Progress on the Next-to-Leading Log framework within High Energy Jets (HEJ)

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- based on work done with members from the HEJ collaboration -

Resummation, Evolution and Factorization 2025

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(One of the many) Roles QCD plays in LHC processes

Both experimental and theoretical advancements made for testing the limits of the Standard Model

$\rightarrow$ **precision era**

From the many processes that occur at the LHC the focus for this talk will be on those that have a t-channel-gluon mediated contribution

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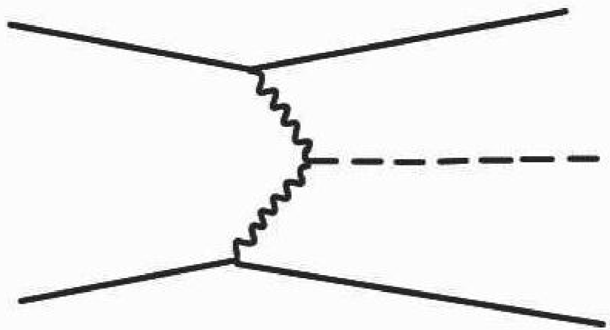
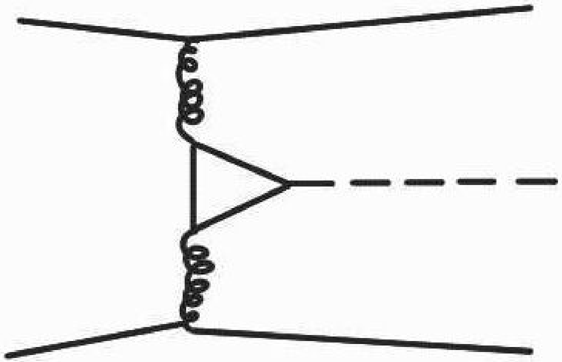
From the many processes that occur at the LHC the focus for this talk will be on those that have a *t*-channel-gluon mediated contribution

As an example: Vector Boson Fusion (VBF) vs *t*-channel gluon fusion (both give Higgs + jets)





What theoretical approach could help one distinguish between such two different origins?

gluon exchange in hard interaction !! → **large perturbative corrections at high energies** → **resummation**

Lessons learnt from Regge theory

In a *2 → 2* scattering process the **Regge limit** means $s >> |t|$

Dominant behaviour in this limit:

asymptotic behaviour of Legendre polynomial of **highest spin *j*** in a ***t*-channel** exchange

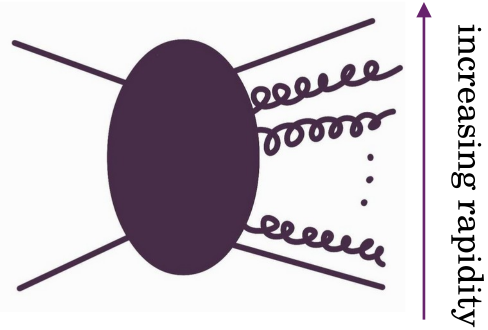
$$\mathcal{M}(s,t) \longmapsto 16\,\pi\;(2j+1)\;a_j(t)\;\left(\frac{s}{t}\right)^j$$

QCD candidate: gluon field exchanged in *t*-channel

For a general *2 → n* scattering

the **Multi-Regge Kinematics (MRK) limit** demands:

$$y_1 \gg y_2 \gg \ldots \gg y_n \quad \text{and} \quad |p_{\perp\, i}|^2 \simeq |p_{\perp\, j}|^2\,,\;\; \forall\, i,j$$



In the *2 → 3* scattering (a gluon emission):

$$\int \frac{d^3p_g}{(2\pi)^3\,2\,E_g}\cdot\left|\mathcal{M}_{2\to3}\right|^2 = \int\frac{d^2p_{\perp\,g}}{(2\pi)^2}\int_{y_3}^{y_1}\frac{dy_g}{4\pi}\cdot\left|\mathcal{M}^{\mathrm{MRK}}_{2\to3}\right|^2 = \Delta y\int\frac{d^2p_{\perp\,g}}{(2\pi)^2}\cdot\left|\mathcal{M}^{\mathrm{MRK}}_{2\to3}\right|^2$$

where $\left|\mathcal{M}^{\mathrm{MRK}}_{2\to3}\right|^2 \propto \dfrac{1}{|p_{\perp\,g}|^2}$ and $\Delta y \underset{\mathrm{MRK}}{\longmapsto} \ln\left(\dfrac{s}{-t}\right)$

High energy logarithms and their resummation

Reorganizing the perturbative series

$$
\begin{aligned}
d\sigma^{pp \to 2j + X} &= c_0^{(1)} \alpha_s^2 \\
&+ \left[c_1^{(2)} \ln\left(-\frac{s}{t}\right) + c_0^{(2)} \right] \alpha_s^3 \\
&+ \left[c_2^{(3)} \ln^2\left(-\frac{s}{t}\right) + c_1^{(3)} \ln\left(-\frac{s}{t}\right) + c_0^{(3)} \right] \alpha_s^4 \\
&+ \left[c_3^{(4)} \ln^3\left(-\frac{s}{t}\right) + c_2^{(4)} \ln^2\left(-\frac{s}{t}\right) + c_1^{(4)} \ln\left(-\frac{s}{t}\right) + c_0^{(4)} \right] \alpha_s^5 \\
&+ \left[c_4^{(5)} \ln^4\left(-\frac{s}{t}\right) + c_3^{(5)} \ln^3\left(-\frac{s}{t}\right) + c_2^{(5)} \ln^2\left(-\frac{s}{t}\right) + c_1^{(5)} \ln\left(-\frac{s}{t}\right) + c_0^{(5)} \right] \alpha_s^6 \\
&+ \cdots\cdots
\end{aligned}
$$

LL **NLL** **NNLL**

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&+ \left[c_2^{(3)}\ln^2\left(-\frac{s}{t}\right) + c_1^{(3)}\ln\left(-\frac{s}{t}\right) + c_0^{(3)}\right]\alpha_s^4 \\
&+ \left[c_3^{(4)}\ln^3\left(-\frac{s}{t}\right) + c_2^{(4)}\ln^2\left(-\frac{s}{t}\right) + c_1^{(4)}\ln\left(-\frac{s}{t}\right) + c_0^{(4)}\right]\alpha_s^5 \\
&+ \left[c_4^{(5)}\ln^4\left(-\frac{s}{t}\right) + c_3^{(5)}\ln^3\left(-\frac{s}{t}\right) + c_2^{(5)}\ln^2\left(-\frac{s}{t}\right) + c_1^{(5)}\ln\left(-\frac{s}{t}\right) + c_0^{(5)}\right]\alpha_s^6 \\
&+ \ldots\ldots
\end{aligned}
$$

Virtual level

Lipatov ansatz for t-channel propagators:

[Kuraev, Lipatov, Fadin (1976)]

$$\frac{1}{t} \longmapsto \frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)}$$

(verified at NNLO by full QCD [hep-ph/0109028])

The Regge trajectory at one-loop is

$$\alpha(t) = 2\, g_s^2\, C_A\, c_\Gamma\, \frac{1}{\epsilon}\left(\frac{\mu_R^2}{-t}\right)^{\epsilon}$$

Real emission level

The phase space measure of the k^{th} emission:

$$\int \frac{d^3 p_k}{(2\pi)^3\, 2\, E_k} \cdot \left|\mathcal{M}_{2\to n}^{\text{MRK}}\right|^2$$

$$= \int \frac{d^2 p_{\perp\, k}}{(2\pi)^2} \int_{y_{k-1}}^{y_{k+1}} \frac{dy_k}{4\pi} \cdot \left|\mathcal{M}_{2\to n}^{\text{MRK}}\right|^2$$

But $\left|\mathcal{M}_{2\to n}^{\text{MRK}}\right|^2$ does not depend on the rapidity

This will result in log of large invariants

The differential cross section in bins of m_{jj}

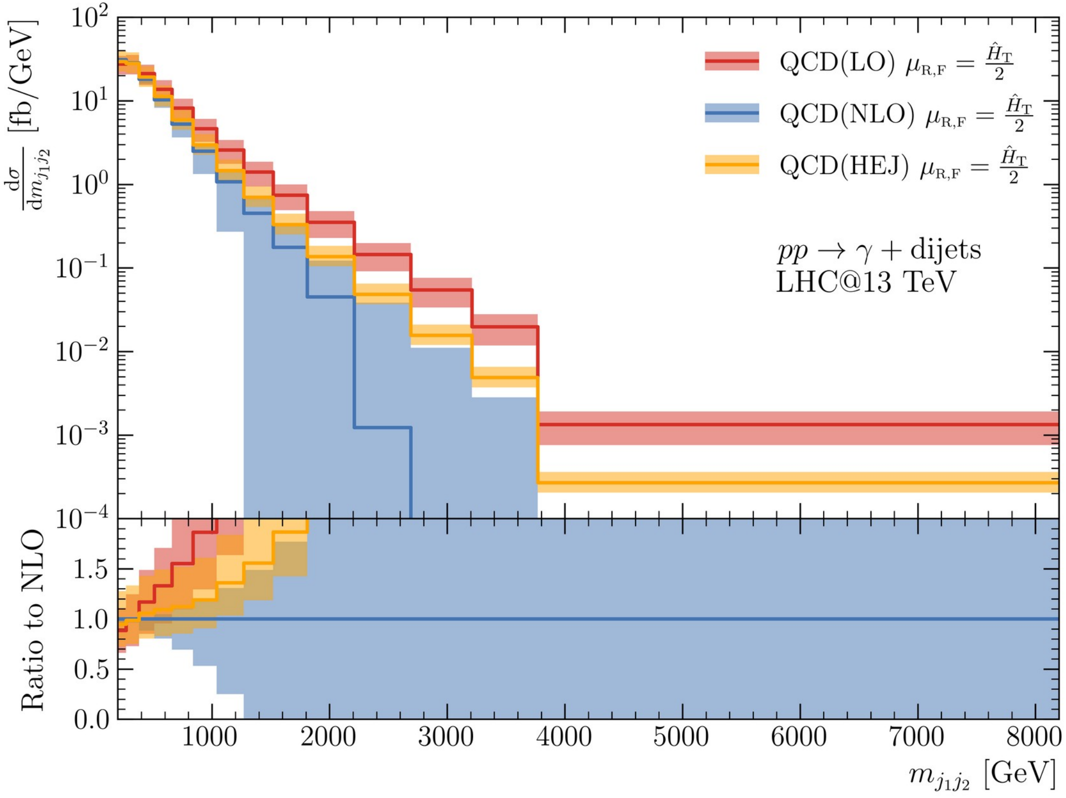
Process studied: $pp \longrightarrow \gamma + 2j$ ... Why?

[hep-ex/2403.02793]

	Final-state event selection					
Production process	p_T^{miss}+jets	$2e$+jets	2μ+jets	e+jets	μ+jets	γ+jets
$Z \to \nu\nu$ + jets	55%	–	–	–	–	–
$Z \to ee$ + jets	–	94%	–	–	–	–
$Z \to \mu\mu$ + jets	–	–	95%	–	2%	–
$W \to e\nu$ + jets	6%	–	–	68%	–	–
$W \to \mu\nu$ + jets	9%	–	–	–	67%	–
$W \to \tau\nu$ + jets	20%	–	–	5%	7%	–
γ + jets	–	–	–	–	–	>99%
Top	7%	3%	2%	25%	21%	–
Multi-boson	3%	3%	3%	2%	3%	<1%

Photon rapidity	$\|y\| \leq 1.37$ or $1.52 \leq \|y\| \leq 2.47$
Leading photon p_T	> 160 GeV
p_T^{recoil}	> 200 GeV
Leading jet p_T	> 80 GeV
Sub-leading jet p_T	> 50 GeV
Leading jet $\|y\|$	< 4.4
Sub-leading jet $\|y\|$	< 4.4
Dijet invariant mass $m_{j_1 j_2}$	> 200 GeV
$\|\Delta y_{j_1 j_2}\|$	> 1
Jets with rapidity in-between hardest two	None with $p_T > 30$ GeV
Jet definition	anti-k_t, $R = 0.4$

[hep-ph/2506.17438]



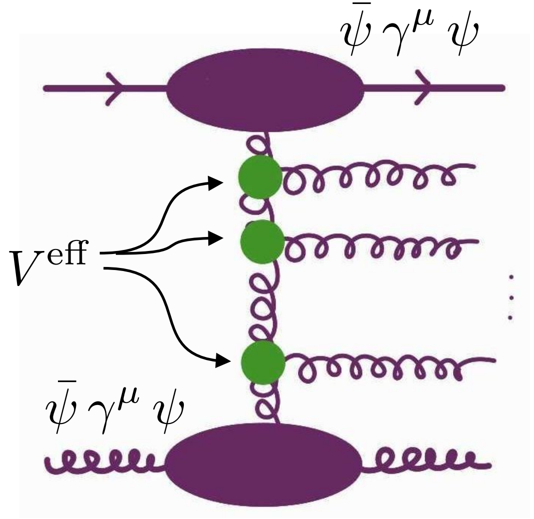
$m_{j_1 j_2}$ is the invariant mass between the two hardest jets (j_1 and j_2)

(A very brief) Introduction to High Energy Jets

HEJ performs a leading log (LL) resummation of high energy logarithms to all orders. It does this with the help of **factorised** amplitudes $\rightarrow$ can construct high multiplicity processes (and includes subleading channels too).

(!! factorised does not necessarily imply that the dependence is only on local momenta)

impact currents and ***t*-channel emissions** create an **effective** diagram (**Lipatov vertices + Lipatov ansatz**)



HEJ works with approximations of amplitudes which preserve:

- **gauge invariance**
- **Lorentz invariance**
- **crossing symmetry**
- **momentum conservation**

$\Longrightarrow$

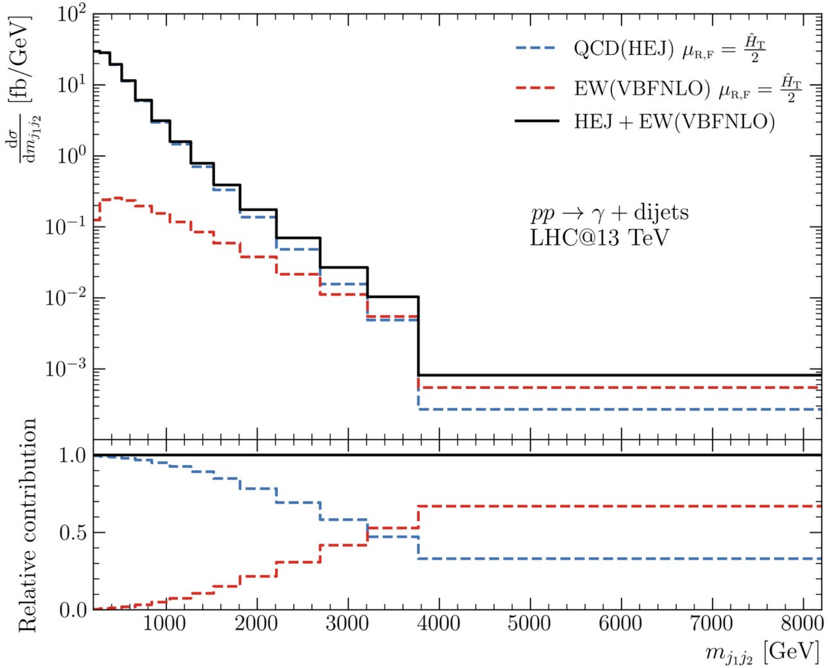
$\Longrightarrow$ retaining these properties can make the predictions for the LHC processes better and richer.

HEJ can also perform matching to full fixed order results.

Some highlights from previous studies

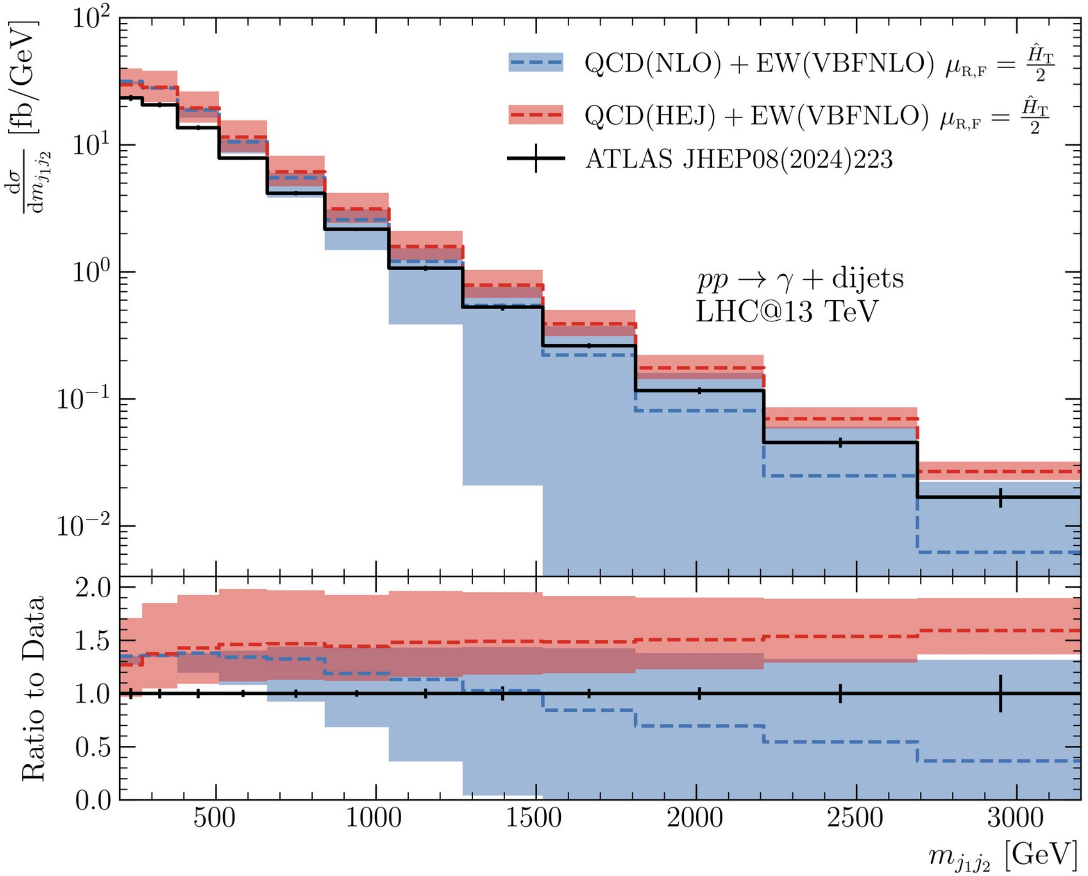
1. Stable HEJ prediction for $\gamma + 2j$

[hep-ph/2506.17438]



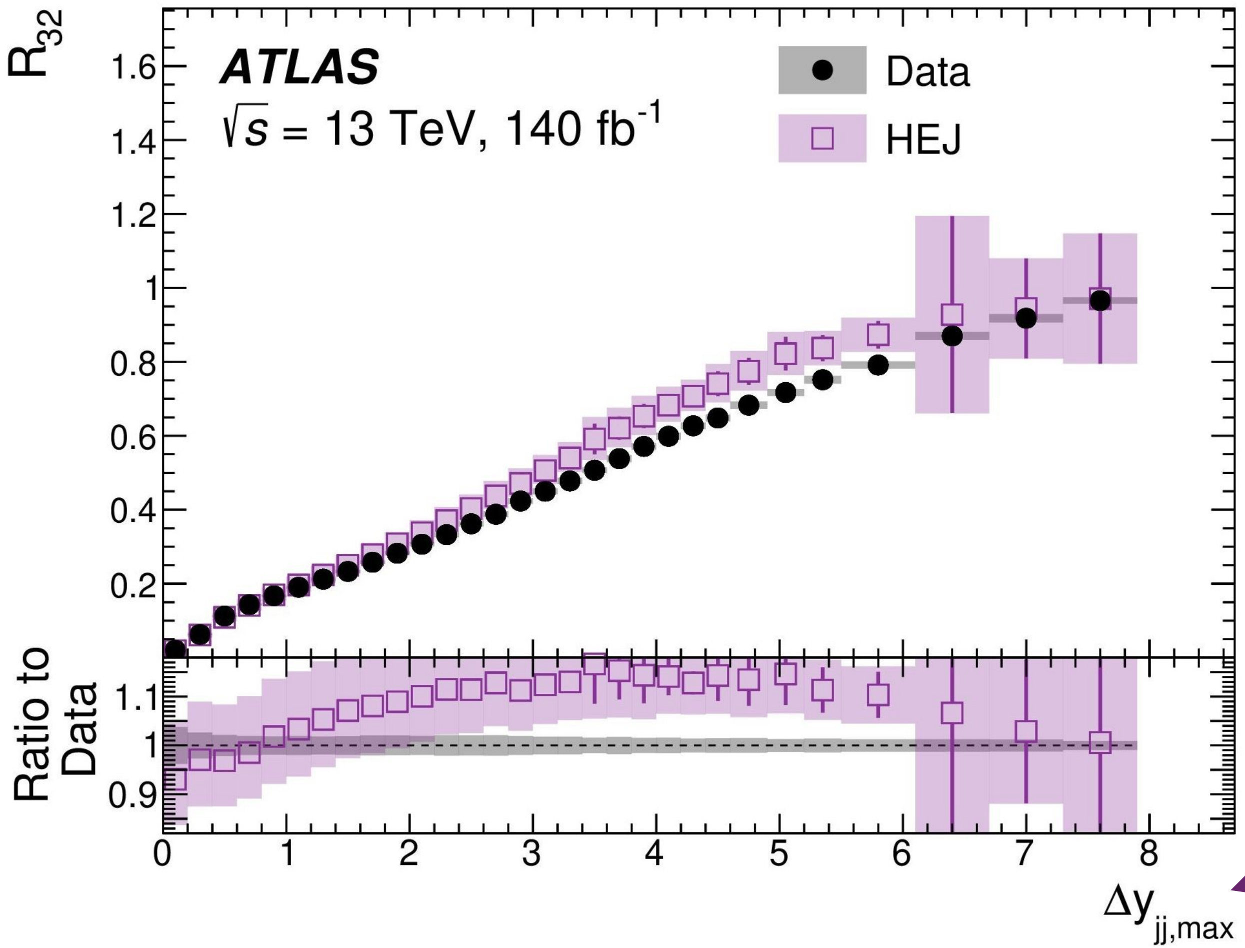
- focus on QCD-dominated m_{jj} range
- jet veto implemented
- photon isolation procedure introduces effects beyond NLO and HEJ → off-set is due to those
- scale variation under control – good indication of perturbative stability

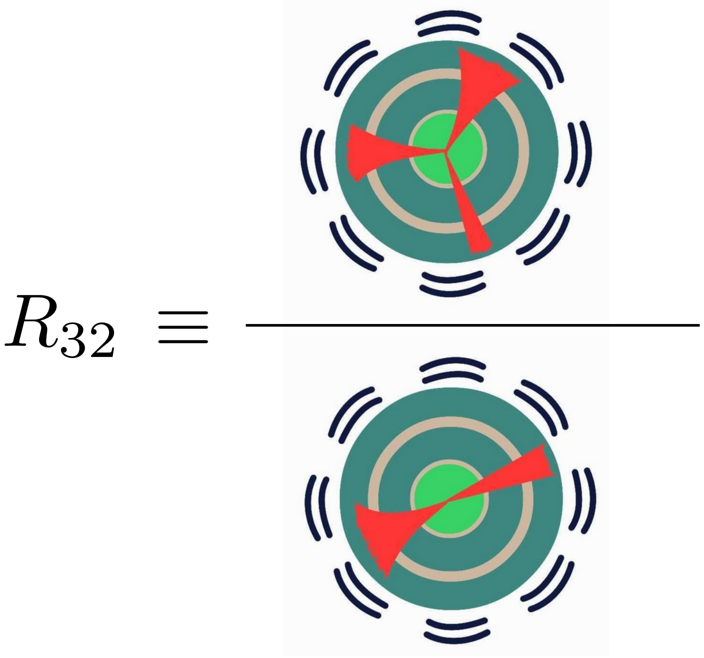
VBFNLO 3.0 - [hep-ph/2405.06990]



2. HEJ prediction for R_{32}

[hep-ex/2405.20206]





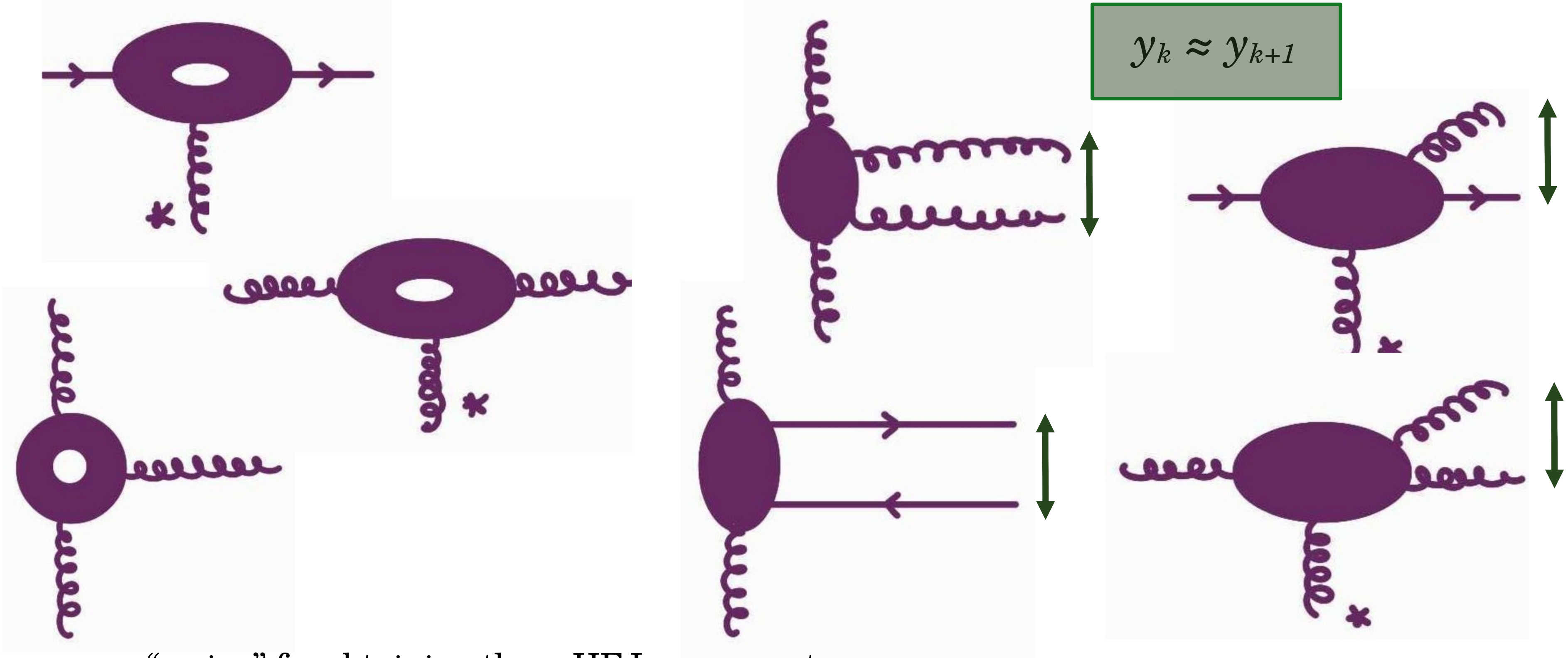
NLL could improve the slope!

absolute value of maximum rapidity difference between most forward and most backward jets

What does it take to do NLL resummation in HEJ?

Next-to-Leading-Log contribution $\Longrightarrow$ α_s corrections to all the LL pieces, meaning:

one-loop corrections and **relaxing** the MRK regime to a **Quasi-Multi-Regge** one instead



There was no “recipe” for obtaining these HEJ components …

...how should one start designing these pieces so as to fulfill all HEJ requirements?

HEJ amplitudes can claim **full NLL accuracy** ***and*** all HEJ amplitudes are **factorised**

the entire relevant phase space is covered

IR structure correctly accounted for and split between currents and resummation region

extend code's MC for the extra phase space

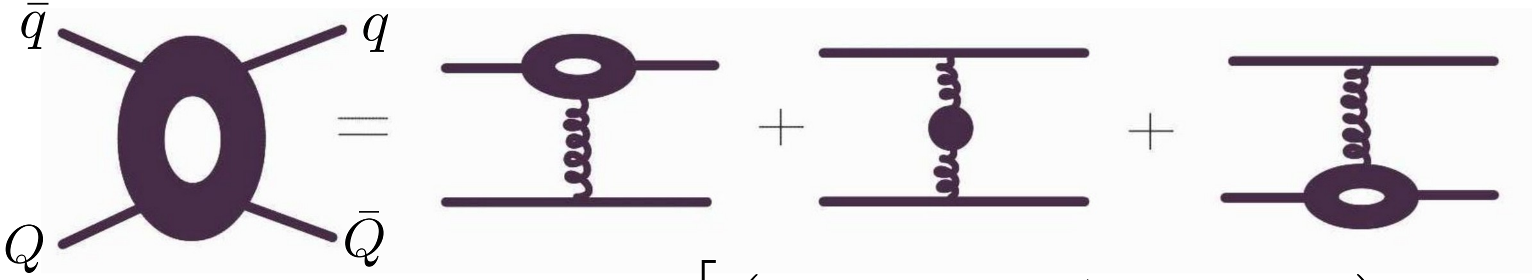
decide on and employ a subtraction scheme

(spoiler: FKS)

are the pieces **universal**?
or should every different partonic channel be treated differently?

One-loop-corrected impact current

HEJ works directly with helicity-spinor objects $\Longrightarrow$ useful to resort to full QCD results of one-loop amplitude given in terms of colour-ordered components **[hep-ph/9305239]**



$$\mathcal{M}^{\text{1-loop}}(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}) = g_s^4 \Bigg[\left(\delta_{\iota_{\bar{q}}\, \iota_Q}\, \delta_{\iota_{\bar{Q}}\, \iota_q} - \frac{1}{N_c}\, \delta_{\iota_{\bar{q}}\, \iota_q}\, \delta_{\iota_{\bar{Q}}\, \iota_Q} \right) a_{4;1}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q})$$
$$+ \left(\delta_{\iota_{\bar{q}}\, \iota_Q}\, \delta_{\iota_{\bar{Q}}\, \iota_q} \right) a_{4;2}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}) \Bigg]$$

The primitive amplitudes are proportional to the **tree level**:

$$a_{4;1}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}) = c_\Gamma\, \boxed{a_{4;0}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q})}\; F_{a;1}^{(\lambda_q,\lambda_Q)}(\epsilon, s_{\bar{q}\bar{Q}}, s_{\bar{q}Q}, s_{\bar{q}q})$$
$$a_{4;2}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}) = c_\Gamma\, \boxed{a_{4;0}\,(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q})}\; F_{a;2}^{(\lambda_q,\lambda_Q)}(\epsilon, s_{\bar{q}\bar{Q}}, s_{\bar{q}Q}, s_{\bar{q}q})$$

One-loop scalar functions for $qQ \to qQ$ scattering

Considering a physical $2 \to 2$ process for which: $s = s_{qQ}\,, \quad t = -s_{q\bar{q}}\,, \quad u = -s_{\bar{q}Q}$

[hep-ph/9305239]

$$F_{a;1}^{(\ominus\,\ominus)} = \left(-\frac{\mu^2}{t}\right)^{\epsilon} \left\{ N_c \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{11}{3\epsilon} - \frac{2}{\epsilon}\ln\left(-\frac{t}{s}\right) + \frac{19}{9} - \frac{2\delta_R}{3} + \pi^2 \right] + N_f \left[-\frac{2}{3\epsilon} - \frac{10}{9} \right] \right.$$

$$\left. -\frac{1}{N_c}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right) - 7 - \delta_R - \frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(-\frac{t}{s}\right) + \pi^2\right) - \frac{t}{s}\ln\left(\frac{t}{u}\right) \right] \right\} - \frac{\beta_0}{\epsilon}$$

$$F_{a;2}^{(\ominus\,\ominus)} = \left(-\frac{\mu^2}{t}\right)^{\epsilon} \left(N_c - \frac{1}{N_c}\right) \left[-\frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right) - \frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(\frac{t}{u}\right) + \pi^2\right) - \frac{t}{s}\ln\left(\frac{t}{u}\right) \right]$$

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[hep-ph/9305239]

$$F^{(\ominus\,\ominus)}_{a;1} = \left(-\frac{\mu^2}{t}\right)^{\epsilon}\Bigg\{N_c\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}+\frac{11}{3\epsilon}-\boxed{\frac{2}{\epsilon}\ln\left(-\frac{t}{s}\right)}+\frac{19}{9}-\frac{2\delta_R}{3}+\pi^2\right]+N_f\left[-\frac{2}{3\epsilon}-\frac{10}{9}\right]$$

$$-\frac{1}{N_c}\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-\frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right)-7-\delta_R-\frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(-\frac{t}{s}\right)+\pi^2\right)-\frac{t}{s}\ln\left(\frac{t}{u}\right)\right]\Bigg\}-\frac{\beta_0}{\epsilon}$$

$$F^{(\ominus\,\ominus)}_{a;2} = \left(-\frac{\mu^2}{t}\right)^{\epsilon}\left(N_c-\frac{1}{N_c}\right)\left[-\frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right)-\frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(\frac{t}{u}\right)+\pi^2\right)-\frac{t}{s}\ln\left(\frac{t}{u}\right)\right]$$

Recall from before that the Lipatov ansatz reads:

$$\frac{1}{t}\left(-\frac{s}{t}\right)^{\alpha(t)} \longmapsto \frac{1}{t}\exp\left[\alpha(t)\,\ln\left(-\frac{s}{t}\right)\right] \quad \text{where} \quad \alpha(t) = 2\,g_s^2\,C_A\,c_\Gamma\,\frac{1}{\epsilon}\left(\frac{\mu_R^2}{-t}\right)^{\epsilon}$$

One-loop scalar functions for $qQ \to qQ$ scattering

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[hep-ph/9305239]

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$$\left. - \frac{1}{N_c} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right) - 7 - \delta_R - \frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(-\frac{t}{s}\right) + \pi^2\right) - \frac{t}{s}\ln\left(\frac{t}{u}\right) \right] \right\} - \frac{\beta_0}{\epsilon}$$

$$F_{a;2}^{(\ominus\ominus)} = \left(-\frac{\mu^2}{t}\right)^{\epsilon} \left(N_c - \frac{1}{N_c}\right) \left[-\frac{2}{\epsilon}\ln\left(-\frac{s}{u}\right) - \frac{1}{2}\frac{t}{s}\left(1+\frac{u}{s}\right)\left(\ln^2\left(\frac{t}{u}\right) + \pi^2\right) - \frac{t}{s}\ln\left(\frac{t}{u}\right) \right]$$

!!! **crossing symmetry** demands distinguishing all invariants.

As a consequence, the **entire IR structure** will be kept.

This enforces the real emission part to account for all IR points as well.

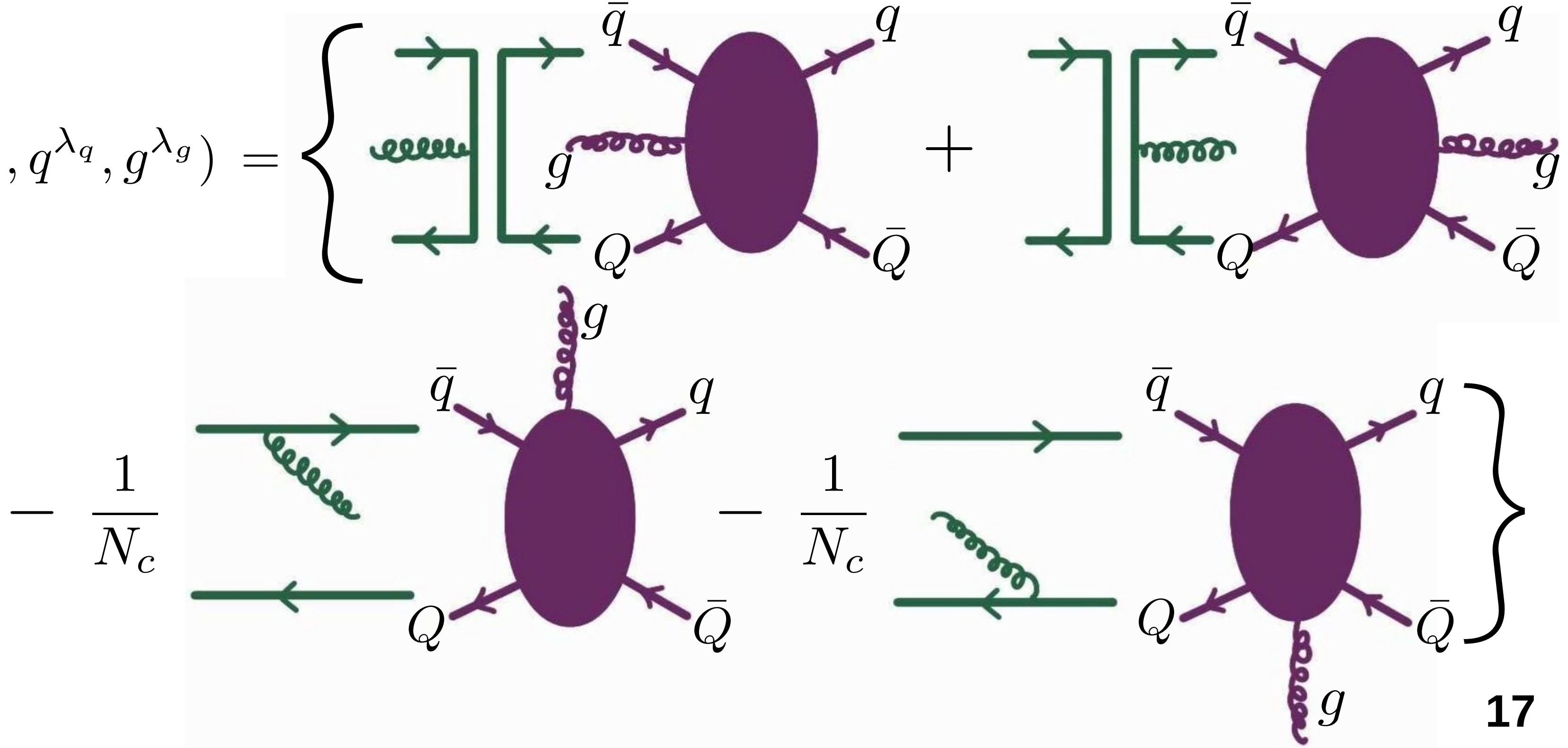
Real emission impact current

1. The full amplitude for $qQ \to qQg$

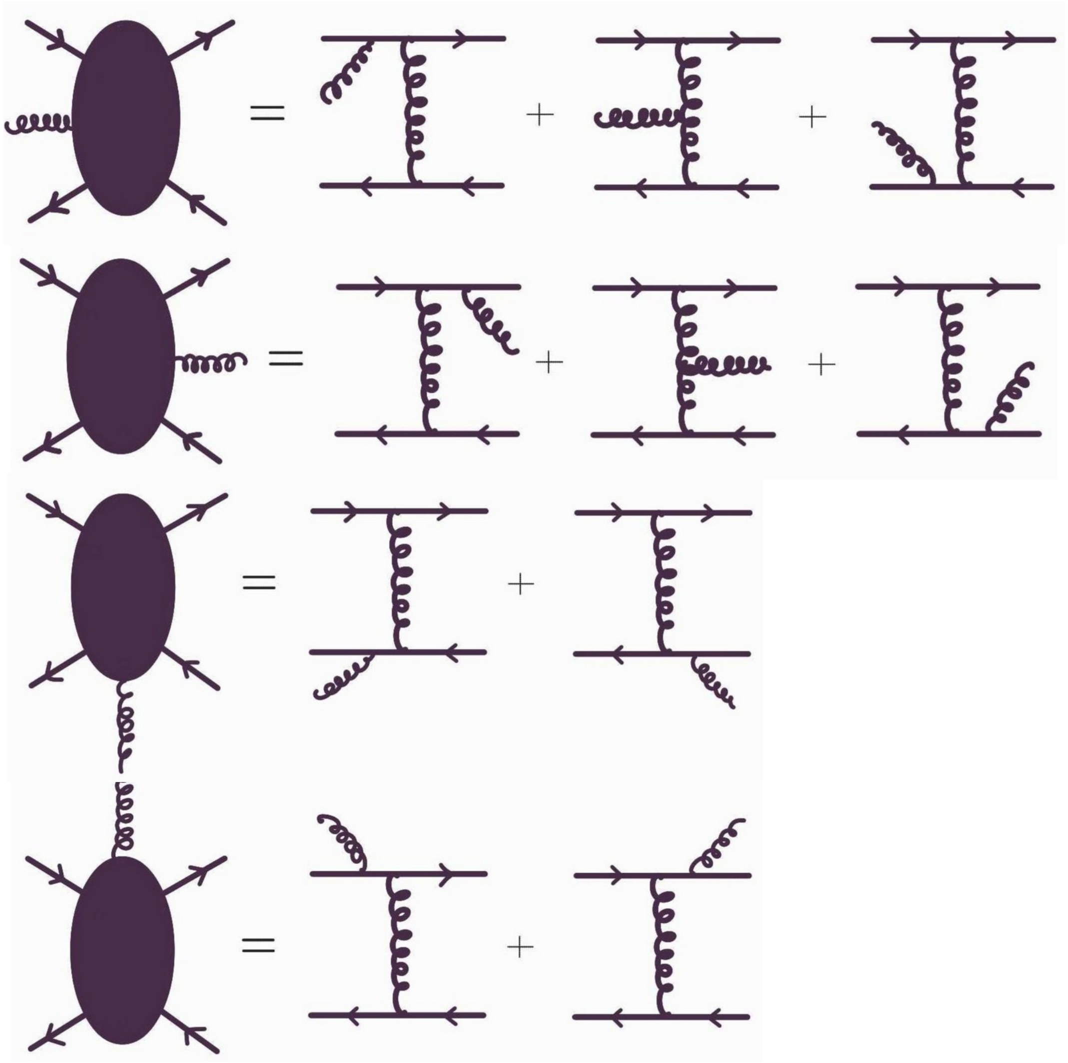
Colour-ordered helicity-dependent amplitudes for this process exist in the literature as well.

[hep-ph/9401294]

$$\mathcal{M}(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) =$$



2. Obtaining the HEJ amplitude



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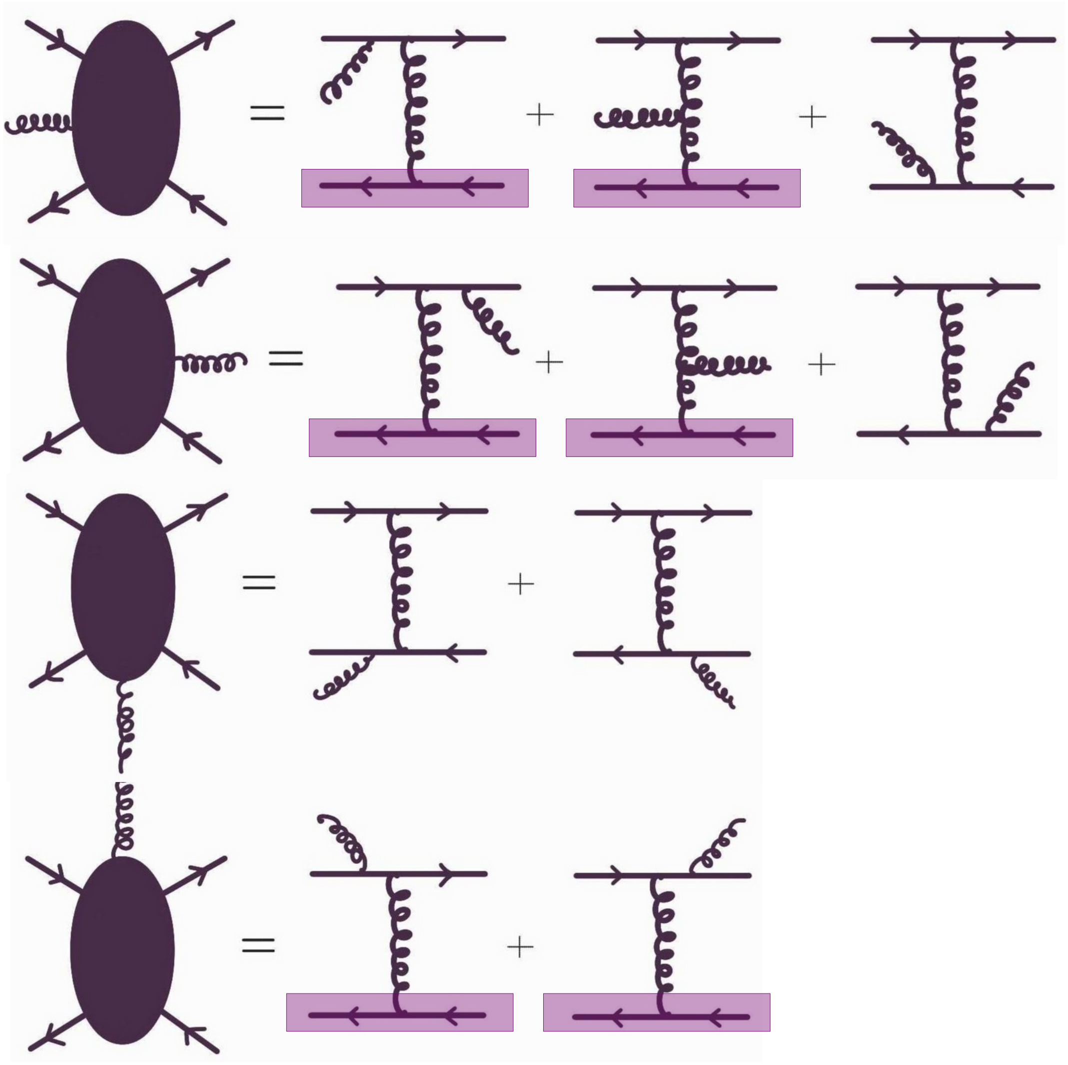
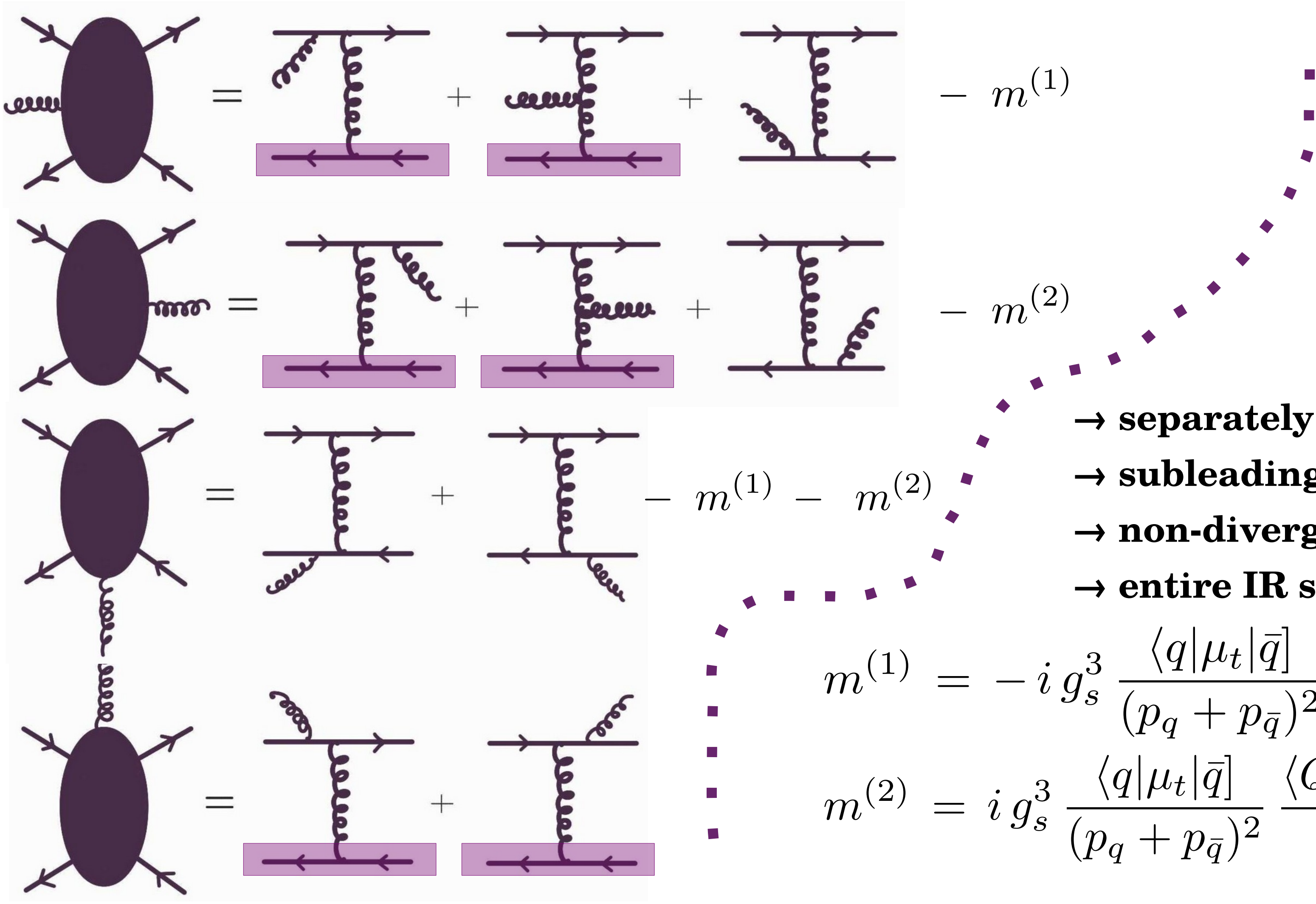


directly factorisable current

2. Obtaining the HEJ amplitude

Effective amplitudes



→ **separately gauge invariant !!**

→ **subleading in QMRK regime**

→ **non-divergent (!)**

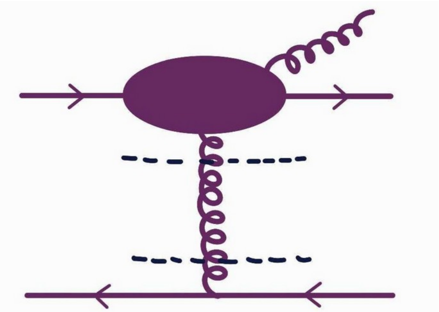
→ **entire IR structure retained**

$$m^{(1)} = -i\, g_s^3 \frac{\langle q|\mu_t|\bar{q}]}{(p_q + p_{\bar{q}})^2} \frac{\langle Q|\mu_g|g]\, \langle g|\mu_t|\bar{Q}]}{(p_g + p_Q)^2}$$

$$m^{(2)} = i\, g_s^3 \frac{\langle q|\mu_t|\bar{q}]}{(p_q + p_{\bar{q}})^2} \frac{\langle Q|\mu_t|g]\, \langle g|\mu_g|\bar{Q}]}{(p_g + p_{\bar{Q}})^2}$$

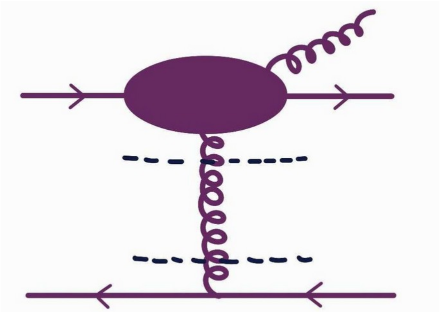
If we want to match the result onto the form:

$$\mathcal{M}^{\mathrm{eff}}(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) \ =$$



If we want to match the result onto the form:

$$\mathcal{M}^{\text{eff}}(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) \; =$$

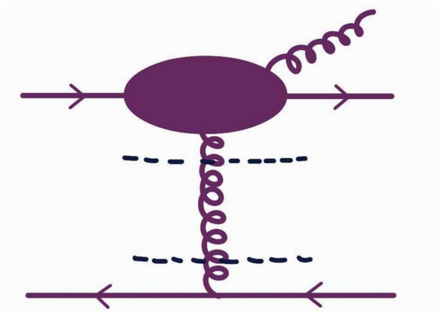


We are getting the expression:

$$
\begin{aligned}
\mathcal{J}^{\text{NLL}}_{q\bar{q}}(\bar{q}^{\lambda_{\bar{q}}}, \bar{Q}^{\lambda_{\bar{Q}}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) \equiv \; & (i\, g_s)^2 \; \epsilon^{\lambda_g}_{\mu_g}(p_g, p_r) \cdot \\
\cdot \Bigg\{ & \delta_{\iota_Q \bar{\iota}_{\bar{q}}} \, (T^{a_g})_{\iota_q \bar{\iota}_{\bar{Q}}} \Bigg[- \frac{\langle q|\, \mu_t \, (p_g + p_{\bar{q}}) \, \mu_g \, |\bar{q}]}{(p_g + p_{\bar{q}})^2} + \frac{2\, p_Q^{\mu_g} \, (p_Q + p_{\bar{Q}})^2 \, \langle q|\mu_{t_2}|\bar{q}]}{(p_g + p_Q)^2 \, (p_q + p_{\bar{q}})^2} \\
& - \frac{\langle q|\mu_{t_1}|\bar{q}]}{(p_q + p_{\bar{q}})^2} \left(2\, \eta^{\mu_g\, \mu_{t_1}} \, p_g^{\mu_{t_2}} \; + \; \eta^{\mu_{t_1}\mu_{t_2}} \, (2p_q + 2p_{\bar{q}})^{\mu_g} \; + \; 2\, \eta^{\mu_{t_2}\, \mu_g} \, p_g^{\mu_{t_1}} \right) \Bigg] \\
& + \delta_{\iota_q \bar{\iota}_{\bar{Q}}} \, (T^{a_g})_{\iota_Q \bar{\iota}_{\bar{q}}} \Bigg[\frac{\langle q|\, \mu_g \, (p_g + p_q) \, \mu_{t_2} \, |\bar{q}]}{(p_g + p_q)^2} - \frac{2\, p_{\bar{Q}}^{\mu_g} \, (p_Q + p_{\bar{Q}})^2 \, \langle q|\mu_{t_2}|\bar{q}]}{(p_q + p_{\bar{q}})^2 \, (p_g + p_{\bar{Q}})^2} \\
& + \frac{\langle q|\mu_{t_1}|\bar{q}]}{(p_q + p_{\bar{q}})^2} \left(2\, \eta^{\mu_g\, \mu_{t_1}} \, p_g^{\mu_{t_2}} \; + \; \eta^{\mu_{t_1}\mu_{t_2}} \, (2p_q + 2p_{\bar{q}})^{\mu_g} \; + \; 2\, \eta^{\mu_{t_2}\, \mu_g} \, p_g^{\mu_{t_1}} \right) \Bigg] \\
& - \frac{1}{N_c} \, \delta_{\iota_q \bar{\iota}_{\bar{q}} (T^{a_g})_{\iota_Q \bar{\iota}_{\bar{Q}}}} \left[\frac{\langle q|\, \mu_g \, (g + q) \, \mu_{t_2} \, |\bar{q}]}{(p_g + p_q)^2} - \frac{\langle q|\, \mu_{t_2} \, (g + \bar{q}) \, \mu_g \, |\bar{q}]}{(p_g + p_{\bar{q}})^2} \right] \\
& - \frac{1}{N_c} \, \delta_{\iota_Q \bar{\iota}_{\bar{Q}}} \, (T^{a_g})_{\iota_q \bar{\iota}_{\bar{q}}} \left[\frac{(p_Q + p_{\bar{Q}})^2}{(p_q + p_{\bar{q}})^2} \, \langle q|\mu_{t_2}|\bar{q}] \cdot \left(\frac{2\, p_{\bar{Q}}^{\mu_g}}{(p_g + p_{\bar{Q}})^2} - \frac{2\, p_Q^{\mu_g}}{(p_g + p_Q)^2} \right) \right] \Bigg\}
\end{aligned}
$$

If we want to match the result onto the form:

$$\mathcal{M}^{\text{eff}}(\bar{q}^{\lambda_{\bar q}}, \bar{Q}^{\lambda_{\bar Q}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) \;=$$



We are getting the expression:

$$\mathcal{J}^{\text{NLL}}_{q\bar q}(\bar{q}^{\lambda_{\bar q}}, \bar{Q}^{\lambda_{\bar Q}}, Q^{\lambda_Q}, q^{\lambda_q}, g^{\lambda_g}) \equiv \; (i\, g_s)^2 \; \epsilon^{\lambda_g}_{\mu_g}(p_g, p_r) \cdot$$

$$\cdot \Bigg\{ \delta_{\iota_Q \bar\iota_{\bar q}} \, (T^{a_g})_{\iota_q \bar\iota_{\bar Q}} \Bigg[- \frac{\langle q|\, \mu_t \, (p_g + p_{\bar q})\, \mu_g \, |\bar q]}{(p_g + p_{\bar q})^2} + \frac{2\, p_Q^{\mu_g} \, (p_Q + p_{\bar Q})^2 \, \langle q|\mu_{t_2}|\bar q]}{(p_g + p_Q)^2 \, (p_q + p_{\bar q})^2}$$

$$- \frac{\langle q|\mu_{t_1}|\bar q]}{(p_q + p_{\bar q})^2} \left(2\, \eta^{\mu_g\, \mu_{t_1}} \, p_g^{\mu_{t_2}} \; + \; \eta^{\mu_{t_1}\, \mu_{t_2}} \, (2p_q + 2p_{\bar q})^{\mu_g} \; + \; 2\, \eta^{\mu_{t_2}\, \mu_g} \, p_g^{\mu_{t_1}} \right) \Bigg]$$

$$+ \, \delta_{\iota_q \bar\iota_{\bar Q}} \, (T^{a_g})_{\iota_Q \bar\iota_{\bar q}} \Bigg[\frac{\langle q|\, \mu_g \, (p_g + p_q)\, \mu_{t_2} \, |\bar q]}{(p_g + p_q)^2} - \frac{2\, p_{\bar Q}^{\mu_g} \, (p_Q + p_{\bar Q})^2 \, \langle q|\mu_{t_2}|\bar q]}{(p_q + p_{\bar q})^2 \, (p_g + p_{\bar Q})^2}$$

$$+ \frac{\langle q|\mu_{t_1}|\bar q]}{(p_q + p_{\bar q})^2} \left(2\, \eta^{\mu_g\, \mu_{t_1}} \, p_g^{\mu_{t_2}} \; + \; \eta^{\mu_{t_1}\, \mu_{t_2}} \, (2p_q + 2p_{\bar q})^{\mu_g} \; + \; 2\, \eta^{\mu_{t_2}\, \mu_g} \, p_g^{\mu_{t_1}} \right) \Bigg]$$

$$- \frac{1}{N_c} \, \delta_{\iota_q \bar\iota_{\bar q} (T^{a_g})_{\iota_Q \bar\iota_{\bar Q}}} \left[\frac{\langle q|\, \mu_g \, (g + q)\, \mu_{t_2} \, |\bar q]}{(p_g + p_q)^2} - \frac{\langle q|\, \mu_{t_2} \, (g + \bar q)\, \mu_g \, |\bar q]}{(p_g + p_{\bar q})^2} \right]$$

$$- \frac{1}{N_c} \, \delta_{\iota_Q \bar\iota_{\bar Q}} \, (T^{a_g})_{\iota_q \bar\iota_{\bar q}} \left[\frac{(p_Q + p_{\bar Q})^2}{(p_q + p_{\bar q})^2} \langle q|\mu_{t_2}|\bar q] \cdot \left(\frac{2\, p_{\bar Q}^{\mu_g}}{(p_g + p_{\bar Q})^2} - \frac{2\, p_Q^{\mu_g}}{(p_g + p_Q)^2} \right) \right] \Bigg\}$$

Improvements over an earlier HEJ current

	(Previous expression) **Unordered impact current**	(Current expression) **Quasi-MRK impact current**
NLL accuracy	fully NLL-accurate for $qQ \to qQg$ process	fully NLL-accurate for $qQ \to qQg$ process
phase space covered	does not have loop corrections so can not cover IR phase space	can be used for fully regulated $qQ \to qQ$ at NLO
factorisability	current contains kinematics and part of the colour	contains the entire colour information of the $2 \to 3$ process
universality	can be applied to any partonic channel	**!!!**

The HEJ Collaboration

(Photo taken during the “HEJ days” in Edinburgh - 2023)

Further steps

- test the new NLL $\emptyset \longrightarrow qqgg^*$ current in a NLO calculation for dijet production

- implement the FKS subtraction scheme for the HEJ calculation of dijet production

- construct the $\emptyset \longrightarrow gggg^*$ impact current at NLL

- fit the LL resummation scheme within the new impact currents

Conclusions

- demonstrated the importance of high energy log resummation for LHC processes

$\rightarrow$ motivation for working towards full NLL

- worked out the first NLL HEJ component

- "recipe" that fulfills all HEJ constraints, accounts for the entire phase space at NLO, amenable to (unmodified) implementation of subtraction scheme (FKS)

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Thank you for your attention!