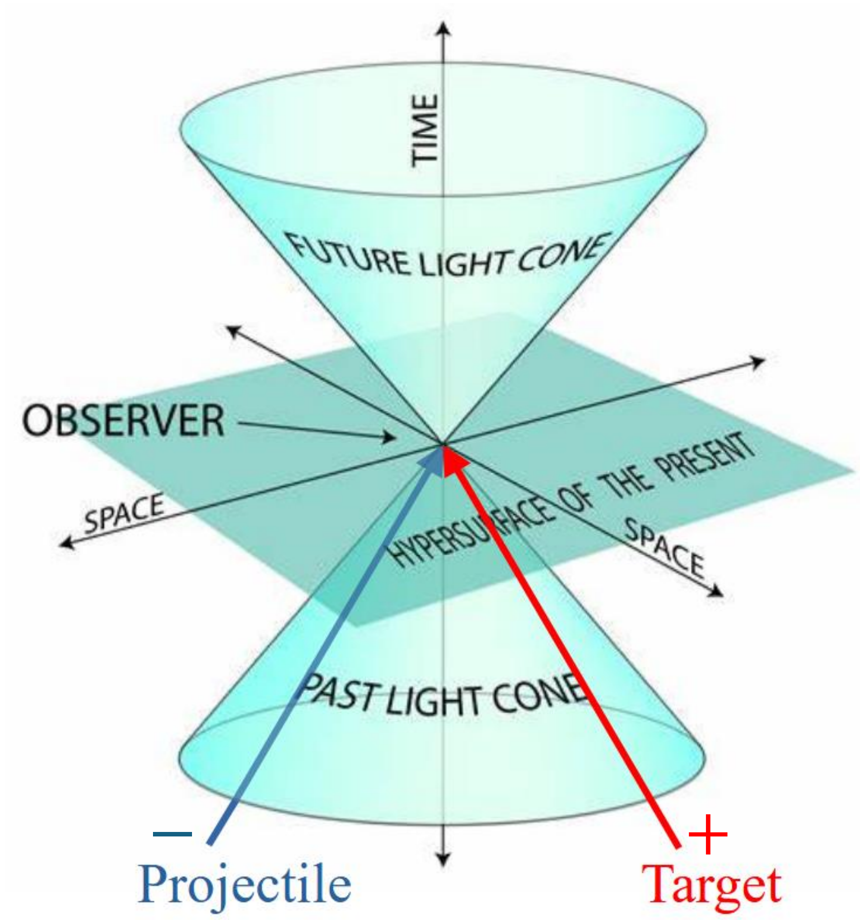


Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- $2 \rightarrow 2$ scattering in the forward limit
- ….



Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

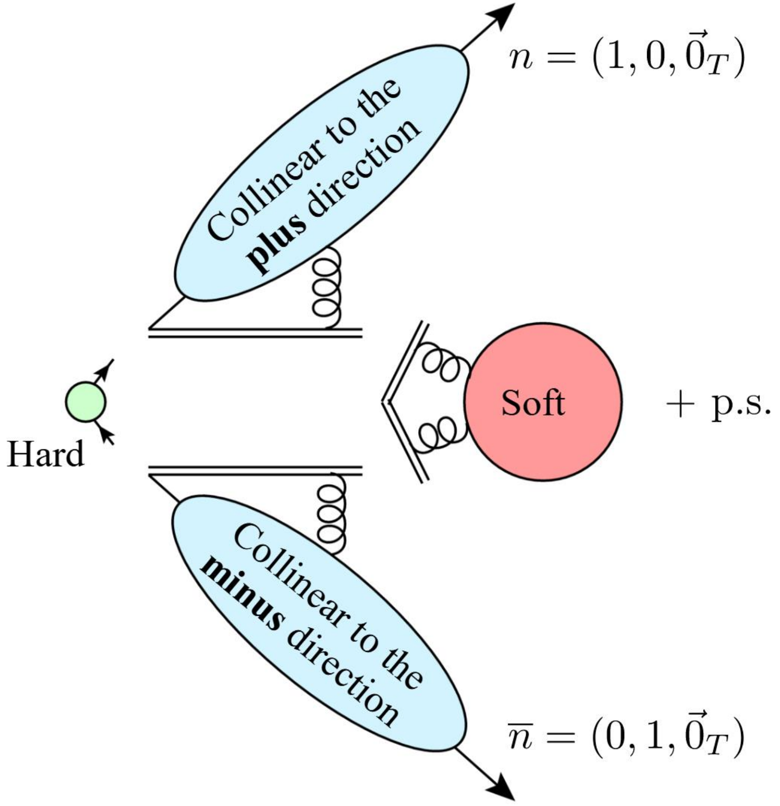
- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- $2 \to 2$ scattering in the forward limit
- ….

A broader meaning of universality

Analogous spacetime structure ⟷ Similar factorization properties



Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

- Di-Jet production from $e^+ \, e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- $2 \to 2$ scattering in the forward limit
- ….

Well established

TMD Factorization
Collins, Soper and Sterman; ‘80s papers

$$H_{process} \underbrace{C_+ \;\; S \;\; C_-}_{F_+^{tmd} F_-^{tmd}}$$

Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- $2 \rightarrow 2$ scattering in the forward limit
- ….

TMD Factorization
Collins, Soper and Sterman; '80s papers

$$H_{process} \underbrace{C_+ \ S \ C_-}_{F_+^{tmd} F_-^{tmd}}$$

Sterman 1987; SCET papers;
our recent work 2502.15033 [hep-ph]

$$H_{DIS@thr} \underbrace{C_+ \ S \ C_-}_{f_+ \ J_-}$$

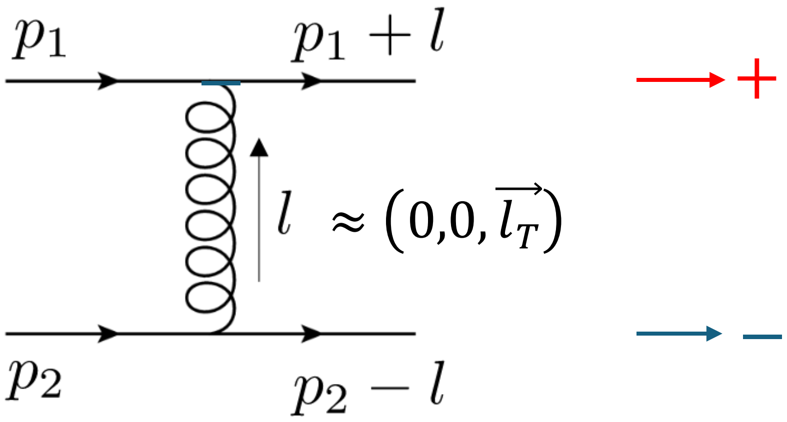
Consider a particularly relevant spacetime configuration dominated by **two opposite light-cone directions**

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- $\boxed{2 \to 2 \text{ scattering in the forward limit}}$
- ….

Color octet gluon exchange in the t-channel in the limit $s \gg |t|$

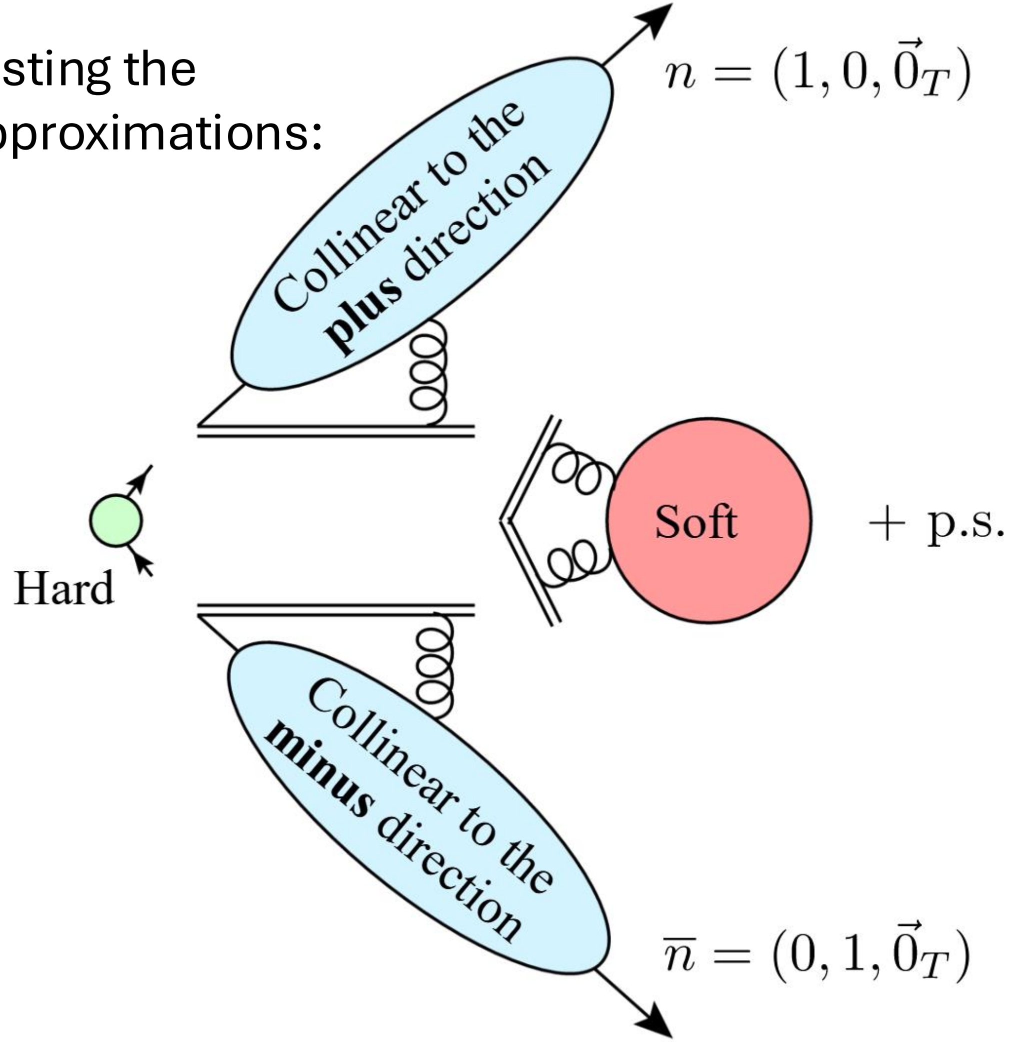


$$M_{ij\to ij} = \underbrace{C_i(t) e^{\log(s/t)\,[1+\omega(t)]} C_j(t) + R_{ij\to ij}}_{???\quad C_+ \, S \, C_-}$$

Can we apply the same perspective to the Reggeon exchange?

Factorization

Testing the approximations:



The Wilson lines lie *exactly* on the light-cone

RAPIDITY DIVERGENCES

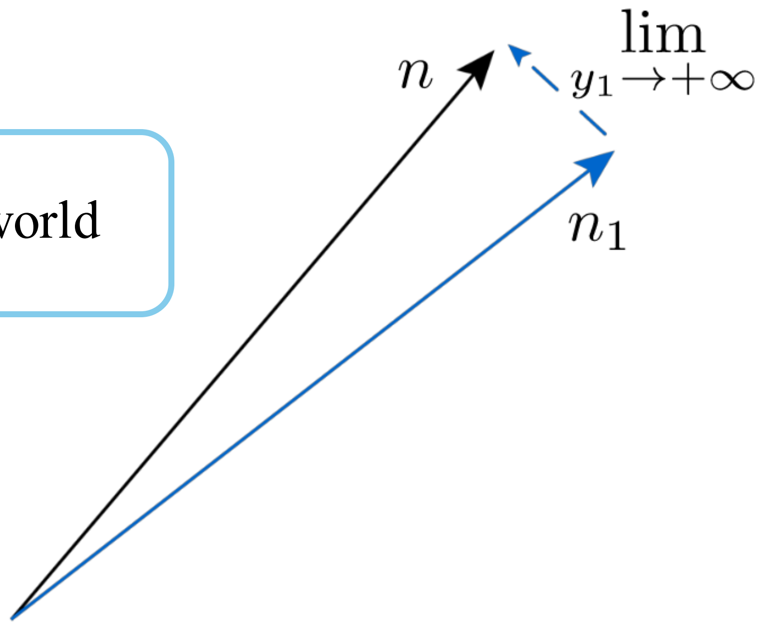
$$\int_0^\infty \frac{dk^+}{k^+}$$

$$H \cdot \boxed{C_+ \cdot S \cdot C_-}$$

Long-distance operators are ill-defined when considered singularly

This is a symptom that we are missing something: too strong approximations?

Off lightcone effects



Deviation from the light-cone $\longleftrightarrow$ Deviation from the idealistic world

How can we parametrize such deviation?

A simple (and realistic) choice is the introduction of tilts:

$$
\begin{cases}
n = (1, 0, \vec{0}_T) \to n_1 = (1, \pm e^{-2y_1}, \vec{0}_T), \\
\bar{n} = (0, 1, \vec{0}_T) \to n_2 = (\pm e^{2y_2}, 1, \vec{0}_T)
\end{cases}
$$

The light-cone limit corresponds to $y_{1,2} \to \pm\infty$

The choice of the sign of the tilts and the orientation (future vs past) is crucial for the validity of factorization

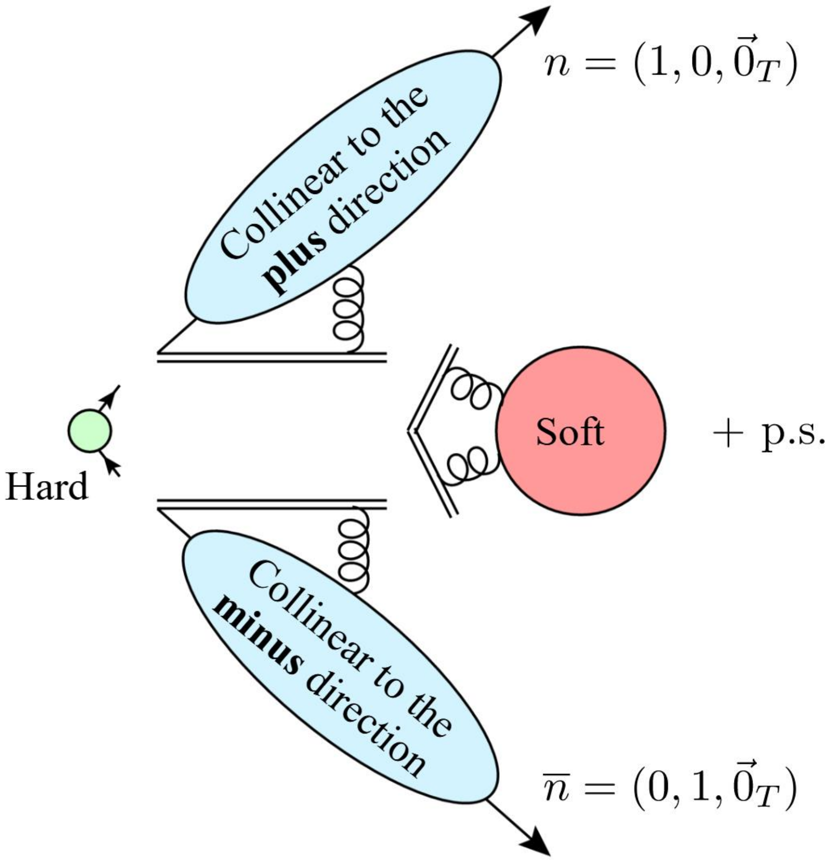
We can now track the effects of going off the light-cone: do they impact leading-power (LP) factorization?

Naively, one might say no. After all, tilts are ultimately mass effects.
However, this conclusion is non-trivial and, most importantly, not guaranteed *a priori*.

Rapidity scale separation

All the operators are now defined off the light-cone:

$$\boxed{H \cdot C_+(y_1) \cdot S(y_1, y_2) \cdot C_-(y_2)}$$



$y \gtrsim y_1$

$y_1 \gtrsim y \gtrsim y_2$

$y_2 \gtrsim y$

There is a clear and transparent separation in rapidity

Universal K-P Decomposition

Each operator in off lightcone factorization:

$$O(\ldots, y_1, y_2) \propto \exp\{K(\ldots, y_1, y_2) + P(\ldots, y_1, y_2) + O(e^{-2\,y_1}, e^{2\,y_2})\}$$

Leading asymptotic behavior in the light-cone limit.
If the tilts are removed, this is the leading rapidity divergent term.

It is related to the **Collins-Soper kernel K** typical of TMD observables.

Sub-Leading asymptotic behavior in the light-cone limit.
If the tilts are removed, this term (might) introduce a sub-leading rapidity divergence.

Light-cone suppressed terms in the light-cone limit.
If the tilts are removed, this terms do not contribute.

Universal K-P Decomposition

Each operator in off light-cone factorization:

$$O(\ldots, y_1, y_2) \propto \exp\{K(\ldots, y_1, y_2) + P(\ldots, y_1, y_2) + O(e^{-2\,y_1}, e^{2\,y_2})\}$$

Two sources of off light-cone effects:

1. The dependence on the tilts

2. The dependence on the P-terms

Three possible scenarios:

Light-cone factorization theorem
Off light-cone effects cancel in both the factorized operators and the cross section.
E.g. DIS at threshold

Operators sensitive to off light-cone effects
Soft and collinear operators are defined off the light-cone, yet the cross section remains independent of off light-cone effects.
E.g. TMD factorized cross sections.

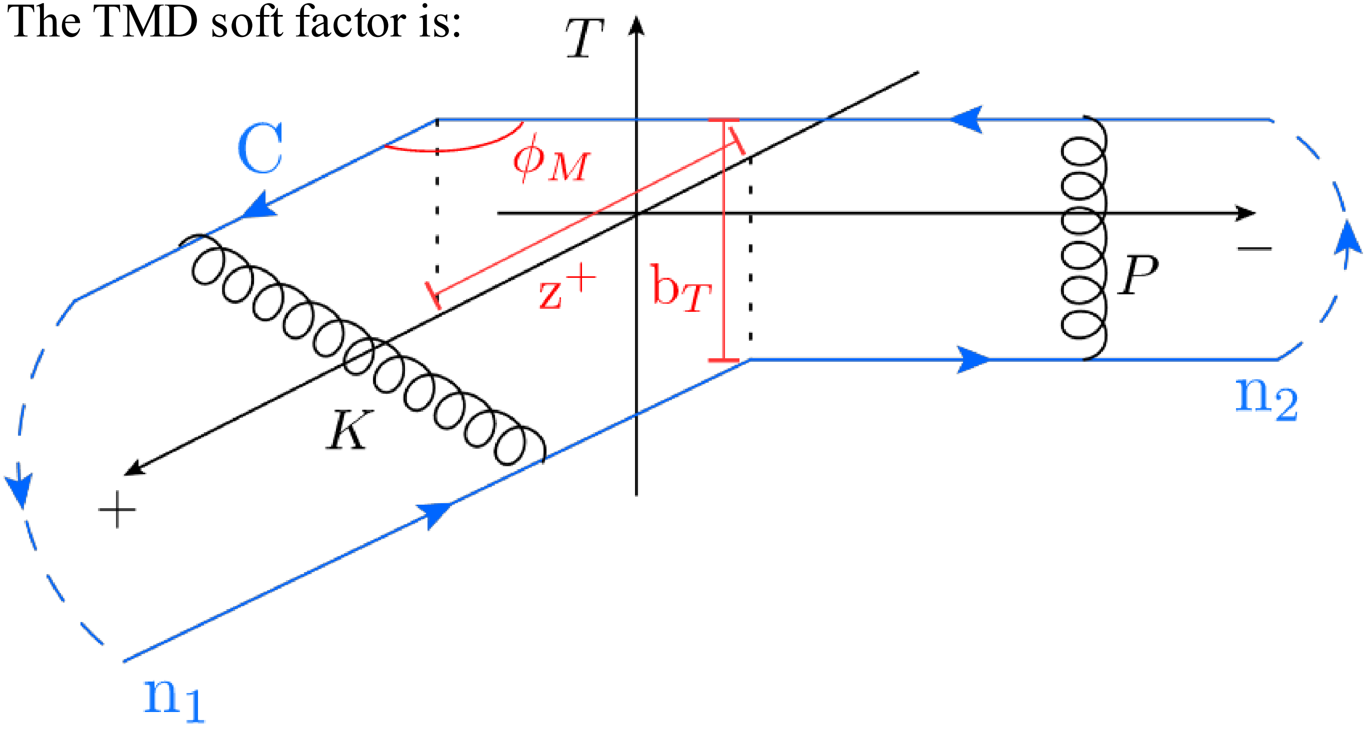
Factorization sensitive to off light-cone effects
Tilts are intimately connected to kinematic variables and do not cancel.
E.g. single inclusive thrust and transverse momentum distribution of e+e- annihiliation (BELLE); **Reggeon exchange?**

On the light-cone

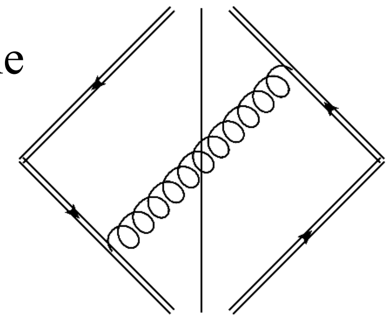
Universal behavior in the light-cone limit

Collins-Soper kernel

The TMD soft factor is:



These kind of diagrams at the lowest order



$$S_{TMD}(\alpha_S, \mu\, b_T; y_1 - y_2) = exp\{\boxed{(y_1 - y_2)K(\alpha_S, \mu\, b_T/c_1)} + P(\alpha_S, \mu\, b_T/c_1) + O\left(e^{-2(y_1-y_2)}\right)\}$$

Rapidity gap between the two directions

Recoil of soft gluons

Collins-Soper kernel

$$S_{TMD}(\alpha_S, \mu\, b_T; y_1 - y_2) = exp\left\{\boxed{(y_1 - y_2)K(\alpha_S, \mu\, b_T/c_1)} + P(\alpha_S, \mu\, b_T/c_1) + O\left(e^{-2(y_1 - y_2)}\right)\right\}$$

$$\frac{d\, K(\alpha_S, \mu\, b_T/c_1)}{d\, \log \mu} = -\gamma_K(\alpha_S) \longrightarrow K(\alpha_S, \mu\, b_T/c_1) = K(\alpha_b, 1) - \int_{\alpha_b}^{\alpha_S} \frac{d\, \alpha}{2\beta(\alpha)} \gamma_K(\alpha)$$

It is a function of the strong coupling evaluated at the scales:

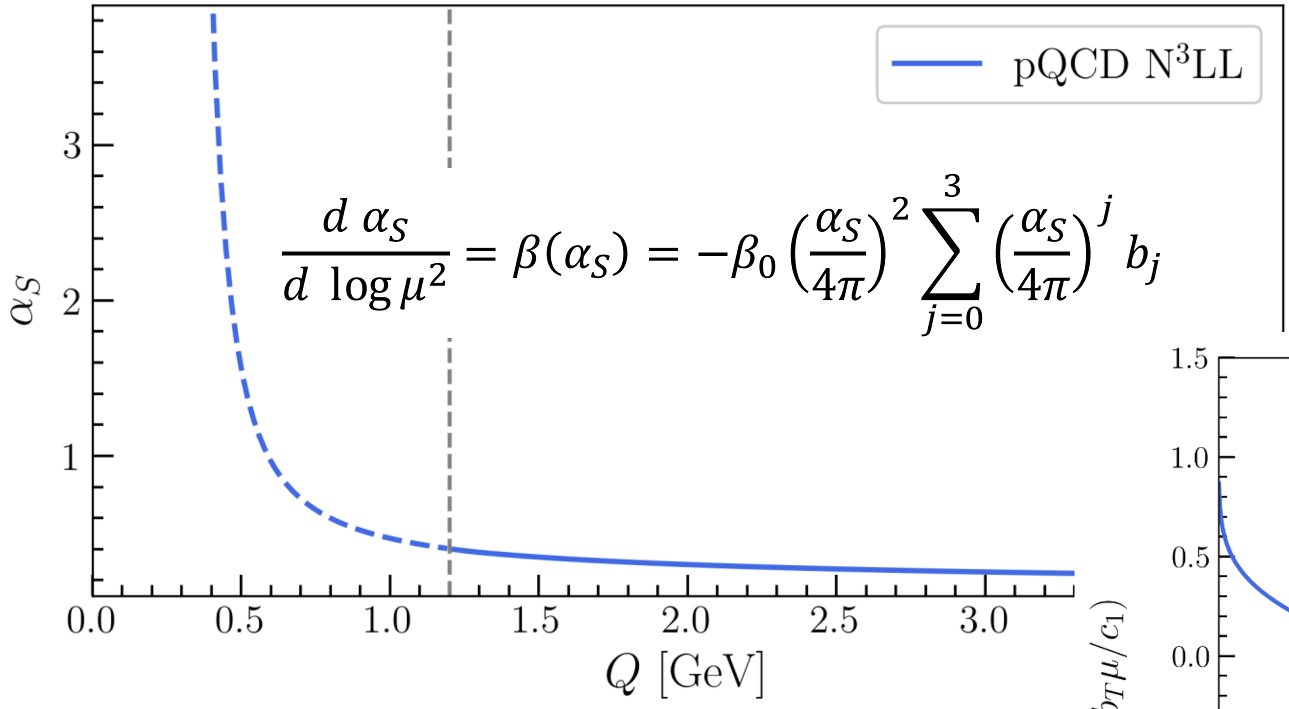
- $\mu \rightarrow \alpha_S(\mu) \equiv \alpha_S$
- $c_1/b_T \rightarrow \alpha_S(c_1/b_T) \equiv \alpha_b$

As long as c_1/b_T is a scale large enough to allow pQCD, the expression above can be safely evaluated, or further (consistently) approximated by trading α_b with logarithms $L_b = \log(\mu\, b_T/c_1)$

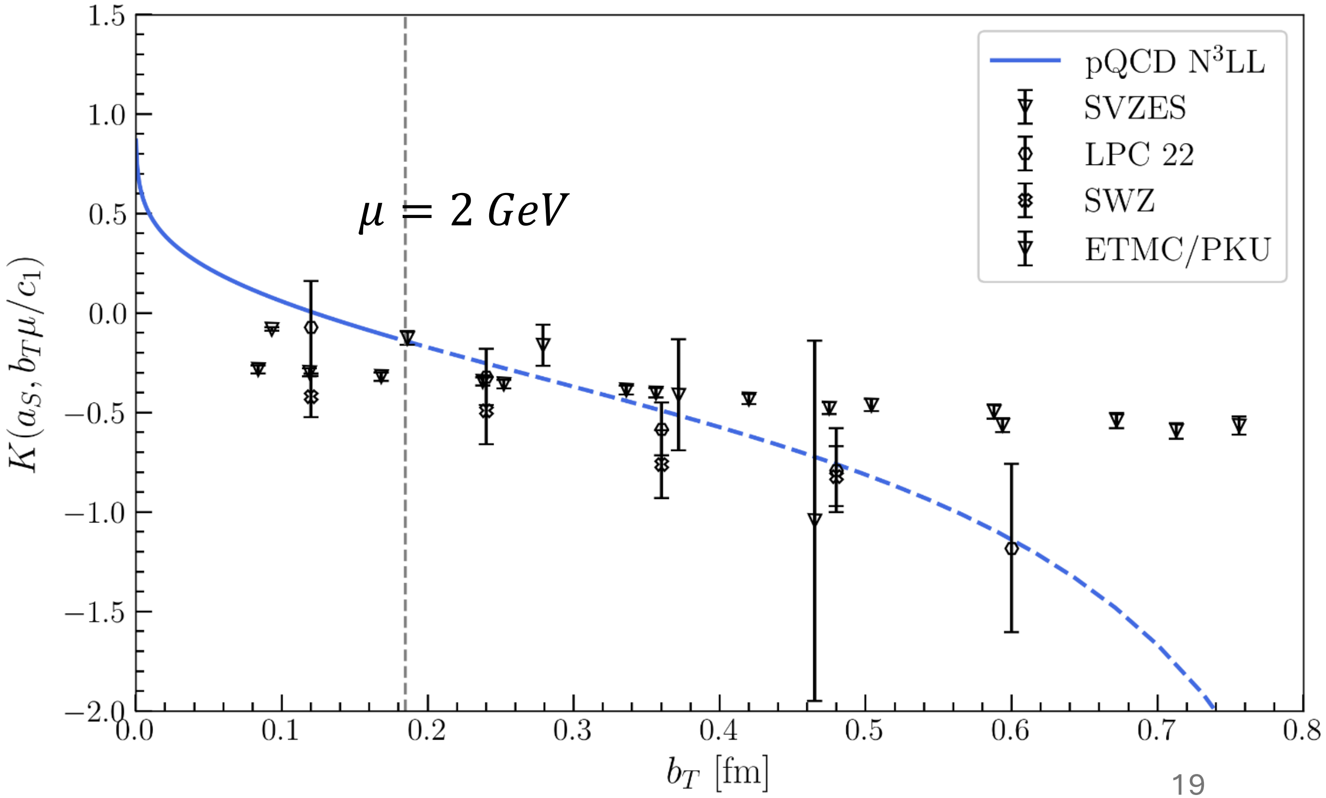
And below $c_1/b_T \approx 1$ GeV?

$$K(\alpha_S, \mu\, b_T/c_1) = g_1(\alpha_S L_b) + \frac{1}{L_b} g_2(\alpha_S L_b) + \frac{1}{L_b^2} g_3(\alpha_S L_b) + \frac{1}{L_b^3} g_4(\alpha_S L_b) + \cdots$$

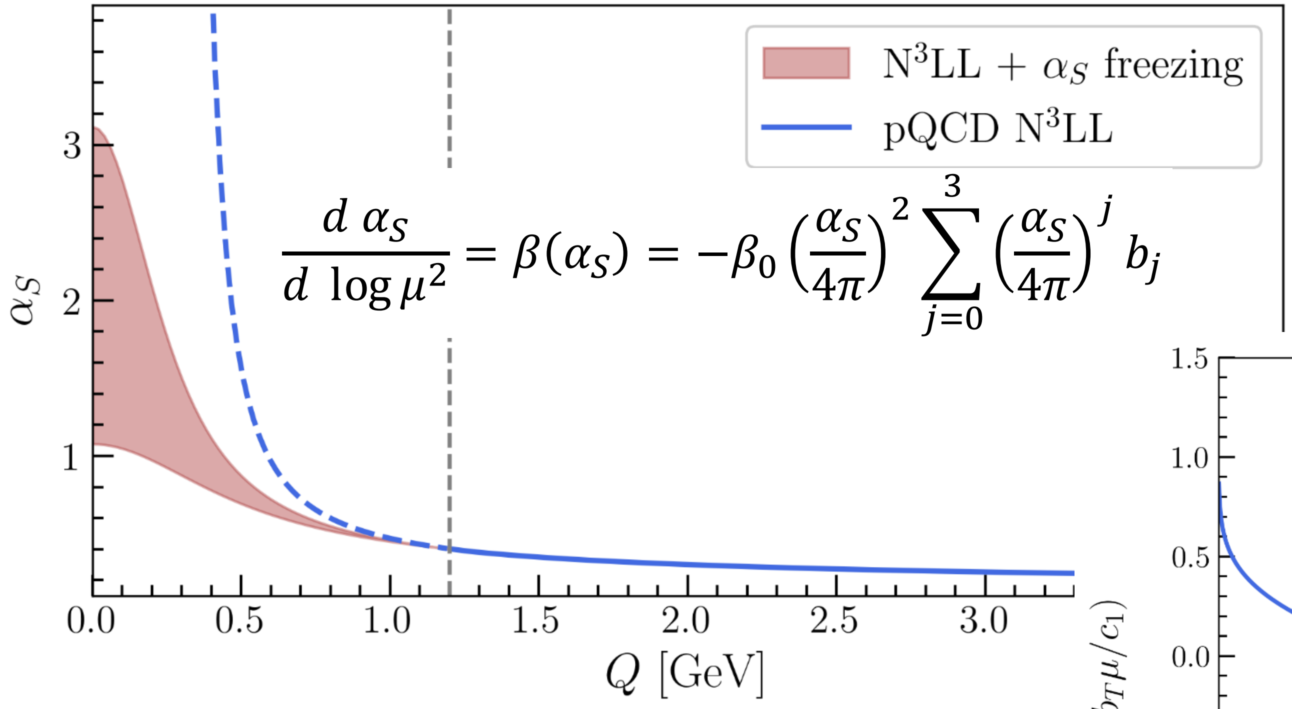
Collins-Soper kernel



$$\frac{d\,\alpha_S}{d\ \log \mu^2} = \beta(\alpha_S) = -\beta_0 \left(\frac{\alpha_S}{4\pi}\right)^2 \sum_{j=0}^{3} \left(\frac{\alpha_S}{4\pi}\right)^j b_j$$



Collins-Soper kernel



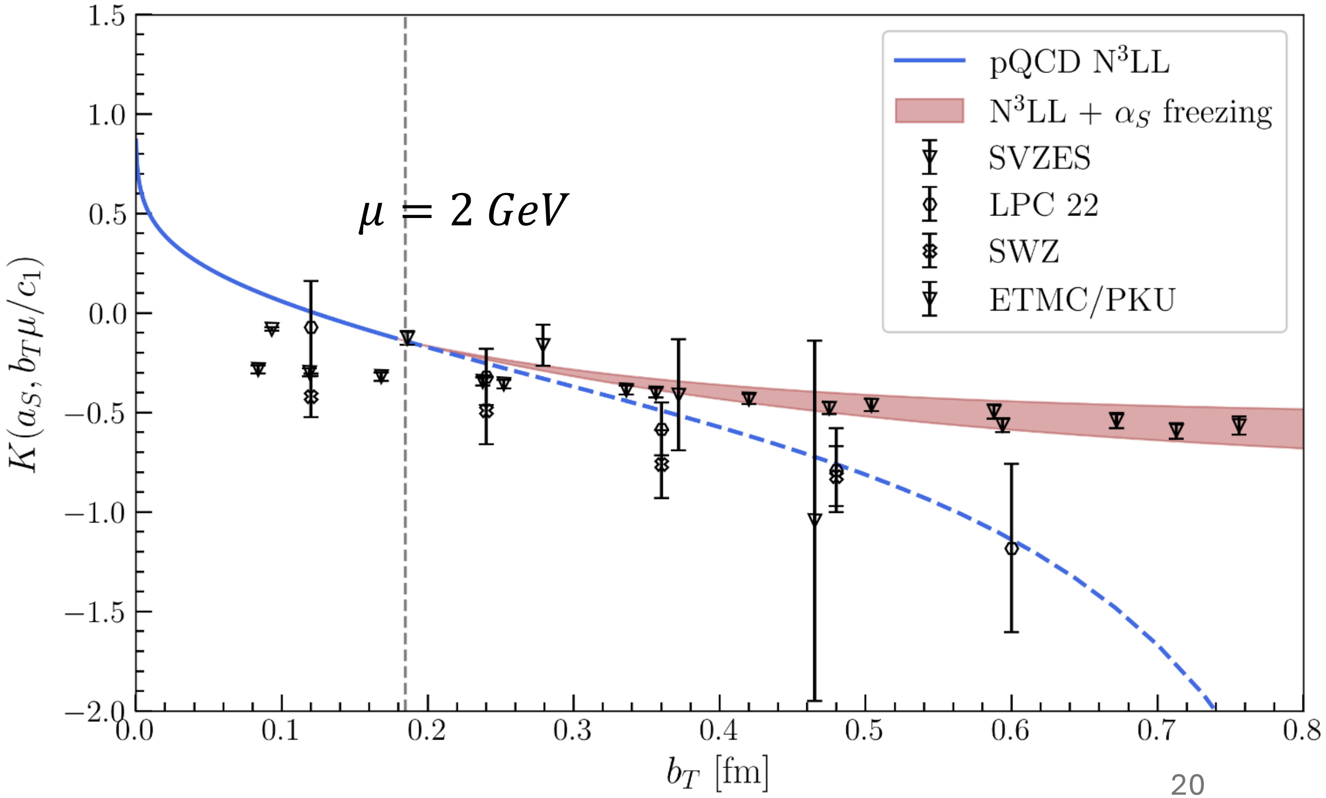
$$\frac{d\ \alpha_S}{d\ \log\mu^2} = \beta(\alpha_S) = -\beta_0\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_{j=0}^{3}\left(\frac{\alpha_S}{4\pi}\right)^j b_j$$

Here we assumed the deep IR model:

- $Q_0 = 1.2\ GeV$
- $450\ MeV \leq m_g \leq 650\ MeV$

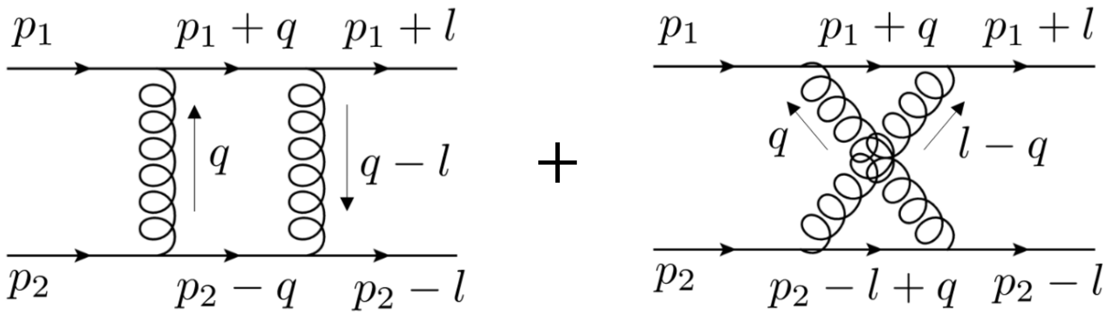
The freezing of alpha strong implies the large b_T behavior:

$$K(\alpha_S, b_T\mu/c_1) = K_0^\infty + \frac{1}{b_T^2}K_1^\infty + \cdots$$



Gluon Regge trajectory

At NLO two graphs contribute:



Rapidity divergence regularized

$$= M^{tree}_{ij\to ij}\left[(y_1 - y_2)\,\frac{\alpha_S}{4\pi}\,\omega_G^{[1]}(\mu^2/t) + O\big(e^{-2(y_1 - y\_2)}\big)\right]$$

The two gluons are soft, but the sum of their momenta is Glauber.

The structure is reminiscent of the NLO K-term of the TMD soft factor, in fact:

$$\omega_G^{[1]}(\mu^2/|t|) = -\frac{1}{2}K_G^{[1]}\big(\mu/\sqrt{|t|}\big) \longleftarrow$$

NLO CS-kernel in adjoint representation under replacing: $\frac{c_1^2}{b_T^2} \longleftrightarrow t$

What can we say beyond NLO?

Gluon Regge trajectory

There is a conjecture verified up to N^{3}LO :

$$\frac{d\,\omega_G(\alpha_S, \mu^2/|t|\,)}{d\ \log\mu} = \frac{1}{2}\gamma_K^{(8)}(\alpha_S)$$

Suppose it holds true.
Then we can define the *RG-invariant* discrepancy operator:

In the perspective of off light-cone factorization this conjecture is totally natural.

The soft factor of 2 → 2 forward scattering amplitude admits a KP-decomposition where the K-term is the Regge trajectory:

$$S_{2\to 2\ forward} = exp\{(y_1 - y_2)\omega_G + \cdots\}$$

$$\Delta_G(\alpha_S, \mu^2/|t|) = \ \omega_G(\alpha_S, \mu^2/|t|) + \frac{1}{2}K_G(\alpha_S, \mu^2/|t|)$$

$$= \left(\frac{\alpha_t}{4\pi}\right)^2 \Delta_G^{[2]} + \left(\frac{\alpha_t}{4\pi}\right)^3 \Delta_G^{[3]} + \cdots$$

Exploiting the RG-invariance we set $\mu = |t|$ and also $\alpha_S(|t|) \equiv a_t$.

Gluon Regge trajectory

The discrepancy operator Δ is the results of *several delicate cancellations*:

$$\omega_G^{[2]}(\mu^2 = t) = N^2\left(\frac{404}{27} - 2\zeta_3\right) - N\, n_f\, \frac{56}{27}$$

$$-\frac{1}{2}K_G^{[2]}(\mu^2 = t) = N^2\left(\frac{404}{27} - 14\zeta_3\right) - N\, n_f\, \frac{56}{27}$$

$$\longrightarrow \quad \Delta_G^{[2]} = N^2 12\, \zeta_3$$

- Large-N (planar contributions)
- Max trascendental degree at 2-loops (rational terms cancel out)

Beyond* the NNLO the trend keeps going:

$$\Delta_G^{[3]}(\alpha_S, |t|) = -N^3\left[\frac{4}{27}\left(125 - 304\frac{n_f}{N} + 44\left(\frac{n_f}{N}\right)^2\right)\zeta_3 + 16\,\zeta_2\zeta_3 + 80\,\zeta_5\right]$$

Gluon Regge trajectory

After even subtler cancellations :

From *Phys.Rev.Lett.* 128 (2022) 13, 13
Falcioni, Gardi,Maher,Milloy,Vernazza

$$\omega_G^{[3]}(\mu^2 = t\,)$$
$$= N^3\left(16\,\zeta_5 + \frac{40}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 - \frac{6664}{27}\zeta_3 - \frac{3196}{81}\zeta_2 + \frac{297029}{1458}\right)$$
$$+ N^2\,n_f\left(\frac{412}{81}\zeta_2 + \frac{2}{3}\zeta_4 + \frac{632}{9}\zeta_3 - \frac{171449}{2916}\right) + N\,n_f^2\left(\frac{928}{729} - \frac{128}{27}\zeta_3\right)$$
$$+ n_f\left(-4\zeta_4 - \frac{76}{9}\zeta_3 + \frac{1711}{108}\right)$$

From *Phys.Rev.Lett. 118 (2017) 2, 022004*
Li, Zhu

$$-\frac{1}{2}K_G^{[3]}(\mu^2 = t)$$
$$= N^3\left(96\,\zeta_5 + \frac{88}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 - \frac{6164}{27}\zeta_3 - \frac{3196}{81}\zeta_2 + \frac{297029}{1458}\right)$$
$$+ N^2\,n_f\left(\frac{412}{81}\zeta_2 + \frac{2}{3}\zeta_4 + \frac{680}{27}\zeta_3 - \frac{171449}{2916}\right) + N\,n_f^2\left(\frac{928}{729} - \frac{16}{27}\zeta_3\right)$$
$$+ n_f\left(-4\zeta_4 - \frac{76}{9}\zeta_3 + \frac{1711}{108}\right)$$

Gluon Regge trajectory

Is there a simpler way to get the Regge trajectory?

$$\omega_G(\alpha_S, \mu^2/|t|) = -\frac{1}{2} K_G(\alpha_S, \mu^2/|t|) \; + \Delta_G(\alpha_S, \mu^2/|t|)$$

Easier than Regge trajectory to compute in full QCD (1 loop less)

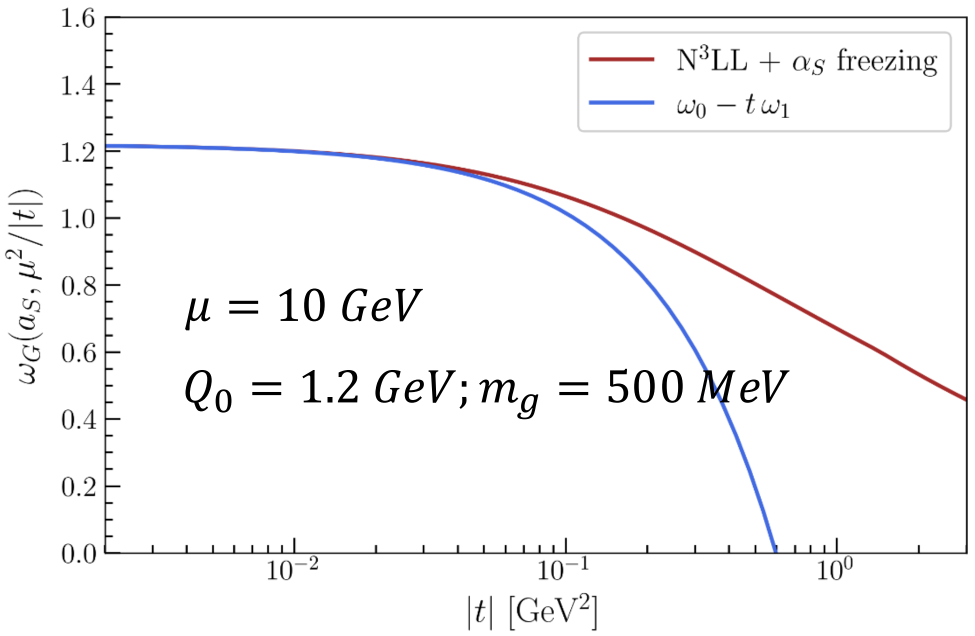
Can it be obtained in some simpler theory? Hints from SYM N=4 in the planar limit?

The two operators starts to be different at N²LL

The IR behavior of the CS-kernel reflects onto the IR behavior of the gluon Regge trajectory

$$\omega_G(\alpha_S, \mu^2/t) = \omega_0 - t\,\omega_1 + \cdots$$

if alpha strong freezes.

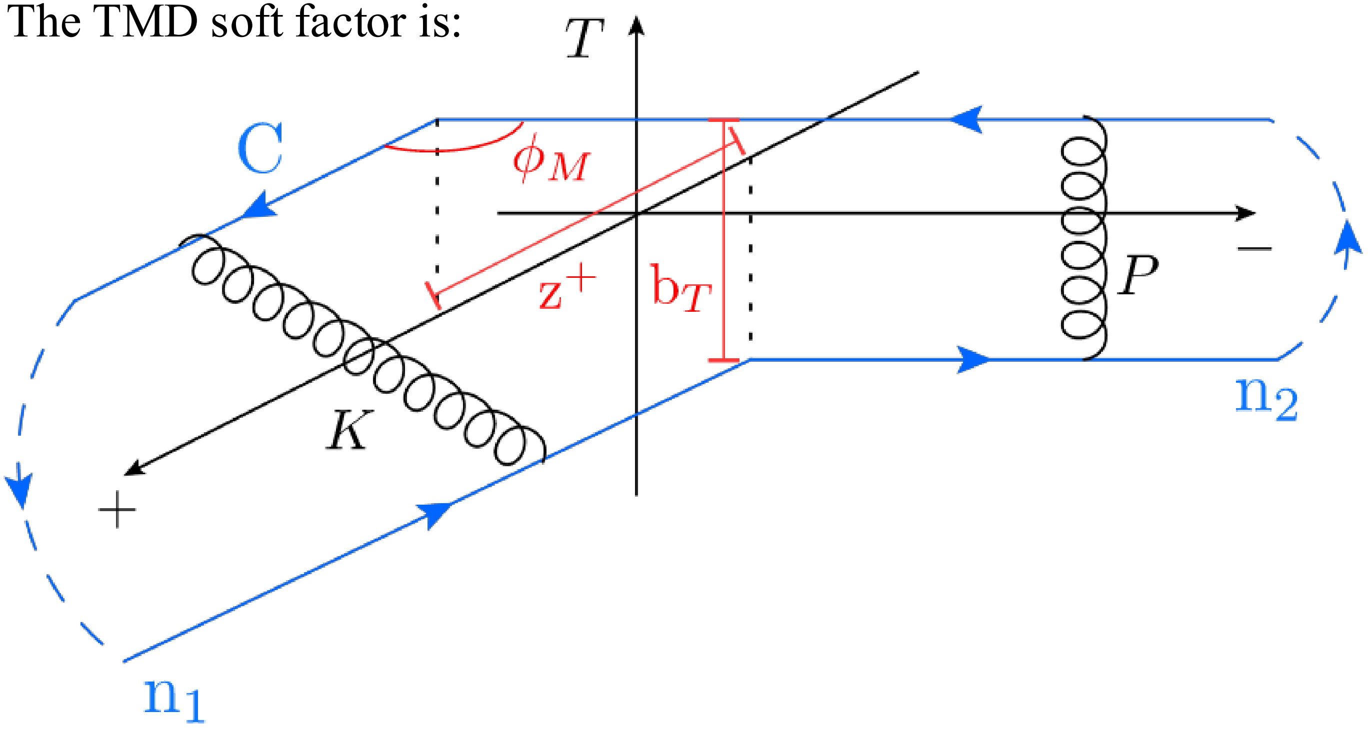


Off the light-cone

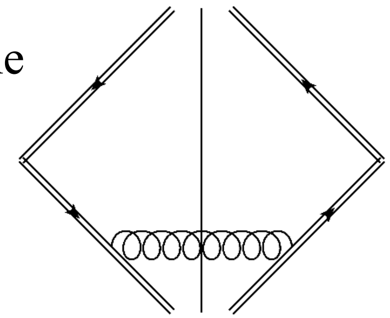
Is the Regge factorization really broken beyond NLL?

TMD P-term

The TMD soft factor is:



These kind of diagrams at the lowest order



$$S_{TMD}(\alpha_S, \mu\, b_T; y_1 - y_2) = exp\{(y_1 - y_2)K(\alpha_S, \mu\, b_T/c_1) + \boxed{P(\alpha_S, \mu\, b_T/c_1)} + O\left(e^{-2(y_1-y_2)}\right)\}$$

These contributions are naively zero, but they are *actually ill-defined on the light-cone.* This means that these effects are genuine off light-cone.

TMD P-term

The P-terms are present at the level of collinear and soft operators, but they cancel out in the cross section:

$$\frac{d\,\sigma}{d\,q_T^2} \sim H\,\bar{F}_+^{tmd}(y_1)S(y_1-y_2)\bar{F}_-^{tmd}(y_2)$$

$$\sim exp\{-(y_1-y_b)K - P/2\}$$

$$\sim exp\{-(-y_b-y_2)K - P/2\}$$

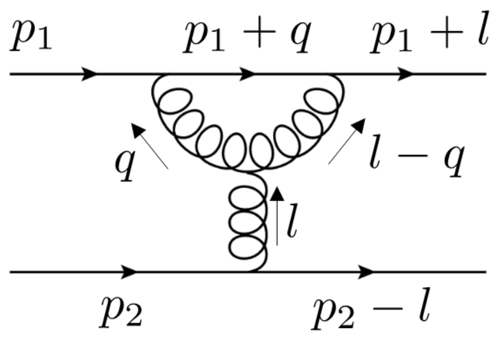
$$\sim exp\{(y_1-y_2)K + P\}$$

This is indeed the property that allows to recast the collinear and soft operators into the usual square-root definition of TMDs, completely without P-terms.

This is also the reason why light-cone rapidity regulators can be safely used in TMD physics.

Forward scattering

Analogous contributions in the forward 2 → 2 amplitudes are diagrams as



Once again, these contributions are ill-defined on the light-cone (naively zero), but perfectly defined once the tilts are introduced

$$\sim \int \frac{d^D q}{(2\pi)^{\mathrm{D}}} \frac{n \cdot \bar{n} q \cdot \bar{n} - \bar{n}^2 q \cdot n}{[q \cdot \bar{n} + i\,0]\,[q^2 + i\,0]\,[2q^+ q^- - v_T^2 + i\,0]} \quad \rightarrow \int d^{2-2\varepsilon} \overrightarrow{q_T} \frac{\log(q_T^2/v_T^2)}{q_T^2 - v_T^2}$$

What is their effect at the level of soft radiation?

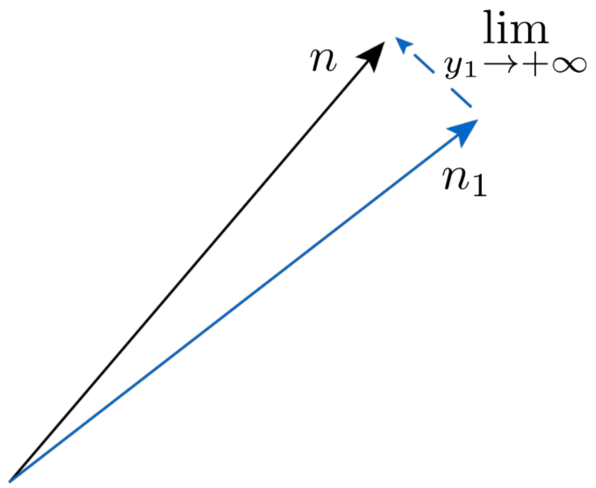
$$S_{2\to 2\ forward} = exp\{(y_1 - y_2)\omega_G + \boxed{\pi_{\mathrm{G}}^{\mathrm{s}}\ \}}$$

KP-decomposition of the soft factor transparent in off light-cone factorization

The impact factors are expected to generate a genuine off light-cone term as well:

$\mathrm{C_i} \propto exp\{-\pi_{\mathrm{G}}^{\mathrm{c}}/2\ \}$ But this time there is no guarantee that $\pi_G^s = \pi_G^c$ (as in TMD case)

Can we rescue factorization?



Suppose that indeed $\pi_G^s = \pi_G^c$. Then:

Starting from $O(\alpha_S^2)$

$$M_{ij\to ij} = C_i(t)e^{\log(s/t)\,[1+\omega(t)]+\Delta_\pi(t)}C_j(t)$$

Can it generate the Regge factorization violating terms $R_{ij\to ij}$?

If this is the case, then the arise of Regge cuts is the manifestation of the breaking of light-cone factorization, or, in other words, the manifestation of *genuine off light-cone effects*.
Perhaps, a consistent factorization still survives off the light-cone!

It will also be the first known situation where these P-terms produce a measurable effect.

Conclusions

- The study of the light-cone deviations in hadronic processes induces a broad sense of universality, in which the operators assume a characteristic KP-decomposition

- The K-term is related to the Collins-Soper kernel of TMD physics. There is an intriguing relation between it and the Regge trajectory of the gluon.

$$\Delta_G(\alpha_S, \mu^2/|t|) = \omega_G(\alpha_S, \mu^2/|t|) + \frac{1}{2} K_G(\alpha_S, \mu^2/|t|)$$

- The P-term encodes genuine off light-cone effects. Can it be responsible for the breaking of Regge factorization?

$$M_{ij\to ij} \overset{?}{=} C_i(t) e^{\log(s/t)\,[1+\omega(t)]+\Delta_\pi(t)} C_j(t)$$

- Towards applications to kT-factorization and unification between CSS and BFKL