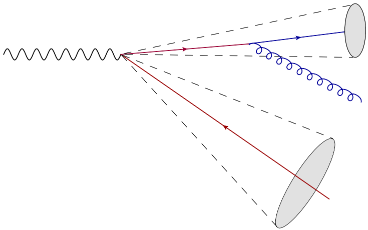
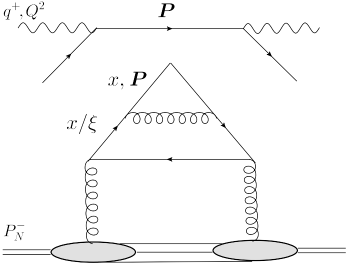
The CGC/TMD correspondence: TMD factorisation and evolution at small x

Edmond Iancu

IPhT, Université Paris-Saclay

with Paul Caucal, Al Mueller, Dionysis Triantafyllopoulos, Shu-Yi Wei, Marcos Morales, Farid Salazar, Feng Yuan... (since 2021)



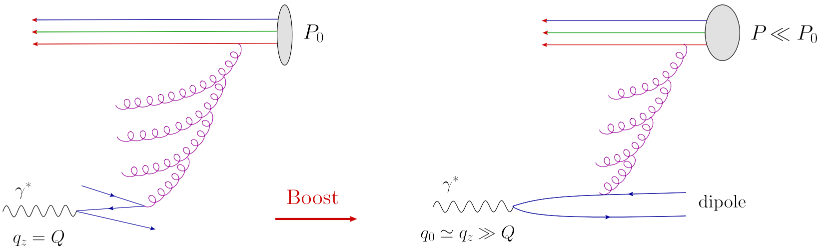


The CGC/TMD correspondence

- TMD factorisation is also useful at small x and for large nuclei $A \gg 1$
  - high parton densities, non-linear phenomena, gluon saturation
  - intrinsic scale: saturation momentum $Q_s^2(x, A) \sim A^{1/3}(1/x)^{0.25}$
- TMD factorisation at small x can be derived from the CGC effective theory
- Processes involving two widely separated transverse momentum scales
  - SIDIS with $Q^2 \gg K_\perp^2$, di-jets with $P_\perp \gg K_\perp$ in eA or pA
- CGC calculations (LO, NLO) for generic kinematics: e.g. $P_\perp \sim K_\perp \sim Q_s$
- $P_\perp \gg K_\perp, Q_s$: TMD factorisation emerges at leading power in $1/P_\perp$
- New types of TMD PDFs: small-x gluons, sea quarks, diffractive TMDs
  - gauge links already at tree-level: Wilson lines, multiple scattering
- NLO corrections preserve TMD factorisation and generate evolutions
  - small-x: BK/JIMWLK & large Q^2 (or $P_\perp$): DGLAP, CSS

DIS in the color dipole picture

- The natural approach to DIS at high energy/small-x: factorisation in time
- Start in the Breit frame: the target $(p,\ A)$ is a left mover: $P_0^- \gg M_N$
- The struck quark: a sea quark at the end of a (BFKL) gluon cascade
- Large boost in the positive z direction: ultrarelativistic photon $q^+ \gg Q$

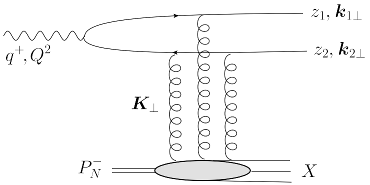


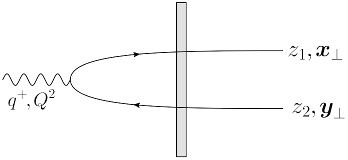
- γ^* fluctuates into a $q\bar{q}$ color dipole, which then scatters off the target

$$x \equiv \frac{Q^2}{2q^+P^-} \ll 1 \iff \Delta t \simeq \frac{2q^+}{Q^2} \gg \frac{1}{P^-}$$

Dijet production in DIS (LO)

- $x \ll 1$ and/or $A \gg 1 \Rightarrow$ dense gluon system $\Rightarrow$ multiple scattering
  - strong Coulomb field $A_a^- \sim 1/g$, localised in x^+ ("shock-wave")
  - eikonal approximation, transverse coordinate representation: $\boldsymbol{x}_\perp$, $\boldsymbol{y}_\perp$
  - Fourier transform to the final transverse momenta $\boldsymbol{k}_{1\perp}$, $\boldsymbol{k}_{2\perp}$





Wilson line: $$V(\boldsymbol{x}_\perp) \equiv \mathrm{T}\exp\left\{ig\int \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}_\perp)t^a\right\}$$

- The scattering amplitude: $V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - 1$
- Random fields A_a^-, to be averaged over in the cross-section ("color glass")

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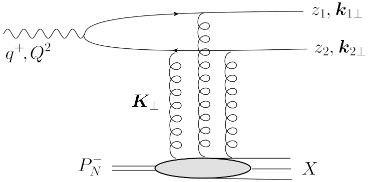
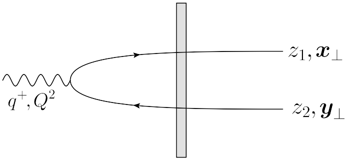




$$\frac{1}{N_c}\Big\langle \mathrm{tr}\big[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - 1\big]\,\big[V^\dagger(\overline{\boldsymbol{x}}_\perp)V(\overline{\boldsymbol{y}}_\perp) - 1\big]\Big\rangle_x$$

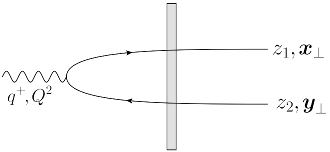
- x-dependence via the high-energy evolution: BK/JIMWLK equations
  - non-linear generalisations of BFKL: powers of $\alpha_s \log(1/x)$ + saturation

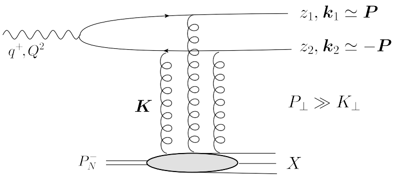
Back-to-back dijets

- Transverse resolution scale set by the dipole size $r \equiv |\boldsymbol{x}_\perp - \boldsymbol{y}_\perp|$
  - scattering is strong when $r \gtrsim 1/Q_s$, but weak for $r \ll 1/Q_s$
- Back-to-back: $\boldsymbol{k}_{1\perp} \simeq -\boldsymbol{k}_{2\perp}$ or, equivalently, $P_\perp \gg K_\perp \gtrsim Q_s$

$$\boldsymbol{P}_\perp \equiv z_2\boldsymbol{k}_{1\perp} - z_1\boldsymbol{k}_{2\perp}\,, \qquad \boldsymbol{K}_\perp \equiv \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

- Small $q\bar{q}$ dipole: $r = |\boldsymbol{x} - \boldsymbol{y}| \sim 1/P_\perp \ll 1/Q_s \implies$ weak scattering (?)





- Multiple scattering still important for the momentum imbalance: $K_\perp \sim Q_s$

$$V_{\boldsymbol{x}} V^\dagger_{\boldsymbol{y}} - 1 \simeq r^j \big(V_{\boldsymbol{b}} \partial^j V^\dagger_{\boldsymbol{b}}\big), \quad \boldsymbol{b} = z_1\boldsymbol{x} + z_2\boldsymbol{y}$$

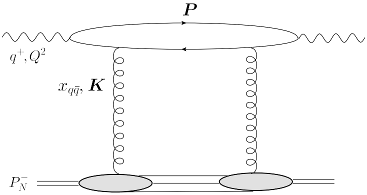
- $r \sim 1/P_\perp$ dependence factorises from the $b \sim 1/K_\perp$ dependence

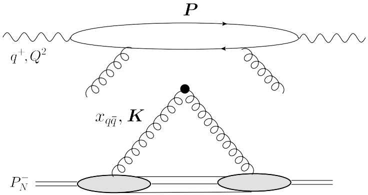
TMD factorisation for inclusive dijets

(Dominguez, Marquet, Xiao and Yuan, arXiv:1101.0715)

$$\frac{\mathrm{d}\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}} = H_{T,L}(z_1,z_2,Q^2,P_\perp^2)\,\mathcal{F}^{(0)}_{WW}(x_{q\bar{q}},\boldsymbol{K})$$

- $x_{q\bar{q}}$: the fraction of P_N^- transferred to the produced dijet





- Hard factor encoding the kinematics of the $q\bar{q}$ pair $(\bar{Q}^2 = z_1 z_2 Q^2)$

$$H_T = \alpha_{em}\alpha_s e_f^2 \delta(1-z_1-z_2)\big(z_1^2+z_2^2\big)\frac{P_\perp^4+\bar{Q}^4}{(P_\perp^2+\bar{Q}^2)^4}$$

  - photon decay $\gamma^* \to q\bar{q}$ (α_{em}) + coupling to the gluon (α_s)

The Weiszäcker-Williams gluon TMD

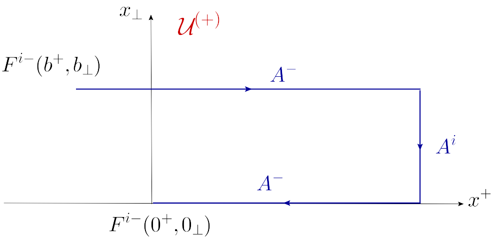
- Gluon occupation number in the dense nuclear target

$$\mathcal{F}^{(0)}_{WW}(x,\boldsymbol{K}) = \int_{\boldsymbol{b},\overline{\boldsymbol{b}}} \frac{\mathrm{e}^{-i\boldsymbol{K}\cdot(\boldsymbol{b}-\overline{\boldsymbol{b}})}}{(2\pi)^4}\,\frac{-2}{\alpha_s}\left\langle \mathrm{tr}\Big[(\partial^i V_{\boldsymbol{b}})V^\dagger_{\boldsymbol{b}}(\partial^i V_{\overline{\boldsymbol{b}}})V^\dagger_{\overline{\boldsymbol{b}}}\Big]\right\rangle_x$$

- The small-x limit of the standard operator definition for the gluon TMD

$$\mathcal{F}^{(0)}_{WW}(x,\boldsymbol{K}) = 2\int \frac{\mathrm{d}b^+\mathrm{d}^2\boldsymbol{b}}{(2\pi)^3}\,\mathrm{e}^{ixb^+P_N^-}\mathrm{e}^{-i\boldsymbol{K}\cdot\boldsymbol{b}}\left\langle \mathrm{tr}\Big[F^{i-}_b\,\mathcal{U}^{(+)}(b,0)F^{i-}_0\,\mathcal{U}^{(+)\dagger}(b,0)\Big]\right\rangle_x$$

  - take $x=0$ in the exponential (power corrections) ....
  - ... but keep x in the CGC average: high-energy evolution, $\alpha_s\ln(1/x)$



The Weizsäcker-Williams gluon TMD

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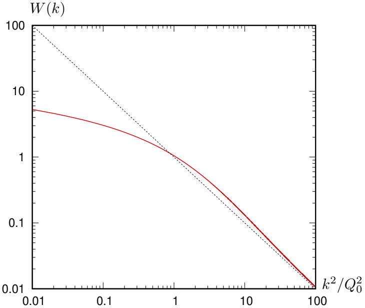
$$\mathcal{F}^{(0)}_{WW}(x, \boldsymbol{K}) = \int_{\boldsymbol{b},\bar{\boldsymbol{b}}} \frac{\mathrm{e}^{-i\boldsymbol{K}\cdot(\boldsymbol{b}-\bar{\boldsymbol{b}})}}{(2\pi)^4}\,\frac{-2}{\alpha_s}\left\langle \mathrm{tr}\Big[(\partial^i V_{\boldsymbol{b}})V^\dagger_{\boldsymbol{b}}(\partial^i V_{\bar{\boldsymbol{b}}})V^\dagger_{\bar{\boldsymbol{b}}}\Big]\right\rangle_x$$

- The gauge link fully materialised already at leading order
  - strong fields, multiple scattering, gluon saturation



- dilute system at $K_\perp \gg Q_s$

$$\mathcal{F}^{(0)}_{WW}(x, \boldsymbol{K}) \sim \frac{Q_s^2(x)}{K_\perp^2}$$

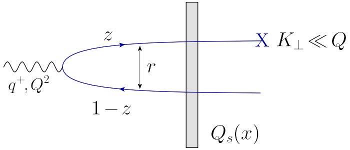
- gluon saturation at $K_\perp \lesssim Q_s$

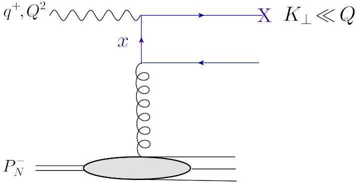
$$\mathcal{F}^{(0)}_{WW}(x, \boldsymbol{K}) \sim \frac{1}{\alpha_s}\ln\frac{K_\perp^2}{Q_s^2(x)}$$

SIDIS: sea quark TMD factorisation

(Marquet, Xiao and Yuan, arXiv:0906.1454)

- Measure a single jet (or hadron) in the "hard" regime $Q^2 \gg K_\perp^2 \gtrsim Q_s^2$
- The integral over z controlled by $z(1-z) \sim \frac{K_\perp^2}{Q^2} \ll 1$: "aligned jets":
- Target picture (Breit frame): photon absorbed by a sea quark



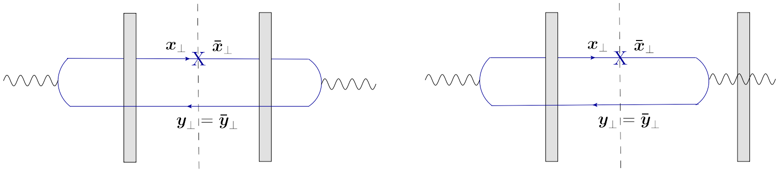


- The struck quark has $z \simeq 1$ and large virtuality $K_\mu K^\mu \sim Q^2 \gg K_\perp^2$

$$\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{K}} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\,\mathcal{F}_q^{(0)}(x,\boldsymbol{K})\left[1+\mathcal{O}\left(\frac{K_\perp^2}{Q^2}\right)\right]$$

- The slow antiquark $(1-z \ll 1)$ can be measured too: *see talk by P. Caucal*

- Wilson lines at $\boldsymbol{x}$, $\bar{\boldsymbol{x}}$ (measured quark) and $\boldsymbol{y} = \bar{\boldsymbol{y}}$ (unmeasured antiquark)
  - left: $\langle \mathrm{tr}[V_{\boldsymbol{x}} V^\dagger_{\bar{\boldsymbol{x}}} V_{\bar{\boldsymbol{y}}} V^\dagger_{\boldsymbol{y}}]\rangle = \langle \mathrm{tr} V_{\boldsymbol{x}} V^\dagger_{\bar{\boldsymbol{x}}}\rangle \equiv N_c \mathcal{D}(\boldsymbol{x}, \bar{\boldsymbol{x}})$: dipole S-matrix
  - colour structure: $\mathcal{D}(\boldsymbol{x}, \bar{\boldsymbol{x}}) - \mathcal{D}(\boldsymbol{x}, \boldsymbol{y}) - \mathcal{D}(\bar{\boldsymbol{x}}, \boldsymbol{y}) + 1$



$$\mathcal{F}_q^{(0)}(x, \boldsymbol{K}) = \frac{N_c}{\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \tilde{\mathcal{D}}(x, \boldsymbol{q}) \left[1 - \frac{\boldsymbol{K}\cdot(\boldsymbol{K}-\boldsymbol{q})}{(K_\perp^2 - (\boldsymbol{K}-\boldsymbol{q})^2)} \ln \frac{K_\perp^2}{(\boldsymbol{K}-\boldsymbol{q})^2}\right]$$

- The integrand involves $\mathcal{F}_D^{(0)}(x, \boldsymbol{q}) \sim \boldsymbol{q}^2 \tilde{\mathcal{D}}(x, \boldsymbol{q})$: the dipole gluon TMD
- Interpretation: a gluon $\boldsymbol{q}$ splits into a pair of sea quarks $\boldsymbol{K}$ and $\boldsymbol{K} - \boldsymbol{q}$

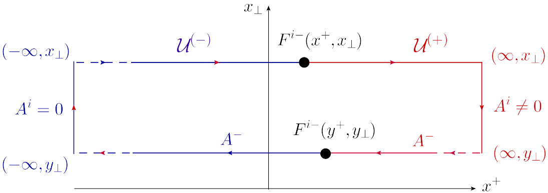
The dipole gluon TMD

$$\mathcal{F}_D^{(0)}(x, \boldsymbol{q}) = \frac{2q_\perp^2}{\alpha_s} \int_{\boldsymbol{x},\boldsymbol{y}} \frac{\mathrm{e}^{-i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})}}{(2\pi)^4} \left\langle \mathrm{tr}\left[V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger\right]\right\rangle_x$$

- It differs from the WW gluon TMD “only” in the choice of the gauge links

$$\mathcal{F}_D^{(0)}(x, \boldsymbol{q}) = 2\int_{x^+,y^+} \int_{\boldsymbol{x},\boldsymbol{y}} \frac{\mathrm{e}^{-i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})}}{(2\pi)^3} \left\langle \mathrm{tr}\left[F_x^{i-}\mathcal{U}^{(-)}(x,y) F_y^{i-}\mathcal{U}^{(+)\dagger}(x,y)\right]\right\rangle_x$$

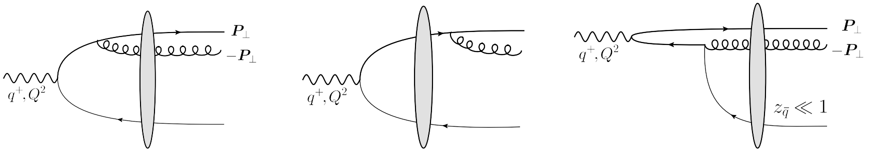
  - both future and past contours: initial-state and final-state interactions



- At LO, the gauge links describe gluon saturation (multiple scattering)
  - different TMDs differ from each other only at low momenta $q_\perp \lesssim Q_s$

Back-to-back quark-gluon jets

- Produce a hard quark-gluon pair: $k_{1\perp} \simeq k_{2\perp} \simeq P_\perp \gg K_\perp$
  - either a hard quark splitting $q \to qg$ ...
  - or a hard photon decay $\gamma^* \to q\bar{q}$
  - in both cases, the unmeasured antiquark is soft: $z_{\bar{q}} \sim K_\perp^2/P_\perp^2 \ll 1$



- The precise value of Q^2 is irrelevant: $P_\perp$ provides the hard scale
- The soft antiquark can be viewed as a sea quark from the target

  Hauksson, E.I., Mueller, Triantafyllopoulos, Wei, 2402.14748 (diffractive)

  Caucal, Guerrero-Morales, E.I., Salazar, Yuan, 2503.16162 (inclusive jets)

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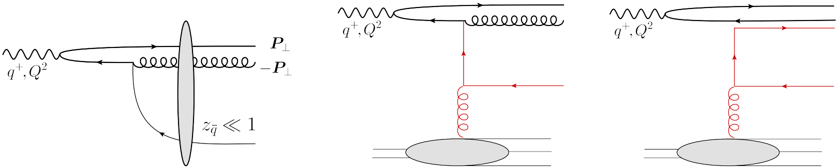


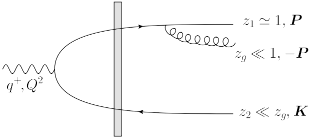
- TMD factorisation for any Q^2: the same sea quark TMD as in SIDIS

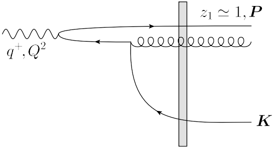
$$\frac{\mathrm{d}\sigma^{\gamma_T^\star + A \to qg + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{K} \mathrm{d}z_q \mathrm{d}z_g} = H_T(P_\perp, Q, z_q, z_g)\, \mathcal{F}_q^{(0)}(x_{qg}, \boldsymbol{K})$$

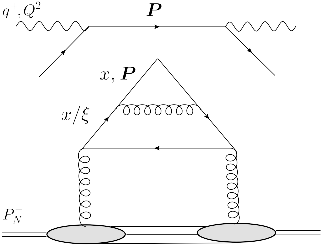
- The color flow is not changed by the gluon in the final state

SIDIS: NLO corrections

- Integrate out both the antiquark and the gluon: NLO corrections to SIDIS
- When $Q^2 \gg P_\perp^2 \gg K_\perp^2,\, Q_s^2$: one recovers TMD factorisation at NLO



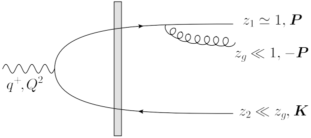


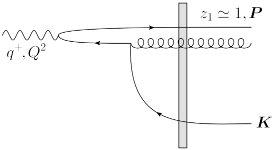


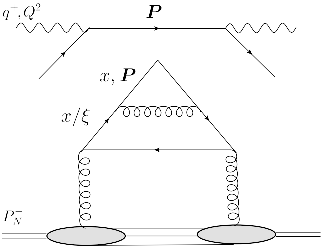
- One step in the DGLAP and CSS evolutions of the sea quark PDF/TMD
  Caucal, E.I., Mueller & Yuan, arXiv:2408.03129
  Hauksson, E.I., Mueller, Triantafyllopoulos, Wei, 2402.14748 (diffraction)
- For the WW gluon TMD (dijets in DIS): *P. Caucal, E.I., arXiv:2406.04238*

SIDIS: NLO corrections

- N.B. The gluon is emitted by the dilute projectile (the $q\bar{q}$ dipole) ... yet it can be interpreted as an evolution of the sea quark TMD in the dense target
  - both $\bar{q}$ and g are soft: $z_2 \ll z_g \ll 1$
  - we measure a jet in the final state: no contribution to FF TMD







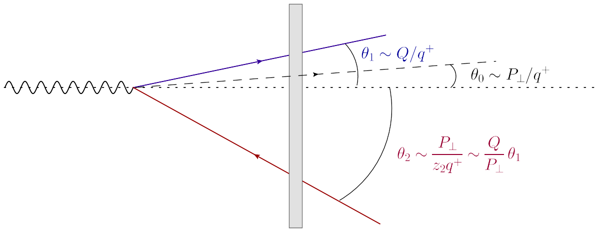
- Change the longitudinal integration variable:
  - from z_g (s-channel gluon) to $\xi \equiv x/x_{qg}$ (t-channel quark)

SIDIS: collinear evolution

- One "real" step in the DGLAP/CSS evolution: $q \to qg$, $P_{qq}(\xi)$

$$x\mathcal{F}_q^{(1)}(x, \boldsymbol{P}_\perp) = \frac{\alpha_s C_F}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{1-\xi_0} \mathrm{d}\xi\, \frac{1+\xi^2}{1-\xi}\, \frac{x}{\xi} f_q^{(0)}\left(\frac{x}{\xi}, P_\perp^2\right)$$

- Rapidity cutoff $\xi_0 \ll 1$ coming from the jet condition
- "Plus prescription" $1/(1-\xi)_+$ (DGLAP) + $\ln(1/\xi_0$ (CSS)
- LO SIDIS: the struck quark has $z_1 \simeq 1$ and large virtuality $Q^2 \gg P_\perp^2$



- It makes a relatively large angle: $\theta_1 \sim \frac{Q}{q^+} \gg \frac{P_\perp}{q^+}$ ("on-shell quark")

SIDIS: collinear evolution

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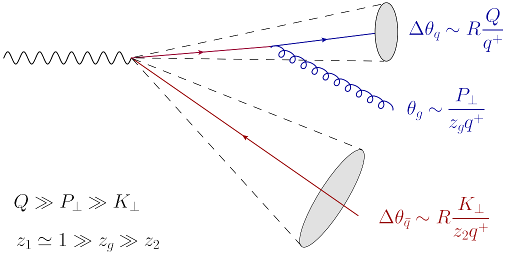
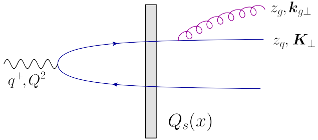
- The gluon must be emitted outside the quark jet: $\theta_g \sim \frac{P_\perp}{z_g q^+} > \theta_1 \sim \frac{Q}{q^+}$

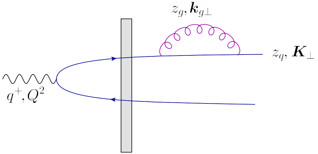


- Upper limit on $z_g < \frac{P_\perp}{Q} \ll 1 \Longrightarrow$ lower limit $\xi_0 = \frac{P_\perp}{Q}$ on $1-\xi$
- The Collins-Soper scale $\zeta = Q^2$ ("gluon rapidity < quark rapidity")

The Sudakov double logarithm

- So far, only the real terms in the DGLAP/CSS equations
- The dominant contribution to CSS comes from uncompensated virtual emissions: the Sudakov double logarithm
- In the dipole picture, they come from soft emissions in the final state





$$\Delta\mathcal{F}_q(x,\boldsymbol{K}) = \frac{\alpha_s C_F}{\pi^2}\int\frac{\mathrm{d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}\int_{k_{g\perp}^2/Q^2}^{k_{g\perp}/Q}\frac{\mathrm{d}z_g}{z_g}\left[\mathcal{F}_q^{(0)}(x,\boldsymbol{K}+\boldsymbol{k}_g) - \mathcal{F}_q^{(0)}(x,\boldsymbol{K})\right]$$

- upper limit on z_g: jet condition; lower limit: separation from JIMWLK
- emissions with $k_{g\perp} < K_\perp$ cancel between real and virtual
- real gluon emissions with $k_{g\perp} > K_\perp$ are suppressed

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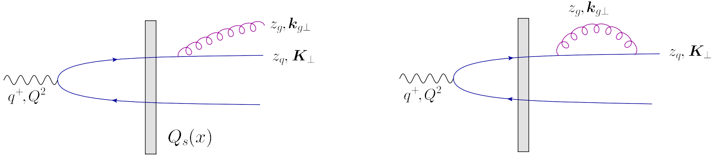
- The dominant contribution to CSS comes from uncompensated virtual emissions: the Sudakov double logarithm
- In the dipole picture, they come from soft emissions in the final state



- Double-log: $K_\perp^2 \ll k_{g\perp}^2 \ll Q^2$ and $k_{g\perp}^2/Q^2 \ll z_g \ll k_{g\perp}/Q$

$$\Delta \mathcal{F}_q(x, \boldsymbol{K}, Q^2) \simeq -\frac{\alpha_s C_F}{4\pi} \ln^2\left(\frac{Q^2}{K_\perp^2}\right) \mathcal{F}_q^{(0)}(x, \boldsymbol{K})$$

- The CSS equation ensures the resummation of the Sudakov logs to all orders

See next talk by Dionysis Triantafyllopoulos !

Conclusions

- TMD factorisation at small x emerges from the CGC effective theory
  - order by order in CGC perturbation theory: so far LO and NLO
- TMDs for small x partons computed (almost) from first principles
- Semi-hard intrinsic scale $Q_s \sim 1$ GeV allowing for weak coupling techniques
- Classical gauge links describing gluon saturation
- High-energy (BK/JIMWLK) and collinear (DGLAP, CSS) evolutions together
- A rich phenomenology in perspective:
  - forward di-jets, photon-jets, Drell-Yan in p+Pb collisions at the LHC
  - SIDIS, di-jets, diffractive jets in e+A collisions at the EIC
  - di-jets in Pb+Pb ultraperipheral collisions at the LHC
- A rapidly growing field: see the next talks by Dionysis, Paul and Misha
- Towards a unified description of multi-scale processes from moderate to small values of x ?

Jet algorithms for SIDIS

- Roughly: 2 partons i and j belong to the same jet provided

$$\Delta\theta_{ij} \le R\theta_{\rm jet}, \qquad \Delta\theta_{ij} \simeq \left|\frac{\boldsymbol{k}_{i\perp}}{z_i q_+} - \frac{\boldsymbol{k}_{j\perp}}{z_j q_+}\right|$$

- The main question: what is $\theta_{\rm jet}$?

- Standard algorithms for jets at the LHC: anti-k_t, Cambridge/Aachen:

$$\theta_{\rm jet} = \frac{k_{\perp,{\rm jet}}}{z_{\rm jet} q^+}, \qquad \boldsymbol{k}_{\perp,{\rm jet}} = \boldsymbol{k}_{i\perp} + \boldsymbol{k}_{j\perp}, \qquad z_{\rm jet} = z_i + z_j$$

  - if applied to the aligned jet in SIDIS, it would select $\boldsymbol{k}_{\perp,{\rm jet}} = P_\perp$

- However, for the aligned jet in SIDIS, $\theta_{\rm jet} = \frac{Q}{q^+}$. The precise condition:

$$M_{ij}^2 \le z_i z_j Q^2 R^2, \quad M_{ij}^2 \equiv (k_i + k_j)^2 \quad : \text{ boost invariant}$$

- Roughly equivalent criterion (CENTAURO) by *Arratia, Makris, Neill, Ringer, Sato, 2006.10751*