Kinematic power corrections in TMD factorization

Based on **S.Piloñeta** and **A.Vladimirov** JHEP12(2024)059 and work in progress

Resummation, evolution and factorization 2025

Sara Piloñeta. October 14th 2025

PID2022-136510NB-C31

The need for power corrections

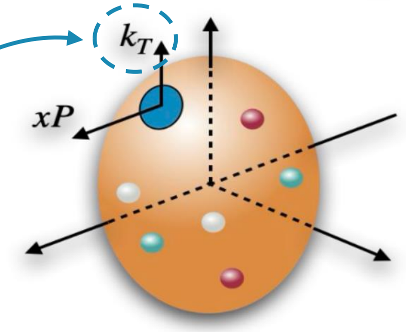
- What do we do? We are interested in studying the structure of nucleons

TMD factorization theorem

→ Transverse Momentum Dependent Distributions (TMDs)

→ Very powerful tool but has some limitations

Parton's transverse momentum



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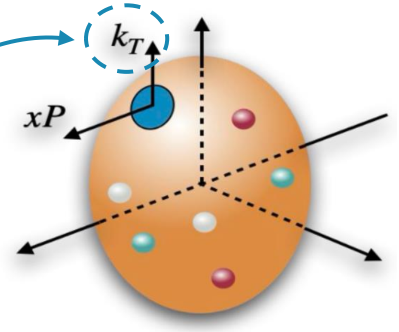
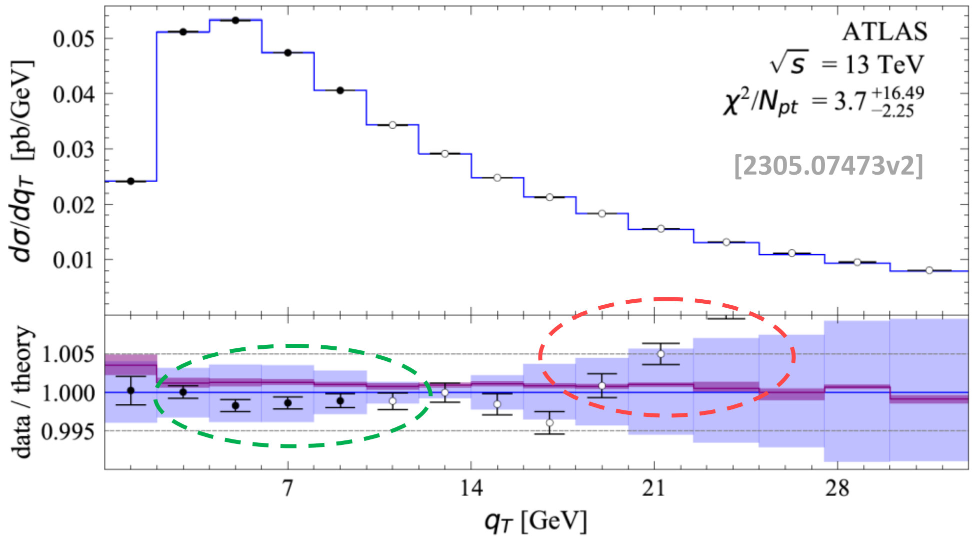


Z-boson production ATLAS measurement comparison



- Very **precise description** at **low q_T**
- **Prediction** systematically **lower** than the data at **larger q_T**

Signal of the need for power corrections

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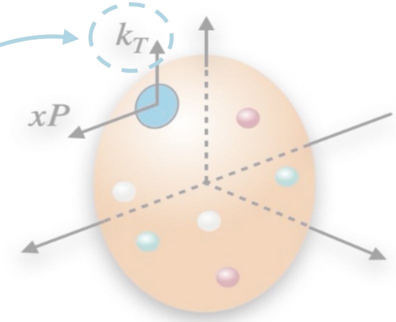
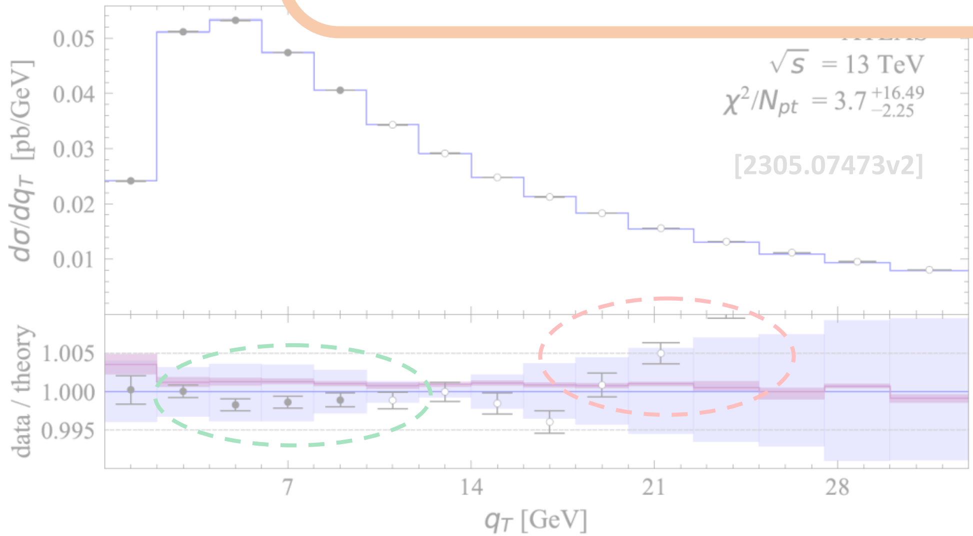


→ Transverse Momentum Dependent Distributions (TMDs)

→ Very powerful tool but has some limitations

This has motivated us to explore the structure of the TMD factorization theorem deeper

Z-boson p



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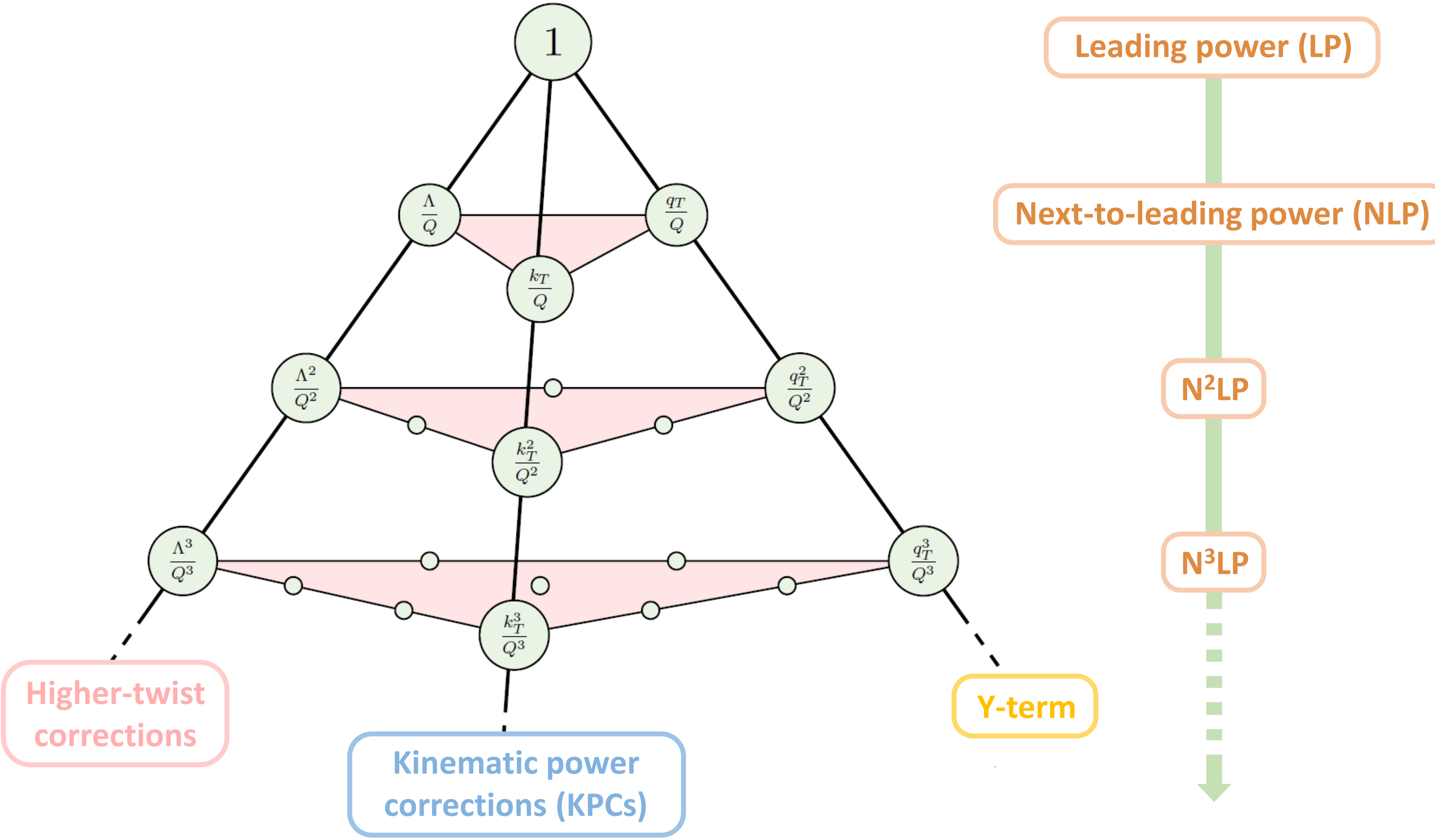
Signal of the need for power corrections

The hierarchy of the TMD factorization theorem

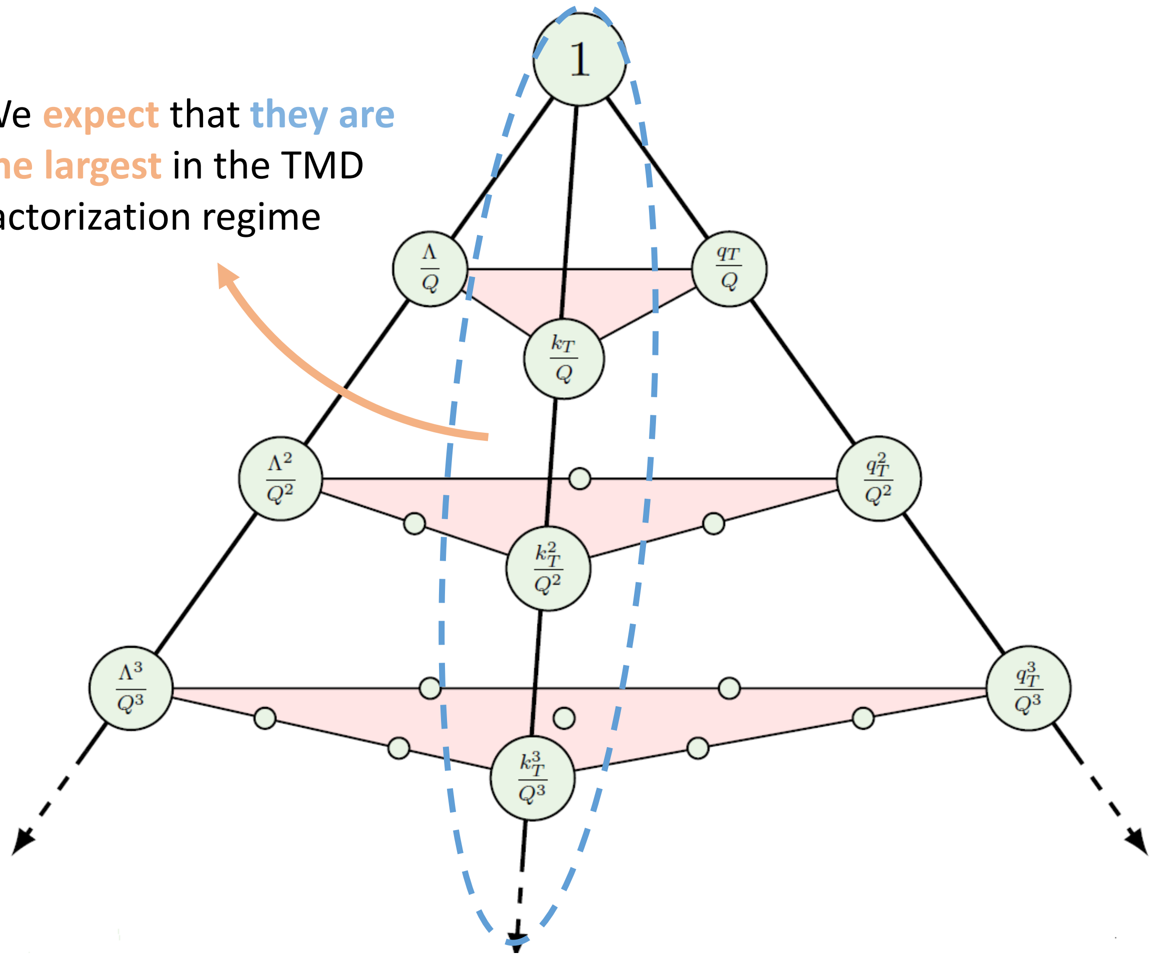
- There is a whole "pyramid" of power corrections



The TMD-with-KPCs factorization theorem

- We focus on the kinematic power corrections (KPCs) that follow the LP term

We expect that they are the largest in the TMD factorization regime



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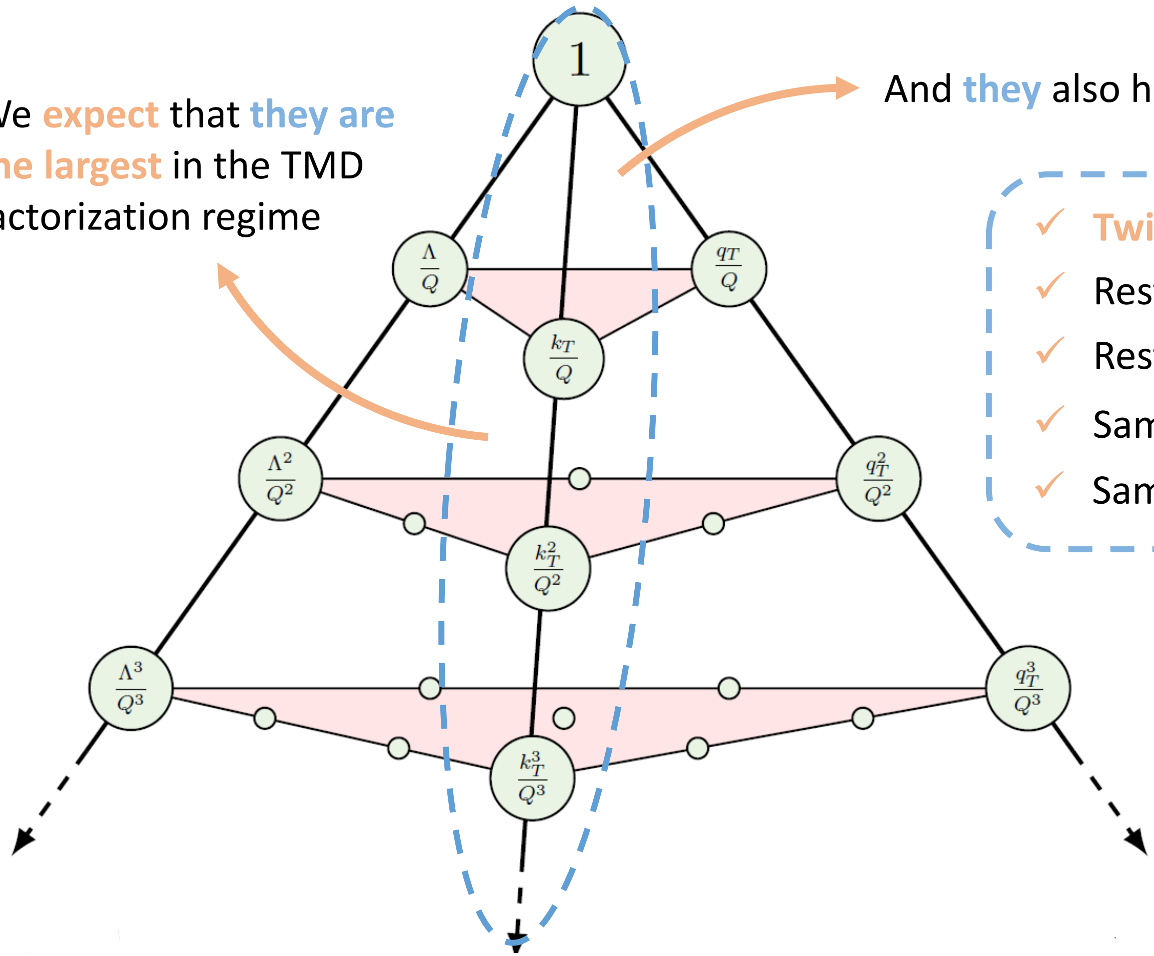

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And they also have nice properties!

- ✓ Twist-2 terms at $q_T \ll Q$
- ✓ Restore EM-gauge invariance
- ✓ Restore frame-invariance
- ✓ Same TMD distributions as LP
- ✓ Same coefficient function as LP

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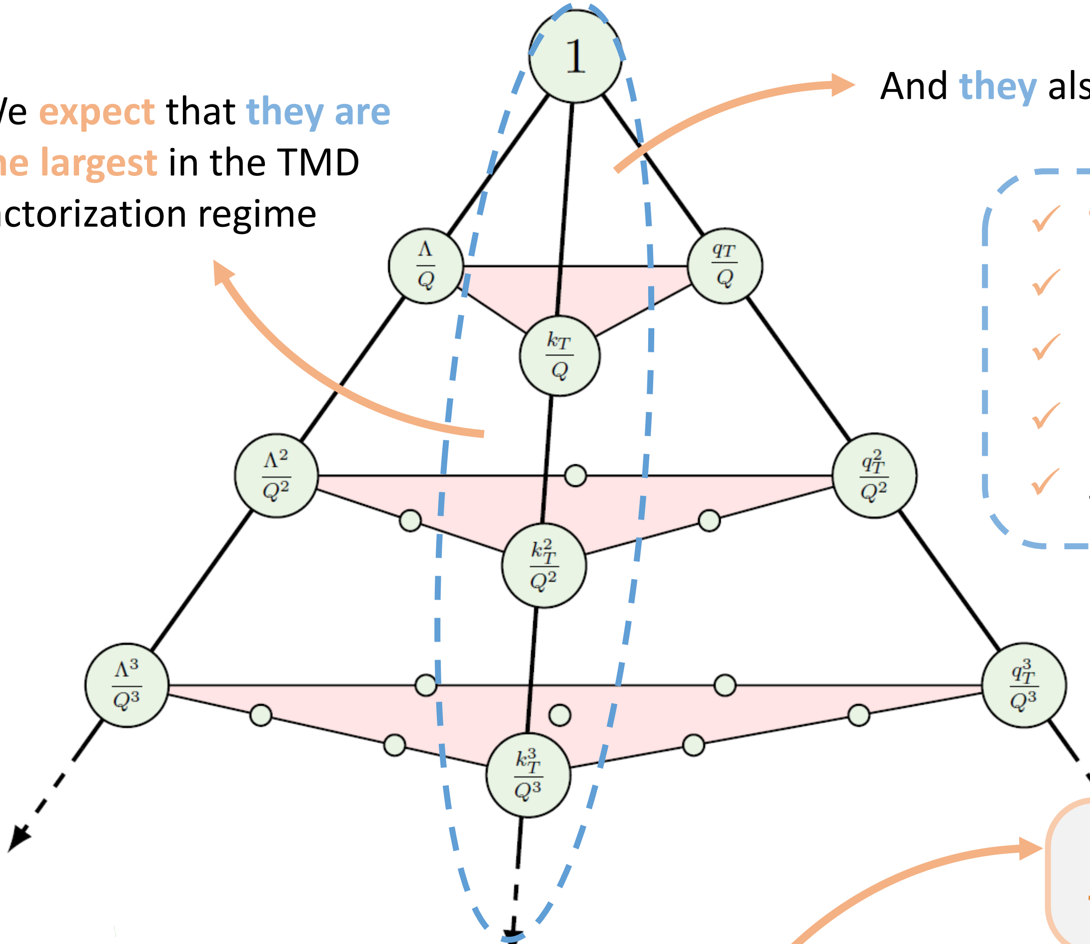

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Theoretical framework including them

TMD-with-KPCs factorization theorem

For details, see [AV, 2307.13054v2]

Angular distributions of Drell-Yan leptons

- **Feasibility** of using the **TMD-with-KPCs factorization** ⟹ unpolarized Drell-Yan as a starting point

  ↳ ***Angular distributions*** *of Drell-Yan leptons in the TMD factorization approach*

    ↳ First practical application of this framework **[S. Piloñeta, AV, 2407.06277]**

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Angular decomposition of the cross-section

$$\frac{d\sigma}{dp_T dy dQ d\Omega} = \frac{3}{16\pi}\boxed{\frac{d\sigma^{U+L}}{dp_T dy dQ}}\Big[(1+\cos^2\theta) + \frac{1-3\cos^2\theta}{2} A_0 + \sin 2\theta \cos\phi A_1 + \frac{\sin^2\theta\cos 2\phi}{2} A_2 + \sin\theta\cos\phi A_3 + \cos\theta A_4 + \sin^2\theta \sin 2\phi A_5 + \sin 2\theta \sin\phi A_6 + \sin\theta\sin\phi A_7\Big]$$

$\boxed{A_U}$ ⟶ Unpolarized angle-integrated cross-section

$\boxed{A_{0,\ldots,7}}$

⟶ Only four at LP $\{A_U, A_2, A_4, A_5\}$

⟶ The rest are power-suppressed

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Angular decomposition of the cross-section

A_U ⟹ **Unpolarized angle-integrated** cross-section

When we incorporate the summed KPCs we can describe all of them!

$$\frac{d\sigma}{dp_T dy dQ d\Omega} = \ldots$$

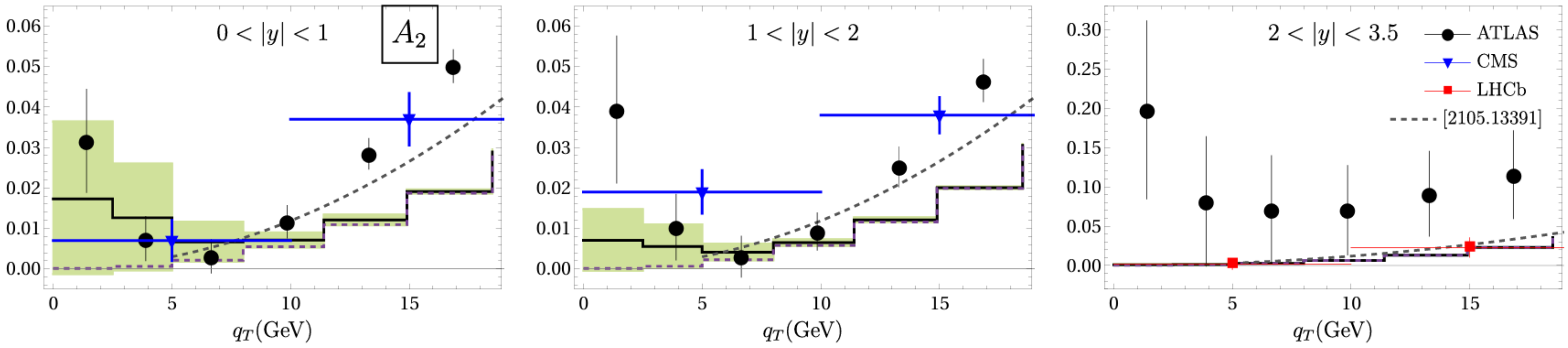
$$\frac{\sin^2\theta\cos 2\phi}{2} A_2 + \sin\theta\cos\phi A_3 + \cos\theta A_4 +$$

$$\sin^2\theta\sin 2\phi A_5 + \sin 2\theta\sin\phi A_6 + \sin\theta\sin\phi A_7 \Big]$$

$A_{0,\ldots,7}$ ⟹ Only **four** at **LP** $\{A_U, A_2, A_4, A_5\}$

⟹ The **rest** are **power-suppressed**

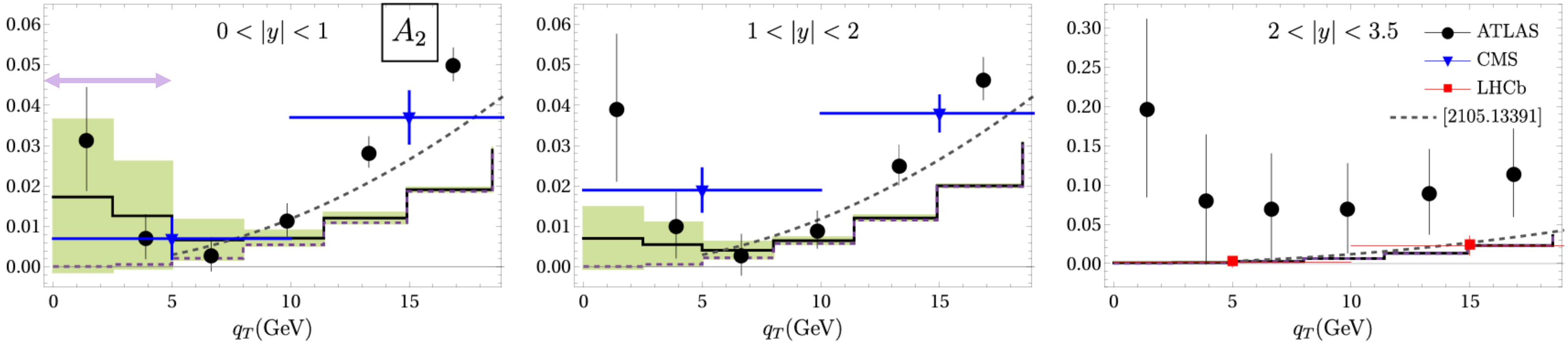
Angular distribution A_2



$$A_2 \sim \left[(v^2 - a^2)_q \frac{h_1^\perp h_1^\perp}{M^2} + (v^2 + a^2)_q \frac{f_1 f_1}{Q^2} \right]$$

- Contains both Boer-Mulders and unpolarized distributions

Angular distribution A_2



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Dominant at $q_T \to 0$

- Contains both Boer-Mulders and unpolarized distributions

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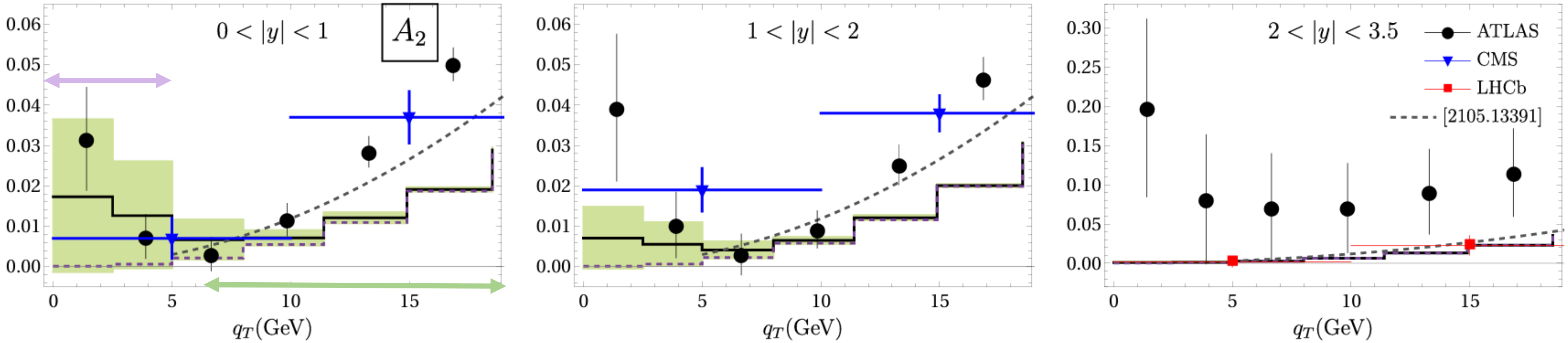

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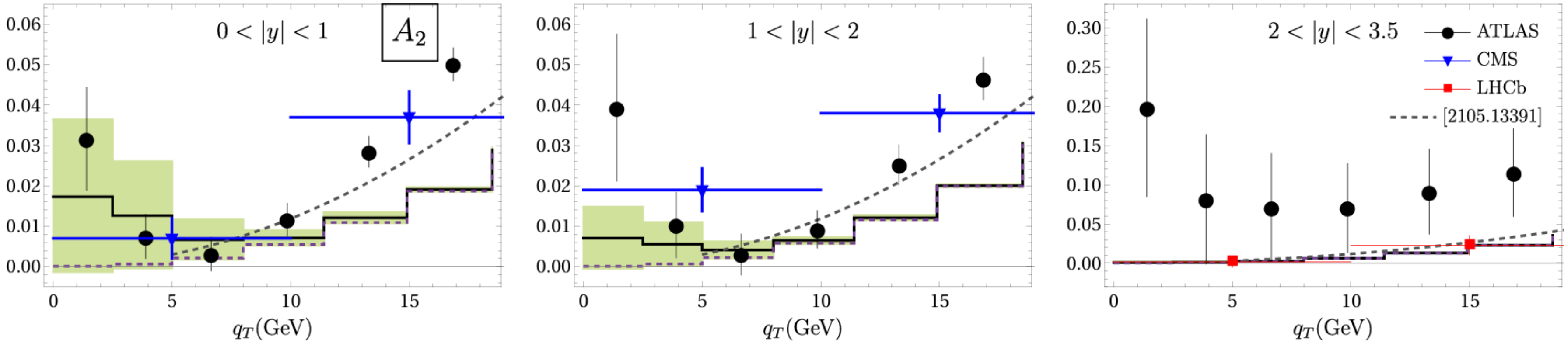
Dominant at larger q_T

Dominant at $q_T \to 0$

- Contains both Boer-Mulders and unpolarized distributions

Angular distribution A_2



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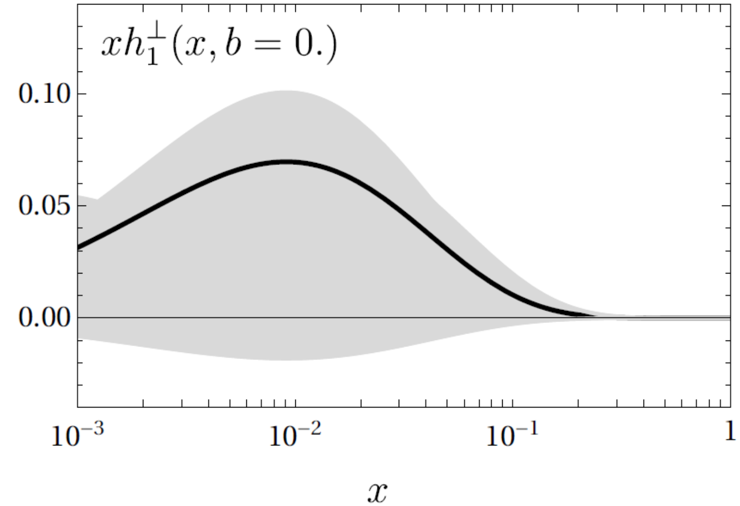
- Boer-Mulders distribution extraction from A_2
  - Very primitive model
    - New fit is required
  - First evidence (?) of twist-3 effects at LHC

$$x|E(-x,0,x)|$$



Lam-Tung relation ($A_0 - A_2$) description

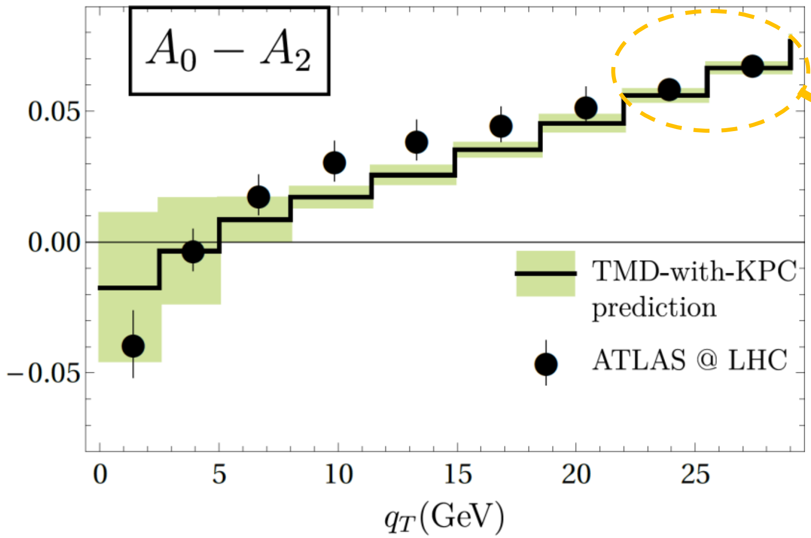
- At leading power, the TMD factorization theorem can not describe it

$$(A_0 - A_2)_{LP} \sim \boldsymbol{k}^2/M^2\, h_1^\perp h_1^\perp$$

The Boer-Mulders is very small!

- If we include KPCs the theoretical expression also contains the unpolarized f_1

This allows us to make a prediction for the Lam-Tung relation



Y-term suppressed by an extra α_S factor

Excellent agreement up to $q_T \sim 30\,\mathrm{GeV}$!

Good description only possible due to KPCs inclusion!

Moving to SIDIS: structure functions computation using KPCs

- Now, let's shift our focus to SIDIS → essential process for probing hadron structure

Cross-section decomposition in terms of structure functions

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,d\boldsymbol{p}_\perp^2} = \frac{\alpha_{\mathrm{em}}^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\Big\{\boldsymbol{F_{UU,T}} + \varepsilon \boldsymbol{F_{UU,L}} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h \boldsymbol{F_{UU}^{\cos\phi_h}}$$
$$+\varepsilon\cos(2\phi_h)\boldsymbol{F_{UU}^{\cos 2\phi_h}} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h \boldsymbol{F_{LU}^{\sin\phi_h}} + \ldots.$$

We want to compute them including KPCs

$$\Big\{\boldsymbol{F_{UU,T}} + \ldots\Big\} = \frac{x}{4z}\frac{1-\varepsilon}{Q^2}L_{\mu\nu}W^{\mu\nu}$$

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1 Lepton tensor conveniently decomposed via the tensors $P^\mu,\ q^\mu,\ p_\perp^\mu$

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2 Hadron tensor computed using the TMD-with-KPCs factorization theorem

- Same coefficient function as LP

$$q_\mu W^{\mu\nu} = 0$$

- Main difference with LP $\longrightarrow$ Convolution integral $\mathcal{C}_{KPC}$

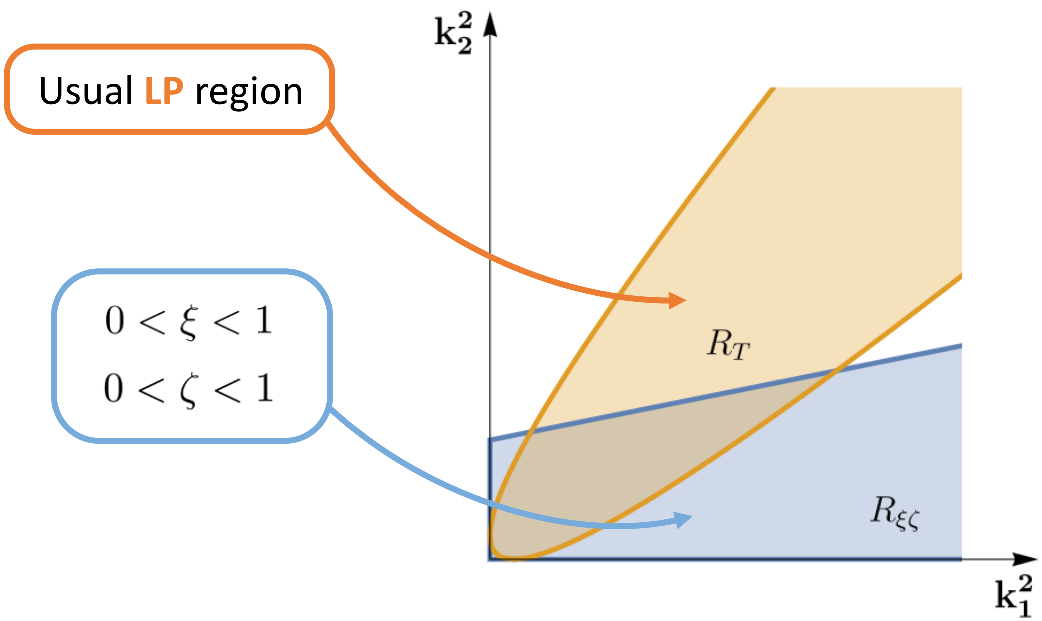
A closer look at the convolution integral

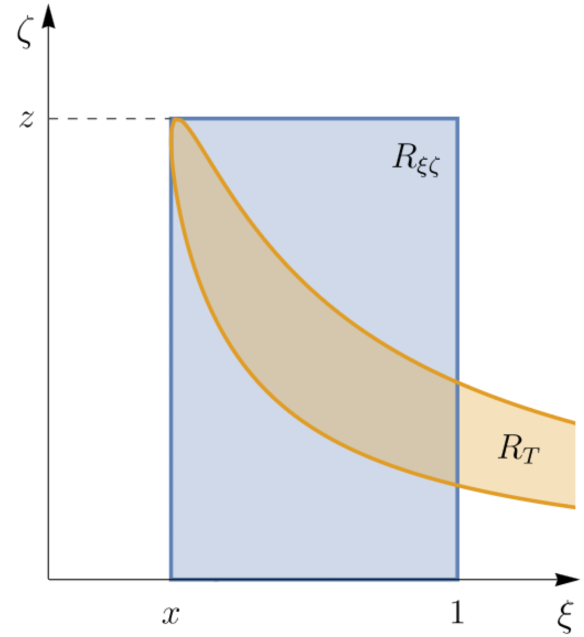
- The convolution integral is more complicated now

LP case $$\mathcal{C}_{LP}[A, f_1 D_1] \sim \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2\, \delta(\boldsymbol{q_T} + \boldsymbol{k_1} - \boldsymbol{k_2})\, f_1(x_1, \boldsymbol{k}_1^2) D_1(z_1, \boldsymbol{k}_2^2)$$

KPCs case $$\mathcal{C}_{KPC}[A, f_1 D_1] \sim \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2\, \delta(\boldsymbol{q_T} + \boldsymbol{k_1} - \boldsymbol{k_2})\, f_1\underbrace{(\xi(x_1, \boldsymbol{k}_{1,2}^2), \boldsymbol{k}_1^2) D_1(\zeta(z_1, \boldsymbol{k}_{1,2}^2)}_{\text{Additional dependance coming from extra } \delta\text{-functions}}, \boldsymbol{k}_2^2)$$

- The integration domain also changes





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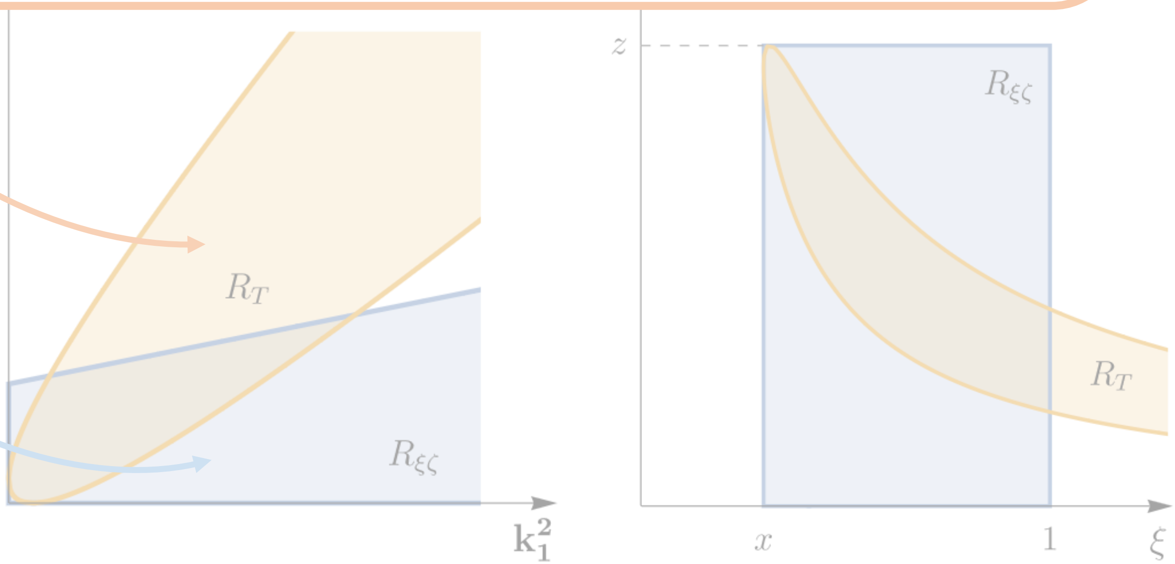
- The i

The convolution integral and the KPCs have been implemented in arTeMiDe, github.com/VladimirovAlexey/artemide-development

Usual LP region

$0 < \xi < 1$
$0 < \zeta < 1$



Focusing on $F_{UU,T}$ and $F_{UU,L}$

- I have obtained the theoretical expressions for all of them using the TMD-with-KPCs factorization
  - But in this talk I am going to focus on $F_{UU,T}$ and $F_{UU,L}$

$$F_{UU,T} = \frac{x}{4z} F_0 (\mathcal{S}_1^{\mu\nu} - \underbrace{\mathcal{S}_0^{\mu\nu}}) W_U^{\mu\nu} \qquad F_{UU,L} = \frac{x}{4z} F_0 (2\underbrace{\mathcal{S}_1^{\mu\nu}}) W_U^{\mu\nu}$$

$$\mathcal{S}_0^{\mu\nu} = g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} \qquad \mathcal{S}_1^{\mu\nu} = \frac{(2xP^\mu + q^\mu)(2xP^\nu + q^\nu)}{(1+\gamma^2)Q^2}$$

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The theoretical expression for $F_{UU,T}$ including KPCs is

$$F_{UU,T} = \frac{x}{2z} Q^2 \, \mathcal{C}_{KPC}[1, f_1 D_1] + \frac{1}{2} F_{UU,L}$$

The inclusion of the KPCs implies that $F_{UU,T}$ is not exactly 1 as it was in the LP case!

$$\underbrace{\mathcal{C}_{LP}[1, f_1 D_1]}_{(F_{UU,T})_{LP}} + \varepsilon F_{UU,L} \;\longrightarrow\; \boxed{\text{Cross-section gets modified}} \;\longrightarrow\; \mathcal{C}_{KPC}[1, f_1 D_1] + \left(\frac{1}{2} + \varepsilon\right) F_{UU,L}$$

$$F_{UU,T} + \varepsilon F_{UU,L}$$

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Cross-section gets modified

$$F_{UU,T} + \varepsilon F_{UU,L}$$

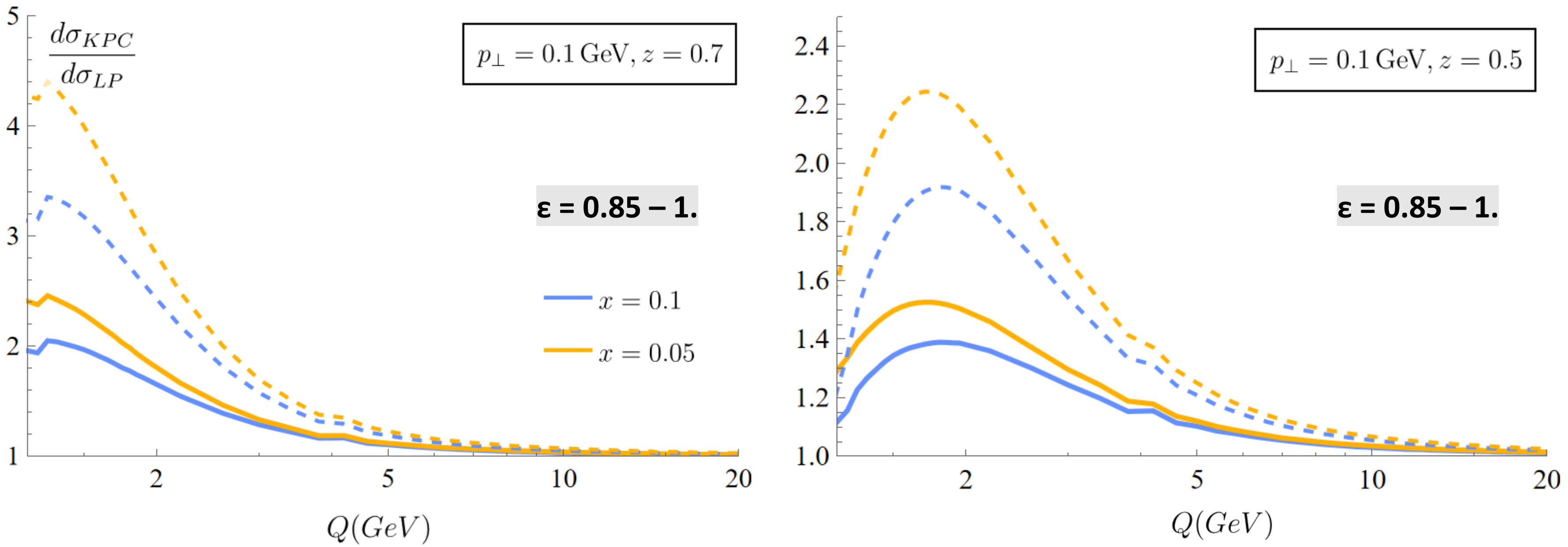
$$\mathcal{C}_{KPC}[1, f_1 D_1] + \left(\frac{1}{2} + \varepsilon\right) F_{UU,L}$$

- I have studied the KPCs impact on
  - The unpolarized SIDIS cross-section $F_{UU,T} + \varepsilon F_{UU,L}$
  - The structure functions $F_{UU,T}$ and $F_{UU,L}$

Ratio of KPCs-summed to LP cross-sections

- I study the ratio $d\sigma_{KPC}/d\sigma_{LP}$ in two different scenarios

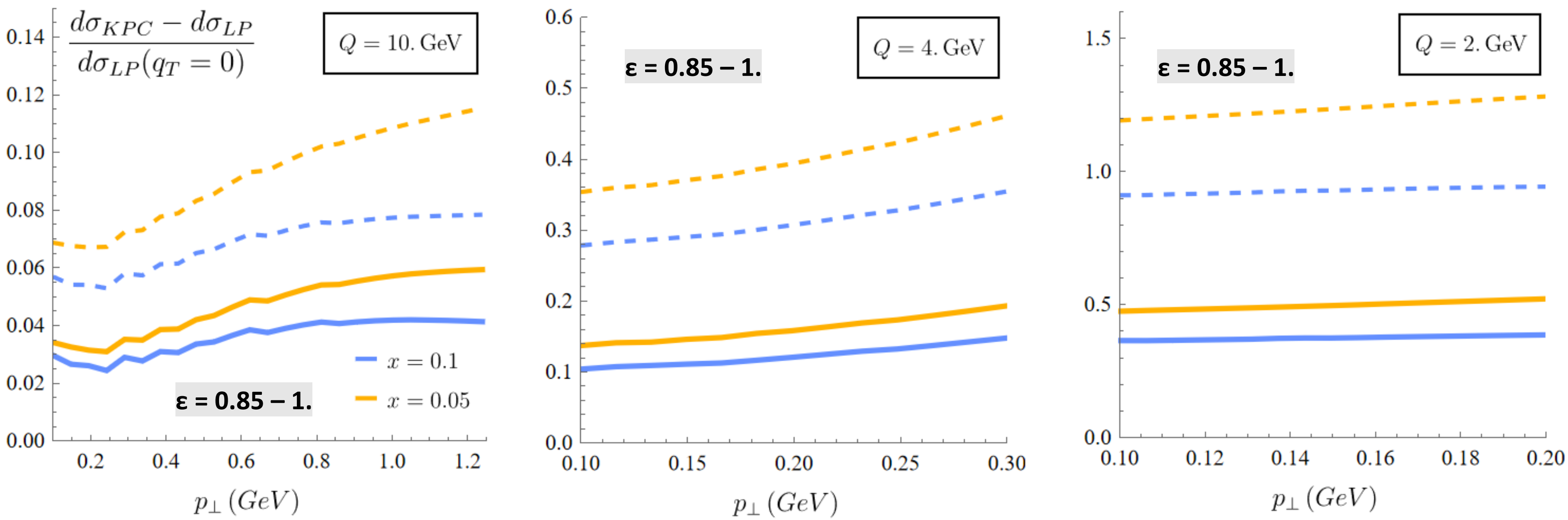
(i) $d\sigma_{KPC} \sim F_{UU,T}$ (ii) $d\sigma_{KPC} \sim F_{UU,T} + \varepsilon F_{UU,L}$



Q = 2 GeV → Corrections comparable to $(F_{UU,T})_{LP}$ both for $F_{UU,T}$ and $F_{UU,L}$

→ As Q grows the corrections become smaller and $\lim_{Q\to\infty} d\sigma_{KPC} = d\sigma_{LP}$

Cross-sections difference relative to the q_T = 0 LP cross-section

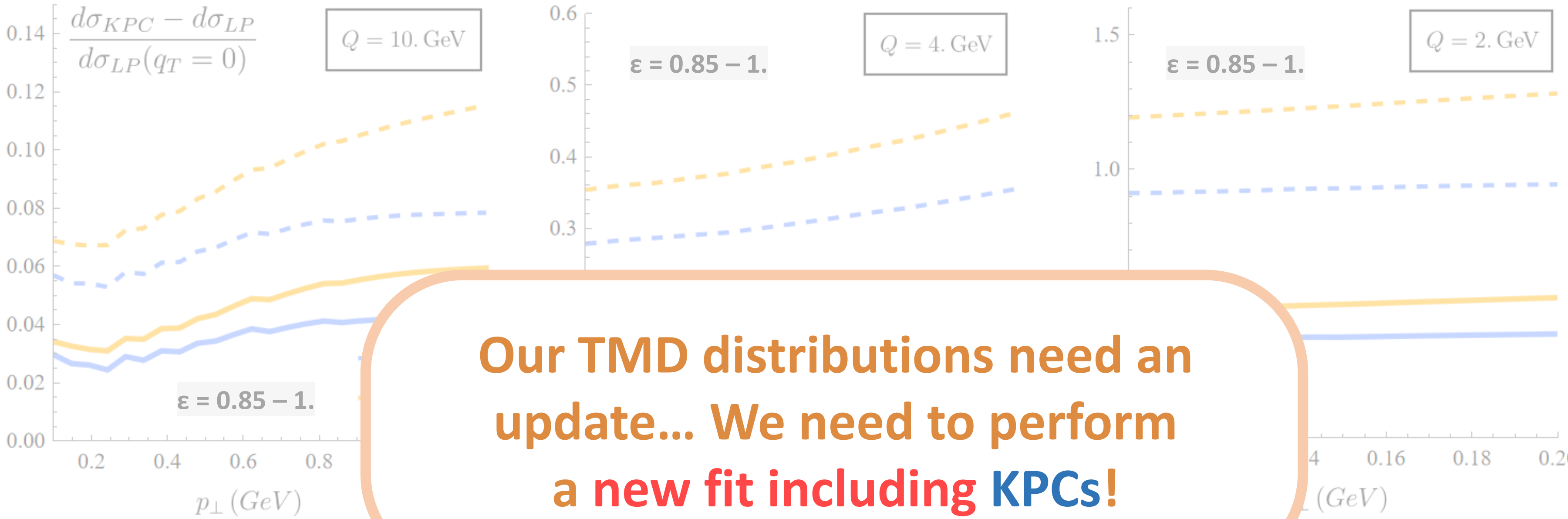


- ✓ **KPCs** very important at low energies

  Q = 2 GeV → About 40-50% $\xrightarrow{+\varepsilon F_{UU,L}}$ Several tens of percents!

- ✓ Almost flat increase of the cross-section → The inclusion of KPCs mostly changes the normalization

Cross-sections difference relative to the $q_T = 0$ LP cross-section



Our TMD distributions need an update… We need to perform a new fit including KPCs!

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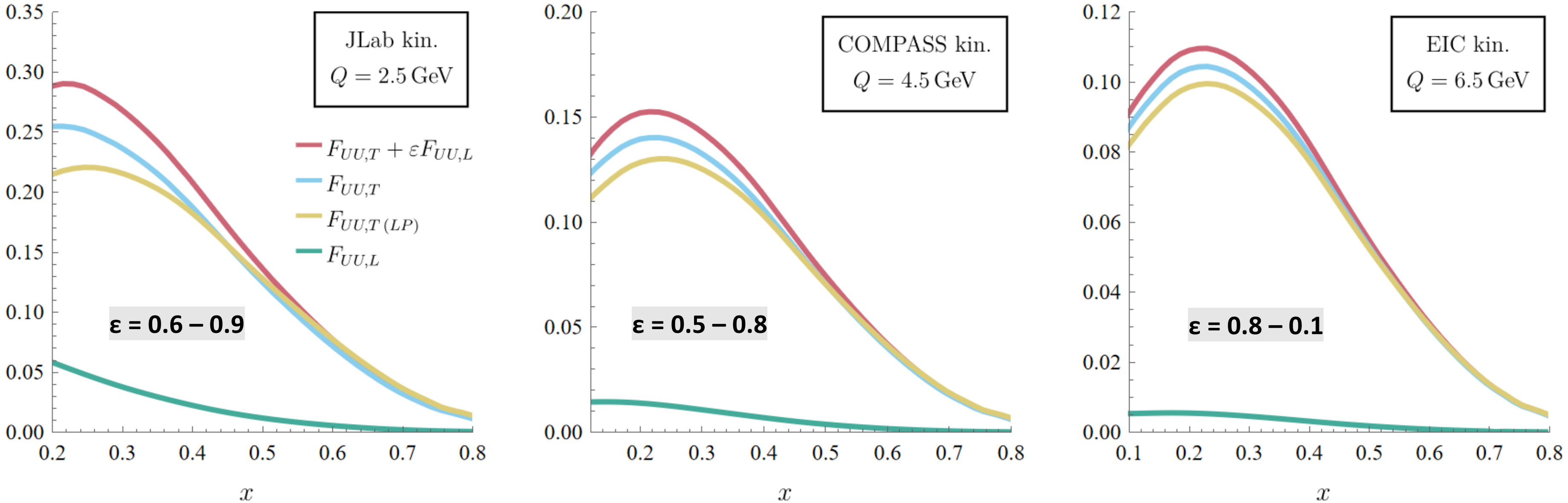
Longitudinal photons effects in different kinematics

- What about the contribution of longitudinal photons ($F_{UU,L}$) for different experiments?



✓ At low energies, the $F_{UU,L}$ contribution is clearly not negligible

JLab → significant difference between $(F_{UU,T})_{LP}$ and $F_{UU,T} + \varepsilon F_{UU,L}$

✓ Less important at higher energies → EIC → smaller but still visible

Conclusions

- The TMD-with-KPCs factorization theorem [AV, 2307.13054v2] has been tested
  - The angular distributions of Drell-Yan leptons can be satisfactorily described
  - It gives a nice description of the Lam-Tung relation

[SP, AV, 2407.06277]

- The subleading $F_{UU,T}$ and $F_{UU,L}$ SIDIS structure functions have been computed using it
  - The unpolarized SIDIS cross-section grows when including KPCs
  - The $F_{UU,L}$ contribution is not negligible at low energies like 2 – 4 GeV

Coming soon!

Our TMD distributions need an update... We need to perform a new fit including KPCs!

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  - The unpola
  - The $F_{UU,L}$

ming soon!

Thank you for your attention! ☺

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BACKUP

Power corrections

The four types

- Corrections to the LP term → suppressed by powers of $Q\,(\sim q^+ \sim q^-)$ large scale
- The power corrections can be categorized into four conceptual types
  - Target-mass corrections $\sim M/Q$ → hadron mass
  - *Higher-twist* power corrections $\sim (\Lambda/Q)^{n-2}$ → TMDs of larger twist *(n = D – S)*
  - q_T/Q power corrections

Position space

$$\sim \frac{1}{bQ}$$

↔

Momentum space

$$\sim \frac{q_T}{Q}$$

  - k_T/Q power corrections → Kinematic Power Corrections (KPCs)

Position space

$$\sim F_1(x_1,b)\,\frac{\partial}{\partial b^\mu}\,F_2(x_2,b)$$

↔

Momentum space

$$\sim \frac{k_{2T}^\mu}{Q}F_1(x_1,k_{1T})F_2(x_2,k_{2T})$$

→ Transverse derivatives of TMD distributions

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{$$

$$\Phi_2 \times \Phi_2 \qquad \text{LP}$$

$$+\frac{1}{Q}\Big(D\ \ \Phi_2\times\Phi_2 + \quad \Phi_2\times\Phi_3\Big) \qquad \text{NLP}$$

$$+\frac{1}{Q^2}\Big(D^2\,\Phi_2\times\Phi_2 + D\ \ \Phi_2\times\Phi_3 + \quad \Phi_3\times\Phi_3 + \quad \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\Big) \qquad \text{NNLP}$$

$$+\frac{1}{Q^3}\Big(D^3\,\Phi_2\times\Phi_2 + D^2\,\Phi_2\times\Phi_3 + D\ \ \Phi_3\times\Phi_3 + D\ \ \Phi_2+\Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + \ldots\Big)$$

$$+\frac{1}{Q^4}\Big(D^4\,\Phi_2\times\Phi_2 + \ldots\Big)$$

$$+\ \ldots \Bigg\}^{\mu\nu}$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \dfrac{\partial}{\partial b^\mu}$ → transverse derivative

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$
\begin{aligned}
W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{ & \Phi_2 \times \Phi_2 \\
& + \frac{1}{Q}\Big(D\ \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \Big) \\
& + \frac{1}{Q^2}\Big(D^2\, \Phi_2 \times \Phi_2 + D\ \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \Big) \\
& + \frac{1}{Q^3}\Big(D^3\, \Phi_2 \times \Phi_2 + D^2\, \Phi_2 \times \Phi_3 + D\ \Phi_3 \times \Phi_3 + D\ \Phi_2 + \Phi_4 + D\frac{\Phi_2 \times \Phi_2}{b^2} + \dots \Big) \\
& + \frac{1}{Q^4}\Big(D^4\, \Phi_2 \times \Phi_2 + \dots \Big) \\
& + \dots \Bigg\}^{\mu\nu}
\end{aligned}
$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \dfrac{\partial}{\partial b^\mu}$ → transverse derivative

Each **column**
- Series of KPCs to the first term
- Unique non-perturbative content
- Independent contributions

The hierarchy of the TMD factorization theorem

- **General structure** of the **TMD factorization theorem**

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \Bigg\{ \Phi_2 \times \Phi_2$$

$$+\frac{1}{Q}\left(D\ \Phi_2\times\Phi_2 + \Phi_2\times\Phi_3\right)$$

$$+\frac{1}{Q^2}\left(D^2\ \Phi_2\times\Phi_2 + D\ \Phi_2\times\Phi_3 + \Phi_3\times\Phi_3 + \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\right)$$

$$+\frac{1}{Q^3}\left(D^3\ \Phi_2\times\Phi_2 + D^2\ \Phi_2\times\Phi_3 + D\ \Phi_3\times\Phi_3 + D\ \Phi_2+\Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + ...\right)$$

$$+\frac{1}{Q^4}\left(D^4\ \Phi_2\times\Phi_2 + ...\right)$$

$$+ \ ... \Bigg\}^{\mu\nu}$$

k_T/Q

k^2_T/Q^2

k^3_T/Q^3

k^4_T/Q^4

Notation

Φ_n → TMD of twist-n

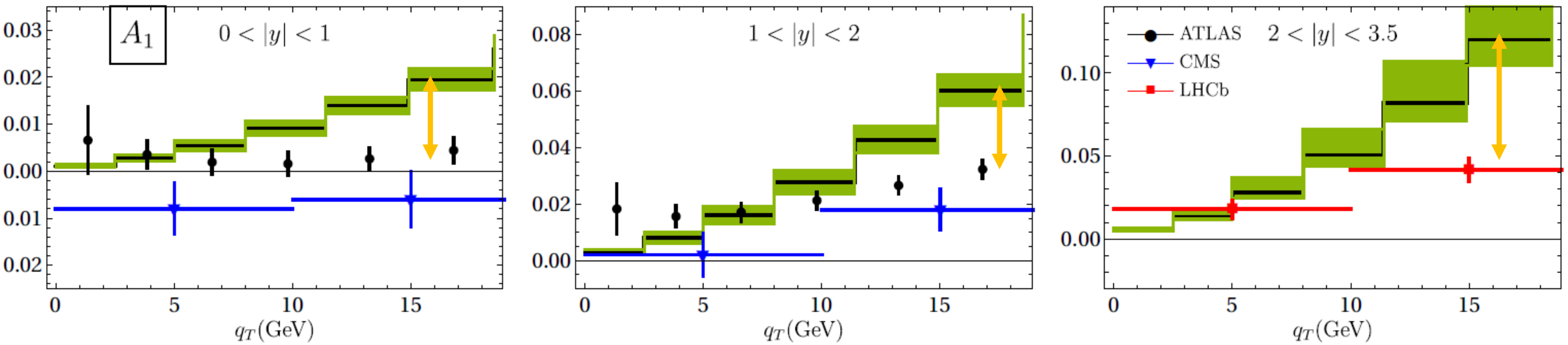
$\times$ → Integral convolution

$D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

KPCs following the **LP term**

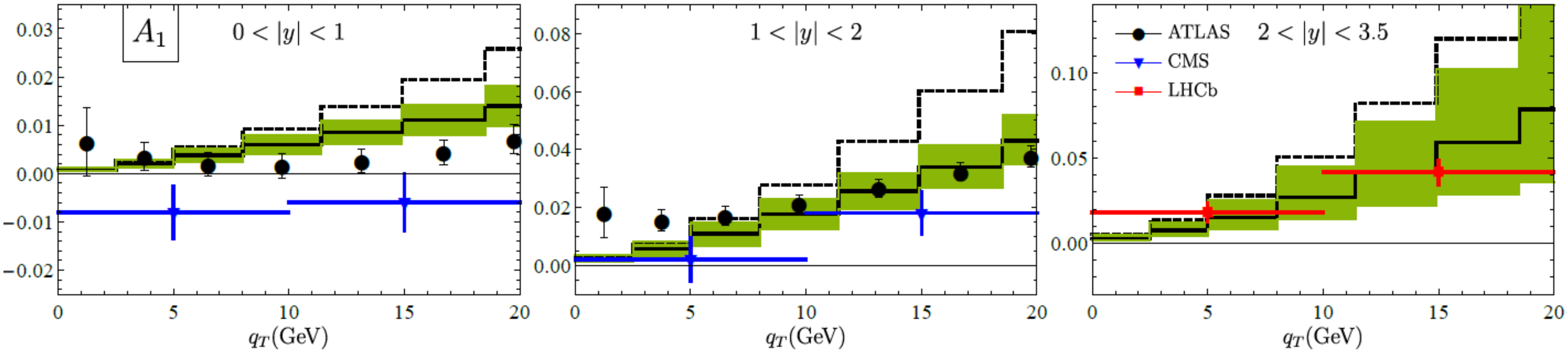
- Same TMDs and perturbative part as LP
- All power-suppressed terms containing twist-2 TMDs
- Restoration of gauge and frame invariance

Angular distribution A_1

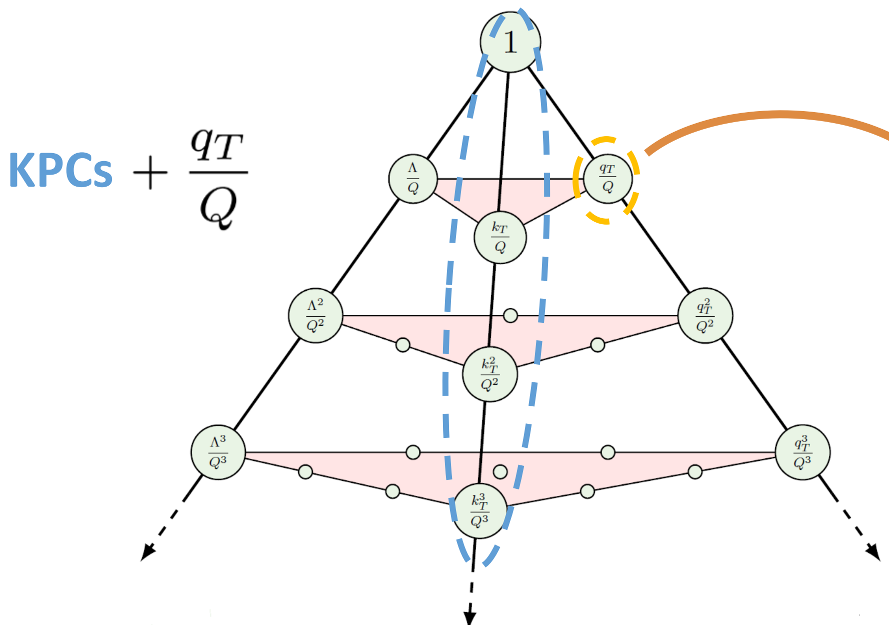


- Problems with the A_1 data description at larger values of q_T

Angular distribution A_1



- Problems with the A_1 data description at larger values of q_T



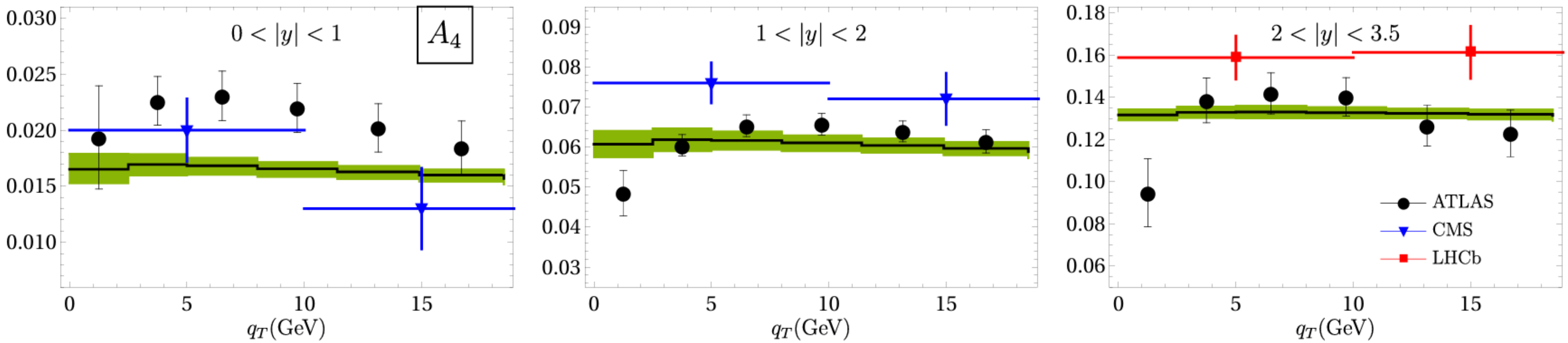
Including the computation of the leading q_T/Q correction fixes the jump!

[A.Arroyo, I.Scimemi, AV, 2503.24336]

Angular distribution A_4

- **Leading power**
- Proportional to the **difference between quark and anti-quark** distributions (**anti-symmetric flavor**)
- **Inclusion** in standard f_1 **extractions**

➢ Comparison as a **function** of q_T with **ATLAS**, **CMS** and **LHCb** measurements

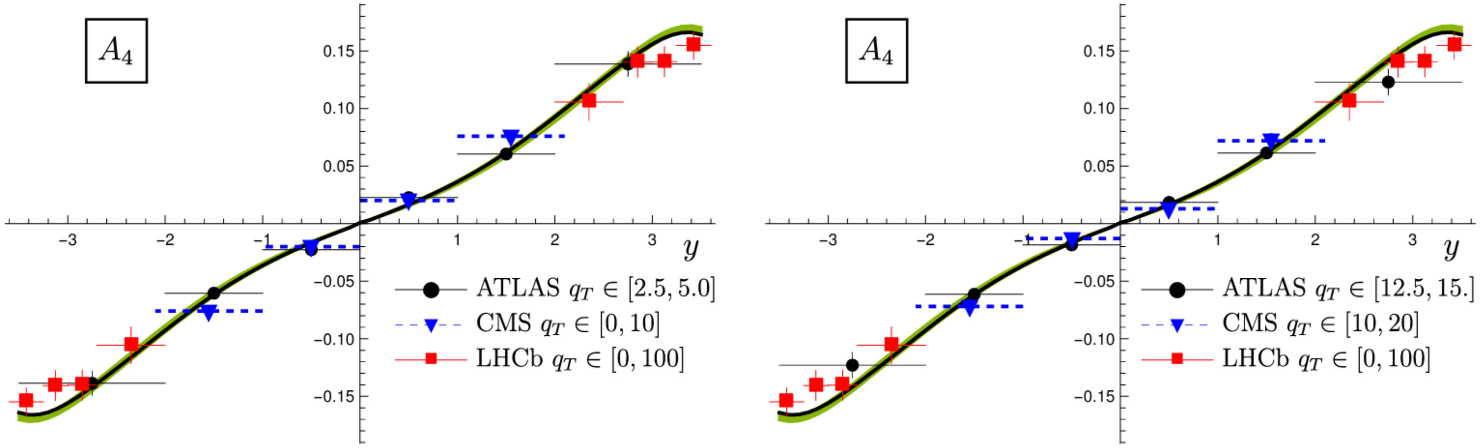


- Theory **prediction agrees** very well **with the measurements**

Angular distribution A_4

- **Leading power**
- Proportional to the **difference between quark and anti-quark** distributions (**anti-symmetric flavor**)
- **Inclusion** in standard f_1 **extractions**

➢ Comparison as a **function** of **y** with **ATLAS**, **CMS** and **LHCb** measurements



- The **agreement** is **even more transparent**